Monetary Models with Nominal Rigidities

by

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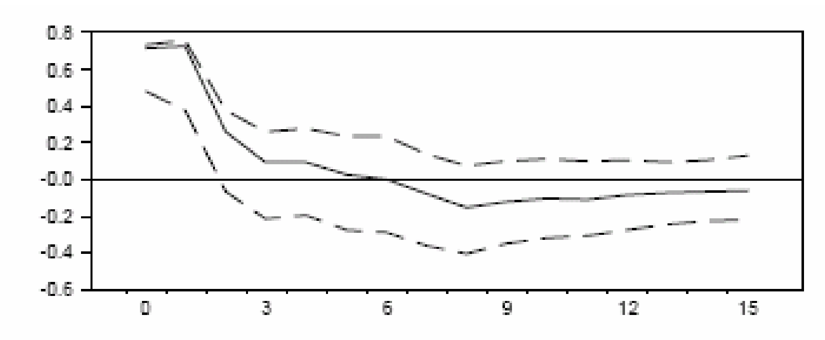
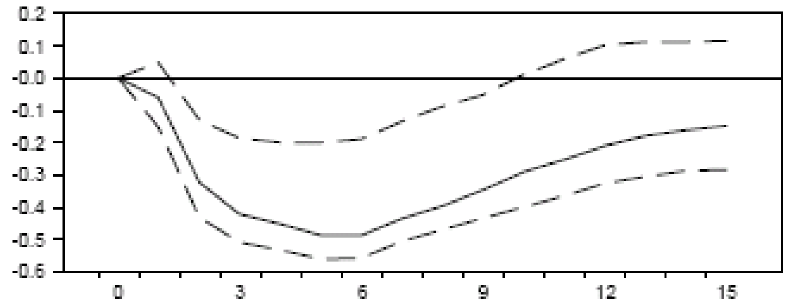
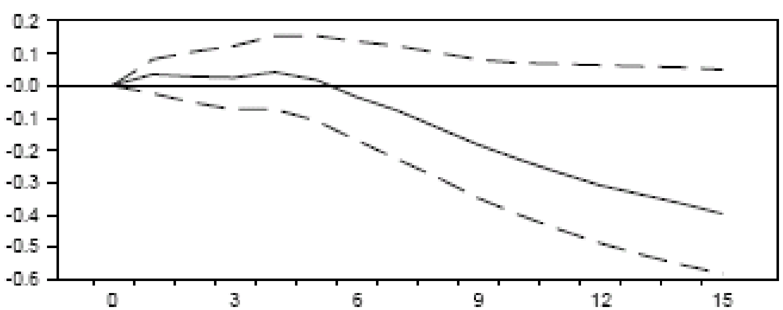
Estimated Dynamic Response to a Monetary Policy Shock



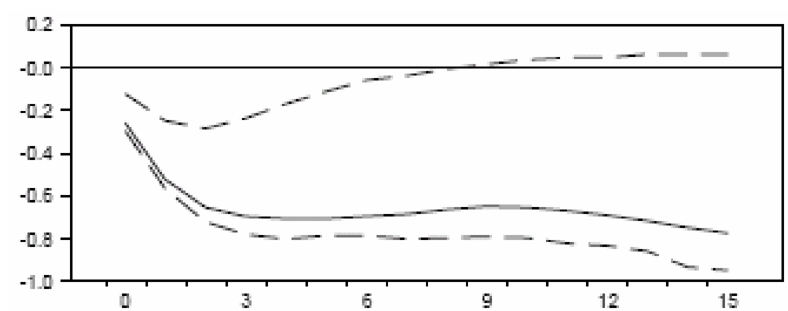
Federal funds rate



GDP



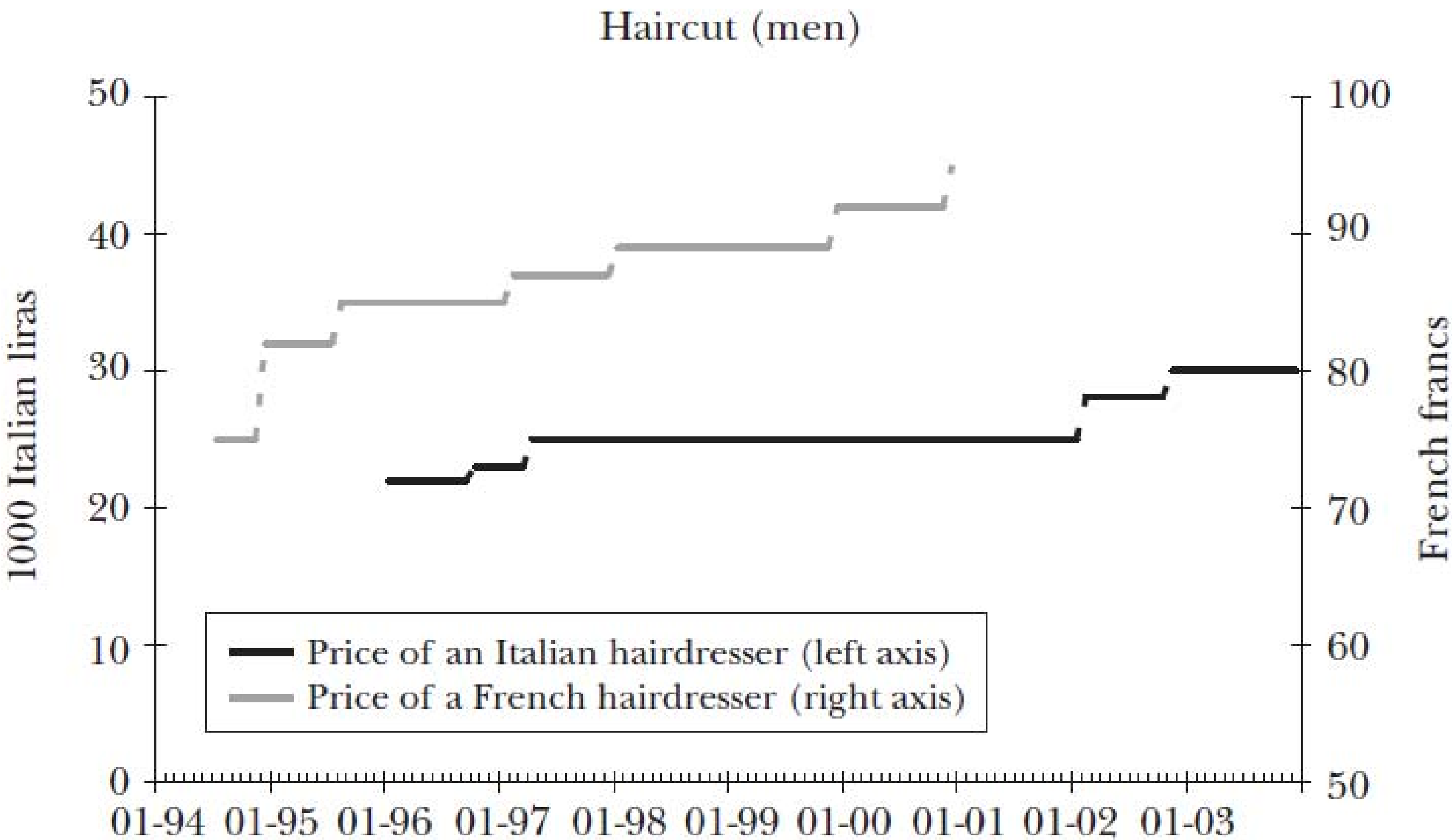
GDP deflator



M2

Source: Christiano, Eichenbaum and Evans (1999)

Micro Evidence on Price Rigidities: An Illustration



Source: Dhyne et al. (JEP 2006)

Motivation

Evidence on Money, Output, and Prices:

- Short Run Effects of Monetary Policy Shocks

  (i) persistent effects on real variables
  (ii) slow adjustment of aggregate price level
  (iii) liquidity effect

- Micro Evidence on Price-setting Behavior: significant price and wage rigidities

Failure of Classical Monetary Models

Introducing Nominal Rigidities

- monopolistic competition
- sticky prices
- competitive labor markets, closed economy, no capital accumulation

Introducing Nominal Rigidities: Price Setting in Advance

Key assumption: firms set prices *before* realization of shocks.

Simplifying assumption: $\alpha = 0$ (constant returns to labor)

Optimal price-setting rule (log-linearized)

$$p_t(i) = \mu + E_{t-1}\{\psi_t(i)\}$$

Symmetric equilibrium $(p_t(i) = p_t,\ \psi_t(i) = \psi_t)$

$$p_t = \mu + E_{t-1}\{\psi_t\}$$

Note that

$$\begin{aligned}\psi_t &= w_t - a_t \\ &= (w_t - p_t) + p_t - a_t \\ &= \sigma c_t + \varphi n_t - a_t + p_t \\ &= (\sigma + \varphi) y_t - (1 + \varphi) a_t + p_t\end{aligned}$$

Thus

$$E_{t-1}\{y_t\} = \frac{1+\varphi}{\sigma+\varphi}\, E_{t-1}\{a_t\} - \frac{\mu}{\sigma+\varphi}$$

Assuming a simple money demand equation

$$m_t - p_t = y_t$$

we have:

$$p_t = E_{t-1}\{m_t\} - \frac{1+\varphi}{\sigma+\varphi}\, E_{t-1}\{a_t\} + \frac{\mu}{\sigma+\varphi}$$

$$
\begin{aligned}
y_t &= m_t - p_t \\
&= (m_t - E_{t-1}\{m_t\}) + \frac{1+\varphi}{\sigma+\varphi}\, E_{t-1}\{a_t\} - \frac{\mu}{\sigma+\varphi}
\end{aligned}
$$

$\Longrightarrow$ monetary non-neutrality: output responds to *unanticipated* changes in the money supply

$\Longrightarrow$ monetary policy shocks have no persistent real effects

$$
\begin{aligned}
n_t &= y_t - a_t \\
&= (m_t - E_{t-1}\{m_t\}) - (a_t - E_{t-1}\{a_t\}) + \frac{1+\varphi}{\sigma+\varphi}\, E_{t-1}\{a_t\} - \frac{\mu}{\sigma+\varphi}
\end{aligned}
$$

$\Longrightarrow$ unanticipated positive technology shocks lower employment, unless accommodated by monetary policy.

The Basic New Keynesian Model

Key Blocks

- *New Keynesian Phillips Curve*

$$\pi_t = \beta \; E_t\{\pi_{t+1}\} + \kappa \; \widetilde{y}_t$$

where $\widetilde{y}_t \equiv y_t - y_t^n$

- *Dynamic IS Equation*

$$\widetilde{y}_t = -\frac{1}{\sigma} \left(i_t - E_t\{\pi_{t+1}\} - r_t^n\right) + E_t\{\widetilde{y}_{t+1}\}$$

- *Monetary Policy Rule*

Example:

$$i_t = \rho + \phi_\pi \; \pi_t + \phi_y \; \widehat{y}_t + v_t$$

Households

Representative household solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \, U\left(C_t, N_t\right)$$

where

$$C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} \, di \right)^{\frac{\epsilon}{\epsilon-1}}$$

subject to

$$\int_0^1 P_t(i) C_t(i) \, di + Q_t B_t \leq B_{t-1} + W_t N_t + D_t$$

for $t = 0, 1, 2, ...$ plus solvency constraint.

1. Optimal allocation of expenditures

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t$$

where

$$P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon} \, di \right)^{\frac{1}{1-\epsilon}}$$

Implication:

$$\int_0^1 P_t(i) C_t(i) \, di = P_t C_t$$

2. Other optimality conditions

$$-U_{n,t} = \frac{W_t}{P_t} \, U_{c,t}$$

$$\frac{Q_t}{P_t} \, U_{c,t} = \beta \; E_t \left\{ U_{c,t+1} \, \frac{1}{P_{t+1}} \right\}$$

Specification of utility:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

implied log-linear optimality conditions (aggregate variables)

$$w_t - p_t = \sigma c_t + \varphi n_t \equiv mrs_t$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - \rho\right)$$

where $Q_t \equiv \exp\{-i_t\}$, $\beta \equiv \exp\{-\rho\}$ and $\pi_t \equiv p_t - p_{t-1}$.

Firms

- Continuum of firms, indexed by $i \in [0, 1]$
- Each firm produces a differentiated good
- Identical technology

$$Y_t(i) = A_t N_t(i)$$

- Constant desired markup $\mathcal{M} \equiv \frac{\epsilon}{\epsilon - 1}$ under flexible prices

$$p_t = \mu + \psi_t$$

- Probability of being able to reset price in any given period: $1 - \theta$, independent across firms (Calvo (1983)).
- $\theta \in [0, 1]$: index of price stickiness
- Implied average price duration $\frac{1}{1-\theta}$

The New Keynesian Phillips Curve

- *Staggered Price Setting*

$$p_t = \theta \; p_{t-1} + (1-\theta) \; p_t^*$$

$$p_t^* = \mu + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k \; E_t\{\psi_{t+k}\}$$

Implying:

$$\pi_t = \beta \; E_t\{\pi_{t+1}\} - \lambda(\mu_t - \mu)$$

where $\mu_t \equiv p_t - \psi_t$ and $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$

- *Average Markup*

$$\mu_t = p_t - (w_t - a_t)$$

- *Labor Market Clearing*

$$w_t - p_t = \sigma \; c_t + \varphi \; n_t$$

$$n_t = y_t - a_t$$

- *Goods Market Clearing*

$$y_t = c_t$$

- *Average Markup and the Output Gap*

$$\mu_t = (1 + \varphi)\ a_t - (\sigma + \varphi)\ y_t$$

Under flexible prices:

$$\mu = (1 + \varphi)\ a_t - (\sigma + \varphi)\ y_t^n$$

Combining both:

$$\mu_t - \mu = -(\sigma + \varphi)\ \widetilde{y}_t$$

- *The New Keynesian Phillips Curve*

$$\pi_t = \beta\ E_t\{\pi_{t+1}\} + \kappa\ \widetilde{y}_t$$

where $\kappa \equiv \lambda\,(\sigma + \varphi)$

- Some properties:

  (i) Forward-looking

$$\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k \; E_t\{\widetilde{y}_{t+k}\}$$

  $\Rightarrow$ no role for past inflation

  (ii) No tradeoff between output gap and inflation stabilization
  $\Rightarrow$ "the Divine Coincidence"
  $\Rightarrow$ costless disinflations

  (iii) Model-based vs. traditional output gap

$$\widehat{y}_t = y_t - f(t)$$

$$\widetilde{y}_t \equiv y_t - y_t^n$$

  $\Rightarrow$ may distort empirical assessments

The Dynamic IS Equation

- Euler equation + goods market clearing

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}\,(i_t - E_t\{\pi_{t+1}\} - \rho)$$

Combined with $\widetilde{y}_t \equiv y_t - y_t^n$

$$\widetilde{y}_t = -\frac{1}{\sigma}\,(i_t - E_t\{\pi_{t+1}\} - r_t^n) + E_t\{\widetilde{y}_{t+1}\}$$

where

$$\begin{aligned} r_t^n &\equiv \rho + \sigma\ E_t\{\Delta y_{t+1}^n\} \\ &= \rho + \frac{\sigma(1+\varphi)}{\sigma+\varphi} E_t\{\Delta a_{t+1}\} \end{aligned}$$

Monetary Policy

- Interest rate rule

  Example:

$$i_t = \rho + \phi_\pi \, \pi_t + \phi_y \, \widehat{y}_t + v_t$$

- The role of monetary aggregates

  - money demand (ad hoc):

$$m_t - p_t = y_t - \eta \, i_t$$

  - implied money growth:

$$\Delta m_t = \pi_t + \Delta y_t - \eta \, \Delta i_t$$

Simulations

- Dynamic effects of monetary policy shock

$$v_t = \rho_v \; v_{t-1} + \varepsilon_t^v$$

- Dynamic effects of a technology shock

$$a_t = \rho_a \; a_{t-1} + \varepsilon_t^a$$

Figure 3.1: Effects of a Monetary Policy Shock (Interest Rate Rule))

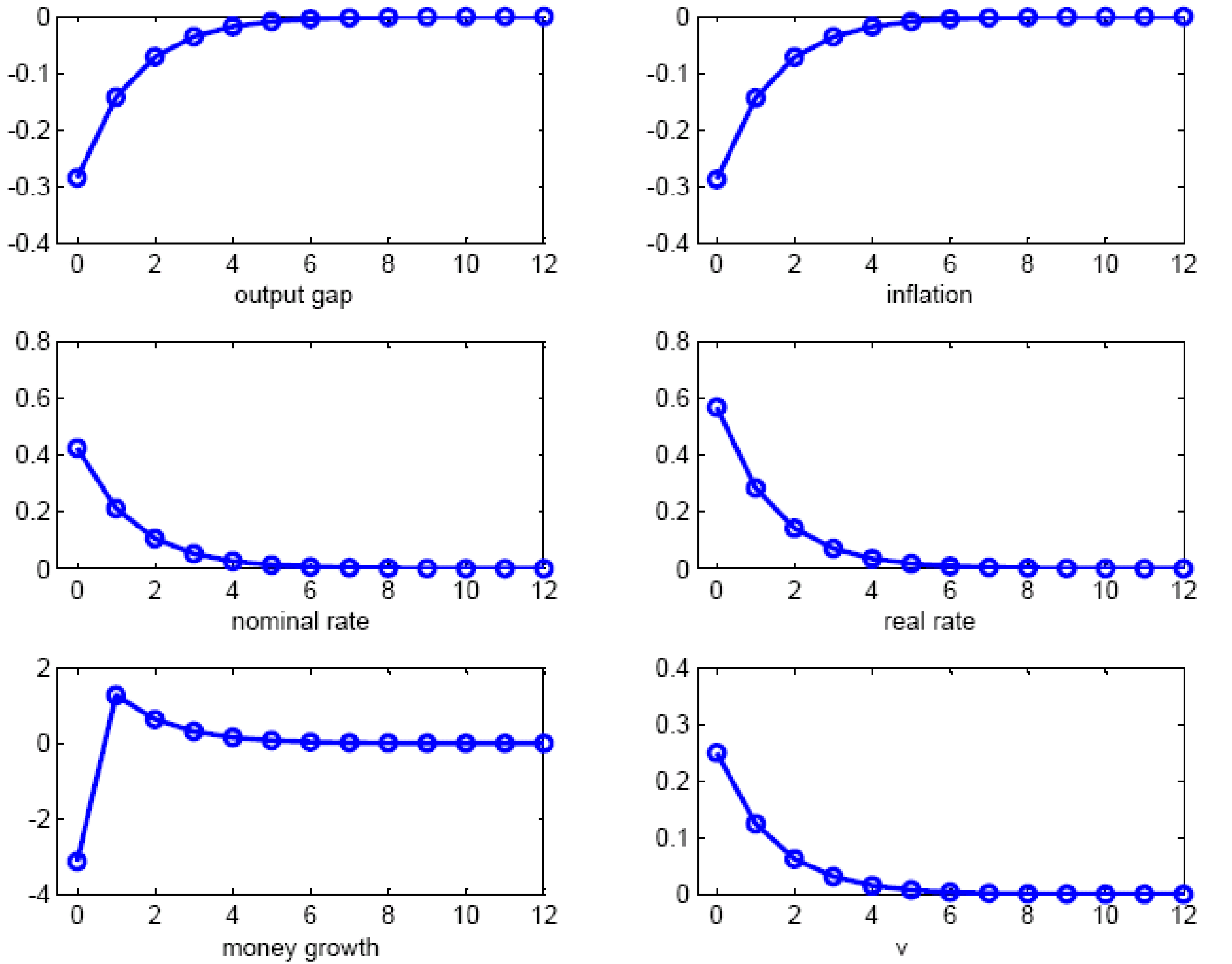
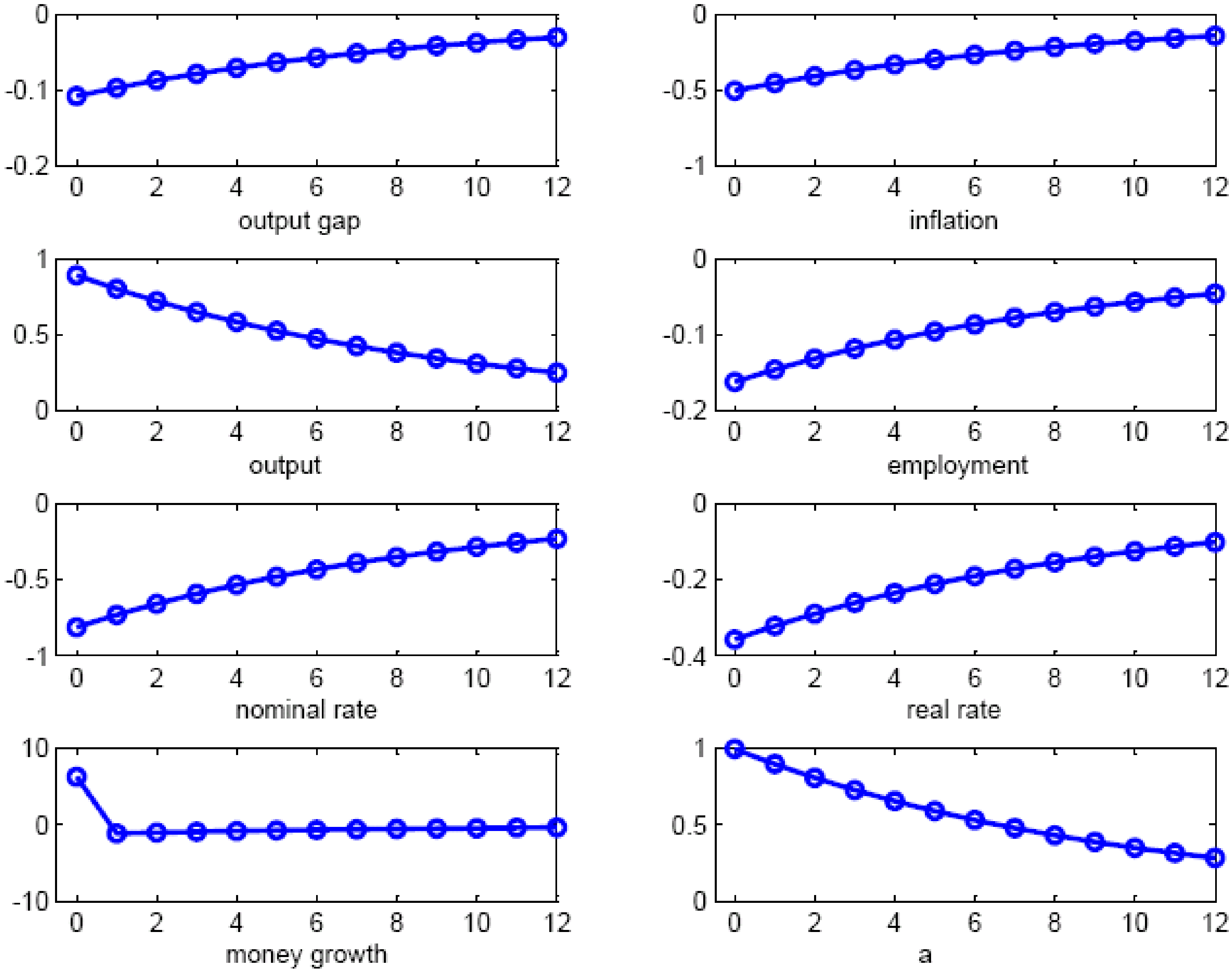


Figure 3.2: Effects of a Technology Shock (Interest Rate Rule)



Monetary Policy Design: The Case of an Efficient Natural Equilibrium

- *Assumption:*

$$y_t^n = y_t^e$$

- *Optimal Policy*

$$\widetilde{y}_t = 0 \quad ; \quad \pi_t = 0$$

- *Implementation*

$$i_t = r_t^n + \phi_\pi \ \pi_t$$

where $\phi_\pi > 1$ (determinacy condition)

- *Evaluation of Alternative Policies*

Welfare losses (second order approx.)

$$\mathbb{L} = \ (\sigma + \varphi) \ \ var(\widetilde{y}_t) + \frac{\epsilon}{\lambda} \ var(\pi_t)$$

Example:

$$i_t = \rho + \phi_\pi \ \pi_t + \phi_y \ \widehat{y}_t$$

Evaluation of the Simple Taylor Rule				
ϕ_π	1.5	1.5	5	1.5
ϕ_y	0.125	0	0	1
$\sigma(\widetilde{y})$	0.55	0.28	0.04	1.40
$\sigma(\pi)$	2.60	1.33	0.21	6.55
$welfare\ loss$	0.30	0.08	0.002	1.92

Monetary Policy Design: The Case of an Inefficient Natural Equilibrium

- *Assumption*: time-varying $y_t^n - y_t^e$
- *Welfare-relevant Output Gap*

$$x_t \equiv y_t - y_t^e$$

- *The New Keynesian Phillips Curve*

$$\pi_t = \beta \ E_t\{\pi_{t+1}\} + \kappa \ x_t + u_t$$

where $u_t \equiv \kappa(y_t^e - y_t^n)$

$\Longrightarrow$ policy trade-off !

- *Dynamic IS Equation*

$$x_t = -\frac{1}{\sigma} \left(i_t - E_t\{\pi_{t+1}\} - r_t^e\right) + E_t\{x_{t+1}\}$$

where $r_t^e \equiv \rho + \sigma \ E_t\{\Delta y_{t+1}^e\} = \rho + \frac{\sigma(\sigma+\varphi)}{1+\varphi} E_t\{\Delta a_{t+1}\}$.

- *A Simple Rule:*

$$i_t = \rho + \phi_\pi \; \pi_t$$

- *Equilibrium Dynamics*

  Assumptions: (i) $\{\Delta a_t\} \sim i.i.d. \longrightarrow r_t^e = \rho$, (ii) $\{u_t\} \sim i.i.d.$

$$\pi_t = \frac{\sigma}{\sigma + \kappa\phi_\pi} \; u_t$$

$$x_t = - \; \frac{\phi_\pi}{\sigma + \kappa\phi_\pi} \; u_t$$

- *Loss Function*

$$\alpha \; var(x_t) + var(\pi_t)$$

- *Optimal Simple Rule*

$$\phi_\pi^* = \frac{\sigma\kappa}{\alpha}$$

- *Utility-based Loss function:* $\alpha \equiv \frac{\lambda(\sigma+\varphi)}{\epsilon} \qquad \Longrightarrow \qquad \phi_\pi^* = \sigma\epsilon$

The New Keynesian Model with Sticky Prices and Wages

- Fraction of households/unions adjusting nominal wage: $1 - \theta_w$
- Constant elasticity of labor demand ϵ_w

  flexible wages $\Rightarrow$ $w_t = \mu^w + mrs_t + p_t$ where $\mu^w \equiv \log(\frac{\epsilon_w}{\epsilon_w - 1})$
- Aggregate wage dynamics

$$w_t = \theta_w \; w_{t-1} + (1 - \theta_w) \; w_t^*$$

- Optimal wage setting rule:

$$w_t^* = \mu^w + (1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k \; E_t\{mrs_{t+k|t} + p_{t+k}\}$$

  where $mrs_{t+k|t} \equiv \sigma \; c_t + \varphi \; n_{t+k|t}$
- Implied wage inflation equation

$$\pi_t^w = \beta \; E_t\{\pi_{t+1}^w\} - \lambda_w \; (\mu_t^w - \mu^w)$$

  where $\mu_t^w \equiv (w_t - p_t) - mrs_t$ and $\lambda_w \equiv \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w \; (1+\varphi\epsilon_w)}$

Some Results

- The New Keynesian Phillips curve revisited

$$\begin{aligned}\mu_t^p &= p_t - (w_t - a_t) \\ &= a_t - (\mu_t^w + \sigma c_t + \varphi n_t) \\ &= (1+\varphi)\ a_t - (\sigma+\varphi)\ y_t - \mu_t^w\end{aligned}$$

implying

$$\widehat{\mu}_t^p = -(\sigma+\varphi)\ \widetilde{y}_t - \widehat{\mu}_t^w$$

which combined with the inflation equation yields

$$\pi_t^p = \beta\ E_t\{\pi_{t+1}^p\} + \kappa_p\ \widetilde{y}_t + \lambda_p\ \widehat{\mu}_t^w$$

$\Rightarrow$ tradeoff between output gap and inflation stabilization

- *Natural* equilibrium allocation is no longer feasible

$$\widehat{\mu}_t^p = \widehat{\mu}_t^w = 0 \quad \Rightarrow \quad \pi_t^p = \pi_t^w = 0 \quad \Rightarrow \quad \text{constant real wage}$$

- Welfare losses (second order approximation)

$$\mathbb{L} = (\sigma + \varphi)\ \ var(\widetilde{y}_t) + \frac{\epsilon_p}{\lambda_p}\ var(\pi_t^p) + \frac{\epsilon_w}{\lambda_w}\ var(\pi_t^w)$$

$\Longrightarrow$ strict price inflation targeting is no longer optimal

- "Composite inflation" and the output gap

$$\pi_t = \beta\ E_t\{\pi_{t+1}\} + \kappa\ \widetilde{y}_t$$

where

$$\pi_t \equiv (1-\vartheta)\ \pi_t^p + \vartheta\ \pi_t^w$$

with $\vartheta \equiv \frac{\lambda_p}{\lambda_p+\lambda_w} \in [0,1]$ and $\kappa \equiv \frac{\lambda_w \kappa_p}{\lambda_p+\lambda_w}$

- Optimal Policy (fig.)
- Evaluation of Alternatives Policies (tab.)

(i) Strict inflation targeting: $\pi_t^x = 0$ (price, wage or composite)

(ii) Flexible inflation targeting : $i_t = 1.5\ \pi_t^x$

Figure 6.4: The Effects of a Technology Shock under the Optimal Policy

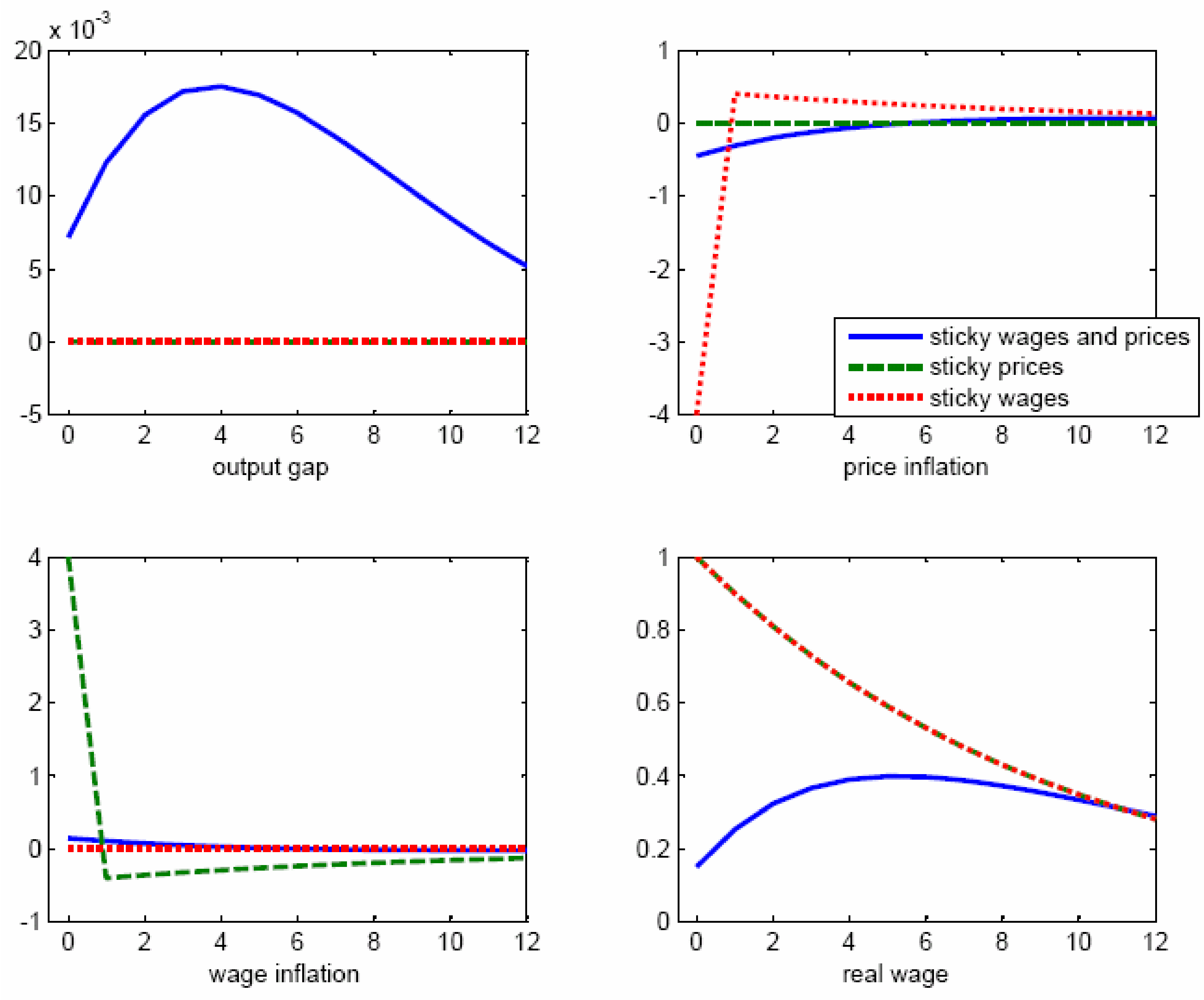


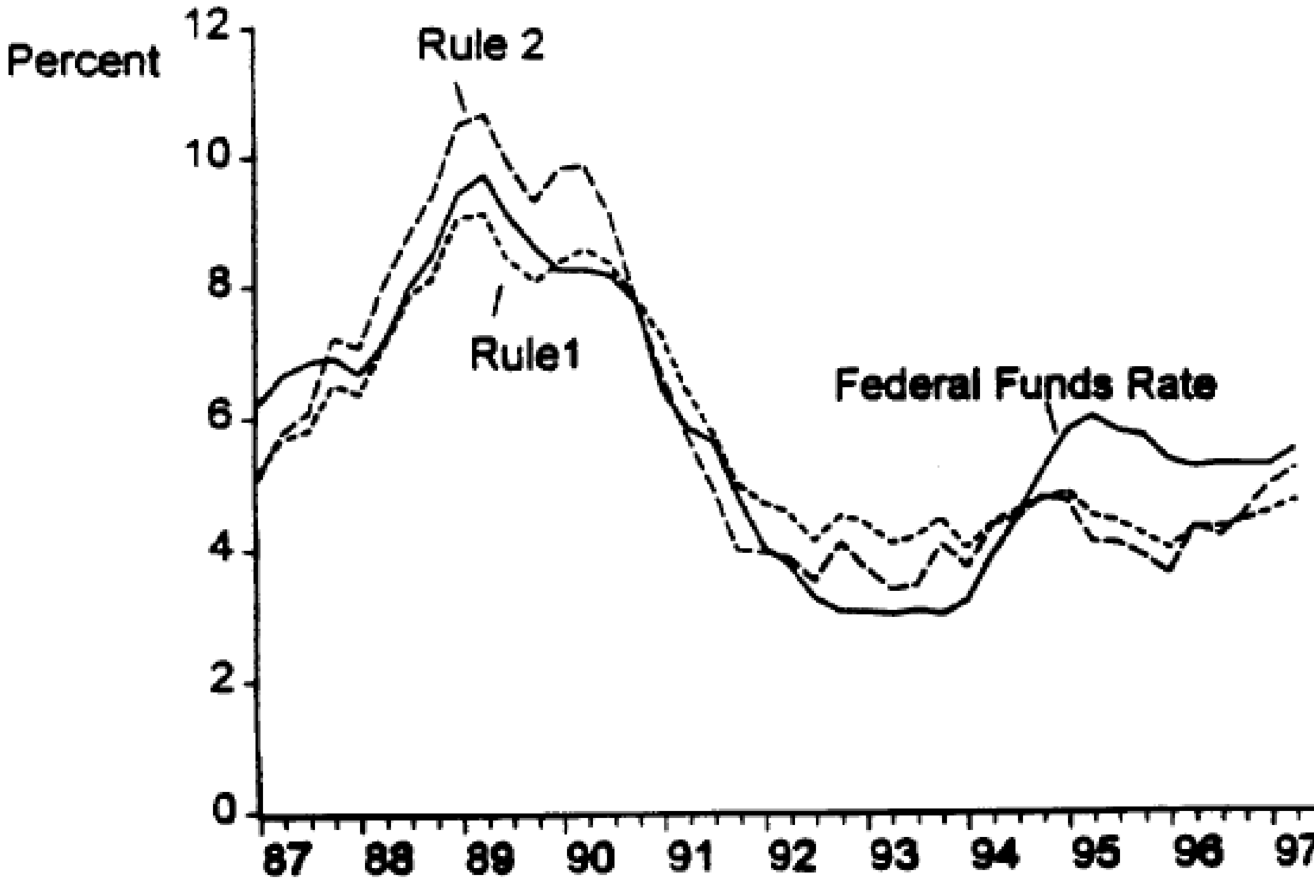
Table 6.1 Evaluation of Simple Rules

		Optimal Policy	Strict Rules			Flexible Rules		
			Price	Wage	Composite	Price	Wage	Composite
$\theta_p = \frac{2}{3}$	$\theta_w = \frac{3}{4}$							
	$\sigma(\pi^p)$	0.64	0	0.82	0.66	1.50	1.08	1.12
	$\sigma(\pi^w)$	0.22	0.98	0	0.19	1.05	0.30	0.42
	$\sigma(\widetilde{y})$	0.04	2.38	0.52	0	0.75	1.16	0.01
	$\mathbb{L}$	0.023	0.184	0.034	0.023	0.221	0.081	0.089
$\theta_p = \frac{2}{3}$	$\theta_w = \frac{1}{4}$							
	$\sigma(\pi^p)$	0.29	0	0.82	0.21	1.40	1.45	1.30
	$\sigma(\pi^w)$	1.24	2.91	0	1.63	1.49	0.98	1.25
	$\sigma(\widetilde{y})$	0.19	0.61	0.52	0	0.29	0.68	0.32
	$\mathbb{L}$	0.010	0.038	0.034	0.012	0.097	0.104	0.083
$\theta_p = \frac{1}{3}$	$\theta_w = \frac{3}{4}$							
	$\sigma(\pi^p)$	1.64	0	1.91	1.75	2.58	2.10	2.10
	$\sigma(\pi^w)$	0.11	0.98	0	0.06	1.47	0.07	0.10
	$\sigma(\widetilde{y})$	0.17	2.38	0.27	0	0.87	0.60	0.58
	$\mathbb{L}$	0.016	0.184	0.021	0.017	0.271	0.030	0.031

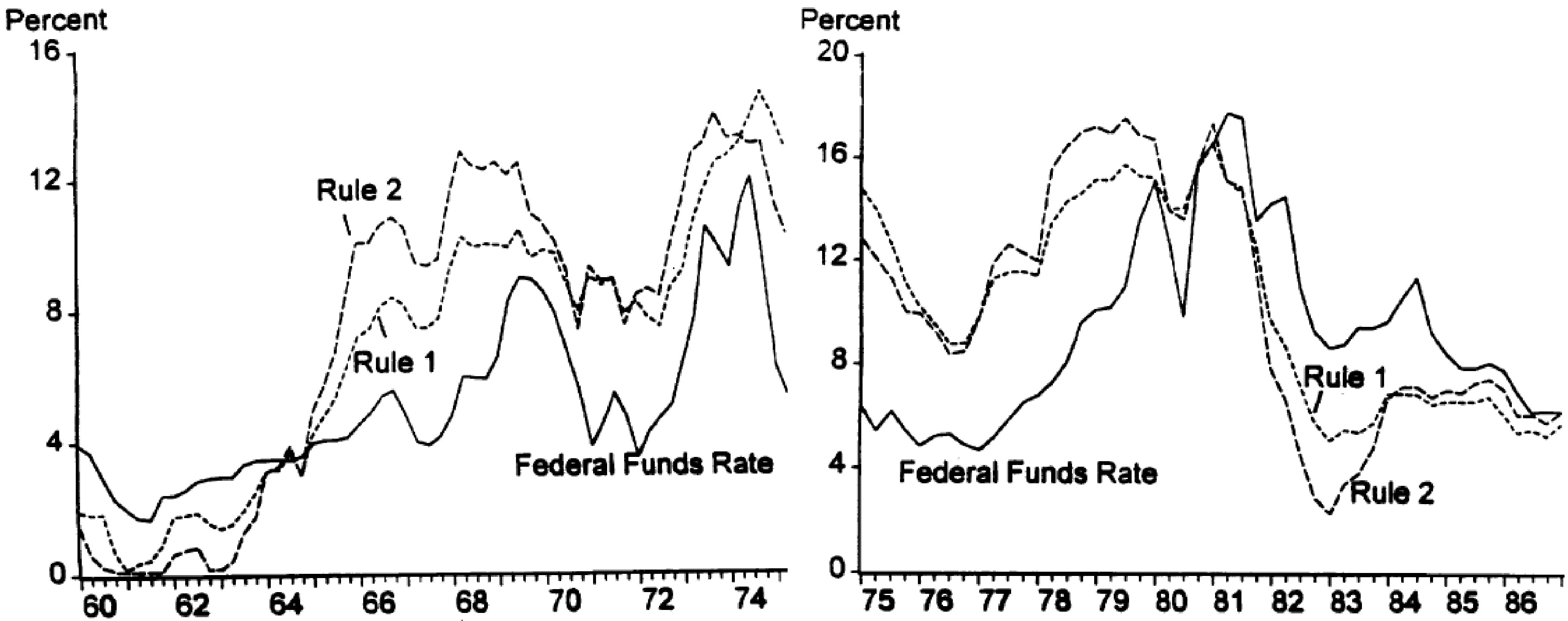
Source: Galí (2008)

The Taylor Rule (Taylor 1993)

$$i_t = 4 + 1.5(\pi_t - 2) + 0.5\, y_t$$

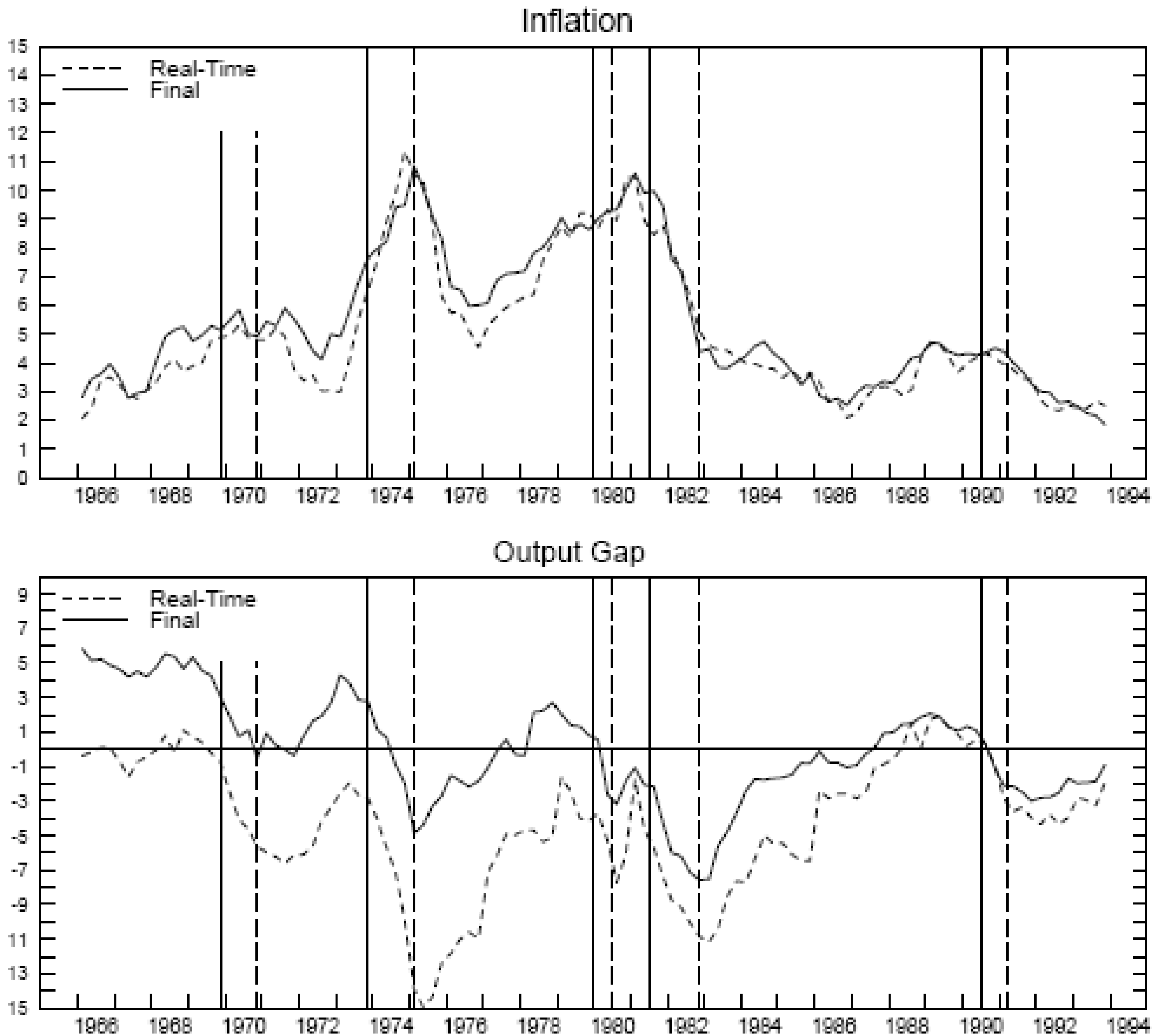


Source: Taylor 1999



Source: Taylor 1999

Orphanides (JME 2003)



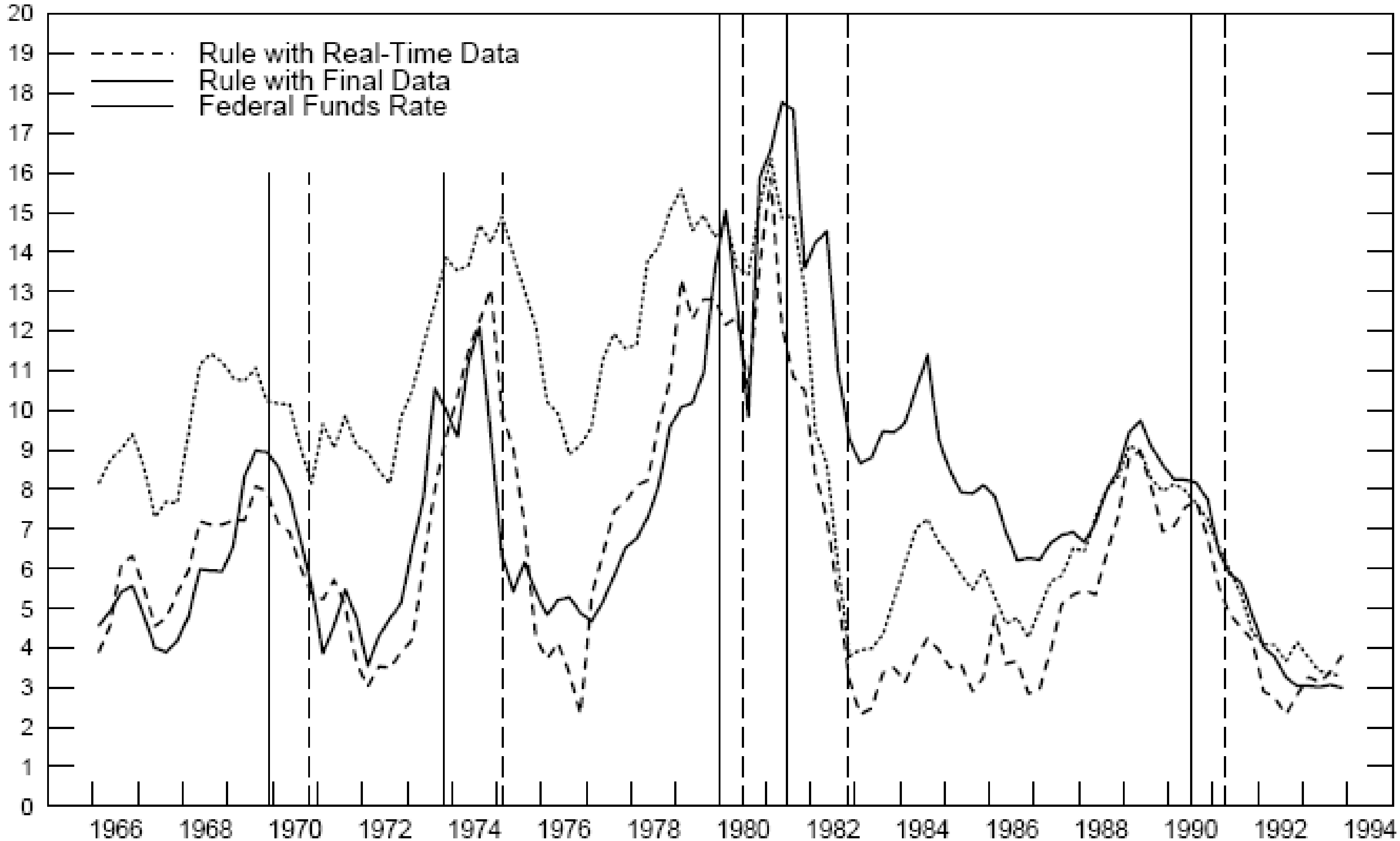


Fig. 5. Then and now: Taylor rule with final and real-time data.

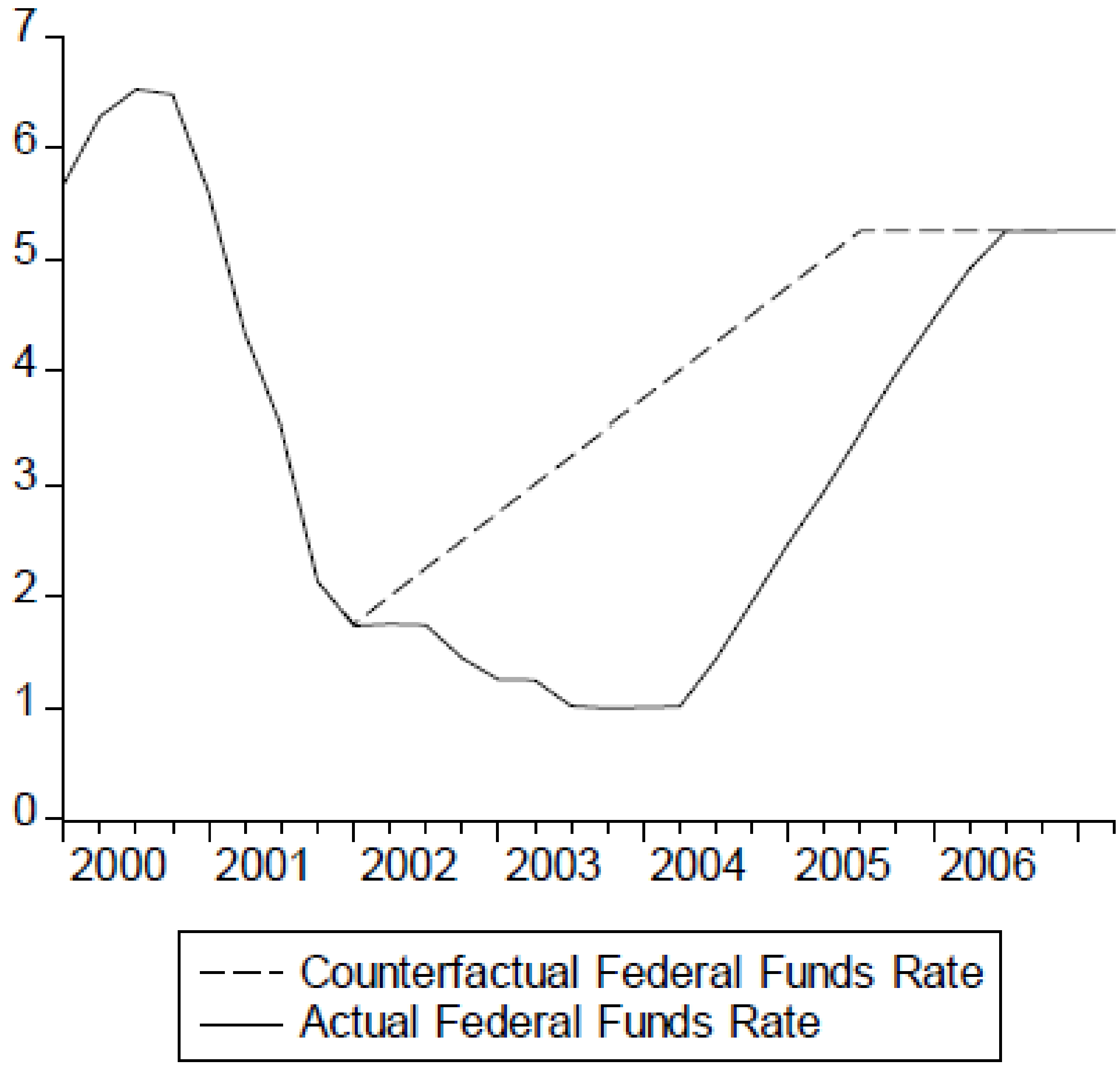
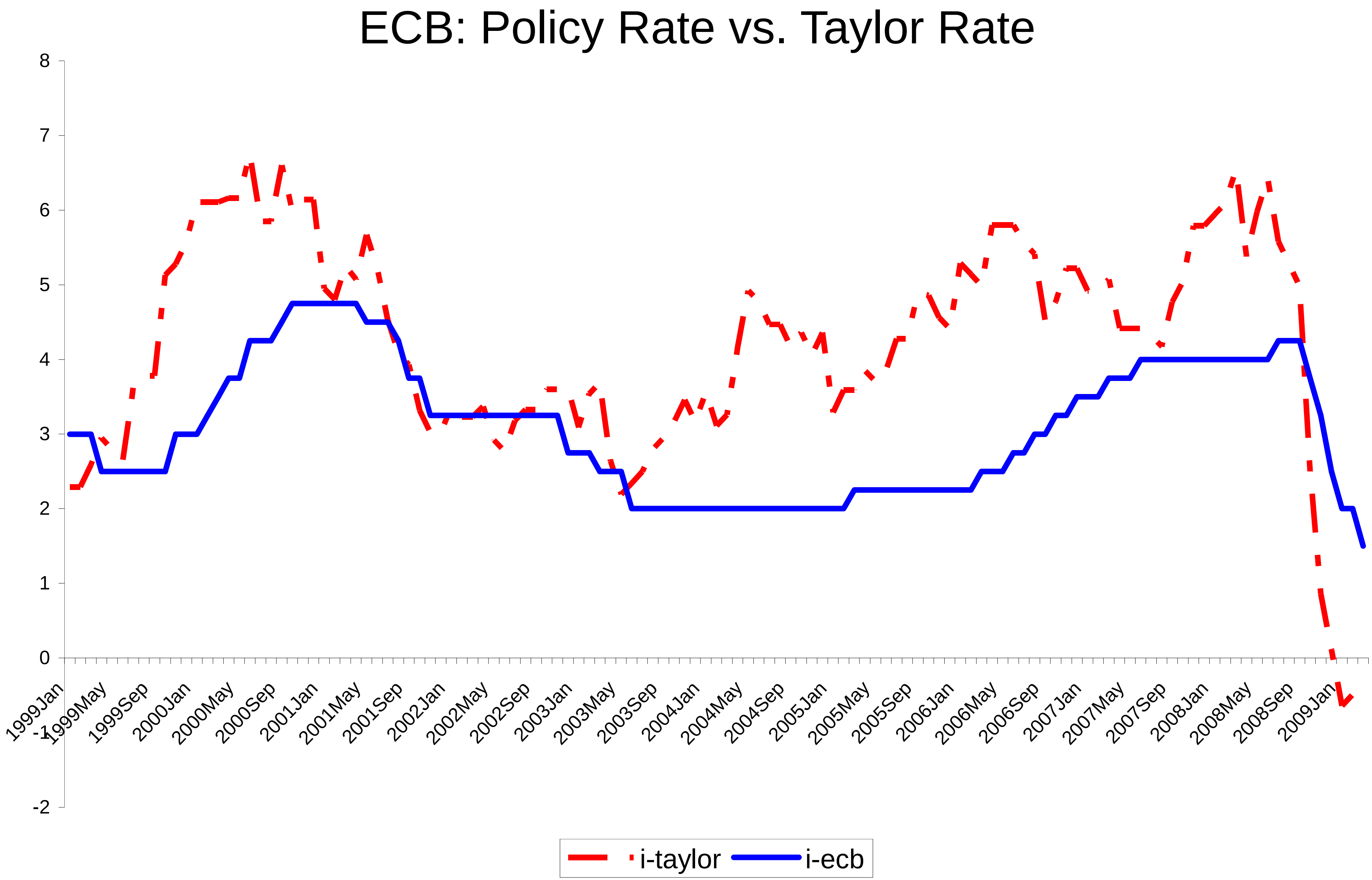

7
6
5
4
3
2
1
0
2000
2001
2002
2003
2004
2005
2006
Counterfactual Federal Funds Rate
Actual Federal Funds Rate

Figure 1



ECB: Policy Rate vs. Taylor Rate
8
7
6
5
4
3
2
1
0
-1
-2
1999Jan
1999May
1999Sep
2000Jan
2000May
2000Sep
2001Jan
2001May
2001Sep
2002Jan
2002May
2002Sep
2003Jan
2003May
2003Sep
2004Jan
2004May
2004Sep
2005Jan
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2008May
2008Sep
2009Jan
i-taylor
i-ecb

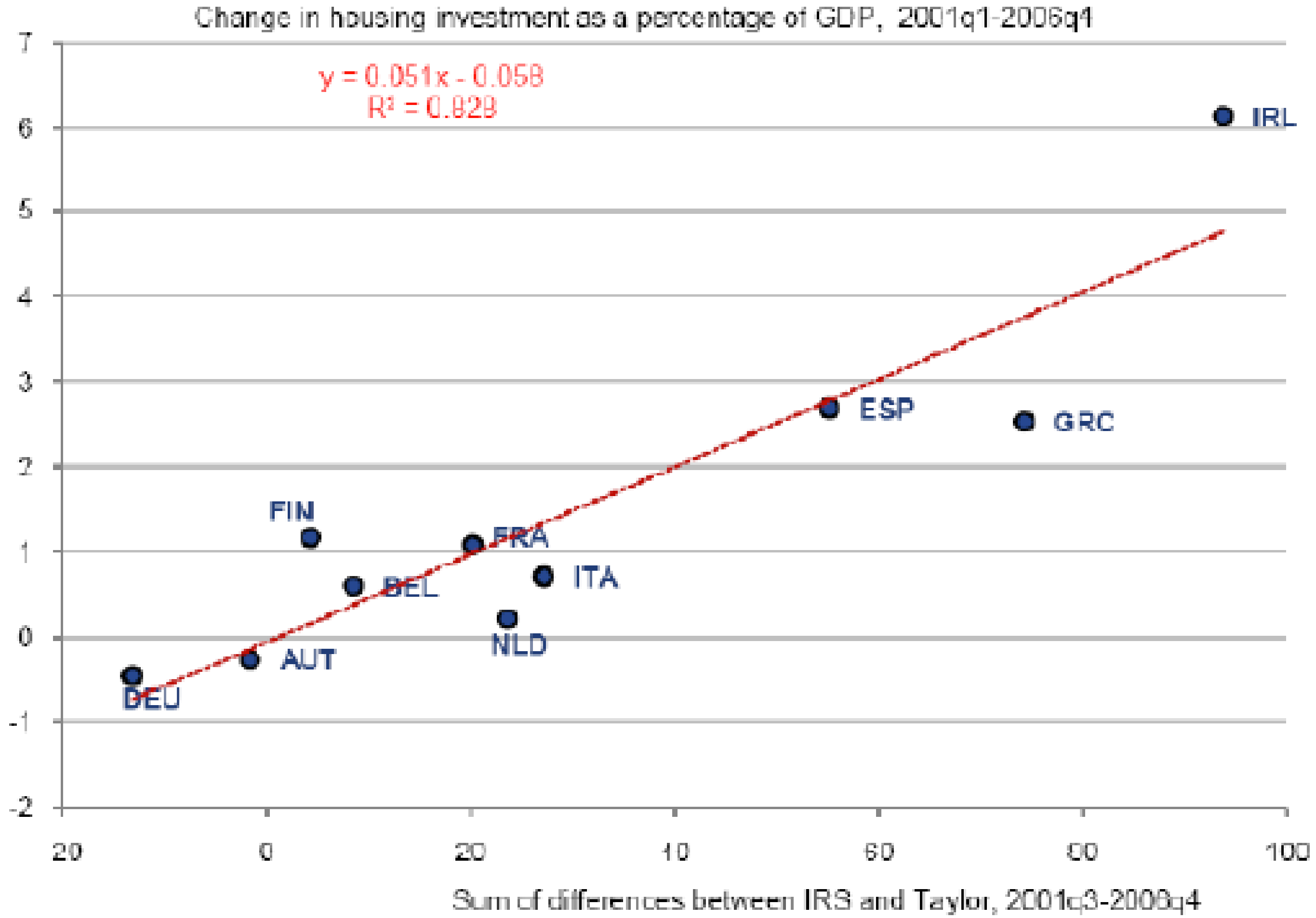

Figure 4. Housing Investment Versus Deviations From the Taylor Rule in Europe (Source: See footnote 3.)