Monetary Policy, Inflation and the Business Cycle

A Classical Monetary Model

by

Jordi Galí

Households

Representative household solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \, U(C_t, N_t) \tag{1}$$

subject to

$$P_t \, C_t + Q_t \, B_t \leq B_{t-1} + W_t \, N_t - T_t \tag{2}$$

for $t = 0, 1, 2, \ldots$ plus solvency constraint.

Optimality conditions

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \tag{3}$$

$$Q_t = \beta \, E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \, \frac{P_t}{P_{t+1}} \right\} \tag{4}$$

Specification of utility:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

implied optimality conditions:

$$\frac{W_t}{P_t} = C_t^{\sigma} \ N_t^{\varphi} \tag{5}$$

$$Q_t = \beta \ E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \ \frac{P_t}{P_{t+1}} \right\} \tag{6}$$

Log-linear versions

$$w_t - p_t = \sigma \ c_t + \varphi \ n_t \tag{7}$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} \ (i_t - E_t\{\pi_{t+1}\} - \rho) \tag{8}$$

where $i_t \equiv -\log Q_t$ and $\rho \equiv -\log \beta$.(interpretation)

Perfect foresight steady state (with zero growth):

$$i = \pi + \rho$$

hence implying a real rate

$$r \equiv i - \pi = \rho$$

Ad-hoc money demand

$$m_t - p_t = y_t - \eta \ i_t$$

Firms

Representative firm with technology

$$Y_t = A_t \; N_t^{1-\alpha} \tag{9}$$

Profit maximization:

$$\max \; P_t Y_t - W_t N_t$$

subject to (9), taking the price and wage as given (perfect competition)

Optimality condition:

$$\frac{W_t}{P_t} = (1-\alpha) \; A_t \; N_t^{-\alpha}$$

In log-linear terms

$$w_t - p_t = a_t - \alpha \; n_t + \log(1-\alpha) \tag{10}$$

Equilibrium

Goods market clearing

$$y_t = c_t \tag{11}$$

Labor market clearing

$$\sigma \ c_t + \varphi \ n_t = a_t - \alpha \ n_t + \log(1 - \alpha)$$

Asset market clearing:

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma} \ (i_t - E_t\{\pi_{t+1}\} - \rho)$$

Aggregate production relationship:

$$y_t = a_t + (1 - \alpha) \ n_t$$

Implied equilibrium values for real variables

$$n_t = \psi_{na}\ a_t + \vartheta_n$$

$$y_t = \psi_{ya}\ a_t + \vartheta_y$$

$$r_t \equiv i_t - E_t\{\pi_{t+1}\} = \rho + \sigma\ E_t\{\Delta y_{t+1}\} = \rho + \sigma\psi_{ya}\ E_t\{\Delta a_{t+1}\}$$

$$\omega_t \equiv w_t - p_t = y_t - n_t + \log(1-\alpha) = \psi_{\omega a}\ a_t + \log(1-\alpha)$$

where $\psi_{na} \equiv \frac{1-\sigma}{\sigma+\varphi+\alpha(1-\sigma)}$; $\vartheta_n \equiv \frac{\log(1-\alpha)}{\sigma+\varphi+\alpha(1-\sigma)}$; $\psi_{ya} \equiv \frac{1+\varphi}{\sigma+\varphi+\alpha(1-\sigma)}$

$$\vartheta_y \equiv (1-\alpha)\vartheta_n \quad ; \quad \psi_{\omega a} \equiv \frac{\sigma+\varphi}{\sigma+\varphi+\alpha(1-\sigma)}$$

$\Longrightarrow$ real variables determined *independently of monetary policy* (neutrality)

$\Longrightarrow$ *optimal policy*: undetermined.

$\Longrightarrow$ specification of monetary policy needed to determine nominal variables

Monetary Policy and Price Level Determination

An Exogenous Path for the Nominal Interest Rate

exogenous stationary process $\{i_t\}$ with mean ρ

$$E_t\{\pi_{t+1}\} = i_t - r_t$$

where $\{r_t\}$ is determined independently of the policy rule.

Any path for the price level which satisfies

$$p_{t+1} = p_t + i_t - r_t + \xi_{t+1}$$

where $E_t\{\xi_{t+1}\} = 0$ for all t is consistent with equilibrium.

Implied path for the money supply:

$$m_t = p_t + y_t - \eta \, i_t$$

and hence it inherits the indeterminacy of p_t.

A Simple Inflation-based Interest Rate Rule

$$i_t = \rho + \phi_\pi \, \pi_t$$

Combined with the definition of the real rate:

$$\phi_\pi \, \pi_t = E_t\{\pi_{t+1}\} + \widehat{r}_t$$

If $\phi_\pi > 1$, unique stationary solution:

$$\pi_t = \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} \, E_t\{\widehat{r}_{t+k}\}$$

If $\phi_\pi < 1$, any process π_t satisfying

$$\pi_{t+1} = \phi_\pi \, \pi_t - \widehat{r}_t + \xi_{t+1}$$

where $E_t\{\xi_{t+1}\} = 0$ for all t is consistent with a stationary equilibrium

$\Longrightarrow$ *price level indeterminacy*

$\Longrightarrow$ illustration of the "Taylor principle" requirement

An Exogenous Path for the Money Supply $\{m_t\}$

Combining money demand and Fisherian equations:

$$p_t = \left(\frac{\eta}{1+\eta}\right) \; E_t\{p_{t+1}\} + \left(\frac{1}{1+\eta}\right) \; m_t + u_t$$

where $u_t \equiv (1+\eta)^{-1}(\eta \, r_t - y_t)$ evolves independently of $\{m_t\}$.
Assuming $\eta > 0$ and solving forward we obtain:

$$p_t = \frac{1}{1+\eta}\sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k \; E_t\,\{m_{t+k}\} + u'_t$$

where $u'_t \equiv \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k \; E_t\,\{u_{t+k}\}$.

In terms of expected future money growth rates

$$p_t = m_t + \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k \; E_t\,\{\Delta m_{t+k}\} + u'_t \tag{12}$$

Implied nominal interest rate:

$$
\begin{aligned}
i_t &= \eta^{-1}\left[y_t - (m_t - p_t)\right] \\
&= \eta^{-1}\sum_{k=1}^{\infty}\left(\frac{\eta}{1+\eta}\right)^{k} E_t\left\{\Delta m_{t+k}\right\} + v_t
\end{aligned}
$$

where $v_t \equiv \eta^{-1}(u_t + y_t)$.is independent of policy.

Example.

$$\Delta m_t = \rho_m \, \Delta m_{t-1} + \varepsilon_t^m$$

Assume no real shocks ($y_t = 0$).

Price response:

$$p_t = m_t + \frac{\eta \rho_m}{1 + \eta(1 - \rho_m)} \, \Delta m_t$$

$$\Longrightarrow \text{large price response}$$

Nominal interest rate response:

$$i_t = \frac{\rho_m}{1 + \eta(1 - \rho_m)} \, \Delta m_t$$

$$\Longrightarrow \textit{no liquidity effect}$$

A Model with Money in the Utility Function

Preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t \, U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

Budget constraint

$$P_t \; C_t + Q_t \; B_t + M_t \leq B_{t-1} + M_{t-1} + W_t \; N_t - T_t$$

Letting $\mathcal{A}_t \equiv B_{t-1} + M_{t-1}$:

$$P_t \; C_t + Q_t \mathcal{A}_{t+1} + (1 - Q_t) \; M_t \leq \mathcal{A}_t + W_t \; N_t - T_t$$

Interpretation: $\qquad (1 - Q_t) = 1 - \exp\{-i_t\} \simeq i_t$

$\Longrightarrow$ opportunity cost of holding money

Optimality Conditions

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

$$Q_t = \beta \ E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\}$$

$$\frac{U_{m,t}}{U_{c,t}} = 1 - \exp\{-i_t\}$$

where marginal utilities evaluated at $\left(C_t, \frac{M_t}{P_t}, N_t\right)$

Two cases:

- utility separable in real balances $\Longrightarrow$ neutrality
- utility non-separable in real balances (e.g. $U_{cm} > 0$) $\Longrightarrow$ non-neutrality

How Important is the implied non-neutrality? (Walsh, ch. 2)

Utility specification:

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \frac{X(C_t, M_t/P_t)^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

where

$$X(C_t, M_t/P_t) \equiv \left[(1-\vartheta)\, C_t^{1-\nu} + \vartheta \left(\frac{M_t}{P_t}\right)^{1-\nu}\right]^{\frac{1}{1-v}} \qquad for\ \nu \neq 1$$

$$\equiv C_t^{1-\vartheta} \left(\frac{M_t}{P_t}\right)^{\vartheta} \qquad for\ \nu = 1$$

Policy Rule: $\Delta m_t = \rho_m\, \Delta m_{t-1} + \varepsilon_t^m$

Calibration: $\nu = 2.56$; $\sigma = 2$ $\Longrightarrow$ $U_{cm} > 0$

Effects of Exogenous Monetary Policy Shock (Fig 2.3 and 2.4)

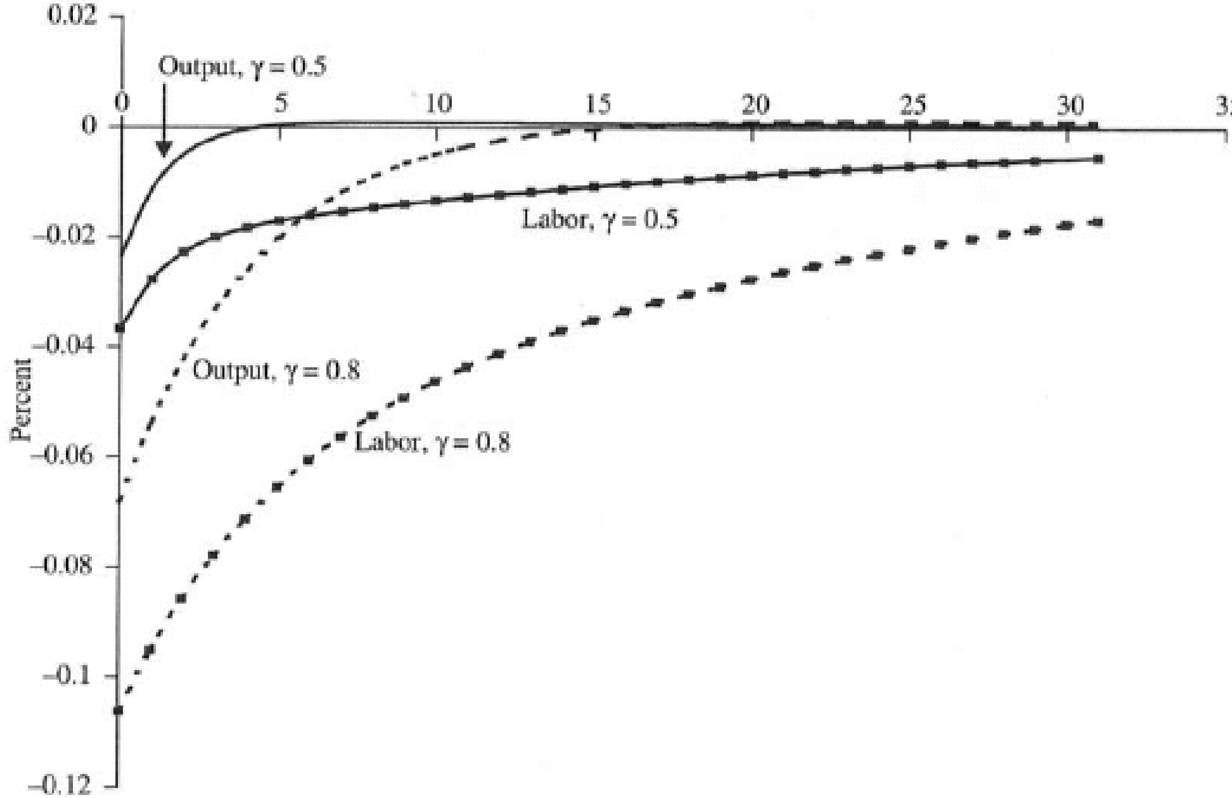

Figure 3.1
Output and Labor Response to a Money Growth Shock

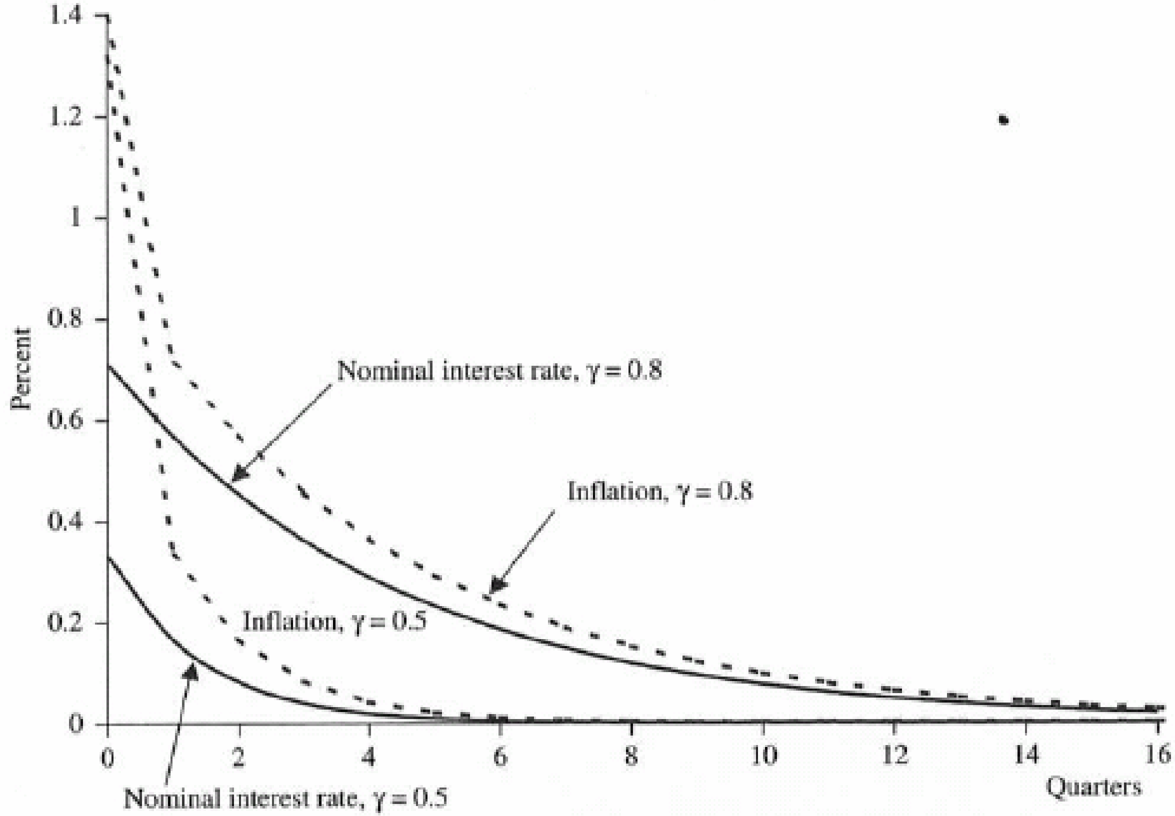

Figure 3.2
Nominal Interest Rate and Inflation Response to a Money Growth Shock (solid lines, nominal interest rate response; dashed lines, inflation response)

Optimal Monetary Policy in a Classical Economy with Money in the Utility Function

Social Planner's problem

$$\max U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

subject to

$$C_t = A_t \; N_t^{1-\alpha}$$

Optimality conditions:

$$-\frac{U_{n,t}}{U_{c,t}} = (1-\alpha)\; A_t N_t^{-\alpha} \tag{13}$$

$$U_{m,t} = 0 \tag{14}$$

Optimal policy (Friedman rule): $i_t = 0$ for all t .

Intuition

Implied average inflation: $\pi = -\rho < 0$

Implementation

$$i_t = \phi \left(r_{t-1} + \pi_t \right)$$

for some $\phi > 1$. Combined with the definition of the real rate:

$$E_t\{i_{t+1}\} = \phi \ i_t$$

whose only stationary solution is $i_t = 0$ for all t.

Implied equilibrium inflation:

$$\pi_t = -r_{t-1}$$