Discussion of Albert Marcet's

"Invertibility and the Virtues of Differencing"

by

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Prepared for the CREI Conference

MACROECONOMICS AND REALITY, 25 YEARS LATER

Barcelona, Spain

1 April 2005

1. Goals

- *Show*

  Given a sufficiently long (finite) lag length, fit a VAR to $y_t \sim I(0)$: (i) $\rho(\mathbf{L})$ is invertible and (ii) recover **true** MA(∞), $A(\mathbf{L})$, asymptotically.

- *Argue*

  Difference until data are rendered stationary, fit VARs, and obtain impulse response functions.

- *Future Research*

  Focus on VARs' small sample properties in levels, differences, or as VECM, rather than debate about which model must be 'correct'.

2. Why Care?

- Is it estimation? consistency? efficiency?
Or is the concern **identification**?

  Sims (1980): orthogonalize VAR errors to identify shocks of economic interest.

- Test theories of Rational Expectations models?

  VARs used to assess validity of competing theories; for example King and Watson (1994, 1997).

- What guides decisions to fit a VAR in levels, differences, or as a VECM?

  Theory often not the source of these decisions.

3. Differencing & the Approximate MA(∞)

- Proposition 1 seems conservative.

  Find VAR(κ) to approximate MA(∞) from sequence of estimates, $\kappa = 1, \ldots, p$.

- Is this only a sufficient condition?
- Is a necessary condition possible?

  Complement to Fernandez-Villáverde, Rubio-Ramírez, and Sargent (2005)?

4. Computing Approximate MA(∞)

- Can filtering affect estimated MA(∞)?

  Sims (1974), Hansen and Sargent (1993), and Maravall and Planas (1998) tie cross-frequency restrictions to filtering.

- As κ increases, find better approximation of MA(∞) when fitting sequence of VAR(κ)s, with κ increasing.

  Is κ a free variable when fitting a VAR? Stopping rule for κ? AIC, HQC, SIC, or LR test?

- Simulation exercises often show κ stops at 20.

  Kapetanios, Pagan, and Scott (2005) find similar results with large scale NKMs. Explore tie-in?

5. VECMs, Cross-Equation Restrictions, and Approximate MA(∞)

- Surprising that VECMs perform poorly.

  What is the source of the specification error?

- Common trends impose cross-equation restrictions useful for recovering shock fundamentals.

  Examples: one sector RBC model of King, Plosser, Stock, and Watson (1991) and Engle and Issler (1995) use of the Long and Plosser (1983) multi-sector RBC model.

A VECM and Its MA(∞)

- Proietti (1997) and Hecq, Palm, and Urbain (2000) present a VECM($p-1$) of $x_t \sim I(1)$, $dim(x_t) = n$

$$\Delta x_t = \alpha\beta' x_{t-1} + \sum_{j=1}^{p-1} \gamma_j \Delta x_{t-j} + \varepsilon_t,$$

- Assume there are d common trends and $r = n - d$ cointegrating relations.

- ε_t is n-dimensional, mean zero, Gaussian innovation process, with respect to the history of x_t.

- The Hecq, Palm, and Urbain VECM-state space is

$$
\begin{aligned}
\Delta x_t &= \mathcal{H}\xi_t, & \mathcal{H} &= [\mathbf{I_n}\ \mathbf{O_{n\times n}} \ldots \mathbf{O_{n\times r}}] \qquad (1)\\
\xi_{t+1} &= \mathcal{F}\xi_t + \upsilon_{t+1}, & \upsilon_{t+1} &= \mathcal{H}'\varepsilon_{t+1} \qquad (2)
\end{aligned}
$$

$$
\xi_{t+1} = [\Delta x_{t+1}\ \ \Delta x_t \ldots \Delta x_{t-p+3}\ \ \alpha\beta' x_t]'
$$

$$
\mathcal{F} = \begin{bmatrix}
\gamma_1 & \gamma_2 & \ldots & \gamma_{p-1} & \alpha \\
\mathbf{I_n} & \mathbf{O_{n\times n}} & \ldots & \mathbf{O_{n\times n}} & \mathbf{O_{n\times r}} \\
\mathbf{O_{n\times n}} & \vdots & \vdots & \vdots & \vdots \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
\beta' & \mathbf{O_{r\times n}} & \vdots & \vdots & \mathbf{I_r}
\end{bmatrix}
$$

- From Propositions 13.1 – 13.4 of Hamilton (1994) or Fernandez-Villáverde, Rubio-Ramírez, and Sargent (2005)

  The state space (1) – (2) yields a VAR(∞) and a MA(∞), given the eigenvalues of $\mathcal{F}$ are inside the unit circle.

- Is it the common trends restrictions that produce the poor performance of VECMs relative to differenced VARs?

  Pagan and Robertson (1998) find that SVARs behave poorly when identified on long-run restrictions.

6. Suggestions

- Take more care with notion of non-invertibility.

  Is it restrictions on VAR coefficients or linear combinations of history of VAR forecast innovations that fail to converge to MA innovations?

- Consider metric for pairwise comparisons of competing models' MA(∞)s?

  Could recent advances on forecast model selection be helpful?

- Tie motivation for differencing and lag length decision to tests of economic models and policy analysis.