

The Basic New Keynesian Model

by

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Motivation and Outline

Evidence on Money, Output, and Prices:

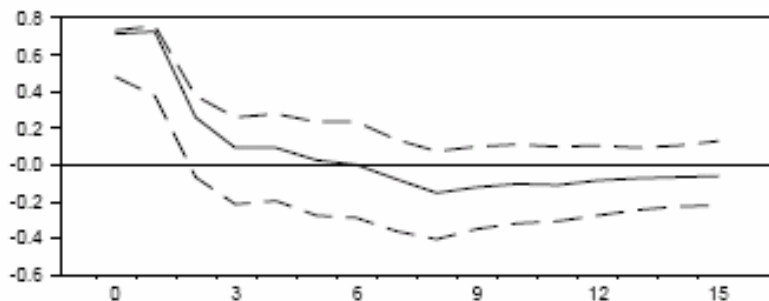
- Short Run Effects of Monetary Policy Shocks
 - (i) persistent effects on real variables
 - (ii) slow adjustment of aggregate price level
 - (iii) liquidity effect
- Micro Evidence on Price-setting Behavior: significant price and wage rigidities

Failure of Classical Monetary Models

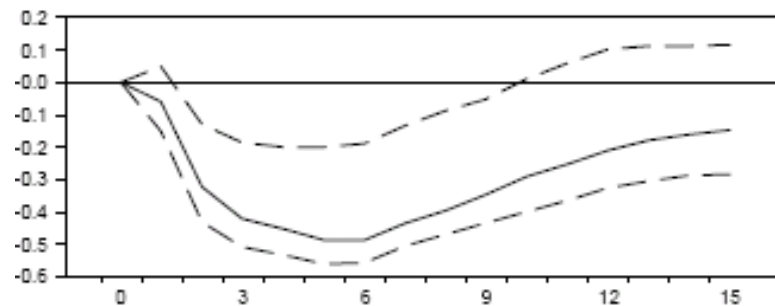
A Baseline Model with Nominal Rigidities

- monopolistic competition
- sticky prices (staggered price setting)
- competitive labor markets, closed economy, no capital accumulation

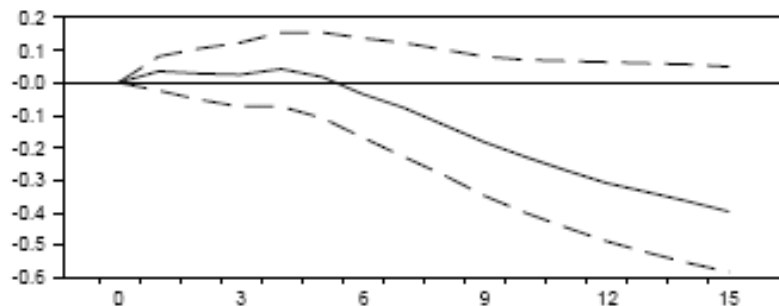
Figure 1. Estimated Dynamic Response to a Monetary Policy Shock



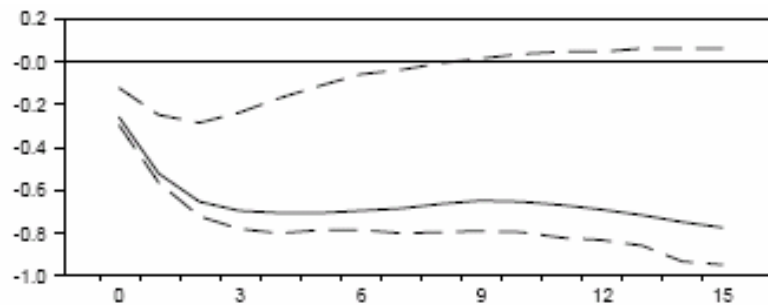
Federal funds rate



GDP

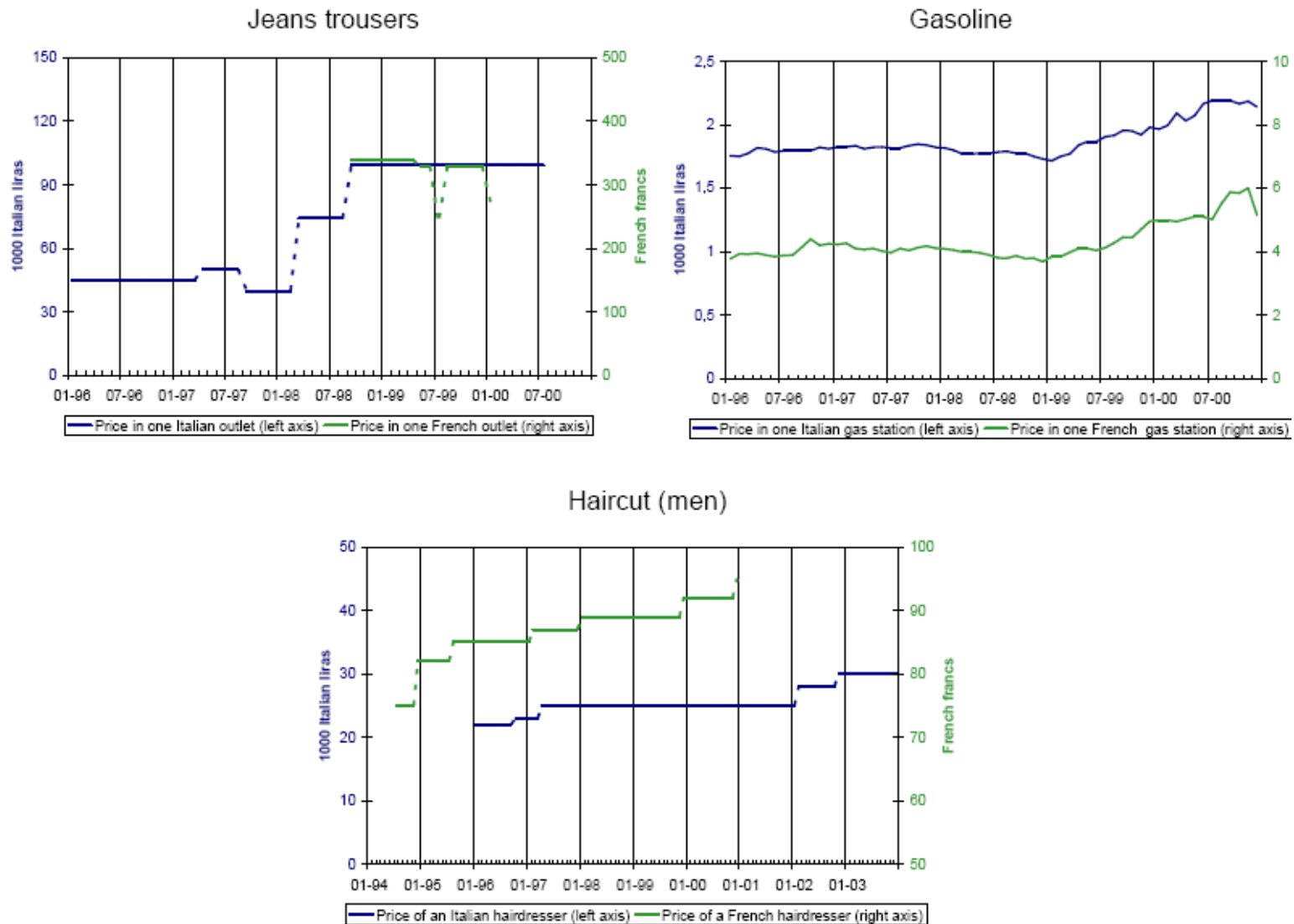


GDP deflator



M2

Figure 1 - Examples of individual price trajectories (French and Italian CPI data)



Note : Actual examples of trajectories, extracted from the French and Italian CPI databases. The databases are described in Baudry *et al.* (2004) and Veronese *et al.* (2005). Prices are in levels, denominated in French Francs and Italian Lira respectively. The dotted lines indicate events of price changes.

Households

Representative household solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

where

$$C_t \equiv \left[\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}$$

subject to

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + W_t N_t - T_t$$

for $t = 0, 1, 2, \dots$ plus solvency constraint.

Optimality conditions

1. *Optimal allocation of expenditures*

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t$$

implying

$$\int_0^1 P_t(i) C_t(i) di = P_t C_t$$

where

$$P_t \equiv \left[\int_0^1 P_t(i)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}}$$

2. *Other optimality conditions*

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

$$Q_t = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\}$$

Specification of utility:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

implied log-linear optimality conditions (aggregate variables)

$$w_t - p_t = \sigma c_t + \varphi n_t$$
$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho)$$

where $i_t \equiv -\log Q_t$ is the *nominal interest rate* and $\rho \equiv -\log \beta$ is the *discount rate*.

Ad-hoc money demand

$$m_t - p_t = y_t - \eta i_t$$

Firms

- Continuum of firms, indexed by $i \in [0, 1]$
- Each firm produces a differentiated good
- Identical technology

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

- Probability of resetting price in any given period: $1 - \theta$, independent across firms (Calvo (1983)).
- $\theta \in [0, 1]$: index of price stickiness
- Implied average price duration $\frac{1}{1-\theta}$

Aggregate Price Dynamics

$$P_t = [\theta(P_{t-1})^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}]^{\frac{1}{1-\epsilon}}$$

Dividing by P_{t-1} :

$$\Pi_t^{1-\epsilon} = \theta + (1-\theta) \left(\frac{P_t^*}{P_{t-1}} \right)^{1-\epsilon}$$

Log-linearization around zero inflation steady state

$$\pi_t = (1-\theta)(p_t^* - p_{t-1}) \tag{1}$$

or, equivalently

$$p_t = \theta p_{t-1} + (1-\theta)p_t^*$$

Optimal Price Setting

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} \left(P_t^* Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}) \right) \right\}$$

subject to

$$Y_{t+k|t} = (P_t^* / P_{t+k})^{-\epsilon} C_{t+k}$$

for $k = 0, 1, 2, \dots$ where

$$Q_{t,t+k} \equiv \beta^k \left(\frac{C_{t+k}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+k}} \right)$$

Optimality condition:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} \left(P_t^* - \mathcal{M} \psi_{t+k|t} \right) \right\} = 0$$

where $\psi_{t+k|t} \equiv \Psi'_{t+k}(Y_{t+k|t})$ and $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$

Equivalently,

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ Q_{t,t+k} Y_{t+k|t} \left(\frac{P_t^*}{P_{t-1}} - \mathcal{M} MC_{t+k|t} \Pi_{t-1,t+k} \right) \right\} = 0$$

where $MC_{t+k|t} \equiv \psi_{t+k|t}/P_{t+k}$ and $\Pi_{t-1,t+k} \equiv P_{t+k}/P_{t-1}$

Perfect Foresight, Zero Inflation Steady State:

$$\frac{P_t^*}{P_{t-1}} = 1 \quad ; \quad \Pi_{t-1,t+k} = 1 \quad ; \quad Y_{t+k|t} = Y \quad ; \quad Q_{t,t+k} = \beta^k \quad ; \quad MC = \frac{1}{\mathcal{M}}$$

Log-linearization around zero inflation steady state:

$$p_t^* - p_{t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{ \widehat{mc}_{t+k|t} + p_{t+k} - p_{t-1} \}$$

where $\widehat{mc}_{t+k|t} \equiv mc_{t+k|t} - mc$.

Equivalently,

$$p_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{ mc_{t+k|t} + p_{t+k} \}$$

where $\mu \equiv \log \frac{\epsilon}{\epsilon-1}$.

Flexible prices ($\theta = 0$):

$$p_t^* = \mu + mc_t + p_t$$

$\implies mc_t = -\mu$ (symmetric equilibrium)

Particular Case: $\alpha = 0$ (constant returns)

$$\implies MC_{t+k|t} = MC_{t+k}$$

Rewriting the optimal price setting rule in recursive form:

$$p_t^* = \beta\theta E_t\{p_{t+1}^*\} + (1 - \beta\theta)\widehat{m}c_t + (1 - \beta\theta)p_t \quad (2)$$

Combining (1) and (2):

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \lambda\widehat{m}c_t$$

where

$$\lambda \equiv \frac{(1 - \theta)(1 - \beta\theta)}{\theta}$$

Generalization to $\alpha \in (0, 1)$ (decreasing returns)

Define

$$\begin{aligned} mc_t &\equiv (w_t - p_t) - mpn_t \\ &\equiv (w_t - p_t) - \frac{1}{1-\alpha}(a_t - \alpha y_t) - \log(1-\alpha) \end{aligned}$$

Using $mc_{t+k|t} = (w_{t+k} - p_{t+k}) - \frac{1}{1-\alpha}(a_{t+k} - \alpha y_{t+k|t}) - \log(1-\alpha)$,

$$\begin{aligned} mc_{t+k|t} &= mc_{t+k} + \frac{\alpha}{1-\alpha}(y_{t+k|t} - y_{t+k}) \\ &= mc_{t+k} - \frac{\alpha\epsilon}{1-\alpha}(p_t^* - p_{t+k}) \end{aligned} \tag{3}$$

Implied inflation dynamics

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \lambda \widehat{mc}_t \tag{4}$$

where

$$\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$$

Equilibrium

Goods markets clearing

$$Y_t(i) = C_t(i)$$

for all $i \in [0, 1]$ and all t .

Letting $Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$,

$$Y_t = C_t$$

for all t . Combined with the consumer's Euler equation:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) \tag{5}$$

Labor market clearing

$$\begin{aligned} N_t &= \int_0^1 N_t(i) \, di \\ &= \int_0^1 \left(\frac{Y_t(i)}{A_t} \right)^{\frac{1}{1-\alpha}} \, di \\ &= \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\epsilon}{1-\alpha}} \, di \end{aligned}$$

Taking logs,

$$(1 - \alpha)n_t = y_t - a_t + d_t$$

where $d_t \equiv (1 - \alpha) \log \int_0^1 (P_t(i)/P_t)^{-\frac{\epsilon}{1-\alpha}} \, di$ (second order).

Up to a first order approximation:

$$y_t = a_t + (1 - \alpha) n_t$$

Marginal Cost and Output

$$\begin{aligned} mc_t &= (w_t - p_t) - mpn_t \\ &= (\sigma y_t + \varphi n_t) - (y_t - n_t) - \log(1 - \alpha) \\ &= \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) y_t - \frac{1 + \varphi}{1 - \alpha} a_t - \log(1 - \alpha) \end{aligned} \quad (6)$$

Under *flexible prices*

$$mc = \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) y_t^n - \frac{1 + \varphi}{1 - \alpha} a_t - \log(1 - \alpha) \quad (7)$$

$$\implies y_t^n = -\delta_y + \psi_{ya} a_t$$

where $\delta_y \equiv \frac{(\mu - \log(1 - \alpha))(1 - \alpha)}{\sigma + \varphi + \alpha(1 - \sigma)} > 0$ and $\psi_{ya} \equiv \frac{1 + \varphi}{\sigma + \varphi + \alpha(1 - \sigma)}$.

$$\implies \widehat{mc}_t = \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) (y_t - y_t^n) \quad (8)$$

where $y_t - y_t^n \equiv \tilde{y}_t$ is the *output gap*

New Keynesian Phillips Curve

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \tilde{y}_t \quad (9)$$

where $\kappa \equiv \lambda \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right)$.

Dynamic IS equation

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma} (i_t - E_t\{\pi_{t+1}\} - r_t^n) \quad (10)$$

where r_t^n is the *natural rate of interest*, given by

$$\begin{aligned} r_t^n &\equiv \rho + \sigma E_t\{\Delta y_{t+1}^n\} \\ &= \rho + \sigma \psi_{ya} E_t\{\Delta a_{t+1}\} \end{aligned}$$

Missing block: description of monetary policy (determination of i_t).

Equilibrium under a Simple Interest Rate Rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t \quad (11)$$

where v_t is exogenous (possibly stochastic) with zero mean.

Equilibrium Dynamics: combining (9), (10), and (11)

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_T(\widehat{r}_t^n - v_t) \quad (12)$$

where

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix} \quad ; \quad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

and $\Omega \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}$

Uniqueness $\iff \mathbf{A}_T$ has both eigenvalues within the unit circle

Given $\phi_\pi \geq 0$ and $\phi_y \geq 0$, (Bullard and Mitra (2002)):

$$\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0$$

is necessary and sufficient.

Effects of a Monetary Policy Shock

Set $\widehat{r}_t^n = 0$ (no real shocks).

Let v_t follow an AR(1) process

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

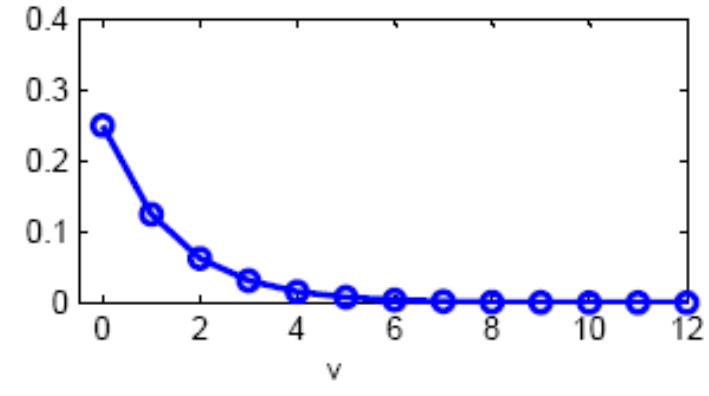
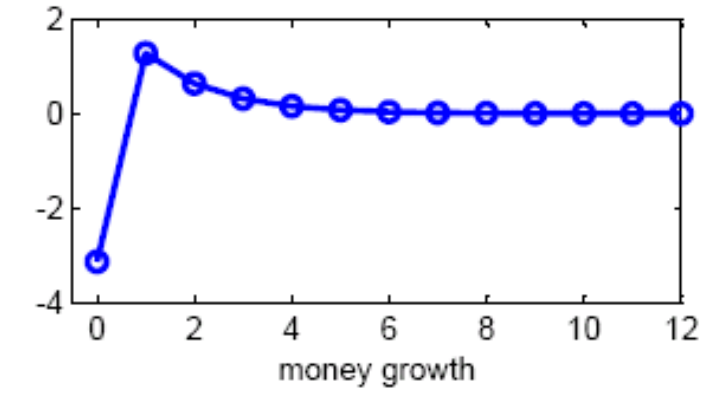
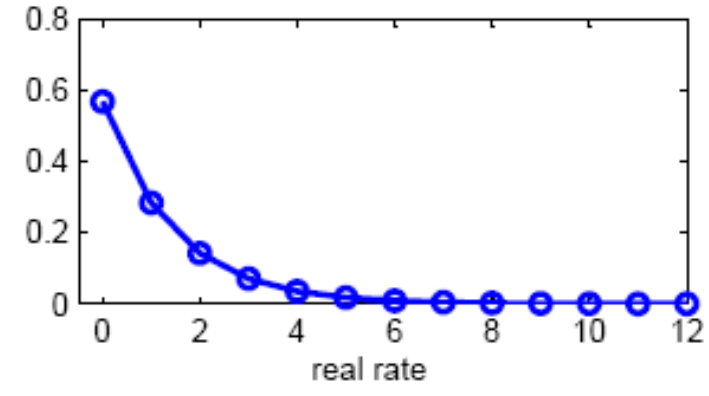
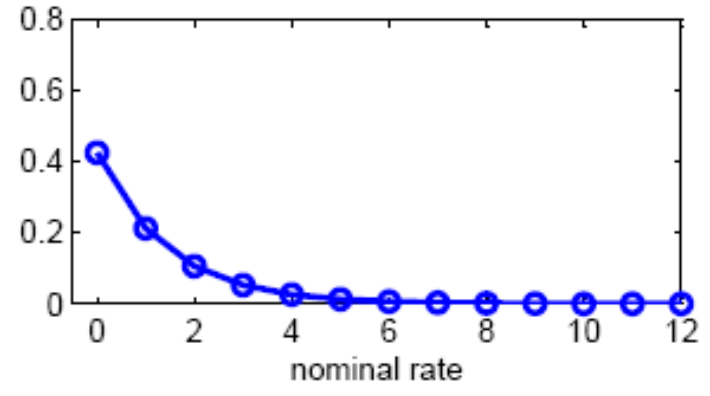
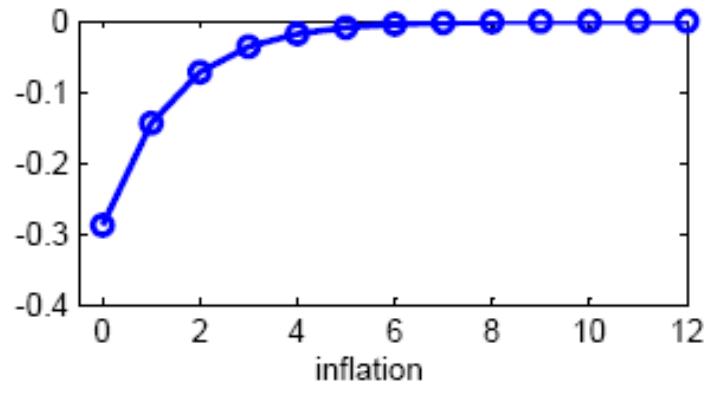
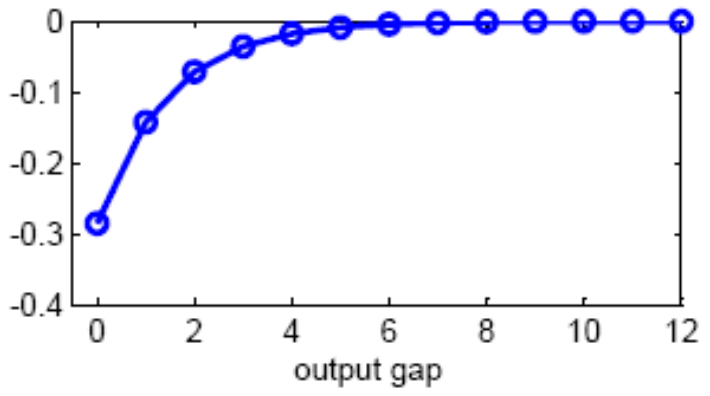
Calibration:

$$\rho_v = 0.5, \phi_\pi = 1.5, \phi_y = 0.5/4, \beta = 0.99, \sigma = \varphi = 1, \theta = 2/3, \eta = 4.$$

Dynamic effects of an exogenous increase in the nominal rate (Figure 1).

Exercise: analytical solution

Figure 3.1: Effects of a Monetary Policy Shock (Interest Rate Rule)



Effects of a Technology Shock

Set $v_t = 0$ (no monetary shocks).

Technology process:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a.$$

Implied natural rate:

$$\widehat{r}_t^n = -\sigma \psi_{ya} (1 - \rho_a) a_t$$

Dynamic effects of a technology shock ($\rho_a = 0.9$) (Figure 2)

Exercise: AR(1) process for Δa_t

Figure 3.2: Effects of a Technology Shock (Interest Rate Rule)

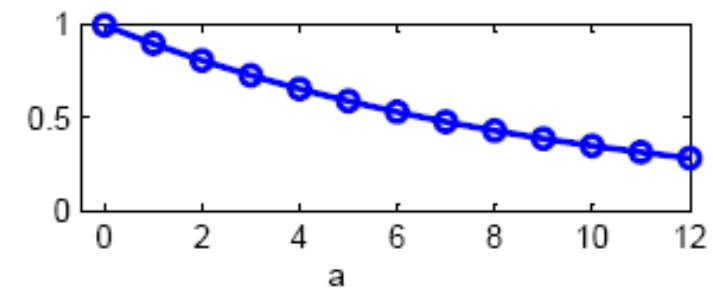
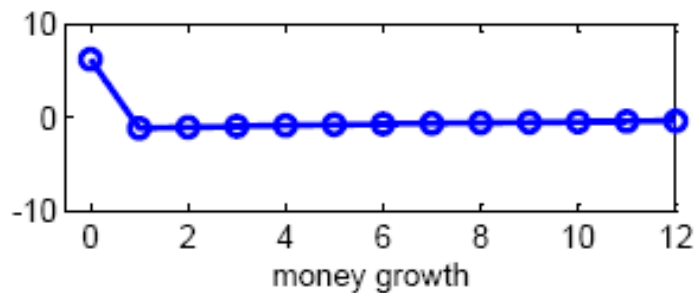
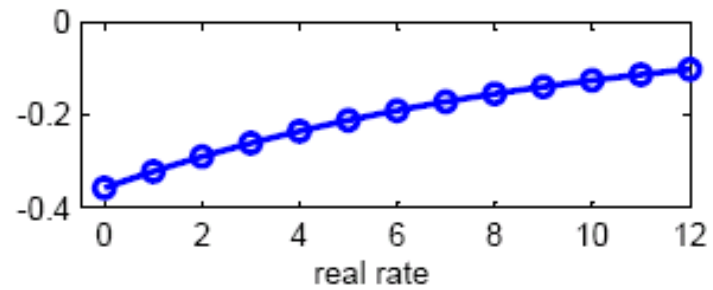
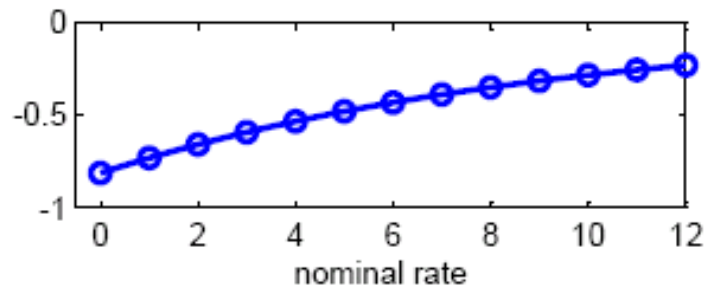
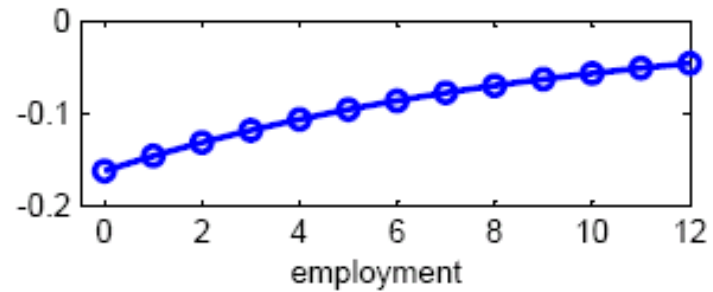
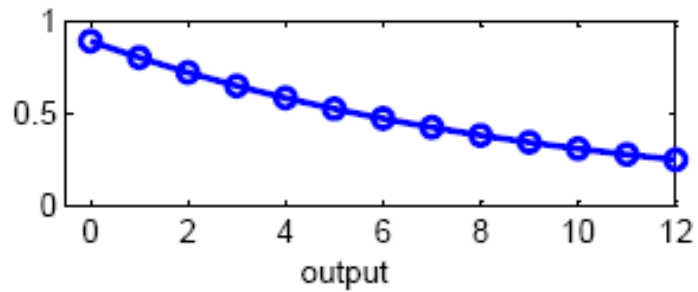
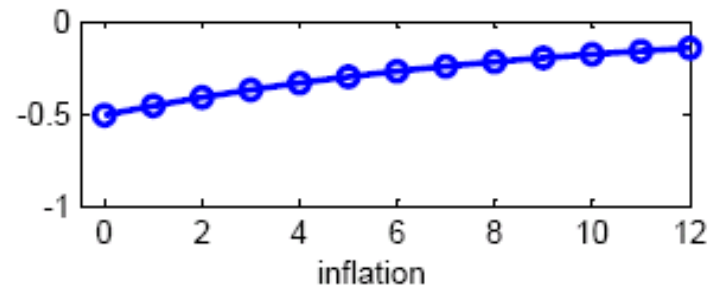
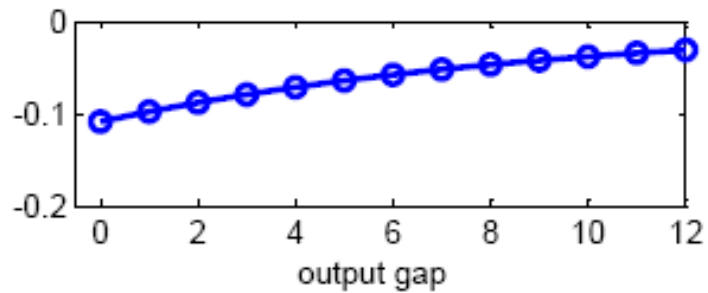


Figure 3.5: Estimated Effects of a Permanent Technology Shock

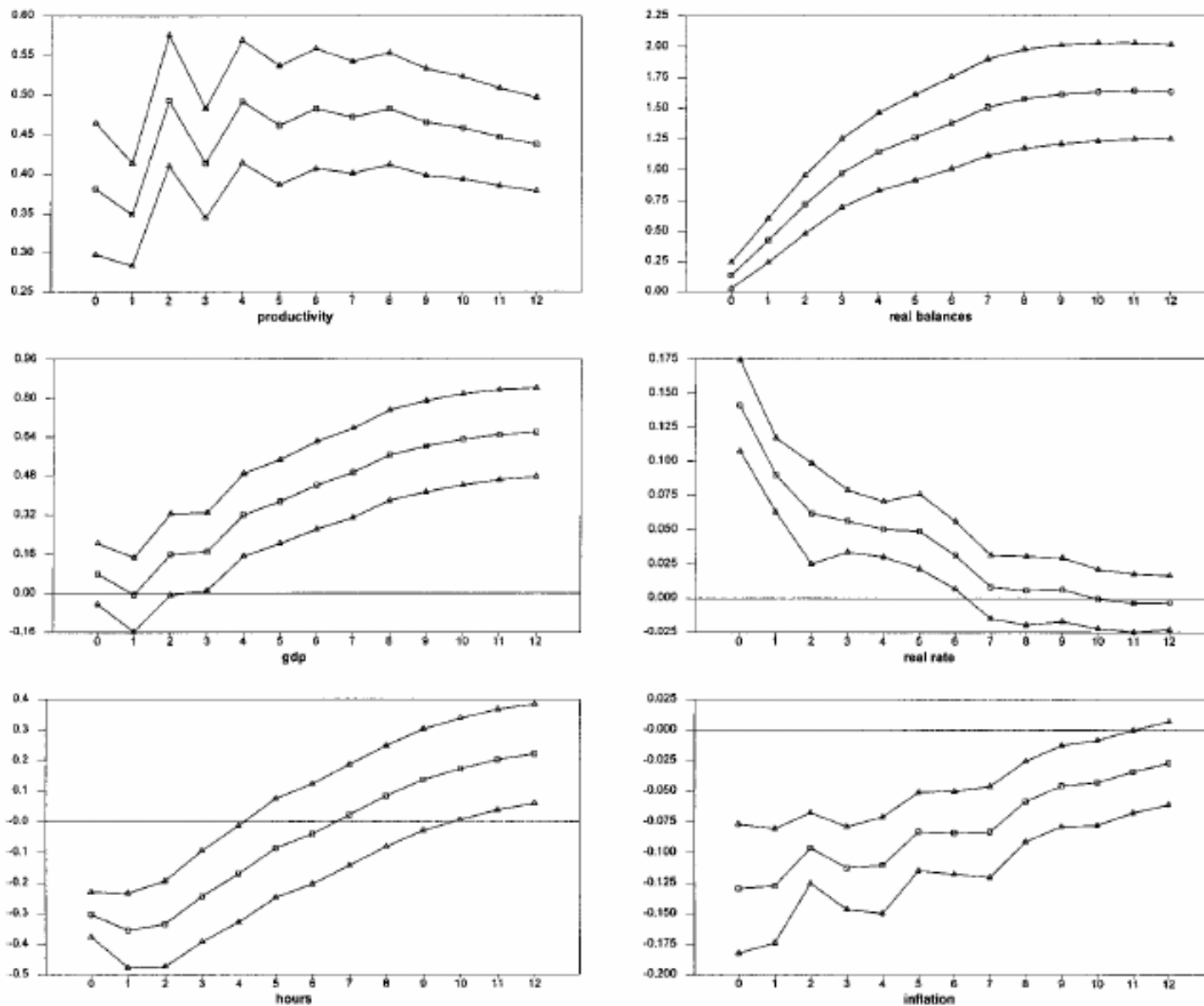


FIGURE 4. ESTIMATED IMPULSE RESPONSES FROM A FIVE-VARIABLE MODEL: U.S. DATA, FIRST-DIFFERENCED HOURS (POINT ESTIMATES AND ± 2 STANDARD ERROR CONFIDENCE INTERVALS)

Source: Galí (1999)

Equilibrium under an Exogenous Money Growth Process

$$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m \quad (13)$$

Money market clearing

$$\hat{l}_t = \hat{y}_t - \eta \hat{i}_t \quad (14)$$

$$= \tilde{y}_t + \hat{y}_t^n - \eta \hat{i}_t \quad (15)$$

where $l_t \equiv m_t - p_t$ denotes (log) real money balances.

Substituting (14) into (10):

$$(1 + \sigma\eta) \tilde{y}_t = \sigma\eta E_t\{\tilde{y}_{t+1}\} + \hat{l}_t + \eta E_t\{\pi_{t+1}\} + \eta \hat{r}_t^n - \hat{y}_t^n \quad (16)$$

Furthermore, we have

$$\hat{l}_{t-1} = \hat{l}_t + \pi_t - \Delta m_t \quad (17)$$

Equilibrium dynamics

$$\mathbf{A}_{\mathbf{M},0} \begin{bmatrix} \tilde{y}_t \\ \pi_t \\ \hat{l}_{t-1} \end{bmatrix} = \mathbf{A}_{\mathbf{M},1} \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ \hat{l}_{t-1} \end{bmatrix} + \mathbf{B}_{\mathbf{M}} \begin{bmatrix} \hat{r}_t^n \\ \hat{y}_t^n \\ \Delta m_t \end{bmatrix} \quad (18)$$

where

$$\mathbf{A}_{\mathbf{M},0} \equiv \begin{bmatrix} 1 + \sigma\eta & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} ; \quad \mathbf{A}_{\mathbf{M},1} \equiv \begin{bmatrix} \sigma\eta & \eta & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} ; \quad \mathbf{B}_{\mathbf{M}} \equiv \begin{bmatrix} \eta & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Uniqueness $\iff \mathbf{A}_{\mathbf{M}} \equiv \mathbf{A}_{\mathbf{M},0}^{-1}\mathbf{A}_{\mathbf{M},1}$ has two eigenvalues inside and one outside the unit circle.

Effects of a Monetary Policy Shock

Set $\widehat{r}_t^n = y_t^n = 0$ (no real shocks).

Money growth process

$$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m$$

where $\rho_m \in [0, 1)$

Figure 3 (based on $\rho_m = 0.5$)

Effects of a Technology Shock

Set $\Delta m_t = 0$ (no monetary shocks).

Technology process:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

Figure 4 (based on $\rho_a = 0.9$).

Empirical Evidence

Figure 3.3: Effects of a Monetary Policy Shock (Money Growth Rule)

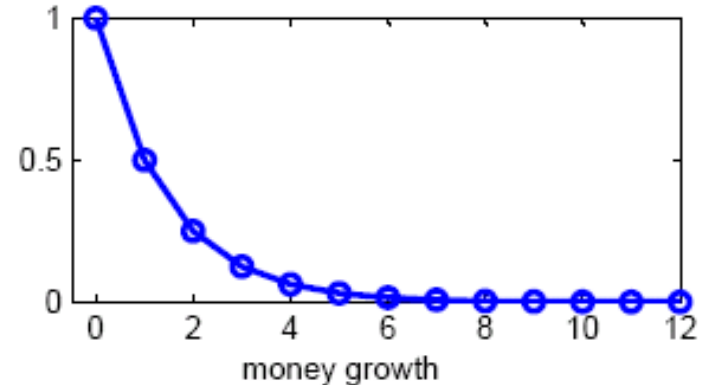
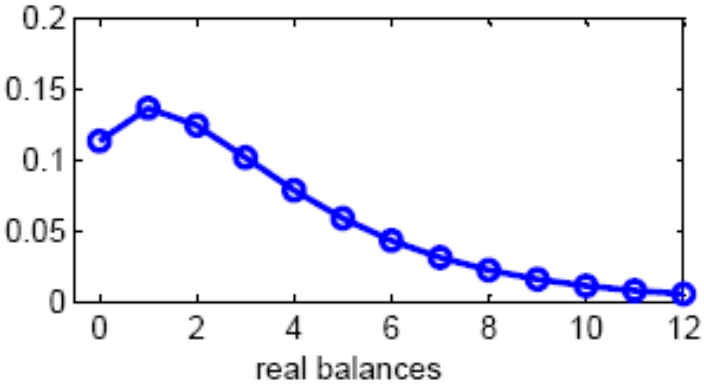
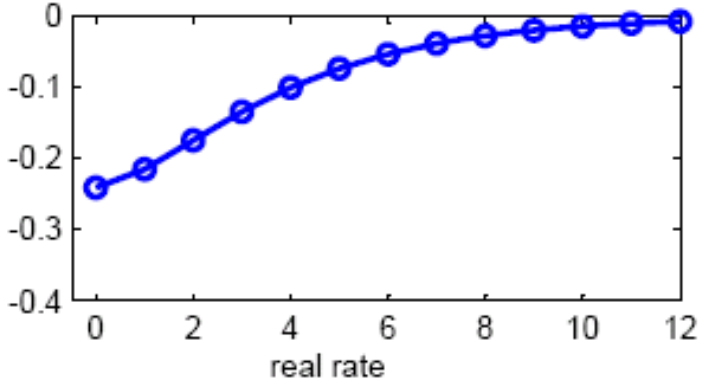
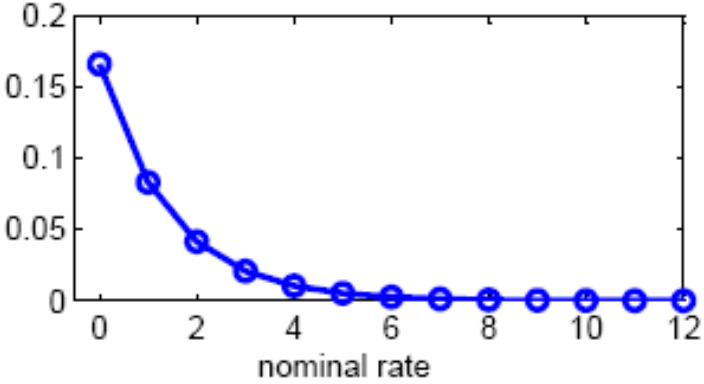
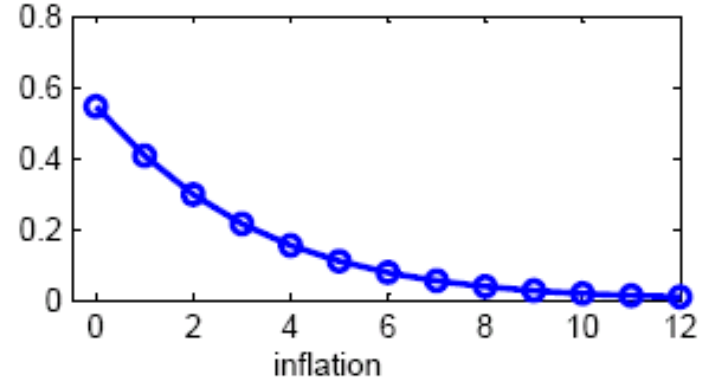
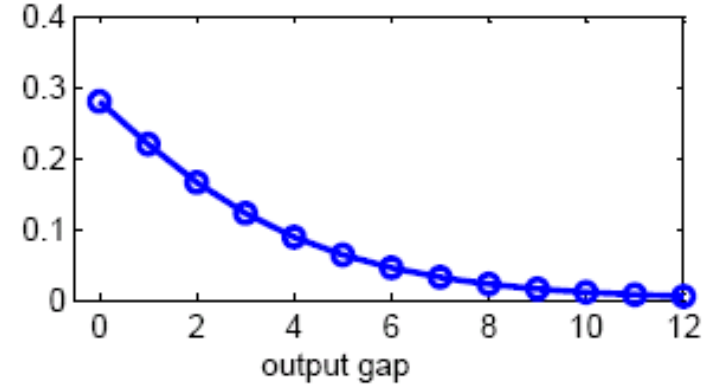
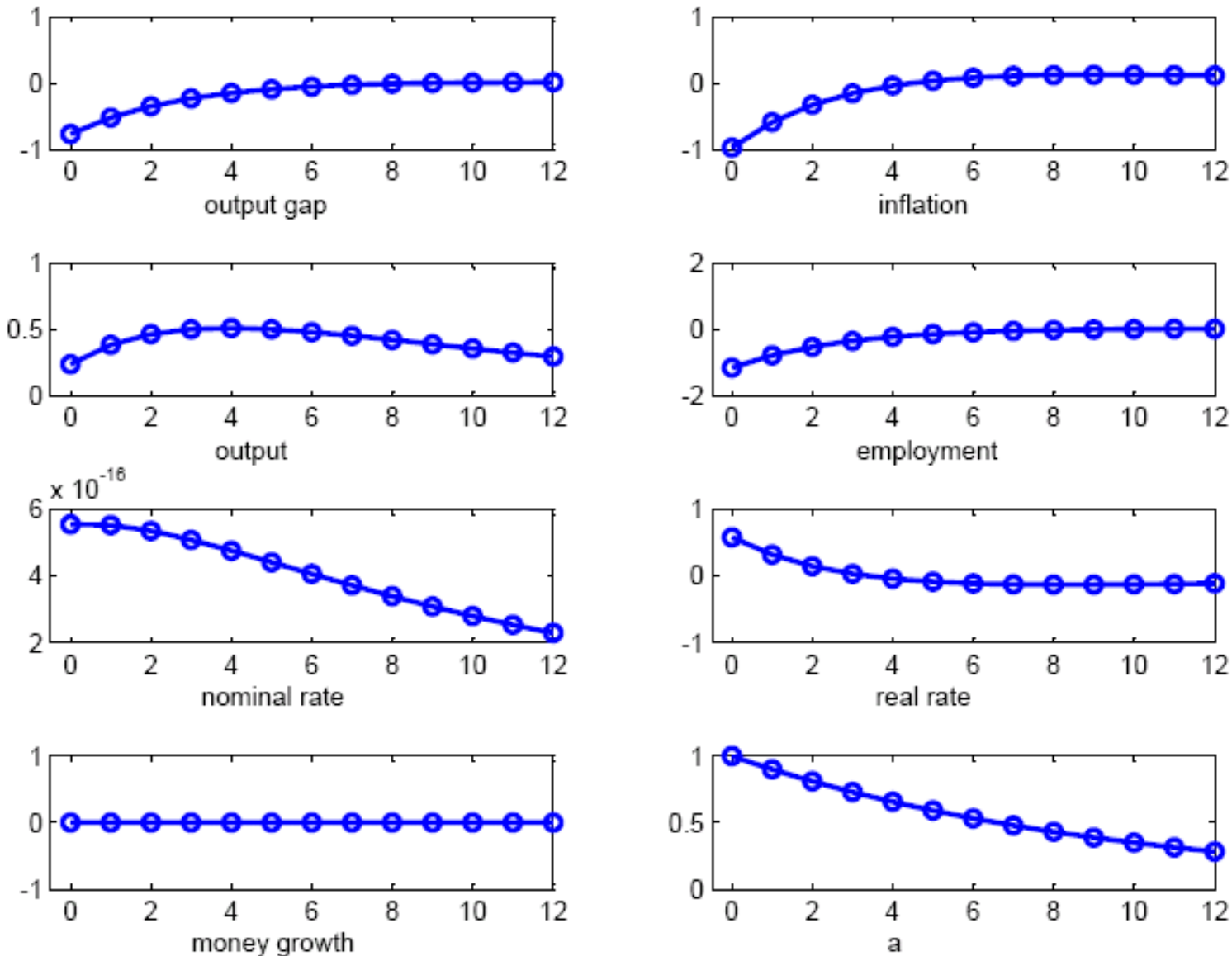


Figure 3.4: Effects of a Technology Shock (Money Growth Rule)



Technical Appendix

Optimal Allocation of Consumption Expenditures

Maximization of C_t for any *given* expenditure level $\int_0^1 P_t(i) C_t(i) di \equiv Z_t$ can be formalized by means of the Lagrangean

$$\mathcal{L} = \left[\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}} - \lambda \left(\int_0^1 P_t(i) C_t(i) di - Z_t \right)$$

The associated first order conditions are:

$$C_t(i)^{-\frac{1}{\epsilon}} C_t^{\frac{1}{\epsilon}} = \lambda P_t(i)$$

for all $i \in [0, 1]$. Thus, for any two goods (i, j) we have:

$$C_t(i) = C_t(j) \left(\frac{P_t(i)}{P_t(j)} \right)^{-\epsilon}$$

which can be plugged into the expression for consumption expenditures to yield

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} \frac{Z_t}{P_t}$$

for all $i \in [0, 1]$. The latter condition can then be substituted into the definition of C_t , yielding

$$\int_0^1 P_t(i) C_t(i) di = P_t C_t$$

Combining the two previous equations we obtain the demand schedule:

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t$$

Log-Linearized Euler Equation

We can rewrite the Euler equation as

$$1 = E_t\{\exp(i_t - \sigma\Delta c_{t+1} - \pi_{t+1} - \rho)\} \quad (19)$$

In a perfect foresight steady state with constant inflation π and constant growth γ we must have:

$$i = \rho + \sigma\gamma + \pi$$

with the steady state real rate being given by

$$\begin{aligned} r &\equiv i - \pi \\ &= \rho + \sigma\gamma \end{aligned}$$

A first order Taylor expansion of $\exp(i_t - \sigma\Delta c_{t+1} - \pi_{t+1} - \rho)$ around that steady state yields:

$$\begin{aligned} \exp(i_t - \sigma\Delta c_{t+1} - \pi_{t+1} - \rho) &\simeq 1 + (i_t - i) - \sigma(\Delta c_{t+1} - \gamma) - (\pi_{t+1} - \pi) \\ &= 1 + i_t - \sigma\Delta c_{t+1} - \pi_{t+1} - \rho \end{aligned}$$

which can be used in (19) to obtain, after some rearrangement of terms, the log-linearized Euler equation

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho)$$

Aggregate Price Level Dynamics

Let $S(t) \subset [0, 1]$ denote the set of firms which do not re-optimize their posted price in period t . The aggregate price level evolves according to

$$\begin{aligned} P_t &= \left[\int_{S(t)} P_{t-1}(i)^{1-\epsilon} di + (1 - \theta)(P_t^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \\ &= [\theta(P_{t-1})^{1-\epsilon} + (1 - \theta)(P_t^*)^{1-\epsilon}]^{\frac{1}{1-\epsilon}} \end{aligned}$$

where the second equality follows from the fact that the distribution of prices among firms not adjusting in period t corresponds to the distribution of effective prices in period $t - 1$, with total mass reduced to θ . Equivalently, dividing both sides by P_{t-1} :

$$\Pi_t^{1-\epsilon} = \theta + (1 - \theta) \left(\frac{P_t^*}{P_{t-1}} \right)^{1-\epsilon} \quad (20)$$

where $\Pi_t \equiv \frac{P_t}{P_{t-1}}$. Notice that in a steady state with zero inflation $P_t^* = P_{t-1}$. Log-linearization around a zero inflation ($\Pi = 1$) steady state implies:

$$\pi_t = (1 - \theta)(p_t^* - p_{t-1}) \quad (21)$$

Price Dispersion

From the definition of the price index:

$$\begin{aligned} 1 &= \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{1-\epsilon} di \\ &= \int_0^1 \exp\{(1 - \epsilon)(p_t(i) - p_t)\} di \\ &\simeq 1 + (1 - \epsilon) \int_0^1 (p_t(i) - p_t) di + \frac{(1 - \epsilon)^2}{2} \int_0^1 (p_t(i) - p_t)^2 di \end{aligned}$$

thus implying the second order approximation

$$p_t \simeq E_i\{p_t(i)\} + \frac{(1 - \epsilon)}{2} \int_0^1 (p_t(i) - p_t)^2 di$$

where $E_i\{p_t(i)\} \equiv \int_0^1 p_t(i) di$ is the cross-sectional mean of (log) prices.

In addition,

$$\begin{aligned}
\int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\epsilon}{1-\alpha}} di &= \int_0^1 \exp \left\{ -\frac{\epsilon}{1-\alpha} (p_t(i) - p_t) \right\} di \\
&\simeq 1 - \frac{\epsilon}{1-\alpha} \int_0^1 (p_t(i) - p_t) di + \frac{1}{2} \left(\frac{\epsilon}{1-\alpha} \right)^2 \int_0^1 (p_t(i) - p_t)^2 di \\
&\simeq 1 + \frac{1}{2} \frac{\epsilon(1-\epsilon)}{1-\alpha} \int_0^1 (p_t(i) - p_t)^2 di + \frac{1}{2} \left(\frac{\epsilon}{1-\alpha} \right)^2 \int_0^1 (p_t(i) - p_t)^2 di \\
&= 1 + \frac{1}{2} \left(\frac{\epsilon}{1-\alpha} \right) \frac{1}{\Theta} \int_0^1 (p_t(i) - p_t)^2 di \\
&\simeq 1 + \frac{1}{2} \left(\frac{\epsilon}{1-\alpha} \right) \frac{1}{\Theta} \text{var}_i\{p_t(i)\} > 1
\end{aligned}$$

where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\epsilon}$, and where the last equality follows from the observation that, up to second order,

$$\begin{aligned}
\int_0^1 (p_t(i) - p_t)^2 di &\simeq \int_0^1 (p_t(i) - E_i\{p_t(i)\})^2 di \\
&\equiv \text{var}_i\{p_t(i)\}
\end{aligned}$$

Finally, using the definition of d_t we obtain

$$d_t \equiv (1-\alpha) \log \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\epsilon}{1-\alpha}} di \simeq \frac{1}{2} \frac{\epsilon}{\Theta} \text{var}_i\{p_t(i)\}$$