Overview

- Simplest model: 2 countries, 2 periods, representative agent, non-stochastic
  - full enforcement
  - strategic enforcement and secondary markets
  - multiple equilibria, welfare, and robustness
  - preference for domestic redistribution
- General result
  - statement
  - crucial assumptions
- Limits to the argument
  - frictions in secondary markets
  - "large" agents
  - commitment
- Final remarks

Barebones model: Debtor-Creditor world

- Two regions: Debtor ($i \in I^D$) and Creditor ($i \in I^C$). Continuum of mass 1 of residents in each region

- Two dates: Today ($t = 0$) and Tomorrow ($t = 1$)

- Preferences: $U(c_{i0}, c_{i1}) = u(c_{i0}) + u(c_{i1})$

- Representative agent: $(y_{i0}, y_{i1}) = \begin{cases} (y - \varepsilon, y + \varepsilon) & \text{if } i \in I^D \\ (y + \varepsilon, y - \varepsilon) & \text{if } i \in I^C \end{cases}$

- Agents can trade in bonds.

- Timeline:

endowments realized	primary markets open	consumption	endowments realized	secondary markets open	payments enforced	consumption

$t = 0$ (first three stages) $\qquad$ $t = 1$ (last four stages)

- Full enforcement: payments always enforced

- No commitment: governments choose ex-post which payments, if any, to enforce so as to maximize $W^D = \int_{i \in I^D} u(c_{i1})$ and $W^C = \int_{i \in I^C} u(c_{i1})$ respectively

Full enforcement

- Since world endowment is constant and payments are enforced, bond prices equal

$$\left(q_0^D\right)^* = \left(q_1^D\right)^* = \left(q_0^C\right)^* = \left(q_1^C\right)^* = 1$$

  and there is consumption smoothing

$$c_{i0}^* = c_{i1}^* = y$$

- Bond holdings after primary markets

$$\left(x_{i0}^D\right)^* = \begin{cases} -\varepsilon & \text{if } i \in I^D \\ +\varepsilon & \text{if } i \in I^C \end{cases}$$

- Bond holdings after secondary markets

$$\left(x_{i1}^D\right)^* = \left(x_{i0}^D\right)^*$$

- Note that full-enforcement bond holdings are in no way unique
  - bond holdings after primary markets satisfy

$$\left(x_{i0}^D\right)^* + \left(x_{i0}^C\right)^* = \begin{cases} -\varepsilon & \text{if } i \in I^D \\ +\varepsilon & \text{if } i \in I^C \end{cases}$$

  - bond holdings after secondary markets are basically unrestricted

Strategic enforcement

- The government of Debtor never enforces payments to creditors
- Yet, the full-enforcement allocation is still an equilibrium!

Strategic enforcement

- Let $e_j^D \in \{1, 0\}$ denote whether the governmet of Debtor enforces payments to residents of j.
- Clearly, it cannot be that $e_C^D = 1$ (under full enforcement $e_C^D = e_D^D = 1$)
- The full-enforcement bond holdings and prices described above cannot be observed in equilibrium
- Assume that, as in the full-enforcement equilibrium,

$$\left(q_0^D\right)^{**} = \left(q_1^D\right)^{**} = \left(q_0^C\right)^{**} = \left(q_1^C\right)^{**} = 1$$

$$c_{i0}^{**} = c_{i1}^{**} = y$$

$$\left(x_{i0}^D\right)^{**} = \begin{cases} -\varepsilon & \text{if } i \in I^D \\ +\varepsilon & \text{if } i \in I^C \end{cases}$$

but debtors repurchase bonds from creditors in secondary markets

$$\left(x_{i1}^D\right)^{**} = 0 \text{ if } i \in I^C \quad \text{and} \quad \left(x_{i1}^D\right)^{**} = \delta_i \text{ if } i \in I^D$$

with $\int_{i \in I^j} \delta_i = 0$. This is feasible because $y_1^D > \int_{i \in I^C} \left(x_{i0}^D\right)^{**}$.
Also assume Debtor's government enforces bond payments between debtors

$$e_D^D = 1$$

- This is an equilibrium because
  - all individuals get repaid, debtors directly and creditors by selling bonds in secondary market
  - the government of Debtor enforces domestic payments (Jensen's inequality)
- The full-enforcement allocation is achieved

Intuition

- Secondary markets "align" asset holdings with the preferences of the enforcers
- This maximizes the price of assets and enforcement
- Secondary markets lead to an ex-post prisoner's dilemma type of situation
  - collectively, it would be better to collude and not purchase domestic securities from foreign creditors
  - individually, it is optimal to repurchase these assets whenever they trade at less than face value
- This ex-post inefficiency is beneficial from an ex-ante point of view

Multiple equilibria, welfare, and robustness

- There is another (pesimistic) equilibrium without enforcement
  - assume creditors expect that if they buy bonds from debtors there will be no enforcement, $e_D^D = 0$
  - as a result, no asset trade takes place Today
- This is an equilibrium because, even if (infinitesimal) individuals deviate and trade, the government of Debtor will be indifferent between enforcing or not
- However, the full enforcement equilibrium is the most robust
- Introduce arbitrarily small *cost of non-enforcement*
  - when average utility is unaffected by enforcement, choose $e_D^D = 1$
  - from pessimistic equilibrium, creditors deviate and buy bonds
  - only the optimistic equilibrium survives this perturbation

Multiple equilibria, welfare, and robustness

- Note that, in general, the full enforcement equilibrium need not be the best one. For example (Risk-Sharing world):
  - two regions: Home (H) and Foreign (F). Two states: s_F and s_H
  - endowment Today: $y_0^H = y_0^F = y$

    endowment Tomorrow: $y_{s_H}^H = y + \varepsilon$ and $y_{s_F}^H = y - \varepsilon$ while $y_{s_H}^F = y - \varepsilon$ and $y_{s_F}^F = y + \varepsilon$
  - the only asset is a non-contingent bond
  - with full enforcement international asset trade is useless
  - there is an equilibrium with strategic enforcement in which $c_0^H = c_0^F = c_{s_H}^H = c_{s_F}^H = c_{s_H}^F = c_{s_F}^F = y$
  - each individual sells ε bonds to and buys ε bonds from individuals in the other region
  - enforcement is $e_{s_H}^H = 1$ and $e_{s_F}^H = 0$ while $e_{s_H}^F = 0$ and $e_{s_F}^H = 1$

Preference for domestic redistribution

- Previous mechanism holds if payments to domestic residents are enforced
- What if there is no representative consumer?
- Assume heterogeneity in:

  (a) ex-ante endowment profiles

  (b) weights in the government objective function
- Partition residents in groups $g \subset I^D$ according to (a) and (b): within-group homogeneity
- Is there an equilibrium outcome of the secondary market such that payments to domestic residents are enforced? Yes
  - individuals $i \in g$ sell all assets from groups $g' \neq g$ in secondary markets and purchase assets issued by other individuals in g (this is feasible because of solvency)
  - at the time of enforcement, all assets issued by individuals $i \in g$ are owned by individuals $i' \in g$
  - all promises are enforced, full enforcement allocation recovered

General case

- J regions: $j \in J \equiv \{1, 2, ..., J\}$
- $T + 1 \leq \infty$ dates: $t \in T \equiv \{0, 1, \ldots, T\}$
- At beginning of t, shock $s_t \in S$ is realized. History $h_t = [s_0, s_1, ..., s_t] \in H_t$. Transition probability $\Pr[h_\tau | h_t] = \pi(h_\tau | h_t)$.
- Endowments: y_{ih_t}
- Preferences: $U_{ih_t} = \sum_{\tau=t}^{T} \beta^{\tau-t} \cdot \int_{h_\tau \in H_\tau} \pi(h_\tau | h_t) \cdot u_{ih_\tau}(c_{ih_\tau})$
- Asset structure: $n \in N$ available assets. Payments of asset n are $d_{h_t n}$. Subset of assets s.t. $d_{h_t n} > 0$ is N_{h_t}.
- Arbitrary restrictions on asset issuance, including spanning and individual and/or history-specific quantity constraints. We assume
  - unique maturity date $N_{h_\tau} \cap N_{h_{\tau'}} = \varnothing$ if $\tau \neq \tau'$
- Particular cases:
  - infinitely-lived representative-agent model
  - overlapping generations
  - bond models
- The government of j at h_t maximizes $W_{h_t}^{j} = \int_{i \in I^j} \phi_i \cdot U_{ih_t}$

Result

- Under assumptions
  - PERFECT SECONDARY MARKETS: no restrictions on retrading of existing assets
  - SMALL AGENTS: if we define groups by endowments, preferences, government weights, and issuance restrictions, individuals are small in their groups
  - NO COMMITMENT: agents can trade in assets at h_t before governments choose enforcement at h_t

  Take a full-enforcement equilibrium: there always exists a strategic-enforcement equilibrium with the same consumption, pre-trade asset holdings and prices. *The only difference is that maturing assets are held by the group of individuals that issued them.*

- There are additional (non-robust) equilibria with arbitrary levels of enforcement

Limits to the argument: Frictions in secondary markets

- **Example (Debtor-Creditor world with transaction costs):** *All assumptions are as in the Debtor-Creditor world, except that buyers and sellers must now pay a proportional or ad valorem transaction cost equal to* t_B *and* t_S*, respectively.*
  - sovereign risk leads to greater trade and thus magnifies the effects of transaction costs on consumption and welfare
  - under full enforcement, threshold beyond which there is no trade

$$(1+t_B)\cdot(1+t_S)=\left(\frac{u'(y-\varepsilon)}{u'(y+\varepsilon)}\right)^2$$

  - under strategic enforcement, threshold is lower

$$(1+t_B)\cdot(1+t_S)=\left(\frac{u'(y-\varepsilon)}{u'(y+\varepsilon)}\right)$$

- Limited participation
  - restrictions on retrading
  - can also limit the effectiveness of secondary markets

Limits to the argument: "Large" agents (I)

- Our result hinges on competitive secondary market: prisoner's dilemma arises because agents are small and behave non-cooperatively

- Large agents (or colluding small agents) might manipulate the price of securities in the secondary market

Limits to the argument: "Large" agents (II)

- **Example (Debtor-Creditor world with large agents):** *Same as Debtor-Creditor world, except that now: (i) there is a continuum of infinitesimal debtors with mass λ^D that make their decisions collectively (i.e., a Debtor Bank), and, (ii) there is a continuum of infinitesimal creditors with mass λ^C that also make their decisions collectively (i.e., a Creditor Bank).*

- If $\lambda^D = 1$ and $\lambda^C = 0$ (i.e., one debtor):
  - Debtor cannot borrow at all
  - Tomorrow, Debtor Bank never repurchases its bonds at any positive price

- If $\lambda^D < 1$ and $\lambda^C = 0$ (i.e., one large and a continuum of small debtors):
  - presence of small debtors make a difference: they demand bonds
  - Debtor Bank now gains $(1 - \left(q_1^D\right)^{**})$ for each bond purchased in the secondary market but loses by raising price of infra-marginal units
  - in the end, $\left(q_1^D\right)^{**} < 1$ and borrowing by Debtor is constrained

- If $\lambda^C = 1$ (i.e., one large creditor):
  - has no effect regardless of λ^D

- Hence:
  - Coordination among debtors might restrict their borrowing
  - Coordination among creditors has no effect

Limits to the argument: Commitment (I)

- We have talked about full-enforcement, NOT commitment. Why?

- In Debtor-Creditor world with transaciont costs: if government could commit to enforce ex-ante it would. Pareto improvement.

- But this world has complete markets and representative agents: in other worlds, full commitment need not lead to full enforcement

- In Risk-Sharing world: if governments can commit ex-ante, they would NOT commit to full enforcement.
  - Commit to enforce if endowment is $(y + \varepsilon)$, to not enforce otherwise
  - This increases asset span and achieves Pareto improvement

- But commitment need not be Pareto improving either
  - Consider Debtor-Creditor world but government likes one half of its residents
  - Commits NOT to enforce payments by the other half
  - Once again, discretion leads to full enforcement, commitment does not

Limits to the argument: Commitment (II)

- What if governments can commit to enforcement before secondary markets open (short-term commitment)

- **Debtor-Creditor world with 'ex-post' inequality:**
  - debtors are subject to idiosyncratic shocks Tomorrow
  - two states Tomorrow, s_1 and s_2, each taking place with probability one half
  - if a given debtor is lucky, he receives $y + \varepsilon + \iota$; otherwise, he receives $y + \varepsilon - \iota$

- Under full commitment (ex-ante) or discretion, everyone consumes y

- Short-term commitment?
  - secondary markets prevent discrimination: either all or no payments enforced
  - enforcement takes place if and only if $\frac{u(y+\varepsilon+\iota)+u(y+\varepsilon-\iota)}{2} \leq u(y)$
  - otherwise, no payments enforced and short-term commitment is detrimental
  - different from situation without secondary markets
  - might maturity play a role?

Limits to the argument: Commitment (III)

- To conclude: subtle relationship between commitment, enforcement and markets
  - if secondary markets work well, commitment can only reduce enforcement
  - with full-commitment, less enforcement might be better if agents are heterogenous or markets incomplete
  - short-term commitment might destroy all international trade in assets

Limits to the argument III.a: Short-run commitment and asset trade

- Two regions: Debtor ($i \in I^D$) and Creditor ($i \in I^C$). Two dates: Today ($t = 0$) and Tomorrow ($t = 1$)
- Debtor has relatively low endowment Today

$$(y_{i0}, y_{i1}) = \begin{cases} (1-\varepsilon, 1+\varepsilon) & \text{if } i \in I^D \\ (1+\varepsilon, 1-\varepsilon) & \text{if } i \in I^C \end{cases}$$

- With no commitment

$$c_0^D = c_1^D = c_0^C = c_1^C = 1$$

  and bond holdings

$$\left(x_{i0}^D, x_{i1}^D\right) = \begin{cases} (-\varepsilon, \delta_i) & \text{if } i \in I^D \\ (+\varepsilon, 0) & \text{if } i \in I^C \end{cases}$$

  with $\int_{i \in I^D} \delta_i = 0$

- With short-run commitment, choosing no enforcement implies that the secondary market price of bonds is zero and no payments to creditors are made. As a result

$$c_1^D = \begin{cases} 1 & \text{if } e_D^D = 1 \\ 1+\varepsilon & \text{if } e_D^D = 0 \end{cases}$$

- This is anticipated by creditors, so no borrowing is possible
- Intuition:
  - the government can preempt the secondary market by committing not to enforce payments
  - this avoids ex-post inefficient payments to creditors
  - of course, this is ex-ante inefficient

Limits to the argument III.b: Short-run commitment and interactions between domestic and international asset trade

- Same as previous case, but with idiosyncratic shocks

$$(y_{i0}, y_{is^1}, y_{is^2}) = \begin{cases} (1-\varepsilon, (1+\varepsilon)\cdot(1+\iota), (1+\varepsilon)\cdot(1-\iota)) & \text{if } i \in l^1 \text{ and } i \in I^D \\ (1-\varepsilon, (1+\varepsilon)\cdot(1-\iota), (1+\varepsilon)\cdot(1+\iota)) & \text{if } i \in l^2 \text{ and } i \in I^D \\ (1+\varepsilon, 1-\varepsilon, 1-\varepsilon) & \text{if } i \in I^C \end{cases}$$

where $\iota \in [0.1]$ is the size of idiosyncratic shocks

- Two assets: asset s^1 (s^2) pays 1 in state s^1 (s^2) and 0 in the other
- With no commitment

$$c_0^D = c_1^D = c_0^C = c_1^C = 1$$

and asset holdings

$$\left(x^D_{i0,s_1}, x^D_{i0,s_2}, x^D_{i1,s_1}, x^D_{i1,s_2}\right) = \begin{cases} (-\varepsilon-(1+\varepsilon)\cdot\iota, -\varepsilon+(1+\varepsilon)\cdot\iota, +\delta_{i,s_1}, +\delta_{i,s2}) & \text{if } i \in l^1 \text{ and } i \in I^D \\ (-\varepsilon+(1+\varepsilon)\cdot\iota, -\varepsilon-(1+\varepsilon)\cdot\iota, +\delta_{i,s_1}, +\delta_{i,s2}) & \text{if } i \in l^2 \text{ and } i \in I^D \\ (+\varepsilon, +\varepsilon, 0, 0) & \text{if } i \in I^C \end{cases}$$

with $\int_{i\in I^D} \delta_{i,s_1} = \int_{i\in I^D} \delta_{i,s_2} = 0$

- With short-run commitment,

$$c_{i1} = \begin{cases} 1 \ \text{ for all of } I^D & \text{if } e_D^D = 1 \\ \begin{cases} (1+\varepsilon)\cdot(1+\iota) & \text{for one half of } I^D \\ (1+\varepsilon)\cdot(1-\iota) & \text{for one half of } I^D \end{cases} & \text{if } e_D^D = 0 \end{cases}$$

- Enforcement trade off

Limits to the argument III.b: Short-run commitment and interactions between domestic and international asset trade

- Intuition:
  - secondary markets after enforcement decision implies non-discrimination
  - committing to enforcing payments between debtors means that creditors alse get repaid by reselling their assets in secondary markets
  - to avoid payments to creditors, government of Debtor must destroy domestic payments too
  - this case is fully analyzed in Broner and Ventura (2006)

Limits to the argument III.c: Short-run commitment and debt maturity

- One small region: Home (H). Three periods: $t \in \{0, 1, 2\}$
- Preferences $U(c_{i0}, c_{i1}, c_{i1}) = \ln(c_{i0}) + \ln(c_{i1}) + \ln(c_{i2})$
- Endowment $(y_0^H, y_1^H, y_2^H) = (1 - \varepsilon, 1 + 2 \cdot \varepsilon, 1 - \varepsilon)$
- Only asset is short-term bond. International interest rate is zero.
- With no commitment $c_0^H = c_1^H = c_2^H = 1$ and bond holdings are

$$\begin{aligned} \left(x_{i0,1}^H, x_{i1,1}^H, x_{i1,2}^H, x_{i2,2}^H\right) &= (-\varepsilon, +\delta_{i,1}, 0, 0) \text{ for all } i \in I^H \\ \left(x_{i0,1}^F, x_{i1,1}^F, x_{i1,2}^F, x_{i2,2}^F\right) &= (0, 0, +\varepsilon, 0) \text{ for all } i \in I^H \end{aligned}$$

  with $\int_{i \in I^D} \delta_{i,1} = 0$
- With short-run commitment, borrowing is not possible, so $c_0^H = 1 - \varepsilon$ and $c_1^H = c_2^H = 1 + 0.5 \cdot \varepsilon$
- Assume now there are two-period bonds
- Full-enforcement outcome $c_0^H = c_1^H = c_2^H = 1$ is achieved with bond holdings

$$\begin{aligned} \left(x_{i0,2}^H, x_{i1,2}^H, x_{i2,2}^H\right) &= (-\varepsilon, +\delta_{i,2}, 0) \text{ for all } i \in I^H \\ \left(x_{i0,2}^F, x_{i1,2}^F, x_{i2,2}^F\right) &= (0, +\varepsilon, 0) \text{ for all } i \in I^H \end{aligned}$$

  with $\int_{i \in I^D} \delta_{i,2} = 0$
  - at $t = 0$, Home residents sell ε bonds that mature at $t = 2$ to foreigners
  - at $t = 1$, Home residents repurchase these bonds and purchase an additional ε foreign bonds
  - at $t = 2$, Home residents resell their foreign bonds to foreigners, keeping their domestic bonds, and the Home and foreing governments enforce payments

Limits to the argument III.c: Short-run commitment and debt maturity

- Intuition:
  - by the time the Home government chooses enforcement Home residents have already repurchased bonds from foreigners
  - just as governments have an ex-post incentive to preemt secondary markets and avoid payments to foreigners
  - so domestic residents have an individual incentive to preemt the government and repurchase the bonds before enforcement is decided

Limits to the argument III.d: Long-run commitment vs. full enforcement

- Two regions: Debtor ($j = D$) and Creditor ($j = C$). Two dates: Today ($t = 0$) and Tomorrow ($t = 1$)
- Preferences $U(c_{i0}, c_{i1}) = \ln(c_{i0}) + \ln(c_{i1})$
- Debtor has relatively low endowment Today

$$(y_{i0}, y_{i1}) = \begin{cases} (1-\varepsilon, 1+\varepsilon) & \text{if } i \in I^D \\ (1+\varepsilon, 1-\varepsilon) & \text{if } i \in I^C \end{cases}$$

- Assume that the government of Debtor only cares about one half of its residents, called allies (A)
- With no commitment $c_0^D = c_1^D = c_0^C = c_1^C = 1$
- If the government of Debtor commits not to enforce the payments of non-allies, consumption is

$$(c_{i0}, c_{i1}) = \begin{cases} \left(\frac{6+\varepsilon^2}{6-2\cdot\varepsilon}, \frac{6+\varepsilon^2}{6+2\cdot\varepsilon}\right) & \text{if } i \in A \text{ and } i \in I^D \\ (1-\varepsilon, 1+\varepsilon) & \text{if } i \notin A \text{ and } i \in I^D \end{cases}$$

- Non-allies are worse off, but allies are better off
- Intuition:
  - not enforcing payments by non-allies improves the terms of trade of allies