Monetary Policy Design in the Basic New Keynesian Model

by

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The Efficient Allocation

$$\max U\left(C_t, N_t\right)$$

where $C_t \equiv \left[\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}$ subject to:

$$C_t(i) = A_t N_t(i)^{1-\alpha}, \ \ all \ i \in [0,1]$$

$$N_t = \int_0^1 N_t(i) di$$

Optimality conditions:

$$C_t(i) = C_t, \ \ all \ i \in [0,1]$$

$$N_t(i) = N_t, \ \ all \ i \in [0,1]$$

$$-\frac{U_{n,t}}{U_{c,t}} = MPN_t$$

where $MPN_t \equiv (1-\alpha) A_t N_t^{-\alpha}$

Sources of Suboptimality of Equilibrium

1. Distortions unrelated to nominal rigidities:

- *Monopolistic competition*: $P_t = \mathcal{M}\frac{W_t}{MPN_t}$, where $\mathcal{M} \equiv \frac{\varepsilon}{\varepsilon-1} > 1$

$$\Longrightarrow \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}} < MPN_t$$

Solution: employment subsidy τ. Under flexible prices, $P_t = \mathcal{M}\frac{(1-\tau)W_t}{MPN_t}$.

$$\Longrightarrow \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}(1-\tau)}$$

Optimal subsidy: $\mathcal{M}(1-\tau) = 1$ or, equivalently, $\tau = \frac{1}{\varepsilon}$.

- *Transactions friction* (economy with valued money): assumed to be negligible

2. Distortions associated with the presence of nominal rigidities:

- *Markup variations* resulting from sticky prices: $\mathcal{M}_t = \frac{P_t}{(1-\tau)(W_t/MPN_t)} = \frac{P_t\mathcal{M}}{W_t/MPN_t}$ (assuming optimal subsidy)

$$\Longrightarrow \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = MPN_t\frac{\mathcal{M}}{\mathcal{M}_t} \neq MPN_t$$

Optimality requires that the average markup be stabilized at its frictionless level.

- *Relative price distortions* resulting from staggered price setting: $C_t(i) \neq C_t(j)$ if $P_t(i) \neq P_t(j)$. Optimal policy requires that prices and quantities (and hence marginal costs) are equalized across goods.

Optimal Monetary Policy in the Basic NK Model

Assumptions:

- optimal employment subsidy

  $\Longrightarrow$ flexible price equilibrium allocation is efficient

- no inherited relative price distortions, i.e. $P_{-1}(i) = P_{-1}$ for all $i \in [0, 1]$

$\Longrightarrow$ the efficient allocation can be attained by a policy that stabilizes marginal costs at a level consistent with firms' desired markup, *given existing prices*:

- no firm has an incentive to adjust its price, i.e. $P_t^* = P_{t-1}$ and, hence, $P_t = P_{t-1}$ for $t = 0, 1, 2, \ldots$(aggregate price stability)
- equilibrium output and employment match their counterparts in the (undistorted) flexible price equilibrium allocation.

Equilibrium under the Optimal Policy

$$\widetilde{y}_t = 0$$

$$\pi_t = 0$$

$$i_t = r_t^n$$

for all t.

Implementation: Some Candidate Interest Rate Rules

Non-Policy Block:

$$\widetilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n) + E_t\{\widetilde{y}_{t+1}\}$$

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

An Exogenous Interest Rate Rule

$$i_t = r_t^n$$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A_O} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}$$

where

$$\mathbf{A_O} \equiv \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} \end{bmatrix}$$

Shortcoming: the solution $\widetilde{y}_t = \pi_t = 0$ for all t is *not* unique: one eigenvalue of $\mathbf{A_O}$ is strictly greater than one.

$\rightarrow$ indeterminacy. (real and nominal).

An Interest Rate Rule with Feedback from Target Variables

$$i_t = r_t^n + \phi_\pi \pi_t + \phi_y \widetilde{y}_t$$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}$$

where

$$\mathbf{A}_T \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi} \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix}$$

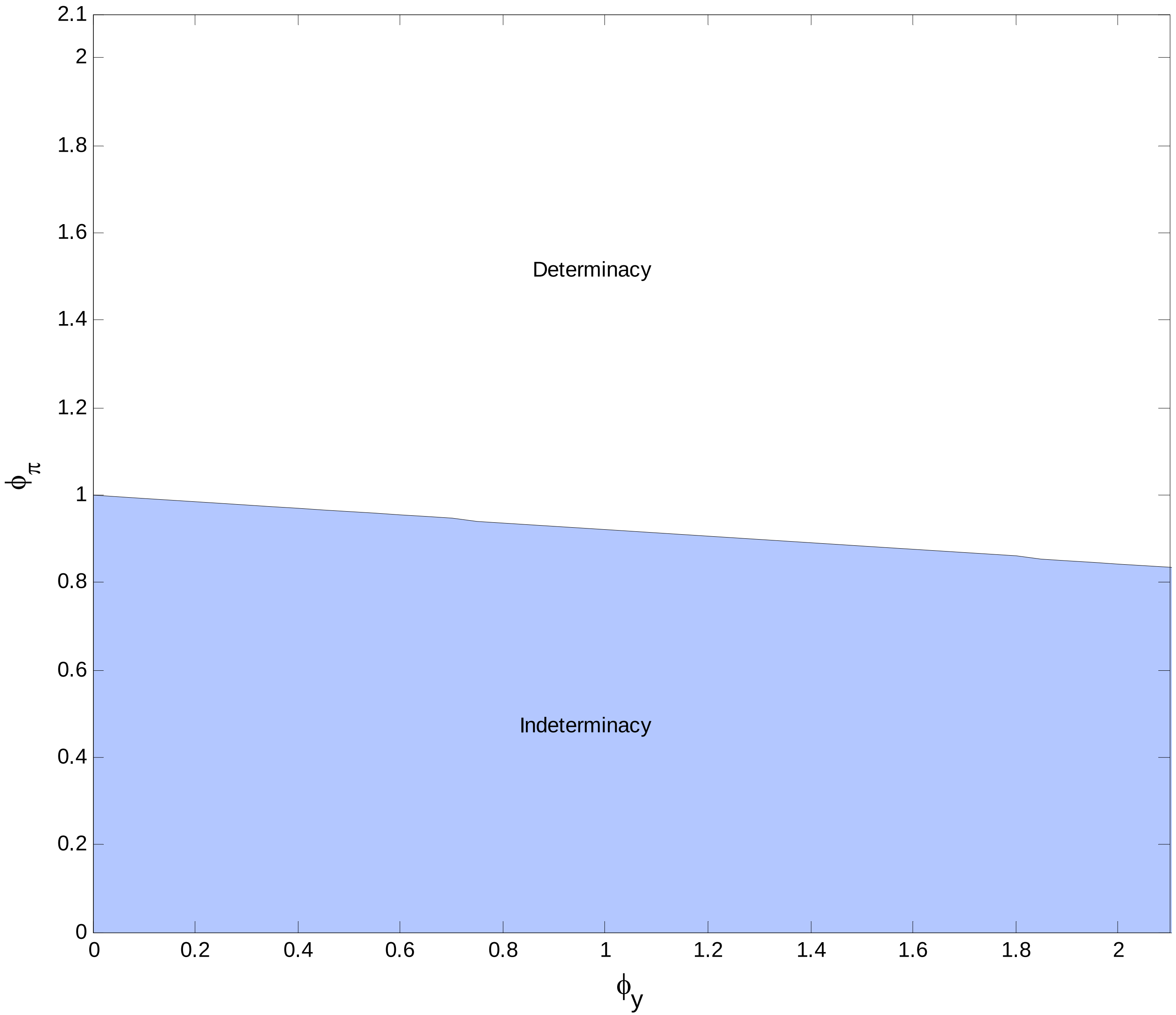
Existence and Uniqueness condition: (Bullard and Mitra (2002)):

$$\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0$$

Taylor-principle interpretation (Woodford (2000)):

$$\begin{aligned} di &= \phi_\pi d\pi + \phi_y d\widetilde{y} \\ &= \left(\phi_\pi + \frac{\phi_y(1-\beta)}{\kappa}\right) d\pi \end{aligned}$$

Figure 4.1


Determinacy
Indeterminacy
ϕ_π
ϕ_y
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.1

A Forward-Looking Interest Rate Rule

$$i_t = r_t^n + \phi_\pi E_t\{\pi_{t+1}\} + \phi_y E_t\{\widetilde{y}_{t+1}\}$$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_F \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}$$

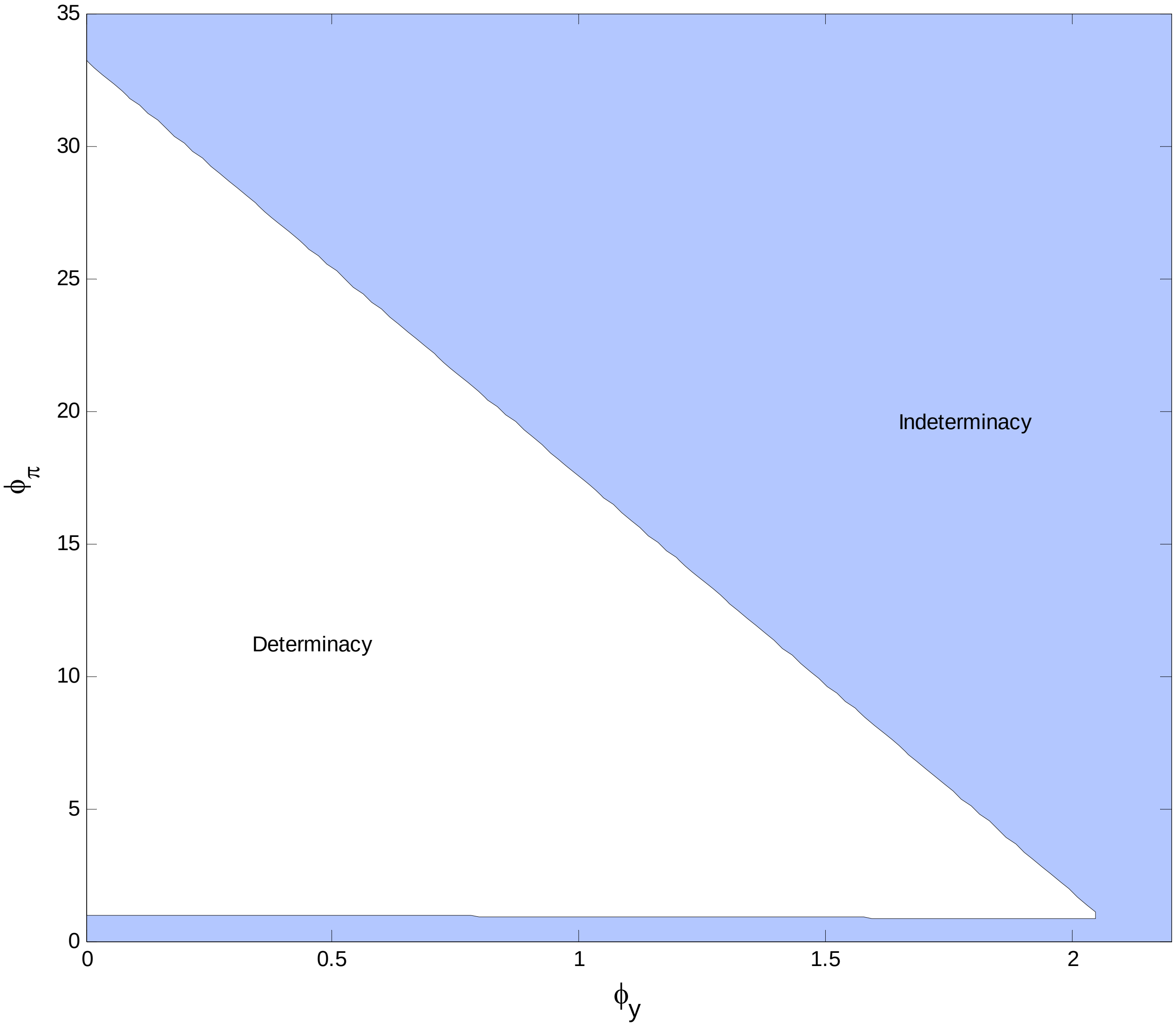
where

$$\mathbf{A}_F \equiv \begin{bmatrix} 1 - \sigma^{-1}\phi_y & -\sigma^{-1}(\phi_\pi - 1) \\ \kappa(1 - \sigma^{-1}\phi_y) & \beta - \kappa\sigma^{-1}(\phi_\pi - 1) \end{bmatrix}$$

Existence and Uniqueness conditions (Bullard and Mitra (2002):

$$\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0$$
$$\kappa(\phi_\pi - 1) + (1 + \beta)\phi_y < 2\sigma(1 + \beta)$$

Figure 4.2



35
30
25
20
15
10
5
0
0
0.5
1
1.5
2
ϕ_π
ϕ_y
Indeterminacy
Determinacy

Shortcomings of Optimal Rules

- they assume observability of the natural rate of interest (in real time).
- this requires, in turn, knowledge of:

  (i) the true model
  (ii) true parameter values
  (iii) realized shocks

Alternative: "simple rules" , i.e. rules that meet the following criteria:

- the policy instrument depends on observable variables only,
- do not require knowledge of the true parameter values
- ideally, they approximate optimal rule across different models

Simple Monetary Policy Rules

Welfare-based evaluation:

$$\mathbb{W} \equiv - E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{U_t - U_t^n}{U_c C} \right) = \frac{1}{2\lambda} E_0 \sum_{t=0}^{\infty} \beta^t \left(\kappa \widetilde{y}_t^2 + \epsilon \pi_t^2 \right)$$

$\Longrightarrow$ expected average welfare loss per period:

$$\mathbb{L} = \frac{1}{2\lambda} \left[\kappa \ var(\widetilde{y}_t) + \epsilon \ var(\pi_t) \right]$$

See Appendix for Derivation.

A Taylor Rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t$$

Equivalently:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widetilde{y}_t + v_t$$

where $v_t \equiv \phi_y \widehat{y}_t^n$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_T(\widehat{r}_t^n - \phi_y \ \widehat{y}_t^n)$$

where

$$\mathbf{A}_T \ \equiv \Omega \ \begin{bmatrix} \sigma & 1-\beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix} \quad ; \quad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

and $\Omega \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}$. Note that $\widehat{r}_t^n - \phi_y \widehat{y}_t^n = -\psi_{ya}^n [\sigma(1-\rho_a) + \phi_y] a_t$

Exercise: $\Delta a_t \sim AR(1)$ + modified Taylor rule $i_t = \rho + \phi_\pi \pi_t + \phi_y \Delta y_t$

Money Growth Peg

$$\Delta m_t = 0$$

money market clearing condition

$$\widehat{l}_t = \widetilde{y}_t + \widehat{y}_t^n - \eta \widehat{i}_t - \zeta_t$$

where $l_t \equiv m_t - p_t$ and ξ_t is a money demand shock following the process

$$\Delta\zeta_t = \rho_\zeta \Delta\zeta_{t-1} + \varepsilon_t^\zeta$$

Define $l_t^+ \equiv l_t - \zeta_t$. $\Longrightarrow$

$$\widehat{i}_t = \frac{1}{\eta} \left(\widetilde{y}_t + \widehat{y}_t^n - \widehat{l}_t^+ \right)$$

$$\widehat{l}_{t-1}^+ = \widehat{l}_t^+ + \pi_t - \Delta\zeta_t$$

Equilibrium dynamics:

$$\mathbf{A_{M,0}} \begin{bmatrix} \widetilde{y}_t \\ \pi_t \\ l_{t-1}^+ \end{bmatrix} = \mathbf{A_{M,1}} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ l_t^+ \end{bmatrix} + \mathbf{B_M} \begin{bmatrix} \widehat{r}_t^n \\ \widehat{y}_t^n \\ \Delta\zeta_t \end{bmatrix}$$

where

$$\mathbf{A_{M,0}} \equiv \begin{bmatrix} 1+\sigma\eta & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad ; \quad \mathbf{A_{M,1}} \equiv \begin{bmatrix} \sigma\eta & \eta & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ; \quad \mathbf{B_M} \equiv \begin{bmatrix} \eta & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

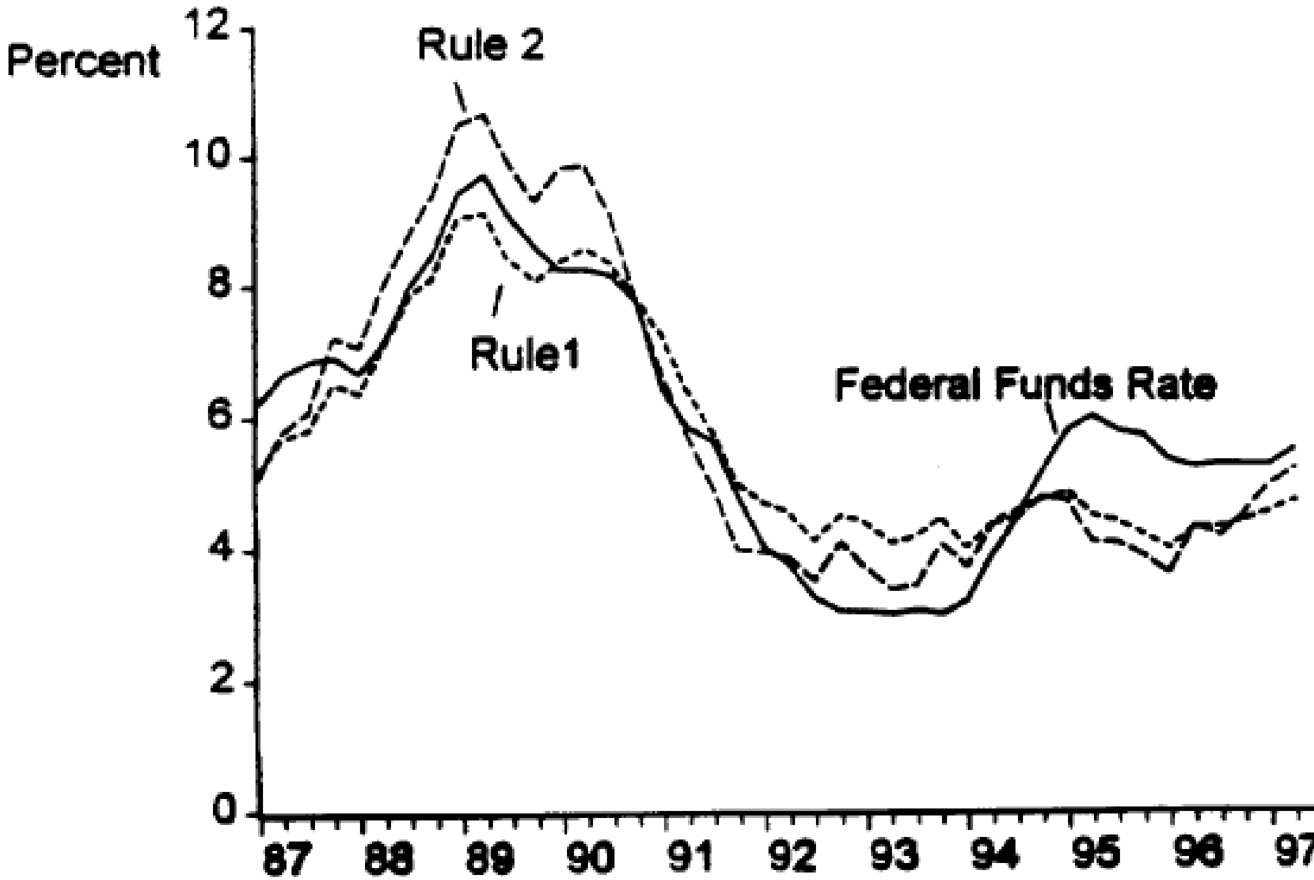
Simulations and Evaluation of Simple Rules

Table 4.1: Evaluation of Simple Monetary Policy Rules

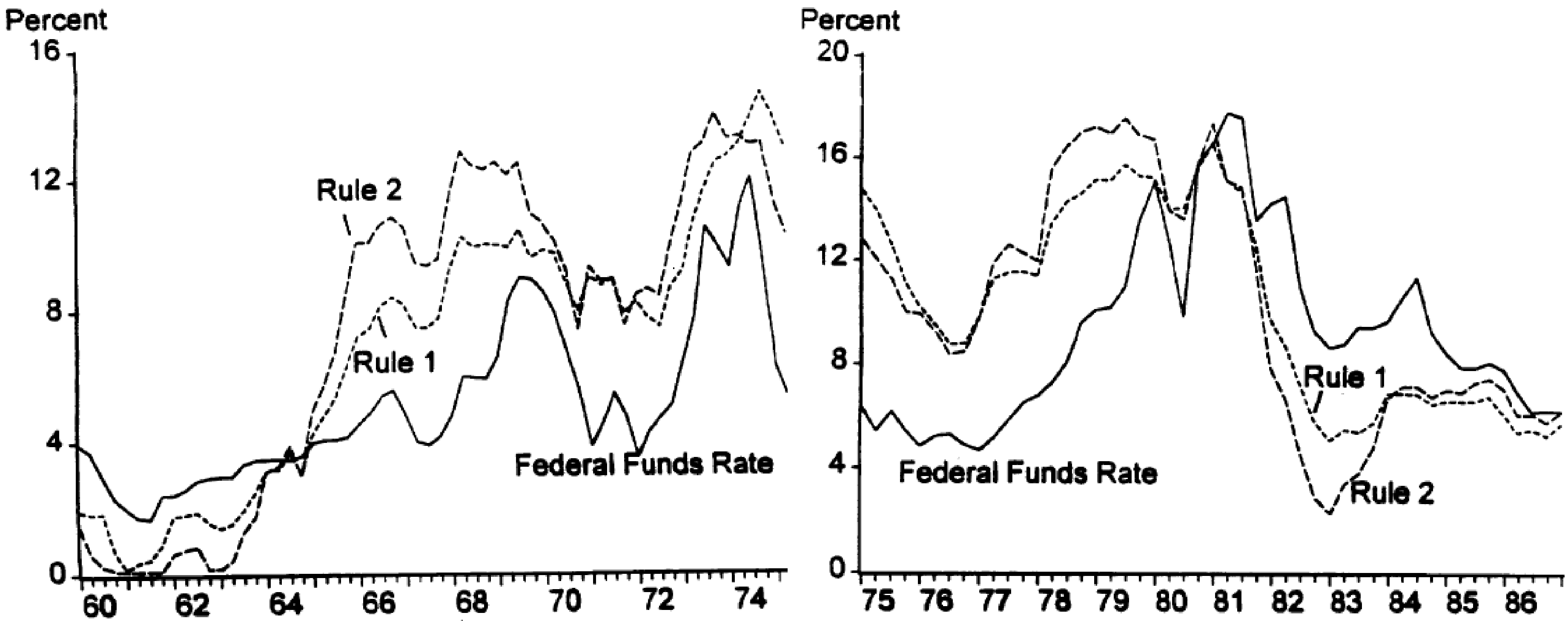
	Taylor Rule				*Constant Money Growth*	
ϕ_π	1.5	1.5	5	1.5	-	-
ϕ_y	0.125	0	0	1	-	-
$(\sigma_\zeta, \rho_\zeta)$	-	-	-	-	$(0, 0)$	$(0.0063, 0.6)$
$\sigma(\widetilde{y})$	0.55	0.28	0.04	1.40	1.02	1.62
$\sigma(\pi)$	2.60	1.33	0.21	6.55	1.25	2.77
$welfare\ loss$	0.30	0.08	0.002	1.92	0.08	0.38

The Taylor Rule (Taylor 1993)

$$i_t = 4 + 1.5(\pi_t - 2) + 0.5\, y_t$$



Source: Taylor 1999



Source: Taylor 1999

Clarida, Galí and Gertler (QJE 2000)

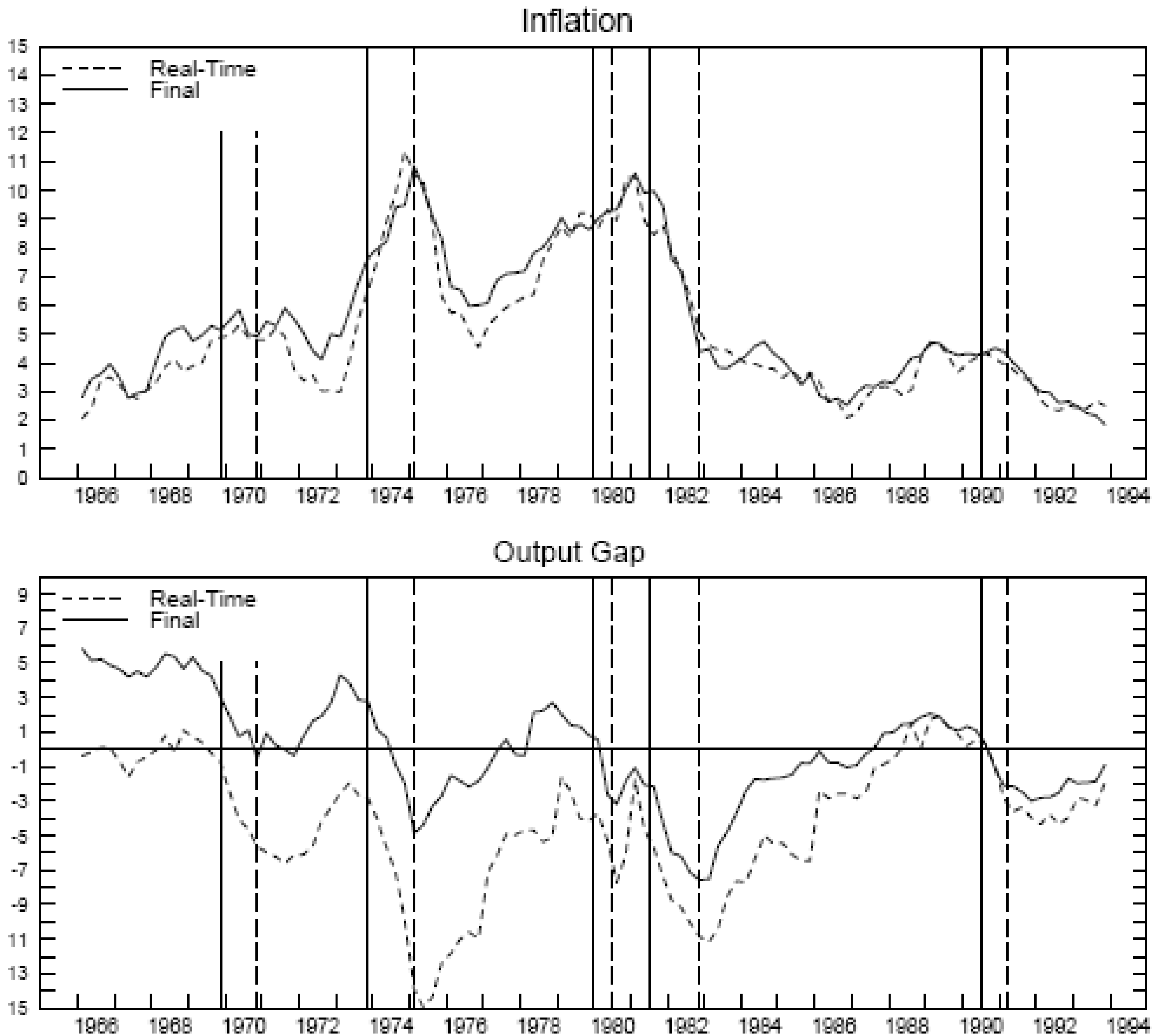
$$i_t = \rho i_{t-1} + (1-\rho)[r + \pi^* + \beta E_t\{\pi_{t+1} - \pi^*\} + \gamma E_t\{y_{t+1} - y^*_{t+1}\}]$$

TABLE II
BASELINE ESTIMATES

	π^*	β	γ	ρ	p
Pre-Volcker	4.24	0.83	0.27	0.68	0.834
	(1.09)	(0.07)	(0.08)	(0.05)	
Volcker-Greenspan	3.58	2.15	0.93	0.79	0.316
	(0.50)	(0.40)	(0.42)	(0.04)	

Standard errors are reported in parentheses. The set of instruments includes four lags of inflation: output gap, the federal funds rate, the short-long spread, and commodity price inflation.

Orphanides (JME 2003)



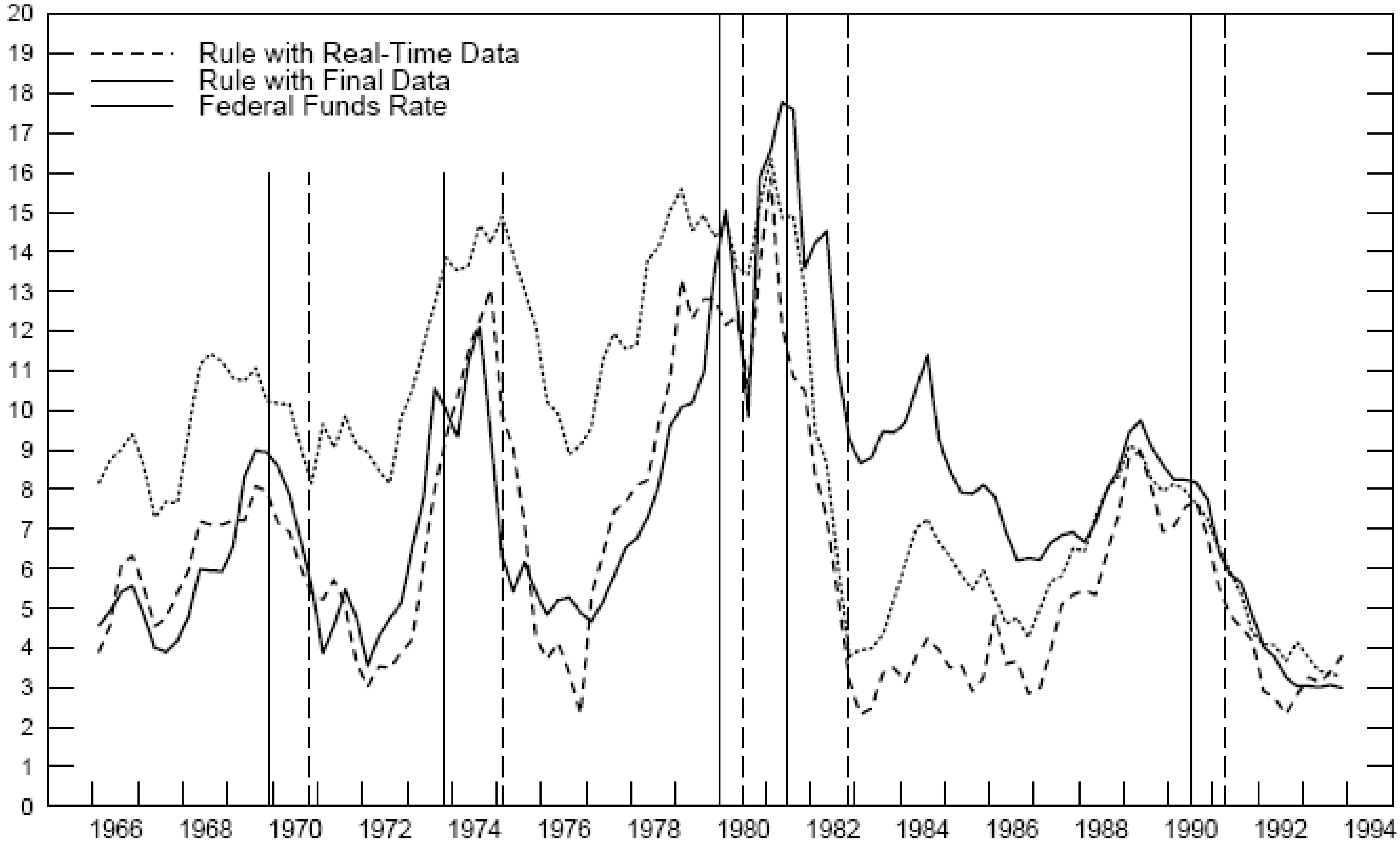


Fig. 5. Then and now: Taylor rule with final and real-time data.

Table 1
Estimated policy rules

	θ_0	θ_i	θ_π	$\theta_{\Delta y}$	θ_y	*see*
Greenbook forecasts						
1969:1–1997:4	–0.42	0.88	0.44	0.27	0.14	1.04
	(0.29)	(0.04)	(0.10)	(0.10)	(0.03)	
1969:1–1979:2	0.53	0.75	0.44	0.14	0.19	0.95
	(0.92)	(0.14)	(0.12)	(0.15)	(0.04)	
1982:3–1997:4	–0.33	0.81	0.52	0.51	0.10	0.56
	(0.32)	(0.06)	(0.13)	(0.17)	(0.03)	
Survey forecasts						
1969:1–2002:4	–0.51	0.84	0.55	0.36	0.17	0.96
	(0.22)	(0.05)	(0.12)	(0.13)	(0.03)	
1969:1–1979:2	0.74	0.91	0.25	0.32	0.21	0.99
	(1.28)	(0.29)	(0.16)	(0.35)	(0.05)	
1982:3–2002:4	–0.66	0.83	0.58	0.53	0.16	0.49
	(0.21)	(0.05)	(0.09)	(0.11)	(0.03)	

Notes: Least-squares estimates of

$$i_t = \theta_0 + \theta_i i_{t-1} + \theta_\pi \pi^a_{t+3} + \theta_{\Delta y}\Delta^a y_{t+3} + \theta_y y_{t-1},$$

where $\pi^a_{t+3} = p_{t+3} - p_{t-1}, y_{t-1} = q_{t-1} - q^*_{t-1}$ and $\Delta^a y_{t+3} = y_{t+3} - y_{t-1} = \Delta^a q_{t+3} - \Delta^a q^*_{t+3}$. All variables dated t and later reflect real-time forecasts formed during quarter t. HAC standard errors in parentheses.

Source: Orphanides (JME 2003 b)