Lectures on Monetary Policy, Inflation and the Business Cycle

Monetary Policy Tradeoffs: Discretion vs. Commitment

by

Jordi Galí

The Monetary Policy Problem

$$\min \; E_0\{\sum_{t=0}^{\infty} \beta^t \; [\alpha_y \, \widetilde{y}_t^2 + \pi_t^2]\} \tag{1}$$

subject to:

$$\pi_t = \beta \; E_t\{\pi_{t+1}\} + \kappa \; \widetilde{y}_t + u_t$$

where $\{u_t\}$ evolves exogenously according to

$$u_t = \rho_u \; u_{t-1} + \varepsilon_t$$

In addition:

$$\widetilde{y}_t = -\frac{1}{\sigma} \; (i_t - E_t\{\pi_{t+1}\} - r_t^n) + E_t\{\widetilde{y}_{t+1}\} \tag{2}$$

Note: utility based criterion requires $\alpha_y = \frac{\kappa}{\epsilon}$

Optimal Policy with Discretion

Each period CB chooses (x_t, π_t) to minimize

$$\alpha_y \, \widetilde{y}_t^2 + \pi_t^2$$

subject to

$$\pi_t = \kappa \, \widetilde{y}_t + v_t$$

where $v_t \equiv \beta \; E_t\{\pi_{t+1}\} + u_t$ is taken as given.

Optimality condition:

$$\widetilde{y}_t = -\frac{\kappa}{\alpha_y} \, \pi_t \tag{3}$$

Equilibrium

$$\pi_t = \alpha_y q \; u_t \tag{4}$$

$$\widetilde{y}_t = -\kappa q \; u_t \tag{5}$$

$$i_t = r_t^n + q \; [\kappa\sigma(1-\rho_u) + \alpha_y \rho_u] \; u_t \tag{6}$$

where $q \equiv \frac{1}{\kappa^2 + \alpha_y(1-\beta\rho_u)}$

Implementation:

$$i_t = r_t^n + \left[(1-\rho_u)\frac{\kappa\sigma}{\alpha_y} + \rho_u\right]\pi_t$$

uniqueness condition: $\frac{\kappa\sigma}{\alpha_y} > 1$ (likely if utility-based: $\sigma\epsilon > 1$)

Alternatively,

$$i_t = r_t^n + q\ [\kappa\sigma(1-\rho_u) + \alpha_y\rho_u]\ u_t + \phi_\pi(\pi_t - \alpha_y q\ u_t)$$

uniqueness condition: $\phi_\pi > 1$.

Optimal Policy with Commitment

State-contingent policy $\{\widetilde{y}_t, \pi_t\}_{t=0}^{\infty}$ that maximizes

$$-\frac{1}{2}\, E_0 \sum_{t=0}^{\infty} \beta^t (\alpha_y\, \widetilde{y}_t^2 + \pi_t^2)$$

subject to the sequence of constraints:

$$\pi_t = \beta\; E_t\{\pi_{t+1}\} + \kappa\; \widetilde{y}_t + u_t$$

Lagrangean:

$$\mathcal{L} = -\frac{1}{2}\, E_0 \sum_{t=0}^{\infty} \beta^t\; [\alpha_y\; \widetilde{y}_t^2 + \pi_t^2 + 2\gamma_t\; (\pi_t - \kappa\; \widetilde{y}_t - \beta\; \pi_{t+1})]$$

First order conditions:

$$\alpha_y\; \widetilde{y}_t - \kappa\; \gamma_t = 0$$

$$\pi_t + \gamma_t - \gamma_{t-1} = 0$$

for $t = 0, 1, 2, \ldots$and where $\gamma_{-1} = 0$.

Eliminating multipliers:

$$\widetilde{y}_0 = -\frac{\kappa}{\alpha_y}\ \pi_0 \tag{7}$$

$$\widetilde{y}_t = \widetilde{y}_{t-1} - \frac{\kappa}{\alpha_y}\ \pi_t \tag{8}$$

for $t = 1, 2, 3, ...$

Alternative representation:

$$\widetilde{y}_t = -\frac{\kappa}{\alpha_y}\ \widehat{p}_t \tag{9}$$

for $t = 0, 1, 2, ...$where $\widehat{p}_t \equiv p_t - p_{-1}$.

Equilibrium

$$\widehat{p}_t = a\ \widehat{p}_{t-1} + a\beta\ E_t\{\widehat{p}_{t+1}\} + a\ u_t$$

for $t = 0, 1, 2, \ldots$where $a \equiv \frac{\alpha_y}{\alpha_y(1+\beta)+\kappa^2}$

Stationary solution:

$$\widehat{p}_t = \delta\ \widehat{p}_{t-1} + \frac{\delta}{(1-\delta\beta\rho_u)}\ u_t \tag{10}$$

for $t = 0, 1, 2, \ldots$where $\delta \equiv \frac{1-\sqrt{1-4\beta a^2}}{2a\beta} \in (0, 1)$.

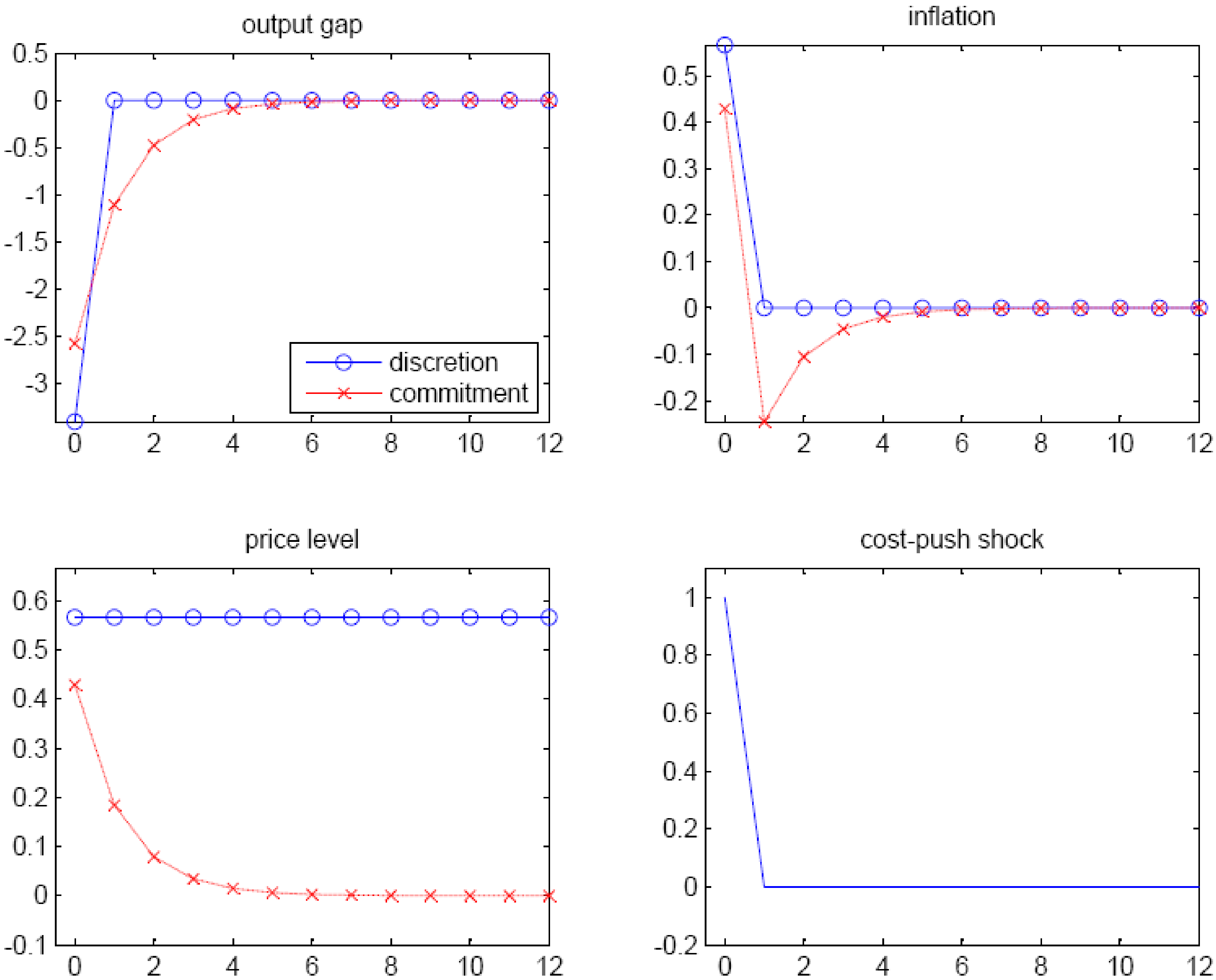
$\rightarrow$ *price level targeting !*

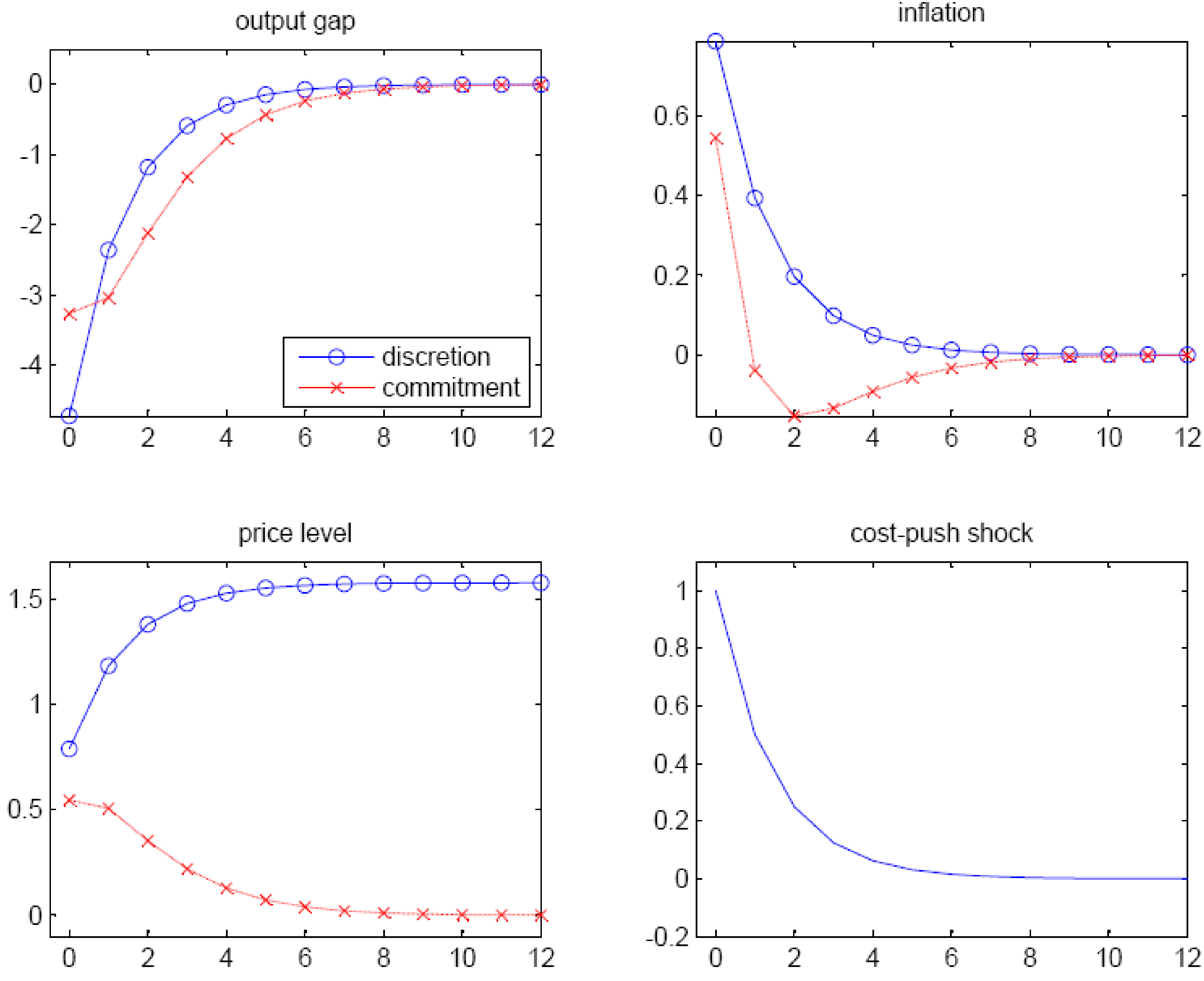
$$\widetilde{y}_t = \delta\ \widetilde{y}_{t-1} - \frac{\kappa\delta}{\alpha_y(1-\delta\beta\rho_u)}\ u_t \tag{11}$$

for $t = 1, 2, 3, \ldots$as well as

$$\widetilde{y}_0 = -\frac{\kappa\delta}{\alpha_y(1-\delta\beta\rho_u)}\ u_0$$

Optimal Monetary Policy: Discretion vs. Commitment





output gap
inflation
price level
cost-push shock
discretion
commitment

Appendix: Sources of Cost Push Shocks

Variations in desired price markups.

Assumption: time varying desired markup: $\mu_t^n \equiv \frac{\epsilon_t}{\epsilon_t - 1}$
Log-linearized optimal price setting rule:

$$
\begin{aligned}
p_t^* &= (1-\beta\theta)\sum_{k=1}^{\infty}(\beta\theta)^k E_t\{\mu_{t+k}^n + mc_{t+k} + p_{t+k}\} \\
&= (1-\beta\theta)\sum_{k=1}^{\infty}(\beta\theta)^k E_t\{\widetilde{mc}_{t+k} + p_{t+k}\}
\end{aligned}
$$

where $\widetilde{mc}_t \equiv mc_t + \mu_t^n$. Thus,

$$
\begin{aligned}
\pi_t &= \beta E_t\{\pi_{t+1}\} + \lambda\, \widetilde{mc}_t \\
&= \beta E_t\{\pi_{t+1}\} + \lambda\, \widehat{mc}_t + \lambda(\mu_t^n - \mu) \\
&= \beta E_t\{\pi_{t+1}\} + \kappa\, (y_t - \overline{y}_t) + \lambda(\mu_t^n - \mu)
\end{aligned}
$$

where $\overline{y}_t$ equilibrium output under a *constant* price markup μ.

Exogenous Variations in Wage Markups

$$\begin{aligned} mc_t &= w_t - a_t \\ &= \mu_{w,t} + mrs_t - a_t \\ &= \mu_{w,t} + (\sigma + \varphi)\ y_t - (1 + \varphi)\ a_t \end{aligned}$$

Thus,

$$\widehat{mc}_t = (\sigma + \varphi)\ (y_t - \overline{y}_t) + (\mu_{w,t} - \mu_w)$$

where $\overline{y}_t$: equilibrium output under a constant price and wage markup.

Implied inflation equation:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa(y_t - \overline{y}_t) + \lambda(\mu_{w,t} - \mu_w)$$