Comments on Invertibility and the Virtues of Differencing

by Albert Marcet

George Tauchen, un Minnesoto de Sims
Duke University

Outline

1. Overview

2. Comments

Overview: Radical Approach

- Difference all of the seemingly non-stationary series
- Run the VAR
- Estimate the MA representation by inverting and accumulating differences
- Appears to work better than running everything in levels or error-correction form

Idea: project

$$y_t = \alpha + \sum_{k=1}^{p} \Gamma_k y_{t-k} + \epsilon_t$$

and then invert

$$\hat{\Gamma}_p(L) = I - \hat{\Gamma}_{p1} L - ... - \hat{\Gamma}_{pp} L^p$$

by doing the usual matrix polynomial long division.

After forming

$$\hat{C}_p(L) = \left[\hat{\Gamma}_p(L)\right]^{-1}$$

accumulate differences as appropriate and then display the cumulated elements of $\hat{C}_p(L)$ as the elements of the MA representation of the non-invertible levels variables.

Comments

- How to impose stability on $\hat{\Gamma}_p(L)$? Include the likelihood of the first p observations. Hard to do in practice.

- Econometric theory: Iterated limits, first let $T \to \infty$, then $p \to \infty$, is a suspect paradigm. Explore local to unity asymptotics.

- Something is amiss with the simulations.

In the **levels** autoregression, say,

$$\Delta y_t = \alpha + \rho y_{t-1} + \psi \Delta y_{t-1} + \epsilon_t$$

it makes a huge difference if the intercept $\alpha = 0$ or not. In the former case the OLS estimate satisfies $T(\hat{\rho} - 1) \xrightarrow{\mathcal{D}}$ DF, while in the latter case $(\alpha \neq 0)$ the OLS estimates $(\hat{\alpha}\, \hat{\rho}\, \hat{\psi})$ are jointly asymptotically $\sqrt{T}$-Gaussian and the usual theory applies. See Hamilton (1994, p. 497, 528).