A Classical Monetary Model

by

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Households

Representative household solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right)$$

subject to

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + D_t$$

for $t = 0, 1, 2, \ldots$ plus solvency constraint.

Optimality conditions

$$U_{n,t} + \frac{W_t}{P_t} U_{c,t} = 0$$

$$\frac{Q_t}{P_t} U_{c,t} = \beta E_t \left\{ U_{c,t+1} \frac{1}{P_{t+1}} \right\}$$

Specification of utility:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

implied log-linear optimality conditions (aggregate variables)

$$w_t - p_t = \sigma \ c_t + \varphi \ n_t$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} \left(i_t - E_t\{\pi_{t+1}\} - \rho\right)$$

where $i_t \equiv -\log Q_t$ is the *nominal interest rate*, $\rho \equiv -\log \beta$ is the *discount rate*, and $\pi_{t+1} \equiv p_{t+1} - p_t$ is the inflation rate.

Firms

Profit maximization:

$$\max \; P_t Y_t - W_t N_t$$

subject to

$$Y_t = A_t N_t^{1-\alpha}$$

and taking the price and wage as given.

Optimality condition

$$\frac{W_t}{P_t} = (1-\alpha) A_t N_t^{-\alpha}$$

log-linear version (ignoring constants):

$$w_t - p_t = a_t - \alpha n_t$$

Equilibrium

Goods market clearing

$$y_t = c_t$$

Labor market clearing

$$\sigma c_t + \varphi n_t = a_t - \alpha n_t$$

Asset market clearing:

$$b_t = 0$$

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho)$$

Aggregate production relationship:

$$y_t = a_t + (1 - \alpha) n_t$$

Implied equilibrium values for real variables

$$n_t = \frac{1-\sigma}{\sigma(1-\alpha)+\varphi+\alpha}\, a_t \quad ; \quad y_t = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}\, a_t$$

$$w_t - p_t = \frac{\sigma+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}\, a_t$$

$$r_t \equiv i_t - E_t\{\pi_{t+1}\} = \rho - \frac{\sigma(1+\varphi)(1-\rho_a)}{\sigma(1-\alpha)+\varphi+\alpha}\, a_t$$

$\Longrightarrow$ real variables determined *independently of monetary policy* (neutrality)

$\Longrightarrow$ *optimal policy*: undetermined.

$\Longrightarrow$ specification of monetary policy needed to determine nominal variables

Monetary Policy and Price Level Determination

A Simple Interest Rate Rule

$$i_t = \rho + \phi_\pi \pi_t + v_t$$

where $v_t = \rho_v v_{t-1} + \varepsilon_t^v$ is an exogenous monetary policy shock.

Combined with the definition of the real rate:

$$\phi_\pi \pi_t = E_t\{\pi_{t+1}\} + \widehat{r}_t - v_t$$

If $\phi_\pi > 1$, unique stationary solution:

$$\begin{aligned} \pi_t &= \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} E_t\{\widehat{r}_{t+k} - v_{t+k}\} \\ &= -\frac{\psi_r}{\phi_\pi - \rho_a}\, a_t - \frac{1}{\phi_\pi - \rho_v}\, v_t \end{aligned}$$

An Exogenous Process for the Money Supply $\{m_t\}$

Money demand

$$m_t - p_t = y_t - \eta i_t$$

Combining money demand and Fisherian equation:

$$p_t = \left(\frac{\eta}{1+\eta}\right) E_t\{p_{t+1}\} + \left(\frac{1}{1+\eta}\right) m_t + u_t$$

where $u_t \equiv (1+\eta)^{-1}(\eta r_t - y_t)$ evolves independently of $\{m_t\}$.

In terms of expected future money growth rates

$$p_t = m_t + \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k E_t\{\Delta m_{t+k}\} + u_t' \qquad (1)$$

Implied nominal interest rate:

$$
\begin{aligned}
i_t &= \eta^{-1}(y_t - (m_t - p_t)) \\
&= \eta^{-1} \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k E_t \left\{ \Delta m_{t+k} \right\} + u_t^{''}
\end{aligned}
$$

where $u_t^{''} \equiv \eta^{-1}(u_t' + y_t)$ is independent of policy.

Example.

$$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m$$

Assume no real shocks ($y_t = r_t = 0$).

Price response:

$$p_t = m_t + \frac{\eta \rho_m}{1 + \eta(1 - \rho_m)} \Delta m_t$$

$\Longrightarrow$ *large price response*

Nominal interest rate response:

$$i_t = \frac{\rho_m}{1 + \eta(1 - \rho_m)} \Delta m_t$$

$\Longrightarrow$ *no liquidity effect*

Introducing Monopolistic Competition

Households

Representative household solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right)$$

where

$$C_t \equiv \left[\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}}$$

subject to

$$\int_0^1 P_t(i) C_t(i)\ di + Q_t B_t \leq B_{t-1} + W_t N_t + D_t - T_t$$

for $t = 0, 1, 2, ...$ plus solvency constraint.

Optimality conditions

1. Optimal allocation of expenditures

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t$$

implying

$$\int_0^1 P_t(i) C_t(i) \, di = P_t C_t$$

where

$$P_t \equiv \left[\int_0^1 P_t(i)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}}$$

2. Other optimality conditions (unchanged)

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

$$Q_t = \beta \; E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \; \frac{P_t}{P_{t+1}} \right\}$$

Firms

Continuum of firms, indexed by $i \in [0,1]$
Each firm produces a differentiated good

Profit maximization:

$$\max_{P_t(i)} \; P_t(i) Y_t(i) - \mathcal{C}_t(Y_t(i))$$

subject to

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t$$

Optimality condition:

$$P_t(i) = \mathcal{M} \Psi_t(i)$$

where $\mathcal{M} \equiv \frac{\epsilon}{\epsilon - 1}$.and $\Psi_t(i) \equiv \mathcal{C}_t'(Y_t(i))$.

Specification of technology

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

Implied marginal cost:

$$\Psi_t(i) = \frac{W_t}{A_t N_t(i)^{-\alpha}}$$

Optimal price setting rule (in logs):

$$\begin{aligned} p_t(i) &= \mu + \psi_t(i) \\ &= \mu + w_t - a_t + \alpha n_t(i) \end{aligned}$$

where $\mu \equiv \log \mathcal{M}$ and $\psi_t(i) \equiv \log \Psi_t(i)$.

Equilibrium

Goods market clearing

$$p_t(i) = p_t, \ all \ i \in [0,1]$$

$$y_t(i) = c_t(i), \ all \ i \in [0,1] \quad \Longrightarrow \quad y_t = c_t$$

Labor market clearing

$$\sigma c_t + \varphi n_t = w_t - p_t = a_t - \alpha n_t - \mu$$

Asset market clearing:

$$b_t = 0$$

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho)$$

Aggregate production relationship:

$$y_t = a_t + (1-\alpha)n_t$$

Implied equilibrium values for real variables

$$n_t = \frac{(1-\sigma)a_t - \mu}{\sigma(1-\alpha)+\varphi+\alpha}$$

$$y_t = \frac{(1+\varphi)a_t - (1-\alpha)\mu}{\sigma(1-\alpha)+\varphi+\alpha}$$

$$w_t - p_t = \frac{(\sigma+\varphi)a_t - \alpha\ \mu}{\sigma(1-\alpha)+\varphi+\alpha}$$

$$r_t \equiv i_t - E_t\{\pi_{t+1}\} = \rho - \frac{\sigma(1+\varphi)(1-\rho_a)}{\sigma(1-\alpha)+\varphi+\alpha}a_t$$

$\Longrightarrow$ real variables determined *independently of monetary policy*

$\Longrightarrow$ *optimal policy*: undetermined.

$\Longrightarrow$ inefficient equilibrium

$\Longrightarrow$ specification of monetary policy needed to determine nominal variables

Technical Appendix

Optimal Allocation of Consumption Expenditures

Maximization of C_t for any *given* expenditure level $\int_0^1 P_t(i)C_t(i)\ di \equiv Z_t$ can be formalized by means of the Lagrangean

$$\mathcal{L} = \left[\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}}\ di\right]^{\frac{\epsilon}{\epsilon-1}} - \lambda\ \left(\int_0^1 P_t(i)\ C_t(i)\ di - Z_t\right)$$

The associated first order conditions are:

$$C_t(i)^{-\frac{1}{\epsilon}}\ C_t^{\frac{1}{\epsilon}} = \lambda\ P_t(i)$$

for all $i \in [0,1]$. Thus, for any two goods (i,j) we have:

$$C_t(i) = C_t(j)\left(\frac{P_t(i)}{P_t(j)}\right)^{-\epsilon}$$

which can be plugged into the expression for consumption expenditures to yield

$$C_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon}\frac{Z_t}{P_t}$$

for all $i \in [0,1]$. The latter condition can then be substituted into the definition of C_t, yielding

$$\int_0^1 P_t(i) C_t(i)\, di = P_t C_t$$

Combining the two previous equations we obtain the demand schedule:

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t$$

Log-Linearized Euler Equation

We can rewrite the Euler equation as

$$1 = E_t\{\exp(i_t - \sigma \Delta c_{t+1} - \pi_{t+1} - \rho)\} \tag{2}$$

In a perfect foresight steady state with constant inflation π and constant growth γ we must have:

$$i = \rho + \sigma\gamma + \pi$$

with the steady state real rate being given by

$$\begin{aligned} r &\equiv i - \pi \\ &= \rho + \sigma\gamma \end{aligned}$$

A first order Taylor expansion of $\exp(i_t - \sigma\Delta c_{t+1} - \pi_{t+1} - \rho)$ around that steady state yields:

$$\begin{aligned} \exp(i_t - \sigma\Delta c_{t+1} - \pi_{t+1} - \rho) &\simeq 1 + (i_t - i) - \sigma(\Delta c_{t+1} - \gamma) - (\pi_{t+1} - \pi) \\ &= 1 + i_t - \sigma\Delta c_{t+1} - \pi_{t+1} - \rho \end{aligned}$$

which can be used in (2) to obtain, after some rearrangement of terms, the log-linearized Euler equation

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\,(i_t - E_t\{\pi_{t+1}\} - \rho)$$