ECONOMIC GROWTH
Lecture Notes
Part II

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Spring 2008

1 Endogenous Technical Change

Endogenous growth theory provides the tools for understanding sustained productivity growth due to technical change. Productivity increases through innovation which is motivated by the prospect of the monopoly rents it generates. These models will be a starting point to study why productivity differs across countries.

1.1 Benchmark model (Romer, 1990)

We study a simplified version of Romer (1990) with no physical capital. See Gancia and Zilibotti (2005). Barro and Sala-I-Martin (2003) also provide an excellent textbook treatment.

Horizontal innovation = introduction of new product variety that does not displace existing varieties. More varieties, in turn translates into higher productivity.

Households

L infinitely lived agents, inelastic labor supply. Consumption path is set to maximize utility:

$$\begin{aligned} \max U &= \int_0^\infty e^{-\rho t} \frac{c_t^{1-\theta} - 1}{1-\theta} dt \\ st &: \dot{b}_t = r_t b_t + w_t - c_t \\ & \quad \text{No-Ponzi condition} \end{aligned}$$

where b is bond holdings, w the wage and r the interest rate. Current value Hamiltonian:

$$H = \frac{c_t^{1-\theta} - 1}{1-\theta} + \mu_t \left[r_t b_t + w_t - c_t \right]$$

FOCs:

$$\begin{aligned} H_c &= 0 \rightarrow c_t^{-\theta} = \mu_t \quad \text{log-differentiate} \quad \frac{\dot{c}_t}{c_t} = -\frac{1}{\theta}\frac{\dot{\mu}_t}{\mu_t} \\ H_b &= -\dot{\mu}_t + \rho\mu_t \rightarrow \frac{\dot{\mu}_t}{\mu_t} = -\left(r_r - \rho\right) \\ & \quad \text{Transversality condition} \end{aligned}$$

Solving yields a standard Euler equation for consumption growth:

$$\frac{\dot{c}_t}{c_t} = \frac{r_t - \rho}{\theta}$$

The production side has two sectors: a competitive sector producing a homogenous final good, and a non-competitive sector producing differentiated intermediate goods.

Final Good Sector (competitive) - FGS

Employs labor and intermediate goods as inputs. Production function:

$$Y_t = L_{y,t}^{1-\alpha} \int_0^{A_t} x_{j,t}^{\alpha}\, dj,$$

where x_j is the quantity of the intermediate good j, A_t is the measure of intermediate goods available at t, L_y is labor and $\alpha \in (0, 1)$.

Note: different inputs are imperfect substitutes and enter symmetrically the production function. No intermediate good is intrinsically better or worse than any other. Also, if $x_{j,t} = K/A_t$ productivity grows with A_t.

Demand for labor and intermediates are found from the profit maximization program of the representative firm:

$$\max \pi_{Y,t} = L_{y,t}^{1-\alpha} \int_0^{A_t} {x_{j,t}}^{\alpha} dj - w_t L_{y,t} - \int_0^{A_t} p_{j,t} x_{j,t} dj.$$

where p_j = price of variety j and the price of Y is one. FOCs:

$$\frac{\partial \pi_{Y,t}}{\partial x_{j,t}} = 0 \rightarrow p_{j,t} = \alpha L_{y,t}^{1-\alpha} (x_{j,t})^{\alpha-1} \qquad \forall j$$
$$\frac{\partial \pi_{Y,t}}{\partial L_{y,t}} = 0 \rightarrow w_{y,t} = (1-\alpha) L_{y,t}^{-\alpha} \int_0^{A_t} x_{j,t}{}^{\alpha} dj$$

Note: demand for $x_{j,t}$ has a constant price elasticity of $\epsilon = \frac{1}{\alpha-1}$.

Intermediate Good Sector (monopolistic) - IGS

Each existing firm holds a patent to produce a single variety j. Technology: one unit of intermediate good requires one unit of final good. Prices are set to maximize profits subject to (isoelastic) demand for $x_{j,t}$. Monopoly pricing:

$$p_{j,t}\left(1 - \frac{1}{|\epsilon|}\right) = \text{MC} \rightarrow p_{j,t} = p_t = \frac{1}{\alpha}$$

where ϵ is the price elasticity of demand. Substituting into demand, we find the quantity:

$$x_{j,t} = x_t = \alpha^{\frac{2}{1-\alpha}} L_{y,t}$$

Hence, profit is:

$$\pi_{j,t} = \pi_t = (p-1)x_t = \frac{1-\alpha}{\alpha}\alpha^{\frac{2}{1-\alpha}} L_{y,t}.$$

Substitution of x_t in the demand for labor yields the wage:

$$w_t = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A_t.$$

Innovation - R&D

Designing a new variety requires a sunk cost of $1/\delta A_t$ units of labor. Thus, the cost of innovation is:

$$\frac{w_t}{\delta A_t}$$

Note the knowledge spillover: productivity in R&D increases with the stock of "knowledge", A_t (i.e., the number of known varieties). The idea is that researchers benefit from past discoveries, obtaining inspiration for new designs ("standing on the shoulders of giants"). The law of motion of A_t follows:

$$\dot{A}_t = \delta A_t L_{RD,t}$$

The rate of technological change is a linear function of total employment in R&D.

Balanced Growth Equilibrium

Guess-and-verify the existence of a balanced growth (BG) equilibrium where c_t, Y_t, w_t and A_t grow at the constant rate, γ. Note that in BG, production and the profits of intermediate firms are constant over time and across industries, $x_t = x$ and $\pi_t = \pi$. By the Euler equation, the interest rate is also constant in BG.

Free entry

For an interior solution with positive growth, the present discounted value (PDV) of profits from innovation has to be equal to the sunk cost of entry:

$$\text{value of innovation } = \frac{\pi}{r} = \frac{w_t}{\delta A_t} = \text{ cost of innovation}$$

Note: both sides are constant in BG due to the knowledge externality. Without it, the cost of innovation would grow over time and technical progress would stop like in the neoclassical model.

Note the role of patents: in the absence of intellectual property rights, free-riding would prevent any innovative activity. If firms could copy, competition would drive ex-post rents to zero. Then, no firms would have an incentive, ex-ante, to pay a sunk cost to design a new input.

The growth rate can be found solving the system:

$$\begin{aligned} \text{Free entry} &: \frac{\pi}{r} = \frac{w_t}{\delta A_t} \\ \text{Euler equation} &: \gamma = \frac{r-\rho}{\theta} \\ \text{Full employment} &: L_y = L - \frac{\gamma}{\delta} \end{aligned}$$

Using π and w_t, and solving:

$$\gamma = \frac{\delta \alpha L - \rho}{\alpha + \theta}$$

Note that an interior solution exists if and only if $\alpha\delta L > \rho$. The growth rate is increasing in the productivity of the research sector (δ), the size of the labor force (L) and the intertemporal elasticity of substitution of consumption ($1/\theta$), while it is decreasing in the elasticity of final output to labor, $(1-\alpha)$, and the discount rate.

The decentralized equilibrium is inefficient (and growth sub-optimally low) for two reasons:

1. Monopoly pricing, higher than marginal cost → underproduction of each variety of intermediates.

2. Ideas produce externalities: innovating firms compare the private cost of innovation, $w_t/(\delta A_t)$, with the present discounted value of profits, π/r and ignore the spillover on the future productivity of innovation.

1.2 **"Lab-Equipment" version (Romer & Rivera Batiz, 1991)**

In the "lab-equipment" model research uses final output instead of labor as a productive input. One innovation requires μ units of Y:

$$\begin{aligned} \text{Free entry} &: \frac{\pi}{r} = \mu \\ \text{Euler equation} &: \gamma = \frac{r-\rho}{\theta} \\ \text{Full employment} &: L_y = L \end{aligned}$$

Together with π and w_t, this yields:

$$\gamma = \frac{1}{\theta}\left[(1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}}\frac{L}{\mu} - \rho\right]$$

Also, $\dot{A}_t = Y_x/\mu$, where Y_x = units of final output devoted to R&D (hence, consumption is $C = Y - Ax - Y_x$). Note that there is no research spillover. Sustained growth is attained by allocating a constant share of production to finance the research activity.

2 Vertical Innovation (Quality Ladder)

Main model: **Aghion & Howitt (1992), Aghion & Howitt (1998)** chapter 2**.** Also, Segerstrom, Anant and Dinopoulos (1990), Barro and Sala-i-Martin (2003) Chapter 7, Grossman and Helpman (1991), chapter 4.

Vertical innovation = higher **quality** good that **replace** an existing good. Key new feature: as emphasized by Schumpeter, innovation generates obsolescence of previous innovations (creative destruction). We discuss here a basic model that abstract from capital accumulation. The structure of the model is similar to those with horizontal innovation (expanding variety).

Preferences:

$$U = \int_0^\infty e^{-\rho t} C_t dt$$

To simplify, agents are risk neutral, $\rho > 0$ is the discount factor.

Technology:

New notation: $i \in \mathbb{N}^+$ indexes the latest generation of innovation. i will increase over time with innovation. We omit the time index when not needed.

Final good sector (competitive):

$$Y(i) = A(i)\, x(i)^\alpha$$

Intermediate good sector (monopolistic):

$$x(i) = L_x$$

R&D (uncertain):

$$\Pr(innov) = \lambda L_{RD} \rightarrow A(i) = \gamma A(i-1)$$

$\Pr(innov)$ is the probability of discovering innovation $i+1$. $\gamma > 1$ is the "size" of the innovation.

Resource constraint (full employment):

$$L = L_x + L_{rd}$$

Static equilibrium ($A(i)$ given)

FGS (competitive):

$$\begin{aligned} \max_x \pi_Y &= A(i)\, x(i)^\alpha - p(i)\, x(i) \\ \text{FOC} &: \; p(i) = \alpha A(i)\, x(i)^{\alpha-1} \end{aligned}$$

Assume $\gamma > \alpha^{-\alpha}$ (drastic innovation - what does that mean?).

IGS (monopoly):

$$\begin{aligned} \max \pi_x &= p(i)\, x(i) - w(i)\, x(i) = \alpha A(i)\, x(i)^\alpha - w(i)\, x(i) \\ \text{FOC} &: \; \alpha^2 A(i)\, x(i)^{\alpha-1} = w(i) \rightarrow x(i)^{\alpha-1} = \frac{w(i)}{\alpha^2 A(i)} \\ p(i) &= \frac{w(i)}{\alpha} \rightarrow \pi(i) = \left(\frac{1}{\alpha} - 1\right) w(i)\, x(i) \end{aligned}$$

Dynamic Equilibrium

Euler equation requires: $r = \rho$

Focus on BG where L_{rd} is constant.

Define $V(i+1)$ the expected present discounted value of the next innovation. $V(i+1)$ is determined by the asset equation:

$$rV(i+1) = \pi(i+1) - \lambda L_{rd} V(i+1)$$

The LHS is the return from investing $V(i+1)$ in bonds, the RHS is the return from buying a new firm. In equilibrium agents should be indifferent about the two types of investment. Note that, with probability λL_{rd}, a new innovation arrives and the value of the firm owning the old technology falls to zero. Solving:

$$V(i+1) = \frac{\pi(i+1)}{r + \lambda L_{rd}}$$

Note: (1) λL_{RD} captures Schumpeterian "creative destruction" and (2) the incumbent does not perform R&D (why? "Arrow effect" the value of an innovation for the incumbent is $V(i+1) - V(i) < V(i+1)$)

FOC for R&D (MB=MC):

$$MB = \lambda V(i+1) = \frac{\lambda \pi (i+1)}{r + \lambda L_{rd}} = w(i) = MC$$

Substitute $w(i) = \alpha p(i) = \alpha^2 A(i) x(i)^{\alpha-1}$, $\pi(i+1) = \left(\frac{1}{\alpha} - 1\right) w(i+1) x(i+1)$ and $x(i) = L - L_{rd}$:

$$\frac{\lambda \left(\frac{1}{\alpha} - 1\right) A(i+1)(L - L_{rd})}{r + \lambda L_{rd}} = A(i)$$

Using $A(i+1) = \gamma A(i)$:

$$\frac{\frac{1-\alpha}{\alpha}\gamma L - \frac{\rho}{\lambda}}{1 + \frac{1-\alpha}{\alpha}\gamma} = L_{rd}$$

Investment in R&D is increasing in γ, λ, L and decreasing in α and ρ. No R&D if L is too low.

Growth Rate

Growth, of course, is proportional to L_{rd}. In particular, note that $Y_{t+dt} = Y_t \cdot \gamma^x$, where x is the number of innovations happening during dt. Thus:

$$\begin{aligned} \ln Y_{t+dt} &= \ln Y_t + x \ln \gamma \rightarrow \ln Y_{t+dt} - \ln Y_t = x \ln \gamma \\ \mathbb{E}(\ln Y_{t+dt} - \ln Y_t) &= \mathbb{E}(x) \ln \gamma \end{aligned}$$

If dt is sufficiently small, the LHS is the expected instantaneous growth rate, while $\mathbb{E}(x) = \lambda L_{rd}$, because innovation is a Poisson process with parameter λL_{rd}. Thus:

$$g = \lambda L_{rd} \ln \gamma$$

- Product market competition α is bad for growth (less profits, lower incentive to innovate)
- Increases in the size of innovation and/or productivity of R&D increase g, both directly and through L_{rd}.

- Impatience discourage growth
- Scale effect (profits are proportional to the size of the economy - the cost of innovation does not depend on the number of people using it)

Social Planner Solution

$$\begin{aligned} U &= \int_0^\infty e^{-\rho t} Y_t dt = \int_0^\infty e^{-\rho t} \mathbb{E}\left[A(\cdot)\right] x(i)^\alpha dt = \\ &= \int_0^\infty e^{-\rho t} \left[\sum_{i=0}^\infty P(i,t) A_0 \gamma^i\right] (L - L_{rd})^\alpha dt \end{aligned}$$

where $P(i,t)$ is the probability that there are exactly i innovations up to time t. Given that innovation is a Poisson process with parameter λL_{rd} we have:

$$P(i,t) = \frac{(\lambda L_{rd} t)^i}{i!} e^{-\lambda L_{rd} t}$$

Substituting:

$$\begin{aligned} U &= \int_0^\infty e^{-\rho t} \left[\sum_{i=0}^\infty \frac{(\lambda L_{rd} t)^i}{i!} e^{-\lambda L_{rd} t} A_0 \gamma^i\right] (L - L_{rd})^\alpha dt = \\ &= \int_0^\infty e^{-\rho t - \lambda L_{rd} t} \left[\sum_{i=0}^\infty \frac{(\lambda \gamma L_{rd} t)^i}{i!}\right] A_0 (L - L_{rd})^\alpha dt \end{aligned}$$

use $\sum_{i=0}^\infty \frac{(z)^i}{i!} = e^z$:

$$U = \int_0^\infty e^{-(\rho + \lambda L_{rd} - \lambda \gamma L_{rd})t} A_0 (L - L_{rd})^\alpha dt = \frac{A_0 (L - L_{rd})^\alpha}{\rho + \lambda L_{rd}(1-\gamma)}$$

Optimal R&D is given by the FOC:

$$\begin{aligned} \frac{\partial U}{\partial L_{rd}} &= 0 \rightarrow \frac{\alpha}{L - L_{rd}} = \frac{\lambda(\gamma - 1)}{\rho + \lambda L_{rd}(1-\gamma)} \\ &= \frac{\lambda(\gamma-1)(L - L_{rd})}{\rho + \lambda L_{rd}(1-\gamma)} = \alpha \end{aligned}$$

Compare this condition with the decentralized solution:

$$\frac{\lambda\gamma\left(1-\alpha\right)\left(L-L_{rd}\right)}{\rho+\lambda L_{rd}}=\alpha$$

Differences:

1. The private discount rate (denominator) is higher than the social discount rate. The planner knows that the benefit of innovation is forever, the private benefit is not. *Intertemporal spillover.* $\rightarrow g^{sp} > g^{d}$

2. The term $(1-\alpha)$ is the *appropriability effect.* The monopolist only appropriate a fraction $(1-\alpha)$ of output and thus does not internalize the entire social value of an innovation. $\rightarrow g^{sp} > g^{d}$

3. *Business stealing effect*: γ instead of $(\gamma-1)$. The social value of an innovation is $\Delta A=\gamma-1$, while the private benefit is proportional to γ. Private innovators are motivated by the benefit of stealing the incumbent's profit, not just by the benefit of improving technology. $\rightarrow g^{sp} < g^{d}$

Depending on which effect dominates, the decentralized growth rate may too low or too high!

Some extensions and applications

- Innovation today is a negative function of expected innovation tomorrow (creative destruction). This opens the possibility to cycles.
- In the basic model, growth comes through discontinuous jumps. The growth rate can be smoothed out by introducing a continuum of final good sectors instead of just one.

Comparison between horizontal and vertical innovation

- The new aspect of vertical innovation is creative destruction

- It introduces the possibility that growth be too high in lassez-faire.
- Horizontal innovation is slightly more tractable.
- Horizontal and vertical innovations are different. Some models combine them. See for example Young (1998).

3 Directed Technical Change

So far, technical progress has been modeled as an increase in total factor productivity (A) that is neutral towards different factors and sectors. For many applications, however, this assumption is not realistic. For example, there is evidence that technical progress has been skill-biased during the last century and that this bias accelerated during the 1980s. What determines the skill-bias of technology? To build a theory for the type and the direction of technical change, we introduce more sectors and factors in a model with horizontal innovation a la Romer (1990). The analysis follows the synthesis in Acemoglu (2002) and Gancia and Zilibotti (2005), Chapter 4. We present the basic model followed by a number of applications.

Aggregate Output

Aggregate output, Y, is a CES function of two goods produced in turn with different factors:

$$Y = \left[Y_L^{(\epsilon-1)/\epsilon} + Y_H^{(\epsilon-1)/\epsilon} \right]^{\epsilon/(\epsilon-1)}.$$

Y_L and Y_H are goods produced with unskilled labor, L, and skilled labor, H, respectively.

Profit maximization gives constant elasticity demand functions:

$$\begin{aligned} & \max_{Y_H, Y_L} \; Y - P_H Y_H - P_L Y_L \\ FOCs \quad : & \quad Y^{-1} Y_H^{\frac{\epsilon-1}{\epsilon}-1} = P_H \\ : & \quad Y^{-1} Y_L^{\frac{\epsilon-1}{\epsilon}-1} = P_L \\ \rightarrow & \quad \frac{P_H}{P_L} = \left[\frac{Y_L}{Y_H} \right]^{1/\epsilon}. \end{aligned}$$

where P_L and P_H are the prices of Y_L and Y_H, respectively.

Aggregate output is chosen as the numeraire. Hence, from $Y = P_H Y_H + P_L Y_L$ and relative demand, we obtain:

$$\begin{aligned}\left(P_L^{1-\epsilon}+P_H^{1-\epsilon}\right)^{1/(1-\epsilon)} &= 1\\ \left[\left(\frac{P_L}{P_H}\right)^{1-\epsilon}+1\right] &= \frac{1}{P_H^{1-\epsilon}}\\ P_H &= \left[\left(\frac{P_H}{P_L}\right)^{\epsilon-1}+1\right]^{\frac{1}{\epsilon-1}}\end{aligned}$$

Sectoral Production

In each sector, production takes place as in models with horizontal innovation (e.g., Romer, 1990). However, the two goods are produced using different technologies:

$$\begin{aligned}Y_L &= L^{1-\alpha}\int_0^{A_L} x_{L,j}{}^{\alpha}dj\\ Y_H &= H^{1-\alpha}\int_0^{A_H} x_{H,j}{}^{\alpha}dj,\end{aligned}$$

where $x_{L,j}$, $j \in [0, A_L]$, are intermediate goods complementing unskilled labor L, whereas $x_{H,j}$, $j \in [0, A_H]$ complement skilled labor H.

Producers of Y_L and Y_H take the price of their output $(P_L, P_H,)$, the price of intermediates $(p_{L,j},\ p_{H,j})$ and wages $(w_L,\ w_H)$ as given. Consider a variety j used in the production of Y_L. Profit maximization gives isoelastic demand:

$$\begin{aligned}& \max_{x_{L,j}}\ P_L Y_L - \int_0^{A_L} p_{L,j}x_{L,j}dj - w_L L\\ FOC\ :\ & \alpha P_L L^{1-\alpha} x_{L,j}{}^{\alpha-1} = p_{L,j}\\ \rightarrow\ & x_{L,j} = \left[\frac{\alpha P_L}{p_{L,j}}\right]^{\frac{1}{1-\alpha}} L,\end{aligned}$$

An equivalent expression gives demand for $x_{H,j}$.

Intermediate good sectors (monopolistic)

Each intermediate is produced by a single monopolist (patent owner). The cost of producing one unit of any intermediate is one unit of the numeraire. As it is well known, monopolists charge a price that is a markup over the marginal cost. Symmetry in demand elasticity and the marginal cost implies that all monopolists charge the same price equal to (verify this yourself):

$$p_L = p_H = 1/\alpha$$

and sell:

$$\begin{aligned} x_L &= \left(\alpha^2 P_L\right)^{\frac{1}{1-\alpha}} L \\ x_H &= \left(\alpha^2 P_L\right)^{\frac{1}{1-\alpha}} L \end{aligned}$$

Profits

The profit flow of each monopolist is:

$$\begin{aligned} \pi_L &= p_L x_L - x_L = (1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}} \left(P_L\right)^{\frac{1}{1-\alpha}} L \\ \pi_H &= p_H x_H - x_H = (1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}} \left(P_H\right)^{\frac{1}{1-\alpha}} H. \end{aligned}$$

Thus, relative profitability in the two sectors is given by:

$$\frac{\pi_H}{\pi_L} = \left(\frac{P_H}{P_L}\right)^{1/(1-\alpha)} \frac{H}{L}.$$

Since profits are the reward of innovation, $\frac{\pi_H}{\pi_L}$ is also the relative profitability of R&D directed to the two sectors. Relative profitability has two components:

1. The first term is the "price effect": there is a greater incentive to invent technologies producing more expensive goods.[1]

2. The second term is the "market size" effect: the incentive to develop a new technology is proportional to the number of workers that will be using it.[2]

[1] The price effect, restated in terms of factor prices, was emphasized by Hicks (1932) and Habakkuk (1962).

[2] Market size was emphasized as a determinant of technical progress by Griliches and Schmookler (1963), Schmookler (1966) and Schumpeter (1950).

Static Equilibrium for a Given Technology

Substituting x_L and x_H, final output in each sector becomes:

$$\begin{aligned} Y_L &= \alpha^{\frac{2\alpha}{1-\alpha}} P_L^{\alpha/(1-\alpha)} A_L L \\ Y_H &= \alpha^{\frac{2\alpha}{1-\alpha}} P_H^{\alpha/(1-\alpha)} A_H H. \end{aligned}$$

Next we can use demand for Y_L and Y_H to solve for **prices** as function of the state of technology:

$$\frac{P_H}{P_L} = \left[\frac{Y_L}{Y_H}\right]^{1/\epsilon} = \left[\frac{A_H}{A_L}\frac{H}{L}\right]^{-(1-\alpha)/\sigma}$$

where $\sigma \equiv 1 + (1-\alpha)(\epsilon - 1)$.

To find relative **wages**, note that the sectorial production function is Cobb-Douglas in wages. Thus, the wage bill in each sector is a constant fraction of total revenue:

$$\frac{w_H}{w_L} = \frac{(1-\alpha) Y_H P_H / H}{(1-\alpha) Y_L P_L / L} = \left[\frac{A_H}{A_L}\right]^{1-1/\sigma} \left[\frac{H}{L}\right]^{-1/\sigma}.$$

It is important to note that:

- σ is the elasticity of substitution between H and L :[3]

$$\sigma = -\frac{d \ln(H/L)}{d \ln(w_H/w_L)}.$$

- The skill premium, w_H/w_L, is decreasing in the relative supply of skilled labor (H/L) and increasing in the skill-bias (A_H/A_L).

Dynamics of Innovation

The development of a new intermediate requires a fixed cost of μ units of the numeraire. An innovator has to decide in advance whether to develop a L- or H-complement innovation. The value of an innovation is the present discounted value of the infinite stream of profits it generates.

[3] This is the short-run elasticity of substitution between L and H, for a given technology A_L and A_H.

Thus, the asset equations for innovation are:

$$\begin{aligned} rV_L &= \pi_L + \dot{V}_L \\ rV_H &= \pi_H + \dot{V}_H \end{aligned}$$

Free-entry implies:

$$\begin{aligned} V_L &\leq \mu \quad \text{with equality if there is L-complement innovation} \\ V_H &\leq \mu \quad \text{with equality if there is H-complement innovation} \end{aligned}$$

Thus, when innovation is positive, we can have three cases:

1. Transition with H-biased innovation only: $V_H = \mu = \frac{\pi_H}{r}$ and $V_L < \mu$.
2. Transition with L-biased innovation only: $V_L = \mu = \frac{\pi_L}{r}$ and $V_H < \mu$.
3. Balanced growth path: $V_H = V_L = \mu = \frac{\pi_H}{r} = \frac{\pi_L}{r}$.

Equilibrium Technology (Balanced Growth)

An equilibrium with a positive rate of innovation in both types of intermediates such that the ratio A_H/A_L remains constant (thus, P_H/P_L, w_H/w_L and π_H/π_L are constant too), requires profit equalization in the two sectors, $\pi_H = \pi_L = \pi$. Imposing this:

$$\begin{aligned} \frac{\pi_H}{\pi_L} &= \left(\frac{P_H}{P_L}\right)^{1/(1-\alpha)} \frac{H}{L} = \left(\frac{A_H}{A_L}\right)^{-\frac{1}{\sigma}} \left(\frac{H}{L}\right)^{1-\frac{1}{\sigma}} = 1 \\ &\rightarrow \frac{A_H}{A_L} = \left[\frac{H}{L}\right]^{\sigma-1}. \end{aligned}$$

Note that, as long as workers are gross substitutes ($\sigma > 1$), an increase in the supply of one factor will induce more innovation directed to that factor. Thus:

- When $\sigma > 1$ the market size effect dominates the price effect and technology is biased towards the abundant factor.
- It is also easy to show that starting from any $\frac{A_H}{A_L}$ the economy will converge to the balanced growth path (why? look at $\frac{\pi_H}{\pi_L}$).

Growth Rate

The growth rate γ of the economy can be found from:

$$\frac{\pi_Z}{r} = \mu, \quad Z \in \{L, H\}.$$

For example, substituting:

$$\begin{aligned}
\pi_H &= (1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}}\,(P_H)^{\frac{1}{1-\alpha}}\,H \\
P_H &= \left[\left(\frac{P_H}{P_L}\right)^{\epsilon-1} + 1\right]^{\frac{1}{\epsilon-1}} \\
\frac{P_H}{P_L} &= \left[\frac{A_H}{A_L}\frac{H}{L}\right]^{-(1-\alpha)/\sigma} \\
\frac{A_H}{A_L} &= \left[\frac{H}{L}\right]^{\sigma-1}
\end{aligned}$$

and using the Euler equation for consumption growth:

$$\gamma = \frac{1}{\theta}\,(r-\rho)$$

we obtain:

$$\gamma = \frac{1}{\theta}\left[\frac{(1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}}}{\mu}\left(L^{\sigma-1} + H^{\sigma-1}\right)^{1/(\sigma-1)} - \rho\right].$$

Using the expression for Y and the resource constraint of the economy, it can be verified that:

$$\gamma = \frac{\dot{A}}{A} = \frac{\dot{Y}}{Y} = \frac{\dot{C}}{C}.$$

4 Directed Technical Change: Applications

The theory of Directed Technical Change (DTC) can help us understand a number of important phenomena.

4.1 Acemoglu (1998): why do new technologies complement skill?

Originally, the first model of DTC was developed to explain why technical progress has been skill-biased in the past decades and its impact on the skill premium. Let's substitute the equilibrium technology bias:

$$\frac{A_H}{A_L} = \left[\frac{H}{L}\right]^{\sigma-1}$$

into relative wages:

$$\frac{w_H}{w_L} = \left[\frac{A_H}{A_L}\right]^{1-1/\sigma} \left[\frac{H}{L}\right]^{-1/\sigma} = \frac{w_H}{w_L} = \left[\frac{H}{L}\right]^{\sigma-2}.$$

The relationship between relative wages and relative labor supply can either be positive or negative and is the result of two opposite forces.

1. On one hand, a large supply of one factor depresses the price of its product. We see this when we change $\frac{H}{L}$ for a given $\frac{A_H}{A_L}$.
2. On the other, a large supply of one factor induces a technology bias in its favor, thereby raising its productivity.

Note that:

- the relative strenght of the two effects depend on σ. A high substitutability between H and L implies a weak price effect of an increase in relative supply, which makes a positive relationship more likely.
- if $\sigma > 2$, the market size effect is so strong that an increase in a factor leads to an increase in the relative reward of that factor.

Implications:

1. The model suggests that technical change has been skill biased in the recent past because of the steady growth in the supply of skilled labor, H.

2. The case $\sigma > 2$ offers an explanation for the fall and rise in the US skill premium during the 1970s and 1980s. In the 1970s, there was a large increase in the supply of skilled labor (H/L). Assuming this shock to be unexpected, the model predicts an initial fall in the skill premium (recall that A_H/A_L is a state variable that does not immediately adjust!), followed by its rise due to the induced skill biased technical change, a pattern broadly consistent with the evidence (see the figure). For this to be the case, we need $\sigma > 2$. Most empirical estimates of this parameter are close to 1.5, but some are indeed above 2.

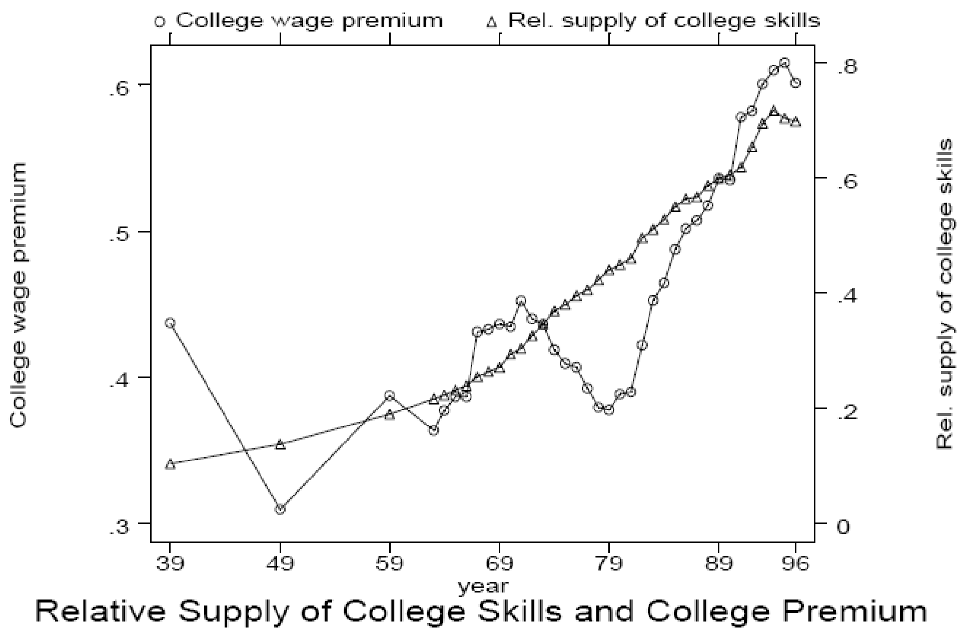


Figure 1: The behavior of the (log) college premium and relative supply of college skills (weeks worked by college equivalents divided by weeks worked of noncollege equivalents) in the U.S. between 1939 and 1996.

4.2 Acemoglu (2003): Patterns of Skill Premia

Next, we can use the model to study how trade between rich and poor countries affects the technological skill-bias. In particular, we study what happens to technology if we allow free trade in Y_L and Y_H between a skill-abundant North and a skill-scarce South. *We assume that technologies developed in the North are not sold to producers in the South (say, because there is no enforcement of IPRs in the South).*

What is the effect of this form of trade on the relative incentives to develop skill-complement innovations? The market size for innovations does not change, because inventors continue to sell their machines in the North only. But trade, at first, will increases the relative price of skill-intensive goods in the North. To see this, note that trade generates a single world market with a relative price depending on the world supply of goods. Since skills are scarcer in the world economy than in the North alone, trade will increase the relative price of skill-intensive goods in the North (the opposite will happen in the South). In particular, for a given $\frac{A_H}{A_L}$ world prices now depend on world endowments:

$$\frac{P_H}{P_L} = \left(\frac{A_H}{A_L}\frac{H^W}{L^W}\right)^{-(1-\alpha)/\sigma} > \left(\frac{A_H}{A_L}\frac{H^N}{L^N}\right)^{-(1-\alpha)/\sigma}.$$

This change in prices, for a given technology, makes skill-complement innovations more profitable and accelerates the creation of skill-complementary machines. Along the BG path, however, both types of innovations must be equally profitable:

$$\begin{aligned}\frac{\pi_H}{\pi_L} &= \left(\frac{P_H}{P_L}\right)^{1/(1-\alpha)}\frac{H^N}{L^N} = 1 \\ &\rightarrow \frac{P_H}{P_L} = \left(\frac{H^N}{L^N}\right)^{\alpha-1}\end{aligned}$$

Thus, skill-biased technical change continues until the relative price of goods has returned to the pre-trade level in the North. Combining these conditions yields the new equilibrium skill bias of technology:

$$\frac{A_H}{A_L} = \frac{L^W}{H^W}\left[\frac{H^N}{L^N}\right]^{\sigma}.$$

Given that $H^N/L^N > H^W/L^W$, the new technology is more skill-biased and skilled workers in the North earn higher wages.

The new skill premium is:

$$\frac{w_H}{w_L} = \left[\frac{A_H}{A_L}\right]^{1-1/\sigma} \left[\frac{H^W}{L^W}\right]^{-1/\sigma} = \left[\frac{H^N}{L^N}\frac{L^W}{H^W}\right]\left[\frac{H^N}{L^N}\right]^{\sigma-2}.$$

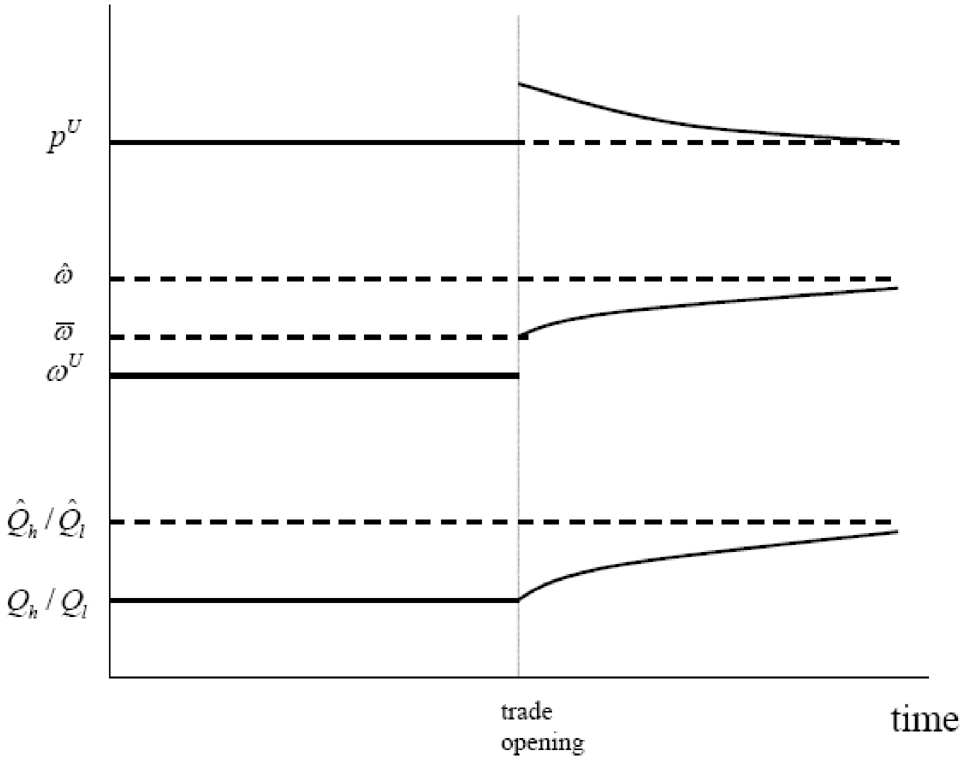
The effect of a move from autarky to free trade on $\frac{w_H}{w_L}$ can be approximated by the elasticity of the skill premium to a change in L^W/H^W computed at $L^W/H^W = L^N/H^N$ (that is, starting from the pre-trade equilibrium).

- In the long run, after technology adjusts, this elasticity is one. Thus, if, for example, L^W/H^W were 4% higher than L^N/H^N, the model would predict trade to raise the skill premium by the same 4%.[4]
- In the short run, without technical change, the elasticity of the skill premium to a change in L/H is $1/\sigma$, less than in the case of endogenous technology as long as $\sigma > 1$, i.e., when skilled and unskilled workers are gross substitutes.

Thus, with directed technical change and $\sigma > 1$, trade increases the skill premium in the North by more than would otherwise be the case: for example, if the elasticity of substitution is 2, the endogenous reaction of technical progress doubles the impact of trade on wage inequality.

The dynamic evolution of the main variables is depicted in Figure 2, where ω stands for the skill premium, p^U is the relative price in the North and Q_h/Q_l is the technology bias $\frac{A_H}{A_L}$. Note that trade opening triggers a period of skill-biased technical change and rising wage inequality.

[4] Borjas, Freeman and Katz (1997) show that 4% is a plausible estimate of the increase in the unskilled labor content of US trade with LDCs between 1980 and 1995. Therefore, this simple exercise may give a sense of how much of the roughly 20% increase in the US skill premium in the same period can be attributed to trade.



4.3 Acemoglu and Zilibotti (2001): Productivity Differences

Directed technical change has interesting implications for the analysis of cross-country income differences. Acemoglu and Zilibotti (2001) show that technologies resulting from directed technical change are optimal for the economic conditions of the markets where they are sold. They analyze the implications of this finding in a North-South model under the following assumptions:

- innovation takes place in the North
- the South can copy technologies from the North and does not enforce IPRs
- there is no trade

Under these assumptions, innovators in the North can only extract rents from selling technologies in the Northern market and innovation does not respond to the factor endowment of the South. As a result, technologies will be too skill-biased for the needs of the South. Through this channel, the model predicts North-South productivity differences, *even when the technology is identical and there are no significant barriers to technology adoption.*

To see this, we use the equilibrium expressions for Y_H and Y_L in the North and South to express relative income as:

$$\frac{Y_N}{Y_S} = \left[\frac{\left(P_{L,N}^{\alpha/(1-\alpha)} A_L L_N\right)^{(\epsilon-1)/\epsilon} + \left(P_{H,N}^{\alpha/(1-\alpha)} A_H H_N\right)^{(\epsilon-1)/\epsilon}}{\left(P_{L,S}^{\alpha/(1-\alpha)} A_L L_S\right)^{(\epsilon-1)/\epsilon} + \left(P_{H,S}^{\alpha/(1-\alpha)} A_H H_S\right)^{(\epsilon-1)/\epsilon}} \right]^{\frac{\epsilon}{\epsilon-1}}$$

To simplify the analysis, we choose convenient value for some parameters. In particular, we set:

$$\begin{aligned} \epsilon &= 2 \\ \lim \alpha &\to 0 \end{aligned}$$

With this parametrization, the model has a reduced form that is almost identical to that of the slightly different model studied in Acemoglu and Zilibotti (2001). In particular, relative output becomes:

$$\frac{Y_N}{Y_S} = \left[\frac{(A_L L_N)^{1/2} + (A_H H_N)^{1/2}}{(A_L L_S)^{1/2} + (A_H H_S)^{1/2}} \right]^2 = \frac{L_N}{L_S} \left[\frac{1 + (a h_n)^{1/2}}{1 + (a h_s)^{1/2}} \right]^2$$

where we have used the notation $a = \frac{A_H}{A_L}$ and $h = \frac{H}{L}$. To study the effects of technology on income differences, we derive with respect to a, i.e., the skill-bias of technology:

$$\begin{aligned} \frac{\partial \ln \frac{Y_N}{Y_S}}{\partial a} &= \frac{a^{-1/2} (h_n)^{1/2}}{1 + (a h_n)^{1/2}} - \frac{a^{-1/2} (h_s)^{1/2}}{1 + (a h_s)^{1/2}} = \\ &= \frac{(h_n)^{1/2} - (h_s)^{1/2}}{a^{1/2} \left(1 + (a h_n)^{1/2}\right) \left(1 + (a h_s)^{1/2}\right)} > 0 \end{aligned}$$

Where the inequality follows the assumption that the North is skill-abundant. Thus:

- The North benefits more than the South from skill-biased technologies
- There will be productivity differences, even if both countries have access to the

same technology

Acemoglu (2002) shows that this result holds in general, without imposing restrictions on parameters. Acemoglu and Zilibotti (2001), instead, show that "inappropriate technologies" can account for about one-third of the total factor productivity gap between the United States and developing countries.

4.4 Acemoglu (2003): Labor- and Capital-Augmenting Technical Change

In Acemoglu (2003), this model is used to study the direction of technical progress when the two factors of production are capital and labor, and capital can be accumulated. The main finding is that, when both capital and labor augmenting innovations are allowed, a balanced growth path still exists and features labor-augmenting technical progress only. The intuition is that, while there are two ways of increasing the production of capital-intensive goods (capital-augmenting technical change and accumulation), there is only one way of increasing the production of labor-intensive goods (labor-augmenting technical progress). Therefore, in the presence of capital accumulation, technical progress must be more labor-augmenting than capital-augmenting. Further, if capital and labor are gross complements (i.e., the elasticity of substitution between the two is less than one), which seems to be the empirically relevant case, it can be shown that the economy converges to the balanced growth path.

5 Competition and Innovation

In the basic model of vertical innovation product market competition discourages growth. It can be shown that the same is true in models of horizontal innovation (such as Romer, 1990). The reason is that the prospect of monopoly profits provide the incentive to innovate. However, the empirical evidence suggests that, in some cases, competition is beneficial for innovation. The most recent studies show that the relationship between competition and innovation is inverted U shaped. That is, innovation first increases with competition, then it starts to decline when competition is very stiff.

TABLE I
Exponential Quadratic: Basic Specification

Dependent variable: citation-weighted patents	(1)	(2)	(3)	(4)
Data frequency	Annual	Annual	5-year averages	Annual
Competition_{jt}	152.80	387.46	819.44	385.13
	(55.74)	(67.74)	(265.63)	(67.56)
$\text{Competition squared}_{jt}$	−80.99	−204.55	−434.43	−204.83
	(29.61)	(36.17)	(141.43)	(36.06)
Significance of: Competition_{jt},	7.60	38.34	9.97	32.59
$\text{Competition squared}_{jt}$	(0.02)	(0.00)	(0.01)	(0.00)
Significance of policy instruments				10.11
in reduced form				(0.002)
Significance of other instruments				5.00
in reduced form				(0.000)
Control functions in regression				4.38
				(4.04)
R^2 of reduced form				0.801
Year effects	Yes	Yes	Yes	Yes
Industry effects		Yes	Yes	Yes
Observations	354	354	67	354

Competition_{jt} is measured by (1-Lerner index) in the industry-year. All columns are estimated using an unbalanced panel of seventeen industries over the period 1973 to 1994. Estimates are from a Poisson regression. Numbers in brackets are standard errors. The standard errors in column (4) have not been corrected for the inclusion of the control function. Significance tests show likelihood ratio test-statistics and P-value from the F-test of joint significance. The fourth column includes a control function. The excluded variables are policy instruments specified in Table II, imports over value-added in the same industry-year, TFP in the same industry-year, output minus variable costs over output in the same industry-year and estimates of markups from industry-country regression [Martins et al. 1996] interacted with time trend, all for the United States and France.

The model by Aghion, Bloom, Blundel, Griffith and Howitt (2005), from which the above table is taken, provides theoretical and empirical support for this hypothesis.

Technology

Final output is a symmetric Cobb-Douglas function of production in a continuum $[0, 1]$ of sectors:

$$\ln y = \int_0^1 \ln x_j dj$$

In each sector, production is done by two firms, a and b:

$$x_j = x_{aj} + x_{bj}$$

Log preferences imply that consumers spend the same amount on each sector j. Using the current expenditure E as the numeraire, we have:

$$p_{aj}x_{aj} + p_{bj}x_{bj} = E = 1$$

Labor is the only input. The production function of firm a in sector j is:

$$x_{aj} = \gamma^{k_{aj}} L_{aj}$$

where $\gamma > 1$ is the size of each innovations and k_{aj} the number of innovations available to firm aj. Firms (a, b) differ in technology, i.e., the number of innovations they master. Innovation will be endogenous and subject to uncertainty, but we make a simplifying assumption on the technology gap between two firms in the same sector.

ASSUMPTION: leaders and followers can at most be separated by one innovation (one-step technology gap) due to a knowledge spillover. That is, technologies two-periods behind the leading edge are common knowledge. This means that a sector can be in one of two possible states:

- unleveled sectors, where one firm is one step ahead
- neck-and-neck sectors, where both firms use the same technology

Unleveled sectors

The leader (subscript 1) is one step ahead of the follower (subscript -1). The leader is γ times more productive that the follower. This means that he cannot charge a markup over the marginal cost higher than γ, or else the follower would be able to compete. Thus, the leader charges a limit price:

$$p_1 = \gamma c$$

where c is the marginal cost of the leader:

$$c = \gamma^{-k} w$$

where k is technology level of the leader.

The follower stays out of the market and thus makes no profits:

$$\pi_{-1} = 0$$

Profits made by the leader instead are:

$$\pi_1 = px - cx = 1 - \frac{c}{p} = 1 - \gamma^{-1}$$

where we used $px = 1$ and $p = \gamma c$.

Note: profits do not depend on the absolute level of technology, only on the gap between firms! This is because of the log functional form.

Neck-and-neck sectors

Both firms use the same technology (subscript 0). In this case, we assume that profits made by each firm are:

$$\pi_0 = (1 - \Delta)\, \pi_1$$

$\Delta \in [0.5, 1]$ measures the level of competition. In fact:

- $\Delta = 0.5$ is perfect collusion, so that the two firms equally split the profits that a single monopolist would make.
- $\Delta = 1$ is Bertrand competition, where profits are driven to zero.
- Note also that:
$$\frac{\pi_1 - \pi_0}{\pi_1} = \Delta$$

That is, the profit gain of becoming leader is increasing in the level of competition.

Innovation

1. Quadratic costs of R&D: by spending $\frac{n^2}{2}$ units of labor, a firm innovates with probability n.
2. Moreover, a follower can move one step ahead with probability h even if it does not invest in R&D (exogenous spillover).

Note: the leader has no incentive to innovate! Why? Cannot gain anything from innovation, because the maximum technology gap is one step and π_1 does not depend on the level of technology. Thus, $n_1 = 0$, but we need to find n_0 and n_{-1}.

Note also that the choice of numeraire implies $w = 1$.

The Bellman equations for the value V of each type of firm are:

$$rV_1 = \pi_1 + (n_{-1} + h)(V_0 - V_1)$$

note that, with probability $(n_{-1} + h)$ the firm looses the lead.

$$rV_{-1} = (n_{-1} + h)(V_0 - V_{-1}) - \frac{n_{-1}^2}{2}$$

note that, with probability $(n_{-1} + h)$ the follower catches up.

$$rV_0 = \pi_0 + n_0^*(V_{-1} - V_0) + n_0(V_1 - V_0) - \frac{n_0^2}{2}$$

where n_0^* is the research intensity of the other competing firm (in equilibrium we will have $n_0^* = n_0$). Note that with probability n_0 the firm becomes leader, with probability n_0^* becomes follower.

The first order conditions for n_{-1} and n_0 are:

$$\begin{aligned}
\frac{\partial V_{-1}}{\partial n_{-1}} &= 0 \to V_0 - V_{-1} = n_{-1} \\
\frac{\partial V_0}{\partial n_0} &= 0 \to V_1 - V_0 = n_0
\end{aligned}$$

The LHS is the marginal benefit of R&D (the gain in firm value if innovation is successful), the RHS is the marginal cost. Substitute into the Bellman equations and impose $n_0^* = n_0$:

$$\begin{aligned}
rV_1 &= \pi_1 - (n_{-1} + h)\, n_0 \\
rV_{-1} &= (n_{-1} + h)\, n_{-1} - \frac{n_{-1}^2}{2} \\
rV_0 &= \pi_0 - n_0 n_{-1} + n_0 n_0 - \frac{n_0^2}{2}
\end{aligned}$$

Escape Competition Effect (neck-and-neck)

To simplify the analysis, consider a limit case when $r \to 0$. Then, subtract the third asset equation from the first:

$$\begin{aligned}
0 &= \Delta\pi_1 - hn_0 - \frac{n_0^2}{2} \\
&\to \; n_0 = \sqrt{h^2 + 2\Delta\pi_1} - h
\end{aligned}$$

NOTE: Δ increases n_0. That is, competition increases innovation in neck-and-neck industries. *Escape competition* effect. The value of innovation in neck-and-neck industries (value of becoming a leader) is higher with more competition. Intuitively, when competition is stiff, the incentive to escape it is strong.

Schmpeterian Effect (unleveled sectors)

To find n_{-1}, substitute n_0 into $rV_1 = 0$:

$$\begin{aligned}
0 &= \pi_1 - (n_{-1} + h)\left(\sqrt{h^2 + 2\Delta\pi_1} - h\right) \\
n_{-1} &= \frac{\pi_1}{\sqrt{h^2 + 2\Delta\pi_1} - h} - h
\end{aligned}$$

Δ decreases n_{-1}. That is, competition reduces innovation in unleveled sectors. *Schum-*

peterian effect. For a follower, more competition discourage innovation, as it means lower profits if innovation is successful.

Competition and Innovation: Inverted U?

Competition fosters innovation in neck-and-neck industries, but lowers innovation in unleveled sectors. Thus, the overall effect of competition on growth will depend on the fraction of leveled versus unleveled sectors. This, fraction, in turn is endogenous. Denote the (steady-state) fraction of **unleveled** industries μ_1.

The flow of industries out of μ_1 is:

$$\mu_1 (n_{-1} + h)$$

The flow of industries into μ_1 is:

$$2 (1 - \mu_1) n_0$$

In steady-state, μ_1 must be constant so that the flows in and out must be equal:

$$\mu_1 (n_{-1} + h) = 2 (1 - \mu_1) n_0$$

thus:

$$\frac{\mu_1}{1 - \mu_1} = \frac{2n_0}{n_{-1} + h}$$

Then, the following possibility arises:

- when competition, Δ, is high, $\frac{n_0}{n_{-1}}$ is high and μ_1 is high too. Since many industries are unleveled, the Schmpeterian effect may dominate. In this case, when competition is already high, more competition can reduce growth.
- when competition, Δ, is low, $\frac{n_0}{n_{-1}}$ is low so that μ_1 is low. Since many industries are leveled, the escape competition effect may dominate. In this case, starting from low competition, more competition fosters growth.

In other words, the relationship between competition and growth may be inverted U shaped. The formal conditions for this to happen are derived in the paper.

Acemoglu & Zilibotti (1997) show that financial underdevelopment (lack of diversification opportunity) may slow down the adoption of more productive, but risky, technology. This can lead to lower TFP, lower speed of convergence and higher volatility at earlier stage of development.

Preferences

Time is discrete. The economy is populated by overlapping generations of two-period lived households. Each cohort has a unit mass ($L = 1$).

Uncertainty is represented by a continuum of equally likely states $s \in [0, 1]$.

Agents consume only in the second period of their lives. Preferences (unit relative risk aversion):

$$E_t U(c_{t+1}) = \int_0^1 \log(c_{t+1}^s)\ ds.$$

Note: utility is state s-dependent.

Final Good Sector

There is a single final good (numeraire). Production requires capital and labor. Output in state s is given by:

$$\begin{aligned} Y_{s,t} &= (K_{s,t})^{\alpha} L^{1-\alpha} \\ K_{s,t} &= x_{s,t-1} + x_{\Phi,t-1} \end{aligned}$$

Capital, $K_{s,t}$, is either produced by:

1. a number n_t of risky projects, producing a state-contingent amount of output (x_s)
2. a separate sector using a "safe technology" (x_Φ).

The number of risky projects, n_t, is determined in equilibrium. Moreover, $n_t \in [0, 1]$, i.e., the set is bounded.

Factor prices (marginal product):

$$\begin{aligned} \text{wage} &: \; w_{s,t} = (1-\alpha) Y_{s,t} \\ \text{return to capital} &: \; \rho_{s,t} = \alpha (K_{s,t})^{\alpha-1} \end{aligned}$$

Timeline:

1. When young, agents work and earn $w_{s,t}$.
2. At the end of the period, they take portfolio decisions. They can: (1) place their savings in a set of risky securities ($\{F_i\}_{i\in[0,n_t]}$), consisting of state-contingent claims to the output of the risky projects or (2) in a safe asset (ϕ), consisting of claims to the output of the safe technology.
3. Uncertainty unravels $\rightarrow$ next period capital is determined. Capital is sold to final sector firms and fully depreciates after use.
4. Old agents consume their capital income and die.

Investment Sectors

Risky Projects. Use final output for production. A project $i \in [0, n_t]$ produces a positive output only if state $s = i$ occurs. In all other states of nature, the project is not productive. Moreover, the i-th project is only productive if it has a minimum size, M_i, where

$$M_i = \max\left\{0, \frac{D}{(1-x)}(i-x)\right\},$$

with $x \in (0,1)$ .Note: projects $i \leq x$ have no minimum size requirement, and for the rest the minimum size requirement increases linearly with the index i.

Projects' outcome:

$$x_{i,s} = \begin{cases} RF_i & \text{if } i = s \text{ and } F_i \geq M_i \\ 0 & \text{otherwise} \end{cases}.$$

The j'th security entitles its owner to a claim to R units of capital in state j (as long as the minimum size constraint is satisfied), and otherwise to nothing.

Safe Technology. Savings invested in the "safe technology" give the return:

$$x_{\Phi,s} = r\phi, \ \forall s \in [0,1],$$

where $r < R$.

Portfolio Decision

Since the risky securities yield symmetric returns, agents will hold a balanced portfolio (containing all available securities in equal amounts):

$$F_i = F, \quad \text{for all } i \in [0, n_t]$$

If $n_t = 1$, a balanced portfolio bears no risk, and dominates the safe investment. However, due to the presence of minimum size requirements, not all projects are in general open. When $n_t < 1$, the inferior technology is safer, and there is a trade-off between risk and productivity. In this case, the optimal investment decision of the representative saver is:

$$\max_{\phi_{t,F_t}} E_t U(c_{t+1}) = n_t \log\left[\rho_{G,t+1}\left(RF_t + r\phi_t\right)\right] + (1-n_t)\log\left[\rho_{B,t+1}\left(r\phi_t\right)\right],$$

subject to:

$$\phi_t + n_t F_t \leq w_t.$$

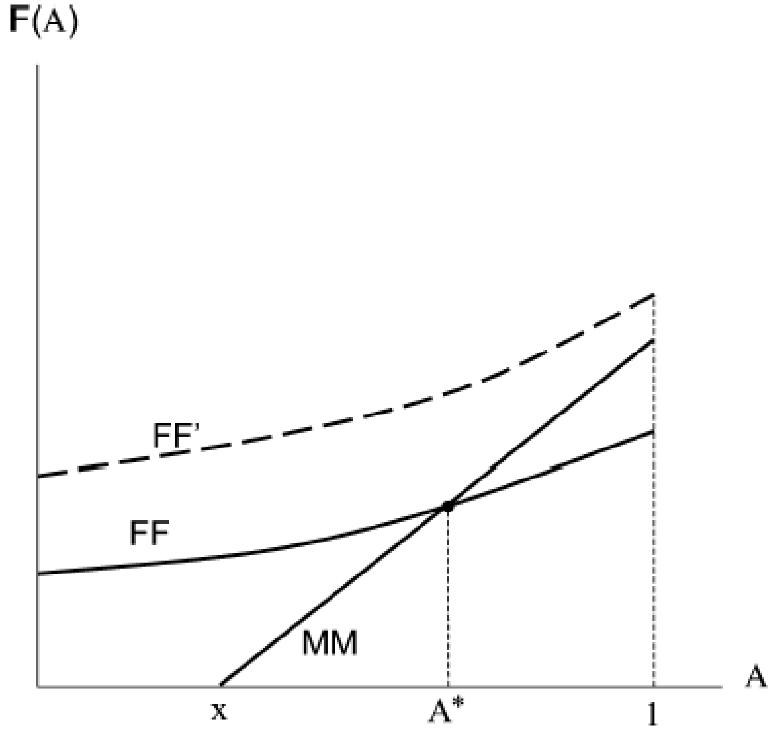
Agents take n_t as parametric.

Substitute from the b.c. $F_t = \frac{w_t - \phi_t}{n_t}$. Then:

$$\frac{\partial E_t U(c_{t+1})}{\partial \phi_t} = 0 \rightarrow \phi_t^* = \frac{(1-n_t)R}{R - rn_t} w_t,$$

Substitute from the b.c. $\phi_t = w_t - n_t F_t$. Then:

$$\frac{\partial E_t U(c_{t+1})}{\partial F_t} = 0 \rightarrow F_{i,t}^* = \begin{cases} F\left(n_t\right) \equiv \frac{R-r}{R-rn_t} w_t, & \forall i \leq n_t \\ 0 & \forall i > n_t \end{cases}$$



Complementarity: the demand for each risky asset, $F(n_t)$ (FF schedule) is a positive function of the number of active projects. This complementarity arises because the more active are intermediate industries, the better is risk-diversification. As n_t increases, savers shift their investments away of the safe asset into high-productivity risky projects. An increase in n creates two effects. (1) investments in projects become safer, which induces complementarity; (2) investments are spread over a larger number of assets, inducing substitution. With sufficiently high risk aversion, the first effect dominates.

The equilibrium n_t^* is determined (as long as $n^* < 1$) by:

$$F\left(n_t^*\right) = M_{n_t^*}.$$

The equilibrium is given by the intersection of schedules FF and MM, where the latter represents the distribution of minimum size requirements across industries: n_t^* is the largest number of projects for which the minimum size can be overcome, subject to the demand of securities.

Growth increases wages and savings over time. This induces an expansion of n_t^*, as the

FF schedule shifts up. Thus, growth triggers financial development.

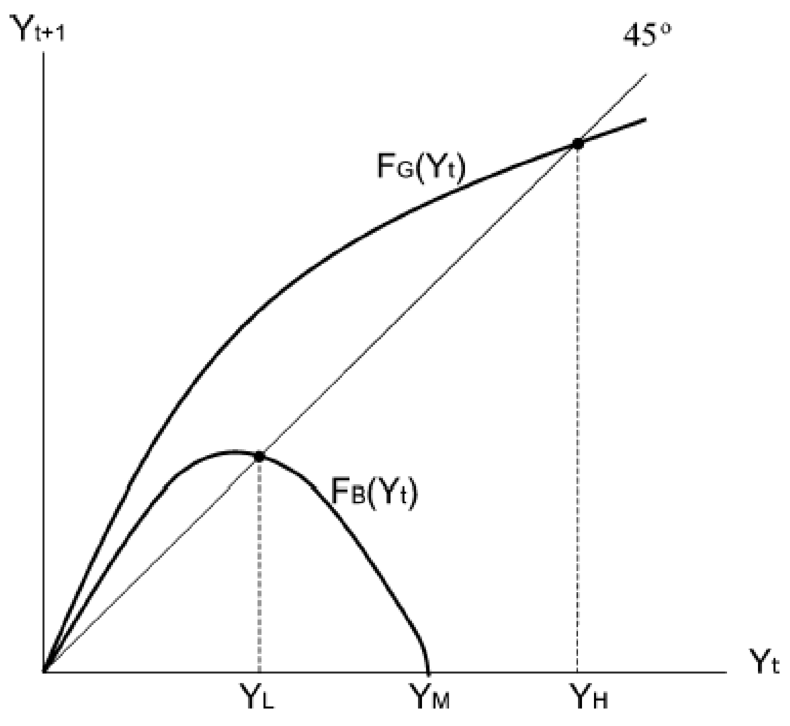
GDP Dynamics

The stochastic dynamics of GDP are:

$$Y_{t+1} = \begin{cases} F_B(Y_t) = \left((1-\alpha)\frac{r(1-n_t^*)}{R-rn_t^*}RY_t\right)^\alpha & prob.\ \ 1-n_t^* \\ F_G(Y_t) = ((1-\alpha)RY_t)^\alpha & prob.\ \ n_t^* \end{cases},$$

where $n_t^* = n(Y_t) \leq 1$ is the equilibrium measure of intermediate industries, and $n' \geq 0$.

- First line: "bad realization" $\rightarrow$ projects do not payoff and capital at time $t+1$ is only given by the return of the safe technology.
- Second line: "good realization" $\rightarrow$ the risky investment paid off at time t and capital and output are relatively large at time $t+1$. **Note:** the probability of a good realization increases with the level of development, since $n'(Y_t) \geq 0$.



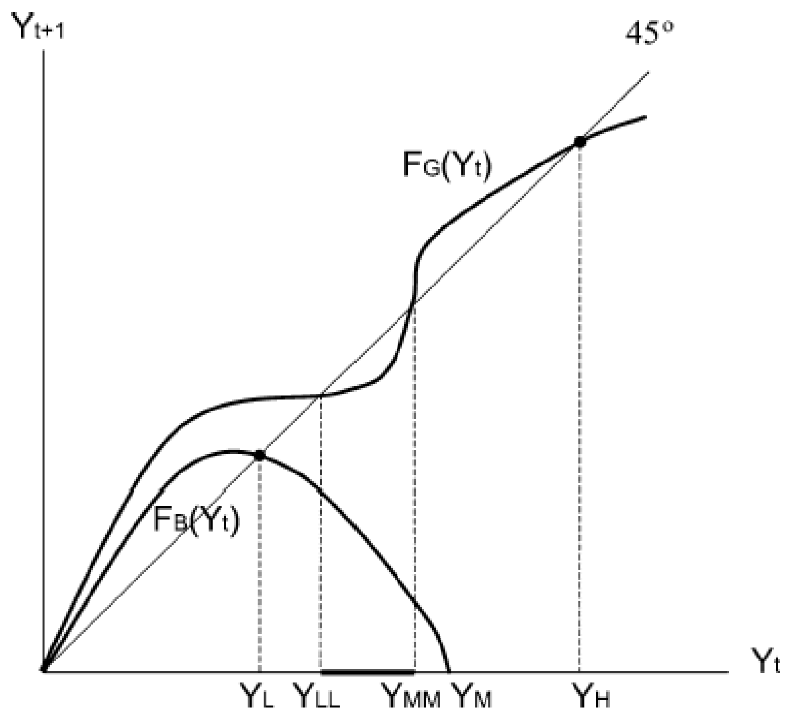
The figure describes the dynamics. The two schedules represent output at time $t+1$ as a function of output at time t conditional on good news ($F_G(Y_t)$) and bad news ($F_B(Y_t)$), respectively.

Stages of Development:

- For $Y \leq Y_L$, the marginal product of capital is very high, which guarantees that growth is positive, even with bad news.
- For $Y \in [Y_L, Y_M]$, growth only occurs if news is good, since $F_B(Y_t) < Y_t < F_G(Y_t)$. The threshold Y_L is not a steady-state; however, it is a point around which the economy will spend some time. When the initial output is below Y_L, the economy necessarily grows towards it. When it is above Y_L, output falls back whenever bad news occurs. So, in this region, the economy is still exposed to undiversified risks, and experiences fluctuations and set-backs.
- For $Y \geq Y_M$, there are enough savings in the economy to overcome all technological non-convexities. When the economy enters this region, all idiosyncratic risks are removed, and the economy deterministically converges to Y_H.

Poverty Traps

In the case studied here the economies "almost surely" converge to a unique steady-state. Different specifications of the model can lead to less optimistic predictions. With higher risk aversion traps can emerge, as in the example described in next figure. An economy starting with a GDP in the region $[0, Y_{MM})$ would never attain the high steady-state Y_H, and would instead perpetually wander in the trapping region $[0, Y_{LL}]$. Conversely, an economy starting above Y_M would certainly converge to the high steady-state, Y_H. Finally, the long-run fate of an economy starting in the region $[Y_{MM}, Y_M]$ would be determined by luck: an initial set of positive draws would bring this economy into the basin of attraction of the good equilibrium. A single set-back, however, would forever jeopardize its future development.



Yt+1
45°
FG(Yt)
FB(Yt)
Yt
YL
YLL
YMM
YM
YH

7 Financial Globalization and Growth

In the standard neoclassical growth framework, financial openness speeds up the process of convergence between countries. This happens because of diminishing returns to capital: in poor countries, capital scarcity implies that the marginal product of capital (the interest rate) is high. Thus, capital has an incentive to flow from developed to developing countries. However, in reality, capital sometimes flows in the opposite direction (Lucas' paradox).

Models incorporating financial frictions can provide an explanation for why capital may flow from poor to rich countries and suggest that, contrary to the standard view, financial openness may promote divergence rather than convergence. This happens because financial frictions introduce a wedge between the interest rate and the marginal product of capital (MPK). Thus, capital scarce countries may not pay high interest rates, despite having a high MPK. Moreover, wealth-dependent borrowing constraints imply that wealthy countries may be more credit worthy and thus able to attract more investment.

7.1 Financial Globalization and Credit Constraints: Matsuyama (2004)

Consider an OLG model where agents work when young and consume only when old. While working, agents earn a wage. Earnings must then be invested.

Each country produces a final good with a Cobb-Douglas technology that combines labor and local capital. We normalize the labor force to one, $L = 1$, so that the production function can be written as:

$$y_t = k_t^{\alpha}, \quad \alpha \in (0, 1)$$

Perfect competition implies that factors are paid their marginal product:

$$\begin{aligned} \text{wage} &: \ w_t = (1-\alpha)\, k_t^{\alpha} \\ \text{return to capital} &: \ \rho_t = \alpha k_t^{\alpha-1} \end{aligned}$$

Capital is produced through investment projects (as in Acemoglu & Zilibotti, 1997) ran by managers. Once produced in a country, capital is installed and cannot be moved internationally. Managers can never start a project abroad (no FDI). After use, capital depreciates fully.

Investment decision

The young at period t have two options to allocate their wealth w_t:

1. lend to managers the rate r_{t+1} (to be determined)
2. become manager: transform ONE unit of y into R units of k_{t+1}. This is the only way to create capital tomorrow. The project is discreet and to run it, the manager needs to borrow $(1 - w_t)$ at the rate r_{t+1}. Two conditions should be satisfied for some young to become manager. First, becoming managers has to be at least equally profitable than lending to others. This is a **Profitability Constraint**:

$$(PC) : \rho_{t+1} R \geq r_{t+1}$$

Second, the manager has to be able to borrow enough. Here is where credit frictions are introduced. The manager can promise to repay only up to a fraction $\lambda \in [0, 1]$ of the project revenue (for example, because he can hide the rest). We can interpret λ as the level of contractual enforcement or financial development. Thus, the total repayment $r_{t+1}(1 - w_t)$ cannot exceed the "collateral" $\lambda \rho_{t+1} R$. This is a **Borrowing Constraint**:

$$\begin{aligned} (BC) \quad &: \quad \lambda \rho_{t+1} R \geq r_{t+1}(1 - w_t) \\ &: \quad \rho_{t+1} R \geq r_{t+1} \left(\frac{1 - w_t}{\lambda} \right) \end{aligned}$$

The young invest iff both PC and BC are satisfied.

- PC is binding when $w_t > 1 - \lambda$
- BC is binding when $w_t < 1 - \lambda$

Note: which constraints binds depends entirely on $w_t = (1 - \alpha) k_t^{\alpha}$.

Intuition: when the borrowing constraint is not binding, because managers are rich enough, the young decide to become managers up to the point of indifference between starting a project or lending $\left(\rho_{t+1}R = r_{t+1}\right)$. When BC is binding $(\lambda\rho_{t+1}R \geq r_{t+1} - r_{t+1}w_t)$, remaining lenders would prefer to become managers $\left(\rho_{t+1}R > r_{t+1}\right)$, but they cannot start new projects as credit is rationed. Note that they cannot obtain loans offering to pay a higher interest (they would like to do so), because such a promise would break the BC and is thus not credible.

Autarky

In autarky the resource constraint must hold:

$$k_{t+1} = Rw_t$$

Why? Because the only way to bring wealth to the second period is through projects (agents can just decide to run them or to finance someone else's project). Thus, all the wealth is used to finance projects. The number of projects is w_t, which is also the fraction of managers in the population.

The dynamics of capital only depend on the supply of credit, not on λ:

$$k_{t+1} = R\left(1-\alpha\right)k_t^{\alpha}$$

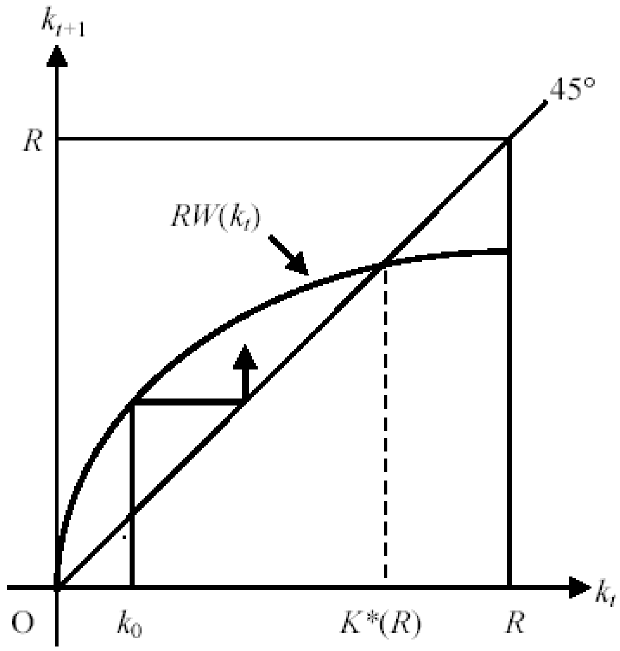
The economy converges to a unique steady-state (due to decreasing returns). Set $k_{t+1} = k_t = k$. Then:

$$\begin{aligned} k &= \left[R\left(1-\alpha\right)\right]^{1/(1-\alpha)} \\ w &= \left(1-\alpha\right)\left[R\left(1-\alpha\right)\right]^{\alpha/(1-\alpha)} \end{aligned}$$

The interest rate depends on λ:

$$\begin{aligned} &\text{if in steady-state PC binds } (w > 1-\lambda) : r = \rho R \\ &\text{if in steady-state BC binds } (w < 1-\lambda) : r = \frac{\lambda\rho R}{1-w} < \rho R \end{aligned}$$

Note: the effect of a binding borrowing constraint is to lower the interest rate (below the MPK).



Financial Openness in a Small Economy

Agents are now allowed to trade intertemporally the final good y. Now the interest rate is fixed by the world market, $r_{t+1} = r$, as agents can lend their wealth to managers abroad. k_{t+1} is not determined anymore by the supply of domestic savings, but rather by the world interest rate. The dynamics of capital depends on which constraint is binding:

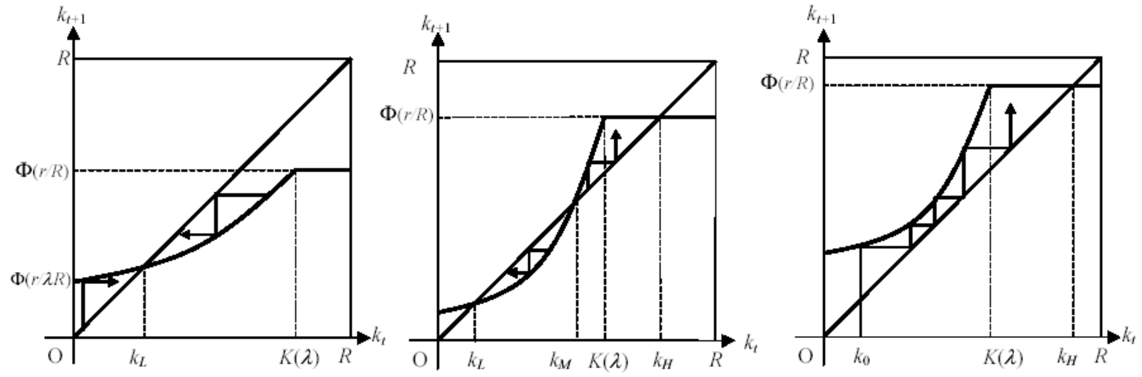
- if PC binds ($w_t > 1 - \lambda$): $r = \rho_{t+1} R \rightarrow$ There is only one k_{t+1} satisfying this:

$$k_{t+1} = \left(\frac{r}{\alpha R}\right)^{\frac{1}{\alpha - 1}}$$

- if BC binds ($w_t < 1 - \lambda$): $\lambda \rho_{t+1} R = r\,(1 - w_t)$. Substituting and rearranging:

$$k_{t+1} = \left(\frac{r}{\lambda \alpha R}\right)^{\frac{1}{\alpha - 1}} \left[1 - (1 - \alpha)\, k_t^{\alpha}\right]^{\frac{1}{\alpha - 1}}$$

Draw k_{t+1} as a function of k_t: the curve has a positive intercept, is increasing and convex and becomes flat (after a kink) when PC becomes binding, that is, $w_t = (1 - \alpha)\, k_t^{\alpha} = 1 - \lambda$. Draw also the 45-degree line.



a) b) c)

There are 3 possible cases for the dynamics.

1. one steady-state, k_L
2. three steady-states, $k_L < k_M < k_H$, the middle one unstable. This is the most interesting case: the effect of financial openness depends on the initial capital stock: if it is high enough, the economy will grow rich, if not it will be impoverished. In the latter case, we may see outflows of capital from LDCs. Thus, poor countries that still have to accumulate a lot may not want to be open. The reason is that a country with low capital has low wages and thus cannot guarantee high repayments on loans. Thus, despite the high marginal product o capital, it may not compete with rich (more credit worthy) countries in financial markets.
3. one steady-state, k_H

The World Economy

Without international financial markets, the world economy is a collection of isolated countries: Each of them will converge to a (unique) steady-state. If countries share the same fundamentals, there will be convergence.

With open financial markets, the world economy is a collection of small open economies and the world interest rate will be determined to equate world saving and world investment. Note that, with identical countries, the autarky equilibrium must also be an equilibrium for the world economy. However, it can be shown that, when there are multiple equilibria, the autarky steady-state corresponds to equilibrium k_M in panel (b). Thus, the autarky steady-state is still a possible steady-state for a financially open country, but it is the unstable one! Thus, even if all countries are symmetric and are in the same steady-state, financial liberalization will polarize the world economy into rich and poor countries.

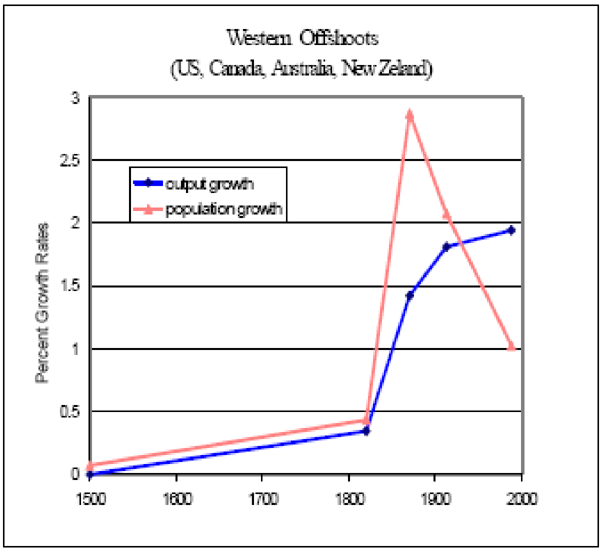
Intuition behind symmetry-breaking: an economy that can finance more projects will have more capital and thus higher wages → higher wages will make the economy better able to guarantee returns on loans and relax the borrowing constraint → even more finance (also from abroad).

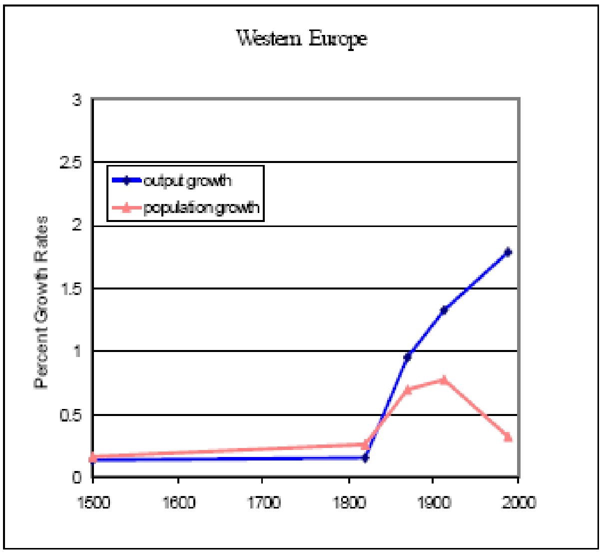
Throughout most of human history (pre 1800):

- Per capita income has remained almost constant.
- Increases in productivity translated into population growth (Malthusian regime).

After 1800:

- Modern economic growth started: sustained growth in per capita income.
- Demographic transition: rise and fall of fertility.





Galor (HEG, 2005) and Galor and Weil (AER, 2000) propose a unified theory to account for these facts.

Technology

In every period, the economy produces a single homogeneous good using effective units of labor (human capital) as the only input. The number of effective units of labor is determined by households' decisions in the preceding period regarding the number and level of human capital of their children.

Output at time t is:

$$Y_t = H_t^{\alpha} A_t^{1-\alpha}$$

where A_t = technology, H_t = efficiency units of labor. In per capita terms:

$$y_t = \frac{Y_t}{L_t} = h_t^{\alpha} \left(\frac{A_t}{L_t}\right)^{1-\alpha}$$

where $h_t = \frac{H_t}{L_t}$ is human capital per person. The wage per efficiency units of labor is equal to output per efficiency unit:

$$w_t = \frac{y_t}{h_t}$$

The growth rate of technology is:

$$g_{t+1} \equiv \frac{A_{t+1} - A_t}{A_t}$$

This rate will be endogenous.

Preferences

In each period t, a generation that consists of L_t individuals with human capital h_t joins the labor force. Each generation works and consumes for one period only. Agents derive utility from consumption c_t, the number of children n_t and the human capital of their children h_{t+1} (i.e., they care about the potential wage of children):

$$u_t = (c_t)^{1-\gamma} (n_t h_{t+1})^{\gamma}$$

where c_t = consumption, n_t = number of children, h_{t+1} = human capital of each child.

ASSUMPTION: consumption cannot fall below a subsistence level $\widetilde{c}$.

Each individual has an endowment of one unit of time that can be devoted to (1) work and (2) child rearing. Raising and educating children require time. The budget constraint is:

$$w_t h_t [1 - n_t (\tau + e_{t+1})] = c_t \geq \widetilde{c}$$

where τ = fraction of time endowment required to raise a child, e_{t+1} = fraction of time endowment devoted to education of each child.

Human capital of the children is a positive concave function of education received from parents and a negative convex function of the growth rate of technical progress:

$$h_{t+1} = h\left(e_{t+1}, g_{t+1}\right)$$

Assumptions on $h(\cdot)$:

$$\begin{aligned} h_e &> 0; \quad h_g < 0; \quad h_{eg} > 0 \\ h_{t+1} &= h(0,0) = h(0,g) = 1 \end{aligned}$$

NOTE: $h_{eg} > 0 \rightarrow$ technology *complements* education in the production of human capital.

Given Cobb-Douglas preferences, we can solve the optimization program in two stages.

Stage I: consumption vs. child rearing

$$n_t\left(\tau + e_{t+1}\right) = \begin{cases} \gamma & \text{if } \; w_t h_t (1-\gamma) > \widetilde{c} \\ 1 - \frac{\widetilde{c}}{w_t h_t} & \text{if } \; w_t h_t (1-\gamma) \leq \widetilde{c} \end{cases}$$

That is, if the constraint $c > \widetilde{c}$ is not binding, individuals devote a constant share $(1-\gamma)$ of their potential income to consumption. Otherwise, they work enough to satisfy subsistence $\widetilde{c}$ and devote the remaining time to child rearing.

Stage II: quantity vs. education

Given the above solution for $n_t\left(\tau + e_{t+1}\right)$, the agents choose n_t and e_{t+1} to maximize:

$$\begin{aligned} & \max_{n_t, e_{t+1}} \quad n_t h\left(e_{t+1}, g_{t+1}\right) \\ \text{st:} \quad & n_t\left(\tau + e_{t+1}\right) = k \end{aligned}$$

and the non-negativity constraints: $h_{t+1} \geq 0$, $n_t \geq 0$. Note that parents care about the human capital of children $h\left(e_{t+1}, g_{t+1}\right)$, but they can only affect their education e_{t+1}. Clearly, $n_t = 0$ is never optimal. On the contrary, we may have a corner solution for education. Substituting n_t from the constraint and taking logs, the problem becomes:

$$\max_{e_{t+1}} \{\ln h\left(e_{t+1}, g_{t+1}\right) - \ln\left(\tau + e_{t+1}\right) + \ln k\}$$

The FOC is:

$$\text{MB} = \frac{h_e}{h} \leq \frac{1}{\tau + e_{t+1}} = \text{MC}$$

with equality if $e_{t+1} > 0$. Thus:

$$e_{t+1} = e\left(g_{t+1}\right) \begin{cases} = 0 \text{ if } h_e \leq \frac{1}{\tau} \Longleftrightarrow g_{t+1} \leq \widetilde{g} \\ > 0 \text{ if } g_{t+1} > \widetilde{g} \end{cases}$$

with $e'\left(g_{t+1}\right) \geq 0$. Intuitively, due to the complementarity between human capital and technical progress, when technical progress is faster, investment in education tend to increase. If technical progress is too low, investment in education does not pay. This follows from the property of the $h\left(e_{t+1}, g_{t+1}\right)$ function.

Population dynamics: $L_{t+1} = L_t n_t$ where

$$n_t = \frac{\text{time in child-rearing}}{\text{time per child}} = \begin{cases} \frac{\gamma}{\tau + e_{t+1}} & \text{if } c > \widetilde{c} \\ \frac{1 - \frac{\widetilde{c}}{w_t h_t}}{\tau + e_{t+1}} & \text{if } c = \widetilde{c} \end{cases}$$

Main results:

- Time spent on child rearing is an increasing function of wage income when the subsistence constraint is binding
- When the subsistence constraint is NOT binding, time devoted to child rearing is constant
- Time spent of child rearing does not depend on the rate of technical progress
- An increase in the rate of technical progress weakly increases education
- An increase in the rate of technical progress weakly decreases the number of children

8.1 Exogenous Technical Progress with $e = 0$

Assume that $g_{t+1} = g < \widetilde{g}$ ($\rightarrow e_{t+1} = 0 \rightarrow h_t = 1$). The growth rate of potential income per capita, $w_t h_t = w_t$, is:

$$\frac{w_{t+1} - w_t}{w_t} = \frac{y_{t+1}}{y_t} - 1 = \left(\frac{A_{t+1}}{A_t} \frac{L_t}{L_{t+1}}\right)^{1-\alpha} - 1 = \left(\frac{1+g}{n_t}\right)^{1-\alpha} - 1$$

Population growth is:

$$n_t = \min\left\{\frac{\gamma}{\tau}, \ \frac{1 - \widetilde{c}/w_t}{\tau}\right\}$$

Recall that w_t is growing if $1 + g > n_t$.

There are two cases:

1. low g : $\left(1 + g < \frac{\gamma}{\tau}\right)$. Initially, w_t and n_t grow, until $n_t = 1 + g$. From that point on, growth of w_t is not sustainable. The economy never escapes from the subsistence level of consumption $\widetilde{c}$ and is trapped in a Malthusian regime.
2. high g : $\left(1 + g > \frac{\gamma}{\tau}\right)$. Initially, w_t and n_t grow, until $n_t = \frac{\gamma}{\tau}$. From that point on, the economy escapes from the subsistence level of consumption $\widetilde{c}$, n_t remains constant, while w_t and c_t grow forever. The economy remains in the Malthusian regime only temporarily.

8.2 Endogenous Technical Progress and Education

We now endogenize technical progress. Assume that technical progress depends positively on education and population size:

$$g_{t+1} = g\left(e_t, L_t\right)$$

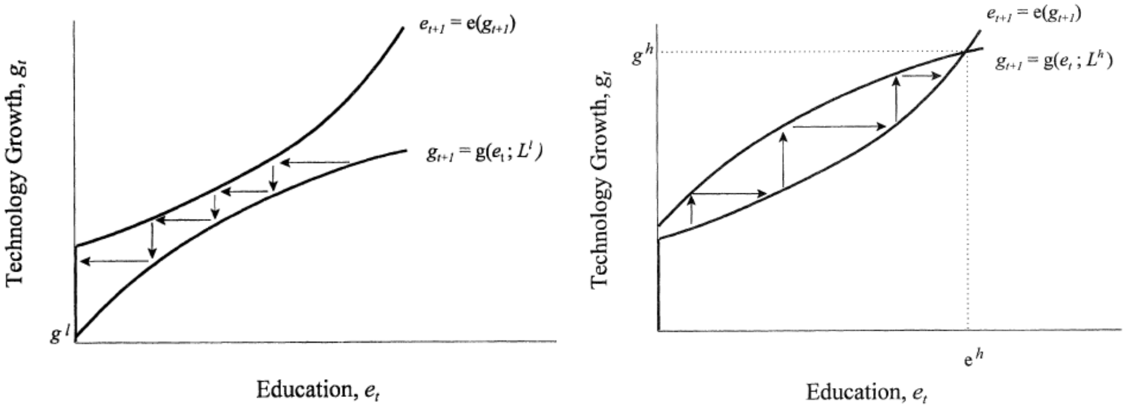
with $g\left(0, L_t\right) > 0$. The scale effect is crucial to eventually escape from the Malthusian regime. For a given L_t, the dynamics of education and technology is governed by an autonomous system of difference equations:

$$\begin{cases} e_{t+1} = e\left(g_{t+1}\right) > 0 \ \text{ if } g_{t+1} > \widetilde{g} \\ g_{t+1} = g\left(e_t\right) \end{cases}$$

As L_t grows, g_{t+1} shifts upwards.

There are three cases:

1. In the left panel, the only steady state is one with no education.
2. If the two loci cross twice, then the are two stable steady states: one with no education and low growth, another with high education and high growth.
3. If the locus $g_{t+1} = g(e_t)$ is high enough (right panel), then there is only a steady state with positive education and high growth.



8.3 Global Dynamics: from Stagnation to Modern Growth

As seen before, the dynamics of e_{t+1} and g_{t+1} are determined by an autonomous system. To fully characterize the dynamic system, we can draw in a phase diagram the coevolution of e_{t+1} and $x_t \equiv \frac{A_t}{L_t}$. The dynamics of x_t are:

$$x_{t+1} = \frac{1 + g_{t+1}(e)}{n_t(x, e)} x_t$$

We draw the following loci.

- Conditional Malthusian Frontier, given g_t :

$$MM_{g_t} = \left\{ (e_t, x_t) : w_t h_t = h_t^{\alpha} (x_t)^{1-\alpha} = \frac{\widetilde{c}}{1-\gamma} \right\}$$

Above it, $\widetilde{c}$ is NOT binding (productivity in high enough). MM_{g_t} is a decreasing, convex function of e_t (with higher education, less productivity is needed to meet the constraint). It shifts up with g_t.

- XX locus where. $x_t = x_{t+1}$.

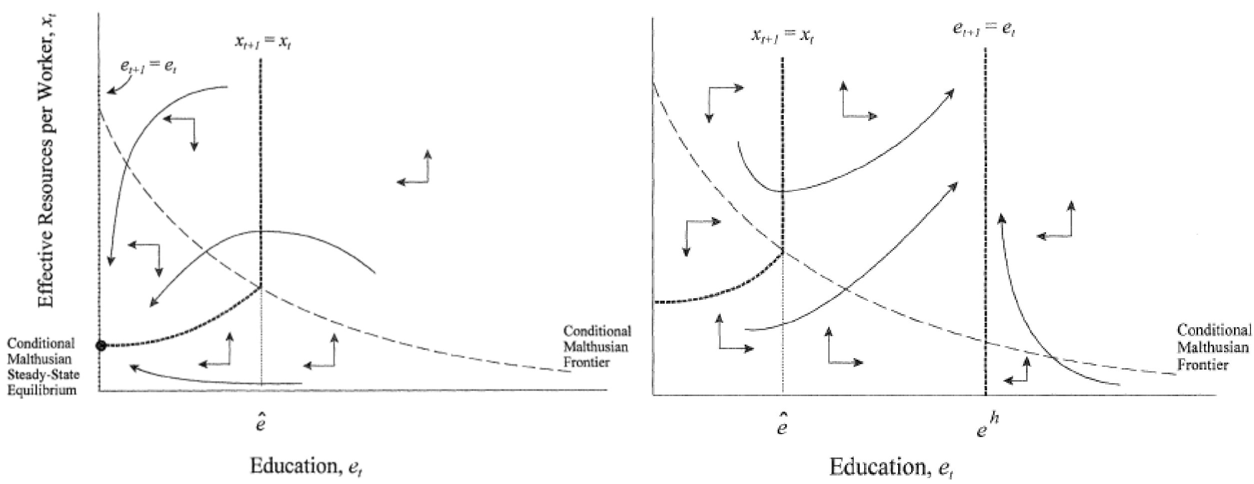
  when $\widetilde{c}$ is not binding, n does not depend on x. Thus, $x_t = x_{t+1} \rightarrow e_t = \widehat{e}$

  when $\widetilde{c}$ is binding, $x_t = x_{t+1}$ requires:

$$1 = \frac{1+g(e)}{n(x,e)}$$

  which is an upward sloping locus in the space (e,x), because $n_x > 0, n_e < 0$

- EE locus where $e_t = e_{t+1}$. This locus only depends on g, e and L. Thus, it is a vertical line in the space (e,x). The dynamics of e_t are those derived in the previous section.



Thus, the economy goes through the following phases:

1. At an early stage, population is small, thereby technical progress is low and there is no investment in education. In this stage, consumption per capita is constant and any shock to technology only translate into a higher population. Given that technical progress is low, fertility is low too. This is the Malthusian regime.

2. Eventually, when L is large enough, the economy escapes from the Malthusian regime and starts the transition to an equilibrium with high levels of education and growth. When $g(e_t, L_t)$ is sufficiently high, agents start to invest in education and this generates a feedback on g. In the Post-Malthusian regime, fertility first increases, as more resources can be devoted to child rearing.

3. When the economy finally escapes the subsistence consumption level, further increases in g leads to lower fertility (total time for child rearing is constant, but more of it is devote to education). The economy finally reaches the stage of Modern Growth, where consumption per capita grows steadily, while education and population may remain constant.

9 Growth and the Environment

Grossman and Krueger (1995) show that pollution (air and water pollution) first increases with income, then it declines. An environmental Kuznets curve. More evidence on growth and the environment is in Brock and Taylor (HEG, 2005). Two key questions:

1. How does growth affect the environment?
2. Do environmental concerns pose a limit to endogenous growth?

To address these questions, we study a simple model taken from Stokey (1998) and discuss some ideas in Aghion and Howitt (1998, Chapter 5)

9.1 Exogenous growth

Consumption goods c and pollution x are joint products of a constant returns to scale technology:

$$\begin{aligned} c &= yz \\ x &= yz^{\beta}, \quad \beta > 1 \end{aligned}$$

where y is potential output (exogenous) and $z \in [0,1]$ is an index of the technology used. Higher z yields more goods, but also more pollution. Thus, z is an index of the emission rate. Note that for fixed potential output, pollution is an increasing and convex function of actual output. Potential output is attained by using the dirtiest technology, $z = 1$.

Preferences over consumption c and pollution x are:

$$U = \frac{c^{1-\sigma} - 1}{1-\sigma} - \frac{x^{\gamma}}{\gamma}, \quad \sigma > 0, \ \gamma > 1$$

Thus, utility is increasing and concave in consumption. The utility cost of pollution is instead convex.

The government can impose emission regulations that set an upper bound on z. In this case, all firms will always operate at the emission standard imposed by the government (why?). Substituting $z = x^{1/\beta} y^{-1/\beta}$, we obtain:

$$c = y^{1-\frac{1}{\beta}} x^{\frac{1}{\beta}}$$

Thus, consumption goods are produced with potential output and total pollution as inputs. By choosing z, the government is implicitly choosing x. Thus, the problem of the government can be expressed as choosing $x \in [0, y]$ in order to maximize utility:

$$\max_{x \in [0,y]} \frac{\left(y^{1-\frac{1}{\beta}} x^{\frac{1}{\beta}}\right)^{1-\sigma} - 1}{1-\sigma} - \frac{x^{\gamma}}{\gamma}$$

FOC:

$$\text{MB} = y^{\frac{(\beta-1)(1-\sigma)}{\beta}} x^{\frac{1-\sigma-\beta}{\beta}} \geq \beta x^{\gamma-1} = \text{MC}$$

with equality if the solution is interior, i.e., $x < y$. That is, if the marginal benefit of pollution is higher that its marginal cost, we set maximum pollution ($z = 1$). Otherwise, the optimal level of pollution is determined by the first order condition with equality:

$$\begin{aligned} \beta^{-\beta} y^{(1-\beta)(\sigma-1)} &= x^{\gamma\beta+\sigma-1} \\ x^* &= a y^{\frac{(1-\beta)(\sigma-1)}{\gamma\beta+\sigma-1}} \end{aligned}$$

where $a = \beta^{\frac{-\beta}{\gamma\beta+\sigma-1}}$. Note that, for an interior solution, pollution x is decreasing in potential output y if $\sigma > 1$. If y is very low, we get to a corner solution, where the dirtiest technology is used. The critical level of y is found imposing $x = y$ and solving: $y = a y^{\frac{(1-\beta)(\sigma-1)}{\gamma\beta+\sigma-1}} \rightarrow y = \widehat{y} = a^{\frac{\sigma+\beta\gamma-1}{\beta(\sigma+\gamma-1)}}$. Thus:

$$\begin{aligned} \text{if } y &< \widehat{y} \rightarrow z^* = 1 \text{ and } x^* = y \\ \text{if } y &> \widehat{y} \rightarrow x^* = a y^{\frac{(1-\beta)(\sigma-1)}{\gamma\beta+\sigma-1}} \end{aligned}$$

If $\sigma > 1$, total pollution increases first with income, but it starts declining after a critical level:

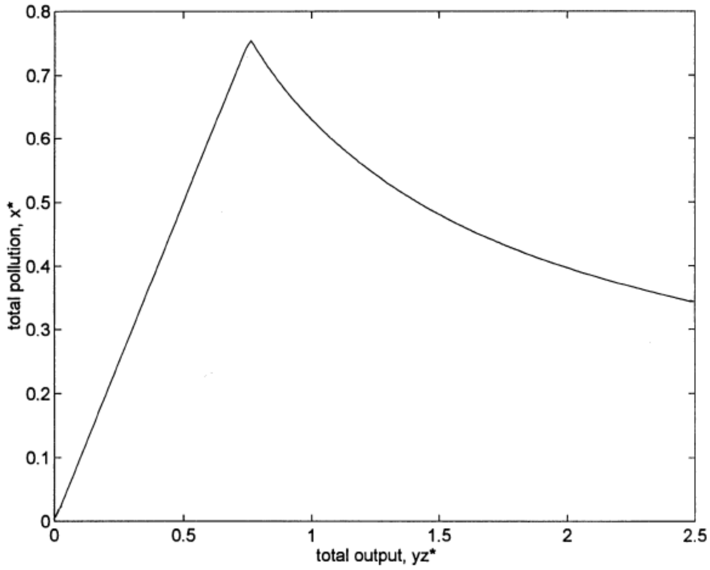


FIGURE 3

ACTUAL OUTPUT AND TOTAL POLLUTION

The intuition for this result is that the marginal cost of pollution does not depend on income (see FOC - this is so because of additive separability in the utility function). On the other hand, the marginal benefit due to higher output declines with income if $\sigma > 1$. Note that the hump shape is a consequence of the elasticity parameter for consumption goods, not for pollution. If $\sigma < 1$ emission standards do get stricter with y, but not fast enough to prevent environmental deterioration.

Consumption increases with potential income:

$$c^{\beta} = y^{\beta-1}x = ay^{\frac{\beta\gamma(\beta-1)}{\sigma+\beta\gamma-1}}$$

For a generic variable x, define its growth rate as $g_x = \dot{x}/x$. Then we have:

$$g_c = \frac{\gamma(\beta-1)}{\beta\gamma+\sigma-1}g_y < g_y$$

the higher σ, the more we are willing to sacrifice consumption growth to live in a cleaner environment.

Suppose now that there is a limit to $x \leq E$, environmental deterioration, that would trigger a natural catastrophe. Thus, the government must take into account that, to prevent the disaster, x cannot grow above E. The question is: will this new constraint impose an additional cost? Will the economy be forced to make additional sacrifices to avoid a natural catastrophe? Of course, we already have the answer: if $\sigma > 1$, it is already optimal to let x fall to zero. Thus, the constraint $x \leq E$ will never be binding and the threat of a catastrophe will not impose additional costs. If $\sigma < 1$, instead, to prevent a natural disaster, the government will have to deviate from the otherwise optimal path and this will have a utility costs.

9.2 Dynamic AK model

We now ask the question: is endogenous growth sustainable in the presence of environmental concerns? To do so, we use a simple Ak model of endogenous growth. Potential output is now Ak, where k can be accumulated. Change of notation:

$$y = Akz$$

y is now current output, which is still a function of potential output, Ak, and environmental standards z. Pollution is still potential output times z^β:

$$x = Akz^\beta, \quad \beta > 1$$

The law of motion of capital is given by the resource constraint (no depreciation):

$$\dot{k} = Akz - c$$

The individual (not the planner) maximization problem is:

$$\max \int_0^\infty e^{-\rho t} \left[\frac{c^{1-\sigma} - 1}{1-\sigma} - \frac{B\left(Akz^\beta\right)^\gamma}{\gamma} \right]$$

subject to the usual constraints. As individual agents take pollution as given, this maximization yields the standard Euler equation:

$$g_c = \frac{1}{\sigma}\left(r - \rho\right)$$

We simplify the planner problem by keeping the catastrophe constraint: $x \leq E$. This implies immediately that, for x to be bounded, z must fall to zero if we have positive long run growth ($g_k > 0$).

Note that sustainable growth is *feasible*. Given any growth rate of capital, it is possible for the emission standards to get stricter at a rate that is fast enough so that total pollution falls but slow enough so that output rises. Then:

$$\begin{aligned} y &= Akz \rightarrow g_y = g_k + g_z > 0 \quad \text{if } \; g_k > -g_z \\ x &= Akz^\beta \rightarrow g_x = g_k + \beta g_z < 0 \quad \text{if } \; \frac{g_k}{\beta} < -g_z \end{aligned}$$

Thus, output rises and pollution declines if:

$$g_k > -g_z > \frac{g_k}{\beta} > 0$$

However, in this model sustainable growth is not *optimal*. To see this, we substitute z into y to obtain:

$$z = \left(\frac{x}{Ak}\right)^{1/\beta} \rightarrow y = (Ak)^{1-\frac{1}{\beta}} \, x^{\frac{1}{\beta}}$$

Next, we find the interest rate as the marginal product of capital:

$$r = \left(1 - \frac{1}{\beta}\right) A^{1-\frac{1}{\beta}} \left(\frac{x}{k}\right)^{\frac{1}{\beta}}$$

using $x = Akz^\beta$:

$$r = \left(1 - \frac{1}{\beta}\right) Az$$

If $z \rightarrow 0$, which is a necessary condition for sustainable growth, then $r \rightarrow 0$. But is incompatible with long run growth (see the Euler equation)! In this model, environmental concerns effectively introduce diminishing returns to accumulation.

Aghion and Howitt (1998) show that models with innovation may overcome this limit, provided that innovation is an activity that is more "green" than other forms of investment. This may be the case, for example, if utility is a positive function of the number or the quality of available goods, so that utility may grow even when physical production is constant.