

Real Wage Rigidities and the New Keynesian Model*

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May 2006 (first draft: April 2005)

Abstract

Most central banks perceive a trade-off between stabilizing inflation and stabilizing the gap between output and desired output. However, the standard new Keynesian framework implies no such trade-off. In that framework, stabilizing inflation is equivalent to stabilizing the welfare-relevant output gap. In this paper, we argue that this property of the new Keynesian framework, which we call the *divine coincidence*, is due to a special feature of the model: the absence of non trivial *real* imperfections.

We focus on one such real imperfection, namely, real wage rigidities. When the baseline new Keynesian model is extended to allow for real wage rigidities, the divine coincidence disappears, and central banks indeed face a trade-off between stabilizing inflation and stabilizing the welfare-relevant output gap. We show that not only does the extended model have more realistic normative implications, but it also has appealing positive properties. In particular, it provides a

* Prepared for the FRB/JMCB conference on "Quantitative Evidence on Price Determination," Washington, D.C., September 29-30, 2005. We have benefited from comments at various seminars and conferences, including CREI-UPF, Bank of England, MIT, LBS-MAPMU Conference, U. of Oslo, NBER Summer Institute, Federal Reserve Board, New York Fed, Boston Fed, NYU, Princeton, Boston University, Harvard, and Boston College. We thank Marios Angeletos, Jeff Fuhrer, Andy Levin, Greg Mankiw, Michael Woodford, our discussants Bob Hall and Julio Rotemberg, two anonymous referees and the editor Ken West for useful comments. We are also grateful to Anton Nakov for excellent research assistance. We thank CREA-Barcelona Economics, MCyT Grant (SEJ 2005-01124, and the NSF, for financial help.

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natural interpretation for the dynamic inflation–unemployment relation found in the data.

JEL Classification: E32, E50

Keywords: oil price shocks, inflation targeting, monetary policy, inflation inertia.

A standard new Keynesian (NK) model has emerged. On the supply side, it consists of Calvo price and/or wage staggering. On the demand side, it is composed of an Euler equation and a Taylor rule. With more explicit microeconomic foundations than its Keynesian ancestor, and more relevance than its RBC predecessor, it has become the workhorse in discussions of fluctuations, policy, and welfare.

A central, albeit controversial, block in this standard framework is the so-called New Keynesian Phillips curve (NKPC), which, in its simple form, has the following representation:

$$\pi = \beta E\pi(+1) + \kappa (y - y^*) \quad (1)$$

where π is inflation, y is (log) output, y^* is (log) natural output, and $(y - y^*)$ is the output gap. The effects of changes in factors such as the price of oil or the level of technology appear through their effects on natural output y^* .

From a welfare point of view, the model implies that it is desirable to stabilize inflation and to stabilize the output gap. Equation (1) implies that the two goals do not conflict: Stabilizing inflation also stabilizes the output gap. Thus, for example, in response to an increase in the price of oil, the best policy is to keep inflation constant; doing so implies that output remains equal to its natural level.¹

This property, which we shall call the *divine coincidence* contrasts with a widespread consensus on the undesirability of policies that seek to fully stabilize inflation at all times and at any cost in terms of output. That consensus underlies the *medium-term* orientation adopted by most inflation targeting central banks.

In this paper, we show that this divine coincidence is tightly linked to a specific property of the standard NK model, namely the fact that the gap between the *natural* level of output and the *efficient* (first-best) level of output is constant and invariant to shocks. This feature implies that stabilizing the *output gap*—the gap between actual and natural output—is equivalent to stabilizing the *welfare*-

relevant output gap—the gap between actual and efficient output. This equivalence is the source of the divine coincidence: The NKPC implies that stabilization of inflation is consistent with stabilization of the output gap. The constancy of the gap between natural and efficient output implies in turn that stabilization of the output gap is equivalent to stabilization of the welfare-relevant output gap.

The property just described can in turn be traced to the absence of non trivial *real* imperfections in the standard NK model. This leads us to introduce one such real imperfection, namely real wage rigidities. The existence of real wage rigidities has been pointed to by many authors as a feature needed to account for a number of labor market facts (see, for example, Hall [2005]). We show that, once the NK model is extended in this way, the divine coincidence disappears. The reason is that the gap between natural and efficient output is no longer constant, and is now affected by shocks. Stabilizing inflation is still equivalent to stabilizing the output gap, but no longer equivalent to stabilizing the welfare-relevant output gap. Thus, it is no longer desirable from a welfare point of view. Stabilization of inflation and stabilization of the welfare-relevant output gap now present the monetary authority with a trade-off. In the face of an adverse supply shocks, in particular, the monetary authority must decide whether to accommodate a higher level of inflation or, instead, keep inflation constant but allow for a larger decline in the welfare-relevant output gap.

While we focus on the implications of real wage rigidities, we see our results as an example of a more general proposition. The optimal design of macroeconomic policy depends very much on the interaction between real imperfections and shocks. In the standard NK model these interactions are limited. In particular, the size of real distortions is either constant or varies over time in response to exogenous shocks to the distorting variables themselves (e.g. tax rates). This has strong implications for policy, one of them being the lack of a trade-off in response to most shocks, including conventional supply shocks. In reality, distortions are likely to interact with shocks, leading to different policy prescriptions. In our model, this interaction works through endogenous variations in wage markups, resulting from the sluggish adjustment of real wages. However, this is not the only possible mechanism: a similar interaction could work, for example, through the

endogenous response of desired price markups to shocks, as in Rotemberg and Woodford (1996). Understanding these interactions should be high on macroeconomists' research agendas.²

At the same time, we see the extension of the NK model to incorporate real wage rigidities as more than an example of this general proposition. We believe that real wage rigidities are high on the list of real imperfections affecting cyclical fluctuations. In fact, as we discuss below, the introduction of real wage rigidities overcomes a well known empirical weakness of the standard NK model (as represented in equation (1)), namely, its lack of *inflation inertia*—which we define as the degree of inflation persistence beyond that inherited from the output gap ($y - y^*$) itself. We show that real wage rigidities are a natural source of inflation inertia, and can account for the good empirical fit of traditional Phillips curve equations.

The rest of the paper is organized as follows. In Section 1 we lay out a baseline new Keynesian model, with staggered price setting and no labor market distortions, and use it to illustrate the shortcomings discussed above. In Section 2 we introduce real wage rigidities, and show how their presence generates a meaningful trade-off between stabilization of inflation and the welfare-relevant output gap. Section 3 looks at the implications of alternative stabilization policies. Section 4 looks at the costs of disinflation. Section 5 relates our results to the literature. Section 6 derives some empirical relations between inflation, unemployment and observable supply shocks implied by our framework, and provides some evidence on its ability to fit the data. Section 7 concludes.

1 The Standard New Keynesian Model

The baseline framework below is standard, with one exception: In order to discuss “supply shocks” we introduce a non-produced input, with exogenous supply M . We interpret shocks to M as supply shocks. For simplicity, we leave out technological shocks, but, in our model, they would have exactly the same implications as supply shocks. The only difference, relevant when going to the data, is that

our supply shocks are directly observable—through changes in the price of the relevant inputs—while technological shocks are not.

Firms. We assume a continuum of monopolistically competitive firms, each producing a differentiated product and facing an isoelastic demand. The production function for each firm is given by:

$$Y = M^\alpha N^{1-\alpha} \quad (2)$$

where Y is output, and M and N are the quantities of the two inputs hired by the firm (in order to keep the notation simple we ignore firm-specific and time subscripts when not strictly needed).

Letting lower case letters denote the natural logarithms of the original variables, the real marginal cost is given by:

$$\begin{aligned} mc &= w - mpn \\ &= w - (y - n) - \log(1 - \alpha) \end{aligned} \quad (3)$$

where w is the (log) real wage, assumed to be taken as given by each firm.

Each good is non-storable and is sold to households, who consume it in the same period. Hence, consumption of each good must equate output.

People. We assume a large number of identical households, with time separable preferences, constant discount factor β , and period utility given by

$$U(C, N) = \log(C) - \exp\{\xi\} \frac{N^{1+\phi}}{1+\phi}$$

where C is composite consumption (with elasticity of substitution between goods equal to ϵ), N is employment, and ξ is a (possibly time-varying) preference parameter.³

The implied marginal rate of substitution (in logs) is given by:

$$mrs = c + \phi n + \xi \quad (4)$$

1.1 Efficient Allocation (First Best)

Let us start by assuming perfect competition in goods and labor markets. In this case we have, from the firms' side:

$$\begin{aligned} w &= mpn \\ &= (y - n) + \log(1 - \alpha) \end{aligned} \tag{5}$$

and, from the consumer-workers' side:

$$\begin{aligned} w &= mrs \\ &= y + \phi n + \xi \end{aligned} \tag{6}$$

where we have imposed the market clearing condition $c = y$. Combining both expressions yields the following expression for the first-best level of employment n_1 (we use the subscript "1" to denote values of variables associated with the first-best—or efficient—allocation)

$$(1 + \phi) n_1 = \log(1 - \alpha) - \xi \tag{7}$$

Note that first-best employment does not depend on the endowment of the non-produced input (because of exact cancellation of income and substitution effects implied by log utility and Cobb-Douglas technology), but it is inversely related to the preference shifter ξ . Given first-best employment, first-best output, denoted by y_1 , is given by

$$y_1 = \alpha m + (1 - \alpha) n_1 \tag{8}$$

which, as expected, depends on both shocks.

1.2 Flexible Price Equilibrium (Second Best)

We maintain the assumption that prices and wages are flexible, but we take into account the monopoly power of firms in the goods market.

From the firms' side, optimal price setting implies $mc + \mu^p = 0$, where $\mu^p \equiv \log(\epsilon/(\epsilon - 1))$ is the markup of price over cost, coming from the monopoly power

of firms. Hence, using (3)

$$w = y - n + \log(1 - \alpha) - \mu^p \quad (9)$$

Henceforth, we use subscript “2” to refer to the *second-best* (or *natural*) levels of a variable, corresponding to the equilibrium with flexible prices. Combining (6) and (9), we can determine second-best employment n_2 :

$$(1 + \phi) n_2 = \log(1 - \alpha) - \mu^p - \xi \quad (10)$$

which, under our assumptions, is independent of m , but may vary as a result of changes in the preference parameter ξ . Second-best output is in turn given by:

$$y_2 = \alpha m + (1 - \alpha) n_2 \quad (11)$$

Note an important implication of this baseline NK model: While both first- and second-best output vary over time, the gap between the two remains *constant* and given by

$$y_1 - y_2 = \frac{\mu^p(1 - \alpha)}{1 + \phi} \equiv \delta \quad (12)$$

That property, which is common to the vast majority of optimizing models with nominal rigidities found in the literature, will play an important role in what follows.

1.3 Staggered Price Equilibrium

Assume now that price decisions are staggered à la Calvo. As is well known, in that case, in a neighborhood of the zero-inflation steady state, the behavior of inflation is described by the difference equation:

$$\pi = \beta E\pi(+1) + \lambda (mc + \mu^p) \quad (13)$$

where $mc + \mu^p$ denotes the log-deviation of real marginal cost from its value in a zero inflation steady state, and $\lambda \equiv \theta^{-1}(1 - \theta)(1 - \beta\theta)$, with θ representing the

fraction of firms not adjusting their price in any given period.⁴

Substituting (6) into (3) and using (10) and (11), we obtain:

$$mc + \mu^p = \left(\frac{1 + \phi}{1 - \alpha} \right) (y - y_2)$$

Combining the previous two equations gives us the new Keynesian Phillips curve (NKPC):

$$\pi = \beta E\pi(+1) + \kappa (y - y_2) \tag{14}$$

where $\kappa \equiv \lambda(1 + \phi)/(1 - \alpha)$.

Inflation depends on expected inflation and the output gap, defined as the (log) distance of output from its natural level. Note that neither supply nor preference shocks appear directly in equation (14). They appear indirectly through their effect on the natural level of output, y_2 , and thus through the output gap $(y - y_2)$.

Equation (14) implies that stabilizing inflation is equivalent to stabilizing the output gap $(y - y_2)$. Now recall from (12) that $(y_1 - y_2) = \delta$. This implies that stabilizing the output gap $(y - y_2)$ is, in turn, equivalent to stabilizing the welfare-relevant (log) distance between output and its first-best level, i.e. $(y - y_1)$. Putting the two steps together implies that stabilizing inflation is equivalent to stabilizing the welfare-relevant (log) distance of output from first best. This is what we referred to as the *divine coincidence* in the introduction.

Note that the divine coincidence is a consequence of the constancy of δ , the (log) distance between the first- and the second-best levels of output. In particular, because an adverse supply shock does not alter δ , it does not create any incentives for the monetary authority to deviate from a policy of constant inflation.⁵

A number of recent papers have introduced a trade-off between stabilization of inflation and stabilization of the distance between output and its first-best level by assuming (explicitly or implicitly) *exogenous* stochastic variations in δ , the gap between first- and second-best output.⁶ We take a different approach. By introducing an additional real imperfection in the model, we get *endogenous* fluctuations in the gap in response to shocks. We defer a discussion of the two

approaches to later.

2 Introducing Real Wage Rigidities

We assume that real wages respond sluggishly to labor market conditions, as a result of some (unmodelled) imperfection or friction in labor markets. Specifically, we assume the partial adjustment model:

$$w = \gamma w(-1) + (1 - \gamma) mrs \quad (15)$$

where γ can be interpreted as an index of real rigidities.⁷

We view equation (15) as an admittedly ad-hoc but parsimonious way of modeling the slow adjustment of wages to labor market conditions, as found in a variety of models of real wage rigidities, without taking a stand on what the "right" model is.⁸ An alternative formalization, explicitly derived from staggering of real wage decisions, is presented in Appendix 2. The algebra is more complex, but the basic conclusions are the same as those presented in the text below. The important assumption underlying equation (15) is that the slow adjustment be the result of distortions rather than preferences, so the first-best equilibrium is unaffected.⁹

Next we examine the implications of real wage rigidities on the equilibrium level of employment and inflation. Once again we find it useful to start by looking at the flexible price case.

2.1 Flexible Price Equilibrium (Second Best)

Assume that prices and wages are flexible, so we solve for the second-best level of employment. Note that the first-best level is the same as before.

From above, using (4), (15), and our assumptions on technology (2), we have, from the wage setting side:

$$\begin{aligned} w &= \gamma w(-1) + (1 - \gamma) (y + \phi n + \xi) \\ &= \gamma w(-1) + (1 - \gamma) [\alpha(m - n) + (1 + \phi)n + \xi] \end{aligned} \quad (16)$$

As before, from the firms' side:

$$\begin{aligned}
w &= mpn - \mu^p \\
&= (y - n) + \log(1 - \alpha) - \mu^p \\
&= \alpha(m - n) + \log(1 - \alpha) - \mu^p
\end{aligned}$$

Putting the two together and rearranging terms, we can solve for second-best output y_2 as a function of first-best output y_1 (which remains unchanged, and given by (7) and (8)) and the two exogenous driving forces:

$$[y_2 - y_1 + \delta] = \Theta [y_2(-1) - y_1(-1) + \delta] + \Theta(1 - \alpha)[\Delta m + (1 + \phi)^{-1}\Delta\xi] \quad (17)$$

where $\Theta \equiv \gamma\alpha/(\gamma\alpha + (1 - \gamma)(1 + \phi)) \in [0, 1]$.

Equation (17) points to the key implication of the introduction of real wage rigidities: The gap between the first and second-best levels of output is no longer constant, fluctuating instead in response to both preference and supply shocks.

Note further that Θ is increasing in γ , implying that the size and persistence of deviations of the gap between second- and first-best output are increasing in the degree of real rigidities. Hence, for instance, the effects of an adverse supply shock (an unexpected decrease in m) are to decrease second-best output by more than first-best output (the gap being increasing in γ). Only gradually does the size of that gap return to its steady state level δ , as the wage adjusts over time. On the other hand, in response to a preference shock that lowers first-best output (an increase in ξ), second-best output falls by less, since the assumed real rigidities prevent the wage from adjusting upward sufficiently to support the lower efficient level of employment and output.

2.2 Staggered Price Equilibrium

We can now solve for the behavior of inflation under the assumption of price staggering à la Calvo. Combining (2), (3), and (16), and rearranging terms we obtain

$$(mc + \mu^p) = \gamma(mc + \mu^p)(-1) + x_2 \quad (18)$$

where

$$x_2 \equiv (1 - \alpha)^{-1}[(1 - \gamma)(1 + \phi)(y - y_2) + \gamma\alpha(\Delta y - \Delta y_2)]$$

is a linear combination of the current and lagged distance of output from second best. Combining (18) with (13) yields:

$$\pi = \beta E\pi(+1) + \frac{\lambda}{1 - \gamma L} x_2 \quad (19)$$

This is the relation between inflation and the output gap implied by our model. Notice that as in the baseline NKPC model, it is still the case that there is an exact relation—although with slightly more complex dynamics—between inflation, expected inflation, and the output gap (now, both level and change). So fully stabilizing inflation is still consistent with fully stabilizing the output gap, the distance of output from its second-best level: In equation (19), a constant π implies a constant $(y - y_2)$.¹⁰

But, in contrast with the baseline NKPC model, the divine coincidence no longer holds, since stabilizing the output gap $(y - y_2)$ is no longer desirable. This is because what matters for welfare is the distance of output not from its second-best level, but from its first-best level. In contrast to the baseline model, the distance between the first- and the second-best levels of output is no longer constant, but is instead affected by the shocks, as we have shown above.

To see what this implies, combine (19) with (17) to obtain the relation between inflation and the distance of employment from its first-best level:

$$\pi = \beta E\pi(+1) + \frac{\lambda}{1 - \gamma L} x_1 - \frac{\lambda\gamma\alpha}{1 - \gamma L} [\Delta m + (1 + \phi)^{-1}\Delta\xi] \quad (20)$$

where now

$$x_1 \equiv (1 - \alpha)^{-1}[(1 - \gamma)(1 + \phi)(y - y_1 + \delta) + \gamma\alpha(\Delta y - \Delta y_1)]$$

involves a linear combination of the current and lagged welfare-relevant output gap.

Inflation depends on expected inflation, a distributed lag of the distance of output

from its first-best level, and a distributed lag of both supply and preference shocks. In other words, there is no longer an exact relation, however complex, between inflation and the welfare-relevant output gap. Thus, there is no way to stabilize both in the presence of either supply or preference shocks, and monetary policy faces a clear trade-off.

To understand the basic source of the trade-off, it is useful to look at the economy's factor price frontier. Let v be the real price of the non-produced input. Then, given the Cobb-Douglas assumption:

$$mc = (1 - \alpha) w + \alpha v + const$$

Thus any shock that induces an increase in the real price of the non-produced input (say, an increase in the price of oil) must lead either to a decrease in real wages or an increase in real marginal cost. Depending on the degree of monetary policy accommodation, the outcome will be reflected in output or inflation. This can be seen clearly by deriving the inflation equation, now expressed in terms of Δv and the distance of output from first-best (see appendix for derivation):

$$\pi = \beta E\pi(+1) + \frac{\lambda}{1 - \Gamma L} [(1 - \Gamma)(1 + \phi)(1 - \alpha)^{-1} (y - y_1 + \delta) + \Gamma\alpha \Delta v] \quad (21)$$

where $\Gamma \equiv \frac{\gamma}{1 - \alpha(1 - \gamma)} \in [0, 1]$ is a monotonic transformation of the index of real wage rigidities γ .

Consider now any shock that drives up the real price of the non produced input, v .¹¹ Stabilizing inflation requires a proportional decline in the real wage; given real wage rigidity, this can only be delivered by a decrease in output relative to first best. Stabilizing instead the distance of output from first best will lead to higher inflation. We explore this trade-off further in the next section.

3 Policy Tradeoffs

To illustrate the implications of the trade off faced by the monetary authority, it is easiest to look at two extreme policies. For convenience, we assume $\xi = 0$ and a random walk process for the non-labor input endowment, so that $\Delta m = \varepsilon_m$ is a white noise process. Note that, in this case, first-best employment is constant.

Suppose that the central bank stabilizes the welfare-relevant output gap (the distance of output from first best) at a level consistent with zero *average* inflation. That requires $y = y_1 - \delta$ at all times. It follows from (20) that:

$$\pi = \beta E\pi(+1) - \frac{\lambda\gamma\alpha}{1 - \gamma L} \varepsilon_m$$

Solving under rational expectations gives:

$$\pi = \gamma \pi(-1) - \frac{\lambda\gamma\alpha}{1 - \beta\gamma} \varepsilon_m$$

So, stabilizing the welfare-relevant output gap implies some accommodation of adverse shocks through inflation, followed by a slow (if γ is high) return to normal.

Suppose instead that the central bank stabilizes inflation, so $\pi = E\pi(+1)$. Then:

$$(1 - \gamma)(1 + \phi)(y - y_1 + \delta) + \gamma\alpha(\Delta y - \Delta y_1) - \gamma\alpha(1 - \alpha) \varepsilon_m = 0$$

Or equivalently:

$$(y - y_1) = -(1 - \Theta)\delta + \Theta [y(-1) - y_1(-1)] + (1 - \alpha)\Theta\varepsilon_m$$

Not surprisingly, this policy replicates the second best, with large fluctuations in output relative to first best.

To the extent that the central bank attaches some weight to stabilization of both inflation and the gap from first best, optimal policy will be somewhere in between. The important point is that, so long as the central bank puts some

weight on activity, it may have to accommodate adverse supply shocks through potentially large and long lasting increases in inflation. How long lasting depends on the degree of real wage rigidity, γ . How large depends also on γ , but also on λ , the degree of nominal rigidity, and α , the share of the non-produced input in production.

To get a sense of the order of magnitude of the implied movements in inflation and the welfare-relevant output gap, we compute the elasticity of those variables with respect to v , which we can interpret as the real price of oil, under the two extreme policies just described (and given the assumption of a random walk process for the endowment m). We assume the following baseline values for the different parameter: $\beta = 0.99$, $\theta = 0.5$ (implying an average price duration of two quarters, consistent with the evidence in Bils and Klenow (2004)), $\alpha = 0.025$ (roughly the share of oil in production), $\phi = 1$ (corresponding to a unit Frisch labor supply elasticity), and $\gamma = 0.9$ (implying a a half-life for the adjustment of the real wage adjustment of about 6 quarters).

Hence, under the policy that fully stabilizes the gap relative to first best, we have

$$\begin{aligned} \frac{d\pi}{dv} &= \frac{d\pi/d\varepsilon_m}{dv/d\varepsilon_m} \\ &= \frac{\lambda\alpha\gamma}{(1-\beta\gamma)(\alpha\gamma+(1-\alpha))} \end{aligned} \tag{22}$$

Under the baseline calibration assumed above we obtain $\frac{\partial\pi}{\partial v} \simeq 0.105$. Hence, stabilizing the welfare-relevant output gap in response to an adverse supply shock which raises the price of oil by, say, 10 percent, implies an (annualized) inflation slightly above 4 percent on impact. Note however that the expression for $\frac{\partial\pi}{\partial v}$ is highly non-linear in γ . Hence, if we set $\gamma = 0.8$ we have $\frac{\partial\pi}{\partial v} = 0.05$, bringing the impact on short run inflation of the same oil price hike down to 2 percent. If $\gamma = 0.5$ then $\frac{\partial\pi}{\partial v} = 0.012$, and inflation is contained below 0.5 percent.¹²

Let us consider next the quantitative impact on the welfare-relevant output gap of the second extreme policy analyzed above, i.e. one that fully stabilizes inflation.

In that case, we have

$$\begin{aligned}
\frac{\partial(y - y_1)}{\partial v} &= \frac{d(y - y_1)/d\varepsilon_m}{dv/d\varepsilon_m} \\
&= -\frac{(1 - \alpha)\Theta}{(1 - \alpha)(1 - \Theta)} \\
&= -\frac{\alpha\gamma}{(1 - \gamma)(1 + \phi)}
\end{aligned} \tag{23}$$

Under our baseline calibration we have $\frac{\partial(y - y_1)}{\partial v} = -0.11$, thus implying a reduction in the welfare-relevant output gap in the first quarter of 1.1 percent in response to a 10 rise in the price of oil resulting from an adverse supply shock, with the persistence of that effect (measured by the AR parameter Θ) being relatively small: 0.10. When we set $\gamma = 0.8$ we obtain a 0.5 percent output gap loss on impact with persistence 0.05. If $\gamma = 0.5$, the corresponding numbers are 0.1 percent and 0.01.

While these numbers are rough and purely illustrative, they suggest a far from quantitatively trivial policy trade-off resulting from the presence of real wage rigidities. There is an important caveat to the analysis above, however, that needs to be raised.¹³ Following much of the NK literature we have chosen to ignore capital as a productive factor. Instead our model assumes, for simplicity, constant returns to labor and the non-produced input. A more realistic model would incorporate firm-specific capital and allow for short-run decreasing returns to labor and the non-produced input. In appendix 3 we show how some of the key equations of our model need to be modified when we assume a production function at the firm-level of the form

$$Y = M^{\alpha_m} N^{\alpha_n}$$

where $\alpha_m + \alpha_n < 1$ and where the quantity of a fixed factor (e.g., capital) has been normalized to one. While none of the qualitative results obtained above are affected by this alternative formulation, some of the quantitative predictions of our model are altered significantly. Hence, assuming $\alpha_m = 0.025$ and $\alpha_n = 2/3$ and a price elasticity $\epsilon = 6$ (while keeping the remaining parameters as in our

baseline calibration), the policy that stabilizes the welfare-relevant output gap in the face of a supply shock implies an impact response of inflation (relative to the change in the price of oil) given by $\frac{\partial \pi}{\partial v} \simeq 0.0284$. Hence, in response to an adverse supply shock which raises the price of oil by, say, 10 percent, stabilization of $y - y_1$ implies an (annualized) inflation slightly above 1.1 percent on impact. On the other hand, full stabilization of inflation requires a decline in the welfare relevant output gap of $\frac{\partial(y-y_1)}{\partial v} = -0.031$, thus implying a reduction in the welfare-relevant output gap in the first quarter of 1.1 percent in response to a 10 rise in the price of oil resulting from an adverse supply shock, with the persistence of that effect (measured by the AR parameter Θ) being relatively small: 0.10.

4 Real Wage Rigidities and the Cost of Disinflation

The analysis above has stressed the role of real wage rigidities in generating a meaningful policy trade-off in response to shocks. Here we briefly discuss their implications for the output cost of disinflations.¹⁴

As is well known, the standard NKPC implies the presence of a long run trade-off, however small, between inflation and the output gap.¹⁵ To illustrate this point, consider a sudden, permanent, unexpected reduction in inflation from, say, $\pi^* > 0$ to zero. For convenience we normalize the natural level of output—which is independent of inflation—to zero (i.e., $y_2 = 0$). In the period before the disinflation, the economy is assumed to be in a steady state, with output is given by

$$y(-1) = \frac{1 - \beta}{\kappa} \pi^* \tag{24}$$

From (14) it follows that, in the standard NK model, disinflation implies a permanently lower level output¹⁶

$$y(+k) = 0$$

for $k = 0, 1, 2, \dots$. Hence, at the time of disinflation (period 0) output declines by $dy(0) = -\frac{1-\beta}{\kappa} \pi^*$, remaining at the lower level thereafter, with no additional transitional dynamics coming into play. In the standard NK model, the real effects

of disinflations mentioned above tend to be small, at least for plausible parameter values. Thus, under the baseline calibration introduced above, $\beta = 0.99$ and $\kappa = 0.176$, implying $dy(+k) \simeq -0.05 \pi^*$. Hence, a permanent reduction in (annualized) inflation of 4 percentage points (i.e. one percentage point in quarterly rates), lowers the level of output at all horizons by only 5 basis points. Even though the implied loss in output is permanent, the decline around the time of disinflation seems substantially smaller than in actual disinflationary episodes, as reported in Ball (1994), among others

Consider next the case with real wage rigidities. Again, in the initial steady state output is given by (24). Attaining price stability requires:

$$mc + \mu^p = 0$$

from period 0 onwards. The evolution of marginal cost must satisfy (18), which we rewrite here for convenience:

$$mc + \mu^p = \gamma(mc + \mu^p)(-1) + (1 - \alpha)^{-1}[(1 - \gamma)(1 + \phi) y + \alpha\gamma \Delta y]$$

where again we normalize $y_2 = 0$. Hence, in period 0—in the transition to price stability—we must have

$$\begin{aligned} 0 &= \gamma\lambda^{-1}(1 - \beta)\pi^* + (1 - \alpha)^{-1}[(1 - \gamma)(1 + \phi) + \alpha\gamma] y(0) - (1 - \alpha)^{-1}\alpha\gamma\kappa^{-1}(1 - \beta) \pi^* \\ &= [(1 - \gamma)(1 + \phi) + \alpha\gamma] y(0) + \gamma\kappa^{-1}(1 - \beta)(1 - \alpha + \phi) \pi^* \end{aligned}$$

Thus,

$$y(0) = - \frac{\gamma(1 - \beta)(1 - \alpha + \phi)}{\kappa[(1 - \gamma)(1 + \phi) + \alpha\gamma]} \pi^* \quad (25)$$

Accordingly, the percent decline in output during the transition to zero inflation is:

$$\Delta y(0) = - \frac{1 - \beta}{\kappa} \left(1 + \frac{\gamma(1 - \alpha + \phi)}{(1 - \gamma)(1 + \phi) + \alpha\gamma} \right) \pi^*$$

which is increasing in γ . Relative to the case with no real rigidities the loss of output in period zero is multiplied by a factor $1 + \frac{\gamma(1 - \alpha + \phi)}{(1 - \gamma)(1 + \phi) + \alpha\gamma}$. Under our baseline

calibration the percent decline in output in the period when inflation is brought down is given by $dy(0) \simeq -0.5 \pi^*$. Hence, a permanent reduction in inflation of 4 percentage points in (annualized) inflation lowers the level of output by roughly 50 basis points in the quarter the policy is implemented, an effect about 10 times larger than in the standard model.

In contrast with the standard model, in the presence of real wage rigidities the decline in output experienced at the onset of the zero inflation period is partly reversed over time, with output converging asymptotically to the same new steady state as in the standard model. Formally, using (18), we see that maintaining zero inflation in subsequent periods implies

$$y(k) = \Theta y(k - 1)$$

for $k = 1, 2, 3, \dots$ where $\Theta \equiv \frac{\alpha\gamma}{(1-\gamma)(1+\phi)+\alpha\gamma}$ determines the speed of convergence to the new steady state. Under our baseline calibration $\Theta \simeq 0.10$, suggesting a relatively fast convergence to the new steady state.

Finally, we show an expression for the *additional cumulative* output losses (relative to the case with no real rigidities) resulting from the disinflation:

$$\frac{\gamma(1-\beta)(1-\alpha+\phi)}{\kappa(1-\gamma)(1+\phi)} \pi^*$$

Under our baseline calibration the previous factor takes a value of roughly 0.5 per percentage point reduction in (quarterly) inflation. Notice also that the cumulative losses converge to infinity as $\gamma \rightarrow 1$ since in that case $\Theta = 1$, and the entire loss of output experienced at the onset of the new low inflation regime becomes permanent.

Finally, it is worth noticing that, as in the analysis of the previous section, the quantitative results above change significantly if we assume the presence of decreasing returns. Hence, under our alternative calibration with decreasing returns, the loss of output at the outset of the disinflation is multiplied by a factor of 4 relative to the case with no real rigidities (compared with a factor of 10 in the case of constant returns). That smaller initial impact coexists with a larger per-

sistence, as given by $\Theta \simeq 0.6$ (compared to $\Theta \simeq 0.10$ in the case of constant returns).

5 Alternative Approaches

We are not the first to confront the divine coincidence and to offer a resolution. There are at least two alternative approaches: distortion shocks and more complex nominal wage and price setting.¹⁷

5.1 Distortion Shocks

A frequently used approach has been to simply append an exogenous disturbance to the NKPC, call it a “cost-push” shock, and thus create a trade-off between inflation stabilization and output gap stabilization (for example, Clarida, Galí and Gertler (1999)).

Taken at face value, this is a fix, not an acceptable solution: One needs to know where this additional disturbance comes from, and why it belongs to the equation. As we have seen, conventional supply shocks do not appear as disturbances in the baseline NKPC.

A number of authors however have shown that a potential justification for this approach is to assume the presence of exogenous “distortion shocks”, for example variations in tax changes, or changes in desired markups by firms (see for example Smets and Wouters (2003), Steinsson (2003), and Clarida, Galí and Gertler (2001)). Such shocks do not affect the relation between inflation and the output gap, the distance of output from its second-best value; they do however affect the distance of second-best output—which is affected by the shock—and first-best output—which, by assumption, is not affected by the shock. Thus, they create a trade-off between stabilization of inflation and stabilization of the welfare-relevant output gap.

Such distortion shocks do probably exist, and, *with respect to these shocks*, the divine coincidence no longer holds. But it still holds with respect to standard

supply shocks, such as movements in the price of oil or technology shocks. Thus the model extended in this way still implies that keeping inflation constant in the face of increases in the price of oil is the right policy—a proposition which, again, seems implausible.

5.2 Alternative Structures of Wage and Price Setting

Yet another approach aimed at removing the divine coincidence has been to explore the implications of alternative structures of wage and price setting. Erceg, Henderson, and Levin (2000; EHL, henceforth) in particular have shown that if both wage decisions and price decisions are staggered à la Calvo, the relation between price inflation and the output gap no longer holds exactly, implying a trade off between price inflation stabilization and output gap stabilization.

To assess their argument, consider the baseline model of Section 1, but now with staggered price and wage setting à la Calvo, as in EHL.

Given our assumptions, *wage inflation* is described by the difference equation:

$$\begin{aligned}\pi^w &= \beta E\pi^w(+1) - \lambda_w(w - mrs - \mu^w) \\ &= \beta E\pi^w(+1) - \lambda_w(w - \alpha m - (1 - \alpha + \phi)n - \xi - \mu^w) \\ &= \beta E\pi^w(+1) - \lambda_w(w - w_2) + \lambda_w \left(1 + \frac{\phi}{1 - \alpha}\right) (y - y_2)\end{aligned}$$

where w_2 and y_2 are, respectively, the natural levels of the real wage and output (now defined as those that would obtain under both flexible prices and wages). μ^w is a constant desired wage markup, and λ_w is a coefficient which is a function of structural parameters, including the Calvo parameter indexing the probability that any individual wage is reset in a given period (see EHL for details).

Price inflation, now denoted by π^p , is given by:

$$\begin{aligned}\pi^p &= \beta E\pi^p(+1) + \lambda_p(mc + \mu^p) \\ &= \beta E\pi^p(+1) + \lambda_p(w - \alpha m + \alpha n - \log(1 - \alpha) + \mu^p) \\ &= \beta E\pi^p(+1) + \lambda_p(w - w_2) + \lambda_p \frac{\alpha}{1 - \alpha} (y - y_2)\end{aligned}$$

Note that neither wage nor price inflation depends only on the output gap. In both cases, inflation depends on the output gap and the distance of the wage from the natural wage. Thus, the divine coincidence does not apply, either with respect to price inflation, or with respect to wage inflation.

Let us define however the composite inflation rate $\pi \equiv (\lambda_w \pi^p + \lambda_p \pi^w)/(\lambda_w + \lambda_p)$. It follows from the definition that:

$$\pi = \beta E\pi(+1) + \kappa (y - y_2) \quad (26)$$

where now $\kappa \equiv \frac{\lambda_w \lambda_p (1+\phi)}{(\lambda_w + \lambda_p)(1-\alpha)}$.

Given that, in the EHL model, the distance between first- and second-best output is given by

$$y_1 - y_2 = \frac{\mu (1 - \alpha)}{1 + \phi} \equiv \delta$$

where $\mu \equiv \mu^p + \mu^w$, we can rewrite (26) as:

$$\pi = \beta E\pi(+1) + \kappa (y - y_1 + \delta)$$

Hence the divine coincidence emerges again, though in a different guise: Stabilizing the distance of output from first best is equivalent to stabilizing a weighted average of wage and price inflation.

What are the policy implications of this weaker form of the divine coincidence? As shown by EHL, the utility-based loss function needed to evaluate alternative policies is a weighted average of the squares of the output gap, price inflation and wage inflation. In this context strict price inflation targeting is generally suboptimal, and often involves welfare losses that are several times larger than other, better designed policies. Interestingly, while strict output gap stabilization (and hence stabilization of the composite inflation index) is exactly optimal only for a specific parameter configuration, EHL conclude that it is nearly optimal for a large range of parameter values (see also Woodford (2003), Chapter 6). Hence, and for all practical purposes, a meaningful policy tradeoff is also missing in the EHL model.

6 The Behavior of Inflation

Beyond its normative implications, our model has implications for the behavior of inflation which appear more consistent with the data than those of the standard NK model. This is the topic of this section.

6.1 Inflation Inertia

Equation (14) implies that inflation will not outlive any variation in the output gap. Closing the latter will be sufficient to stabilize inflation *fully* and *with no delay*.

This is no longer the case under our proposed modification of the NK model. As equation (19) shows, in the presence of real wage rigidities ($\gamma > 0$) any change in the output gap, even if purely transitory, will have persistent effects on inflation. The reason is simple: Any change in the workers' reservation wage resulting from a change in output (and thus a change in employment), will affect the real wage (and hence real marginal cost) only gradually, with that effect outliving the eventual return of output to its natural level.

To illustrate this point, consider the limiting case where the output gap follows a white noise process (so $\rho_y = 0$). Equation (19) implies that inflation will follow:

$$\pi = \frac{\lambda(1 + \phi)}{1 - \alpha} \varepsilon_y + \lambda \left(1 + \frac{\phi}{1 - \alpha} \right) \sum_{k=1}^{\infty} \gamma^k \varepsilon_y(-k)$$

Note how inflation inertia is increasing in γ . For $\gamma = 0$ —that is, in the absence of real wage rigidities—inflation is white noise, just like the output gap. For γ close to one, inflation exhibits considerable inertia, despite the lack of serial correlation in the output gap.¹⁸

The previous point can also be illustrated by using a representation of inflation dynamics more closely linked to the empirical inflation equations found in the literature. Multiplying both sides of equation (19) by $(1 - \gamma L)$ and rearranging

terms, we get:

$$\pi = \frac{\gamma}{1 + \beta\gamma} \pi(-1) + \frac{\beta}{1 + \beta\gamma} E\pi(+1) + \frac{\lambda}{1 + \beta\gamma} x_2 + \zeta \quad (27)$$

where $\zeta \equiv [(\beta\gamma)/(1 + \beta\gamma)] (\pi - E(\pi | -1))$ is white noise and x_2 is defined as above.

Note that this equation takes a form very similar to the *hybrid NKPC* specifications used in many empirical and policy analysis applications, and which allow for both backward-looking and forward-looking inflation terms (with coefficients whose sum is close to one, as is the case here).¹⁹ In our model, the relative weight of lagged inflation is tightly linked to the degree of real wage rigidities. Thus, as γ increases from 0 to 1, the coefficient on past inflation rises from 0 to $1/(1 + \beta)$, which is slightly greater than $1/2$; the coefficient on expected inflation declines from β to $\beta/(1 + \beta)$, which is slightly less than $1/2$.

The previous discussion provides a potential explanation for the significance of lagged inflation in estimates of hybrid versions of the NKPC. Yet, we do not feel fully comfortable in relating equation (27) to much of the existing empirical work. The reason is that (27) is not directly estimable since the natural level of output, and by implication the output gap, is not observable (a point stressed by Galí and Gertler [1999]). We view the ad-hoc measures of the output gaps used in the literature (which approximate the natural levels by some smooth function of time) with some suspicion.

Fortunately, our model implies a representation of the inflation equation that can be taken to the data. Perhaps surprisingly, the representation turns out to be fairly close to the traditional Phillips curve equation relating inflation to lagged inflation, the unemployment rate, and a supply shock indicator (as estimated for example by Gordon (1997) among others.)

6.2 Inflation and Unemployment

In order to derive the inflation-unemployment relation implied by our model we proceed in two steps:

The first is to explicitly introduce unemployment. To do so, let n_s be implicitly defined by

$$w = y + \phi n_s + \xi$$

Note that n_s measures the quantity of labor households would want to supply given the current wage and marginal utility of income.²⁰ Accordingly, define the (involuntary) rate of unemployment, u , as the (log) deviation between the desired supply of labor and actual employment:

$$u \equiv n_s - n$$

Clearly, in the absence of wage rigidities ($\gamma = 0$) there is no involuntary unemployment as the wage is always equal to the marginal rate of substitution of households. For $\gamma > 0$, the previous definition and some algebraic manipulation of (15) give:

$$\Delta w = -\frac{(1-\gamma)\phi}{\gamma} u$$

Hence, in our model with real wage rigidities, a rate of unemployment above (below) some constant (implicitly normalized to zero) induces a downward (upward) adjustment of real wages. This adjustment goes on as long as the unemployment rate deviates from zero, with the size of the implied response being inversely related to γ , our index of real rigidities, but positively related to ϕ , the slope of the labor supply.

The second step consists in rewriting the inflation equation (19) in terms of unemployment, and of the price rather than the quantity of the non-produced input. Some manipulation gives (the algebra is given in the appendix):

$$\pi = \frac{1}{1+\beta}\pi(-1) + \frac{\beta}{1+\beta}E\pi(+1) - \frac{\lambda(1-\alpha)(1-\gamma)\phi}{\gamma(1+\beta)} u + \frac{\alpha\lambda}{1+\beta} \Delta v + \zeta \quad (28)$$

Inflation is a function of past and expected future inflation, of the unemployment rate, and of the change in the real price of the non-produced input. As in equation (27) earlier, the term ζ is proportional to $(\pi - E(\pi| - 1))$ and so is white noise, orthogonal to all variables at $t - 1$.²¹ Except for the presence of expected future inflation, this specification is indeed quite close to traditional specifications of

the Phillips curve, which typically include changes in the price of oil and other supply side factors in addition to unemployment on the right hand side.²²

Equation (28) can be estimated using instrumental variables. To do so, we use annual U.S. data on inflation (measured by the percent change in the GDP deflator), the civilian unemployment rate, and the percent change in the PPI raw materials index (relative to the GDP deflator). Our instrument set consists of four lags of the previous three variables, with Newey-West correction. The sample period is 1960-2004. The resulting estimated equation is (standard errors in brackets, constant not reported):

$$\pi = \underset{(0.17)}{0.66}\pi(-1) + \underset{(0.14)}{0.42}E\pi(+1) - \underset{(0.09)}{0.20}u + \underset{(0.01)}{0.018}\Delta v + \zeta$$

which accords, at least qualitatively, with (28). In particular, all the estimated coefficients have the right sign and, with the exception of the coefficient on Δv , are statistically significant at the 5 percent level. Furthermore, the theoretical restriction that the sum of coefficients on lagged and expected inflation equals one cannot be rejected at the 5 percent level (though not by much). When we impose this restriction the resulting estimated equation is:

$$\pi = \underset{(0.13)}{0.45}\pi(-1) + \underset{(0.13)}{0.54}E\pi(+1) - \underset{(0.05)}{0.05}u + \underset{(0.01)}{0.014}\Delta v + \zeta$$

Again the signs of all the estimated coefficients match well the theoretical specification, though the estimated coefficients on unemployment now is insignificant. The coefficients on past and expected future inflation imply a value for β slightly above, but insignificantly different from, unity. The other structural coefficients are not identified and cannot be recovered (they would be if we estimated the full model, something we have not done).

6.3 Alternative approaches

Here again, we are not the first to offer potential solutions to the lack of inflation inertia implied by the standard NK model. Three sets of alternative explanations, different from ours, can be found in the literature:²³

Lagged indexation. A simple way of generating inflation inertia has been to simply append a lagged inflation term to the NKPC, thus giving rise to what has been known as the *hybrid* NKPC. Taken at face value, this is again just a fix, not an acceptable solution. Christiano, Eichenbaum, and Evans (2005), Smets and Wouters (2003), and Steinsson (2004), among others, have shown however that such a formulation can be derived from automatic indexation of prices to past inflation by the firms that are not re-optimizing prices in any given period. We also see this as an unconvincing fix, with little basis in fact: none of the existing micro-studies of price setting uncover any form of mechanical indexation to past inflation; instead the prices of individual goods appear to remain unchanged for several months, even quarters in some cases (see, e.g. Dhyne et al. [2005]).²⁴

Relative wage concerns. In a well known paper, Fuhrer and Moore (1995) have argued that one gets inflation persistence in a model of wage staggering if one assumes that wage setters care about their real wage relative to the real wage of other workers over the period during which their own nominal wage is fixed. Holden and Driscoll (2003) have shown however that inflation persistence in the Fuhrer-Moore specification comes in fact from the assumption that workers care about the real wages of workers in the previous period. In this sense, the results from Fuhrer and Moore come from an assumption similar to ours, the assumption that real wages this period depend, to some extent, on real wages last period.

Sticky information: Mankiw and Reis (2002) have proposed a model in which firms adjust prices every period, according to a pre-specified plan, but revise that plan infrequently. Accordingly, current inflation is the result of decisions based on news about future demand and cost conditions obtained in previous periods, in addition to current news. A consequence of that “distributed lag” property is the emergence of inertia in inflation. Again, we view that explanation as one at odds with the micro evidence: Firms do not seem to adjust their prices continuously according to a pre-specified plan; instead they typically keep prices unchanged for long periods of time. Furthermore, recent survey-based evidence suggests that firms review their prices more often than they change them, exactly the opposite to what is assumed by sticky information models (see Fabiani et al. (2004)).

7 Conclusions

The standard new Keynesian framework is often criticized for its lack of a trade-off between inflation and output gap stabilization and the radical normative implications that arise from it. In the present paper we have argued that the introduction of real wage rigidities is a natural way to overcome that shortcoming. We have proposed a tractable modification of the new Keynesian framework that incorporates these real wage rigidities and generates more realistic policy trade-offs.

Our model can also account for some aspects of the empirical behavior of inflation that are often viewed as inconsistent with the standard NK model. In particular, we show how the presence of real wage rigidities becomes a source of inflation inertia, implying a degree of inflation persistence beyond that inherited from output gap fluctuations. In addition, our model yields a simple representation of inflation as a function of lagged and expected inflation, the unemployment rate and the change in the price of non-produced inputs. When we estimate that relation using annual postwar US data, we find that it fits the data pretty well. The latter result may not be very surprising given that the resulting empirical equation is not too different from some of the ad-hoc specifications that other authors have used in the older Phillips curve literature. A key difference lies in the relevance (confirmed empirically) of a forward-looking component in the determination of inflation, a feature emphasized by the more recent literature. In a sense, our framework helps bridge the gap between the old and new Phillips curve literatures in a way that is consistent with the micro-evidence on price setting.

There are several avenues that we plan to pursue, building on the framework developed in the present paper.

On the theoretical front, we want to dig deeper and explore microfoundations for real wage rigidity, and their implications for optimal monetary policy. Micro foundations, based for example on shirking (Felices 2005), on rule-of-thumb wage setters (Rabanal 2004), or on bargaining (Hall 2005) may lead to more substantial departures from the NK benchmark than we have allowed for here. The formalization offered by Hall for example implies modifying not only the real

wage equation, but also the specification of labor demand: In his model, the real wage rigidity affects only the rents going to workers and firms, and thus only the rate of job creation.

On the empirical front, we plan to estimate a model of joint wage and price inflation dynamics that combines both nominal and real rigidities in wage setting. We also plan to conduct a quantitative analysis of the optimal monetary policy response to a change in the price of oil or other raw materials, as a function of alternative assumptions about the degree of real wage rigidities and the persistence of the shock.

More generally, we hope our paper contributes to raise macroeconomists' awareness of the likely interactions between aggregate shocks and distortions, and of the implications of these interactions for optimal macroeconomic policy.

Notes

¹See, e.g., Goodfriend and King (1997) for discussion of the case for price stability associated with the new Keynesian model.

²See Galí, Gertler, and López-Salido (2005) for some evidence suggesting the presence of important cyclical variations in the size of aggregate distortions.

³While our focus is on supply shocks, we introduce preference shocks for two reasons. First, our (simplifying) assumption of a closed economy with no capital accumulation implies that first best employment is invariant to supply shocks (but not to preference shocks). Thus, in the absence of preference shocks, the employment gap would coincide with employment. Such nonrobust property could be misleading, especially in empirical applications. Secondly, and as discussed below, the implications for output of strict inflation targeting policies vary considerably depending on whether preference or supply shocks are the relevant disturbance at any point in time.

⁴See, e.g. Galí and Gertler (1999) for a derivation. An identical representation obtains under the assumption of quadratic costs of price adjustment, as in Rotemberg (1982). In the latter case coefficient λ is related to the magnitude of price adjustment costs. See Roberts (1995).

⁵As this section ends, it may be useful to restate the semantic conventions we use in the paper. We refer to the level of output which would prevail in the absence of imperfections as the “efficient” or “first-best” level of output. We refer to the level of output which would prevail in the absence of nominal rigidities as the “natural” or “second-best” level of output. We refer to the log distance between the actual level of output and the second-best level of output as the “output gap.” We refer to the log distance between the actual level of output and the first-best level of output as the “welfare-relevant output gap”. Similar statements apply to employment and employment gaps.

⁶See Woodford (2004, section 4.5) for a general discussion of that approach.

⁷In principle one would want to guarantee $w \geq mrs$ at all times, to prevent workers from working more than desired, given the wage (as would be the case for example in a model where wages set in bargaining vary over time, but always remain above the workers’ reservation wage). This would be easily achieved by introducing a (sufficiently large) positive steady state wage markup, as in

$$w = \gamma w(-1) + (1 - \gamma)(\mu_w + mrs)$$

without altering any of the conclusions below, though at the cost of burdening the notation.

⁸Other authors have adopted a similar assumption in order to model real wage rigidities. Hence, Christoffel and Linzert (2005) propose an analogous partial adjustment model in the context of a matching model in order to account for the response of unemployment and inflation

to monetary policy shocks. Alternative, but related, formalizations of real wage rigidities can be found in Felices (2005), in a model with variable effort and shirking, and Rabanal (2004), in a model with rule-of-thumb wage setters. Other authors have adopted a similar assumption in order to model real wage rigidities.

⁹Danthine and Kurman (2005) develop a model in which a real wage rigidity of a very similar form arises as a result of non-standard preferences, which make the disutility of effort a function of some "norm" which depends on the level and the change in the real wage. The normative implications of such a model would be clearly different from ours.

¹⁰More accurately, it implies a constant x_2 , which in turn implies an asymptotically constant $y - y_2$.

¹¹Note that v is endogenous in our framework, and so an increase in v may come from either a negative supply shock (a drop in m) or a preference shock that brings down the marginal disutility of labor (a drop in ξ).

¹²Notice that the expression we derive corresponds to the short run multiplier, and does not take into account subsequent changes in v , though the latter are small (more specifically, $dv(+k)/d\varepsilon_m = -\alpha(1 - \gamma)\gamma^k$ for $k = 1, 2, \dots$)

¹³We are indebted to Julio Rotemberg for raising the issue and encouraging us to explore its implications.

¹⁴We are grateful to Greg Mankiw for raising the question that stimulated the present analysis.

¹⁵See King and Wolman (1996) for a detailed discussion of the steady state in the standard new Keynesian model.

¹⁶Intuitively, the reduction in output comes from the increase in average markups. Calvo price-setting implies a negative relationship between the average markup and the rate of inflation in the steady state. Thus, lowering inflation permanently requires that average markups increase. Accordingly, real wages must decrease, with labor supply and output following suit.

¹⁷As discussed in the introduction, models of endogenous desired markups along the lines of those developed by Rotemberg and Woodford (1996, 1998) would also eliminate the divine coincidence, very much for the same reason our real wage rigidity assumption does. Rotemberg and Woodford restricted their analysis to the real implications of endogenous markups, without exploring their consequences for inflation and monetary policy in the presence of nominal frictions.

¹⁸The result that, in the absence of real rigidities, the persistence of inflation is equal to that of the output gap is not a general proposition. A counterexample is given in Rotemberg (2005), whom we thank for correcting us on this point. What is true in general is that persistence increases with γ .

¹⁹See, e.g. Buiter and Jewitt (1989), Fuhrer and Moore (1997), or Orphanides and Williams (2005).

²⁰Note that the two conditioning variables y and n generally differ from what they would be in a first-best equilibrium, which explains why, in general, $n_s \neq n_1$

²¹Note that, in contrast with the representation (27), the coefficients on lagged and expected inflation are independent of the degree of real wage rigidities, as measured by γ . Instead, that parameter has an influence on the size of the unemployment coefficient. Of course, in a full fledged general equilibrium model it would have an influence on the path of unemployment itself.

²²See, for example, Gordon (1997), or Blanchard and Katz (1999).

²³A fourth set of papers adopt an approach closer to ours in showing how different types of labor market imperfections affect the dynamics of inflation. A non-exhaustive list includes Trigari (2004), Walsh (2004), Rabanal (2004), Danthine and Kurmann (2004), Felices (2005), Krause and Lubik (2005), among others. The analysis in those papers typically focuses on the effects of labor market frictions (of different types) on the persistence of the inflation and output responses to an exogenous monetary policy shock.

²⁴The version of the hybrid NKPC proposed by Galí and Gertler (1999) which assumes a fraction of rule-of-thumb, backward-looking price setters overcomes that problem, since those firms are assumed to adjust their prices only occasionally, as conventional Calvo firms.

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Appendix 1: Derivation of Alternative Representations of Inflation Dynamics in the Presence of Real Wage Rigidities

Throughout this appendix we assume the marginal cost and wage setting schedules:

$$mc + \mu^p = w - mpn + \mu^p$$

$$w = \gamma w(-1) + (1 - \gamma) mrs$$

as well as the inflation equation implied by Calvo staggered price setting:

$$\pi = \beta E\pi(+1) + \lambda(mc + \mu^p) \quad (29)$$

Representation #1: in terms of the gap between actual and second-best output.

Combining the wage schedule with (2) and (4)

$$w = \gamma w(-1) + (1 - \gamma)(\alpha m + (1 - \alpha + \phi)n + \xi)$$

which can be combined with the marginal cost schedule to yield

$$mc + \mu^p = \gamma(mc(-1) + \mu^p) + (1 - \gamma)(\xi - \log(1 - \alpha)) + (1 - \gamma)(1 + \phi)n - \gamma\alpha(\Delta m - \Delta n)$$

Setting $mc = mc(-1) = -\mu^p$ (flexible price assumption):

$$0 = (1 - \gamma)(\xi - \log(1 - \alpha)) + (1 - \gamma)(1 + \phi)n_2 - \gamma\alpha(\Delta m - \Delta n_2)$$

which can be subtracted from the equation immediately above to yield:

$$mc + \mu^p = \gamma(mc(-1) + \mu^p) + (1 - \gamma)(1 + \phi)(n - n_2) + \gamma\alpha(\Delta n - \Delta n_2)$$

Finally, we combine the previous difference equation for real marginal cost with

the Calvo equation (29) and rewrite in terms of output to obtain:

$$\pi = \beta E\pi(+1) + \frac{1}{1-\gamma L} \frac{\lambda}{1-\alpha} [(1-\gamma)(1+\phi)(y-y_2) + \gamma\alpha(\Delta y - \Delta y_2)]$$

or, equivalently,

$$\pi = \frac{\gamma}{1+\gamma\beta} \pi(-1) + \frac{\beta}{1+\beta\gamma} E\pi(+1) + \frac{\lambda}{1+\beta\gamma} x_2 + \zeta$$

where $x_2 \equiv \frac{1}{1-\alpha} [(1-\gamma)(1+\phi)(y-y_2) + \gamma\alpha(\Delta y - \Delta y_2)]$ and $\zeta \equiv \frac{\beta\gamma}{1+\gamma\beta} (\pi - E(\pi|-1))$.

Representation #2: in terms of the gap between actual and first-best output, and underlying exogenous shocks.

Combining the wage schedule with (2) and (4)

$$w = \gamma w(-1) + (1-\gamma)(\alpha m + (1-\alpha + \phi)n + \xi)$$

which can be combined with the marginal cost schedule to yield

$$mc + \mu^p = \gamma(mc(-1) + \mu^p) + (1-\gamma)(\xi - \log(1-\alpha)) + (1-\gamma)(1+\phi)n - \gamma\alpha(\Delta m - \Delta n)$$

Using the expression for first-best employment, we can rewrite the previous difference equation in terms of the welfare relevant employment gap and the underlying shocks:

$$mc + \mu^p = \gamma(mc(-1) + \mu^p) + (1-\gamma)(1+\phi)(n - n_1 + \delta) + \gamma\alpha(\Delta n - \Delta n_1) - \gamma\alpha[\Delta m + (1+\phi)^{-1}\Delta\xi]$$

Combined with (29) we obtain:

$$\pi = \beta E\pi(+1) + \frac{\lambda}{1-\gamma L} \{(1-\gamma)(1+\phi)(n - n_1 + \delta) + \gamma\alpha(\Delta n - \Delta n_1) - \gamma\alpha[\Delta m + (1+\phi)^{-1}\Delta\xi]\}$$

or, equivalently,

$$\begin{aligned}\pi &= \frac{\gamma}{1 + \gamma\beta}\pi(-1) + \frac{\beta}{1 + \beta\gamma}E\pi(+1) \\ &+ \frac{\lambda}{1 + \beta\gamma}(1 - \alpha)^{-1}[(1 - \gamma)(1 + \phi)(y - y_1 + \delta) + \gamma\alpha(\Delta y - \Delta y_1)] \\ &- \frac{\lambda}{1 + \beta\gamma}\gamma\alpha(\Delta m + (1 + \phi)^{-1}\Delta\xi) + \zeta\end{aligned}$$

Representation #3: in terms of the gap between actual and first-best output, and the real price of non-produced input.

Combining the wage schedule with (2) and (4)

$$w = \gamma w(-1) + (1 - \gamma)(\alpha(m - n) + (1 + \phi)n + \xi)$$

Using the fact that $m - n = (w - v) + \log(\alpha/1 - \alpha)$, as implied by cost minimization:

$$w = \gamma w(-1) + (1 - \gamma)(\alpha(w - v) + \alpha \log(\alpha/1 - \alpha) + (1 + \phi)n + \xi)$$

Rearranging terms:

$$w = \Gamma w(-1) + (1 - \Gamma)(1 - \alpha)^{-1} (\alpha \log(\alpha/1 - \alpha) - \alpha v + (1 + \phi)n + \xi) \quad (30)$$

where $\Gamma \equiv \frac{\gamma}{1 - \alpha(1 - \gamma)} \in [0, 1]$ is a monotonic transformation of the index of real wage rigidities γ .

Note also that

$$\begin{aligned}mc + \mu^p &= w - \alpha(m - n) - \log(1 - \alpha) + \mu^p \\ &= w - \alpha(w - v) - \alpha \log \alpha - (1 - \alpha) \log(1 - \alpha) + \mu^p \\ &= (1 - \alpha) w + \alpha v - \alpha \log \alpha - (1 - \alpha) \log(1 - \alpha) + \mu^p \quad (31)\end{aligned}$$

which is a version of the factor price frontier (allowing for variable markups). Thus we see that an increase in the real price of the endowment input (independently

of the source, in principle any shock may do), will create both downward pressure on real wages and upward pressure on marginal costs and, hence, inflation.

Combining (30) and (31) we obtain (after some algebra):

$$mc + \mu^p = \Gamma (mc(-1) + \mu^p) + (1 - \Gamma)(1 + \phi) (n - n_1 + \delta) + \Gamma\alpha \Delta v \quad (32)$$

We can now rewrite inflation as

$$\pi = \beta E\pi(+1) + \frac{\lambda}{1 - \Gamma L} [(1 - \Gamma)(1 + \phi) (n - n_1 + \delta) + \Gamma\alpha \Delta v]$$

or, equivalently,

$$\pi = \frac{\Gamma}{1 + \beta\Gamma} \pi(-1) + \frac{\beta}{1 + \beta\Gamma} E\pi(+1) + \frac{\lambda(1 - \Gamma)(1 + \phi)}{(1 + \beta\Gamma)(1 - \alpha)} (y - y_1 + \delta) + \frac{\lambda\Gamma\alpha}{1 + \beta\Gamma} \Delta v + \zeta$$

where $\Gamma \equiv \frac{\gamma}{1 - \alpha(1 - \gamma)} \in [0, 1]$

Representation #4: in terms of the unemployment rate and the real price of the non-produced input.

First we derive a simple relationship between marginal cost, the unemployment rate (defined as above), and the employment gap:

$$\begin{aligned} mc + \mu^p &= w - (y - n) - \log(1 - \alpha) + \mu^p \\ &= (y + \phi n_s + \xi) - (y - n) - \log(1 - \alpha) + \mu^p \\ &= \phi (u - u_n) + (1 + \phi)(n - n_1 + \delta) \end{aligned}$$

We use the previous expression to substitute for $(n - n_1 + \delta)$ in the expression for real marginal cost in (32):

$$mc + \mu^p = \Gamma (mc(-1) + \mu^p) + (1 - \Gamma) [mc + \mu^p - \phi u] + \Gamma\alpha \Delta v$$

After rearranging terms we obtain the difference equation:

$$mc = mc(-1) - \frac{(1-\gamma)(1-\alpha)\phi}{\gamma} u + \alpha \Delta v$$

which is well defined only if $\gamma > 0$ (notice that as γ approaches 0 so does u).

Combining the previous equation with (29), we obtain:

$$\pi = \frac{1}{1+\beta} \pi(-1) + \frac{\beta}{1+\beta} E\pi(+1) - \frac{\lambda(1-\alpha)(1-\gamma)\phi}{\gamma(1+\beta)} u + \frac{\alpha\lambda}{1+\beta} \Delta v + \zeta$$

Appendix 2: A Model with Staggered Real Wage Setting

Consider an economy with staggered wage setting and full indexation. Each period a fraction $1 - \gamma$ of workers, drawn randomly from the population, reset their wage. The log-linearized law of motion for the aggregate real wage is thus given by

$$w_t = \gamma w_{t-1} + (1 - \gamma)w_t^*$$

where w_t^* denotes the newly set wage in period t . Utility maximization implies the following wage setting rule (up to first order, and ignoring a constant term):

$$\begin{aligned} w_t^* &= (1 - \beta\gamma) \sum_{k=0}^{\infty} (\beta\gamma)^k E_t\{mrs_{t+k|t}\} \\ &= (1 - \beta\gamma) \sum_{k=0}^{\infty} (\beta\gamma)^k E_t\{mrs_{t+k} - \epsilon_w \phi(w_t^* - w_{t+k})\} \\ &= \beta\gamma E_t\{w_{t+1}^*\} + \frac{1 - \beta\gamma}{1 + \epsilon_w \phi} (mrs_t + \epsilon_w \phi w_t) \end{aligned}$$

where $mrs_{t+k|t}$ denotes the marginal rate of substitution for a worker who last set its wage in period t .²⁵

Combining both equations we obtain

$$w = \Phi w(-1) + \Phi\beta E_t\{w(+1)\} + \Lambda mrs \quad (33)$$

where $\Phi \equiv \frac{(1 + \epsilon_w \phi)\gamma}{1 + \gamma[\epsilon_w \phi(1 + \beta) + \beta\gamma]}$ and $\Lambda \equiv \frac{(1 - \beta\gamma)(1 - \gamma)}{1 + \gamma[\epsilon_w \phi(1 + \beta) + \beta\gamma]}$. Notice that $1 = \Phi(1 + \beta) + \Lambda$ thus implying that in the steady state $w = mrs$.

Let $\tilde{w} \equiv w_1 - w_2$ and $\widetilde{mrs} \equiv mrs_1 - mrs_2$. Using the fact that $w_1 = mrs_1$ we can write

$$\tilde{w} = \Phi \tilde{w}(-1) + \Phi\beta E\tilde{w}(+1) + \Lambda \widetilde{mrs} + \Phi z \quad (34)$$

where

$$\begin{aligned} z &\equiv (1 + \beta)w_1 - w_1(-1) - \beta Ew_1(+1) \\ &= \Delta w_1 - \beta E\Delta w_1(+1) \end{aligned}$$

and where

$$\begin{aligned}
\Delta w_1 &= \Delta m p n_1 \\
&= \alpha(\Delta m - \Delta n_1) \\
&= \alpha[\Delta m - (1 + \phi)^{-1} \Delta \xi]
\end{aligned}$$

is a function of the exogenous shocks (and as a result so is z).

Notice also that $\tilde{w} = m p n_1 - m p n_2 = -\frac{\alpha}{1-\alpha} \tilde{y}$ and $\widetilde{mrs} = (1 + \frac{\phi}{1-\alpha}) \tilde{y}$, where $\tilde{y} \equiv y_1 - y_2$. Using these results we can rewrite (34) as:

$$\tilde{y} = \alpha \Phi \Upsilon \tilde{y}(-1) + \alpha \beta \Phi \Upsilon E \tilde{y}(+1) - (1 - \alpha) \Phi \Upsilon z$$

where $\Upsilon \equiv \frac{1}{\alpha + \Lambda(1 - \alpha + \phi)}$. Thus, to the extent that real wage rigidities are present ($\gamma > 0$), we will have $\Phi > 0$ and hence fluctuations in the gap between first and second best output in response to shocks.

Next we derive the implied inflation equation. Notice that real marginal cost is given by

$$\begin{aligned}
mc &= w - (y - n) - \log(1 - \alpha) \\
&= w + \frac{\alpha}{1 - \alpha} (y - m) - \log(1 - \alpha)
\end{aligned}$$

In terms of deviations from the flexible price equilibrium we have:

$$mc + \mu = (w - w_2) + \frac{\alpha}{1 - \alpha} (y - y_2)$$

Notice that we can also rewrite (33) in terms of deviations from the flexible price equilibrium values:

$$w - w_2 = \Phi (w - w_2)(-1) + \Phi \beta E(w - w_2)(+1) + \Lambda \left(1 + \frac{\phi}{1 - \alpha}\right) (y - y_2)$$

Collecting results we obtain

$$mc + \mu = \Phi (mc + \mu)(-1) + \Phi\beta E(mc + \mu)(+1) + f(y - y_2)$$

where

$$\begin{aligned} f(y - y_2) \equiv & [\Lambda (1 + (\phi/1 - \alpha)) + (\alpha/1 - \alpha)] (y - y_2) \\ & - \Phi \frac{\alpha}{1 - \alpha} (y - y_2)(-1) - \Phi\beta \frac{\alpha}{1 - \alpha} E(y - y_2)(+1) \end{aligned}$$

Combined with inflation equation (13) we obtain:

$$\begin{aligned} \pi &= \beta E\pi(+1) + \lambda E(1 - \Phi L - \beta\Phi L^{-1})f(y - y_2) \\ &= \beta E\pi(+1) + \lambda E(1 - \Phi L - \beta\Phi L^{-1})f(y - y_1) + u \end{aligned}$$

where $u \equiv \lambda E(1 - \Phi L - \beta\Phi L^{-1})f(y_1 - y_2)$.

Notice that there is no trade-off between stabilization of the output gap $y - y_2$ and stabilization of inflation. However, when $\gamma > 0$, u will fluctuate in response to shocks, making it impossible to stabilize both inflation and the welfare-relevant output gap.

Appendix 3: The Case of Decreasing Returns

In the present appendix we present some of the key equations of our model, derived under the alternative assumption of a production function at the firm level given by

$$Y = M^{\alpha_m} N^{\alpha_n}$$

where $\alpha_m + \alpha_n < 1$.

The efficient or first-best output is given by $y_1 = \alpha_m m + \alpha_n n_1$, where $(1 + \phi) n_1 = \log \alpha_n - \xi$.

The dynamics of natural or second-best output (relative to first best), in the presence of real wage rigidities are given by

$$[y_2 - y_1 + \delta] = \Theta [y_2(-1) - y_1(-1) + \delta] + \Theta \alpha_n [\alpha_m (1 - \alpha_m)^{-1} \Delta m + (1 + \phi)^{-1} \Delta \xi]$$

where $\Theta \equiv \frac{\gamma(1-\alpha_n)}{\gamma(1-\alpha_n)+(1-\gamma)(1+\phi)}$ and $\delta \equiv \frac{\mu^p \alpha_n}{1+\phi}$.

The coefficient on marginal cost in inflation equation (13) under staggered price setting is now given by $\lambda = \frac{(1-\theta)(1-\beta\theta)(1-\alpha_k)}{\theta [1-\alpha_k + \alpha_k \epsilon]}$, where $\alpha_k \equiv 1 - (\alpha_m + \alpha_n)$ is an index of the extent of decreasing returns and $\epsilon > 1$ is the price elasticity of product demand. Notice that λ is decreasing in α_k , implying a smaller response of inflation to variations in real marginal costs in the presence of decreasing returns. Furthermore, the relationship between marginal cost and the output gap is now given by $mc + \mu^p = \left(\frac{1+\phi}{\alpha_n}\right) (y - y_2)$.

Equation (19) carries over to the case of decreasing retruns, with x_2 now defined as

$$x_2 \equiv \alpha_n^{-1} [(1 - \gamma)(1 + \phi)(y - y_2) + \gamma(1 - \alpha_n)(\Delta y - \Delta y_2)]$$

whereas (20) now takes the form

$$\pi = \beta E\pi(+1) + \frac{\lambda}{1 - \gamma L} x_1 - \frac{\lambda \gamma}{1 - \gamma L} [\alpha_m \Delta m + (1 - \alpha_n)(1 + \phi)^{-1} \Delta \xi]$$

where $x_1 \equiv \alpha_n^{-1} [(1 - \gamma)(1 + \phi)(y - y_1 + \delta) + \gamma(1 - \alpha_n)(\Delta y - \Delta y_1)]$.

The factor frice frontier in the presence of decreasing returns needs to be modified

as follows

$$mc = (1 - \alpha_m) w + \alpha_m v + \alpha_k n - const$$

implying a representation of inflation in terms of the welfare-relevant output (corresponding to (21)) of the form

$$\pi = \beta E\pi(+1) + \frac{\lambda}{1 - \Gamma L} \left[\frac{(1 - \Gamma)(1 + \phi)}{\alpha_n} (y - y_1 + \delta) + \Gamma(\alpha_k \Delta n + \alpha_m \Delta v) \right]$$

The formulas for the impact of a supply shock under the two extreme policies considered in the text, and corresponding to equations (22) and (23), now take the form, respectively,

$$\frac{d\pi}{dv} = \frac{\gamma \lambda \alpha_m}{(1 - \beta \gamma)(1 - \alpha_m + \gamma \alpha_m)}$$

and

$$\frac{d(y_2 - y_1)}{dv} = - \frac{\Theta \alpha_n \alpha_m}{\alpha_m \Theta [1 + (1 - \gamma)(\alpha_n + \phi)] - (1 - \alpha_n)(1 - (1 - \gamma)\alpha_m)}$$

The expression determining the output costs of disinflation, corresponding to (25), is now given by

$$y(0) = - \frac{\gamma(1 - \beta)(\alpha_n + \phi)}{\kappa[(1 - \gamma)(1 + \phi) + (1 - \alpha_n)\gamma]} \pi^*$$

with the adjustment dynamics given again by $y(k) = \Theta y(k - 1)$.

Finally, the representation of inflation dynamics in terms of unemployment is given by

$$\begin{aligned} \pi = & \frac{1}{1 + \beta} \pi(-1) + \frac{\beta}{1 + \beta} E\pi(+1) - \frac{\lambda(1 - \alpha_n)(1 - \gamma)\phi(1 + \phi)}{\gamma(1 + \beta)(1 - \alpha_k + \phi)} u \\ & - \frac{\lambda\phi\alpha_k}{(1 + \beta)(1 - \alpha_k + \phi)} \Delta u + \frac{\lambda(1 + \phi)\alpha_m}{(1 + \beta)(1 - \alpha_k + \phi)} \Delta v + \zeta \end{aligned}$$