Monetary Policy Tradeoffs

by

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November 2010

Policy Tradeoffs and the New Keynesian Phillips Curve

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa(y_t - y_t^n)$$

Criticism: no policy tradeoffs, optimality of strict inflation targeting

Implicit assumption: $\quad y_t^e - y_t^n = \delta$

Alternative: time-varying $y_t^e - y_t^n$ gap

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa x_t + u_t$$

where $x_t \equiv y_t - y_t^e$ and $u_t \equiv \kappa(y_t^e - y_t^n)$

The Monetary Policy Problem

$$\min \; E_0\{\sum_{t=0}^{\infty} \beta^t \left(\alpha_x x_t^2 + \pi_t^2\right)\} \tag{1}$$

subject to:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa x_t + u_t$$

where $\{u_t\}$ evolves exogenously according to

$$u_t = \rho_u u_{t-1} + \varepsilon_t$$

In addition:

$$x_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^e) + E_t\{x_{t+1}\} \tag{2}$$

Note: utility based criterion requires $\alpha_x = \frac{\kappa}{\epsilon}$

Optimal Discretionary Policy

Each period CB chooses (x_t, π_t) to minimize

$$\alpha_x x_t^2 + \pi_t^2$$

subject to

$$\pi_t = \kappa x_t + v_t$$

where $v_t \equiv \beta E_t\{\pi_{t+1}\} + u_t$ is taken as given.

Optimality condition:

$$x_t = -\frac{\kappa}{\alpha_x}\pi_t \tag{3}$$

Equilibrium

$$\pi_t = \alpha_x \Psi u_t \tag{4}$$

$$x_t = -\kappa \Psi u_t \tag{5}$$

$$i_t = r_t^e + \Psi[\kappa\sigma(1-\rho_u) + \alpha_x \rho_u]u_t \tag{6}$$

where $\Psi \equiv \frac{1}{\kappa^2 + \alpha_x(1-\beta\rho_u)}$

Implementation:

$$i_t = r_t^e + [(1-\rho_u)\frac{\kappa\sigma}{\alpha_x} + \rho_u]\pi_t$$

uniqueness condition: $\frac{\kappa\sigma}{\alpha_x} > 1$ (likely if utility-based: $\sigma\epsilon > 1$)

Alternatively,

$$i_t = r_t^e + \Psi[\kappa\sigma(1-\rho_u) + \alpha_x\rho_u]u_t + \phi_\pi(\pi_t - \alpha_x\Psi u_t)$$

uniqueness condition: $\phi_\pi > 1$.

Optimal Policy with Commitment

State-contingent policy $\{x_t, \pi_t\}_{t=0}^{\infty}$ that minimizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\alpha_x x_t^2 + \pi_t^2\right)$$

subject to the sequence of constraints:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa x_t + u_t$$

Lagrangean:

$$\mathcal{L} = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t [\alpha_x \ x_t^2 + \pi_t^2 + 2\gamma_t \ (\pi_t - \kappa x_t - \beta \pi_{t+1})]$$

First order conditions:

$$\alpha_x x_t - \kappa \gamma_t = 0$$

$$\pi_t + \gamma_t - \gamma_{t-1} = 0$$

for $t = 0, 1, 2, ...$and where $\gamma_{-1} = 0$.

Eliminating multipliers:

$$x_0 = -\frac{\kappa}{\alpha_x}\pi_0 \tag{7}$$

$$x_t = x_{t-1} - \frac{\kappa}{\alpha_x}\pi_t \tag{8}$$

for $t = 1, 2, 3, ...$

Alternative representation:

$$x_t = -\frac{\kappa}{\alpha_x}\widehat{p}_t \tag{9}$$

for $t = 0, 1, 2, ...$where $\widehat{p}_t \equiv p_t - p_{-1}$

Equilibrium

$$\widehat{p}_t = a\widehat{p}_{t-1} + a\beta E_t\{\widehat{p}_{t+1}\} + au_t$$

for $t = 0, 1, 2, ...$where $a \equiv \frac{\alpha_x}{\alpha_x(1+\beta)+\kappa^2}$

Stationary solution:

$$\widehat{p}_t = \delta\widehat{p}_{t-1} + \frac{\delta}{(1-\delta\beta\rho_u)}u_t \tag{10}$$

for $t = 0, 1, 2, ...$where $\delta \equiv \frac{1-\sqrt{1-4\beta a^2}}{2a\beta} \in (0, 1)$.

$\rightarrow$ *price level targeting !*

$$x_t = \delta x_{t-1} - \frac{\kappa\delta}{\alpha_x(1-\delta\beta\rho_u)}u_t \tag{11}$$

for $t = 1, 2, 3, ...$as well as

$$x_0 = -\frac{\kappa\delta}{\alpha_x(1-\delta\beta\rho_u)}\ u_0$$

Figure 5.1: Optimal Responses to a Transitory Cost-Push Shock

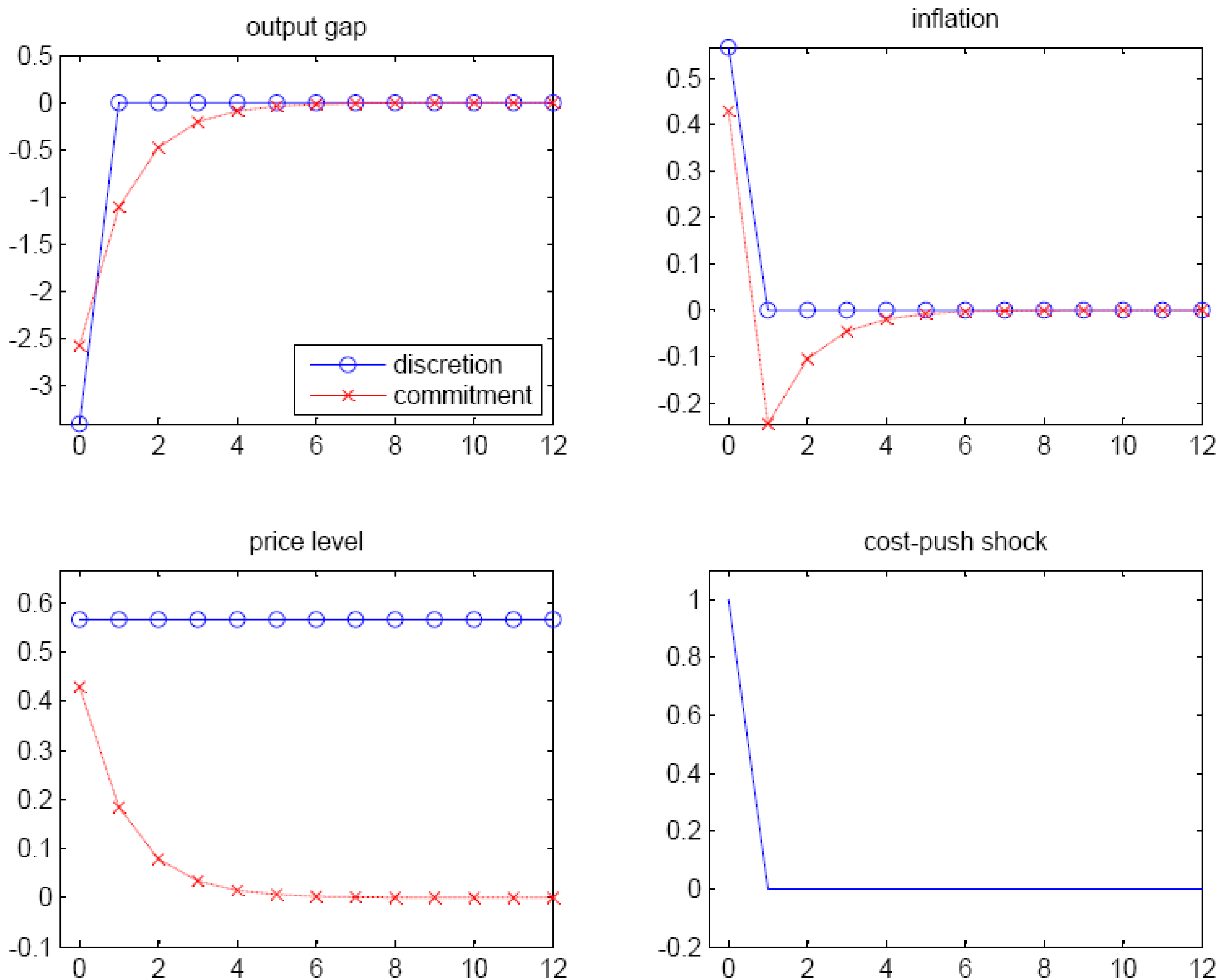

Figure 5.2 : Optimal Responses to a Persistent Cost Push Shock