Lectures on Monetary Policy, Inflation and the Business cycle

Monetary Policy Design in the Basic New Keynesian Model

by

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The Efficient Allocation

$$\max U\left(C_t, N_t\right)$$

where $C_t \equiv \left[\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}}\, di\right]^{\frac{\epsilon}{\epsilon-1}}$ subject to:

$$C_t(i) = A_t\ N_t(i)^{1-\alpha},\ \ all\ i \in [0,1]$$

$$N_t = \int_0^1 N_t(i)\ di$$

Optimality conditions:

$$C_t(i) = C_t,\ \ all\ i \in [0,1]$$

$$N_t(i) = N_t,\ \ all\ i \in [0,1]$$

$$-\frac{U_{n,t}}{U_{c,t}} = MPN_t$$

where $MPN_t \equiv (1-\alpha)\ A_t N_t^{-\alpha}$.

Sources of Suboptimality of Equilibrium

1. Distortions unrelated to nominal rigidities:

- *Monopolistic competition*: $P_t = \mathcal{M}\ \frac{W_t}{MPN_t}$, where $\mathcal{M} \equiv \frac{\varepsilon}{\varepsilon-1} > 1$

$$\Longrightarrow \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}} < MPN_t$$

Solution: employment subsidy τ. Under flexible prices, $P_t = \mathcal{M}\ \frac{(1-\tau)W_t}{MPN_t}$.

$$\Longrightarrow \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}(1-\tau)}$$

Optimal subsidy: $\mathcal{M}(1-\tau) = 1$ or, equivalently, $\tau = \frac{1}{\varepsilon}$.

- *Transactions friction* (economy with valued money): assumed to be negligible

2. Distortions associated with the presence of nominal rigidities:

- *Markup variations* resulting from sticky prices: $\mathcal{M}_t = \frac{P_t}{(1-\tau)(W_t/MPN_t)} = \frac{P_t \mathcal{M}}{W_t/MPN_t}$ (assuming optimal subsidy)

$$\Longrightarrow \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = MPN_t \, \frac{\mathcal{M}}{\mathcal{M}_t} \neq MPN_t$$

Optimality requires that the average markup be stabilized at its frictionless level.

- *Relative price distortions* resulting from staggered price setting: $C_t(i) \neq C_t(j)$ if $P_t(i) \neq P_t(j)$. Optimal policy requires that prices and quantities (and hence marginal costs) are equalized across goods. Accordingly, markups should be identical across firms/goods at all times.

Optimal Monetary Policy in the Basic NK Model

Assumptions:

- optimal employment subsidy

  $\Longrightarrow$ flexible price equilibrium allocation is efficient

- no inherited relative price distortions, i.e. $P_{-1}(i) = P_{-1}$ for all $i \in [0,1]$

$\Longrightarrow$ the efficient allocation can be attained by a policy that stabilizes marginal costs at a level consistent with firms' desired markup, *given existing prices*:

- no firm has an incentive to adjust its price, i.e. $P_t^* = P_{t-1}$ and, hence, $P_t = P_{t-1}$ for $t = 0, 1, 2, ...$As a result the aggregate price level is fully stabilized and no relative price distortions emerge.

- equilibrium output and employment match their counterparts in the (undistorted) flexible price equilibrium allocation.

Equilibrium under the Optimal Policy

$$\widetilde{y}_t = 0$$

$$\pi_t = 0$$

$$i_t = r_t^n$$

for all t.

Implementation: Some Candidate Interest Rate Rules

Non-Policy Block:

$$\widetilde{y}_t = -\frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - r_t^n\right) + E_t\{\widetilde{y}_{t+1}\}$$

$$\pi_t = \beta\ E_t\{\pi_{t+1}\} + \kappa\ \widetilde{y}_t$$

An Exogenous Interest Rate Rule

$$i_t = r_t^n$$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A_O} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}$$

where

$$\mathbf{A_O} \equiv \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} \end{bmatrix}$$

Shortcoming: the solution $\widetilde{y}_t = \pi_t = 0$ for all t is *not* unique: one eigenvalue of $\mathbf{A_O}$ is strictly greater than one.

$\rightarrow$ indeterminacy. (real and nominal).

An Interest Rate Rule with Feedback from Target Variables

$$i_t = r_t^n + \phi_\pi \; \pi_t + \phi_y \; \widetilde{y}_t$$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \; \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}$$

where

$$\mathbf{A}_T \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi} \; \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix}$$

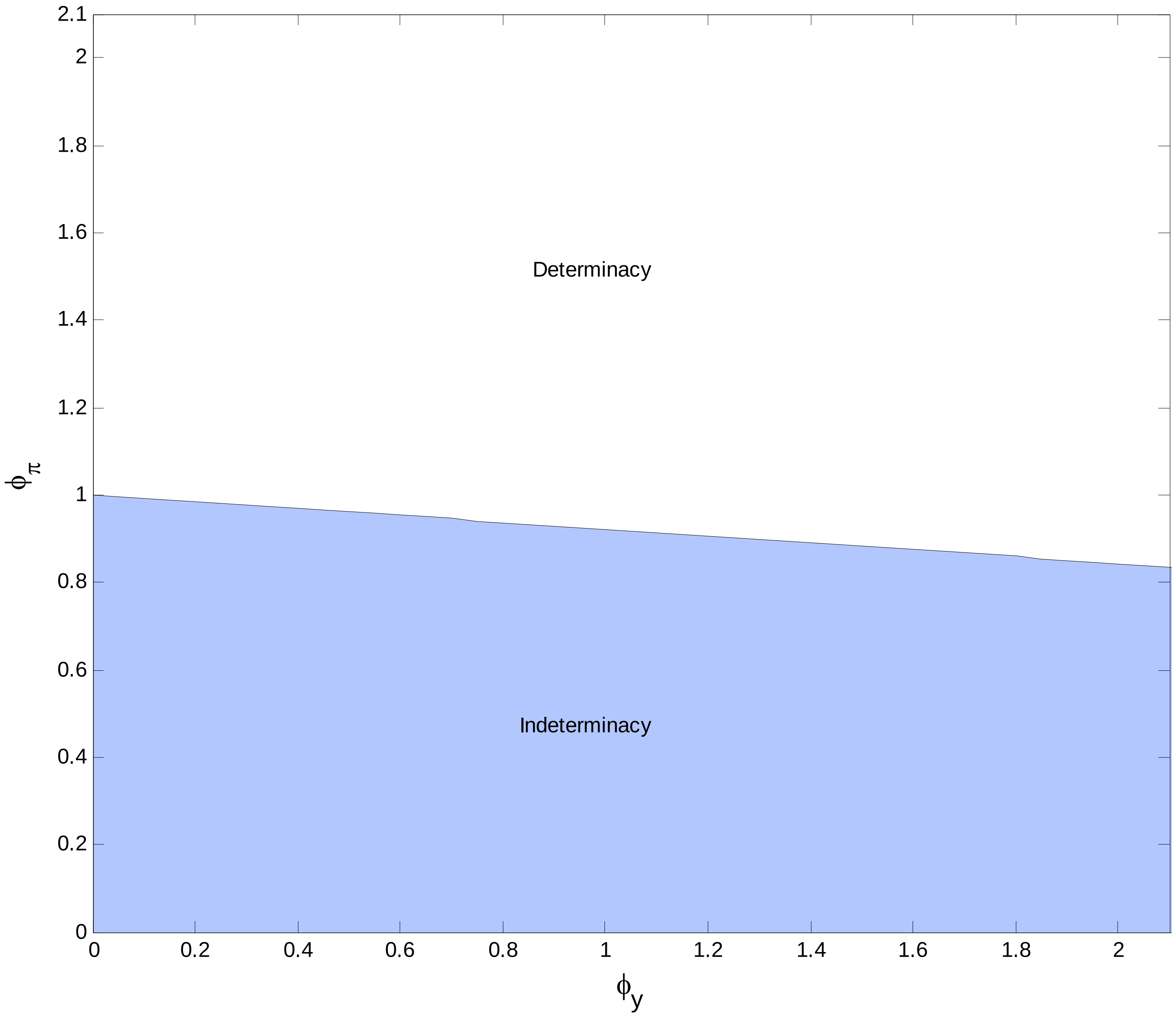
Existence and Uniqueness condition: (Bullard and Mitra (2002)):

$$\kappa\ (\phi_\pi - 1) + (1-\beta)\ \phi_y > 0$$

Taylor-principle interpretation (Woodford (2000)):

$$\begin{aligned} di &= \phi_\pi\ d\pi + \phi_y\ d\widetilde{y} \\ &= \left(\phi_\pi + \frac{\phi_y\ (1-\beta)}{\kappa}\right)\ d\pi \end{aligned}$$

Figure 4.1



Determinacy
Indeterminacy
ϕ_π
ϕ_y
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
2.1

A Forward-Looking Interest Rate Rule

$$i_t = r_t^n + \phi_\pi \; E_t\{\pi_{t+1}\} + \phi_y \; E_t\{\widetilde{y}_{t+1}\}$$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_F \; \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}$$

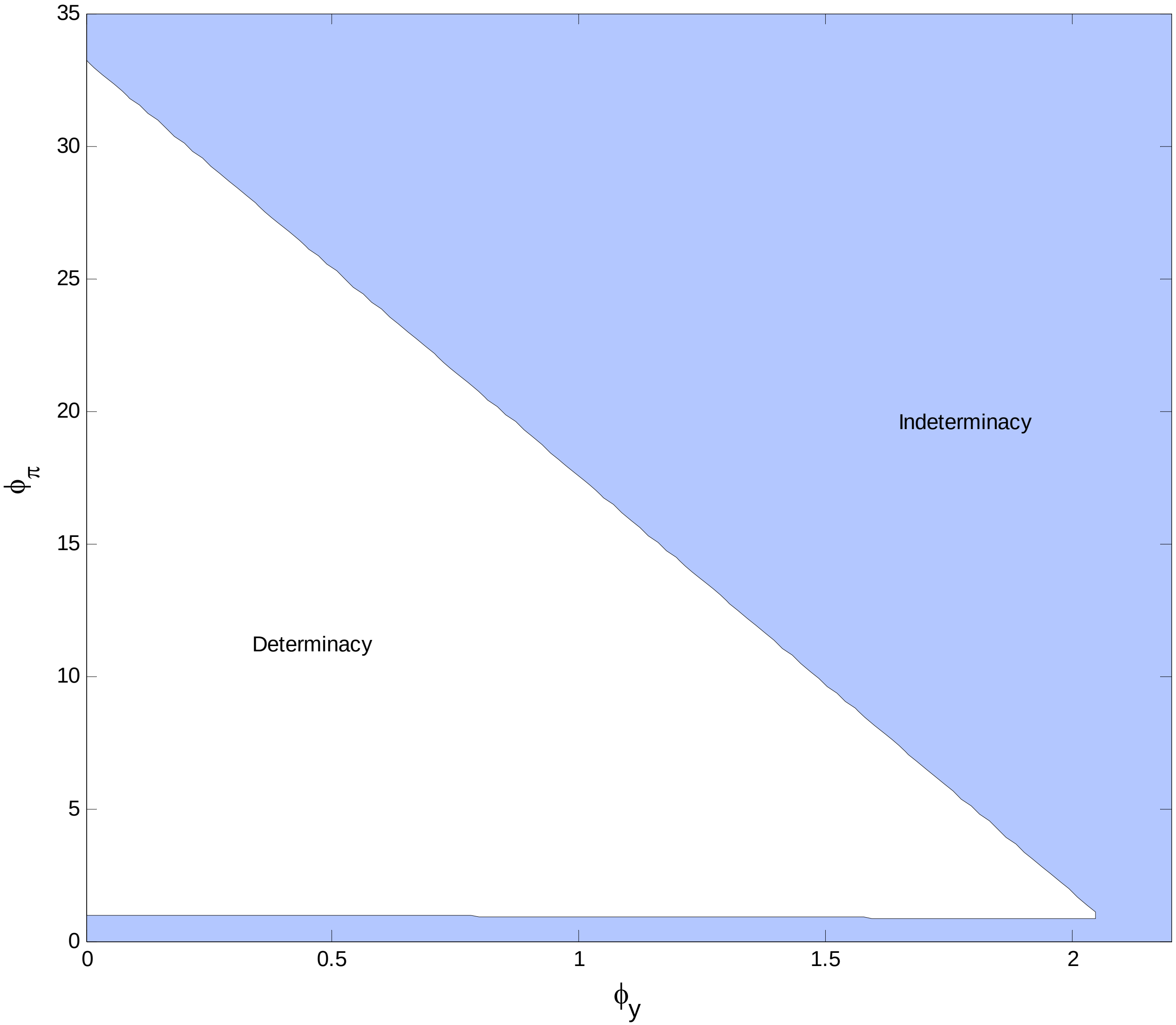
where

$$\mathbf{A}_F \; \equiv \begin{bmatrix} 1 - \sigma^{-1}\phi_y & -\sigma^{-1}(\phi_\pi - 1) \\ \kappa(1 - \sigma^{-1}\phi_y) & \beta - \kappa\sigma^{-1}(\phi_\pi - 1) \end{bmatrix}$$

Existence and Uniqueness conditions (Bullard and Mitra (2002):

$$\kappa \; (\phi_\pi - 1) + (1 - \beta) \; \phi_y \; > \; 0$$
$$\kappa \; (\phi_\pi - 1) + (1 + \beta) \; \phi_y \; < \; 2\sigma(1 + \beta)$$

Figure 4.2



35
30
25
20
15
10
5
0
ϕ_π
Indeterminacy
Determinacy
0
0.5
1
1.5
2
ϕ_y

Shortcomings of Optimal Rules

- they assume observability of the natural rate of interest (in real time).
- this requires, in turn, knowledge of:

  (i) the true model
  (ii) true parameter values
  (iii) realized shocks

Alternative: "simple rules" , i.e. rules that meet the following criteria:

- the policy instrument depends on observable variables only,
- do not require knowledge of the true parameter values
- ideally, they approximate optimal rule across different models

Simple Monetary Policy Rules

Welfare-based evaluation:

$$\mathbb{W} \equiv - E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{U_t - U_t^n}{U_c C} \right) = \frac{1}{2\lambda} \, E_0 \sum_{t=0}^{\infty} \beta^t \left(\kappa \, \widetilde{y}_t^2 + \epsilon \, \pi_t^2 \right)$$

$\Longrightarrow$ expected average welfare loss per period:

$$\mathbb{L} = \frac{1}{2\lambda} \, \left[\kappa \, var(\widetilde{y}_t) + \epsilon \, var(\pi_t) \right]$$

See Appendix for Derivation.

A Taylor Rule

$$i_t = \rho + \phi_\pi \; \pi_t + \phi_y \; \widehat{y}_t$$

Equivalently:

$$i_t = \rho + \phi_\pi \; \pi_t + \phi_y \; \widetilde{y}_t + v_t$$

where $v_t \equiv \phi_y \; \widehat{y}_t^n$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_T \; (\widehat{r}_t^n - \phi_y \; \widehat{y}_t^n)$$

where

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix} \quad ; \quad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

and $\Omega \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}$. Note that $\widehat{r}_t^n - \phi_y \; \widehat{y}_t^n = -\psi_{ya}^n \; [\sigma(1 - \rho_a) + \phi_y] \; a_t$

Exercise: $\Delta a_t \sim AR(1)$ + modified Taylor rule $i_t = \rho + \phi_\pi \; \pi_t + \phi_y \; \Delta y_t$

Money Growth Peg

$$\Delta m_t = 0$$

money market clearing condition

$$\widehat{l}_t = \widetilde{y}_t + \widehat{y}_t^n - \eta\, \widehat{i}_t - \zeta_t$$

where $l_t \equiv m_t - p_t$ and ξ_t is a money demand shock following the process

$$\Delta\zeta_t = \rho_\zeta\, \Delta\zeta_{t-1} + \varepsilon_t^\zeta$$

Define $l_t^+ \equiv l_t - \zeta_t$. $\Longrightarrow$

$$\widehat{i}_t = \frac{1}{\eta}\,(\widetilde{y}_t + \widehat{y}_t^n - \widehat{l}_t^+)$$

$$\widehat{l}_{t-1}^+ = \widehat{l}_t^+ + \pi_t - \Delta\zeta_t$$

Equilibrium dynamics:

$$\mathbf{A}_{\mathbf{M},\mathbf{0}} \begin{bmatrix} \widetilde{y}_t \\ \pi_t \\ l_{t-1}^{+} \end{bmatrix} = \mathbf{A}_{\mathbf{M},\mathbf{1}} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ l_t^{+} \end{bmatrix} + \mathbf{B}_{\mathbf{M}} \begin{bmatrix} \widehat{r}_t^n \\ \widehat{y}_t^n \\ \Delta\zeta_t \end{bmatrix}$$

where

$$\mathbf{A}_{\mathbf{M},\mathbf{0}} \equiv \begin{bmatrix} 1+\sigma\eta & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad ; \quad \mathbf{A}_{\mathbf{M},\mathbf{1}} \equiv \begin{bmatrix} \sigma\eta & \eta & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ; \quad \mathbf{B}_{\mathbf{M}} \equiv \begin{bmatrix} \eta & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Simulations and Evaluation of Simple Rules

Table 4.1: Evaluation of Simple Monetary Policy Rules

	Taylor Rule				*Constant Money Growth*	
ϕ_π	1.5	1.5	5	1.5	-	-
ϕ_y	0.125	0	0	1	-	-
$(\sigma_\zeta, \rho_\zeta)$	-	-	-	-	$(0, 0)$	$(0.0063, 0.6)$
$\sigma(\widetilde{y})$	0.55	0.28	0.04	1.40	1.02	1.62
$\sigma(\pi)$	2.60	1.33	0.21	6.55	1.25	2.77
$welfare\ loss$	0.30	0.08	0.002	1.92	0.08	0.38

Technical Appendix: Derivation of Second-Order Approximation of Welfare around the Undistorted Flexible Price Equilibrium Allocation

We derive a second order approximation of utility around the efficient equilibrium allocation. Under our assumptions the latter corresponds to the flexible price equilibrium allocation. All along we assume that utility is separable in consumption and hours (i.e., $U_{cn} = 0$). In order to lighten the notation we define $U_t \equiv U(C_t, N_t)$, $U_t^n \equiv U(C_t^n, N_t^n)$, and $U \equiv U(C, N)$.

A second order Taylor of expansion of U_t yields:

$$\begin{aligned} U_t - U_t^n &= U_{c,t}^n C_t^n \left(\frac{C_t - C_t^n}{C_t^n} \right) + U_{n,t}^n N_t^n \left(\frac{N_t - N_t^n}{N_t^n} \right) \\ &+ \frac{1}{2} U_{cc,t}^n (C_t^n)^2 \left(\frac{C_t - C_t^n}{C_t^n} \right)^2 + \frac{1}{2} U_{nn,t}^n (N_t^n)^2 \left(\frac{N_t - N_t^n}{N_t^n} \right)^2 \end{aligned}$$

Letting $\widetilde{x}_t \equiv \log \left(\frac{X_t}{X_t^n} \right)$ denote log-deviations from flexible price equilibrium values, we can write:

$$U_t - U_t^n = U_{c,t}^n \ C_t^n \ \left(\widetilde{c}_t + \frac{1-\sigma}{2} \ \widetilde{c}_t^2 \right) + U_{n,t}^n N_t^n \ \left(\widetilde{n}_t + \frac{1+\varphi}{2} \ \widetilde{n}_t^2 \right)$$

where we use the approximation

$$\frac{X_t - X_t^n}{X_t^n} \simeq \widetilde{x}_t + \frac{1}{2} \ \widetilde{x}_t^2$$

The next step consists in rewriting $\widetilde{n}_t$ in terms of the output gap. Using the fact that $N_t = \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\frac{\epsilon}{1-\alpha}} \ di$, we have

$$(1-\alpha) \ \widetilde{n}_t = \widetilde{y}_t + d_t$$

where $d_t \equiv (1-\alpha)\log\left[\int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}} di\right]$. The following lemma shows that d_t is proportional to the cross-sectional variance of relative prices and, hence, of second order.

Lemma 1: up to a second order approximation, $d_t = \frac{\epsilon}{2\Theta}\ var_i\{p_t(i)\}$, where $\Theta \equiv \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$. See proof at the end of the present appendix.

Accordingly we have:

$$U_t - U_t^n = U_{c,t}^n C_t^n \left(\widetilde{y}_t + \frac{1-\sigma}{2}\ \widetilde{y}_t^2\right) + \frac{U_{n,t}^n N_t^n}{1-\alpha}\left(\widetilde{y}_t + \frac{1+\varphi}{2(1-\alpha)}\ \widetilde{y}_t^2 + \frac{\epsilon}{2\Theta}\ var_i\{p_t(i)\}\right)$$

where we have made use of the market clearing condition $\widetilde{c}_t = \widetilde{y}_t$ for all t.

Finally, we recall that when the optimal subsidy is in place, the flexible price allocation is efficient, thus implying

$$-U_{n,t}^n N_t^n = U_{c,t}^n C_t^n (1-\alpha)$$

Hence, up to second order, we have

$$U_t - U_t^n = -\frac{1}{2} U_{c,t}^n C_t^n \left(\frac{\sigma+\varphi+\alpha(1-\sigma)}{1-\alpha}\ \widetilde{y}_t^2 + \frac{\epsilon(1+\alpha(\epsilon-1))}{1-\alpha}\ var_i\{p_t(i)\}\right)$$

Next we derive a first order approximation to $U_{c,t}^n C_t^n$ about the steady state:

$$\begin{aligned} U_{c,t}^n C_t^n &= U_c C + (U_{cc} C + U_c)\left(\frac{C_t^n - C}{C}\right) \\ &= U_c C + U_c C\ (1-\sigma)\ \widehat{c}_t^n \end{aligned}$$

It follows that, up to second order,

$$U_t - U_t^n = -\frac{1}{2} U_c C \left(\frac{\sigma + \varphi + \alpha(1-\sigma)}{1-\alpha} \, \widetilde{y}_t^2 + \frac{\epsilon(1+\alpha(\epsilon-1))}{1-\alpha} \, var_i\{p_t(i)\} \right)$$

Accordingly, we can write a second order approximation to the consumer's *welfare losses* resulting from deviations from the efficient allocation, expressed as a fraction of steady state consumption (or output), as:

$$\mathbb{W} \equiv - \, E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{U_t - U_t^n}{U_c C} \right) = \frac{1}{2} \, E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\sigma + \varphi + \alpha(1-\sigma)}{1-\alpha} \, \widetilde{y}_t^2 + \frac{\epsilon(1+\alpha(\epsilon-1))}{1-\alpha} \, var_i\{p_t(i)\} \right)$$

Lemma 2: up to second order and additive term independent of policy,,

$$\sum_{t=0}^{\infty} \beta^t \; var_i\{p_t(i)\} = \frac{\theta}{(1-\beta\theta)(1-\theta)} \sum_{t=0}^{\infty} \beta^t \; \pi_t^2$$

Proof: Woodford (2003, chapter 6)

Combining the previous lemma with the expression above we get

$$\mathbb{W} = \frac{1}{2\lambda} \, E_0 \sum_{t=0}^{\infty} \beta^t \left(\kappa \; \widetilde{y}_t^2 + \epsilon \; \pi_t^2 \right)$$

Hence the average period welfare loss will be given by:

$$\mathbb{L} = \frac{1}{2\lambda} \; [\kappa \; var(\widetilde{y}_t) + \epsilon \; var(\pi_t)]$$

Proof of Lemma 1

Let $\widehat{p}_t(i) \equiv p_t(i) - p_t$. Notice that,

$$\begin{aligned}\left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon} &= \exp\{(1-\epsilon)\,\widehat{p}_t(i)\} \\ &= 1+(1-\epsilon)\,\widehat{p}_t(i)+\frac{(1-\epsilon)^2}{2}\,\widehat{p}_t(i)^2\end{aligned}$$

Furthermore, from the definition of P_t, we have $1=\int_0^1\left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon}di$. Hence, a second order approximation to this expression implies

$$E_i\{\widehat{p}_t(i)\}=\frac{(\epsilon-1)}{2}\,E_i\{\widehat{p}_t(i)^2\}$$

In addition, a second order approximation to $\left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}}$ yields:

$$\left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}}=1-\left(\frac{\epsilon}{1-\alpha}\right)\widehat{p}_t(i)+\frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)^2\widehat{p}_t(i)^2$$

Combining the two previous results, it follows that

$$\begin{aligned}\left[\int_0^1\left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}}di\right] &= 1+\frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)\frac{1}{\Theta}\,E_i\{\widehat{p}_t(i)^2\} \\ &= 1+\frac{1}{2}\left(\frac{\epsilon}{1-\alpha}\right)\frac{1}{\Theta}\,var_i\{p_t(i)\}\end{aligned}$$

where $\Theta\equiv\frac{1-\alpha}{1-\alpha+\alpha\epsilon}$ and where the second equality holds up to second order, given that $(E_i\{\widehat{p}_t(i)\})^2$ is of higher order. Thus, we have $u_t=(1-\alpha)\log\left[\int_0^1\left(\frac{P_t(i)}{P_t}\right)^{-\frac{\epsilon}{1-\alpha}}di\right]\simeq\frac{\epsilon}{2\Theta}\,var_i\{p_t(i)\}$, up to a second order approximation. QED.