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MONEY AND BANKING IN AN ECONOMY WITH VILLAGES

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Abstract

This is a model of money, credit, and banking in a framework with competitive search in the goods's market. Sometimes, individuals interact with acquaintances, so financial contracts, such as those found in banking, are viable. Other times, individuals interact with strangers with whom proving ones' identity is costly, so there is a role for cash. Banks are financial intermediaries that offer loans and deposits. At a cost deposits are transferrable via cheques or other instruments. In an equilibrium with multiple means of payment, cash specializes on small ticket items and deposit transfers specialize on large ticket items. Theoretically, the demand for cash is interest elastic for three reasons: High interest rates speed up the circulation of cash, reduce the fraction of goods paid with cash, and cut down the demand for these goods. A calibrated version of the model accounts for the observed interest elasticity of the velocity of cash. In this calibration, the only quantitatively significant source of elasticity is the endogenous determination of the fraction of goods paid with cash.

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1 Introduction

To rigorously capture the main features of a modern monetary system, this paper incorporates a banking sector into a monetary search model. Search models have provided sound foundations for monetary economics. In these models, money is essential because a double coincidence of wants is rare and individuals are anonymous in the market place. Unfortunately, if unqualified, anonymity prevents the existence of credit contracts that are essential to the roles that banks play. These roles include the intermediation of credit between borrowers and lenders. It also includes the transfer of bank deposits, which may be made using cheques, debit cards, or direct orders. This paper enriches the environment typically assumed in a monetary search model by qualifying the anonymity of individuals in such a way that banks playing the roles just described coexist with cash.

In the model of this paper, individuals belong to villages. Inside each village, individuals know each other so financial contracts are viable. In each village, some individuals, to be denoted as banks, have the means to keep a record of the credit histories of their fellow villagers. Therefore, credit contracts supporting bank loans and bank deposits can be enforced. When individuals go shopping outside their village of origin, they are not known by the sellers of the goods they seek to purchase, so a credit arrangement between these two parties is not feasible. At a cost, an individual can reveal his identity and arrange for a transfer from his bank account to that of the trading partner. Alternatively, the individual can carry cash to make the payment. Transfer of deposits and cash compete as the means of payment. The two means coexist if cash is preferable for some transactions, while a transfer is preferable for some others.

The partition of individuals by villages, where individuals know each other, is a useful device not only to introduce credit, but also to achieve the tractability of monetary search models with divisible money. To this end, the model assumes that individuals observe the trading opportunities of their fellow villagers. As a result, individuals of the same village can insure one another against risks on their trading opportunities. The combination of credit and insurance leads to a tractable model of divisible money in a search environment, even though preferences are not quasi-linear as in Lagos and Wright (2002). As explained in Faig (2004) the role of financial markets inside the village replaces the role of the large household assumption in Shi (1997).

This is not the first paper to attempt the introduction of banks in a monetary search framework. However, it is the first to my knowledge to introduce banking in a model with divisible money. The closest contribution to the present paper is that of He, Huang, and Wright (2003). Both papers have banks that issue transferable deposits and are able to make loans. Also, in both papers there are equilibria where cash and deposits coexists. However, the two papers differ in many respects. First, in He, Huang, and Wright money is indivisible with a unit storage capacity, while in the present contribution money is divisible without storage limits. Also, in He, Huang, and Wright the coexistence of cash and deposits is achieved by assuming that cash has a higher probability of being stolen than deposits. As a result, the coexistence of cash and deposits requires that the equilibrium return on cash is higher than the equilibrium return on deposits. In contrast, in this paper cash realistically earns a lower rate of return than deposits when both means of payment coexist.

Other monetary search models with banks are those in Cavalcanti and Wallace (1999a b), Cavalcanti, Erosa, and Temzelides (2000), and Wallace and Zhu (2003). These papers study models where the history of banks is known but the history of non-banks is not. This allows for equilibria where banks issue private notes, but it does not allow for the type of contracts supporting bank loans.

There have also been many monetary search contributions where money coexists with some form of private credit. For example, Diamond (1990), Shi (1996), and Jin and Temzelides (2004) study models where the repeated interaction of a pair of individuals or the frequent meetings of a group of individuals allow for a form of private credit. Also, Kocherlakota and Wallace (1998) study the coexistence of money and credit in a setup where public record-keeping is incomplete because it is updated with a lag. Finally, Williamson (1999) presents a model with banks that make loans and issue private money. However, in this model money is not needed to solve a double-coincidence of wants problem, so even if there is search in the model, the logic of the model is quite different from the rest of the papers referenced in this paragraph.

The model studied in this paper is similar to the one studied in Faig (2004). The key difference between the two models is that in here credit inside the village is intermediated by banks that at a cost can transfer deposits to banks in other villages. Also, in Faig (2004) Nash bargaining determines the terms of trade in the goods' market, while in this paper competitive search does this job.

Competitive search is very attractive for the purpose of this paper for several reasons. To start with, competitive search is more tractable than Nash bargaining. More importantly, competitive search allows for competitive pressures to determine the split of the trade surplus between a buyer and a seller instead this split being governed by an arbitrary bargaining weight as in Nash bargaining. Recently, Rocheteau and Wright (2003) has introduced this equilibrium concept into the monetary search literature and promises to be a standard concept of equilibrium in future work.[1]

In summary, the findings of the paper are the following. In the environment described, cash and banking arise as complementary institutions of the monetary system. In equilibrium, cash and transfers of deposits may coexist. When this happens, buyers use cash to pay for small purchases, while they use transfers for large purchases. (This is reminiscent of Prescott, 1987.) A higher rate of inflation reduces the demand for money for three reasons. It speeds up the circulation of cash because the congestion of buyers in the market falls. It cuts down the consumption of goods purchased with cash because inflation increases the opportunity cost of purchasing these goods. Finally, it reduces the fraction of goods purchased with cash. A numerical implementation of the model shows that this last source of elasticity is quantitatively dominant for reasonable parameters.

The rest of the paper is organized as follows. Section 2 describes a simple version of the model in which banks cannot transfer deposits to other banks outside the village. Section 3 allows for costly transfers among banks located at different villages. Also, it assumes that buyers are subject to preference shocks that determine the size of the purchases buyers intent to do. In this extension, cash and transfers can coexist as alternative means of payment. Section 4 studies numerically a generalized version of the models in Sections 2 and 3. Section 5 concludes.

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[1] See also Faig and Xiuhua (2004) and Lagos and Rocheteau (2003) for papers using competitive search in a monetary search environment.

2 Money and Banking

The economy is composed of a continuum of measure one of individuals who live in a large number of symmetric villages. All individuals have identical preferences and production abilities. All of them are able to produce a nondurable good specific to their village. Also, they all get utility from the goods produced in other villages, but they have no desire for their own village's good. Consequently, individuals must trade with individuals from other villages to consume.

Time is a discrete succession of days. The horizon is infinite.

Production must take place around the village of origin. Therefore, at the beginning of each day an individual has to choose to either stay around their own village as a seller or visit other villages as a buyer. Each day, some individuals are buyers while the others are sellers. Over time, each individual typically alternates between these two roles because selling goods is the primary source of the income needed to buy them.

The interaction between buyers and sellers takes place every afternoon in a competitive search market. In this market, sellers post a contract detailing the terms at which they commit to trade. Buyers observe all the posted contracts, pick one of the them, and direct their search to all sellers posting that particular contract. Buyers that are indifferent between two contracts play mixed strategies by randomizing between the two. The set of individuals that includes the sellers posting the same contract and the buyers that direct their search towards them constitute a submarket. Within each submarket, traders are matched randomly. Consequently, if θ^i is the fraction of individuals that are sellers in submarket i, the probability that a buyer meets a seller in this submarket is θ^i, and the probability that a seller meets a buyer is $1 - \theta^i$. (The superscript i is dropped when there is no ambiguity about the submarket that is being referred.)

Because of the randomness in matching, the trading opportunities of buyers and sellers are risky. However, individuals can observe or infer the trading opportunities encountered by the other individuals from the same village. Consequently, in the morning, prior to the opening of the goods market, individuals can insure with their fellow villagers against the risk of meeting or failing to meet a trading partner during the day. At a fair premium, buyers can purchase a contract for the delivery of μ^b dollars tomorrow contingent on meeting a seller today. For a buyer searching in submarket i, the insurance premium is $\mu^b \theta^i$. Likewise, sellers can purchase a contract for the delivery of μ^s dollars tomorrow contingent upon failing to meet a buyer today. For a seller in submarket i, the fair premium to acquire this contract is $\mu^s \left(1 - \theta^i\right)$. Insurance premiums are payable the day after the purchase of the contract.

In a village, some individuals, to be denoted banks, have the ability to keep a record of the histories of their fellow villagers. Furthermore, all the individuals in the village know the histories of banks. Therefore, contracts supporting bank loans and bank deposits among members of the same village are enforceable. For simplicity, I assume that there is no cost in managing bank contracts. Therefore, competition among banks closes the spread between the lending and borrowing interest rates. Outside their village all the individuals, banks or not, are anonymous, so bank deposits cannot be transferred outside the village. As a result, the only means of payment outside the village is cash. (This assumption is relaxed in the next section.)

Cash is an intrinsically useless, perfectly divisible, and storable asset. Units of cash are called dollars. The supply of cash grows at a constant factor γ, so

$$M_{+1} = \gamma M, \tag{1}$$

where M is the supply of cash per individual. The subscript t is omitted in most expressions of the paper, so, for example, M stands for M_t and M_{+1} stands for M_{t+1}. New cash is injected via a lump-sum transfer τ to all individuals at the beginning of each day. For cash to grow at the rate γ, this transfer must satisfy:

$$\tau = (\gamma - 1) M. \tag{2}$$

The timing of markets in the course of a day is the following. In the morning, after the cash transfers have been received, financial markets open. During this time, individuals manage the accounts they have with the banks of their village. Also, they redeem and purchase insurance contracts against risks on trading opportunities. In the afternoon, financial markets are closed, but the search market for goods is open.

The ex-post one period utility of an individual is:

$$\mathcal{U}(q^b, q^s) = U(q^b) - C(q^s); \tag{3}$$

where q^b and q^s are respectively quantities of goods consumed and produced. Because of the environment where individuals interact q^b and q^s cannot be simultaneously positive. The objective of individuals is to maximize their expected utilities:

$$E \sum_{t=0}^{\infty} \beta^t \mathcal{U}\left(q_t^b, q_t^s\right) \tag{4}$$

where $\beta \in (0,1)$ is the discount factor. The discount and money growth factors obey $\gamma > \beta$. There is a maximum daily output that an individual can produce to be denoted $\overline{q}$. The functions U and C are continuously differentiable and increasing. The function U is concave and C is convex. Moreover, $U(0) = C(0) = 0$, $C'(0) = 0$, and $U'(0) = \infty$. Finally, $\overline{q} \geq q^*$ where q^* is defined as $C'(q^*) = U'(q^*)$, and $C(\overline{q}) \geq U(\overline{q})$.

Financial markets are assumed to be perfectly competitive, while there is competitive search in the goods market. In equilibrium, individuals make optimal choices in the environment where they live. This environment includes the sequence of nominal interest rates on bank loans and deposits and the sequence of market conditions in the active submarkets where the individual can trade goods. For simplicity, I focus on symmetric and stationary equilibria where all individuals follow the same strategies and real allocations are constant over time.

To find an equilibrium, I adopt the following widely used method. First, I solve for the optimal behavior of one individual in a conjectured stationary environment where the individual operates. Then, I check that when individuals behave optimally and symmetrically the conjectured environment is consistent with competitive search in the goods' market and perfect competition in the credit and insurance markets.

2.1 Optimal Behavior of a Representative Individual

Consider an individual in the following environment:

1. There is a single active submarket where goods are exchanged. In this submarket, the fraction of individuals that are sellers is θ. All these sellers post a contract that consists

of a price schedule and the promise to sell whatever amount of output the buyer desires for the dollars specified in the price schedule. The price schedule posted has the following form:

$$Z(q) = \frac{M_{+1}}{\beta}\left[c + C(q)\right], \tag{5}$$

where c is a constant and $C(q)$ is the utility cost of producing the amount of output traded q.

2. Regardless of their wealth, the expected trade surplus of buyers in the active submarket has the functional form:

$$S^b\left(q, z, \theta\right) = \theta\left[U\left(q\right) - \beta\frac{z}{M_{+1}}\right] - r\beta\frac{z}{M_{+1}}. \tag{6}$$

   where z is the payment in dollars in a trade. The equilibrium value of this function is common to all buyers and denoted by $\overline{S}^b$.

3. The nominal interest rate on bank loans and bank deposits is

$$r = \frac{\gamma - \beta}{\beta}. \tag{7}$$

   Since good prices are proportional to M_{+1}, which grows at the factor γ, (7) implies that the real interest rate is equal to the subjective discount rate: $\beta^{-1} - 1$.

The rest of this subsection analyzes the optimal behavior of an individual when faced with this environment.

Each period, the individual freely chooses to be a buyer or a seller. Therefore, the utility value V of the individual at the beginning of the day obeys:

$$V\left(\frac{A}{M}\right) = \max\left\{V^b\left(\frac{A}{M}\right), V^s\left(\frac{A}{M}\right)\right\}; \tag{8}$$

where A is the initial wealth of the individual in dollars, and V^b and V^s are respectively the value functions conditional on being buyer or a seller during the day. The money supply is used to deflate nominal quantities. This deflator is appropriate because (1) and (5) imply that in the environment where the individual interacts good prices increase proportionately with M. The ratio A/M will be called initial real wealth and it will be denoted as a.

Conditional on being a buyer, each morning the individual must choose the insurance coverage μ^b, the allocation of wealth between cash m^b and deposits d^b, and the projected consumption q^b if a trade meeting in the afternoon takes place. These choices solve the following optimization program:

$$V^b(a) = \max_{\{m^b, d^b, q^b, \mu^b\}} \theta \left[U\left(q^b\right) + \beta V\left(a_{+1}^{b1}\right) \right] + (1-\theta)\beta V\left(a_{+1}^{b0}\right) \tag{9}$$

subject to

$$a_{+1}^{b1} = \frac{m^b + d^b(1+r) + \tau - Z(q^b) + \mu^b(1-\theta)}{M_{+1}}, \tag{10}$$

$$a_{+1}^{b0} = \frac{m^b + d^b(1+r) + \tau - \mu^b\theta}{M_{+1}}, \tag{11}$$

$$\frac{m^b + d^b}{M} = a, \text{ and} \tag{12}$$

$$m^b \geq Z(q^b). \tag{13}$$

As a buyer, the individual faces two possible outcomes: either he meets a seller and buys q^b paying $Z(q^b)$, or he does not meet a seller and does not purchase anything. The respective probabilities of these two outcomes are θ and $(1-\theta)$. If the buyer meets a seller, next period's real wealth is a_{+1}^{b1} and satisfies (10). If the buyer does not meet a seller, next period's real wealth is a_{+1}^{b0} and satisfies (11). The choice on how to allocate wealth between cash and deposits must satisfy the budget constraint (12). Finally, the payment for the goods acquired is constrained by the cash carried to the goods' market because the buyer is anonymous outside his village (13). Since deposits earn positive interest while cash does not, the cash constraint (13) is always binding.

Conditional on being a seller, each morning the individual must choose how to allocate his wealth, how much insurance to purchase, and the contract to post in the goods market. The contract posted will determine the submarket at which the seller will search for trading opportunities. In this submarket, the fraction of sellers, θ^s, must be such that buyers attain the trading surplus $\overline{S}^b$ with the posted price schedule. This applies even if the seller deviates from other sellers and instead of posting (5) posts an original price schedule. In this case, the individual is the unique seller in the submarket. Buyers are going to direct their search to this seller until θ^s is such that each buyer attains a trading surplus $\overline{S}^b$. The fraction θ^s does not need to be the inverse of an integer because buyers can play mixed strategies.

Since all buyers share the same functional form of the expected trading surplus (6), there is no loss of generality in restricting the seller to post a contract that consists of a pair (q^s, z^s) specifying the output and the payment to be exchanged in a trade. In equilibrium, this contract is going to be equivalent to the price schedule (5), but at this point I do not artificially restrict the seller to a schedule of that particular form. Therefore, the maximization program solved by a seller is:

$$V^s(a) = \max_{\{m^s, d^s, \mu^s, \theta^s q^s, z^s\}} (1-\theta^s)\left[\beta V\left(a_{+1}^{s1}\right) - C(q^s)\right] + \theta^s \beta V\left(a_{+1}^{s0}\right) \tag{14}$$

subject to

$$a_{+1}^{s1} = \frac{d^s(1+r) + m^s + z^s - \mu^s\theta^s + \tau}{M_{+1}}, \tag{15}$$

$$a_{+1}^{s0} = \frac{d^s(1+r) + m^s + \mu^s(1-\theta^s) + \tau}{M_{+1}}, \tag{16}$$

$$\frac{m^s + d^s}{M} = a, \tag{17}$$

$$\theta^s\left[U(q^s) - \beta\frac{z^s}{M_{+1}}\right] - r\beta\frac{z^s}{M_{+1}} = \overline{S}^b, \text{ and} \tag{18}$$

$$m^s \geq 0. \tag{19}$$

The individual as a seller also faces two possible outcomes: either he meets a buyer and sells q^s for z^s dollars, or he does not meet a buyer and sells nothing. The probabilities of these two outcomes are respectively $(1-\theta^s)$ and θ^s. The real wealth achieved in either outcome is described in (15) and (16). The budget constraint is (17) . The seller realizes that to attract buyers to the submarket s those must attain at least a expected trading surplus $\overline{S}^b$. Furthermore the seller realizes, that θ^s will adjust endogenously to the contract (q^s, z^s), so if submarket s is active, then buyers in this submarket will attain an expected trading surplus $\overline{S}^b$. This is constraint (18). Finally, cash cannot be negative (19). Since $r > 0$, the inequality constraint (19) is always binding, so sellers carry no cash to the goods' market.

In addition to all constraints specified above for buying and selling activities, the individual faces an endogenous lower bound on tomorrow's wealth because he must be able to repay the amounts borrowed from the bank with probability one without reliance to unbounded borrowing (No-Ponzi game condition):

$$a^i_{+1} \geq a_{\min}, \text{ for } i \in \{b1, b0, s1, s0\}. \tag{20}$$

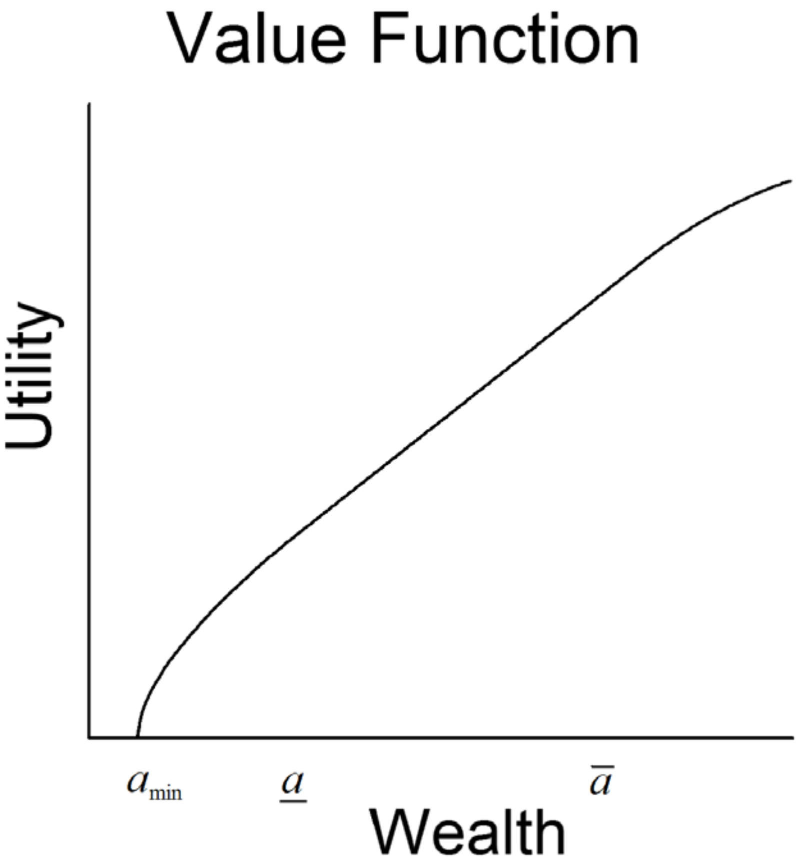
The lower bound $a_{\min}$ is minus the present discounted value of the maximum guaranteed income that the individual as a seller can obtain. See the Appendix for the characterization of $a_{\min}$.

Recursive methods are useful to solve the individual's optimization program described in equations (8) to (20). As shown in the Appendix, standard arguments show that V is a well defined value function that depends on a. Furthermore, V is concave with a linear segment as displayed in Figure 1. That is, in an interval of real wealth $[\underline{a}, \overline{a}]$ V has the following linear form:

$$V(a) = v_0 + a, \text{ for } a \in [\underline{a}, \overline{a}]; \tag{21}$$

where v_0 is a term independent from the initial real wealth, and the bounds $\underline{a}$ and $\overline{a}$ will be determined below. The linearity of V specified in (21) is ascertained using the method of undetermined coefficients. The Appendix provides a formal argument supporting the validity of this method in this context.

Figure 1



The value function V displayed in Figure 1 is consistent with the following behavior. At $a = a_{\min}$, the No-Ponzi game condition (20) is binding. That is, the individual has such an enormous amount of debt that the only way that it can be serviced for sure is being a fully insured seller offering a contract with the maximum output $\overline{q}$ forever. If a belongs to the lower interval $(a_{\min}, \underline{a})$, the individual has still a large pile of debt that is served being a fully insured seller forever. However, the individual does not need to sell the maximum output $\overline{q}$. The output q^s that the individual sells decreases with a. Therefore, the convexity of $C(q^s)$ implies the strict concavity of V in the interval $(a_{\min}, \underline{a})$. At the other extreme, if a belongs to the upper interval $[\overline{a}, \infty)$, an individual is sufficiently rich to be able to afford being a fully insured buyer forever. In this interval, both m^b and q^b increase with a. Therefore, the strict concavity of $U\left(q^b\right)$ implies the strict concavity of V. Finally, if a belongs to the middle interval $[\underline{a}, \overline{a}]$, the individual chooses each period the trading role to be played. In equilibrium, the individual is indifferent between being a buyer or a seller as long as $a_{+1} \in [\underline{a}, \overline{a}]$ with probability one. Moreover, the interval $[\underline{a}, \overline{a}]$ is absorbing because it is not optimal to make choices that lead to $a_{+1} \notin [\underline{a}, \overline{a}]$ with positive probability. As long as $a \in [\underline{a}, \overline{a}]$, the quantities consumed as a buyer or produced as a seller are independent from wealth. Inside the bracket $[\underline{a}, \overline{a}]$ richer individuals expect to consume more often and produce less often during the rest of their lives, but the quantities consumed or produced when they do so are not affected by wealth. Since utility is linear on the times an individual consumes and produces, the value function is linear. This argument is developed in more detail in the following paragraphs.

If $a \in [\underline{a}, \overline{a}]$, then optimal behavior implies that a_{+1} must also belong to $[\underline{a}, \overline{a}]$. To see this, suppose there is a positive probability that $a_{+1} \notin [\underline{a}, \overline{a}]$. Since outside the interval $[\underline{a}, \overline{a}]$ V is strictly concave, optimal behavior implies full insurance of trading risks. With full insurance, the Euler equation of program (8) is

$$V'(a) = \frac{\beta(1+r)M}{M_{+1}} V'(a_{+1}). \tag{22}$$

Given (1) and (7), this equation is violated if $V'(a) \neq V'(a_{+1})$, so a_{+1} must remain in the interval $[\underline{a}, \overline{a}]$. Consequently, if a is close to $\underline{a}$ being a seller is strictly preferred to avoid $a_{+1} < \underline{a}$, while if a is close to $\overline{a}$ being a buyer is optimal to avoid $a_{+1} > \overline{a}$.

As long as the next period real wealth remains in $[\underline{a}, \overline{a}]$ with probability one, the functional form (21) implies that the optimization problem for buyers (9) and sellers (14) simplifies to:

$$V^b(a) = \max_{q^b} \left\{ \theta \left[U\left(q^b\right) - \beta \frac{Z(q^b)}{M_{+1}} \right] - r\beta \frac{Z(q^b)}{M_{+1}} \right\} + \beta \left(v_0 + \frac{\gamma - 1}{\gamma} \right) + a, \text{ and} \tag{23}$$

$$V^s(a) = \max_{\{q^s, z^s, \theta^s\}} \left\{ (1 - \theta^s) \left[\beta \frac{z^s}{M_{+1}} - C(q^s) \right] \text{ s.t. } (18) \right\} + \beta \left(v_0 + \frac{\gamma - 1}{\gamma} \right) + a. \tag{24}$$

This simplification uses (7), (1), (2), and the constraints (13) and (19) with equality. The expressions in brackets in (23) and (24) are the expected buyer's and seller's surplus in a trade meeting. Both expected surpluses are independent from the initial wealth a. Consequently, the quantity of output the individual carries to the market as a buyer and the contract it posts as a seller are independent from a. The insurance coverages μ^b and μ^s drop from (23) and (24) entirely because of the linearity of V. That is, individuals are indifferent between insuring their trading risks or not if there is no chance that $a_{+1} \notin [\underline{a}, \overline{a}]$. However, as explained in the previous paragraph, insurance must be purchased when it is necessary to do so to ensure $a_{+1} \in [\underline{a}, \overline{a}]$.

The first order condition to the maximization program in (23) characterizes the quantity q^b consumed in a trade meeting:

$$U'\left(q^b\right) = \beta \frac{Z'(q^b)}{M_{+1}} \left(1 + \frac{r}{\theta}\right) \tag{25}$$

Since the cash constraint (13) holds with equality, (25) also characterizes the cash the individual carries to the market as a buyer.

The maximization program in (24) characterizes the contract posted by the seller. The following proposition shows an intuitive and simple way of characterizing an optimal contract.

Proposition 1 *An optimal contract posted by the seller is equivalent to committing a price schedule that is proportional to a flat fee c plus variable cost $C(q^s)$.*

$$Z^s(q^s) = \frac{M_{+1}}{\beta}\left[c + C(q^s)\right]. \tag{26}$$

The seller also commits to let the buyer pick the quantity purchased. Finally, the equilibrium values of q^s, θ^s, *and c solve the following maximization program:*

$$S^b = \max_{\{q^s, \theta^s, c\}} \theta^s \left[U\left(q^s\right) - c - C(q^s)\right] - r\left[c + C(q^s)\right] \tag{27}$$

subject to

$$(1 - \theta^s)\, c = S^s, \text{ and} \tag{28}$$

$$q^s \geq 0, \ \theta^s \in [0, 1]. \tag{29}$$

where S^s is the expected seller's surplus attained by the individual. (The proof is in the Appendix.)

For $r > 0$, the program (27) is not concave because of the interaction between the opportunity cost of carrying cash and the endogenous probability θ^s. As a result, if r is large, the second order conditions for an interior solution are violated. Fortunately, for reasonable parameters this requires very high values of r. For r reasonably small, the second order conditions to (27) are satisfied (see Appendix) and the solution of the program is characterized by the following first order conditions:

$$U'\left(q^{s}\right)=C'\left(q^{s}\right)\left(1+\frac{r}{\theta^{s}}\right),\text{ and} \tag{30}$$

$$1-\theta^{s}=\frac{c\left(1+r\right)}{U(q^{s})-C(q^{s})}. \tag{31}$$

Given the price schedule (26), equation (30) is the equivalent to the first order condition (30) that characterizes the optimal choice of consumption by a buyer. This reflects that the mechanism described in Proposition 1 is suitable to implement the optimum program of a seller. Condition (31) characterizes the optimal trade off between using prices (c) and probabilities of trade (θ^{s}) to attract buyers. This equation is the analog to Hosios (1990) condition modified to take into account the cost of carrying cash.

2.2 Equilibrium

This subsection characterizes a symmetric monetary stationary equilibrium where all individuals have initial wealth in the interval $\left[\underline{a},\overline{a}\right]$.

In equilibrium, individuals must behave optimally and their expectations must be consistent. Therefore, the optimal behavior of buyers characterized in (25) must be consistent with the optimal behavior of sellers characterized by (26) to (31) with $q^{b}=q^{s}=q$ and $\theta^{s}=\theta$. In addition, some individuals must choose to be buyers while other must choose to be sellers, so $S^{b}=S^{s}$. Combining all these equations yields the following system that characterizes the equilibrium values of q,θ, and c :

$$U'\left(q\right)=C'\left(q\right)\left(1+\frac{r}{\theta}\right), \tag{32}$$

$$1-\theta=\frac{c\left(1+r\right)}{U(q)-C(q)},\text{ and} \tag{33}$$

$$\theta\left[U(q)-c-C(q)\right]-r\left[c+C(q)\right]=\left(1-\theta\right)c. \tag{34}$$

Equation (32) is the condition for an optimal choice of q. Equation (33) is the condition for optimal pricing. Finally, (34) is the condition for the coexistence of buyers and sellers in the market. This characterization of an equilibrium assumes that r is sufficiently small so the second order condition (80) is satisfied.

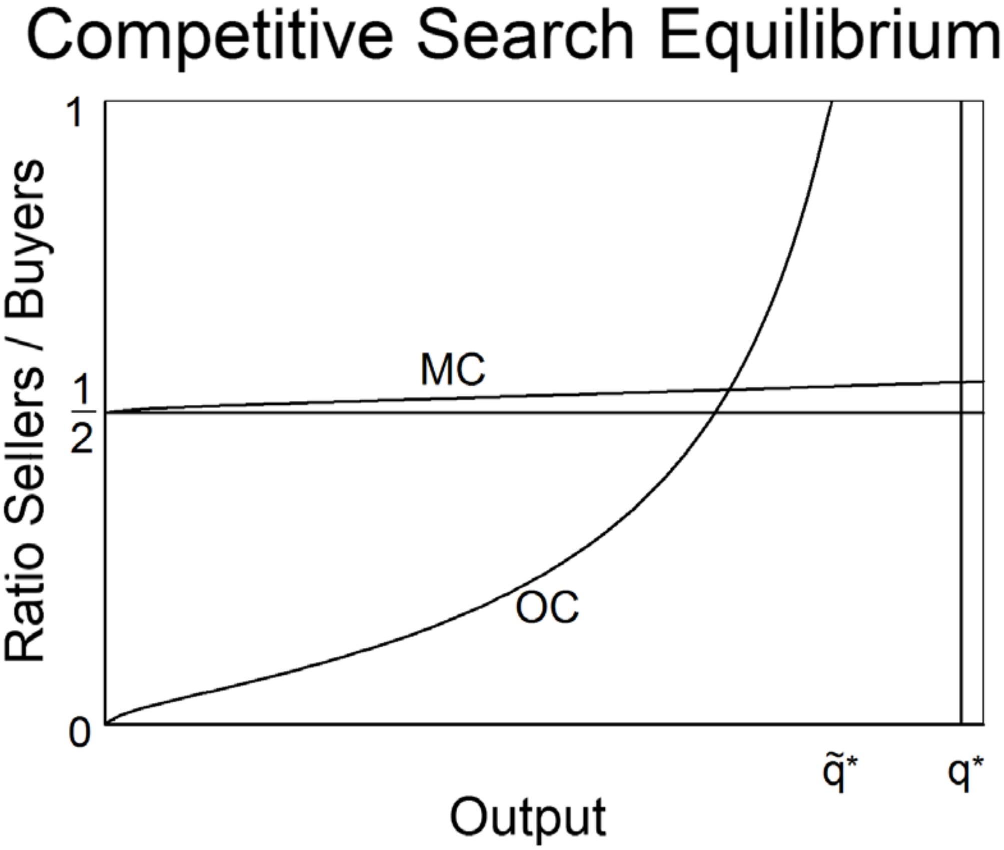
The conjectured environment where the representative individual was assumed to be operating in the previous subsection is consistent with the equilibrium described by (32) to (34). Indeed, as proved in Proposition 1, the optimal contract posted with a seller is implemented by posting the conjectured price schedule (5). The buyer's surplus derived in (23) has the form conjectured in (6). Finally, the demand for loans and deposits is perfectly elastic at (7), so the credit market clears.

The system of equations (32) to (34) characterizing an equilibrium can be simplified using (33) to eliminate c in (34) to obtain:

$$2\theta - 1 = \frac{rC(q)}{U(q) - C(q)} \tag{35}$$

With this simplification, the two key variables q and θ determining the workings of the goods' market are determined by the two equations (32) and (35). These two equations are represented in Figure 2.

Figure 2



The line OC (output condition) in Figure 2 represents equation (32), which specifies the optimal choice of q in a transaction. This line is vertical if $r \to 0$ and upward sloping if $r > 0$. If $r \to 0$, the output traded is q^*, which is the efficient output that equates the marginal utility of output to the marginal cost of its production. If $r > 0$, the output traded is lower than q^*. In this case, the marginal utility of output is equated to the sum of the marginal cost of producing output plus the opportunity cost of carrying the cash needed to pay for this cost. An increase in θ shortens the average time cash is carried before it is spent. Hence, the opportunity cost of the cash carried in a transaction is decreasing in θ. This results in buyers purchasing larger amounts of output as θ increases. As θ approaches zero, buyers on average must carry cash for a very long time before they find a seller, so $q \to 0$ if $\theta \to 0$. At the other extreme, at θ is equal to 1 the buyer finds the seller in just one period. Still there is the opportunity cost of carrying money during that period, so

$q = \widetilde{q}^* < q^*$ if $\theta = 1$.

Equation (35) is derived from the two conditions that determine the density of sellers in the market. These two conditions are the following: (i) the optimal pricing (33), and (ii) the equality of expected returns from being a buyer and a seller (34). The line representing this equation is denoted as MC (market composition) in Figure 2. This line is horizontal at $\theta = 1/2$ if $r \to 0$ and upward sloping if $r > 0$. If $r \to 0$, the fraction sellers in the market maximizes the measure of trading matches. With random matching between all traders, this implies $\theta = 1/2$. This outcome, together with $q = q^*$, implies that the competitive search equilibrium is efficient, that is it maximizes the joint expected trading surplus of all individuals if $r = 0$. In contrast, if $r > 0$, buyers face a positive opportunity cost of carrying cash. To reduce this cost and still maintain the equality of trading surpluses, θ must be above 1/2. The opportunity cost of carrying cash increases with the amount of cash that buyers carry and so with q. As a result, the positive gap between θ and 1/2 increases with q. Graphically, equation (35) is an upward sloping line.

As represented in Figure 2, the equilibrium values of q and θ are in the region $q < q^*$ and $\theta > 1/2$. Inflation above the Friedman rule, $r > 0$, reduces the output exchanged in a trading meeting and increases the fraction of sellers in the market relative to the outcomes with $r \to 0$. An increase in r shifts the two equilibrium lines upward and to the left. Since both (32) and (35) are upward sloping this graphical analysis does not determine the sign of the derivatives of q and θ with respect to r. Using calculus (see the Appendix), the derivative of q with respect to r is unambiguously negative as long as (80) is satisfied. The derivative of θ with respect to r is positive for low values of r. For high values of r, the derivative of θ with respect to r is theoretically ambiguous, but numerical simulations show that for reasonable parameterization of the model θ is a monotonically decreasing function of r. In summary, an increase in r increases the opportunity cost of buying goods. As a result, markets respond by reducing the output exchanged and, as long as r is reasonably low, increasing the speed at which buyers meet sellers.

Existence of an equilibrium requires that the two lines representing equations (32) and (35) in Figure 2 cross. The necessary and sufficient condition that ensures this is the following (see the Appendix):

$$\max_{q} \{U(q) - C(q)\,(1+r)\} \geq 0. \tag{36}$$

This condition puts an upward bound on r.

2.3 Velocity, the Demand for Cash, and the Welfare Cost of Inflation

The utilitarian measure of welfare in this model is

$$W = \theta\,(1-\theta)\,[U\,(q) - C\,(q)]\,. \tag{37}$$

At $r \to 0$, W attains its maximum because both the number of trades $\theta\,(1-\theta)$ are maximized at $\theta = 1/2$, and the joint surplus per trade $[U\,(q) - C\,(q)]$ is maximized at $q = q^{*}$. Inflation generated by a faster rate of growth of the money supply increases the nominal interest rate r as implied by (7). Therefore, inflation is costly from a welfare perspective because it distorts two margins: it reduces the number of trades (extensive margin) and it reduces the amount of output exchanged in each trade (intensive margin). Next section, it will focus on a third margin: the choice of the means of payment.

The fraction of the money supply that changes hands each period is equal to the fraction of buyers that meet with a seller. Hence, the velocity of circulation of cash is θ. The fact that θ is sensitive to changes in r implies that velocity is interest elastic. The real aggregate demand for cash is :

$$M^{d} = \frac{(1-\theta)m^{b}}{M} = (1-\theta)\,(1+r)\,[c + C(q)]\,. \tag{38}$$

Using (34), we obtain a simple relationship between the real demand for cash and welfare:

$$M^{d} = W + (1-\theta)\,C(q). \tag{39}$$

Consequently,

$$\frac{dM^{d}}{dr} = \frac{dW}{dr} + (1-\theta)\,C'(q)\frac{dq}{dr} - C(q)\frac{d\theta}{dr} \tag{40}$$

Equation (40) shows the intimate relationship between the welfare cost of inflation and the effect of r on the demand for money. The three terms determining the derivative of M^{d}

with respect to r in (40) are negative if, as is typically the case, $d\theta/dr < 0$.

2.4 A General Matching Technology

The preceding analysis assumes that all traders in a submarket are randomly matched. This is a strong assumption if we are thinking of cash being used in retail trade. Fortunately, the preceding analysis can be replicated with a general constant returns matching technology. To this end, consider a general matching technology $\Pi(\theta)$ that maps the fraction of sellers in the market θ onto a measure of trading matches. The function Π is assumed to be twice continuously differentiable, strictly concave, and $\Pi(0) = \Pi(1) = 0$. These assumptions imply that there is a $\theta^* \in (0,1)$ where Π attains its maximum value. At θ^*, $\Pi'(\theta^*) = 0$. Constant returns in matching implies that the probabilities of a buyer and a seller with a trading partner are respectively:

$$\pi^b = \frac{\Pi(\theta)}{1-\theta} \quad \text{and} \quad \pi^s = \frac{\Pi(\theta)}{\theta}. \tag{41}$$

Random matching among all traders is a special case in which $\Pi(\theta) = \theta(1-\theta)$. With a general matching technology, matching may proceed along many alternative rules. These rules, exogenous to the model, may allow for various degrees of efficiency in the way buyers and sellers are matched.

With a general matching technology, the counterpart to the key maximization program (27) that determines a competitive search equilibrium is:

$$S^b = \max_{\{q,\theta,c\}} \pi^b [U(q) - c - C(q)] - r[c + C(q)] \tag{42}$$

subject to

$$\pi^s c = S^s, \tag{43}$$

where π^b and π^s obey (41). The first order conditions to this program together with the free choice of acting as a buyer or a seller can be written as:

$$U'(q) = C'(q)\left[1 + \frac{r(1-\theta)}{\Pi(\theta)}\right], \tag{44}$$

$$[1+\eta(\theta)]\,[U(q)-C(q)]=\frac{c}{1-\theta}\left(1+r\frac{\theta(1-\theta)}{\Pi(\theta)}\left[\frac{1-\theta}{\theta}-\eta(\theta)\right]\right),\ \text{and} \tag{45}$$

$$[U(q)-C(q)]=\frac{c}{1-\theta}\left(1+r\frac{\theta(1-\theta)}{\Pi(\theta)}+r\frac{C(q)}{c}\right), \tag{46}$$

where

$$\eta(\theta)\equiv\frac{(1-\theta)\,\Pi'(\theta)}{\Pi(\theta)}. \tag{47}$$

Combining (45) and (43), one obtains

$$1+\eta(\theta)=\frac{1+r\frac{\theta(1-\theta)}{\Pi(\theta)}\left[\frac{1-\theta}{\theta}-\eta(\theta)\right]}{1+r\frac{\theta(1-\theta)}{\Pi(\theta)}+r\frac{C(q)}{c}}. \tag{48}$$

The main results obtained with a the random matching technology generalize as follows:

Proposition 2 *In a competitive search equilibrium with $r\to 0$, the output traded in a meeting and the fraction of sellers in the market are efficient: $q=q^*$ and $\theta=\theta^*$. Let r_0 be a positive interest rate and suppose that $\theta^*\geq 1/2$ and the second order condition for an interior solution to (42) is satisfied for all $r\leq r_0$. Then, at the equilibrium with r_0 the output traded in a meeting is inefficiently low, $q<q^*$, and the fraction of sellers in the market is inefficiently high, $\theta>\theta^*$. (See the Appendix for the proof.)*

3 Cash and Transfer of Deposits as Means of Payment

This section relaxes the assumption that banks cannot transfer deposits across villages. Banks from different villages are now assumed to have access to a national clearance system through which funds deposited at different villages can be transferred at a cost. In a stationary equilibrium, the transfers into a village balance those going out of the village. Therefore, the national clearance system does not require the actual movement of cash or goods across villages. Instead, it requires the record keeping of all interbanking transfers. The cost of running the clearance system is assumed proportional to the number of transfers realized. As a result, competitive banks charge a fee, ϕ, for each transfer a depositor requests. The mechanism for realizing these transfers could be writing a cheque, using an electronic debit card, or giving a direct order to the bank. The actual mechanism used is immaterial to the model. However, since buyers and sellers do not know each other, a form of secure communication between depositors and their banks, such as those in an electronic debit card system, fits bests the framework of the model. For short, these transfers of deposits are referred as transfers.

Apart from the possibility of using transfers as a means of payment, the model from the previous section is enriched with a preference shock, which individuals experience at the beginning of each day. Therefore, preferences are now:

$$\mathcal{U}(q^b, q^s, \varepsilon) = \varepsilon U(q^b) - C(q^s), \tag{49}$$

where ε is a random variable that takes values in a positive interval $[\underline{\varepsilon}, 1]$ and has a distribution function $F(\varepsilon)$. Because of this preference shock, buyers are going to make purchases of different sizes and those are going to be increasing in ε. For simplicity, the lower bound $\underline{\varepsilon}$ is assumed to be sufficiently high so all buyers seek to make purchases regardless of the realized value of ε.

To concentrate the analysis on the margin of choice cash versus transfers, this section assumes that matching in a submarket takes place in such a way that the short side of the market is always served:

$$\Pi(\theta) = \min(\theta, 1 - \theta). \tag{50}$$

This "all-served" or efficient matching technology does not satisfy the differentiability assumption made in the previous section. However, it is very tractable because in a competitive search equilibrium $\theta = 1/2$, and $\pi^b = \pi^s = 1$. With efficient matching, buyers and sellers know that if they go to the market they are going to find a trading partner. However, individual traders in a submarket are still assigned randomly to a complementary pair, so long term relationships between one buyer and one seller that would make credit contract feasible are ruled out.

The timing of events during a day is the following. As in the model of the previous section, financial markets open in the morning and goods markets open in the afternoon. Even if the goods' market does not open until the afternoon, individuals must decide early in the morning if they are going to be a buyer or a seller that day. After this decision is made and before the financial markets close, individuals observe the idiosyncratic preference shock ε. Based on the realization of ε, they decide the amount of cash to carry to the goods' market. In this decision, they must ponder if the ϕ is sufficiently low to use a transfer to pay for their purchases or instead they prefer paying with cash.

As in the model of the previous section, the simple analytical tractability that results when all individuals share a common marginal value of wealth is achieved thanks to the existence of financial contracts inside each village. In the model of this section, insurance contracts are not always needed to achieve this tractability. Thanks to the efficient matching technology (50), individuals face no uncertainty on their transaction opportunities, so there is no need to insure against this type of risk. The only source of uncertainty that individuals face is the realization of preference shock ε. However, this risk does not necessarily imply the need for insurance to ensure a common marginal value of wealth. As in the previous section, the choice between being a buyer or a seller generates a linear interval in the value function. As long as wealth remains inside this interval with probability one, individuals are risk neutral so insurance contracts are redundant and the marginal value of wealth is constant. The role of insurance in the model of the previous section was to allow individuals to remain in the linear interval. In the model of this section, individuals can be certain of earning income if they go to the market as sellers. So they can avoid their wealth drifting below the linear interval by choosing to be a seller as their wealth approaches the lower limit. Furthermore, as long as the lower bound $\underline{\varepsilon}$ is sufficiently high, all buyers spend a positive amount bounded away from zero. So individuals can also avoid their wealth drifting above the linear interval by choosing to be buyer as their wealth approaches the upper limit. Consequently, with the special assumptions adopted in this section insurance contracts are redundant and unnecessary for generating equilibria where all individuals share a common marginal value of wealth.

Because ε is known in advance of the matching in the goods market, there is a different submarket for each realization of ε. An analogous argument to the one presented in the previous section implies that the pricing schedule in all submarkets has the form:

$$Z^{\varepsilon}(q_{\varepsilon}) = \frac{M_{+1}}{\beta}\left[c_{\varepsilon} + C(q_{\varepsilon})\right]. \tag{51}$$

Buyers pay the marginal cost of production plus a flat fee c_ε. This flat fee is the trade surplus of the seller. In principle, the flat fee could differ across submarkets. However, the frictionless matching technology (50) implies that $\pi^s = 1$ in equilibrium. Moreover, sellers can choose the submarket where they want to be active, so they all must receive the same expected surplus S^s. Therefore, in all submarkets: $c_\varepsilon = S^s$.

Before financial markets close, buyers must decide if they plan to make a purchase that afternoon, and if so which means of payment to use. The outcome of these decisions depends on the realized value of ε. If ε were very low, buyers would obtain a negative trade surplus because of the flat fee in (51). Therefore, it would be optimal for them to stay at home and remain inactive in the goods markets during that period. The assumption that $\underline{\varepsilon}$ is sufficient high avoids this unnecessary complication. In this instance, the key decision that buyers confront is the means of payment they plan to use in the afternoon market. This decision is made by comparing the trade surplus they expect to attain conditional on using cash or a transfer.

The same line of reasoning that leads to (27) in the previous section implies that submarkets where cash is used as the means of payment the output exchange in a trade meeting q^c_ε solves the following program:

$$\psi^c(\varepsilon, S^s) = \max_{q^c_\varepsilon} \left[\varepsilon U(q^c_\varepsilon) - S^s - C(q^c_\varepsilon)\right] - r\left[S^s + C(q^c_\varepsilon)\right]. \tag{52}$$

The first order condition that characterizes an optimum is:

$$\varepsilon U'(q^c_\varepsilon) = (1+r)\,C'(q^c_\varepsilon). \tag{53}$$

An increase in r reduces q^c_ε. Analogously, submarkets for which buyers pay with transfers q^d_ε solves:

$$\psi^d(\varepsilon, S^s) = \max_{q^d_\varepsilon} \left[\varepsilon U\left(q^d_\varepsilon\right) - S^s - C(q^d_\varepsilon)\right] - \phi. \tag{54}$$

The first order condition is:

$$\varepsilon U'\left(q^d_\varepsilon\right) = C'(q^d_\varepsilon). \tag{55}$$

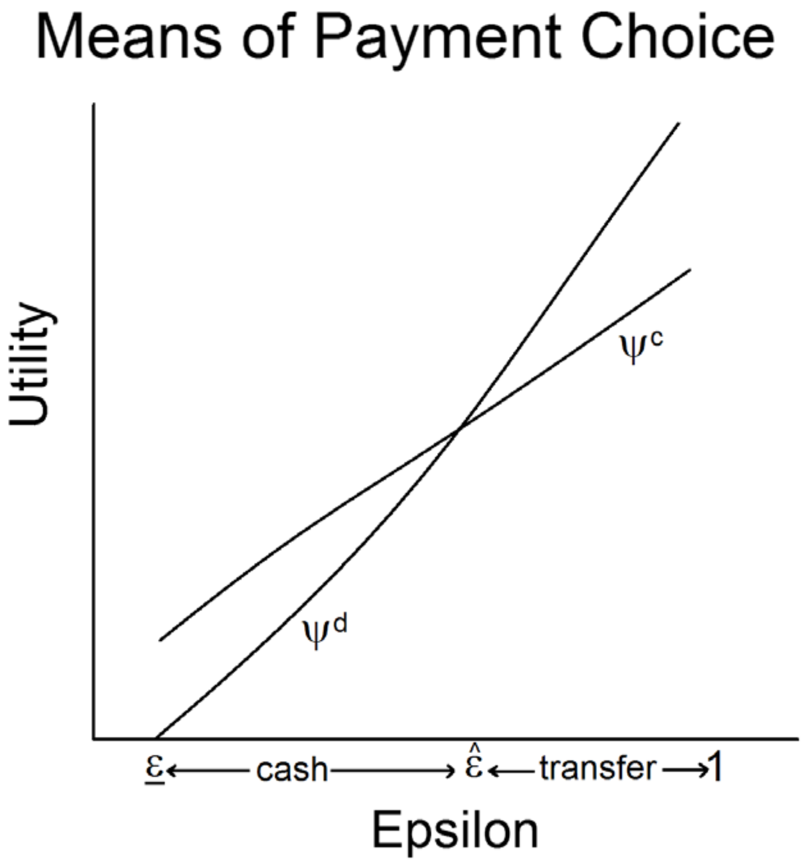
In this case, the output exchanged in a trade meeting q^d_ε is independent from r. The comparison between (53) and (55) implies that for the same ε the amount of output exchanged is greater if a transfer is used as a means of payment instead of cash: $q_\varepsilon^d \geq q_\varepsilon^c$.

Buyers choose the means of payment brings to them the maximum expected trading surplus given the preference shocks they have experienced, so

$$S^b(\varepsilon, S^s) = \max\left\{\psi^c(\varepsilon, S^s), \psi^d(\varepsilon, S^s)\right\}. \tag{56}$$

The choice between using cash or a transfer is represented in Figure 3. Using the Envelope Theorem, the derivatives of ψ^c and ψ^d are equal to $U(q_\varepsilon^c)$ and $U\left(q_\varepsilon^d\right)$ respectively. Since $q_\varepsilon^d \geq q_\varepsilon^c$ and U is an increasing function, ψ^d is a steeper function of ε than ψ^c. This implies that if ψ^d and ψ^c cross, they cross only once. Moreover, if both means of payments coexist, cash is preferred for low values of ε and a transfer is preferred for high values of ε.

Figure 3



The optimal choice of the means of payment is summarized in the following proposition:

Proposition 3 *As long as $\underline{\varepsilon}$ is sufficiently close to 1, all buyers are active for all realizations of ε. There is a positive demand for cash if $\psi^c(\underline{\varepsilon}, S^s) > \psi^d(\underline{\varepsilon}, S^s)$. Transfers are used to pay for some goods if $\psi^c(1, S^s) < \psi^d(1, S^s)$. If cash and transfers coexists as active means of payment, then there is a break-point $\widehat{\varepsilon} \in [\underline{\varepsilon}, 1]$ such that cash is used for $\varepsilon \in [\underline{\varepsilon}, \widehat{\varepsilon})$ and a transfer is used for $\varepsilon \in (\widehat{\varepsilon}, 1]$. At the break-point $\varepsilon = \widehat{\varepsilon}$, a buyer is indifferent between using cash or a transfer.*

In equilibrium, an individual chooses to be a buyer if and only if

$$S^b(\varepsilon, S^s) \geq S^s. \tag{57}$$

Therefore, the values $\widehat{\varepsilon}$ and S^s in an equilibrium where cash and transfers coexist as active means of payment are characterized by the following system of equations:[2]

$$\psi^c(\widehat{\varepsilon}, S^s) = \psi^d(\widehat{\varepsilon}, S^s), \text{ and} \tag{58}$$

$$\int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} \psi^c(\widehat{\varepsilon}, S^s)\, dF(\varepsilon) + \int_{\widehat{\varepsilon}}^{1} \psi^c(\widehat{\varepsilon}, S^s)\, dF(\varepsilon) = S^s. \tag{59}$$

This system implies the following comparative statics (see the Appendix):

$$\frac{d\widehat{\varepsilon}}{dr} < 0\ ,\ \frac{d\widehat{\varepsilon}}{d\phi} > 0\ ,\ \frac{dS^s}{dr} < 0, \text{ and } \frac{dS^s}{d\phi} < 0 \tag{60}$$

An increase in the nominal interest rate r increases the opportunity cost of carrying cash. As a result, cash becomes less attractive as a means of payment so the fraction of cash goods decreases. In contrast, an increase in the cost of transferring deposits ϕ makes this means of payment more onerous, so the fraction of cash goods increases. Finally, increasing the cost of carrying cash or the cost of realizing transfers reduces the equilibrium trading surplus of sellers, which in equilibrium is equal the expected trading surplus of buyers. Therefore, the expected utility of all individuals is decreasing in r and ϕ.

3.1 Velocity of Circulation of Money

In this model, the fundamental velocity of circulation of cash, defined as the fraction of cash that changes hands each period, is constant and equal to one. The annual fundamental

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[2] If some buyers remain inactive because $\underline{\varepsilon}$ is low, the extra equation $\psi^d(\varepsilon_0, S^s) = S^s$ determines the lowest realization of the preference shock ε_0 for which a buyer is active. Comparative static results about $\widehat{\varepsilon}$ and S^s are robust to this extension.

velocity of circulation of cash is simply equal to the number of periods in a year and so inversely proportional to the length of a period. However, the apparent velocity of circulation of cash, defined as the ratio of nominal spending over the money supply is endogenous and interest elastic. Using its definition, the apparent velocity of circulation of cash is equal to

$$\zeta = \frac{\int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} \left[S^s + C\left(q_\varepsilon^c\right)\right] dF(\varepsilon) + \int_{\widehat{\varepsilon}}^{1} \left[S^s + C\left(q_\varepsilon^d\right)\right] dF(\varepsilon)}{\int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} \left[S^s + C\left(q_\varepsilon^c\right)\right] dF(\varepsilon)} \tag{61}$$

$$\zeta = 1 + \frac{S^s\left[1 - F(\widehat{\varepsilon})\right] + \int_{\widehat{\varepsilon}}^{1} C\left(q_\varepsilon^d\right) dF(\varepsilon)}{S^s F(\widehat{\varepsilon}) + \int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} C\left(q_\varepsilon^c\right) dF(\varepsilon)} \tag{62}$$

The velocity of circulation ζ is monotonically increasing in r for the following reasons. An increase in r reduces the fraction of cash goods $[\underline{\varepsilon}, \widehat{\varepsilon}]$, while it expands the fraction of transfer goods $[\widehat{\varepsilon}, 1]$. Furthermore, the increase in r reduces q_ε for cash goods, while it does not affect q_ε for transfer goods. Both of these effects increase the numerator of the fraction in (62) and reduce the denominator. In addition, these two effects are reinforced by the fall of S^s as r increases because ζ is a decreasing function of S^s. The quantitative importance of these effects is investigated in the next section.

4 Calibration

The previous two chapters have stressed three different primary sources of interest elasticity of the demand for cash. In the model of section 2, an increase in the nominal interest rate accelerates the velocity of circulation of cash by increasing the density of sellers in the market (market composition effect). This increased density raises the probability a buyer is matched, so it takes shorter for the buyer to spend the cash that he carries.[3] In the model of section 3, an increase in the nominal interest rate reduces both the output exchanged of the goods paid with cash (intensive output effect) and the fraction of goods that use cash as the means of payment (means of payment effect). This section combines all these effects in a single model and calibrates it to United States data.

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[3] This channel has received much attention in previous monetary search model since it was introduced by Li (1994) in models with indivisible money and Shi (1997) in models with divisible money. Altought, Rocheteau and Wright (2003) have recently questioned the robustness of this channel in ex-post bargaining models.

In this section, the model of Section 3 is reinterpreted and generalized. The preference shock ε is reinterpreted as signifying the type of good that buyers desire to purchase. That is, buyers sometimes desire to buy a large ticket item (a car) and other times they desire to buy a small ticket item (a candy). With this reinterpretation, it makes sense to assume that the retail costs differ with ε : selling a car requires much more labor than selling a candy. In the model, this is captured by assuming that the matching technology is dependent on ε. With efficient matching, a match requires 1 buyer and a team of k_ε sellers to perform a trade. The parameter k_ε is not restricted to be an integer, so it is implicitly assumed that a seller can participate in more than one match each period. This section also studies a model that combines the random matching used in section 2 with the multiple means of payment model in section 3. With random matching, buyers and teams of k_ε sellers are randomly matched.

The analysis of the generalized model is analogous to those from the previous section. The equations analogous to (52), (54), (58), and (59) characterizing an equilibrium are the following:

$$\psi^c\left(\varepsilon, S^s\right) = \max_{\{q_\varepsilon, \theta_\varepsilon\}} \pi^b \left[\varepsilon U\left(q_\varepsilon\right) - \frac{k_\varepsilon S^s}{\pi^s} - C(q_\varepsilon)\right] - r\left[\frac{k_\varepsilon S^s}{\pi^s} + C(q_\varepsilon)\right], \tag{63}$$

$$\psi^d\left(\varepsilon, S^s\right) = \max_{\{q_\varepsilon, \theta_\varepsilon\}} \pi^b \left[\varepsilon U\left(q_\varepsilon\right) - \frac{k_\varepsilon S^s}{\pi^s} - C(q_\varepsilon) - \phi\right], \tag{64}$$

$$\psi^c\left(\widehat{\varepsilon}, S^s\right) = \psi^d\left(\widehat{\varepsilon}, S^s\right), \text{ and} \tag{65}$$

$$\int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} \psi^c\left(\widehat{\varepsilon}, S^s\right) dF(\varepsilon) + \int_{\widehat{\varepsilon}}^{1} \psi^c\left(\widehat{\varepsilon}, S^s\right) dF(\varepsilon) = S^s; \tag{66}$$

where $\pi^b = \pi^s = 1$ with efficient matching, and $\pi^b = \theta_\varepsilon$ and $\pi^s = 1 - \theta_\varepsilon$ with random matching.

The main challenge to calibrate this generalized model is that a large fraction of cash in United States, as well as in other industrialized countries, is not used for regular transactions. The average official measure of cash by the Federal Reserve during 2000 was \$2572 for each American adult. However, we all know that almost nobody carries such a large amount on their pockets. In response to this challenge, the following calibration corrects the official measures of cash to take into account that only a small fraction of cash is used to conduct regular transactions. Faig and Li (2004) estimate these fractions by studying the response of the amounts of cash in circulation to the sharp increases in retail sales occurring each December. For the period 1979-2000, they estimate that only around 8 percent of the cash in circulation was used to conduct regular transactions. This implies that American adults carried around \$200 on average at the turn of the millennium. A much more plausible amount than over \$2500.

The calibration of the model adopts the following functional forms:

$$U(q_{\varepsilon}) = \frac{q_{\varepsilon}^{1-\sigma}}{1-\sigma}, \ \sigma \in (0,1)\,, \tag{67}$$

$$C(q_{\varepsilon}) = q_{\varepsilon}, \tag{68}$$

$$F(\varepsilon) = \frac{\underline{\varepsilon}^{-\eta} - \varepsilon^{-\eta}}{\underline{\varepsilon}^{-\eta} - 1}, \quad \eta \in [-1, \infty) \text{ and} \tag{69}$$

$$k_{\varepsilon} = \left(\underline{\varepsilon}^{-\eta} - 1\right) \frac{1-\eta\sigma}{\eta\sigma} \varepsilon^{\frac{1}{\sigma}}. \tag{70}$$

The utility and the cost functions, (67) and (68), have respectively the standard isoelastic and linear forms employed in this literature. The distribution function (**??**) of the preference shocks yields a tractable isoelastic density function and allows for the distribution of transactions to be decreasing with ε.[4] For Finally, the teams of sellers are assumed to be increasing in ε. The particular functional form in (70) is assumed to yield constant commercial margins as a fraction of sales in the absence of transfer costs. Also, the level of k_{ε} is adjusted so on average a buyer meets with one seller. In this way, the equilibrium ratio of buyers and sellers is approximately one as empirical estimates imply (see Faig and Jerez, 2004).

With the functional forms (67) to (70), there are 5 parameters to calibrate:

1. Length of the period that converts annual interest rates into period interest rates:

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[4] At $\eta = -1$, the distribution is uniform. For $\eta \geq -1$, the density function is decreasing with ε.

T.

2. Utility function curvature: σ.
3. Per unit transfer cost: ϕ.
4. Dispersion parameter of the distribution the ε shocks: η.
5. Lowest value of $\varepsilon : \underline{\varepsilon}$.

These parameters are calibrated to match the following empirical estimates:

1. The payment period (fundamental velocity of cash) is 2 weeks.
2. The ratio from the highest to the lowest output exchanged in a transaction is 100,000.
3. The average commercial margin in retail trade is 28 percent.
4. The apparent velocity of circulation of cash at 5 percent annual interest is 6.3.
5. The apparent velocity of circulation of cash at 9 percent annual interest is 8.6.

The length of the payment period is set to 2 weeks which is the most typical frequency blue collar workers are paid and white collar workers fetch cash from the bank. The ratio from the highest to the lowest transaction sizes is set to 100,000. This roughly matches the ratio from the cost of a good quality car and a candy bar. The Bureau of the Census calculates that the average commercial margin in retail trade has fluctuated with little variation around 28 percent from 1993 to 2002.[5] The velocity of circulation of cash is defined as the ratio of biweekly consumption expenditures to the cash used in transactions. For the period 1979-1989, the average 3-month Treasure Bill rate was 9 percent. For the same period, the ratio of biweekly consumption expenditures to the official measure of cash for the period was 0.6284 on average. Since during this period, Faig and Li (2004) calculate that 7.3 percent of cash was used perform regular transactions, the biweekly velocity was $0.6282/0.073 = 8.6$. Analogous figures for the period 1990-2000 were 5 percent interest rate, 0.5433 biweekly consumption over cash, and 8.58 percent of cash being used to perform regular transactions. This implies a biweekly velocity of 6.3.

The results of the calibration for a model with efficient matching and another with random matching are reported in Table 1. Two remarkable facts stand from this table. The first remarkable fact is that both models have almost identical parameters. The reason

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[5] Bureau of the Census, Service Sector Statistics Division http://www.census.gov/svsd/www/artstbl.html

for this similarity is that even though with random matching the probability of a match is endogenous, this probability is very insensitive to changes in interest rates. The second remarkable fact is that the calibrated cost of making a transfer payment is quite small. On average, this cost represents 0.12% of the total commercial margin for transfer goods. As discussed below, both facts are a consequence of the realistically short payments period.

Table 1

CALIBRATION

Average payments period	2 weeks	
Velocity at 5% annual interest rate	6.3 biweekly	
Velocity at 9% annual interest rate	8.6 biweekly	
Average commercial margin	28 percent	
Ratio highest to lowest q	100,000	
Calibrated parameters:	Efficient Matching	Random Matching
T	26	52
σ	0.4380	0.4377
ϕ	7.806×10^{-5}	7.788×10^{-5}
η	1.104	1.106
$\underline{\varepsilon}$	6.456×10^{-3}	6.477×10^{-3}

Table 2 reports the effect of increasing the annual nominal interest rate from 3 to 13 percent. This represents going from 0 to 10 percent inflation assuming that the annual subjective discount rate is 3 percent. With 10 percent inflation, the opportunity cost of carrying cash for 2 weeks remains very small. Consequently, 10 percent inflation has a minor effect on the composition of traders in the market and the output exchanged in a transaction. As Table 2 reports, the effect of 10 percent inflation on the probability that a buyer meets a seller is negligible. Also, for both types of matching, the output exchanged in transactions paid with cash falls by only 0.8 percent as inflation escalates from 0 to 10 percent. If the fraction of cash goods were exogenous these effects would increase the apparent velocity from 4.82 to 4.85. That is, an almost unnoticeable change. However, despite the minor composition and output effects, the apparent velocity of circulation of cash is very sensitive to changes in interest rates because of the large means of payment effect. As the nominal interest rate climbs from 3 to 13 percent, the fraction of goods paid with cash falls quite strongly. The break point $\widehat{\varepsilon}$ that separates the goods paid with cash and transfers from 0.266 to 0.14. This induces, almost single-handedly, a more than doubling of the apparent velocity, which climbs from 4.82 to 10.47.

The welfare cost of inflation is low relative to the equivalent variation of consumption it represents, but it is high relative to the seigniorage it raises. The welfare cost of inflation measured as the equivalent variation of consumption is 0.161 percent in both models. This is much smaller than the estimates reported by Lucas and Lagos and Wright (2003). The main reason for this discrepancy resides on the different measures of the quantity of money. Both Lucas and Lagos and Wright use M1to calculate their welfare costs. Instead this paper uses the fraction of cash that is used to perform regular transactions. This measure of cash was around 4 percent of M1 in the period 1990-2000. However, even if the welfare cost of 10 percent inflation represents only a 0.161 percent of the equivalent variation of consumption, it represents 106 percent of the seigniorage it raises. To understand this, remark that almost all of the welfare cost of inflation consists of avoidable deposit transfer fees. Because cash is quite interest sensitive, raising inflation from 0 to 10 percent does not increase seigniorage by much, yet the volume of transfer fees increases considerably.

The welfare cost of inflation calculated in Table 2 refers only to the distortions it generates in the payments mechanism. Because a large fraction of cash is held for reasons not modelled here, or in most of monetary theory,[6] this paper is mute about the overall welfare cost of inflation. If most of "missing" cash is used to shelter illegal activities, inflation may be a welfare enhancing law enforcement tool. In contrast, if most of the "missing" cash is used as a saving vehicle by marginalized people who do not have bank accounts, in the present paper villages without banks, the welfare cost of inflation may be quite high.

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[6] For an exception see Camera ().

Table 2

EFFECT OF 10 PERCENT INFLATION

	Efficient Matching	Random Matching
Equilibrium at 3 % annual interest rate:		
Biweekly apparent velocity	4.82	4.82
Average commercial margin	0.280	0.280
Break point $\widehat{\varepsilon}$	0.266	0.266
Average one period utility	1.398×10^{-3}	1.389×10^{-3}
Output cash goods	0.997ε	0.997ε
Output transfer goods	ε	ε
Probability a buyer meets a seller in one period	1	0.5
Equilibrium at 13 % annual interest rate:		
Biweekly apparent velocity	10.47	10.47
Average commercial margin	0.280	0.280
Break point $\widehat{\varepsilon}$	0.140	0.140
Average biweely utility	1.396×10^{-3}	1.388×10^{-3}
Output cash goods	0.989ε	0.989ε
Output transfer goods	ε	ε
Probability a buyer meets a seller in one period	1	0.5
Welfare cost of 10 inflation:		
Equivalent variation of consumption in percent	0.161	0.161
Extra DWL over extra seignoirage in percent	106	106

5 Conclusion

This paper makes two contributions. The first contribution is to advance a model of money, credit, and banking in a search theoretic framework with divisible money. In this model, cash and transfers of bank deposits can coexists as alternative means of payments. Cash earns no interest while deposits earn positive interest as long as inflation is above the Friedman rule. However, individuals are willing to use cash to purchase small ticket items because there is a small cost of transferring deposits.

The second contribution of the paper is to evaluate the quantitative importance of the various theoretical sources of the interest elasticity of the velocity of circulation of cash. When care is taken to adjust the official measure of cash for the fact that only a small fraction of cash is used in regular transactions, the apparent velocity of cash is high and the payment period is short. Consequently, inflation at a 10 percent annual rate has very small effects on the matching probabilities of buyers and sellers and the output exchanged in transactions paid with cash. Therefore, these effects accelerate the velocity of cash very little. In contrast, the effect of 10 percent inflation on the fraction of goods that are paid with cash is large, so it induces a realistically high interest elasticity of velocity of the circulation of cash.

Appendix

Characterization of $a_{+1\min}$

When the initial real wealth is $a_{+1\min}$ the individual must pay to the bank the maximum income from government transfers and from insured selling activities to maintain next period's real wealth at $\omega_{\min}$. Let ϑ be the real value tomorrow of the maximum cash obtainable as a fully insured seller. The lower bound on real wealth must obey:

$$a_{+1\min}M_{+1} = \tau + \vartheta M_{+1} + a_{+1\min}M\left(1+r\right) \tag{71}$$

Dividing both sides of this equation by M_{+1} and simplifying with the help of () and (7), we obtain

$$a_{+1\min} = -\frac{\frac{\gamma-1}{\gamma}+\vartheta}{\beta^{-1}-1}. \tag{72}$$

The value of ϑ is obtained by solving the following program:

$$\vartheta M_{+1} = \max_{\{q,\theta\}}\left(1-\theta\right)z \tag{73}$$

subject to

$$\theta\left[U\left(q\right)-\frac{\beta z}{M_{+1}}\right]-\frac{\beta z}{M_{+1}}r = S^{b} \tag{74}$$

$$q \leq \overline{q} \tag{75}$$

Solving for z in (74) and substituting in (73), we obtain an objective that is increasing in q. Therefore, the output sold is at the corner $q=\overline{q}$, and

$$\vartheta = \left\{\max_{\theta}\left(1-\theta\right)\frac{\theta U\left(\overline{q}\right)-S^{b}}{\theta+r}\right\}\frac{1}{\beta} \tag{76}$$

Proof of Proposition 1: Consider an arbitrary submarket 0 with $\left\{q^{0},z^{0},\theta^{0}\right\}$ that satisfies (18). Let S_{0}^{s} be the expected seller's surplus in submarket 0. Consider now submarket 1 where the seller posts a pricing schedule $Z^{1}\left(q^{1}\right)$ that satisfies (26) and lets the buyer choose q^{1}. The constant c in (26) is such that (18) holds for the same fraction of sellers θ^{0} as in submarket 0. Combining (18) and the definition of expected seller's surplus in (24), we obtain

$$S_0^s = \left(1-\theta^0\right)\left(r+\theta^0\right)\left\{\theta^0\left[U\left(q^0\right)-C\left(q^0\right)\right]-rC\left(q^0\right)-\overline{S}^b\right\}, \text{ and} \tag{77}$$

$$S_1^s = \left(1-\theta^0\right)\left(r+\theta^0\right)\left\{\theta^0\left[U\left(q^1\right)-C\left(q^1\right)\right]-rC\left(q^1\right)-\overline{S}^b\right\} \tag{78}$$

In submarket 1, the buyer maximizes the expression inside brackets in (78). Therefore, $S_1^s \geq S_0^s$.

Problem (27) is the dual of the seller's optimization problem upon substitution of (26)■

Second order conditions of (27)

Substituting the value of c obtained from (28 into (27), gives an objective that depends on two variables q^s and θ^s. The Hessian of this objective is:

$$\begin{bmatrix} \theta^s\left[U''\left(q^s\right)-C''\left(q^s\right)\right]-rC''\left(q^s\right) & \left[U'\left(q^s\right)-C'\left(q^s\right)\right] \\ \left(1-\theta^s\right)^2\left[U'\left(q^s\right)-C'\left(q^s\right)\right] & -2\left(1-\theta^s\right)\left[U\left(q^s\right)-C\left(q^s\right)\right] \end{bmatrix} \tag{79}$$

For the second order conditions of (27) to be satisfied, the matrix (79) must be negative semi-definite. Using (30) to simplify, this requires that for interior values of q^s and θ^s the following inequality holds:

$$2\left[U\left(q^s\right)-C\left(q^s\right)\right]\left\{\theta^s\left[U''\left(q^s\right)-C''\left(q^s\right)\right]-rC''\left(q^s\right)\right\}+\left(1-\theta^s\right)\left[\frac{rC'\left(q^s\right)}{\theta^s}\right]^2 \leq 0 \tag{80}$$

For all $q^s \in (0,\bar{q})$ and $\theta^s \in (0,1)$, first addend in (80) is negative while the second addend is positive. This second addend converges to zero as $r \rightarrow 0$, while first addend does not. Consequently, (80).

Equilibrium System (32) and (35)

The comparative statics of the system (32) and (35) are described by:

$$\begin{bmatrix} \theta\left(U''-C''\right)-rC'' & U'-C' \\ (2\theta-1)\left(U'-C'\right)-rC' & 2(U-C) \end{bmatrix}\begin{bmatrix} d \\ d\theta \end{bmatrix} = \begin{bmatrix} C'dr \\ Cdr \end{bmatrix} \tag{81}$$

Using (35) to simplify, the determinant of the 2x2 matrix in (81) is equal to:

$$\Delta_1 = 2(U-C)\left[\theta\left(U''-C''\right)-rC''\right]+(1-\theta)\left(U'-C'\right)^2 \tag{82}$$

The determinant Δ is negative if (80) is satisfied. Solving (81) and using (32) and (35) to simplify, we obtain

$$\frac{dq}{dr} = \frac{C' \left[2\theta \left(U - C\right) - rC\right]}{\Delta_1 \theta}, \text{ and} \tag{83}$$

$$\frac{d\theta}{dr} = \frac{\theta C \left[\theta \left(U'' - C''\right) - rC''\right] + \left(1 - \theta\right) r C'^2}{\Delta_1 \theta}. \tag{84}$$

The numerator in (83) is positive as long as the buyer's trading surplus is positive. The numerator in (84) is negative if r is sufficiently small.

Proof that (36) ensures that OC and MC cross in Figure 2

At $\theta = 1/2$, the output condition line OC is to the right of the market composition line MC. Therefore, the two lines cross if OC at $\theta = 1$ is not below MC. The output that solves (36) is by definition $\widetilde{q}^*$, which is the value of q on line OC at $\theta = 1$. Let $(\theta_1, \widetilde{q}^*)$ be the point of the line MC at $q = \widetilde{q}^*$. For OC and MC to cross, θ_1 must be not greater than 1. The inequality $\theta_1 \leq 1$ is equivalent to $2\theta_1 - 1 \leq 1$. Using (35), this is satisfied if and only if $U(\widetilde{q}^*) \geq C(\widetilde{q}^*)\left(1 + r\right)$, which is a restatement of (36).

Proof of Proposition 2

At $r \to 0$, equations (44) and (48) imply $q = q^*$ and $\theta = \theta^*$. If $r > 0$, equation (44) implies $0 < q < q^*$. The result $\theta > \theta^*$ is proved by contradiction. Suppose there is a positive interest rate r_0 for which $\theta_0 < \theta^*$. Given our assumptions about Π, this is equivalent to $\eta\left(\theta_0\right) > 0$. If the second order condition for an interior optimum of the program (42) is satisfied for all $r \in \left[0, r_0\right]$, θ is a continuous function of r in this interval. Therefore, there is an interest rate $r_1 \in [0, r_0]$ for which $\theta_1 \in [1/2, \theta^*]$ and $\eta\left(\theta_1\right) \geq 0$. This contradicts (48).

Equilibrium System (58) and (59)

The comparative statics of the system (58) and (59) are described by:

$$\begin{bmatrix} U\left(q^d_{\widehat{\varepsilon}}\right) - U\left(q^c_{\widehat{\varepsilon}}\right) & r \\ 0 & -\left[2 + F\left(\widehat{\varepsilon}\right)\right] \end{bmatrix} \begin{bmatrix} d\widehat{\varepsilon} \\ dS^s \end{bmatrix} = \begin{bmatrix} -\left[S^s + C\left(q^c_{\widehat{\varepsilon}}\right)\right] dr + d\phi \\ \left[\int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} \left[S^s + C\left(q^c_{\varepsilon}\right)\right] dF(\varepsilon)\right] dr + \left[1 - F\left(\widehat{\varepsilon}\right)\right] d\phi \end{bmatrix} \tag{85}$$

After some minor simplifications, the solution to this system is

$$\frac{d\widehat{\varepsilon}}{dr} = -\frac{r \int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} \left[C\left(q^c_{\widehat{\varepsilon}}\right) - C\left(q^c_{\varepsilon}\right)\right] dF(\varepsilon) + 2\left[S^s + C\left(q^c_{\widehat{\varepsilon}}\right)\right]}{\left[U\left(q^d_{\widehat{\varepsilon}}\right) - U\left(q^c_{\widehat{\varepsilon}}\right)\right]\left[2 + F\left(\widehat{\varepsilon}\right)\right]} < 0, \tag{86}$$

$$\frac{dS^s}{dr} = -\frac{\int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} \left[S^s + C\left(q^c_\varepsilon\right)\right] dF(\varepsilon)}{2 + F\left(\widehat{\varepsilon}\right)} < 0, \tag{87}$$

$$\frac{d\widehat{\varepsilon}}{d\phi} = \frac{2 + F\left(\widehat{\varepsilon}\right) + r\left[1 - F\left(\widehat{\varepsilon}\right)\right]}{\left[U\left(q^d_{\widehat{\varepsilon}}\right) - U\left(q^c_{\widehat{\varepsilon}}\right)\right]\left[2 + F\left(\widehat{\varepsilon}\right)\right]} > 0, \tag{88}$$

$$\frac{dS^s}{d\phi} = -\frac{1 - F\left(\widehat{\varepsilon}\right)}{2 + F\left(\widehat{\varepsilon}\right)} < 0. \tag{89}$$

Derivative of ζ with respect to S^s

$$\frac{d\zeta}{dS^s} = \frac{\left[1 - F(\widehat{\varepsilon})\right] \int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} C\left(q^c_\varepsilon\right) dF(\varepsilon) - F(\widehat{\varepsilon}) \int_{\widehat{\varepsilon}}^{1} C\left(q^d_\varepsilon\right) dF(\varepsilon)}{\left[S^s F(\widehat{\varepsilon}) + \int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} C\left(q^c_\varepsilon\right) dF(\varepsilon)\right]^2} \tag{90}$$

Using the function C is increasing, and both q^c_ε and q^d_ε are increasing functions of ε implies:

$$\frac{d\zeta}{dS^s} \leq \frac{\left[1 - F(\widehat{\varepsilon})\right] F(\widehat{\varepsilon}) \left[C\left(q^c_{\widehat{\varepsilon}}\right) - C\left(q^d_{\widehat{\varepsilon}}\right)\right]}{\left[S^s F(\widehat{\varepsilon}) + \int_{\underline{\varepsilon}}^{\widehat{\varepsilon}} C\left(q^c_\varepsilon\right) dF(\varepsilon)\right]^2} \leq 0 \tag{91}$$

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