The 'Origin-Margin', the Rise of Trade Volumes, and Per Capita Income

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Abstract

Recent empirical literature documents the existence of an "origin-margin" in international trade: countries import the same goods from an increasing number of supplier countries. This qualitative change in the trade patterns parallels the remarkable rise in world trade volumes. The present paper proposes per capita income as a joint determinant for both observations. It develops a model where varieties - defined as goods differentiated by origin - are non-essential in an otherwise standard love-for-variety utility. In presence of transportation costs consumers demand varieties from neighboring countries only; yet the set of supplier countries expands as per capita income and total expenditure grow. This expansion of the origin-margin partly compensates the decreasing marginal utility from the import bundle. The bundle of domestic varieties lacks this effect. Consequently, income growth shift expenditure shares towards imports and trade volumes rise through the increase of imported varieties. Predictions concerning per capita income in the gravity framework are derived and tested. A calibration exercise replicates the dynamics of US trade shares and the number of imported goods and varieties.

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1 Introduction

It is well known and widely acknowledged that any increase in trade volumes decomposes into a rise in the quantity of previously traded goods (the intensive margin) and an expansion of the set of traded goods (the extensive margin). In addition to these two margins recent empirical evidence documents a third, an "origin-margin": importers can expand their basket of imports by purchasing the same good from a larger number of source countries. Using disaggregated trade data Broda and Weinstein (2004) and (2004a) illustrate the nominal expansion of this margin and its welfare implications. The authors show that all of the 20 economies with largest import volumes have extended their origin-margin between 1972 and 1997 and increased the number of source countries of the average good between 21% and 326% (with an unweighted country average of 76%), implying "enormous welfare gains for many countries" with an average of roughly 5% in their conservative specification. In an earlier paper Haveman and Hummels (1999) illustrate that the origin-margin is far from being exhausted. Using the standard definition of varieties as goods differentiated by origin, the authors document that "importers purchase a very small fraction of available varieties".

The impressive expansion along the origin-margin relates positively to the considerable growth of aggregate trade volumes. Figure 1 illustrates both developments for US trade between 1972 and 2000: the increase in non-petrol manufacturing trade share of about 150 percent was paralleled by a steady rise in the number of imported goods and the average number of source countries per good - i.e. the average number of varieties per good. This remarkably parallel development hints at some common determinants of both time series and suggests a joint analysis of both phenomena.

Despite its quantitative significance and possibly substantial welfare implications, none of the standard trade models is able to map the observed origin-margin in international trade. Two-country models fail to address this issue ab initio. Referring to the probably best-known class of multicountry models, which is typically used in the gravity theory Anderson and van Wincoop (2003) point out that an "important drawback of the existing theory is that all countries import all varieties from all countries". The same criticism applies to a second class of multicounty models with setup cost of exporting and firm heterogeneity, where standard assumptions about the productivity distribution imply positive bilateral trade volumes in any sector and for every pair of countries (Eaton and Kortum (2002)).

The present paper addresses these shortcomings and develops a multi-country model that accounts for the origin-margin of international trade. Following standard assumptions and terminology, goods are differentiated into varieties by virtue of their origin, i.e. output of two different countries in one and the same good sector constitute two distinct varieties. These varieties are, moreover, imperfect substitutes for consumers. The model departs in one crucial point from the standard by assuming, in the spirit of Stokey (1988), that varieties are non-essential to consumers in an otherwise standard love-for-variety preference structure. In presence of non-negligible trade costs, countries then import from close neighbors only with a marginal trade partner defining a maximal shipping distance of imports. When consumption expenditure increases, the subset of imported varieties endogenously expands and the maximal shipping distance increases.

Trade volumes and per capita incomes are positively related as an immediate consequence of the origin-margin. This margin constitutes an additional dimension to allocate an extra dollar of expenditure, but is present in the imported bundle only. This implies that the utility derived from imports exhibit milder decreasing returns than the utility derived from domestic bundle varieties and consequently, income growth translates into a rising marginal propensity of expenditure on imported goods. This finally causes an increase in the trade share.

In a nutshell, per capita income drives trade volumes through the extra margin of origin. The model thus suggests per capita income as a joint determinant of the parallel dynamic of trade volume and the number of the imported varieties illustrated in Figure 1.

While the driving forces of the variety-growth have been widely neglected, those of world trade volumes received considerable attention over the past decade. Ever since Krugman (1995) observed that the causes for the dramatic growth in post-WWII trade volumes were surprisingly disputed, trade economists searched to identify the underlying forces that drive trade volumes. With income convergence, trade policies and transport costs being the most popular explanations, recent empirical evidence suggests that tariff cuts have been the main contributors to the rise in world trade. In an important empirical contribution, Baier and Bergstrand (2001) show that variation in tariff explains the bigger share of the increase in world trade between the late 1950ies and the late 1980ies - with tariff cuts as the main explanation, transport cost far less important, and the impact of income convergence negligible.

Yet, this insight recreated new questions: Yi (2003) states that the implied import elasticity in standard trade models must be excessively high when the rise in trade volumes is to be explained mainly by tariff reductions. In an attempt to solve this puzzle, he shows that under realistic substitution elasticities between home and foreign goods, a modest fall in tariffs causes strong responses in trade volumes if it generates international vertical specialization: the value of one and the same good enters the trade statistics more than once as it is shipped back and forth at different production stages. With a calibrated model, Yi (2003) is able to explain up to half of the rise in world trade volumes. Cuñat and Maffezzoli (2005), on the other hand, point out that trade liberalization has a dynamic effect on trade volumes that works through time-intensive factor accumulation and adds to the ordinary static effects. In a standard Heckscher-Ohlin setup, tariff reductions change the incentives for factor accumulation, lead to diverging factor endowments, and thus raise trade volumes. The cumulated effects of moderate but gradual trade liberalization are shown to generate large long-run responses that replicate the growth of US trade volumes.

Unless both of these explanations, the present paper recognized that a large and increasing share of world trade is of the intra-industry type. Its closer predecessors are models of Markusen (1986) and Bergstrand (1990) who formalize and investigate on the so called Linder hypothesis, which predicts rising intra-industry trade shares with per capita growth. These models assume that consumer cover minimum level of an homogenous, domestically produced good before demanding imported "luxury" goods. The present model does not rely on this set of strict and questionable assumptions. Instead, it builds on the assumption that all varieties enter the consumers' utility symmetrically and only transportation costs create asymmetries in demand via their effect on consumer prices. This implies qualitatively different responses to per capita income at the low end of the world income distribution.

To investigate upon the quantitative performance of the model, a basic calibration exercise is executed, which shows that the model can mimic the time series plotted in Figure 1 reasonably well. An important aspect of the calibration is to target the import elasticities. To get this variable right is particularly important since first, import elasticities play a key role in the "quantity puzzle" of international trade and second, a bounded marginal utility generally implies unbounded ranges of the price elasticities.

To further strengthen the argument, a simple version of the models is used to derive predictions and test them in the framework of the gravity equation. In an approximation around the symmetric equilibrium of the model, the usual elasticities of trade volumes to GDP are slightly reduced. More importantly, the gravity equation is augmented by per capita income of the importer and exporter country. A subsequent test rejects the null hypothesis of zero impact of per capita incomes on bilateral trade flows with the "right" positive signs and within the predicted range.

The remainder of the paper is organized in the following way. Section 2 develops the model and derives its key implications, including the augmented gravity equation, section 3 provides a simple test of gravity equation and presents some calibration results for the US trade data, and section 4 concludes.

2 The Model

There is a continuum of countries, located on a circle C. Country $i \in C$ is populated by identical individuals, which inelastically supply labor L_i to the domestic labor market. For the time being and until specified otherwise, there is only one good. This good is differentiated by the origin of production (the Armington assumption); following standard terminology, output from two distinct source countries will represent two different varieties of the good.

The good produced in country i is further differentiated by different types V_i. Deviating slightly from the standard terminology, these types are equally called varieties throughout the paper. Thus, varieties are defined along two dimensions, within the country by types and by geographical origin.

Production

Country i produces competitively its varieties V_i with labor as the sole factor according to the linear production function

$$x_{ik} = a_i L_{ik} \qquad\qquad k \in V_i \quad i \in C \tag{1}$$

Note that productivity a_i is allowed to differ by country but not by variety. Finally, the varieties V_i are exogenously partitioned in tradables and nontradables. In particular, there is one single tradable and the set N_i of nontradables[2] with mass α. Thus, the total set of varieties available to consumers in country i can be identified with $N_i \cup C$.

Preferences

Consumers value all - tradables and nontradables - varieties symmetrically. The preferences are summarized by the utility function

$$U_i = \exp\left(\int_{N_i \cup C} \ln(c_{ik} + 1)\, dk \right) \tag{2}$$

where c_{ik} is the quantity of variety k consumed by an individual in country i. As a most important deviation from a standard setup, this utility departs from the familiar Cobb-Douglas structure in the unit constant added to consumption level[3]. Consequently, marginal utility of each variety has a finite upper bound. Accordingly. consumers refrain from consumption of goods whose prices exceed a maximum.

Note also that utility (2) is separable in the parts derived from the imported and the nontradable varieties:

$$U_i = \exp\left(u_N\right) \cdot \exp(u_C) \tag{3}$$

with the sub-utilities

$$u_X = \int_X \ln(c_{ik} + 1)\, dk \qquad\qquad X = N, C \tag{4}$$

such that utility maximization can be divided to a first stage involving sub-utilities (4) given expenditure on imports and domestic varieties, and a second stage where

[2] This is an artificial but innocent assumption. It guarantees that the trade share of the atomistic countries is strictly smaller that one. Technically speaking, the set of tradables and nontraded types must have equal cardinality.

[3] See e.g. Young (1991) for a motivation of the deviation from the standard.

these expenditure levels are determined by maximizing (3). Before looking at consumers' optimal choices, however, the description of the model's structure is completed.

Prices

Producer price p_j in country j and the corresponding consumer price q_{ij} in country i and are assumed to differ sue to positive trade costs that drive a wedge between them. In particular, the relation between q_{ij} and p_j is assumed to be of the following functional form

$$q_{ij} = \begin{cases} p_j T_{ji}(1 + \delta r_{ij}^{\gamma}) & \text{if } i \neq j \\ p_i & \text{if } i = j \end{cases}$$

where r_{ij} is the distance between country i and country j. The positive parameters γ and δ imply that the standard "iceberg" transportation cost is increasing with the distance. The gross border cost $T_{ji} \geq 1$ on goods exported from j to i can be thought of as a cumulative effect of tariffs, delays at borders, exchange rate risk and the like. Some of these costs enter prices additively, while others effectively work as a tax on consumer prices, multiplying prices. In the current setup, T is assumed multiply prices - a convenient but inessential assumption. Tariff revenues, as far as they are generated, are burned.

For tractability of the model assume symmetry across countries "most" countries. In particular, it will prove useful to focus the analysis on one single country (indexed with 0) that differs in its parameters from the set of identical rest of the world countries (ROW, indexed by $i \in C\backslash\{0\}$). Consequently, prices and quantities that country 0 imports can be indexed conveniently by the distance r between a ROW-country and country 0

$$q_r = \begin{cases} p^* T(1 + \delta r^{\gamma}) & \text{if } r > 0 \\ p & \text{if } r = 0 \end{cases} \tag{5}$$

where here and in the following, ROW-variables will be marked with an asterisk.

1^{st} Stage Optimization

The set of imported varieties and optimal bundle generally depends on all world prices. If trade costs are negligible ($T = 1$, $\delta = 0$) all imports to country 0 have the same price and every consumer in country 0 purchases the full bundle of varieties available on the world market. Yet, when trade is costly, consumer prices of varieties differ due to the trade costs. Let the representative individual in country 0 spend the total amount I_m (in domestic currency unites) on imported varieties. The optimal bundle of imports then maximizes u_C from (4) subject to the budget constraint

$$\int q_r c_r \, dr = I_m \tag{6}$$

under the price schedule (5). The optimality conditions give relative demand for the varieties imported from two countries at distances r and r'

$$\frac{c_r + 1}{c_{r'} + 1} = \frac{q_{r'}}{q_r} = \frac{1 + \delta r'^{\gamma}}{1 + \delta r^{\gamma}} \tag{7}$$

whenever c_r and $c_{\tilde{r}}$ are positive.

For country 0 import prices are strictly increasing with distance and there is a "marginal exporter" at distance $\overline{r}$, from which country 0 imports a negligible amount. Equation (7) can be rewritten in terms of $\overline{r}$ such that for all foreign varieties domestic consumption is

$$c_r = \max\left\{\delta \frac{\overline{r}^{\gamma} - r^{\gamma}}{1 + \delta r^{\gamma}}, 0\right\} \tag{8}$$

The value $\overline{r}$ is equal to half of the mass of different varieties that country 0 actually imports and will turn out to be a central variable in the following. $\overline{r}$ can be expressed in terms of the expenditure I_m, producer prices p^*, and the parameters by using equations (5), (6), and (8)

$$\overline{r} = \left(\frac{I_m(1+\gamma)}{Tp^* 2\gamma\delta}\right)^{1/(1+\gamma)} \tag{9}$$

Equation (9) shows that the $\overline{r}$ is an increasing function of I_m, exhibiting already the model's basic mechanism: at constant world prices, the mass of imported varieties increases with the expenditure on tradables varieties. Not only more of the same varieties, but also new varieties are imported when the expenditure on imports rises. In other words, the bundle of traded varieties has a non-trivial extensive margin.

For the derivation of this result, it was assumed that $\overline{r}$ is smaller than half the circumference of the circle C - i.e. the maximal distance a variety is shipped is less than the maximal distance between two countries on the circle. This assumption will be made for the rest of the paper.

With the help of equation (8), the sub-utility of the imported bundle (4) can be expressed in terms of $\overline{r}$:

$$u_C = 2\overline{r}\ln(1 + \delta\overline{r}^{\gamma}) - 2\int_o^{\overline{r}} \ln(1 + \delta r^{\gamma})\, dr$$

such that $du_C/d\overline{r} = 2\gamma\delta\overline{r}^{\gamma}/(1 + \delta\overline{r}^{\gamma})$. This, together with (9) leads to the marginal sub-utility derived from expenditure on the optimal tradable bundle

$$\frac{du_C}{dI_m} = \frac{du_C}{d\overline{r}}\frac{d\overline{r}}{dI_m} = \frac{1}{Tp^*\left(1 + \delta\left(\frac{I_m(1+\gamma)}{Tp^* 2\gamma\delta}\right)^{\gamma/(1+\gamma)}\right)} \tag{10}$$

It is worth noting that the expenditure on tradables I_m enters the denominator of the marginal sub-utility with the power of $\gamma/(1+\gamma)$, such that the decreasing returns to I_m is milder, the smaller is the parameter γ.

The optimal bundle of nontradables is easily determined. When I is total per capita expenditure, the expenditure of the representative individual on domestic varieties is $I = I_m$. As the mass of nontradbles are symmetric, one has $\alpha p c_d = I - I_m$ where c_d is the consumption level of each of the domestic varieties. Thus, the individuals' utility (4) from nontradables is $u_N = \alpha \ln(1 + c_d)$ and

$$\frac{du_N}{d(I - I_m)} = \frac{\alpha}{\alpha p + I - I_m} \tag{11}$$

It is worth to note a qualitative difference between marginal utility from expenditure on tradable (10) and nontradable (11) varieties. The respective expenditures enter the denominator of du_D/dI_d linearly, but sublinearly the denominator of du_C/dI_d. In other words, the marginal utility of imported bundles is milder decreasing in expenditure than marginal utility of the domestic bundle. This feature stems, of course, from the extensive margin in the imported varieties and, as will be shown shortly, that the import share rises with per capita income.

2^{nd} Stage Optimization

All income is consumed period by period, such that within each period total income of an individual in country 0 equals its its total expenditure is $I = pa$. Writing e as the share of income spent on imported varieties one has $I_m = pae$ and $I - I_m = pa(1 - e)$ and thus

$$c_d = a(1 - e)/\alpha \tag{12}$$

Note that e is also the trade share of country 0 since the value of total imports over value of total output in country 0 is $I_m L/(paL) = e$. It will be useful to rewrite (9) and express the trade share in terms of $\bar{r}$

$$e = \frac{2\gamma}{\gamma + 1}\delta T\pi\frac{\bar{r}^{\gamma+1}}{a} \tag{13}$$

where here and in the following $\pi = p^*/p$ stands for the inverse of country 0's terms of trade.

Now, the 2^{nd} stage optimization requires $du_N/dI_d = du_C/dI_m$ whenever trade volumes are positive. With equations (10), and (11) is is quick to check that e is positive whenever condition

$$T\pi - 1 < a/\alpha \tag{14}$$

holds. This will be assumed in the following. By imposing the optimality condition $du_N/d(I - I_m) = du_C/dI_m$ and using equations (9) - (13), $\bar{r}$ is implicitly determined as a function of world prices and the underlying parameters by

$$\alpha T\pi\left(1+\delta\bar{r}^{\gamma}\right)+\bar{r}^{\gamma+1}T\pi\delta\frac{2\gamma}{\gamma+1}-\alpha-a=0 \tag{15}$$

Equations (8), (12) with $I_m = pae$, (13), and (15) implicitly determine optimal consumption as a function of world prices and the parameters of the model. The model is then closed when the relative world prices π are determined. This is particularly simple in a symmetric setting.

2.1 The Symmetric Equilibrium

The model is particularly simlpe to solve in the the symmetric case of identical countries. In that case, relative world prices are one ($\pi = 1$) and it is straight forward to derive from (13) and (15) the following

Proposition 1 *In a symmetric world economy the trade share e and the number of imported varieties $2\bar{r}$ are decreasing in trade costs T and δ, constant in labor L, and increasing in productivity a.*

This proposition states that, just as in standard models (Ricardian as in Dornbusch Fischer and Samuelson (1977), Heckscher-Ohlin, or monopolistic competition as in Helpman and Krugman (1985)), there is no scale effect in the trade share in the sense that multiplying the labor force L of every country in the world, the world trade share remains unaffected. Very much unless those standard models, however, the trade share increases the present model with labor productivity a. This relation reflects the core mechanism of the paper: higher labor productivity translates into higher wages and shifts out the individuals' budget set. As pointed out above, the decrease in the marginal sub-utility from imports is milder than that of the marginal sub-utility from domestic varieties. Consequently, the marginal propensity of expenditure on imports rises with an increase in income, driving up the trade share.

Parallel to the rise in the trade share comes the increase of the number of imported varieties of the representative country. This is the "origin-margin" that stems from the consumer's willingness to pay a higher price for foreign varieties as her total income increases. The basket of purchased foreign varieties thus grows with per capita income.

Proposition 1 states that per capita income drives up the trade share and the amount of imported varieties, defined as goods differentiated by origin. It shows that the model can qualitatively replicate the parallel growth of the trade share and the number of traded varieties per good shown in Figure 1. Yet, imposing symmetry among countries was a cheap way to close the model by imposing constant terms of trade. The next step will be to analyze to what extend the Proposition 1 survives when departing from the strong assumption of symmetry and conducting comparative statics on the individual country parameters.

2.2 An Asymmetric Equilibrium

Equation (13) and (15) determine the variables of the country 0 that differs from the mass of identical ROW-countries. To close the model by solving for the prices, the corresponding variables of the ROW-countries are to be determined. Now notice that the deviation of country 0's characteristics also introduces heterogeneity among the ROW-countries through the location each of the ROW-countries has relative to country 0. This could in general introduce complicated heterogeneity among the ROW-countries. Yet, in the present setup of a continuum of countries, country 0's exports to any of the ROW-countries has zero weight in the respective import bundle. Thus, the ROW-countries are still identical in their relevant variables: the trade share e^* and the maximal trade distance among each other r^*. Both variables can be determined for all ROW countries by replacing the terms of trade $1/\pi = 1$ in the equivalent of (13) and (15).

$$\alpha^* T^* \left(1 + \delta(\bar{r}^*)^\gamma\right) + (\bar{r}^*)^{\gamma+1} T^* \frac{2\gamma\delta}{\gamma+1} - \alpha^* - a = 0 \tag{16}$$

and

$$e^* = \frac{2\gamma}{\gamma+1} \delta T^* \frac{(\bar{r}^*)^{\gamma+1}}{a^*} \tag{17}$$

The trade share of country 0 and the maximal distance over which country 0's exports are shipped, however, differ from these expressions and have to be determined separately. Let the latter distance be denoted by $\bar{\rho}$. Using the generic optimality condition (7), one calculates the relation between $\bar{r}^*$ and $\bar{\rho}$ to be

$$T^* p^* (1 + \delta(\bar{r}^*)^\gamma) = Tp(1 + \delta\bar{\rho}^\gamma) \tag{18}$$

assuming $\bar{r}^*$ and $\bar{\rho}$ are positive. Now notice that total demand for country 0's exports is $L^* 2 \int_0^{\bar{\rho}^*} T(1 + \delta r^\gamma) c^*_{r,o} dr$, while its exports are Lae. When country 0's export market clears one gets thus

$$L^* T \delta \frac{2\gamma}{1+\gamma} \bar{\rho}^{1+\gamma} = aeL$$

and with (13)

$$\bar{\rho}^{1+\gamma} = \pi\lambda\bar{r}^{1+\gamma} \tag{19}$$

where λ stands for the relative country size $\lambda = L/L^*$. Notice that symmetric border cost border cost are assumed here, i.e. goods shipped form ROW to country 0 and the other way round face the same border cost T.

Solving now for relative prices $\pi = p^*/p$ in (15) and combining (18) and (19) gives

$$T^2 \frac{\alpha(1+\delta\bar{r}^\gamma) + \frac{2\gamma\delta}{1+\gamma}\bar{r}^{1+\gamma}}{a+\alpha} + \delta\bar{r}^\gamma T^{\frac{2+\gamma}{1+\gamma}} \lambda^{\frac{\gamma}{1+\gamma}} \left(\frac{\alpha(1+\delta\bar{r}^\gamma) + \frac{2\gamma\delta}{1+\gamma}\bar{r}^{1+\gamma}}{a+\alpha} \right)^{1/(1+\gamma)} = M^* \tag{20}$$

where $M^* = T^*(1+\delta(\bar{r}^*)^\gamma)$ is a constant in country 0's variables. It is immediate to see that the left hand side is decreasing in a and increasing in T, λ, and $\bar{r}$. On the other hand, $M^* = T^*(1+\delta(\bar{r}^*)^\gamma)$ is constant in these variables but increasing in T^* and a^*. (Use (16) to check this.)

Finally, with relative prices π from (15) one can rearrange (13) to get

$$e = \frac{2\gamma}{1+\gamma} \frac{a+\alpha}{a} \frac{1}{\frac{2\gamma}{1+\gamma} + \alpha\bar{r}^{-1} + \alpha\bar{r}^{-(1+\gamma)}/\delta} \tag{21}$$

showing that the trade share e is decreasing in $\bar{r}$. Together, the identities (20) and (21) lead to the following

Proposition 2 *Country 0's trade share, e, and the number of its imported varieties, $2\bar{r}$, are increasing in a^*, and T^* and decreasing in T and λ. Further, $2\bar{r}$, is increasing in a.*

Proposition 2 establishes an unambiguously positive relation between a the number of imported varieties and productivity (per capita income) of a country. Just as in the symmetric equilibrium an individual country's trade share proves to increase in its productivity due to its increased inclination to buy imported goods, caused by rising income. Note that its terms of trade is moving against it, but the demand effect is stronger and overcompensates the adverse terms of trade effect.

The Proposition further points at the the positive effect of the ROW-income a^* on country 0's trade share and the number of its imported varieties This reflects appreciation of country 0's terms of trade due to increased demand for its exports. Similarly, an increase in the relative size of ROW countries (a drop in λ) positively affects country 0's trade through this standard channel.

The relation between country 0's trade variables and the border cost between ROW countries, T^*, is positive. This obviously is the case since an increase in T^* deviates ROW demand for imports from ROW countries towards country 0. Consequently, country 0 experiences an appreciation of its terms of trade, which leads to and increase in its trade share and the number of its imported varieties. Conversely, an increase in the trade cost T reduces country 0's trade share and the number of its imported varieties.

While Proposition 2 shows that the number of varieties imported by country 0 is increasing in its productivity a, no statement has been made about a's effect on trade share e. In fact, as can be seen easily, an increase of a can have ambiguous effects on e. Consider the following extreme case where country 0 is very unproductive and produces a negligible amount of its varieties while at the same time its rich ROW neighbors have a high demand for country 0's varieties. In that case, the equilibrium price of country 0's varieties can then exceed the price of its imports. As in country 0 individuals consume the cheapest varieties only, its trade share then equals unity. Naturally, country 0's trade share then falls as it catches up with the ROW in terms of individual income, and its trade share starts falling as individuals in country 0 begin to consume also the relatively expensive domestic varieties.

To rule out this extreme case one may want to impose that, in domestic prices, country 0's domestic varieties are less expensive than the varieties of its next neighbors at marginal output levels ($a \to 0$). Using (14), this means $T\pi \geq 1$, which is consistent with optimal consumption of ROW if (compare (18))

$$T^* p^* (1 + \delta (\bar{r}^*)^\gamma) < T p (1 + \delta \bar{\rho}^\gamma)$$

holds at $\bar{\rho} = 0$, implying i.e. $\pi M^* \leq T$. Both conditions together give $T^2 > M^*$. It turns out that this latter condition is already sufficient for the trade share e to be increasing in productivity a and one can prove the following

Proposition 3 *If condition $T^2 > M^*$ holds, country 0's trade share e is increasing in its productivity a.*

Proof. See Appendix. ■

Both, Proposition 1 and Proposition 3 stress the role of absolute productivity and per capita income as a determinant of trade share and trade volumes. Together with the dependence of $2\bar{r}$ on per capita income form Proposition 2, the present model suggests income per capita as a joint determinant that can explain the dynamics reported in Figure 1 qualitatively. With the behavior qualitative in the right direction, a quantitative assessments will be performed below, assessing how much of the observed rise can possibly be attributed to the rise in per capita income. In order to do so, the present model will on one hand be simplified by assuming linear transport cost and on the other hand be extended by introducing a continuum of goods (as opposed to varieties).

2.3 Linear Transport Cost and a Continuum of Goods

The assumption of linear transport cost (i.e. $\gamma = 1$) allows the derivation of close form solutions for the central variables e and $\bar{r}$ since equations (13) and (15) can now be written as

$$e(a) = 1 + \frac{1}{a}\left(b - \alpha\sqrt{T\pi\delta}\sqrt{a+c}\right) \qquad (22)$$

$$\bar{r}(a) = \sqrt{\frac{a+c}{\delta T \pi}} - \alpha/2 \qquad (23)$$

with the shorthand $b = \alpha + \alpha T\pi\,(\alpha\delta/2 - 1)$ and $c = \alpha + \alpha T\pi\,(\alpha\delta/4 - 1)$. These explicit expressions will prove useful in the following.

The decision to extend the model by a nontrivial goods margin is motivated by the observation That not only the number of imported varieties but the number of improted goods increases substantially as well in the course of expansion in trade volumes. This is reported by Broda and Weinstein (2004) and Kehoe and Ruhl (2002) and shown in Figure 1. As the present paper set out to provide an explanation for the rise in imported varieties *per* good, fixing the number of goods arbitrarily to a constant then biases the model in favor of replicating the results. After all, a mechanism that explains the dynamics of a ratio (R/G) by artificially setting the denominator (G) to a constant is not particularly convincing.

Thus, the model is now extended by a continuum of goods of total mass one. All goods are differentiated by domestic and foreign varieties as described above and the production of varieties takes place as in (1) for each variety and alike in all countries

$$x_{ikn} = a_i L_{ikn} \qquad\qquad n \in [0,1] \quad k \in V_i \quad i \in C$$

where n indices the goods.

Just as the single good in the previous section, each of these goods is either traded or not depending whenever condition (14) is satisfied on the goods level, i.e. when

$$\alpha(T\pi - 1) < h_n$$

holds, where h_n is the expenditure on the good considered. Clearly, a country with terms of trade π only imports varieties of goods that have an expenditure exceeding the threshold $h_o = \alpha(T\pi - 1)$. This threshold gives an endogenous partition of the set of goods into traded goods with high expenditure and nontraded goods with low expenditure.

According to these definitions, all goods are identical except for their expenditure share, which will be assumed to be constant in prices and income[4]. The distribution of expenditure can be described by a function $\varphi : (0, \infty) \to [0, \infty)$, where $\varphi(h)$ determines the mass of goods with expenditure $h > 0$. Since the total mass of goods is normalized, $\varphi(h)$ integrates to one and represents a density function. With the help of the function $\varphi(h)$ the total number of imported goods, total expenditure on foreign goods, and the total number of imported varieties, of a representative

[4] In other words, there is some Cobb-Douglas aggregator over the goods.

country can be written, respectively, as

$$G = \int_{\alpha(T\pi-1)}^{\infty} \varphi(h)\, dh$$
$$E = \int_{\alpha(T\pi-1)}^{\infty} he(h)\varphi(h)\, dh \qquad (24)$$
$$R = \int_{\alpha(T\pi-1)}^{\infty} 2\bar{r}(h)\varphi(h)\, dh$$

where $e(\,.\,)$ and $\bar{r}(\,.\,)$ are the functions from (22) and (23).

With the adequate choice of the density φ it is possible to generate arbitrary responses of each of the trade volume E (or G or R) to increases in per capita income[5]. Thus, φ must be carefully chosen in accordance to the data. It will turn out that the following truncated Power-distribution of φ captures a strong regularity in the data

$$\varphi(h) = \begin{cases} D{\cdot}h^{-\theta} & if \quad h \in [\bar{h}, d\bar{h}] \\ 0 & else \end{cases} \qquad (25)$$

where $d \in (1, \infty)$. The normalization of the mass of goods ($\int \varphi(h)dh = 1$) and the requirement that total expenditure equals income ($\int h\varphi(h)dh = a$) determine the constants

$$D = \frac{1-\theta}{1-d^{1-\theta}}\bar{h}^{\theta-1} \qquad \text{and} \qquad \bar{h} = \frac{\theta-2}{\theta-1}\frac{1-d^{1-\theta}}{1-d^{2-\theta}}a$$

while the parameters θ and d govern the shape of the distribution φ. Note the linear relationship between the $\bar{h}$ and a. This implies that comparative statics with respect to a can be performed by varying the parameter $\bar{h}$ while holding θ and d constant.

In the following, I will again consider the symmetric equilibrium characterized by $\pi = 1$. Under the assumption that not all goods are traded, i.e. when

$$\bar{h} < \alpha(T-1) < d\bar{h} \qquad (26)$$

holds, one checks with definitions (25) that the trade share and number of goods and varieties are increasing in world per capita income. In particular, one can prove the following

Proposition 4 *Assume (26) holds. In a symmetric world economy the trade share, E/a, and the number of varieties, R, are decreasing in trade costs, T and δ, constant in labor, L, and increasing in world per capita income, a. The number of goods, G, is decreasing in T, constant in δ and L, and increasing in a. Moreover, the average number of varieties per good, R/G, is increasing in world per capita income a.*

[5] In that sense, the funcion φ is the equivalent to the A-line of comparative advantage in Dornbusch Fischer and Samuelson (1977), the choice of which implies arbitrarily large responses to reductions off trade costs.

Proof. Using (22) - (24) statements concerning E/a, G, and R are immediate. The ratio R/G is increasing in $\bar{h}$ (and therefore in a) since

$$\frac{d}{d\bar{h}}\frac{R}{G} = d\left\{\frac{2\bar{r}(d\bar{h})}{R} - \frac{1}{G}\right\}\frac{R}{G}\varphi(d\bar{h}) > 0$$

and $R < \int 2\bar{r}(d\bar{h})\varphi(h)\,dh = 2\bar{r}(d\bar{h})\cdot G$. ■

The proposition states that an increase in absolute expenditure in all good categories makes some of the previously nontraded goods jump the critical level of expenditure (14) and become traded goods, increasing the number traded goods. Moreover, the number of traded varieties increases because for every traded good, the number of traded varieties increases (compare equation (23)) and on top of that, new good with new varieties are traded. By these two extensive margins and the intensive margin, the total trade share also increases.

The rise in the number of traded varieties per good, R/G, in response to an increase in a is the less intuitive result. In fact, there is an effect that tends to lead to a reduction of the average number of varieties per good: the marginal non-traded good has the mass $\varphi(\alpha(T-1)/a)$, which is larger than any traded good. Now, an incremental increase in a adds these goods with a large mass and a low number of varieties per good to the traded basket, which tends to reduce the average R.

Yet, to see that this effect does not impact the ratio R/G, take two expenditure shares h_1 and h_2 and observe that the *relative* mass of goods with the respective expenditure is constant in $\bar{h}$. Thus, the relative weights on the values $\bar{r}(h)$ do not change - except that at the upper end of the distribution the mass of goods $\varphi(d\bar{h})$ with the maximal expenditure $d\bar{h}$ are added. This margin a the upper end in fact creates the rise in the ration R/G, the average number of varieties per good.

With the model augmented by the continuum of goods, exhibiting the extensive amrgin and the "origin-margin" along the dimensions of goods and varieties, respectively, the calibration will be performed in next section. In addition, the key implication regarding per capita income is tested in order to further assess the performance the model's prediction.

3 Calibration and the Gravity Equation

The aim of this section is to evaluate the model's performance when bringing it to the data. With the help of a simple calibration exercise the first part shows that the dynamics of the US trade share and the number of imported goods and varieties can be mimicked quite well. With a log-linear approximation of the basic model, a second part derives a version of the gravity equation, augmented by per capita income. In a consecutive test, the "new" variables per capita incomes add explanatory power to the regression and the key predictions cannot be rejected.

3.1 A Calibration exercise

In the following exercise, the symmetric version of the model is used to calibrate the dynamics of US trade share, the number of imported goods and varieties. The free parameters are α, d, and the output units while the value θ and time series for T and δ are chosen in accordance to US data. Finally, US per capita output growth approximates expenditure growth in the symmetric model. By choosing world parameters of trade trade cost according to US data I follow Yi (2003). The choice of the symmetric version of the model clearly constitutes a strong simplification, yet as the growth performance of the US and the rest of the world is reasonably parallel over the period considered here (see Figure 2) the resulting error should be moderate.

The Parameter θ of the Distribution φ

In the present model the distribution of expenditure shares across goods has important consequences for the relation between income growth and trade share and the number of traded goods and varieties. The function φ must therefore be carefully chosen in accordance with the relevant data. Figure 1 has been generated with US trade data disaggregated by 22,000 goods categories classified by the "Harmonized System" (see Feenstra et al (2002)). Ideally, φ is calibrated according to the expenditure shares of these goods, but unfortunately the according data of US expenditure shares at these disaggregation levels are not available. One way to bypass this problem is to infer the underlying expenditure structure from the trade data . To this goal, observe with (22) that $e(h) \to 1$ as h grows large. Making use of this observation, the nature of φ can be read from the upper end of the distribution of the expenditure shares $e(h)$. Figure 3 plots the nominal import volumes of the HS-classification ordered by descending size[6] for the years 1972 and 2001 on a log-log scale. Apart from an upward shift of the schedule for 2001 (due to real import growth and inflation) the two curves have a similar shape and are very well approximated by a straight line with slope -1 (the solid line on top) for left part of the distribution, i.e. the goods with the larger expenditure[7]. Thus, assuming that h_n is large such that $e(h_n)$ is close enough at unity at this upper end of the distribution, the relation between the rank κ and expenditure h can inferred to satisfy approximately $\kappa \sim h^{-1}$. Consequently, the probability that the expenditure of a good, which is randomly chosen out of the pool of tradeable exceeds $\bar{h}$ is proportional to $\bar{h}^{-1}$ ($P(e(h) > \bar{h}) \sim \bar{h}^{-1}$). This implies a parameter of approximately $\theta = 2$ in the density function (25).

With this parameter choice, one can use (23) and (22) to calculate the variables defined in (24). Under the assumption that (26) holds, the total number of imported

[6] The order is different for the two years.

[7] A regression of the log expenditure on log rank and a constant of the 5% traded categories with highest expenditure a coefficient of -0.93 (-.92) with a t-value of -397.71 (-196.85) and an R^2 of 0.99 (0.99) for the year 2000 (1972). This shows thatthe approximation is not perfect but reasonably good.

varieties becomes[8]

$$R = D\left\{-\frac{1}{\sqrt{T\pi\delta}}\left[\frac{1}{\sqrt{c}}\tanh^{-1}\left(\sqrt{\frac{h}{c}+1}\right)+\frac{\sqrt{h+c}}{h}\right]+\frac{\alpha}{2h}\right\}_{\alpha(T-1)}^{d\bar{h}} \tag{27}$$

while total expenditure on tradables is

$$E = D\left\{\ln(h)-\frac{b}{h}+\alpha\sqrt{T\pi\delta}\left[\frac{1}{\sqrt{c}}\tanh^{-1}\left(\sqrt{\frac{h}{c}+1}\right)+\frac{\sqrt{h+c}}{h}\right]\right\}_{\alpha(T-1)}^{d\bar{h}} \tag{28}$$

(Remember $b = \alpha + T\pi\alpha\left(\alpha\delta/2-1\right)$ and $c = \alpha + T\pi\alpha\left(\alpha\delta/4-1\right)$. Finally, the number of imported goods is

$$G = D\cdot\left[1/(\alpha(T-1)) - 1/(d\bar{h})\right] \tag{29}$$

Trade costs T and δ

In an recent article Anderson and van Wincoop (2004) give an estimation and a decomposition of broadly defined trade costs. The authors estimate the border cost for industrialized countries to amount to a tariff equivalent of about 44% , less that 5% of which actually stems from tariffs. To these roughly 40% border cost ($T = 1.4$) the time series of tariff derived from data of Feenstra et al (2002) are added.

A standard measure for the raw border-to-border transportation costs is the cif over fob ratio (see e.g. Baier and Bergstrand (1999)) which can be derived from IMF data. However, this measure does not coincide with the per unit transportation costs δ in the present model. In fact, using (5) and (8) the cif/fob measure, when derived from one good only, depends on the parameter δ in the following way

$$\frac{cif_n}{fob_n} = \frac{\int T(1+\delta r)c_r\,dr}{\int Tc_r\,dr} = \frac{(\delta\bar{r})^2}{2\left((1+\delta\bar{r})\ln(1+\delta\bar{r})-\delta\bar{r}\right)}$$

which, with (23), is a function of the parameter of the model and can be shown[9] to be decreasing in T, constant in L, increasing in a and increasing and concave in δ. In particular, the relation between the cif/fob measure and the "true" δ is a nonlinear one. The cif/fob measure derived from all imports, is

$$\frac{cif}{fob} = \frac{\int_{\alpha(T-1)}^{d\bar{h}}(\delta\bar{r})^2\varphi(h)\,dh}{\int_{\alpha(T-1)}^{d\bar{h}}2\left((1+\delta\bar{r})\ln(1+\delta\bar{r})-\delta\bar{r}\right)\varphi(h)\,dh} \tag{30}$$

[8] Use $\int\frac{\sqrt{x+c}}{x^2}dx = -\frac{\sqrt{x+c}}{x} - \frac{1}{\sqrt{c}}\tanh^{-1}(\sqrt{x/c+1})$.

[9] Use Proposition 1 and $d\ln(cif/fob)/d\xi = -(\xi - \ln(1+\xi))/(\xi((1+\xi)\ln(1+\xi)-\xi))$ where $\xi = \delta\bar{r}$ for the statement on T, L, and a.

For fix T, a, and α and for a given data point of cif/fob, this identity determines implicitly the value of δ.

Figure 4 shows for the case for parameter values as in the "full" specification below. There is a highly non-linear and concave relation between the cif/fob measure and parameter δ - meaning that the standard cif/fob measure of transportation cost is biased upwards at low values of δ and biased downwards for high values. As a direct consequence, the measured response in the cif/fob ratio to a change in the per unit transport cost δ differs strongly according to the level of δ. In addition to this bias, border costs T and per capita income a enter the cif/fob measure. These systematic measurement problems may be worth to analyze in detail. In particular, evaluating their impact on empirical studies like Baier and Bergstrand (1999) seems to be in order. Yet, this analysis is not within the scope of the present paper and will not be further pursued here.

Instead, in the present calibration the cif/fob measure is simply scaled up to match in the year 2000 the value of 0.12 measured by Anderson and van Wincoop (2004) and then used to numerically infer the parameter δ form (30) for given per capita income, α and border costs T.

The Import Elasticity

The rise in trade volumes constitutes is considered to be a puzzle since standard models fail to explain it under realistic import elasticities. Any attempt to quantitatively address the issue must therefore consider the implied price elasticity of imports. Yi (2003) argues that the interval [2,3] constitutes a reasonable range for import elasticities and these values will be the reference in the calibration.

In the framework of the present paper's model, the important elasticity deserves special attention since for each single variety the price elasticity is endogenous and ranges in the open interval $(1, \infty)$. It is thus neither constant nor bounded[10]. It is easy to see that a varieties' price elasticities change with quantity and thereby depend, among others on the exporter's distance r and thus on their origin. Moreover, the elasticities of goods, as an aggregate of the varieties, generally differ with the expenditure level such that there is no unique definition of an import elasticity in this model.

Before the calibration, careful definition of import elasticity is therefore needed. A relatively simple and natural candidate for the definition is the elasticity derived from a uniform change in terms of trade the highest aggregation level. This gives the definition

$$\mathcal{E} = -\frac{\partial_\pi \ln(E)}{\partial_\pi \ln(Q)} \tag{31}$$

where $Q = E/C$ is the ideal price index of imports, E is expenditure on imports from (28),and C is the summation of all units of imported varieties. To derive the

[10] The demand elasticity is $-(\ln(Q))'/(\ln(P)')$ and grows unbounded as price Q approaches the level at which demand C drops to zero.

elasticity, Q and C are to be calculated. The total imported quantity is

$$C = 2\int_{\alpha(T-1)}^{d\bar{h}} \int_{0}^{\bar{r}} c_r \, dr d\varphi(h) = 2\int_{\alpha(T-1)}^{d\bar{h}} \int_{0}^{\bar{r}} \frac{\delta(r-\bar{r})}{1+\delta r} dr d\varphi(h)$$

The expression for the import elasticity is rather cumbersome and is presented in the appendix. The values for $\mathcal{E}$ are reported here along with the other calibration results.

The Calibration Targets

The three free parameters for a calibration the model are (α, d, m). α is the mass of home-produced varieties, d governs the truncation of the distribution φ, and m represent the price of consumption units in 1996 dollars. These parameters are jointly used to target the US trade volume in 1972 and 2000 and the import elasticity. Further, as the number of goods and varieties imported are represented by a continuum in the model, the actual numbers of the variables are normed to match the data at the start of the period, i.e. in the year 1972.

Calibration Results

The a basic calibration all trade costs are fixed at their 1972 values $(T, cfi/fob) = (1.45, 0.12)$ and only US per capita income growth is fed into in the model. Figure 5a illustrates the results for the calibration with $(\alpha, d, m) = (20.1, 2750, 10^6)$. The top panel shows that the trade share predicted by the model replicates the dynamics of the data very well and parallels the swings of the time series. Further, the middle and the bottom panel show model's fit regarding the number of goods and the number of varieties. While the latter is rather on the higher side towards the end of the period considered, the general fit is satisfactory. It is worth remembering that the free parameters are used to match the trade shares and import elasticity such that the successful fit of the growth in the number of goods and varieties is not result of calibration these time series. The import elasticity, however, rages between 3.1 and 4.5, which is less than some empirical estimates the literature provides estimates (Baier and Bergstrand (1999) estimate an elasticity around 6) but above the target interval [2,3].

The "full specification" now incorporates the time series of tariffs and the cif/fob measure. Figure 5b graphically reports the calibration results with $(\alpha, d, m) = (18.4, 5.7 \cdot 10^6, 10^9)$ regarding the trade share, and the number of imported goods and varieties. Again, the model is very successful in calibrating the trade share and the number of imported goods while it overstates the number of imported varieties. The import elasticity, on the other hand, drops considerably in the full specification and ranges now between 2.4 and 3.05, which I consider as a successful calibrations to the targeted range of [2,3].

With the parameters thus calibrated one can disentangle the effects of, respectively, the per capita growth, the tariff and the trade cost reduction on the trade share. This is done by freezing two of the three parameters to their 1972 level and feeding only the data of the third into the model. This exercise shows that the model attributes 4.05 percentage points (or 60.5 % of the total) of the rise in the trade share to the increase in per capita income, 2.05 percentage points (or 30.6 % of the total) to the fall in transport cost, and only 0.33 percentage points (or 4.9 % of the total) to tariff reductions.

This latter result is particularly noteworthy as it starkly contrasts the finding of Baier and Bergstrand (2004) who estimate in the framework of the gravity equation that the impact of tariff reduction by far dominates the impact of the reduction in transport cost. This calibration most obviously cannot replace a full econometric analysis, but the above findings hint at the possibility that Baier and Bergstrand's (2004) results may be sensitive to the introduction of per capita income in their estimation and to a more careful treatment of the transportation cost.

Necessary for a rigorous evaluation is the introduction of these changes into the standard empirical framework used to assess bilateral trave volumes, the gravity equation. A fist step in that direction will be done next.

3.2 Towards the Gravity Equation

Anderson and van Wincoop (2003) describe the gravity equation as "one of the most empirically successful in economics". It relates the bilateral trade volume of any pair of countries to trade costs and their respective size and it represents an authoritative framework to test theories regarding trade flows. Empirically successful international trade models must, as a minimum requirement, be consistent with the gravity equation, and, if possible, give testable predictions in its context. The next paragraphs derive a version of the gravity equation from the model developed above. In order to do so, return to the stylized and tractable setup with one single good and a continuum of varieties produced by the different countries. Further, the assumption of linear transport cost ($\gamma = 1$) will be kept.

Up to this stage, all countries but a single one (country 0) were assumed to be identical. This assumption made the model tractable and allowed a convenient formulation of its key variables. In particular, taking country 0's terms of trade ($1/\pi$) as given its trade share was described by equations (22) and (23) and generally differed from that of the homogeneous rest of the world countries. In order to derive meaningful predictions concerning bilateral trade flows, however, the model must allow for more variation in the country parameters. In particular, the country characteristics of two different countries (exporter and importer) must be able to deviate from the average.

In order to enrich the model in that respect the setup previous setup is mildly generalized by introducing a subset Z of countries with zero measure that differ form the otherwise identical ROW. This generalization allows exporter and importer prices to vary from the average while at the same time the equations that led to

(16) - (19) still go through unchanged for every country $i \in Z$, where prices p_i, productivity a_i, and population L_i have to be marked the country index $i \in Z$ now.

With the additional restriction $\gamma = 1$ the relevant equations (18) and (19) now become

$$T^*(1+\delta\bar{r}^*)\pi_i = T_i(1+\delta\bar{\rho}_i) \qquad \text{and} \qquad \bar{\rho}_i^2 = \pi_i\lambda_i\bar{r}_i^2$$

where T_i represents the symmetric border cost between country i and the ROW and distances $\bar{\rho}_i$ and $\bar{r}_i$ are those of country i relative to the ROW (i.e. $\bar{r}_i = \sup_k\{\, r_{ik} \,|\, c_{ik} > 0 \quad k \in C\backslash Z\,\}$). These two equations can be combined to

$$T_i/\pi_i + T_i\delta\sqrt{\lambda_i/\pi_i}\bar{r}_i - T^*(1+\delta\bar{r}^*) = 0 \tag{32}$$

and $\bar{r}_i$ is determined by (compare (23))

$$T_i\pi_i\left(\delta\bar{r}_i^2 + \delta\alpha\bar{r}_i + \alpha\right) - a - \alpha = 0 \tag{33}$$

Finally, the value of imports of a country $i \in Z$ form any other country $j \in C\backslash\{i\}$, measured in ROW-prices will be labeled B_{ij}. To determine B_{ij}, use the generic optimality condition (7) to get

$$B_{ij} = L_i q_{ij} c_{ij} = L_i(T_i\,(1+\delta\bar{r}_i) - T_{ij}p_j(1+\delta r_{ij})) \tag{34}$$

Remember that q_{ij} and p_i are the respective consumer and producer prices as defined in (5). To estimate equation (34) with standard techniques, one can (log-) linearize this equation around the average world variables for the variables a_i, a_j, L_i, and L_j and around $\bar{r}/2$ for r.

In a first step use (33) to get

$$\begin{aligned}\frac{\partial\bar{r}_i}{\partial\pi_i} &= -\frac{1}{\pi_i}\frac{\bar{r}_i^2+\alpha\bar{r}_i+\alpha/\delta}{2\bar{r}_i+\alpha}\\ \frac{\partial\bar{r}_i}{\partial T_i} &= -\frac{1}{T_i}\frac{\bar{r}_i^2+\alpha\bar{r}_i+\alpha/\delta}{2\bar{r}_i+\alpha}\\ \frac{\partial\bar{r}_i}{\partial a_i} &= \frac{1}{\delta T_i\pi_i}\frac{1}{2\bar{r}_i+\alpha}\end{aligned}$$

This gives with (32) the derivatives of π_i w.r.t. a_i, λ_i, and T_i around the symmetric equilibrium characterized by $a_i = \lambda_i = 1$ and $\bar{r}/2$, this is

$$\begin{aligned}\frac{d\pi_i}{da_i} &= \frac{2}{T}\frac{1}{4\bar{r}(\delta\bar{r}+1)+\alpha(3\delta\bar{r}+4)}\\ \frac{d\pi_i}{d\lambda_i} &= \frac{\delta\bar{r}(2\bar{r}+\alpha)}{4\bar{r}(\delta\bar{r}+1)+\alpha(3\delta\bar{r}+4)}\\ \frac{d\pi_i}{dT_i} &= \frac{2\bar{r}}{T}\frac{\delta\bar{r}+2}{4\bar{r}(\delta\bar{r}+1)+\alpha(3\delta\bar{r}+4)}\end{aligned} \tag{35}$$

With the help of these derivatives one gets the log-linearization (34) around the symmetric equilibrium as

$$\widehat{\ln(B_{ij})} \approx \theta_a \left(\widehat{\ln(a_i)} + \widehat{\ln(a_j)}\right) + \theta_L \left(\widehat{\ln(L_i)} + \widehat{\ln(L_j)}\right) + \theta_T \widehat{\ln(T_{ij})} - \widehat{\ln(r_{ij})} \quad (36)$$

with the coefficients

$$\begin{aligned}
\theta_a &= \frac{2}{T\delta\bar{r}} \frac{\delta\bar{r}+2}{4\bar{r}(\delta\bar{r}+1)+\alpha(3\delta\bar{r}+4)} a \\
\theta_L &= \frac{(2\bar{r}+\alpha)(\delta\bar{r}+2)}{4\bar{r}(\delta\bar{r}+1)+\alpha(3\delta\bar{r}+4)} \\
\theta_T &= -\frac{\delta\bar{r}+2}{\delta\bar{r}}
\end{aligned}$$

The hat in equation (36) stand for the deviation from the ROW-variables ($\widehat{\ln(X_{ij})} = \ln(X_{ij}) - \ln(X_{ROW})$).

The linearized model suggests with equation (36) a testable version of the gravity equation augmented by productivities a_i. For practical reasons, however, it is convenient to rewrite (36) in the variables available in the data and typically used in trade regressions, i.e. GDP and GDP per capita. To do so, consider changes in GDP due to the size of the labor force only and note that $\ln(\widehat{GDP_k}) = \ln(\widehat{p_k a L_k}) = \widehat{\ln(p_k)} + \widehat{\ln(L_k)}$. Use further $p_k = 1/\pi_k$ and (35) to confirm

$$\widehat{\ln(p_k)} = -\frac{\delta\bar{r}(2\bar{r}+\alpha)}{4\bar{r}(\delta\bar{r}+1)+\alpha(3\delta\bar{r}+4)} \widehat{\ln(L_k)}$$

such that

$$\theta_L \widehat{\ln(L_k)} \approx \frac{\bar{r}+\alpha/2}{\bar{r}+\alpha} \ln(\widehat{GDP_k})$$

Parallel calculations for the variation in a_k lead to the identity

$$\theta_a \widehat{\ln(L_k)} = \frac{\delta\bar{r}+2}{\delta\bar{r}} \frac{T(\delta\bar{r}^2+\alpha\delta\bar{r}+\alpha)-\alpha}{T(\delta\bar{r}^2+(2+\alpha\delta/2)\bar{r}+\alpha)+\alpha} \ln(\widehat{gdp_k})$$

where $gdp_k = p_k a_k$ stands for per capita GDP in country k. As total GDP picks up any variation in per capita GDP, a econometric test will estimate the following equation[11]

$$\widehat{\ln(B_{ij})} = \theta_1 \left(\ln(\widehat{GDP_i}) + \ln(\widehat{GDP_j})\right) + (\theta_2 - \theta_1) \left(\ln(\widehat{gdp_i}) + \ln(\widehat{gdp_j})\right) - \widehat{\ln(r_{ij})} \quad (37)$$

[11] In (36) and te following test the variation in border costs T is suppressed for simplicity.

with the coefficients

$$\theta_1 = \frac{\bar{r} + \alpha/2}{\bar{r} + \alpha}$$
$$\theta_2 = \frac{\delta\bar{r} + 2}{\delta\bar{r}} \frac{T(\delta\bar{r}^2 + \alpha\delta\bar{r} + \alpha) - \alpha}{T(\delta\bar{r}^2 + (2 + \alpha\delta/2)\bar{r} + \alpha) + \alpha}$$

It is quick to check that the parameters are within the following range $\theta_1 \in (1/2, 1)$ and $\theta_2 \geq 1$ such that $\theta_2 - \theta_1 \geq 1/2$.

Thus, the econometric model derived from the theory is

$$\widehat{\ln(B_{ij})} = \beta_0 + \beta_1 \left(\widehat{\ln(gdp_i)} + \widehat{\ln(gdp_j)} \right) ... \quad (38)$$
$$... + \beta_2 \left(\ln(\widehat{GDP_i}) + \ln(\widehat{GDP_j}) \right) - \beta_3 \widehat{\ln(r_{ij})} + \varepsilon_{ij}$$

where ε_{ij} is interpreted as a measurement error. Equation (38) is estimated with standard OLS using data of nominal bilateral trade volumes on a yearly basis between 1962 and 2000 for 150 countries (for a description of the data see Feenstra et al (2005)). This gives about a quarter of a million observations. There are six specifications of (38) reported below, that differ slightly from each other. All estimations include time dummies.

Estimation Results

When reviewing the regression results two predictions on the coefficients β_i differ from the standard models and therefore deserve particular attention. These predictions are, first, that the impact of per capita income is positive and between one half and uity ($\beta_1 \in (0.5, 1)$), second, that the coefficient on total GDP is positive but may be smaller than unity ($\beta_2 \geq 1/2$). A third prediction concerns the parameter on distance, which equals one $\beta_3 = 1$. It is noteworthy that this prediction is independent of the actual transport cost δ. For comparison the "traditional" gravity equation, i.e.

$$\ln(B_{ij}) = (\ln(GDP_i) + \ln(GDP_j)) - \ln(r_{ij})$$

are also reported.

Table 1 shows the estimates of equation (38). Compared with that traditional gravity equation (column 1) the inclusion of the per capita incomes (column 3) raises the R^2 from 0.53 to 0.59. The first interesting observation is that the estimated coefficients of the sum of log per capita incomes are significant on all conventional levels and, as predicted, larger than one half and smaller than unity. As second observation, the estimated coefficients on total national incomes drop significantly when introducing per capita income and are much lower than unity. This contradicts the traditional gravity equation but is within the range predicted by the current model. Finally, as usual in the gravity regressions, the estimate of the coefficient on distance is close to unity and the null that it is one cannot be rejected on the

5% confidence level in specifications 4 and 6. Moreover, columns 5 and 6 report estimations of the augmented gravity equation including per capita income and total GDP of the source country, showing that the estimates of the individual country variables are very close to each other. In fact, the estimated coefficient of per capita income of the source country can only be statistically distinguished from zero if landlocked-dummies are included.

This paper started out with the aim to jointly explain the rise in US trade volumes and the number of traded goods and varieties. The motivation was further based on the observation by Haveman and Hummels (1999) that countries purchase only a small fraction of varieties available on the world market. The model developed here could account for this observation by introducing a mild modification of standard consumer preferences, which gave rise to some novel results. Alternative approaches to address this observation can be based on the now popular assumptions of fixed cost to enter export markets (see e.g. Melitz (2003)), which generally implies that strict subsets of available varieties are purchased and consumed in each country. In this class of models, however, the set of imported varieties is determined by trade costs and the size of the export market (i.e. the importer's GDP); per capita income becomes entirely inessential (see Chaney (2005)). The fact that the coefficient on the per capita income of the importing country is estimated to be positive and highly significant (in both, statistical and economic terms) can be viewed as supportive to the approach pursued in the present paper.

4 Conclusion

The present paper has simultaneously addressed the substantial expansion along the "origin-margin" - i.e. the increase in the number of source countries per imported good - and the impressive growth in aggregate trade volumes over the past decades. Per capita income growth has been suggested as a joint determinant for both observations. The paper's mechanism is based on the central assumption that varieties are nonessential in the consumers' utility function. Under ex-ante identical preferences, consumers minimize trade cost and import form neighboring countries only. Richer consumers, however, are willing to incur higher transport costs and purchase varieties from a larger set of trade partners, such that the set of imported varieties grows with per capita income. This "origin-margin" of imports implies then that the marginal utility of imports exhibits milder decreasing returns than the marginal utility of domestic goods. Consequently, growth in per capita income raises the trade share of a given country via the effects of the origin-margin, inherently linking both variables, aggregate trade volumes and imported varieties per good, which thus move together.

Testing the effects of per capita income on the bilateral trade flows within the framework of the gravity equation provided support for the derived hypothesis. Finally, the calibrated model is able to essentially replicate the dynamics of US imports, and the number of imported goods and varieties between 1972 and 2000.

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A Appendix

The Import Elasticity The ideal price index for the bundel of imported goods is

$$Q = E/C \tag{A1}$$

where E is country 0's expenditure on imports (28) and C is the total sum of imported quantities. The price change defining the price elasticity of imports is assumed to stem from a terms of trade shock that is uniform across exporting countries. When $d/d\pi$ is denoted by a prime of variables in question, this means that the price elasticity of imports is $\mathcal{E} = -(\ln(C))'/(\ln(Q))'$ or

$$\mathcal{E} = \frac{\ln(C)'}{\ln(C)' - \ln(E)'}$$

Using (28), derivatives of E are quickly calculated and

$$\frac{1}{D}\frac{dE}{d\pi} = \left[-\frac{4T}{h} + \sqrt{T}\left\{\tanh^{-1}(\sqrt{h/4+1}) + 2\frac{\sqrt{h+4}}{h}\right\}\right]_{4(T-1)}^{\bar{h}} \ldots$$
$$\ldots - 4T\left\{\ln(4(T-1)) + 1 + 2\sqrt{T}\tanh^{-1}(\sqrt{T})\right\}$$

Now define the quantity of total imports as the sum over all goods and varieties:

$$C = 2\int_{4(T-1)}^{\bar{h}}\int_{0}^{\bar{r}(h)} c(r,h)\,dr\,d\varphi(h) \tag{A3}$$

With the expression of $\bar{r}$ from (23), one can take the derivative of C with respect to import prices π at the symmetric equilibrium ($\pi = 1$)

$$\frac{dC}{d\pi} = 2\int_{4(T-1)}^{\bar{h}}\int_{0}^{\bar{r}(h)} \frac{dc(r,h)}{d\bar{r}}\frac{d\bar{r}}{d\pi}\varphi(h)\,dr\,d\varphi(h)\ldots$$
$$\ldots + 2\int_{4(T-1)}^{\bar{h}} c(\bar{r},h)\frac{d\bar{r}}{d\pi}\,d\varphi(h) - 4T\int_{0}^{\bar{r}(4(T-1))} c(r,h)\,dr\,\varphi(4(T-1))$$

The two last terms equal zero such that, with $d\bar{r}/d\pi$ from (23) and $\varphi(h)$ from (25)

$$\frac{dC}{d\pi} = D\int_{4(T-1)}^{\bar{h}} \frac{\sqrt{(h+4)/T}}{h^2}\ln\left(\sqrt{(h+4)/T} - 1\right)dh \tag{A4}$$

Now, use (7) and (23) to get the import quanitiy (A3) for the symmetric equilibrium

$$C = 2D \int_{4(T-1)}^{\bar{h}} \frac{1}{h^2} \left[(1+\bar{r})\ln(1+\bar{r}) - \bar{r}\right] dh$$

$$= 2D \int_{4(T-1)}^{\bar{h}} \frac{1}{h^2} \left[\left(\sqrt{\tfrac{h+4}{T}} - 1\right) \ln\left(\sqrt{\tfrac{h+4}{T}} - 1\right) - \sqrt{\frac{h+4}{T}} + 2\right] dh$$

$$= 2\frac{dC}{d\pi} - 2D \int_{4(T-1)}^{\bar{h}} \frac{\ln\left(\sqrt{\tfrac{h+4}{T}} - 1\right)}{h^2} dh - 2D \int_{4(T-1)}^{\bar{h}} \frac{1}{h^2}\sqrt{\frac{h+4}{T}} dh - 4D \left[\frac{1}{h}\right]_{4(T-1)}^{\bar{h}}$$

The two integrals can be solved and the whole expeession becomes

$$C = 2\frac{dC}{d\pi} - 4D\left[\frac{1}{h}\right]_{4(T-1)}^{\bar{h}} \ldots$$

$$-2D\frac{1}{4-T}\left[\frac{\sqrt{T}}{2}\tanh^{-1}\left(\sqrt{\tfrac{h}{4}+1}\right) - \frac{1}{2}\ln(h) \ldots\right.$$

$$\left. \ldots + \ln(\sqrt{h+4} - \sqrt{T}) - \frac{4-T}{h}\ln\left(\sqrt{\frac{h+4}{T}} - 1\right)\right]_{4(T-1)}^{\bar{h}}$$

$$\ldots + 2D\left[\frac{\sqrt{h+4}}{h} + \frac{1}{2}\tanh^{-1}(\sqrt{h/4+1})\right]_{4(T-1)}^{\bar{h}}$$

Unfortunately, there is no close form solution for the derivative $dC/d\pi$ in (A4). It can, however be written as

$$\frac{dC}{d\pi} = 2D\left\{\left(\ln\left(\sqrt{\tfrac{h+4}{T}} - 1\right) - 1\right)\left(\sqrt{\tfrac{h+4}{T}} - 1\right) \ldots\right.$$

$$\ldots + \ln\left(\sqrt{\tfrac{h+4}{T}} - 1\right) 2\left(\tanh^{-1}(\sqrt{h/4+1}) - \tanh^{-1}(\sqrt{T}/2)\right) \ldots$$

$$\left. \ldots + \mathrm{Li}_2\left(\frac{\sqrt{h+4} - \sqrt{T}}{\sqrt{T} - 2}\right) - \mathrm{Li}_2\left(\frac{-\sqrt{h+4} + \sqrt{T}}{\sqrt{T} + 2}\right)\right\}_{4(T-1)}^{\bar{h}}$$

where Li_n is the Polylogarithmic function $\mathrm{Li}_n(x) = \sum_{k=1}^{\infty} z^k/k^n$. Combining finally E, $dE/d\pi$, C, and $dC/d\pi$ gives the price elasticity of imports $\mathcal{E}$.

Proof of Proposition 3. Show: for $T^2/M^* \geq 1$ the trade share e of country 0 is increasing in a. Use (18) and (19) to write (15) as

$$a = T\left[\frac{2\gamma}{\gamma+1}\frac{\delta}{\lambda}\bar{\rho}^{\gamma+1} + \frac{\alpha\delta}{\lambda^{\gamma/(\gamma+1)}}\bar{\rho}^{\gamma}\left(\frac{T}{M}(1+\delta\bar{\rho}^{\gamma})\right)^{\frac{1}{\gamma+1}} + \alpha\delta\bar{\rho}^{\gamma}\frac{T}{M^*}\right] + \alpha\left(\frac{T^2}{M^*}-1\right)$$

As the left hand sinde is increasing in $\bar{\rho}$, this means that $\bar{\rho}$ is increasing in a. Further, one gets

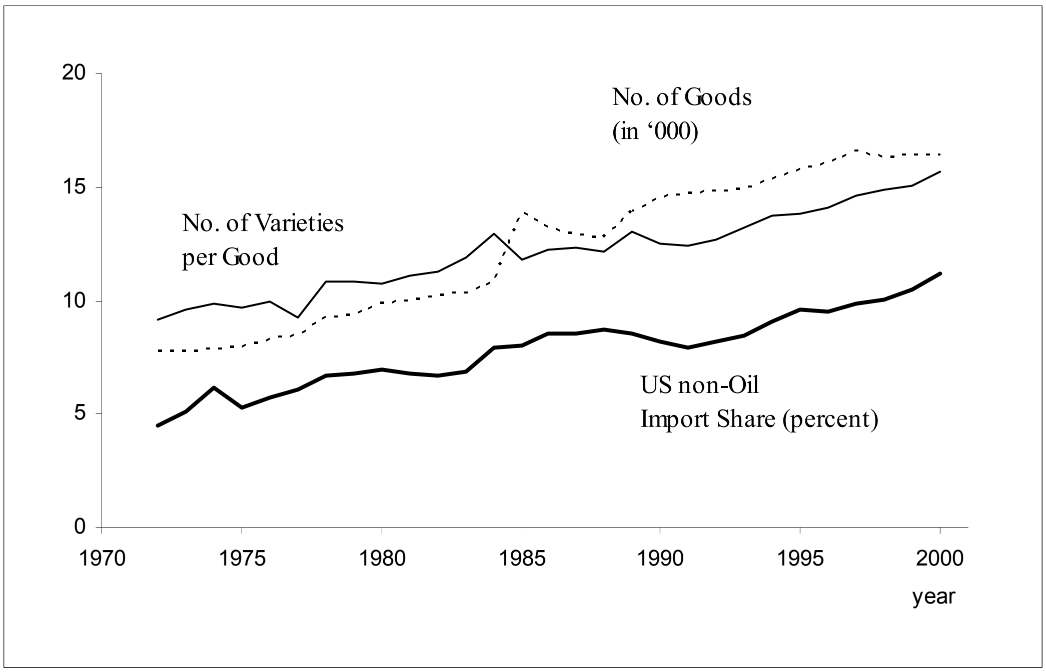
$$\frac{\bar{\rho}^{\gamma+1}}{a} = \left\{T\left[\frac{2\gamma}{\gamma+1}\frac{\delta}{\lambda} + \frac{\alpha\delta}{\lambda^{\frac{\gamma}{\gamma+1}}}\left(\frac{\frac{T}{M}(1+\delta\bar{\rho}^{\gamma})}{\bar{\rho}^{\gamma+1}}\right)^{\frac{1}{\gamma+1}} + \frac{\alpha\delta}{\bar{\rho}}\frac{T}{M^*}\right] + \frac{\alpha}{\bar{\rho}^{\gamma+1}}\left(\frac{T^2}{M^*}-1\right)\right\}^{-1}$$

Since $T^2 \geq M^*$, the expression on the left hand side is increasing in $\bar{\rho}$ and therefore in a. Rewriting finally (13) with (19) as

$$e = \frac{2\gamma}{\gamma+1}\delta T\frac{\bar{\rho}^{\gamma+1}}{\lambda a}$$

this proves the statement in Proposition 3.

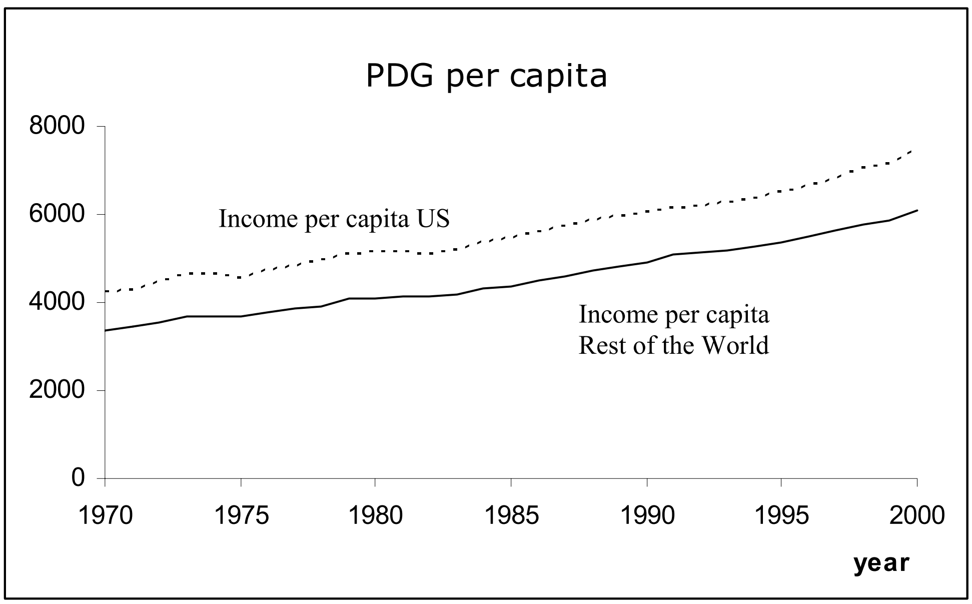
Figure 1



20
15
10
5
0
1970
1975
1980
1985
1990
1995
2000
year
No. of Goods
(in '000)
No. of Varieties
per Good
US non-Oil
Import Share (percent)

Figure 2

Income per capita in US and the Rest of the World



Source: PWT 6.1

Figure 3

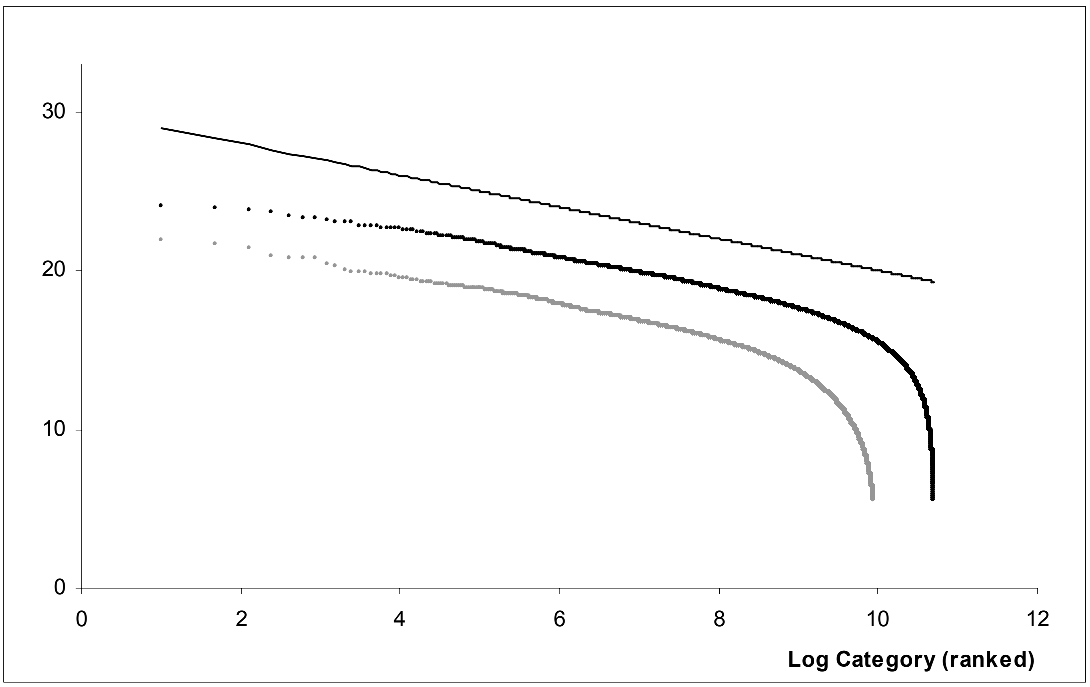


Figure 4

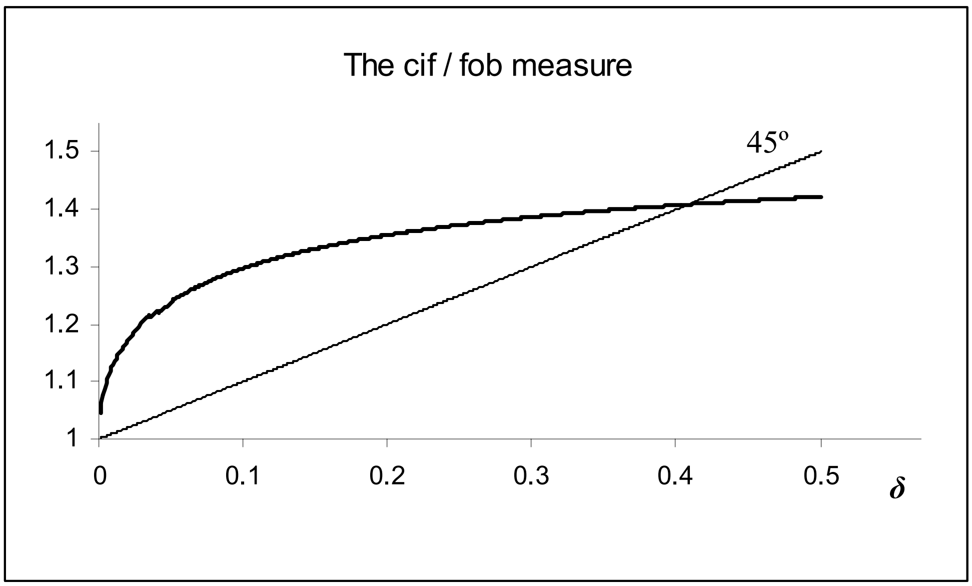
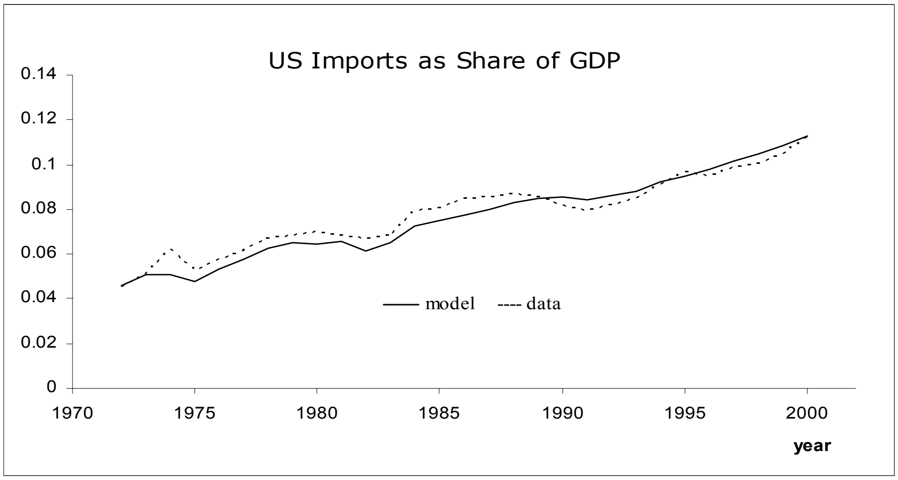
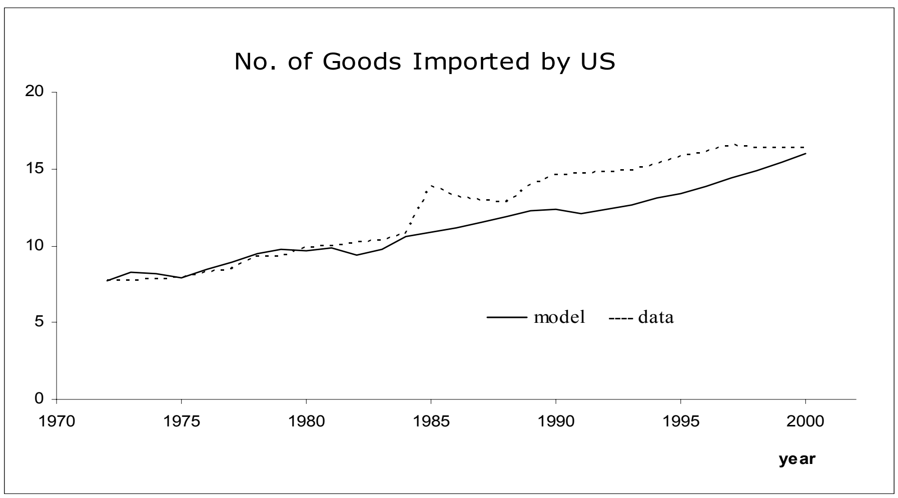


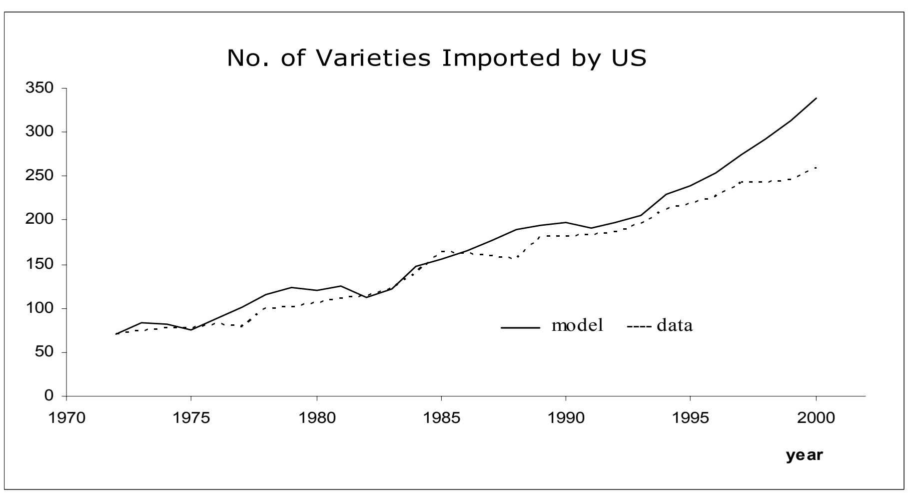
Figure 5a



US Imports as Share of GDP
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1970
1975
1980
1985
1990
1995
2000
year
model
---- data

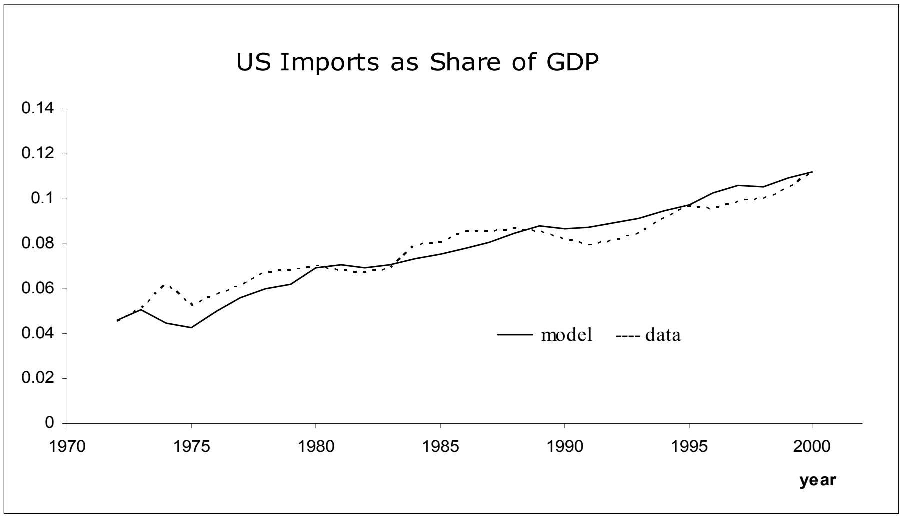


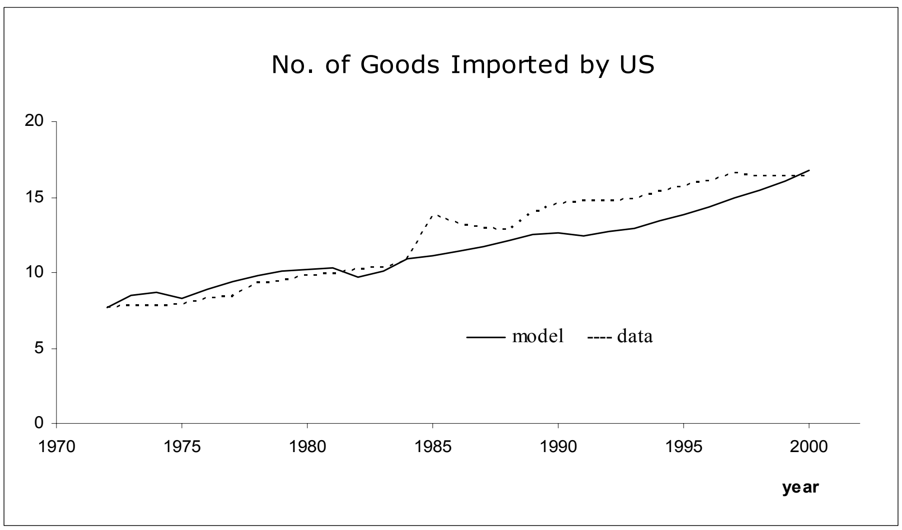
No. of Goods Imported by US
20
15
10
5
0
1970
1975
1980
1985
1990
1995
2000
year
model
---- data



No. of Varieties Imported by US
350
300
250
200
150
100
50
0
1970
1975
1980
1985
1990
1995
2000
year
model
---- data

Figure 5b


US Imports as Share of GDP
0.14
0.12
0.1
0.08
0.06
0.04
0.02
0
1970
1975
1980
1985
1990
1995
2000
year
model
---- data


No. of Goods Imported by US
20
15
10
5
0
1970
1975
1980
1985
1990
1995
2000
year
model
---- data

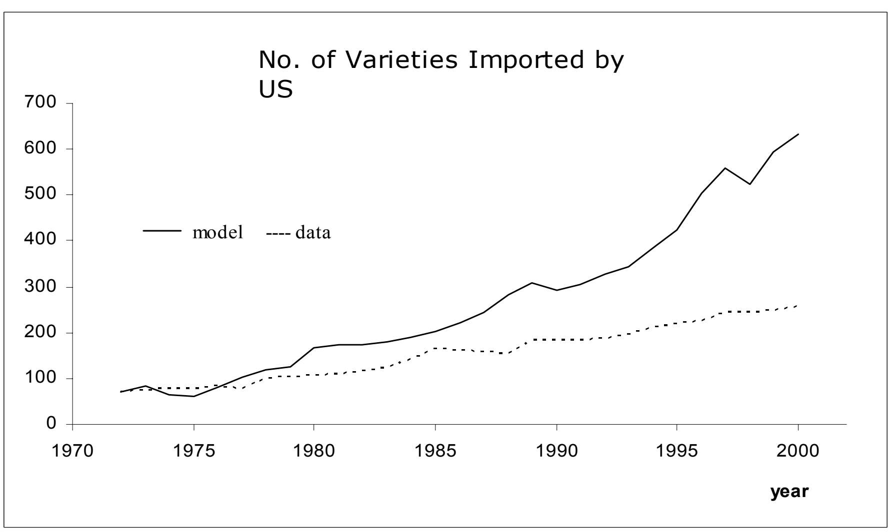

No. of Varieties Imported by US
700
600
500
400
300
200
100
0
1970
1975
1980
1985
1990
1995
2000
year
model
---- data

Table 1

	(1)	(2)	(3)	(4)	(5)	(6)
gdp i + gdp j		0.61	0.58	0.57	0.58	0.56
		(176.88)	(179.69)	(175.79)	(132.8)	(123.98)
GDPi + GDPj	0.93	0.77	0.81	0.80	0.77	0.76
	(501.45)	(381.79)	(430.61)	(424.65)	(309.37)	(303.89)
dist ij	-1.02		-0.98	-0.99	-0.98	-1.00
	(-199.84)		(-202.48)	(-205.2)	(-203.5)	(-206.34)
llock i				-0.37		-0.44
				(-31.09)		(-36.62)
llock j				-0.23		-0.15
				(-19.01)		(-12.45)
gdp p.c. j					0.01	0.02
					(1.52)	(3.57)
GDP j					0.08	0.09
					(24.97)	(26.63)
const	-17.35	-30.74	-23.21	-22.53	-23.28	-22.61
	(-217.43)	(-381.91)	(-282.68)	(-269.94)	(-285.86)	(-272.27)
t-Dummies	YES	YES	YES	YES	YES	YES
N	263431	263431	263431	263431	263431	263431
Rsq adj.	0.54	0.53	0.59	0.59	0.59	0.59

Dependent variable: Log(imports) of country i from country j.
All explenatory variables are in Logs; t-values are in brackets below