A Monetary Model with Sticky Wages and Prices

by

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June 2008

Firms

Continuum of firms, each producing a differentiated good.

Technology

$$Y_t(i) = A_t \; N_t(i)^{1-\alpha}$$

$$N_t(i) \equiv \left[\int_0^1 N_t(i,j)^{1-\frac{1}{\epsilon_w}} \; dj \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}$$

Cost minimization:

$$N_t(i,j) = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} \; N_t(i) \tag{1}$$

for all $i, j \in [0,1]$, where

$$W_t \equiv \left[\int_0^1 W_t(j)^{1-\epsilon_w} \; dj \right]^{\frac{1}{1-\epsilon_w}}$$

In addition,

$$\int_0^1 W_t(j) N_t(i,j) \; dj = W_t N_t(i).$$

Optimal price setting (as in baseline sticky price model)

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta_p^k \ E_t \left\{ Q_{t,t+k} \ \left(P_t^* \ Y_{t+k|t} - \Psi_{t+k}(Y_{t+k|t}) \right) \right\}$$

subject to

$$Y_{t+k|t} = (P_t^*/P_{t+k})^{-\epsilon_p} \ C_{t+k}$$

Aggregation:

$$\pi_t^p = \beta \ E_t\{\pi_{t+1}^p\} - \lambda_p \ \widehat{\mu}_t^p \tag{2}$$

where $\widehat{\mu}_t^p \equiv \mu_t^p - \mu^p = -\widehat{mc}_t$, $\mu^p \equiv \log \frac{\epsilon_p}{\epsilon_p - 1}$, and $\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$.

Households

Continuum of households, each specialized in the supply of a differentiated labor service, $j \in [0, 1]$.

Probability of adjusting nominal wage in any given period: $1 - \theta_w$

Optimal Wage Setting

$$\max_{W_t^*} \sum_{k=0}^{\infty} (\beta\theta_w)^k \; E_t\{U(C_{t+k|t}, N_{t+k|t})\}$$

subject to:

$$N_{t+k|t} = (W_t^*/W_{t+k})^{-\epsilon_w} \; N_{t+k}$$

$$P_{t+k} \; C_{t+k|t} + E_{t+k}\{Q_{t+k,t+k+1} D_{t+k+1|t}\} \leq D_{t+k|t} + W_t^* \; N_{t+k|t} - T_{t+k}$$

where $N_t \equiv \int_0^1 N_t(i) \; di$.

Optimality condition:

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k \, E_t \left\{ N_{t+k|t} \ \left(U_c(C_{t+k|t}, N_{t+k|t}) \, \frac{W_t^*}{P_{t+k}} + \mathcal{M}_w \ U_n(C_{t+k|t}, N_{t+k|t} \right) \right\} = 0$$

where $\mathcal{M}_w \equiv \frac{\epsilon_w}{\epsilon_w - 1}$

Complete markets: $C_{t+k|t} = C_{t+k}$ for $k = 0, 1, 2, ...$

Letting $MRS_{t+k|t} \equiv -\frac{U_n(C_{t+k}, N_{t+k|t})}{U_c(C_{t+k}, N_{t+k|t})}$

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k \, E_t \left\{ N_{t+k|t} \ U_c(C_{t+k|t}, N_{t+k|t}) \left(\frac{W_t^*}{P_{t+k}} - \mathcal{M}_w \ MRS_{t+k|t} \right) \right\} = 0 \tag{3}$$

Full wage flexibility ($\theta_w = 0$):

$$\frac{W_t^*}{P_t} = \frac{W_t}{P_t} = \mathcal{M}_w \ MRS_{t|t}$$

Zero inflation steady state:

$$\frac{W^*}{P} = \mathcal{M}_w \ MRS$$

Log-linearization (after factoring out $\mathcal{M}_w \; MRS_{t+k|t}$ in (3)):

$$w_t^* = \mu^w + (1-\beta\theta_w)\sum_{k=0}^{\infty}(\beta\theta_w)^k \; E_t\left\{mrs_{t+k|t} + p_{t+k}\right\} \tag{4}$$

where $\mu^w \equiv \log \frac{\epsilon_w}{\epsilon_w - 1}$.

With isoelastic separable utility $\Longrightarrow mrs_{t+k|t} = \sigma \; c_{t+k} + \varphi \; n_{t+k|t}$.

Average marginal rate of substitution: $\quad mrs_{t+k} \equiv \sigma \; c_{t+k} + \varphi \; n_{t+k}$

$$\begin{aligned} mrs_{t+k|t} &= mrs_{t+k} + \varphi \; (n_{t+k|t} - n_{t+k}) \\ &= mrs_{t+k} - \epsilon_w \varphi \; (w_t^* - w_{t+k}) \end{aligned}$$

Hence,

$$
\begin{aligned}
w_t^* &= \frac{1-\beta\theta_w}{1+\epsilon_w\varphi}\sum_{k=0}^{\infty}(\beta\theta_w)^k\ E_t\left\{\mu_w + mrs_{t+k} + \epsilon_w\varphi\ w_{t+k} + p_{t+k}\right\} \\
&= \frac{1-\beta\theta_w}{1+\epsilon_w\varphi}\sum_{k=0}^{\infty}(\beta\theta_w)^k\ E_t\left\{(1+\epsilon_w\varphi)\ w_{t+k} - \widehat{\mu}_{t+k}^w\right\}
\end{aligned}
$$

where $\mu_t^w \equiv w_t - p_t - mrs_t$ is the average wage markup and $\widehat{\mu}_t^w \equiv \mu_t^w - \mu^w$

More compactly:

$$
w_t^* = \beta\theta_w\ E_t\{w_{t+1}^*\} + (1-\beta\theta_w)\ \left(w_t - (1+\epsilon_w\varphi)^{-1}\ \widehat{\mu}_t^w\right) \qquad (5)
$$

Wage Inflation Dynamics

$$W_t = \left[\theta_w W_{t-1}^{1-\epsilon_w} + (1-\theta_w) W_t^{*\,1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}}$$

Log-linearization:

$$w_t = \theta_w \ w_{t-1} + (1-\theta_w) \ w_t^* \tag{6}$$

Combining (5) and (6):

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} - \lambda_w \ \widehat{\mu}_t^w \tag{7}$$

where $\lambda_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)}$.

Additional Optimality Condition

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} \left(i_t - E_t\{\pi_{t+1}^p\} - \rho\right)$$

Equilibrium

Define *real wage gap:*

$$\widetilde{\omega}_t \equiv \omega_t - \omega_t^n$$

Price markups vs. output and real wage gaps:

$$\begin{aligned} \widehat{\mu}_t^p &= (mpn_t - \omega_t) - \mu^p \\ &= (\widetilde{y}_t - \widetilde{n}_t) - \widetilde{\omega}_t \\ &= -\frac{\alpha}{1-\alpha}\,\widetilde{y}_t - \widetilde{\omega}_t \end{aligned} \tag{8}$$

Combining (2) and (8):

$$\pi_t^p = \beta\ E_t\{\pi_{t+1}^p\} + \kappa_p\ \widetilde{y}_t + \lambda_p\ \widetilde{\omega}_t \tag{9}$$

where $\kappa_p \equiv \frac{\alpha\lambda_p}{1-\alpha}$.

Wage markups vs. output and real wage gaps:

$$\begin{aligned}\widehat{\mu}_t^w &= \omega_t - mrs_t - \mu^w \\ &= \widetilde{\omega}_t - (\sigma \widetilde{y}_t + \varphi \widetilde{n}_t) \\ &= \widetilde{\omega}_t - \left(\sigma + \frac{\varphi}{1-\alpha}\right) \widetilde{y}_t \end{aligned} \tag{10}$$

Combining (7) and (10):

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} + \kappa_w \, \widetilde{y}_t - \lambda_w \, \widetilde{\omega}_t \tag{11}$$

where $\kappa_w \equiv \lambda_w \left(\sigma + \frac{\varphi}{1-\alpha}\right)$.

Wage gap identity:

$$\widetilde{\omega}_{t-1} \equiv \widetilde{\omega}_t - \pi_t^w + \pi_t^p + \Delta\omega_t^n \tag{12}$$

Dynamic IS equation

$$\widetilde{y}_t = -\frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}^p\} - r_t^n\right) + E_t\{\widetilde{y}_{t+1}\} \tag{13}$$

Interest Rate Rule:

$$i_t = \rho + \phi_p\ \pi_t^p + \phi_w\ \pi_t^w + \phi_y\ \widetilde{y}_t + v_t \tag{14}$$

Dynamical system:

$$\mathbf{x}_t = \mathbf{A}_w \; E_t\{\mathbf{x}_{t+1}\} + \mathbf{B}_w \; \mathbf{z}_t \tag{15}$$

where

$$\mathbf{x}_t \equiv [\widetilde{y}_t, \; \pi_t^p, \; \pi_t^w, \; \widetilde{\omega}_{t-1}]'$$

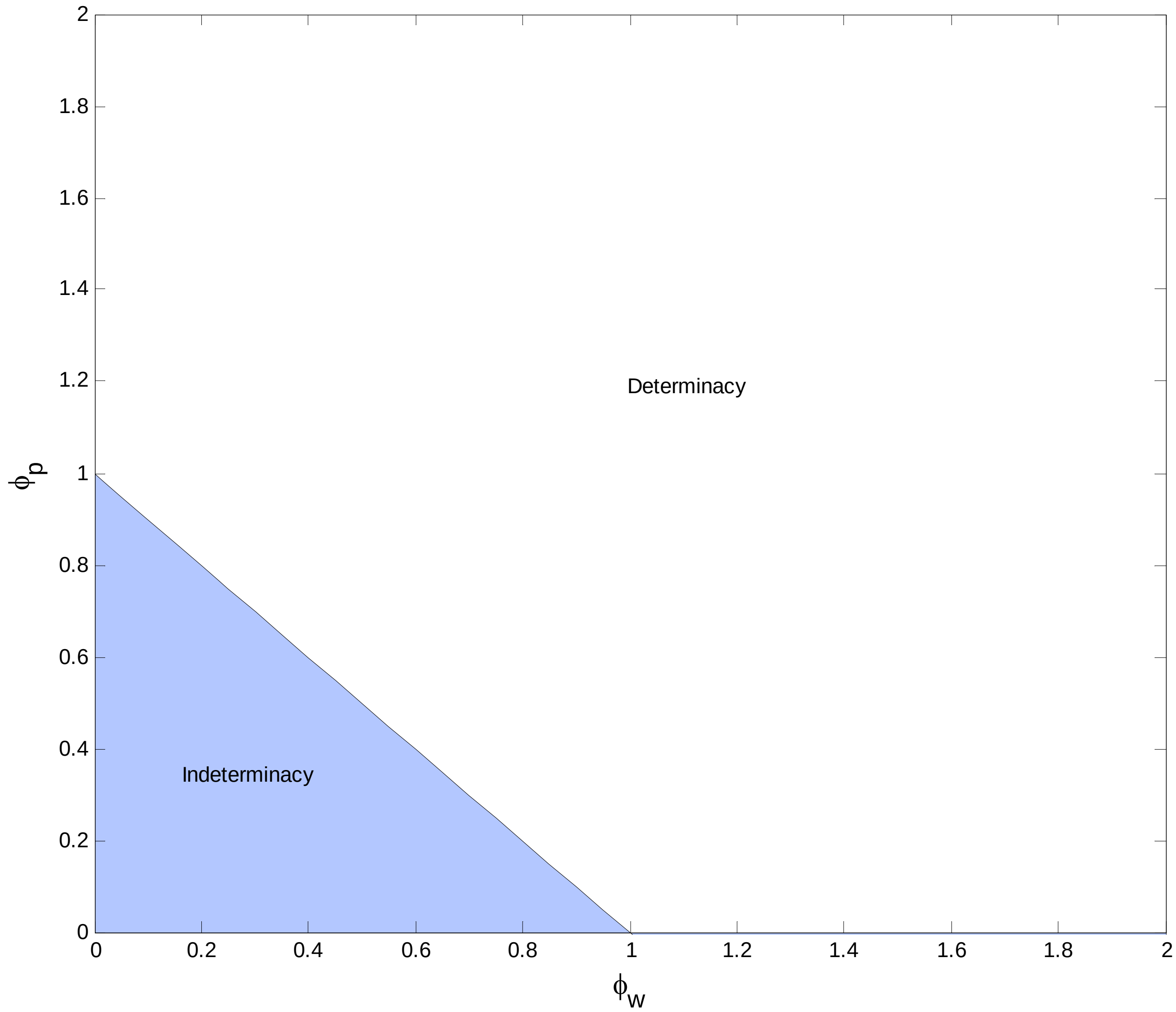
$$\mathbf{z}_t \equiv [\widehat{r}_t^n - v_t, \; \Delta\omega_t^n]'$$

Conditions for uniqueness of the equilibrium

Particular case ($\phi_y = 0$):

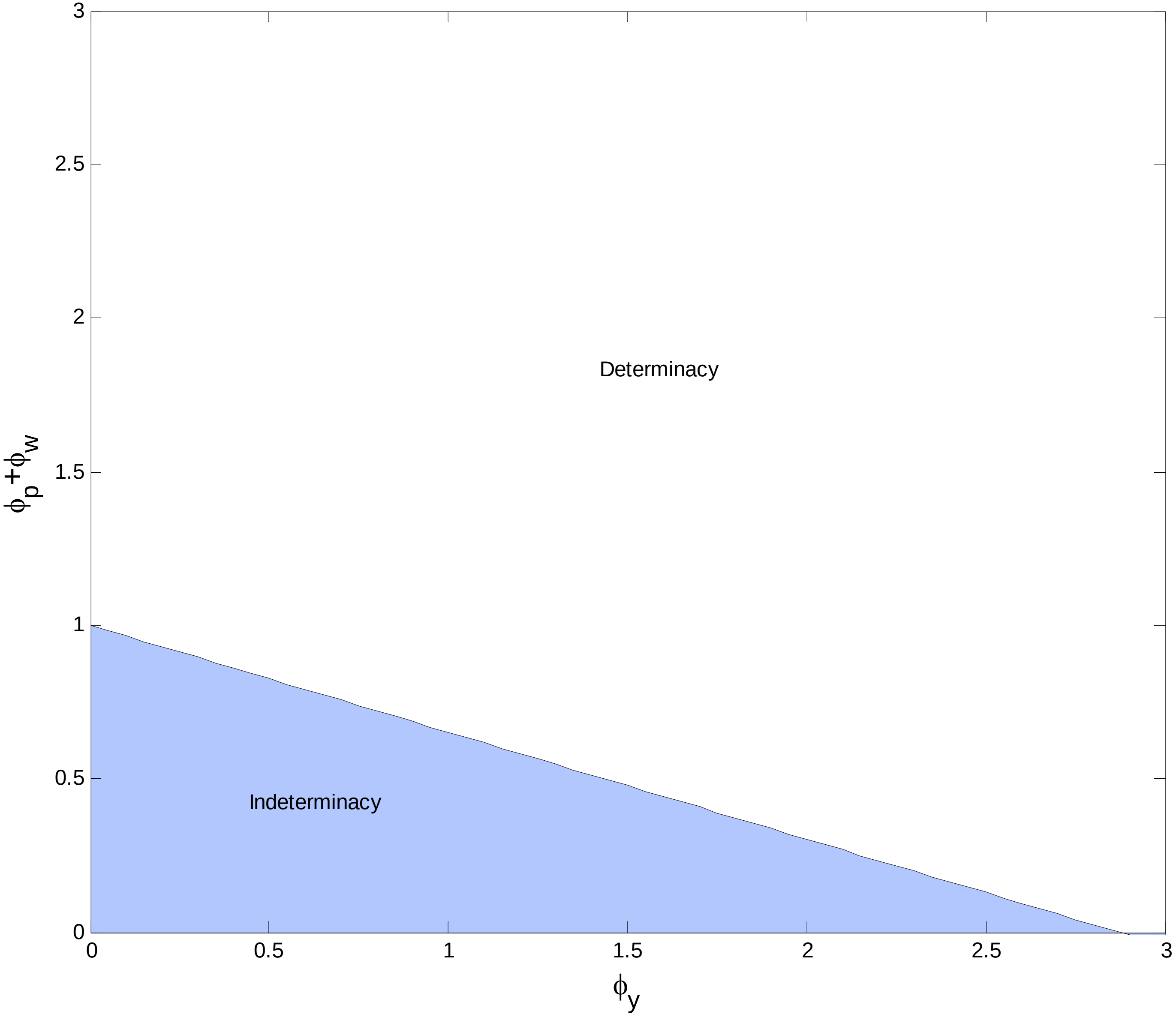
$$\phi_p + \phi_w > 1$$

Figure 6.1



Determinacy
Indeterminacy
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
ϕ_p
ϕ_w

Figure 6.2



3
2.5
2
1.5
1
0.5
0
0
0.5
1
1.5
2
2.5
3
Determinacy
Indeterminacy
φp+φw
φy

Dynamic Responses to a Monetary Policy Shock

Interest rate rule: $\phi_p = 1.5$, $\phi_y = \phi_w = 0$, $\rho_v = 0.5$

Three calibrations:

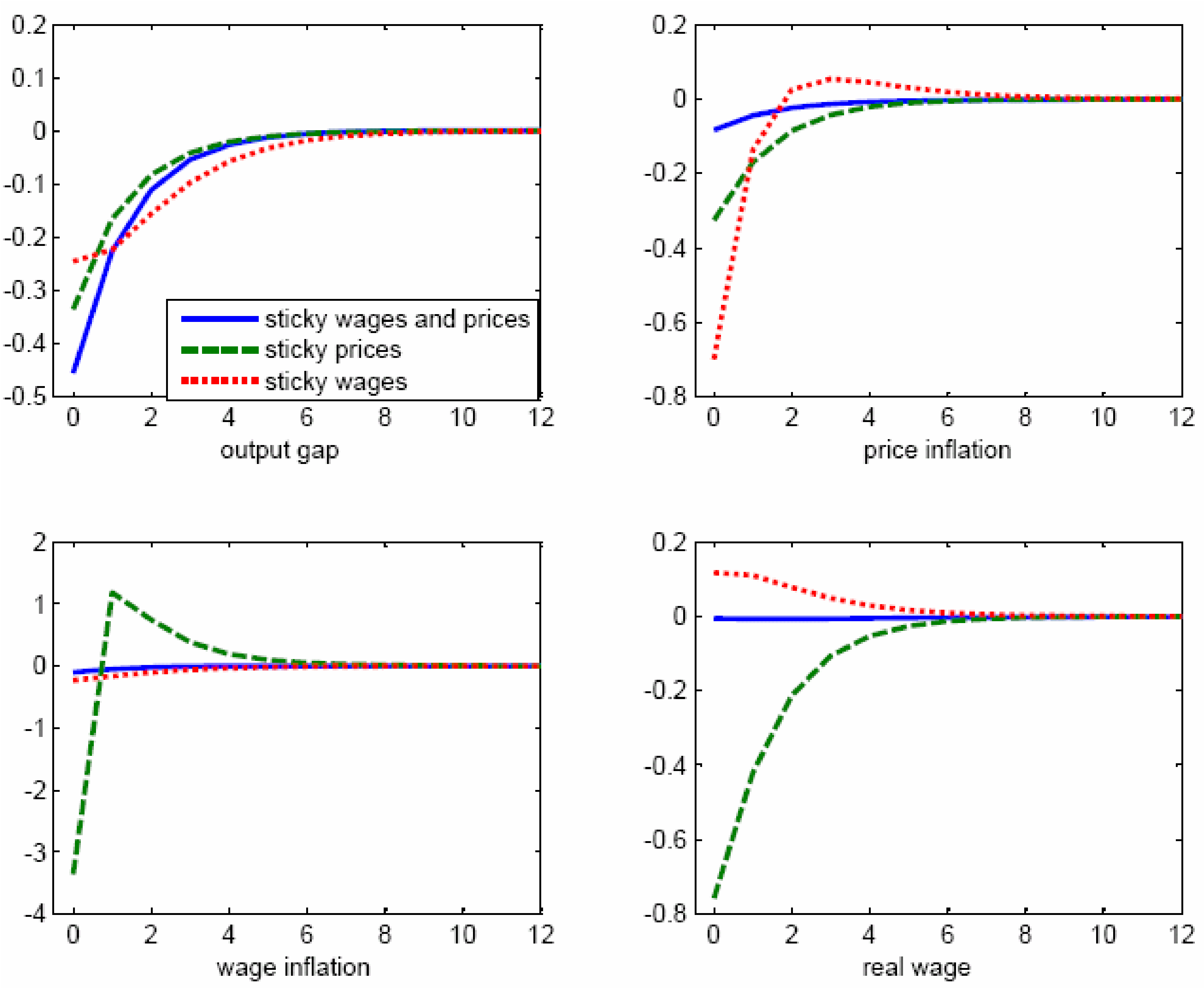
Baseline: $\theta_p = 2/3$, $\theta_w = 3/4$

Flexible wage: $\theta_p = 2/3$, $\theta_w = 0$

Flexible price: $\theta_p = 0$, $\theta_w = 3/4$

Figure 6.3

Figure 6.3: Sticky Wages and the Effects of a Monetary Policy Shock



Monetary Policy Design with Sticky Wages and Prices

Assumption: optimality of the natural equilibrium allocation.

Remark: replication of natural equilibrium is not feasible

Proof: $\widetilde{y}_t = \pi_t^p = \pi_t^w = 0$ is not a solution, unless ω_t^n is constant.

Second Order Approximation to Welfare Losses

$$\mathbb{W} = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \widetilde{y}_t^2 + \frac{\epsilon_p}{\lambda_p} (\pi_t^p)^2 + \frac{\epsilon_w (1 - \alpha)}{\lambda_w} (\pi_t^w)^2 \right) + t.i.p.$$

$$\mathbb{L} = \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) var(\widetilde{y}_t) + \frac{\epsilon_p}{\lambda_p} var(\pi_t^p) + \frac{\epsilon_w (1 - \alpha)}{\lambda_w} var(\pi_t^w)$$

- Optimal monetary policy
- Evaluation of alternative simple rules

Optimal Monetary Policy

$$\min E_0 \sum_{t=0}^{\infty} \beta^t \left(\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \widetilde{y}_t^2 + \frac{\epsilon_p}{\lambda_p} (\pi_t^p)^2 + \frac{\epsilon_w (1 - \alpha)}{\lambda_w} (\pi_t^w)^2 \right)$$

subject to

$$\pi_t^p = \beta \; E_t\{\pi_{t+1}^p\} + \kappa_p \; \widetilde{y}_t + \lambda_p \; \widetilde{\omega}_t$$

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} + \kappa_w \; \widetilde{y}_t - \lambda_w \; \widetilde{\omega}_t$$

$$\widetilde{\omega}_{t-1} \equiv \widetilde{\omega}_t - \pi_t^w + \pi_t^p + \Delta \omega_t^n$$

Optimality conditions:

$$\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) \ \widetilde{y}_t + \kappa_p \ \xi_{1,t} + \kappa_w \ \xi_{2,t} = 0 \tag{16}$$

$$\frac{\epsilon_p}{\lambda_p} \ \pi_t^p - \Delta \xi_{1,t} + \xi_{3,t} = 0 \tag{17}$$

$$\frac{\epsilon_w (1 - \alpha)}{\lambda_w} \ \pi_t^w - \Delta \xi_{2,t} - \xi_{3,t} = 0 \tag{18}$$

$$\lambda_p \ \xi_{1,t} - \lambda_w \ \xi_{2,t} + \xi_{3,t} - \beta E_t\{\xi_{3,t+1}\} = 0 \tag{19}$$

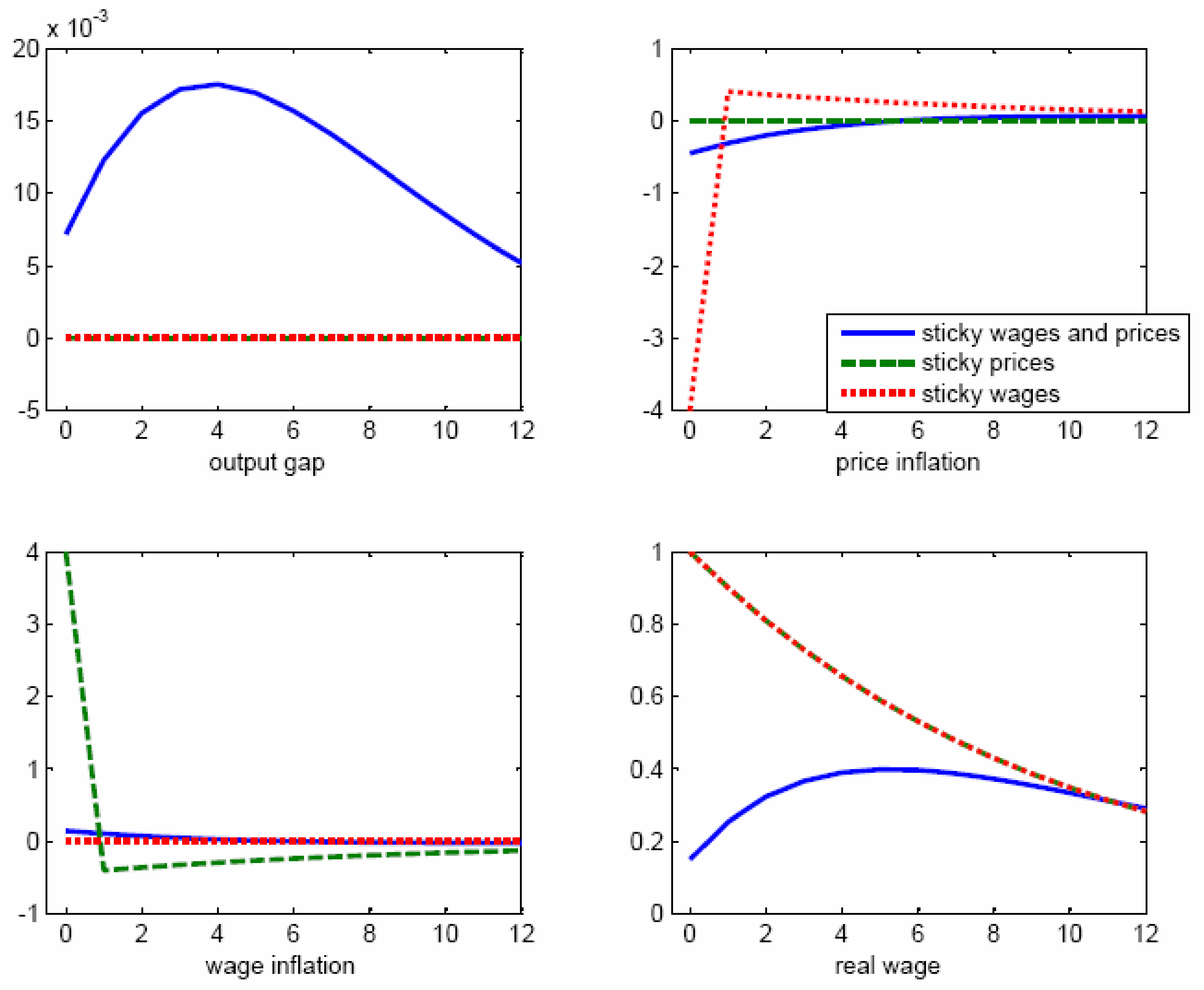
Combined with (9), (11), and (12):

$$\mathbf{A}_0^* \, \mathbf{x}_t = \mathbf{A}_{\mathbf{1}}^* \, E_t\{\mathbf{x}_{t+1}\} + \mathbf{B}^* \, \Delta a_t$$

where $\mathbf{x}_t \equiv [\widetilde{y}_t, \; \pi_t^p, \; \pi_t^w, \, \widetilde{\omega}_{t-1}, \, \xi_{1,t-1}, \; \xi_{2,t-1}, \; \xi_{3,t}]'$

Dynamic Responses to a Technology Shock (Figure 6.4)

Figure 6.4: The Effects of a Technology Shock under the Optimal Policy



A Special Case with an Analytical Solution

Define:

$$\pi_t \equiv (1 - \vartheta)\ \pi_t^p + \vartheta\ \pi_t^w \tag{20}$$

where $\vartheta \equiv \frac{\lambda_p}{\lambda_p + \lambda_w} \in [0, 1]$

Note that (9) and (11) imply:

$$\pi_t = \beta\ E_t\{\pi_{t+1}\} + \kappa\ \widetilde{y}_t \tag{21}$$

where $\kappa \equiv \frac{\lambda_p \lambda_w}{\lambda_p + \lambda_w}\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right)$

- no trade-off !
- when is it optimal to fully stabilize π_t (and the output gap)?

Assumptions: $\kappa_p = \kappa_w$; $\epsilon_p = \epsilon_w(1-\alpha) \equiv \epsilon$

Then, (16)-(18) simplify to:

$$\lambda_w \ \pi_t^p + \lambda_p \ \pi_t^w = -\frac{\lambda_p}{\epsilon} \ \Delta\widetilde{y}_t$$

for $t = 1, 2, 3, ...$ and $\lambda_w \ \pi_0^p + \lambda_p \ \pi_0^w = -\frac{\lambda_p}{\epsilon} \ \widetilde{y}_0$ for period 0

Equivalently,

$$\pi_t = -\frac{\vartheta}{\epsilon} \ \Delta\widetilde{y}_t$$

for $t = 1, 2, 3, ...$, and $\pi_0 = -\frac{\vartheta}{\epsilon} \ \widetilde{y}_0$ in period 0.

In levels:

$$\widehat{q}_t = -\frac{\vartheta}{\epsilon} \ \widetilde{y}_t \tag{22}$$

where $\widehat{q}_t \equiv q_t - q_{-1}$, and $q_t \equiv (1-\vartheta) \ p_t + \vartheta \ w_t$.

Combining (22) and (21) (using $\pi_t \equiv \widehat{q}_t - \widehat{q}_{t-1}$):

$$\widehat{q}_t = a\ \widehat{q}_{t-1} + a\beta\ E_t\{\widehat{q}_{t+1}\} = 0$$

for $t = 0, 1, 2, ...$where $a \equiv \frac{\vartheta}{\vartheta(1+\beta)+\kappa\epsilon}$.

Stationary solution:

$$\widehat{q}_t = \delta\ \widehat{q}_{t-1}$$

where $\delta \equiv \frac{1-\sqrt{1-4\beta a^2}}{2a\beta} \in (0,1)$ for $t = 0, 1, 2, ...$

Given that $\widehat{q}_{-1} = 0$, the optimal policy requires:

$$\pi_t = 0$$

$$\widetilde{y}_t = 0$$

for $t = 0, 1, 2, ...$

Evaluation of Simple Rules under Sticky Wages and Prices

Six rules:

- strict price inflation targeting ($\pi_t^p = 0$, all t)
- strict wage inflation targeting ($\pi_t^w = 0$, all t)
- strict composite inflation targeting ($\pi_t = 0$, all t)
- flexible price inflation targeting ($i_t = \rho + 1.5\ \pi_t^p$)
- flexible wage inflation targeting ($i_t = \rho + 1.5\ \pi_t^w$)
- flexible composite inflation targeting ($i_t = \rho + 1.5\ \pi_t$)

Three scenarios

- baseline: $\theta_p = 2/3$; $\theta_w = 3/4$
- low wage rigidities: $\theta_p = 2/3$ and $\theta_w = 1/4$
- low price rigidities: $\theta_p = 1/3$ and $\theta_w = 3/4$

Table 6.1: Evaluation of Simple Rules

	Optimal	*Strict Rules*			*Flexible Rules*		
	Policy	Price	Wage	Composite	Price	Wage	Composite
$\theta_p = \frac{2}{3}$ $\theta_w = \frac{3}{4}$							
$\sigma(\pi^p)$	0.64	0	0.82	0.66	1.50	1.08	1.12
$\sigma(\pi^w)$	0.22	0.98	0	0.19	1.05	0.30	0.42
$\sigma(\widetilde{y})$	0.04	2.38	0.52	0	0.75	1.16	0.01
$\mathbb{L}$	0.023	0.184	0.034	0.023	0.221	0.081	0.089
$\theta_p = \frac{2}{3}$ $\theta_w = \frac{1}{4}$							
$\sigma(\pi^p)$	0.29	0	0.82	0.21	1.40	1.45	1.30
$\sigma(\pi^w)$	1.24	2.91	0	1.63	1.49	0.98	1.25
$\sigma(\widetilde{y})$	0.19	0.61	0.52	0	0.29	0.68	0.32
$\mathbb{L}$	0.010	0.038	0.034	0.012	0.097	0.104	0.083
$\theta_p = \frac{1}{3}$ $\theta_w = \frac{3}{4}$							
$\sigma(\pi^p)$	1.64	0	1.91	1.75	2.58	2.10	2.10
$\sigma(\pi^w)$	0.11	0.98	0	0.06	1.47	0.07	0.10
$\sigma(\widetilde{y})$	0.17	2.38	0.27	0	0.87	0.60	0.58
$\mathbb{L}$	0.016	0.184	0.021	0.017	0.271	0.030	0.031