Does the Time Inconsistency Problem Make Flexible Exchange Rates Look Worse Than You Think?*

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Abstract

Lack of commitment in monetary policy leads to the well known Barro-Gordon inflation bias. We show that, absent commitment, independent monetary policy can also induce expectation traps—welfare ranked multiple equilibria—and perverse policy responses to real shocks, i.e. the equilibrium policy response is welfare inferior to policy inaction. Both possibilities imply that macroeconomic volatility is larger under flexible exchange rates than under fixed exchange rates.

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1 Introduction

Arguably, the most influential case for fixed exchange rates has rested exclusively on the celebrated inflation bias of Barro and Gordon (1983).[1] A monetary authority lacking the credibility to commit to a policy, the logic goes, can peg its currency and import the monetary policy of another country with more credible institutions. Of course, this argument assumes that the exchange rate regime is a credible commitment even if monetary policy is not. The textbook argument against fixed exchange rates follows the lines of the classic Mundell-Flemming analysis. A fixed exchange rate means no independent monetary policy and therefore no ability to ease real macroeconomic volatility. This roughly summarizes the conventional wisdom about the costs and benefits of an exogenous commitment to a fixed exchange rate.[2]

We argue that two phenomena associated with the time inconsistency problem have been overlooked in the exchange rate debate. First, the lack of commitment can induce *expectation traps*, i.e. welfare ranked multiple equilibria, even in finite horizon economies.[3] Second, we show that real shocks can exacerbate the time inconsistency problem. As a result, the equilibrium policy response to these shocks can be worse than policy inaction—we label this as *perverse policy response*.

Expectation traps and perverse policy responses are the reasons why we answer positively to the question posed in the title. Contrary to the standard view, flexible exchange rates may feature larger macroeconomic volatility than fixed exchange rates. First, in the presence of expectation traps, independent monetary policy may react unnecessarily to shifts in expectations. Second, the Mundell-Flemming argument does not necessarily hold if the monetary authority lacks credibility. The possibility of perverse policy responses implies that not only may an independent monetary policy fail to ease macroeconomic volatility, but it may even magnify it.

We also show that expectation traps can be ruled out with a soft exchange rate peg with appropriately chosen bands. However, in order to avoid perverse policy responses a hard exchange rate peg is required.

We present a tractable model of a small open economy built upon Armenter and Bodenstein (2004). Nominal rigidities introduce a role for active monetary policy. Combined with

[1] See Obstfeld and Rogoff (1996) and references herein.

[2] Of course there are other persuasive macroeconomic arguments in favor of fixed exchange rates, as the well known “fear of floating” of Calvo and Reinhart (2002). See also Arellano and Heathcote (2003) who argues that dollarization can provide better access to financial markets.

[3] Chari, Christiano and Eichenbaum (1998) originally introduced the term. There is a growing literature on multiple equilibria with discretionary monetary policy, e.g., Albanesi, Chari and Christiano (2003), Armenter (2004), King and Wolman (2004), Armenter and Bodenstein (2004), and Siu (2004).

the monopoly distortion, nominal rigidities also set the stage for optimal monetary policy to be time inconsistent. We define three policy equilibrium concepts, where monetary policy is endogenously determined as the outcome of a benevolent policymaker. In the analysis of flexible exchange rates, we focus on Markov equilibria featuring independent monetary policy without commitment.[4] We are also interested in the optimal monetary policy with commitment, which is formalized as the Ramsey equilibrium. Finally, we define policy equilibria under the constraint of an arbitrary exchange rate regime. The policymaker takes the exchange rate regime as given and it therefore constitutes an exogenous commitment device for monetary policy.

We show that there are expectation traps in an economy calibrated to match several stylized facts on inflation and openness.[5] We find two Markov equilibria, which we label low and high inflation equilibrium. Hence, under flexible exchange rates, the monetary authority can unwillingly be caught in a high inflation equilibrium for long periods, and shifts in expectations can induce unnecessary macroeconomic volatility.

Expectation traps increase the costs of the lack of commitment by a significant amount. In our calibrated economy, the welfare loss of a shift in expectations from the low to the high inflation equilibrium is about three times the welfare change from implementing the optimal monetary policy in place of the low inflation equilibrium. A soft exchange rate peg with appropriately chosen bands is sufficient to rule out expectation traps without hindering the ability of the monetary authority to respond to macroeconomic shocks.

We illustrate the perverse policy response phenomenon with a negative terms of trade shock. The shock contracts the sector of tradeables, which makes the whole economy less competitive and therefore it increases the time inconsistency problem. The heightened monopoly distortion raises the incentives of the monetary authority to inflate. In equilibrium, private sector inflation expectations rise, leading monetary policy away from the optimal response to the shock.

In our calibrated economy, the policy response in a Markov equilibrium overshoots the optimal response by a factor of ten. Households prefer no policy response—the outcome of a fixed exchange rate—to the Markov equilibrium policy response. Hence, a flexible exchange rate fails to provide the macroeconomic stability which is presumed to be its main upside. Due to concavity, a positive terms of trade shock does not outweigh the welfare losses of a negative shock under the flexible exchange rate regime.

We do not attempt to establish, theoretically or empirically, that fixed exchange rates are welfare superior. Indeed, a definitive welfare ranking of exchange rate regimes may be more

[4] We label this equilibrium Markov because we focus on equilibria sustainable in finite horizon economies. This rules out equilibria based in trigger strategies.

[5] Markov equilibrium multiplicity is quite robust. Indeed, Armenter (2004) shows that the conditions for the existence of expectation traps in monetary policy are very general.

elusive than ever. However, we see clear implications for the exchange rate policy debate. For example, the recent literature on dollarization has dealt with the time inconsistency problem.[6] However, to the best of our knowledge, none considered expectation traps or perverse policy responses. This omission renders any welfare analysis incomplete.

The results of this paper also imply that we should treat with caution some of the arguments made lately in favor of flexible exchange rates. For example, the observed fall of inflation rates worldwide should not be taken as conclusive evidence that "the credibility consideration seems to be less compelling than it once was for emerging markets" as Chang and Velasco (2000) claim. The multiplicity of equilibria implies that all what is needed is a shift in expectations to be back into high inflation.

Moreover, larger real volatility does not necessarily make a stronger case for flexible exchange rate regimes. Summarizing the state of the debate, Frankel (1998) assert that "if the country is subject to many external disturbances, [...] then it is more likely to want to float its currency." Chang and Velasco (2000) also conclude the case for exchange-rate flexibility is "especially strong for countries that are often hit by large real shocks from abroad." In the light of our analysis, it is necessary to check that the relevant real shocks do not induce perverse policy responses. Otherwise, more real volatility makes the case for hard exchange rate pegs stronger—indeed, our example with terms of trade shocks is suggestive of this possibility in developing economies.

The remainder of the paper is organized as follows. In Section 2 we present our model and define the equilibrium concepts. Section 3 discusses expectation traps and Section 4 takes upon the possibility of perverse policy responses. Section 5 concludes. An Appendix, containing several proofs as well as calibration details, is included.

2 The Economy

First, we characterize the private sector equilibrium, which includes a detailed description of the economy. Then we define the different policy equilibrium concepts considered: Markov equilibrium, Ramsey equilibrium and Exchange Rate policy equilibrium.

2.1 Private Sector Equilibrium

This infinite-horizon small open economy is populated by a representative household, a representative final good firm, a continuum of intermediate good firms and a monetary authority.

[6] A small sample are Chang and Velasco (2002), Cooley and Quadrini (2001) and Mendoza (2001).

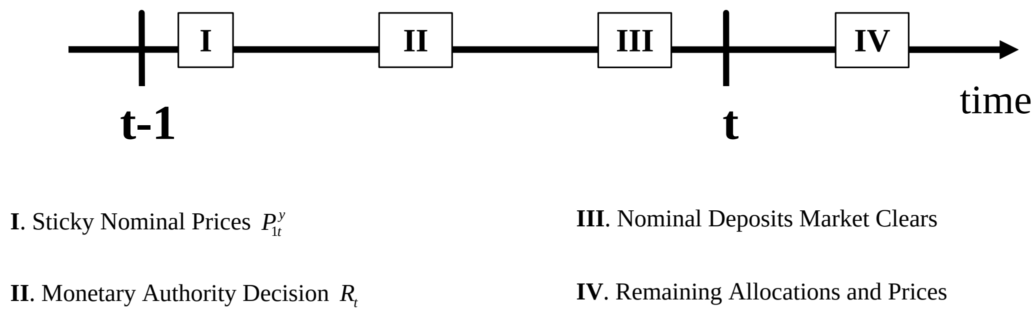

Figure 1: Timing of Relevant Decisions for Date t

Figure 1 illustrates the timing of the model. Several of the relevant decisions for period t are taken one period in advance. First, a fraction of the intermediate good firms—the sticky price firms—set their nominal price for period t, P^y_{1t}, at the beginning of period $t-1$. The monetary authority then chooses its policy to maximize the representative household's welfare taking P^y_{1t} as given.

We assume that the monetary policy instrument is the nominal interest rate, R_t, to be paid at date t on nominal deposits carried from period $t-1$. Both the supply and the demand of nominal deposits respond to the monetary policy decision. In particular, the nominal interest rate is implemented by means of a monetary transfer, X_{t-1}, that clears the nominal deposits market. We show below that in our model the monetary authority can implement any desired inflation rate at date t, π_t, within some feasibility bounds.[7] From now on, we will think of inflation as the policy instrument.

All remaining allocations and prices are determined at date t. Monetary policy as well as nominal sticky prices set at date $t-1$ are taken as given by private agents at date t.

Sticky price firms set their nominal price P^y_{1t} according to an expectation about inflation in period t, denoted $\hat{\pi}_t$, and given the aggregate price index at date $t-1$, P_{t-1}. In the private sector equilibrium, real prices and allocations at date t are fully determined by $s_t = (\hat{\pi}_t, \pi_t)$. Note that neither past nor future policy decisions are included and there is no physical state

[7] Below we detail the monetary authority's problem. The lower feasibility bound is given by the zero nominal interest bound. There is also an upper bound which we prove to never be binding.

variable in the economy. This enables us to study the monetary authority's decision as a sequence of static problems.

We do not model money directly. Implicitly, nominal deposits are as good as cash balances. This feature of the model allows us to abstract from money demand considerations and to focus on nominal frictions on the supply side of the economy.[8]

2.1.1 Households

Household preferences at date t are given by

$$\sum_{j=t}^{\infty} \beta^{j-t} u\left(c_j, n_j\right)$$

with $0 < \beta < 1$, and

$$u\left(c, n\right) = c + h\left(1 - n\right).$$

where h is a strictly increasing, concave function that satisfies the usual Inada conditions. The separability of leisure as well as the linearity in consumption are instrumental for tractability.

We express the household problem in recursive form

$$v(D, s) = \max_{c,n,D'} u\left(c, n\right) + \beta v\left(D', s'\right) \tag{1}$$

subject to

$$\begin{aligned} c &\geq 0 \\ 0 &\leq n \leq 1 \end{aligned}$$

and

$$P\left(s\right) c + D' \leq R\left(s\right) D + W\left(s\right) n + T^f\left(s\right) \tag{2}$$

where D are nominal deposits, which pay a nominal interest rate R, and T^f are lump sum transfers from firms. Nominal deposits D are the unique asset holdings of the household and $s = (\hat{\pi}, \pi)$ is the economy-wide state.

Because both sticky prices and the actual policy choice are set one period in advance, the household does not need to create any belief about s'—the actual next period state is common knowledge. For tractability, we assume that no intertemporal claims are traded internationally. Following the recursive formulation, time subscripts are dropped for the rest of the paper.

[8] In the spirit of the cashless economies discussed in Woodford (2003).

In order to pin down nominal prices, we normalize the last period final good price index to 1. Hence, the price level is equal to the inflation rate, $P(s) = \pi$.

The equilibrium labor supply is characterized by the first order condition

$$-\frac{u^n(s)}{u^c(s)} = w(s)$$

which implies

$$h'(1 - n(s)) = w(s) \tag{3}$$

where $w(s) = \frac{W}{P}$. Note that the concavity of h implies that the labor supply has a positive slope everywhere.

For the nominal deposit decision, we have the following arbitrage condition

$$R' = \frac{1}{\beta}\pi'$$

where $\pi' = \frac{P'}{P}$ and we used the envelope theorem and established that $\frac{dv}{dR}(D, s) = R$. This is a simple Fischer equation. It includes the actual rather than the expected inflation rate because households can adjust nominal deposits after the nominal interest rate has been set. Cash-credit good models—as Lucas and Stokey (1983)—share this feature. In our model, though, the nominal interest rate is an intertemporal price.

Note that the policy choice for date t is bound by the relationship between the nominal interest rate to be paid at date t with the rate of inflation at date t given by

$$R(s) = \frac{1}{\beta}\pi \tag{4}$$

Expression (4) will belong to the date t private sector equilibrium. It follows that the monetary policy can think of inflation as its policy choice.

2.1.2 Firms

There is a continuum $I = [0, 1]$ of intermediate goods. A **representative final good firm** combines a continuum $[0, 1 - \mu_x]$ of intermediate inputs $y(i)$ to produce the final good $y(s)$ according to

$$y(s) = \left[\int_0^{1-\mu_x} y_i(s)^\eta \, di\right]^{\frac{1}{\eta}} \tag{5}$$

with $\eta < 1$.

The final good firm profit-maximization problem is

$$\max_{c,\{y_i\}_0^{1-\mu_x}} P(s)\,c - \int_0^{1-\mu_x} P_i^y(s)\,y_i di$$

subject to (5).

Using the first order conditions, we compute the demand good y_i

$$p_i^y(s) = \left(\frac{y(s)}{y_i(s)}\right)^{1-\eta} \tag{6}$$

where, again, $p_i^y(s) = \frac{P_i^y(s)}{P(s)}$.

There are several types of intermediate goods. There is a fraction α of non-tradeable inputs and a fraction $(1-\alpha)$ of tradeable inputs. Each fraction subdivides in different types of firms, as detailed below.

Non-tradeable intermediate goods are produced in a classic monopolistic competition setting. Each good is produced by a single firm i according to a simple linear technology

$$y_i(s) = \theta_i n_i(s)\,.$$

There are three types of intermediate good firms in the non-tradeable input sector. Let μ_i denote the measure of firms of type i, with $\mu_1 + \mu_2 + \mu_3 = \alpha$. We assume that, within each type, the firm's decisions are symmetric.

Firms type $i = 1$ are sticky price setters. As discussed earlier, the nominal price of the intermediate good produced by firms $i = 1$ must be set before the monetary authority sets its policy choice. As a consequence, the nominal price $P_1^y(\hat{\pi})$ is a function of the private sector inflation expectations, $\hat{\pi}$, but not of the actual inflation π. As all intermediate good firms, the sticky price firms take in account the demand function for its own good, y_i.

Given our specification for the demand of each good i, (6), profit maximization results in the classic markup over marginal cost equal price rule

$$P_1^y(\hat{\pi}) = \frac{1}{\eta}\frac{\hat{W}}{\theta_1}$$

where $\hat{W}$ is the expected nominal wage. Rational expectations require that $\hat{W}$ is the equilibrium nominal wage under the expectation that $\hat{\pi}$ is the actual policy choice, i.e.

$$P_1^y(\hat{\pi}) = \frac{1}{\eta}\frac{w(\hat{\pi},\hat{\pi})}{\theta_1}\hat{\pi} \tag{7}$$

where $w(\hat{\pi}, \hat{\pi})\hat{\pi}$ is the nominal wage—recall that the last period price index is normalized to 1 so $P(s) = \pi$.

Firms type $i = 2$ are flexible price setters as they can set the nominal price $P_2^y(s)$ after the monetary authority decision, and hence it is a function of both π and $\hat{\pi}$. Firms of type $i = 2$ are financially constrained: we assume they have to borrow the nominal wage bill Wn at nominal rate $R(s)$. These firms provide the demand side for the household deposits.[9] Their optimal pricing rule is

$$p_2^y(s) = \frac{1}{\eta} R(s) \frac{w(s)}{\theta_2} \tag{8}$$

so the fact that their marginal cost is augmented by $R(s)$ is reflected in the real price. The principal feature is that the financial friction is increasing with actual inflation. We conjecture there are several alternatives to our modelling choice with the same key feature.

Finally, firms type $i = 3$ are flexible price setters and financially unconstrained. Therefore we have

$$p_3^y(s) = \frac{1}{\eta} \frac{w(s)}{\theta_3}. \tag{9}$$

Note that if the expectation and the actual inflation rate are the same, $\hat{\pi} = \pi$, and productivity parameters are identical across firm types, (7) and (9) imply that prices are the same across sticky and non-financially constrained flexible price firms, i.e., $p_1^y(\pi, \pi) = p_3^y(\pi, \pi)$ and as a result $y_1(s) = y_3(s)$. Moreover, if $R(\pi, \pi) = 1$, then prices and production for all firms are equal. Because the production function for the final good (5) is concave, symmetry across firm types is a necessary condition for production efficiency. In other words, $R(s) > 1$ and $\hat{\pi} \neq \pi$ introduce costly price distortions.

The **tradeable intermediate good sector** is composed of export and import firms. There is a measure μ_x of export firms, which produce domestically and they supply exclusively to the world markets. We assume that the country's export goods are not differentiated and the export price is determined in the world markets. Hence, the industrial organization in the export sector is characterized by perfect competition.

Their production function is

$$y_x(s) = \theta_x n_x(s).$$

The optimization condition equates the marginal cost to the price

$$p_x(s) = \frac{w(s)}{\theta_x}. \tag{10}$$

[9] Note that the deposit demand is also determinated with knowledge of the actual policy choice π.

In addition, the law of one price equates the domestic nominal price to the world market price for x, P_x^*,

$$P_x(s) = \varepsilon(s) P_x^*$$

where $\varepsilon(s)$ is the nominal exchange rate. In terms of real prices,

$$p_x(s) = q(s) p_x^* \tag{11}$$

where $q(s) = \frac{\varepsilon(s)P^*}{P(s)}$, where P^* is the world price for the final good. We follow the same normalization than for the domestic final good price as we set the last period world final good price equal to one. Hence

$$q(s) = \frac{\varepsilon(s)\pi^*}{\pi} \tag{12}$$

where π^* is the world rate of inflation.

Import firms do not produce domestically: they simply buy $y_m(s)$ from the world markets. Import prices are taken as given, hence

$$p_m(s) = q(s) p_m^*. \tag{13}$$

Imports constitute a measure μ_m of total tradeable inputs, with $\mu_x + \mu_m = 1 - \alpha$.

Because there is no trade in intertemporal assets with the rest of the world, the value of imports and exports must be equated every period,

$$\mu_m y_m(s) = \mu_x tt y_x(s) \tag{14}$$

where $tt = \frac{P_x^*}{P_m^*}$ are the terms of trade.

2.1.3 Market Clearing Conditions and Private Sector Equilibrium Definition

The aggregate resource constraint is

$$c(s) = \left[\sum_{i=1}^{3} \mu_i (\theta_i n_i(s))^\eta + \mu_m (\theta_m n_m(s))^\eta\right]^{\frac{1}{\eta}} \tag{15}$$

where (5) has been combined with each intermediate good production technology and $c(s) = y(s)$. The market clearing condition for the labor market is

$$n(s) = \sum_{i=1}^{3} \mu_i n_i(s) + \mu_x n_x(s) \tag{16}$$

Equations (3)-(16) are sufficient to solve for all real prices and allocations as function of expected and actual inflation. This confirms our conjecture that $s = (\hat{\pi}, \pi)$ fully characterizes the economy. We proceed now to define a Private Sector Equilibrium (PSE) given $\hat{\pi}$ as a collection of allocation and price functions defined over π and a sticky nominal price $P_1^y(\hat{\pi})$.

Definition 1 *Given an inflation rate expectation $\hat{\pi}$, a* ***Private Sector Equilibrium*** *is a number, $P_1^y(\hat{\pi})$, and a collection of functions, $\{p_i^y(s), y_i(s), n_i(s)\}_{i\in I}$, $R(s)$, $w(s)$, $n(s)$, $c(s)$, $\varepsilon(s)$, $q(s)$ and $y(s)$, such that*

1. *The household optimal conditions, (3) and (4), are satisfied.*
2. *Firm maximize profits, i.e.,(7)-(10) are satisfied.*
3. *Markets clear, i.e., (5), (6) and (12)-(16) hold.*

A ***Private Sector Equilibrium outcome*** *indexed by $s = (\hat{\pi}, \pi)$ is the collection of allocations and prices result of a PSE given $\hat{\pi}$ evaluated at π.*

Our definition of the PSE is sufficient to characterize the monetary authority problem. Note that nominal prices, deposits and monetary transfers are not included in the PSE. Now we show how to characterize these and why they are not relevant for the monetary authority problem.

It is straightforward to recover all nominal prices, as under our normalization, $\pi = P(s)$. The nominal deposit market clearing condition is

$$D = W(s) \int_{I_2} n_i(s)\, di - X(D, s) \tag{17}$$

where $X(D, s)$ are monetary transfers by the monetary authority. For any level of nominal deposits D and state s, there is $X(D, s)$ that clears the nominal deposits market. Hence for any D and $\hat{\pi}$, the monetary authority can implement its policy decision in terms of inflation by setting $X(D, s)$ correspondingly.

Finally, the household budget constraint (2) gives a law of motion for nominal deposits, $D' = R(s)\, D$. Because of the zero nominal interest rate bound and a feasibility upper bound on inflation, the sequence for nominal deposits is well defined for every t with no impact on the private sector equilibrium allocations.

2.1.4 Solving for the PSE

We start by taking P_1^y, a number, as given. Then we solve for the PSE functions mappings the actual inflation rate π into allocations and prices. Using these PSE functions, we will characterize the sticky price firms decision as function of the expected inflation rate, $P_1^y(\hat{\pi})$.

From the Fischer equation (4), we trivially link the nominal interest rate and inflation:

$$R(s) = \frac{\pi}{\beta}.$$

The relative price of sticky price firm goods is given by

$$p_1^y(s) = \frac{P_1^y(\hat{\pi})}{\pi}$$

Recall we normalize $P_{-1} = 1$, so $\pi = P$, to resolve the level nominal indeterminacy.

Next we solve for relative prices,

$$\frac{y_i(s)}{y_j(s)} = \left[\frac{p_j^y(s)}{p_i^y(s)}\right]^{\frac{1}{1-\eta}}$$

combining the demand function (6) for two given goods i and j.

Using the pricing formulas (7)-(13),

$$\begin{aligned}
\frac{y_1(s)}{y_3(s)} &= \left[\frac{1}{\eta\theta_3}\frac{w(s)}{p_1^y(s)}\right]^{\frac{1}{1-\eta}} \\
\frac{y_2(s)}{y_3(s)} &= \left[\frac{\theta_2}{\theta_3}R(s)^{-1}\right]^{\frac{1}{1-\eta}} \\
\frac{y_x(s)}{y_3(s)} &= \left[\frac{\theta_x}{\eta\theta_3}\right]^{\frac{1}{1-\eta}} \\
\frac{y_m(s)}{y_3(s)} &= \frac{\mu_x}{\mu_m}tt\frac{y_x(s)}{y_3(s)}
\end{aligned}$$

where the latest equality is derived using (14).[10] We combine the expressions with (5) to obtain

$$\frac{y(s)}{y_3(s)} = \left[\mu_3 + \mu_2\left[\frac{\theta_2}{\theta_3 R(s)}\right]^{\frac{\eta}{1-\eta}} + \mu_1\left[\frac{w(s)}{\eta\theta_3 p_1^y(s)}\right]^{\frac{\eta}{1-\eta}} + \mu_m\left(\frac{\mu_x}{\mu_m}tt\right)^{\eta}\left[\frac{\theta_x}{\eta\theta_3}\right]^{\frac{\eta}{1-\eta}}\right]^{\frac{1}{\eta}}. \tag{18}$$

[10] It can also be calculated by solving for the real exchange rate using (12).

Next, we use the pricing formula and demand for the intermediate good $i = 3$,

$$\left[\frac{w(s)}{\eta\theta_3}\right]^{\frac{\eta}{1-\eta}} = \mu_3 + \mu_2\left[\frac{\theta_2}{\theta_3 R(s)}\right]^{\frac{\eta}{1-\eta}} + \mu_1\left[\frac{w(s)}{\eta\theta_3 p_1^y(s)}\right]^{\frac{\eta}{1-\eta}} + \mu_m\left(\frac{\mu_x}{\mu_m}tt\right)^{\eta}\left[\frac{\theta_x}{\eta\theta_3}\right]^{\frac{\eta}{1-\eta}}$$

where the real wage rate can be explicitly solved for

$$w(s) = \eta\left[\frac{\tilde{\mu}_3 + \tilde{\mu}_2 R(s)^{\frac{\eta}{\eta-1}} + \tilde{\mu}_m\eta^{\frac{\eta}{\eta-1}}}{1 - \mu_1 p_1^y(s)^{\frac{\eta}{\eta-1}}}\right]^{\frac{1-\eta}{\eta}} \tag{19}$$

where

$$\tilde{\mu}_i = \mu_i\theta_i^{\frac{\eta}{1-\eta}}$$

for $i = 1, 2, 3$ and

$$\begin{aligned}\tilde{\mu}_m &= \mu_m\left(\frac{\mu_x}{\mu_m}tt\right)^{\eta}\theta_x^{\frac{\eta}{1-\eta}},\\ \tilde{\mu}_x &= \mu_x\theta_x^{\frac{\eta}{1-\eta}}.\end{aligned}$$

This expression is the key to solve for the PSE. With knowledge of $w(s)$, the rest of equilibrium allocations and prices follow. Labor $n(s)$ is given by (3). To pin down output, use (16) to derive

$$\frac{n(s)}{y_3(s)} = \frac{\mu_3}{\theta_3} + \frac{\mu_2}{\theta_2}\left[\frac{\theta_2}{\theta_3 R(s)}\right]^{\frac{1}{1-\eta}} + \frac{\mu_1}{\theta_1}\left[\frac{w(s)}{\eta\theta_3 p_1^y(s)}\right]^{\frac{1}{1-\eta}} + \frac{\mu_x}{\theta_x}\left[\frac{\theta_x}{\eta\theta_3}\right]^{\frac{1}{1-\eta}}$$

and combining the last expression with (18)

$$y(s) = \varphi(s)\,n(s)$$

where

$$\varphi(s) = \frac{\left[\tilde{\mu}_3 + \tilde{\mu}_2 R(s)^{\frac{\eta}{\eta-1}} + \mu_1\left[\frac{w(s)}{\eta p_1^y(s)}\right]^{\frac{\eta}{1-\eta}} + \tilde{\mu}_m\eta^{\frac{\eta}{\eta-1}}\right]^{\frac{1}{\eta}}}{\tilde{\mu}_3 + \tilde{\mu}_2 R(s)^{\frac{1}{\eta-1}} + \frac{\mu_1}{\theta_1}\left[\frac{w(s)}{\eta p_1^y(s)}\right]^{\frac{1}{1-\eta}} + \tilde{\mu}_x\eta^{\frac{1}{\eta-1}}}.$$

The numerator is also equal to $\left[\frac{w(s)}{\eta}\right]^{\frac{1}{1-\eta}}$.

To close the PSE, it is still needed to solve for $P_1^y(\hat{\pi})$. Given expectations $\hat{\pi}$, (7) implies that $P_1^y(\hat{\pi})$ will satisfy $p_1^y(s) = \frac{\theta_3}{\theta 1} p_3^y(s)$. This allows to write the real wage rate when $\pi = \hat{\pi}$ as

$$w(\hat{\pi}, \hat{\pi}) = \eta \left[\tilde{\mu}_3 + \tilde{\mu}_2 R(s)^{\frac{\eta}{\eta-1}} + \tilde{\mu}_1 + \tilde{\mu}_m \eta^{\frac{\eta}{\eta-1}} \right]^{\frac{1-\eta}{\eta}} \tag{20}$$

and hence, using (7) again,

$$P_1^y(\hat{\pi}) = \hat{\pi} \frac{w(\hat{\pi}, \hat{\pi})}{\eta \theta_1}.$$

It can be easily shown that $P_1^y(\hat{\pi})$ is increasing in the expected inflation. The real wage is never depressed by inflation so much that more expected inflation leads to lower nominal sticky prices.

2.2 Policy Equilibria

We view the policy decision as an equilibrium object in this economy. We entertain three different policy equilibrium concepts: the Markov equilibrium, the Ramsey equilibrium and the Exchange Rate Policy equilibrium.

In the Markov equilibrium, the monetary authority maximizes household welfare taking sticky prices as given and validates the private sector expectations on inflation. A feature of the Markov equilibrium is that equilibria featuring trigger strategies, which arise in a Nash equilibrium, are ruled out.

The Ramsey equilibrium characterizes the optimal monetary policy with commitment. A formal definition is given below but the reader can think of the Ramsey equilibrium as the result of switching the timing: the monetary policy is set once and for all before the sticky price decisions.

Finally, the Exchange Rate Policy (ERP) equilibrium is designed to capture the possibility that the monetary authority decision is constrained by some exchange rate policy. The latter is taken as an exogenous commitment.

2.2.1 The Markov Equilibrium

The monetary authority problem consists of choosing the inflation rate that maximizes household welfare. The monetary authority takes the nominal price $P_1^y(\hat{\pi})$ as given: it can not manipulate the sticky price firm expectations.

The choice of the inflation rate is constrained as follows. First, the nominal interest rate is bounded below by one, i.e., $R(s) \geq 1$. This bound is implied by arbitrage between nominal bonds and cash balances, which are not explicitly modelled. Using (4), the lower bound for inflation is the intertemporal discount rate, $\pi \geq \beta$.

The existence of a PSE outcome also imposes an upper bound $\bar{\pi}(\hat{\pi})$ on the inflation rate as well. This upper bound depends on the private sector inflation expectation. Basically, as π approaches $\bar{\pi}$, the sticky price firms have unbounded losses as their real price depreciates and the real wage rate explodes.[11]

Proposition 2 *For any $\hat{\pi} \geq \beta$, a PSE outcome exists for all π such that*

$$\pi < \bar{\pi}(\hat{\pi}) = \hat{\pi} P_1^y(\hat{\pi}) \mu_1^{\frac{\eta-1}{\eta}}.$$

Proof. As long as we have a finite, strictly positive real wage rate, a PSE outcome exists. From (19), $B \geq w(s) > 0$ implies that

$$\left(1 - \mu_1 p_1^y(s)^{\frac{\eta}{\eta-1}}\right)^{\frac{\eta-1}{\eta}} > 0$$

The above restriction can be rewritten

$$p_t^y(s) > \mu_1^{\frac{1-\eta}{\eta}}$$

or in terms of π and $\hat{\pi}$,

$$\pi < \bar{\pi}(\hat{\pi}) = \frac{P_1^y(\hat{\pi})}{\mu_1^{\frac{1-\eta}{\eta}}}$$

■

In Armenter and Bodenstein (2004), we show that the policy choice set can be defined without any loss of generality as

$$\beta \leq \pi \leq \bar{\pi}(\hat{\pi}) - \varepsilon$$

for an arbitrarily small $\varepsilon > 0$. First, the upper bound will never be binding. Second, we prove that the policy choice set is never empty as $\bar{\pi}(\hat{\pi}) > \beta$ for all $\hat{\pi} \geq \beta$.

Because a PSE outcome fully determines the household period welfare, we can state the monetary authority problem as an intratemporal optimization problem. In the Markov equilibrium, the monetary authority maximizes household welfare taking the private sector expectation, $\hat{\pi}$, as given. The formalization of the problem is

$$\max_{\beta \leq \pi < \bar{\pi}(\hat{\pi})} u(c(s), n(s)) \tag{21}$$

where $c(s)$ and $n(s)$ belong to a PSE given $\hat{\pi}$.

All is set for the definition of a Markov equilibrium.

[11] Whether we allow for negative profits or not has no impact in our results. Ruling out negative profits will impose a tighter upper bound on the inflation rate. In practice, the monetary authority decision π can be characterized by looking in a small neighborhood of expected inflation $\hat{\pi}$. See Armenter and Bodenstein (2004) for a discussion of the issue.

Definition 3 *A* ***Markov equilibrium*** *is an inflation rate* π *and a PSE given private sector expectations* $\hat{\pi}$ *such that* π *solves (21) given* $\hat{\pi}$ *and private sector expectations are rational*

$$\hat{\pi} = \pi.$$

The definition is of a *period* Markov equilibrium. It is straightforward to extend it to the infinite horizon economy and we will dispense the reader of the definition.

2.2.2 The Ramsey Equilibrium

The Ramsey equilibrium is the correspondent equilibrium concept with commitment. The monetary authority pins down private sector expectations with the policy decision. Because the monetary authority is a benevolent policymaker, the Ramsey equilibrium also characterizes the optimal monetary policy.

Definition 4 *A Ramsey Equilibrium is an inflation rate* π^r *and a PSE given* π^r *such that for all* π*,*

$$u\left(c\left(\pi^r, \pi^r\right), n\left(\pi^r, \pi^r\right)\right) \geq u\left(c\left(\pi, \pi\right), n\left(\pi, \pi\right)\right)$$

where $c\left(s\right)$ *and* $n\left(s\right)$ *are respective PSE functions.*

Not surprisingly, the Ramsey equilibrium turns out to be characterized by the Friedman rule. By setting the nominal interest rate $R\left(s\right) = 1$, all relative price distortions are zeroed. The distortion arising from the monopolistic price setting remains—but there is nothing a deterministic monetary policy rule can do to curtail the market power of the intermediate good firms. The labor supply remains inefficiently low.

Proposition 5 *The Ramsey equilibrium features* $R\left(s\right) = 1$*.*

Proof. Consider functions $\tilde{\varphi}\left(\pi\right) = \varphi\left(\pi, \pi\right)$ and $\tilde{w}\left(\pi\right) = w\left(\pi, \pi\right)$. Simple algebra shows that $\tilde{\varphi}$ and $\tilde{w}$ are decreasing in π, and $\tilde{\varphi}\left(\pi\right) \geq \tilde{w}\left(\pi\right)$ for all $\pi \geq \beta$. Next we show that the household welfare is increasing in φ and w. Let

$$\tilde{u}\left(\varphi, w\right) = \varphi \tilde{n}\left(w\right) + h\left(1 - \tilde{n}\left(w\right)\right)$$

where $\tilde{n}\left(w\right)$ is given by (3). It is clear that $\tilde{u}$ is increasing in $\tilde{\varphi}$. Moreover,

$$\frac{d\tilde{u}}{dw} = \left(\varphi - w\right) \frac{d\tilde{n}}{dw}$$

so given that $\varphi > w$ and the labor supply has an upward slope, household welfare is also increasing in the wage. Hence any policy choice $\pi > \beta$ is welfare dominated by $\pi = \beta$ ■

Does the Friedman Rule constitute a Markov Equilibrium? Assume private sector expectations are $R(\hat{\pi}, \hat{\pi}) = 1$, i.e., sticky nominal prices were set under the belief that the Friedman Rule would be chosen by the monetary authority. Ex-post, the monetary authority can choose to set $\pi > \beta$ and cut the markup of the sticky price firms. However, such a move creates price distortions. The price difference between the sticky and flexible price firm goods is welcome as it reflects the improved efficiency in the sticky price firms good production. However, there is an additional price distortion, as financially constrained firms have their marginal cost augmented by $R(s)$. This implies lower efficiency in the production of financially constrained firm goods. Hence, at least on the margin, whether the Friedman Rule is a Markov Equilibrium depends on the relative weight of each distortion, as indexed by the relative size of the sticky price firm sector to the financially constrained firm sector.

2.2.3 The Exchange Rate Policy Equilibrium

In the ERP equilibrium, the monetary authority takes the private sector expectations as given and maximizes household welfare—as in the Markov equilibrium—but its choice is exogenously constrained by a condition on the nominal exchange rate.

The exchange rate policy is formalized as a constraint on the set of feasible nominal exchange rate, Σ. A fixed exchange rate regime reduces Σ to a singleton $\bar{\varepsilon}$, $\Sigma = \{\bar{\varepsilon}\}$. A soft exchange rate peg, within some bands, would be formalized as $\Sigma = [\bar{\varepsilon} - \delta_0, \bar{\varepsilon} + \delta_1]$. As we focus our analysis in commonly observed exchange rate regimes, we do not consider the possibility that the set Σ is history dependent.

The correspondent version of (21) is

$$\max_{\beta \leq \pi < \bar{\pi}(\hat{\pi})} u(c(s), n(s)) \tag{22}$$

subject to

$$\varepsilon(s) \in \Sigma$$

where $c(s)$, $n(s)$ and $\varepsilon(s)$ belong to a PSE given $\hat{\pi}$.

Definition 6 *An* ***Exchange Rate Policy*** Σ ***equilibrium*** *is an inflation rate π and a PSE given private sector expectations $\hat{\pi}$ such that π solves (22) given $\hat{\pi}$ and private sector expectations are rational*

$$\hat{\pi} = \pi.$$

Implicitly, we assumed that, ex-post, it is not possible to review the exchange rate policy decision. In other words, policymakers are able to commit to an exchange rate regime but not to a monetary policy.

3 Expectation Traps

The absence of commitment can lead to costly volatility due to self-fulfilling private sector expectations. Expectation traps, i.e. multiple Markov equilibria, are a robust property of this economy.[12]

3.1 Understanding Expectation Traps

To understand expectation traps, we first discuss the monetary authority's decision given private sector expectations. In this economy, the costs and benefits of inflation are driven by the very different impact of inflation across firm types. Inflation hinders overall efficiency by distorting relative prices and by increasing the cost of working capital for financially constrained firms. On the other hand, unexpected inflation erodes the markup of sticky price firms thereby improving efficiency. In a Markov equilibrium, costs and benefits from unexpected inflation are balanced.

Expectations traps arise because expected inflation changes the composition of the intermediate good sector. While the measure of each firm type is constant, inflation alters the relative output of sticky price firms and financially constrained firms.

Sector composition is a key determinant of the monetary authority's decision. When expected inflation is low, each type of firm operate at a similar scale. Production efficiency gains from unexpected inflation are almost zero: any cut in the markup of the sticky price firms is roughly offset by an increase in the marginal cost of the financially constrained firms. As a result, there are little net efficiency gains to outweigh the costs of price distortion. A low inflation equilibrium exists where the marginal cost of price distortion is low.

When the private sector expects high inflation, financially constrained firms operate at a reduced scale because of the large costs of nominal working capital. There are considerable net efficiency gains from unexpected inflation because the sticky price sector is relatively large compared to the financially constrained sector. These large efficiency gains can balance the higher marginal cost of price distortion. Hence, the monetary authority will find it optimal to validate the high inflation expectations.

Figure 2 displays the policy best response π^* as a function of private sector expectation $\hat{\pi}$. The 45-degree line (dashed) is the set of points where actual inflation equals expected inflation, $\pi = \hat{\pi}$. This is the rational expectation locus. Crossings of the policy best response function with the 45-degree line indicate Markov equilibria. We calibrated the economy displayed in figure 2 to match some stylized facts of the U.S. economy.[13] There are two Markov

[12] See Armenter and Bodenstein (2004) for an exhaustive exploration of expectation traps in a closed economy version of the model.

[13] In the Appendix we provide the details of our calibration.

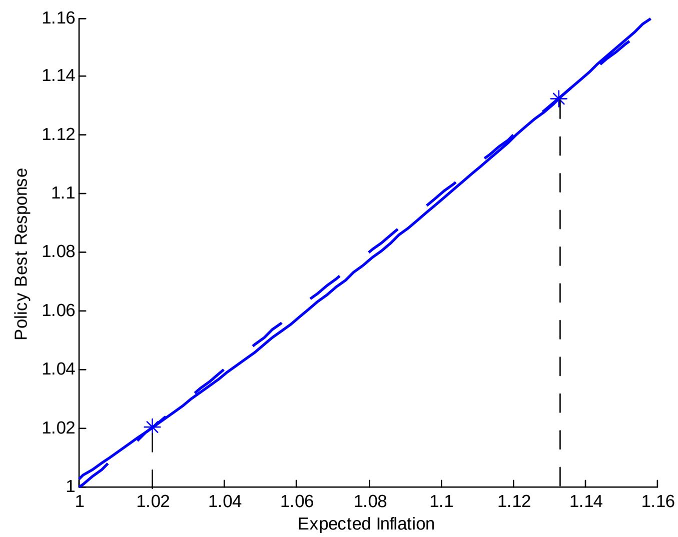

Figure 2: Expectation Traps in a Calibrated Economy

equilibria with inflation rates of 2% and 13.2%. As argued in Armenter and Bodenstein (2004), this matches the very different inflation experience of U.S. before and after the 80s.

The policy best response is close to linear but also very close to the 45-degree line. This explains why the economy features expectation traps even with small nominal frictions. In our calibration, over 80% of total firms experience no nominal friction.[14] Figure 2 shows that linear approximations of the policy best response function will deliver misleading results: the high inflation Markov equilibrium will be overlooked despite the high accuracy of the linear approximation.

An additional feature to note is that each Markov equilibrium is locally unique. This is easy to see from Figure 2. There will be non-locally unique equilibria only when the best policy response exactly coincides with the 45-degree line—quite obviously a non-generic case. Armenter (2004) shows that Markov equilibria are, generically, locally unique under very general conditions.

We argued that the changes in the composition of the intermediate good sector are behind the expectation traps. This is illustrated in Figure 3. Firm output for sticky price firms and

[14] There is an unfortunate flip side to this feature. The model is particularly sensitive to changes in the ratio of sticky price firms to financially constrained firms, μ_1/μ_2. For parametrizations with small nominal frictions, this means that even small changes in μ_2 can lead to large shifts in equilibrium inflation rates. See Armenter and Bodenstein (2004) for a more complete robustness analysis of the model.

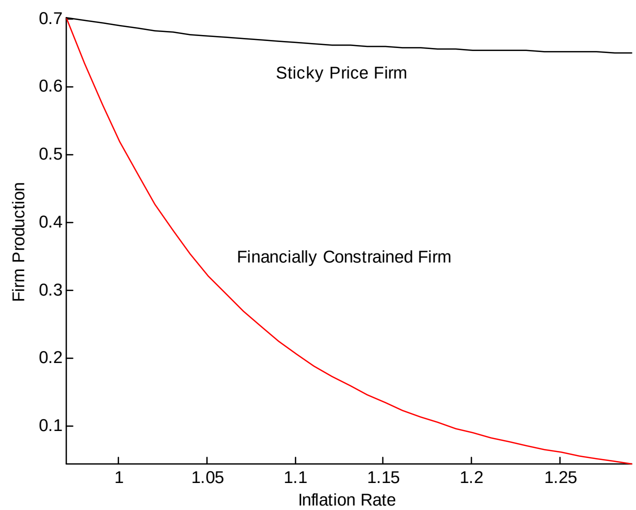

Figure 3: Intermediate Good Output for Firms $i = 1, 2$.

financially constrained firms is plotted along the rational expectations locus, i.e., $\hat{\pi} = \pi$, for different values of inflation π. Firm output is similar across firm types when inflation is low.[15] High inflation disproportionately reduces the production of financially constrained firms—sticky price firm production also falls because aggregate demand is reduced by the price distortion.

The welfare implications of expectation traps dwarf the classic inflationary bias by Barro and Gordon (1983). Table 1 documents this claim for several economies calibrated to match different equilibrium inflation rates. This is achieved by varying the measure of sticky price firms and financially constrained firms in the economy—see the Appendix for details.

For each economy we compute the welfare implications of several experiments. First, we reduce inflation from the low inflation π_1 to the Ramsey equilibrium π^r. This would be equivalent to correct the classic inflationary bias in an economy with a single equilibrium. Second, we evaluate the shift from the high inflation π_2 to the low inflation equilibrium π_1. The last two columns report the welfare change per period as given by the equivalent consumption change in percentage points evaluated at the low inflation equilibrium π_1.

In our baseline calibration, with a low inflation of 2%, the welfare impact of an equilibrium shift is about three times the welfare gains of removing the classic inflationary bias. The

[15] Indeed, firm production is identical when inflation is equal to β, the optimal monetary policy, as firms do not differ in productivity in this numerical illustration.

Low Inflation	High Inflation	Welfare Change per period	
π_1	π_2	*From* π_1 *to* π^r	*From* π_2 *to* π_1
1.5 %	14.4 %	.11 %	.43 %
2 %	13.2 %	.12 %	.36 %
2.5 %	12.2 %	.13 %	.31 %
3 %	11.5 %	.14 %	.27 %

Welfare changes computed as percentage points of consumption at equilibrium inflation π_1. See the Appendix for calibration details.

Table 1: **Welfare Implications: Several Calibrations.**

overall magnitude of welfare losses is significant but not large. The situation is similar for alternative calibrations.

Armenter and Bodenstein (2004) perform an exhaustive characterization of expectation traps in a closed economy version. The model is quite sensitive to the ratio of sticky price firms to financially constrained firms. However, we emphasize that it is very difficult to find parametrizations such that there is a unique and interior Markov equilibrium. One can find parameters such that there is no Markov equilibria or the Friedman rule is time consistent. For the former, just set the share of financially constrained firms to 0 and there is no cost to unexpected inflation. For the latter, set the share of sticky price firms to 0 and there are no gains from unexpected inflation.

3.2 The Case for Soft Exchange Rate Pegs

As expectation traps are, generically, locally unique, they are not in conflict with flexibility. With respect to exchange rate policy, a soft peg with appropriately chosen bands is sufficient to rule out expectation traps, yet it allows to the monetary authority to react to real shocks. Moreover, short term shifts in the world inflation rate can be smoothed out.

To see this, consider an inflation cap $\bar{\pi}$ strictly below the high inflation equilibrium, $\bar{\pi} < \pi_2$, but strictly above the low inflation equilibrium rate, $\pi_1 < \bar{\pi}$. Such a cap exists because the Markov equilibria are locally unique. Under the inflation cap the monetary authority cannot validate high inflation expectations even if it would like to. Therefore the low inflation equilibrium π_1 becomes the unique Markov equilibrium of the economy.[16]

[16]The inflation cap does not constitute a Markov equilibrium by itself because, for all $\hat{\pi} \in (\pi_1, \pi_2)$, $\pi^* (\hat{\pi}) < \hat{\pi}$, i.e., the policy best response is always below inflation expectations. This property is specific of a two Markov equilibria economy.

Next we show how to implement a given inflation cap $\bar{\pi}$ with an exchange rate policy Σ. Combining (10) with (12), we obtain

$$\frac{w(s)\pi}{\theta_x} = \varepsilon(s) P^*.$$

In any Markov equilibrium, $\hat{\pi} = \pi$, and using (20) and some algebra,

$$\frac{\eta}{\theta_x}\left[\left(\tilde{\mu}_1 + \tilde{\mu}_3 + \tilde{\mu}_m \eta^{\frac{\eta}{\eta-1}}\right)\pi^{\frac{\eta}{1-\eta}} + \tilde{\mu}_2 \beta^{\frac{\eta}{1-\eta}}\right]^{\frac{1-\eta}{\eta}} = \varepsilon(s)\pi^* \tag{23}$$

where we used the normalization that $P^* = \pi^*$. The left hand side is an increasing function in inflation. Hence, there is a one-to-one relationship between inflation and the nominal exchange rate for given π^*. Thus, it is possible to implement any inflation cap $\bar{\pi}$ with the proper choice of the exchange rate policy $\Sigma = \{\varepsilon : \varepsilon \leq \bar{\varepsilon}\}$, where

$$\bar{\varepsilon} = \frac{\eta}{\pi^* \theta_x}\left[\left(\tilde{\mu}_1 + \tilde{\mu}_3 + \tilde{\mu}_m \eta^{\frac{\eta}{\eta-1}}\right)\bar{\pi}^{\frac{\eta}{1-\eta}} + \tilde{\mu}_2 \beta^{\frac{\eta}{1-\eta}}\right]^{\frac{1-\eta}{\eta}}$$

and $\pi_1 < \bar{\pi} < \pi_2$.

Note that a soft exchange rate regime improves welfare even if it does not correct the classic inflationary bias, i.e. it does not implement the optimal monetary policy. First, the monetary authority can not be caught in the high inflation equilibrium. Second, there will be no volatility arising from expectation shifts.

Moreover, the low and the high equilibrium inflation rates are far apart. Hence, the exchange rate bands can be broad enough to allow complete flexibility. In our calibration, the difference between low and high equilibrium inflation rates is about ten percentage points. This leaves plenty of room for plausible policy responses to real shocks. Hence, absent any other consideration and leaving the inflationary bias unchanged, the classic textbook argument a la Mundell-Flemming favours broad bands to a hard exchange rate peg. We challenge this view in the next section.

4 Perverse Policy Responses

The textbook argument against fixed exchange rates builds on the classic Mundell-Flemming analysis. A fixed exchange rate regime means no independent monetary policy. The monetary authority loses its ability to react to real shocks and ends up "importing" the foreign monetary policy. The loss of flexibility is often seen as the flip side of the gains that commitment to fixed exchange rate can provide.

We argue that the Mundell-Flemming argument does not hold for the analysis of monetary policy without commitment. We show that the policy response to certain real shocks can be perverse, i.e. worse than inaction, as the shocks exacerbate the time inconsistency problem. An independent monetary policy is no guarantee for less macroeconomic volatility.

The intuition behind a perverse policy response is quite general. A real shock can increase the marginal gains from a unexpected inflation. Consequently, firms anticipate higher inflation. The monetary authority reacts, rightfully, to the real shock but also reacts, unnecessarily, to the induced change in private sector expectations. If the latter dominates, the equilibrium policy response leads to a worsening of the inflationary bias and welfare inferior allocations.

We focus here on a negative terms of trade shock because of its appeal for developing economies, where the case for fixed exchange rates is often built upon time inconsistency issues.[17] A negative terms of trade shock contracts the open intermediate sector, which is characterized by perfect competition. As a result, the economy is less competitive, the distortion from monopolistic competition is larger and so is the temptation to cut markups with unexpected inflation.

To see this, we compute an "aggregate" markup κ by dividing the final good price by the aggregate marginal cost of production. In the Appendix, we detail how we construct the aggregate markup and we show that

$$\kappa = \frac{\left[\left(\frac{\mu_1 y_1}{y} + \frac{\mu_2 y_2}{y} R^{\frac{1}{1-\eta}} + \frac{\mu_3 y_3}{y}\right)\left(\frac{1}{\eta}\right)^{\frac{1}{1-\eta}} + \frac{\mu_m y_m}{y}\right]^{1-\eta}}{\left[\left(\frac{\mu_1 y_1}{y} + \frac{\mu_2 y_2}{y} R^{\frac{1}{1-\eta}} + \frac{\mu_3 y_3}{y}\right) + \frac{\mu_m y_m}{y}\right]^{1-\eta}}.$$

We set each firm's productivity equal for simplicity. The aggregate markup is a geometric average of the monopolistic competitive sectors, with a markup of $\frac{1}{\eta} > 1$, and the perfect competitive sectors, with no markup.

In response to a negative terms of trade shock, the import sector output contracts in relative terms, i.e. $\frac{y_m}{y}$ falls as the relative price of imports goes up.[18] The aggregate markup increases as the competitive sector is weighted less. In the Appendix we show that the markup is decreasing in $\frac{y_m}{y}$.

The fact that the tradeable sector is competitive may seem arbitrary. However, the key implicit assumption is that the country's exports are not differentiated and hence export

[17] There are other real shocks which induce a perverse policy response in our model, like a productivity shock in the export sector or a change in the trade costs.

[18] The measure of firms μ_m is an exogenous parameter and stays constant. However, production y_m is endogenous and it adjusts to the shock.

prices p_x are set in the world market. This particularly suits a developing economy framework. For the import sector, the law of one price implies that importing firms are perfectly competitive from the point of view of the domestic economy. Whether the world price of imports p_m is above marginal cost is irrelevant: it is a wedge which is out of reach for the monetary authority.

We illustrate the perverse policy phenomenon in a calibrated version of our model—the details of the calibration are provided in the Appendix. We compare a fixed and flexible exchange rate regime in the event of an unanticipated and permanent negative terms of trade shock. The fixed exchange rate regime is modelled as an Exchange Rate Policy equilibrium with $\Sigma = \{\bar{\varepsilon}\}$. For the flexible exchange rate regime, we use our concept of Markov equilibrium. Since there are usually multiple Markov equilibria, we pick the one with lowest inflation.[19]

In order to abstract from the classic inflationary bias argument, we set the world inflation rate π^* such that the flexible and the fixed exchange rate regime deliver the same allocations in the pre-shock economy, i.e. at date 0. In other words, there are no "level" gains in terms of inflation under a fixed exchange rate regime as the world inflation rate is set equal to the inflation rate π_1 in the low inflation Markov equilibrium.

We model the terms of trade shock as unforeseen and permanent. By having the shock totally unanticipated, we choose the best scenario for active monetary policy. Our welfare computations are always per period; hence the assumption that the shock is permanent has no impact beyond providing us with at least one period where the shock is anticipated by the sticky price firms.

The timing of the shock is as follows. At date 0, the economy is in the original steady state. The terms of trade deteriorate by 1% after firms $i = 1$ have set their sticky price for date 1 but before the monetary authority policy decision. Hence, there is a role for a monetary policy response. At date 2, sticky price firms are correctly aware that the shock is permanent and their prices are set accordingly. Prices and allocations reach the new steady state at date 2.[20]

Figure 4 displays the response of several prices and allocations. The solid line corresponds to the Markov equilibrium and the dashed line to the ERP equilibrium with $\Sigma = \{\bar{\varepsilon}\}$. The

[19] Alternatively, the reader can think of a comparison of a soft versus a hard exchange rate regime, where the former is characterized by exchange rate bands chosen to rule out the high inflation equilibrium and to allow enough flexibility, as documented in the previous section.

[20] We need to be more precise about our terms of trade shock because, given a change in the relative price of exports and imports, there are infinite possible changes in the price levels. We pick the change in the price levels such that, given a constant monetary policy, the ratio of domestic and world inflation is left intact. In other words, we abstract from simultaneous real exchange rate movements which may naturally accompany a terms of trade shock, if they are not policy induced.

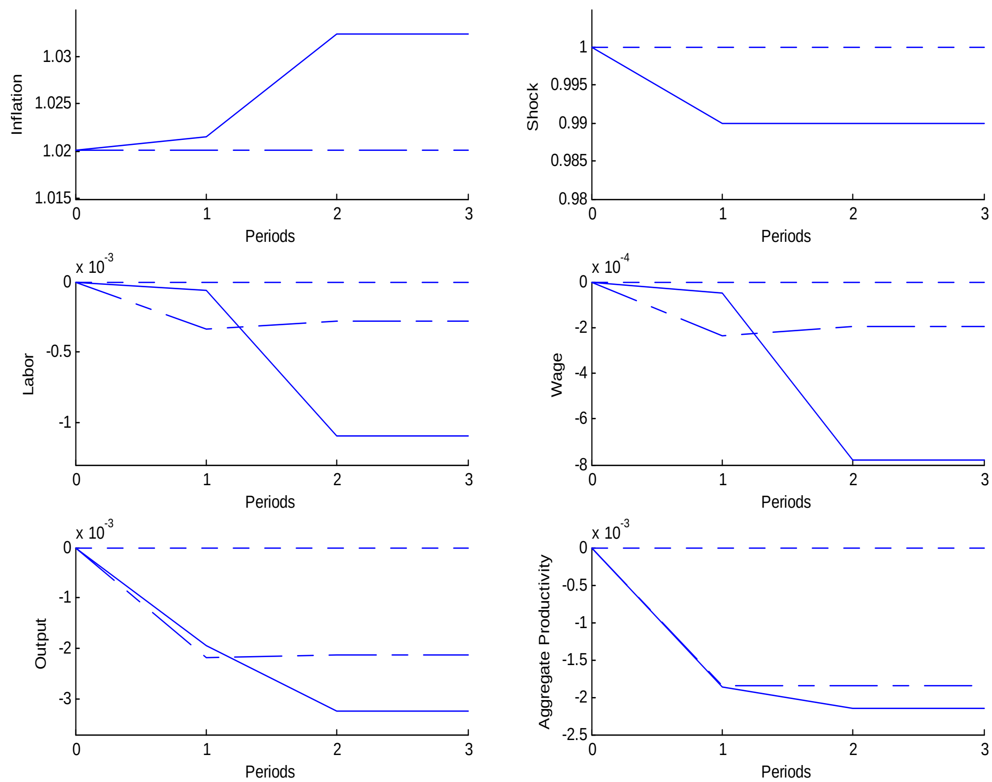

Figure 4: **Equilibrium Response to a Negative Terms of Trade Shock**. Solid line correspond to the low inflation Markov equilibrium. Dashed line corresponds to a fixed exchange rate regime. See text for details.

Period	*Date $t = 0$ Inflation Rate*		
	$\pi = 2.0$	$\pi = 2.5$	$\pi = 3.0$
Date $t = 0$	0	0	0
Negative Shock			
Date $t = 1$	0.00015	0.00016	0.00017
Date $t = 2$	-0.0393	-0.0597	-0.0912
Positive Shock			
Date $t = 1$	0.00014	0.00016	0.00017
Date $t = 2$	0.0241	0.0296	0.0348

Welfare changes reported as percentage points of consumption under non-stochastic economy. Values per period.

Table 2: **Welfare Comparison: Markov Equilibrium versus Fixed Exchange Rate.**

most important graph is in the upper left corner and it displays the inflation rate. Under the fixed exchange rate inflation is constant. Under independent monetary policy, inflation increases in two steps. At date 1, there is a small inflation increase. This is the optimal response induced by the presence of nominal frictions.[21] However, at date 2 inflation jumps by a large amount in the Markov equilibrium—when there is no longer any role for monetary policy to ease the real shock. From date $t = 2$ onwards, high inflation is only reflecting higher sticky prices. This response is clearly welfare reducing.

Prices and allocations tell the same story. At date 1, the policy response in the Markov equilibrium keeps the wage and labor close to the steady state values despite the shock, while under the fixed exchange rate there is no smoothing. However, from date 2 onwards, the impact is more pronounced under flexible exchange rate regime. Higher expected inflation brings wage, labor and output below their counterparts under the fixed exchange rate regime.

Table 2 compares the welfare properties of both exchange rate regimes. We report the per period consumption compensation, in percentage points, for a shift from the fixed exchange rate regime to the Markov equilibrium. A negative number means that households are willing to pay to keep the fixed exchange rate regime *ex-post*. We included several calibrations, indexed by the inflation rate at date $t = 0$. In each calibration, the world inflation rate is set such that the flexible and the fixed exchange rate have the same welfare properties in the pre-shock economy.

[21] This is the *ex-post* optimal response: the monetary authority is a benevolent policymaker. The Ramsey policy in a stochastic economy would not necessarily look alike. First, the response would be evaluated around the Friedman rule, which is the optimal level of inflation. Second, if the terms of trade shock had a positive probability of occurring, the Ramsey policy would have *ex-ante* considerations.

After the negative terms of trade shock at date $t = 1$, the Markov equilibrium dominates the fixed exchange rate, but the welfare difference is small. From date $t = 2$ onwards, the fixed exchange rate equilibrium welfare dominates. However, the welfare comparison is far from ambiguous. The welfare gains from a fixed exchange rate at date $t = 2$ is about three orders of magnitude superior to the losses at date $t = 1$. Recall these are per period welfare changes. The welfare ranking for date $t = 2$ stands as long as the terms of trade do not reverse to the initial level.

Summarizing, Table 2 clearly speaks in favour of fixed exchange rates in the event of a real shock—a scenario usually associated with the costs of the loss of independent monetary policy. It is important to emphasize that the impact of a positive terms of trade shock is not symmetric. Table 2 also reports welfare in the aftermath of a positive shock to the terms of trade. A simple average of date $t = 2$ numbers makes clear that positive and negative shocks do not cancel each other in welfare terms. This is true even after considering that the Markov equilibrium dominates at date $t = 1$ in both events. This is a direct consequence of the concavity of the policy problem. In Table 2, a positive terms of trade brings welfare gains which are only slightly above half the welfare loss under a negative terms of trade shock.

5 Conclusion

This paper contributes to the exchange rate debate but by no means settles it. Indeed, a definitive welfare ranking of exchange rate regimes seems more elusive than ever. Expectation traps and perverse policy responses increase the complexity of any welfare evaluation of exchange rate regimes—yet any such evaluation would be incomplete without considering both phenomena.

For example, we should not conclude from the observed downward trend in inflation that commitment problems are a thing of the past and consequently that fixed exchange rate regimes are no longer appealing. A low inflation country may be only a shift in expectations away from high inflation. Moreover, large real volatility does not necessarily make a stronger case for a flexible exchange rate. We have to ask what is the impact of the relevant real shocks on the time inconsistency problem and how likely it is that independent monetary policy reacts perversely.

During our analysis we have abstracted from the time consistency of the exchange rate policy itself. We view implementation issues as unavoidable and they may corner a country into extreme and costly solutions such as dollarization. In particular for developing economies, intermediate arrangements may not be a possibility. Trying to sustain a fixed exchange rate absent commitment can lead to currency crises, as discussed in Obstfeld (1996).

Yet, we also note that it is often the case that fixed exchange rates are brought down by fiscal rather than monetary crises. We do not view fixed exchange rates as a solution to fiscal problems. The ongoing skepticism about fixed exchange rate credibility arises very much from using the wrong tool for the wrong problem.

References

Albanesi, S., Chari, V. and Christiano, L. J.: 2003, Expectation traps and monetary policy, *The Review of Economic Studies* **70(4)**(245), 715–742.

Arellano, C. and Heathcote, J.: 2003, Dollarization and financial integration. Working Paper, Georgetown University.

Armenter, R.: 2004, A general theory (and some evidence) of expectation traps in monetary policy. Working Paper, Federal Reserve Bank of New York.

Armenter, R. and Bodenstein, M.: 2004, Can the u.s. monetary policy fall (again) in an expectation trap? Working Paper, Federal Reserve Bank of New York.

Barro, R. J. and Gordon, D.: 1983, A positive theory of monetary policy in a natural rate model, *Journal of Political Economy* **91**, 589–610.

Calvo, G. A. and Reinhart, C. M.: 2002, Fear of floating, *Quarterly Journal of Economics* **117**(2), 379–408.

Chang, R. and Velasco, A.: 2000, Exchange-rate policy for developing countries, *The American Economic Review* **90**(2), 71–75.

Chang, R. and Velasco, A.: 2002, Dollarization: Analytical issues, *in* E. Levy Yeyati and F. Sturzenegger (eds), *Dollarization*, MIT Press Book, Cambridge, Mass.

Chari, V., Christiano, L. J. and Eichenbaum, M.: 1998, Expectation traps and discretion, *Journal of Economic Theory* **81**(2), 462–492.

Cooley, T. F. and Quadrini, V.: 2001, The costs of losing monetary independence: The case of mexico, *Journal of Money, Credit and Banking* **33**(2).

Frankel, J. A.: 1998, *No Single Currency Regime is Right for All Countries or at All Times*, number 215, Princeton University Press, Princeton. Essays in International Finance.

King, R. G. and Wolman, A. L.: 2004, Monetary discretion, pricing complementarity and dynamic multiple equilibria, *Quarterly Journal of Economics* **199**.

Lucas, R. E. J. and Stokey, N. L.: 1983, Optimal fiscal and monetary policy in an economy without capital, *Journal of Monetary Economics* **12**, 55–93.

Mendoza, E. G.: 2001, The benefits of dollarization when stabilization policy lacks credibility and financial markets are imperfect, *Journal of Money, Credit and Banking* **33**(2), 440–474.

Obstfeld, M.: 1996, Models of currency crises with self-fulfilling features, *European Economic Review* **40**, 1037–48.

Obstfeld, M. and Rogoff, K.: 1996, *Foundations of International Macroeconomics*, The MIT Press, Cambridge, Massachusetts.

Siu, H. E.: 2004, Time consistent monetary policy with endogenous price rigidity. Working Paper, University of British Columbia.

Woodford, M.: 2003, *Interest and Prices*, Princeton University Press, Princeton.

A Appendix

A.1 Calibration

We start by setting the preferences parameters to standard values. The inverse of β is the real interest rate in our economy: we set it equal to 3%, $\beta = .9709$, evaluating the model at the annual frequency.

Our choice for leisure preferences is

$$h\left(1-n\right) = \psi_0 \frac{\left(1-n\right)^{1-\psi}}{1-\psi}$$

and then we set our parameters on the labor supply to match a Frisch labor elasticity of 1 and the Aristotelian proportion of leisure and work in the first best, $n = \frac{1}{2}$.

We set the share of nontradeable goods at 60% and the share of export firms at 12%. The last of the pre-set parameters is η, which is set to replicate a 12% markup.

We calibrate the measure of firms of type $i = 1$ and $i = 2$ to match the diverse US inflation experience. Our target for the low inflation equilibrium is in a $1.5\% - 3\%$ range, and for the high inflation equilibrium, it is between $11\% - 15\%$. We find that we can match these numbers with reasonable parameter values. The share of firms with sticky prices is about 12%, and the measure of financially constrained firms is just above 5%.

Our parameter choices are summarized in Table 3.

Parameter	*Notation*	*Value*
Intertemporal Discount Rate	β	.9709
Leisure-Consumption	ψ_0	.5
Inverse Frisch Labor Elasticity	ψ	1
Share of Non Tradeables	α	.6
Inverse Markup	η	1.12^{-1}
Measure of Firms $i = 1$	μ_1	.12
Measure of Firms $i = 2$	μ_2	.0552
Measure of Firms $i = x$	μ_x	.12

Table 3: **Baseline Calibration.**

A.2 The Aggregate Markup

In order to provide some insight on the perverse policy response phenomenon, we compute an aggregate markup. From the final good firm profit maximization problem, we write the correspondent cost minimization problem:

$$\min_{\{y_i\}^{1-\mu_x}} \int_0^{1-\mu_x} p_i^y(s)\, y_i di$$

subject to

$$y \leq \left[\int_0^{1-\mu_x} y_i^\eta di\right]^{\frac{1}{\eta}}.$$

The first order condition is

$$p_i^y(s)^{\frac{1}{1-\eta}} \frac{y_i}{y} = \lambda^{\frac{1}{1-\eta}}$$

where λ is the Lagrangian multiplier associated with the technological constraint, which equals the marginal cost of unit of final good. The previous condition is necessary for all $y_i > 0$. Hence,

$$\left[\int_0^{1-\mu_x} p_i^y(s)^{\frac{1}{1-\eta}} \frac{y_i}{y} di\right]^{1-\eta} = \lambda$$

gives the marginal cost as a geometric weighted average of each intermediate good price.

We compute the markup in a Markov equilibrium. We substitute the pricing formula for each price to obtain:

$$\lambda = \left[\mu_1 \frac{y_1}{y}\left(\frac{w}{\eta\theta_1}\right)^{\frac{1}{1-\eta}} + \mu_2 \frac{y_2}{y}\left(\frac{wR}{\eta\theta_2}\right)^{\frac{1}{1-\eta}} + \mu_3 \frac{y_3}{y}\left(\frac{w}{\eta\theta_3}\right)^{\frac{1}{1-\eta}} + \mu_m \frac{y_m}{y}\left(q(s)\, p_m^*\right)^{\frac{1}{1-\eta}}\right]^{1-\eta}$$

which is long and not very helpful. We assume there are no differences in productivity $\theta_i = 1$ for all goods.

We factor out the wage

$$\lambda = w \left[\left(\mu_1 \frac{y_1}{y} + \mu_2 \frac{y_2}{y} R^{\frac{1}{1-\eta}} + \mu_3 \frac{y_3}{y} \right) \left(\frac{1}{\eta} \right)^{\frac{1}{1-\eta}} + \mu_x \frac{y_m}{y} \right]^{1-\eta}.$$

We do not want to mix the price distortions and the markup distortion. The marginal cost of producing one unit of the final good, given the current price distortions, is

$$\tilde{\lambda} = w \left[\left(\mu_1 \frac{y_1}{y} + \mu_2 \frac{y_2}{y} R^{\frac{1}{1-\eta}} + \mu_3 \frac{y_3}{y} \right) + \mu_x \frac{y_m}{y} \right]^{1-\eta}.$$

Since the final good producer is competitive, λ is also the final good price. We set our aggregate markup definition as $\kappa \equiv \lambda/\tilde{\lambda}$.

To see that κ is decreasing with y_m/y, note that $\left(\frac{1}{\eta}\right)^{\frac{1}{1-\eta}} > 1$ and apply simple differential calculus.