The Return of the Wage Phillips Curve *

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Abstract

The standard New Keynesian model with staggered wage setting is shown to imply a simple dynamic relation between wage inflation and unemployment. Under some assumptions, that relation takes the same form as the original Phillips (1958) curve, and may thus be viewed as providing some theoretical foundations to the latter. The structural wage equation derived here is shown to account reasonably well for the comovement of wage inflation and the unemployment rate in the U.S. economy, even under the assumption of a constant natural rate of unemployment. In addition, simulations of a calibrated version of a standard New Keynesian model suggest that staggered nominal wage setting may, in itself, be an significant source of observed unemployment fluctuations.

Keywords: staggered nominal wage setting, New Keynesian model, unemployment fluctuations.
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1 Introduction

The past decade has witnessed the emergence of a new popular framework for monetary policy analysis, the so called New Keynesian (NK) model. The new framework combines some of the ingredients of Real Business Cycle theory (e.g. dynamic optimization, general equilibrium) with others that have a distinctive Keynesian flavor (e.g. monopolistic competition and nominal rigidities).

Many important properties of the NK model hinge on the specification of its wage-setting block. While basic versions of that model, intended for classroom exposition, assume fully flexible wages and perfect competition in labor markets, the larger, more realistic versions (including those developed in-house at different central banks and policy institutions) typically assume staggered nominal wage setting, modeled in a way symmetric to price setting. The degree of nominal wage rigidities and other features of wage setting play an important role in determining the response of the economy to monetary and other shocks. Furthermore, and as argued in Erceg, Henderson and Levin (2000), the coexistence of price and wage rigidities has important implications for the optimal design of monetary policy. Yet, and despite the central role of the wage-setting block in the NK model, the amount of work aimed at assessing its empirical relevance has been surprisingly scant.[1] This is in stark contrast with the recent but already large empirical literature on price

[1] A recent exception is Sbordone (2006). Bob Gordon (1998) claims that such an omission has been "deliberate." In his words, "...[t]he earlier fixation on wages was a mistake. The relation of prices to wages has changed over time...The Fed's goal is to control inflation, not wage growth, and models with separate wage growth and price markup equations do not perform as well as the [price inflation] equation...in which wages are only implicit...."

inflation dynamics and firms' pricing patterns, which has been motivated to a large extent by the desire to evaluate the price-setting block of the NK model.[2]

One of the main objectives of the present paper is to fill part of that gap, by providing evidence on the NK model's ability to account for the observed patterns of wage inflation in the U.S. economy. In order to do so, I reformulate the standard version of the NK wage equation in terms of the (suitably defined) unemployment rate. The main advantage of that reformulation is the observability of the associated driving force (the unemployment rate), which contrasts with the inherent unobservability of the wage markup or the output gap, which are the driving forces in standard formulations of the NK wage inflation equation.

A valuable byproduct of the reformulation proposed here lies in the explicit introduction of unemployment in the *standard* NK model. This opens the door for an analysis of the model's qualitative and quantitative implications regarding a variable which, despite its central role in the policy debate, has been largely ignored in the monetary economics literature until recently. It is worth noticing in that regard that, in contrast with the latest batch of models combining nominal rigidities with search frictions in the labor market, unemployment in the present model arises exclusively as a result of assumptions on workers' market power and staggered wage setting already embedded in standard versions of the NK model. This should in principle allow one to

[2] See, e.g. Galí and Gertler (1998), Galí, Gertler and López-Salido (2001), Sbordone (2002) and Eichenbaum and Fisher (2007) for examples of papers using aggregate data. Micro evidence on price-setting patterns and its implications for aggregate models can be found in Bils and Klenow (2004), Nakamura and Steinsson (2008), and Mackowiak and Smets (2008), among others.

examine how far the model can go in accounting for the observed behavior of unemployment in the absence of search frictions and, accordingly, to evaluate the importance of introducing those frictions into the NK model (which requires a more substantial modification of the latter).

The main contributions of the paper can be summarized as follows:

- The staggered wage setting model à la Calvo (1983) embedded in standard versions of the New Keynesian framework is shown to imply a simple dynamic relation between wage inflation and the unemployment rate. Under certain assumptions, that relation takes the same form as the original equation of Phillips (1958) or a wage equation specification often used in applied work. The analysis developed here can thus be seen as providing some theoretical foundations for those specifications, as well as a structural interpretation to its coefficients.

- The implied structural wage equation is shown to account reasonably well for the behavior of wage inflation in the U.S. economy, even under the strong assumption of a constant natural rate of unemployment.

- Simulations of a calibrated version of the standard New Keynesian model suggest that staggered nominal wage setting may, in itself, be a significant source of large and persistent unemployment fluctuations.

The remainder of the paper is organized as follows. Section 2 describes the basic model of staggered nominal wage setting. Section 3 introduces the measure of unemployment latent in that model, and reformulates the wage inflation equation in terms of that variable. Section 4 provides an empirical assessment of the model's implied relation between wage inflation and unemployment using postwar U.S. data. Section 5 concludes.

2 Staggered Wage Setting and Wage Inflation Dynamics

This section introduces a variant of the staggered wage setting model originally developed in Erceg, Henderson and Levin (2000; henceforth, EHL). That model (and extensions thereof) constitutes one of the key building blocks of the monetary DSGE frameworks that have become part of the toolkit for policy analysis in both academic and policy circles. The variant presented here assumes that labor is indivisible, with all variations in hired labor input taking place at the extensive margin (i.e. in the form of variations in employment). The assumption of indivisible labor leads to a definition of unemployment consistent with its empirical counterpart.

The model assumes a (large) representative household with a continuum of members represented by the unit square and indexed by a pair $(i, j) \in [0, 1] \times [0, 1]$. The first dimension, indexed by $i \in [0, 1]$, represents the type of labor service in which a given household member is specialized. The second dimension, indexed by $j \in [0, 1]$, determines his disutility from work. The latter is given by $\chi_t \ j^{\varphi}$ if he is employed and zero otherwise, where $\varphi \geq 0$ determines the elasticity of the marginal disutility of work, and $\chi_t > 0$ is an exogenous preference shifter. Household utility is assumed to take the form

$$E_0 \sum_{t=0}^{\infty} \beta^t \ U(C_t, \{N_t(i)\}, \chi_t)$$

where C_t denotes household consumption, and $N_t(i) \in [0, 1]$ is the fraction of members specialized in type i labor who are employed in period t. Note that I am implicitly assuming full risk sharing of consumption among household members, as in Andolfatto (1996) and Merz (1995).

Period utility is assumed to be given by

$$\begin{aligned} U(C_t, \{N_t(i)\}, \chi_t) &\equiv \log C_t - \chi_t \int_0^1 \int_0^{N_t(i)} j^{\varphi} \, dj \, di \\ &= \log C_t - \chi_t \int_0^1 \frac{N_t(i)^{1+\varphi}}{1+\varphi} \, di \end{aligned}$$

As in EHL, and following the formalism of Calvo (1983), workers supplying a labor service of a given type (or a union representing them) get to reset their (nominal) wage with probability $1-\theta_w$ each period. That probability is independent of the time elapsed since they last reset their wage, in addition to being independent across labor types. Thus, a fraction of workers θ_w keep their wage unchanged in any given period, making that parameter a natural index of nominal wage rigidities.

When reoptimizing their wage in period t, workers choose a wage W_t^* in order to maximize household utility (as opposed to their individual utility), subject to a sequence of isoelastic demand schedules for their labor type, and the usual sequence of household flow budget constraints.[3] The first order condition associated with that problem can be written as:

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k \, E_t \left\{ \frac{N_{t+k|t}}{C_{t+k}} \left(\frac{W_t^*}{P_{t+k}} - \mathcal{M}_w \, MRS_{t+k|t} \right) \right\} = 0$$

where $N_{t+k|t}$ denotes the quantity demanded in period $t+k$ of a labor type whose wage is being reset in period t, $MRS_{t+k|t} \equiv \chi_t C_t N_{t+k|t}^{\varphi}$ is the rel-

[3] Details of the derivation of the optimal wage setting condition can be found in EHL (2000).

evant marginal rate of substitution between consumption and employment in period $t+k$, and$\mathcal{M}_w \equiv \frac{\epsilon_w}{\epsilon_w - 1}$ is the desired (or flexible wage) markup, with ϵ_w denoting the (constant) wage elasticity of demand for the services of each.labor type.

Log-linearizing the above optimality condition around a perfect foresight zero inflation steady state, and using lower case letters to denote the logs of the corresponding variable, we obtain the approximate wage setting rule

$$w_t^* = \mu^w + (1-\beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k \; E_t \left\{ mrs_{t+k|t} + p_{t+k} \right\} \tag{1}$$

where $\mu^w \equiv \log \mathcal{M}_w$. Note that in the absence of nominal wage rigidities $(\theta_w = 0)$ we have $w_t^* = w_t = \mu^w + mrs_t + p_t$, implying a constant markup μ^w of the wage w_t over the price-adjusted marginal rate of substitution, $mrs_t + p_t$. When nominal wage rigidities are present, new wages are set instead as a constant markup μ^w over a weighted average of current and expected future price-adjusted marginal rates of substitution.

Letting $mrs_t \equiv c_t + \varphi \, n_t + \xi_t$ denote the economy's average (log) marginal rate of substitution, where $\xi_t \equiv \log \chi_t$, we can write

$$\begin{aligned} mrs_{t+k|t} &= mrs_{t+k} + \varphi \; (n_{t+k|t} - n_{t+k}) \\ &= mrs_{t+k} - \epsilon_w \varphi \; (w_t^* - w_{t+k}) \end{aligned} \tag{2}$$

Furthermore, log-linearizing the expression for aggregate wage index around a zero inflation steady state we obtain

$$w_t = \theta_w \; w_{t-1} + (1-\theta_w) \; w_t^* \tag{3}$$

As in EHL (2000), we can combine equations (1) through (3) and derive the baseline wage inflation equation

$$\pi_t^w = \beta \ E_t\{\pi_{t+1}^w\} - \lambda_w \ (\mu_t^w - \mu^w) \tag{4}$$

where $\pi_t^w \equiv w_t - w_{t-1}$ is wage inflation, $\mu_t^w \equiv w_t - p_t - mrs_t$ denotes the (average) wage markup, and $\lambda_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)} > 0$. In words, wage inflation depends positively on expected one period ahead wage inflation and negatively on the deviation of the average wage markup from its desired value.[4] Equivalently, and solving (4) forward, we have

$$\pi_t^w = -\lambda_w \sum_{k=0}^{\infty} \beta^k \ E_t\{(\mu_{t+k}^w - \mu^w)\} \tag{5}$$

i.e. wage inflation is proportional to the discounted sum of expected deviations of current and future average wage markups from their desired levels. Intuitively, if average wage markups are below (above) their desired level, workers that have a chance to reset their wage will tend to adjust it upward (downward), thus generating positive (negative) wage inflation.

Estimated versions of the model above found in the literature generally allow for automatic indexation to inflation of the wages that are not reoptimized in any given period. An indexation rule often assumed in the literature (see, e.g., Smets and Wouters (2003)) is given by

$$w_{t+k|t} = w_{t+k-1|t} + \gamma\pi_{t-1}^p + (1-\gamma)\pi \tag{6}$$

for $k = 1, 2, 3, ...$where $w_{t+k|t}$ is the period $t + k$ (log) wage for workers who last re-optimized their wage in period t (with $w_{t|t} \equiv w_t^*$), $\pi_{t-1}^p \equiv p_{t-1} - p_{t-2}$ is

[4] Note that the previous equation is the wage analog to the price inflation equation resulting from a model with staggered price setting à la Calvo. See Galí and Gertler (1998) and Sbordone (2000) for a derivation and empirical assessment.

the lagged price inflation, and where π denotes steady state inflation (which is common to wages and prices, in the absence of secular productivity growth). In that case the following wage inflation equation can be derived:

$$\pi_t^w = \alpha + \beta \; E_t\{\pi_{t+1}^w\} + \gamma(\pi_{t-1}^p - \beta \pi_t^p) - \lambda_w \; (\mu_t^w - \mu^w) \tag{7}$$

where $\alpha \equiv (1-\gamma)(1-\beta)\pi$.

3 Wage Inflation and Unemployment

Consider household member (i, j), specialized in type i labor and with disutility of work $\chi_t j^{\varphi}$. Using household welfare as a criterion, and *taking as given* current labor market conditions (as summarized by the prevailing wage for his labor type), he will find it optimal to supply his labor services in period t if and only if

$$\frac{W_t(i)}{P_t} \geq \chi_t C_t \; j^{\varphi}$$

Thus, the marginal supplier of type i labor (employed or unemployed), which I denote by $N_t^*(i)$, is implicitly given by

$$\frac{W_t(i)}{P_t} = \chi_t C_t \; N_t^*(i)^{\varphi}$$

Taking logs and integrating over i we obtain

$$w_t - p_t = c_t + \varphi \; n_t^* + \xi_t \tag{8}$$

where $n_t^* \equiv \int_0^1 n_t^*(i) \; di$ can be interpreted as the model's implied (log) aggregate participation rate.

I define the unemployment rate u_t as

$$u_t \equiv n_t^* - n_t \tag{9}$$

Combining (8) and (9) with the expression for the average wage markup $\mu_t^w \equiv (w_t - p_t) - (c_t + \varphi n_t + \xi_t)$, one can easily obtain the following simple linear relation between the wage markup and the unemployment rate

$$\mu_t^w = \varphi \; u_t \tag{10}$$

Let us define the *natural* rate of unemployment, u_t^n, as the rate of unemployment that would prevail in the absence of nominal wage rigidities. It follows from the assumption of a constant desired wage markup that u_t^n is constant and given by

$$u^n = \frac{\mu^w}{\varphi} \tag{11}$$

Finally, combining (4), (10), and (11) we obtain the following New Keynesian Wage Phillips curve (NKWPC, for short):

$$\pi_t^w = \beta \; E_t\{\pi_{t+1}^w\} - \lambda_w \varphi \; (u_t - u^n) \tag{12}$$

Note that, like the original Phillips curve, the NKWPC establishes a relationship between wage inflation and the unemployment rate. But a number of differences with respect to Phillips' (1958) original curve (and some of its subsequent amendments) are worth emphasizing.

Firstly, (12) is a microfounded equilibrium relation between wage inflation and unemployment, with coefficients that are functions of parameters that have a structural interpretation. In particular, the steepness of the slope of the implied wage inflation-unemployment curve (given expected wage inflation) is decreasing in the the degree of wage rigidity θ_w (which is inversely related to λ_w). In the limit, as θ_w approaches zero (the case of full wage flexiblity), the curve becomes vertical. Also, the slope of the (π^w, u) relation is decreasing in the size of the Frisch labor supply elasticity (which corresponds to the inverse of φ).

Secondly, note that (12) implies that wage inflation is a forward looking variable, which is inversely related to current unemployment but also to its expected future path. This feature, which reflects the forward looking nature of wage setting, is immediately seen by solving (12) forward to obtain

$$\pi_t^w = -\lambda_w \varphi \sum_{k=0}^{\infty} \beta^k \ E_t\{(u_{t+k} - u^n)\} \tag{13}$$

Under the assumption that unemployment unemployment fluctuations about its natural level can be approximated by an AR(1) process with autoregressive coefficient ρ_u we can rewrite (13) as

$$\pi_t^w = \alpha_0 - \alpha_1 \ u_t \tag{14}$$

where $\alpha_0 \equiv \frac{\lambda_w \mu^w}{1-\beta} > 0$ and $\alpha_1 \equiv \frac{\lambda_w \varphi}{1-\beta\rho_u} > 0$. Note that (14) establishes a simple linear inverse relationship between wage inflation and the unemployment rate, analogous to that proposed by Phillips (1958), providing the latter with some theoretical underpinnings: in the NK framework, wage inflation arises as a result of a misalignment between current and anticipated wage markups and the desired markup, with the current unemployment rate being a sufficient statistic for that misalignment under the AR(1) assumption made here.

Note also that the simple linear relation between the wage markup and unemployment derived in this section holds irrespectively of the details of the wage setting process. In particular, it also holds in the presence of wage indexation as described in equation (6). In that case the resulting wage Phillips curve is given by:

$$\pi_t^w = \alpha + \beta \ E_t\{\pi_{t+1}^w\} + \gamma(\pi_{t-1}^p - \beta\pi_t^p) - \lambda_w\varphi \ (u_t - u^n) \tag{15}$$

Once again, consider the special case of an AR(1) process for the unemployment rate. In that case the solution to (15) can be written as

$$\pi_t^w = \alpha_0 - \alpha_1 \ u_t + \gamma \ \pi_{t-1}^p \tag{16}$$

i.e. wage inflation is a linear function of lagged price inflation and the current unemployment rate. Note that (16) is consistent with a common specification of the wage equation found in the empirical literature (e.g., Blanchard and Katz (1999)), as well as in mainstream undergraduate textbooks.

Finally, I end this section by considering the case, analyzed by Smets and Wouters (2003, 2008) among others, of a non-constant desired wage markup, denoted by $\overline{\mu}_t^w$. The wage inflation equation (without indexation) is now given by

$$\pi_t^w = \beta \ E_t\{\pi_{t+1}^w\} - \lambda_w(\mu_t^w - \overline{\mu}_t^w)$$

while the corresponding NKWPC now takes the form

$$\pi_t^w = \beta \ E_t\{\pi_{t+1}^w\} - \lambda_w\varphi \ (u_t - u_t^n)$$

where $u_t^n \equiv \frac{\overline{\mu}_t^w}{\varphi}$ denotes the (now time-varying) natural rate of unemployment. Variations in the latter variable, resulting from changes in desired wage markups, could thus potentiallly shift the relation between wage inflation and the unemployment rate.[5]

[5] Equivalently, we can write

Having derived the basic relation between wage inflation and unemployment implied by the baseline NK model, we now turn to a preliminary assessment of its empirical relevance.

4 Empirical Assessment: A First Pass

Next I provide a preliminary assessment of the empirical merits of the NKWPC developed in the previous section. More specifically, I want to evaluate to what extent a version of the NKWPC with a constant natural rate can account for the joint behavior of unemployment and wage inflation in the U.S. economy. First, I use simple statistics and graphical tools to seek evidence of a prima facie negative relationship between wage inflation and unemployment of the sort predicted by the theory above. Secondly, I compare the observed behavior of wage inflation with that predicted by an estimated version of the model above, conditional on the unemployment rate.[6] Finally, I embed the relation between the wage markup and the unemployment rate derived above in a calibrated version of the standard New Keynesian model to evaluate the potential role of wage stickiness in accounting for the observed volatility and persistence of unemployment.

$$\pi_t^w = \beta\ E_t\{\pi_{t+1}^w\} - \lambda_w \varphi\ u_t + v_t$$

$v_t \equiv \lambda_w \overline{\mu}_t^w$. In contrast with the representation of the wage equation found in Smets and Wouters (2003, 2007), the error term in the wage inflation formulation proposed here captures exclusively "wage markup shocks," and not preference shocks (even though the latter have been allowed for in the model above). This feature should in principle allow one to overcome the basic identification problem raised by Chari, Kehoe and McGrattan (2008) in their critique of current New Keynesian models.

[6] The observability of the unemployment rate, may be viewed as an advantage of the present framework relative to Sbordone (2006), who focuses instead on a parameterized version of (4).

I use quarterly U.S. data drawn from the Haver database, including the civilian unemployment rate (LR), average hourly earnings of production and nonsupervisory workers (LEPRIVA), and the consumer price index (CPU)..The sample period analyzed, constrained by wage data availability, is 1964Q1 - 2007Q4.

4.1 A Quick Glance at the Data

Figure 1 displays a scatterplot of wage inflation and the unemployment rate for the U.S. economy. In this and subsequent figures–though not in the formal econometric work below–wage inflation is measured as the centered four-quarter difference of the log nominal wage expressed in percent terms (i.e., 100*($w_{t+2} - w_{t-2}$)), in order to smooth the high volatility associated with quarter-to-quarter log-differences. The scatterplot reveals the absence of a stable negative relation between the two variables. Similar graphs, though typically focusing on price inflation, have often been used to demonstrate "the empirical failure of the Phillips curve." This is also reflected in the correlation between the two series, which is as low as 0.03.

Figure 2 displays the evolution of the same two variables over time. While no stable relation seems evident at a first glance, a more careful examination points to a strong inverse relation starting sometime around the mid-1980s and prevailing up to the end of the sample. That relation becomes even more evident in Figure 3, which displays wage inflation and (minus) the unemployment rate, while zooming in on the post-1986 period. The simple correlation between the two variables over the post-1986 period is -0.76, a large negative value.

Figure 4 adds a temporal dimension to the scatterplot of Figure 1. It suggests that the paths of U.S. wage inflation and unemployment have completed a full circle, returning in recent years to the same downward locus that characterized the 1960s. The evidence thus points to the presence of a stable negative relation between wage inflation and unemployment during periods of low and stable price inflation. That relation is broken during transitions from low to high inflation (late 60s and early 70s), or from high to low inflation (the early 80s), leading to an overall lack of correlation, as suggested by Figure 1. This is clearly illustrated by Figure 5, which shows a scatterplot of wage inflation and the unemployment rate, restricted to quarters with a year-on-year inflation below 4 percent.

Thus, while the basic NKWPC model seems to be at odds with the behavior of wage inflation and unemployment during the long 1970-1985 episode, it is possible that the extension of that model which allows for partial indexation to price inflation may be consistent with the evidence. I explore that avenue in the next subsection.

Why has the re-emergence of a stable negative relation between wage inflation and unemployment over the past two decades gone unnoticed among academic economists? A possible explanation lies in the focus on price inflation and away from wage inflation in much of the empirical research of recent years, combined with a lack of a significant empirical relation between price inflation and unemployment.[7] This is illustrated in Figure 6, which dis-

[7] Bob Gordon (1998) claims that such an omission has been "deliberate." In his words, "...[t]he earlier fixation on wages was a mistake. The relation of prices to wages has changed over time...The Fed's goal is to control inflation, not wage growth, and models with separate wage growth and price markup equations do not perform as well as the [price inflation] equation...in which wages are only implicit...."

plays once again (minus) the unemployment rate over the post-1986 period, but now against a measure of *price* inflation (the four-quarter difference in log CPI). The correlation between those two series is low and insignificant, and has the wrong sign (0.09). Of course, the theory developed above has nothing to say, by itself, about the relation between price inflation and the unemployment rate, since that relation is likely to be influenced by factors other than wage setting, including features of price setting and the evolution of labor productivity, among others.[8]

4.2 Can the New Keynesian Wage Phillips Curve Account for the Observed Fluctuations in Wage Inflation?

As shown above, the wage setting assumptions underlying the standard New Keynesian model make wage inflation a function of the current and expected gaps between the unemployment rate and its natural counterpart, as well as of past price inflation (in the version with wage indexation). In the present subsection I provide an empirical assessment of the model's ability to account for the observed fluctuations in wage inflation, under the maintained assumption of a constant natural rate of unemployment.

In the spirit of Campbell and Shiller's (1987) proposed assessment of present value relations, I start by defining the following measure of "fundamental" or "model-based" wage inflation:

$$\widetilde{\pi}_t^w(\Theta) \equiv \gamma \pi_{t-1}^p - \lambda_w \varphi \sum_{k=0}^{\infty} \beta^k \ E\{u_{t+k} | \ \mathbf{z}_t\}$$

[8] See, e.g. Blanchard and Galí (2009) for an analysis of the relation between price inflation and unemployment in a model with labor market frictions.

where vector $\Theta \equiv [\gamma, \theta_w, \beta, \epsilon_w, \varphi]$ collects the exogenous parameters of the model and where $\mathbf{z}_t = [u_t, \pi_t^w - \gamma \pi_{t-1}^p, ..., u_{t-q}, \pi_{t-q}^w - \gamma \pi_{t-q-1}^p]$ for some finite q. Under the null hypothesis that the model is correct, it is easy to check that $\widetilde{\pi}_t^w(\Theta) = \pi_t^w$ for all t. Next I estimate $\widetilde{\pi}_t^w(\Theta)$ and plot it against actual wage inflation, to evaluate the extent to which the simple model developed here can explain obseved fluctuations in that variable.

I assume that the joint dynamics of unemployment and wage inflation are well captured by the first order vector autoregressive model

$$\mathbf{z}_t = \mathbf{A}\ \mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_t$$

where $E\{\boldsymbol{\varepsilon}_t |\ \mathbf{z}_{t-1}\} = 0$ for all t. Thus, and letting $\mathbf{e}_i$ denote the i^{th} unit vector in $\mathbb{R}^{2q}$, we have $E\{u_{t+k}|\ \mathbf{z}_t\} = \mathbf{e}_1' \mathbf{A}^k \mathbf{z}_t$, implying

$$\widetilde{\pi}_t^w(\Theta) = \gamma \pi_{t-1}^p - \lambda_w \varphi\ \mathbf{e}_1' (\mathbf{I} - \beta \mathbf{A})^{-1}\ \mathbf{z}_t \tag{17}$$

I exploit the previous result to construct a time series for fundamental inflation $\widetilde{\pi}_t^w(\Theta)$ using a minimum distance estimator. Since not all structural parameters in Θ are separately identified, I calibrate three of them $(\beta, \epsilon_w, \varphi)$ and estimate the remaining two (γ, θ_w). Note that the latter define two key aspects of the wage setting process: the degree of rigidities and indexation. I set $\beta = 0.99$, a value commonly used in the business cycle literature. Note that φ is the inverse of the wage elasticity of the labor supply, a controversial parameter. In my baseline calibration I choose $\varphi = 5$, which is somewhere in between the very low values found in the micro literature and the higher values often assumed in the calibration of macro models. Finally, I set $\epsilon_w = 4.52$, which is consistent with a natural rate of unemployment of 5 percent,

–roughly the average unemployment rate over the sample period considered– given (11) and my baseline calibration for φ.[9]

I estimate the two remaining parameters, θ_w and γ, by minimizing $\sum_{t=0}^{T}(\pi_t^w - \widetilde{\pi}_t^w(\Theta))^2$ subject to (17), over all possible values $(\theta_w, \gamma) \in [0,1] \times [0,1]$, and given the calibrated values for $(\beta, \epsilon_w, \varphi)$ and the OLS estimate for matrix $\mathbf{A}$ (with $q = 4$). That procedure yields an estimate of of 0.60 for the indexation parameter γ (with a standard error equal to 0.029), and of 0.81 for the stickiness parameter θ_w (with a standard error of 0.01).[10] The previous estimates seem quite reasonable. In particular, the point estimate for θ_w implies an average wage duration of 5 quarters, which is just slightly longer than the mode duration uncovered by micro studies.[11] Interestingly, my estimates for γ and θ_w are very close to those obtained by Smets and Wouters (2007) using a very different approach (one that does not use information on the unemployment rate, among other differences): 0.58 and 0.7, respectively.

The model implies some restrictions that can be subject to formal testing. In particular, note that if the model holds exactly, we must have

$$\mathbf{e}_2' + \lambda_w \varphi\ \mathbf{e}_1' = \beta \mathbf{e}_2' \mathbf{A}$$

Unfortunately the previous set of restrictions is rejected at very low significance levels for our sample and baseline calibration. This may not be surprising, given the simplicity of the model. But, following Campbell and Shiller (1987), I seek a more informal evaluation of the model by comparing actual and fundamental wage inflation. Those are shown in Figure 7 both

[9] Note that $\epsilon_w = (1 - \exp\{-\varphi u^n\})^{-1}$.

[10] Standard errors are obtained by drawing from the empirical distribution of $\mathbf{A}$, and re-estimating θ_w and γ for each draw.

[11] See, e.g., Taylor (1999).

expressed in year-on-year terms. While the fit is far from perfect, it is clear that the model-based series captures pretty well the bulk of the low and medium frequency fluctuations in actual wage inflation, with the correlation between the two series being 0.83. The fact that such a good fit is obtained using a model for wage inflation that assumes a constant natural rate of unemployment makes that finding perhaps even more surprising.

Figure 8 displays the estimates of θ_w (together with ± 2 standard deviations confidence band) as a function of the calibrated value for the inverse labor supply elasticity φ, for a range of the latter between 1 and 10.[12] It is worth noticing that despite the large variation in φ considered, the estimates of the Calvo parameter θ_w remain within a relatively small (and plausible) range, corresponding to an average wage duration between 3 and 6 quarters.[13]

In order to evaluate its stability over time I have re-estimated the model above using post-1984 data, thus focusing on the so-called Great Moderation period. Under the baseline calibration for the inverse labor supply elasticity ($\varphi = 5$) the estimate of the Calvo parameter θ_w is 0.84, with a standard error of 0.09. That value is close to the one obtained for the full sample, though it is now considerably less precisely estimated. The point estimate for the indexation parameter γ using the post-84 sample is much lower, 0.15, and it is estimated with very little precision: the standard error is 0.35. This is undoubtedly a consequence of the shorter sample, as well as the little variability of price inflation during this period, which makes its role as a determinant of wage inflation harder to identify.

[12] Note that the estimate of γ is not affected by our calibration of φ, since it is separately identified.

[13] Note that the estimate of γ is independent of the calibration of φ.

I finish this subsection by comparing the time series for fundamental wage inflation obtained above with the reduced form OLS projection of wage inflation on the unemployment rate and lagged price inflation. As argued above, the latter is consistent with a specification of wage inflation often found in the literature. It would also be consistent with the NKWPC if the unemployment rate followed an exogenous $AR(1)$ process, though that assumption is formally rejected by the data.

The estimated equation for the full sample period is given by:

$$\pi_t^w = \underset{(0.101)}{0.855} - \underset{(0.018)}{0.039}\, u_t + \underset{(0.036)}{0.453}\, \pi_{t-1}^p$$

with an R^2 of 0.42 (and with standard errors in brackets). Note that the coefficients on both unemployment and lagged inflation are significant and have the expected sign. Figure 9 plots the implied wage inflation projection, together with the actual and fundamental wage inflation measures shown earlier. As the Figure makes clear, the difference between the model-based measure of wage inflation and the corresponding fitted measure from the simple OLS regression is pretty small. As to measures of fit, they are slightly better for fundamental inflation: its correlation with the actual wage series is 0.83 (against 0.78 for the OLS projection), whereas the corresponding root mean squared error is 0.369 (against 0.383). Such small differences in goodness-of-fit are related to the fact that lagged values of unemployment, price inflation and wage inflation have little forecasting power for future unemployment rates beyond that contained in the current unemployment rate. All in all, those findings provide some justification for the use of the simple reduced form specification in some applied work, though one should remain aware that its coefficients are not structural.

4.3 Nominal Wage Rigidities and Unemployment Fluctuations

Next I analyze a version of the New Keynesian model with staggered wage and price setting originally developed by Erceg, Henderson and Levin (2000), augmented to allow for partial wage indexation, and allowing for both technology and monetary policy shocks as exogenous driving forces. The objective of the exercise presented below is a relatively narrow one, namely, to explore the properties of unemployment fluctuations generated by that model, and to understand how those properties are affected by some key parameters.

The equilibrium dynamics of the New Keynesian model with staggered price and wage setting are described by the following equations:[14]

$$\pi_t^p = \beta \ E_t\{\pi_{t+1}^p\} + \kappa_p \ \widetilde{y}_t + \lambda_p \ \widetilde{\omega}_t \tag{18}$$

$$\pi_t^w = \beta \ E_t\{\pi_{t+1}^w\} + \gamma(\pi_t^p - \beta\pi_{t-1}^p) + \kappa_w \ \widetilde{y}_t - \lambda_w \ \widetilde{\omega}_t \tag{19}$$

$$\widetilde{\omega}_{t-1} \equiv \widetilde{\omega}_t - \pi_t^w + \pi_t^p + \Delta\omega_t^n \tag{20}$$

$$\widetilde{y}_t = - \ (i_t - E_t\{\pi_{t+1}^p\} - r_t^n) + E_t\{\widetilde{y}_{t+1}\} \tag{21}$$

$$i_t = \phi_p \ \pi_t^p + \phi_y \ y_t + v_t \tag{22}$$

Equations (18) and (19) describe the evolution of price and wage inflation (respectively), as a function of the output gap $\widetilde{y}_t \equiv y_t - y_t^n$ and the real wage gap $\widetilde{\omega}_t \equiv \omega_t - \omega_t^n$, where y_t^n and ω_t^n denote the natural levels of output and the natural wage, respectively.[15] Equation (20) is an identity linking

[14] Details of the derivation of (18)-(22) can be found in Galí (2008, chapter 6)

[15] The natural levels are defined as the equilibrium values under flexible prices and wages. In the model considered here, we have $y_t^n = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} \ a_t$ and $\omega_t^n = \frac{1-\alpha\psi_{ya}^n}{1-\alpha} \ a_t$. See Galí (2008, chapter 6) for details.

the real wage gap to price and wage inflation and the change in the natural wage ω_t^n..Equation (21) is a dynamic IS equation, with i_t denoting the short term nominal rate, and where $r_t^n \equiv \frac{\sigma(1+\varphi)}{\sigma+\varphi} E_t\{\Delta a_{t+1}\}$ is the natural rate of interest, which depends on an exogenous technology process $\{a_t\}$. Finally, (22) is an interest rate rule, with v_t an exogenous monetary policy shock. The exogenous monetary and technology shifters v_t and a_t are assumed to follow independent AR(1) processes. Given a choice of parameter values, the previous dynamical system can be solved and used to determine the model's implied statistical properties of all the endogenous variables.

Given the equilibrium path for the output and wage gaps we can determine the implied equilibrium unemployment rate using the relation

$$u_t - u^n = \frac{\widehat{\mu}_t^w}{\varphi} \tag{23}$$

$$= \frac{1}{\varphi}\left(\widetilde{\omega}_t - \left(1 + \frac{\varphi}{1-\alpha}\right)\widetilde{y}_t\right) \tag{24}$$

where the second equality follows from the definition of the wage markup and the assumption of an aggregate technology of the form $y_t = a_t + (1-\alpha)\, n_t$.

Table 1 summarizes the baseline values assumed for the different parameters, other than the degree of wage rigidities, θ_w, and the autoregressive coefficients, ρ_v and ρ_a, for the two shock processes, for which I consider several alternative values. Most of the assumed values are standard and drawn from Galí (2008), with the exception of the elasticity of substitution among labor types, ϵ_w , which is chosen so that the steady state unemployment is 5 percent, given $\varphi = 5$.

Table 2 reports measures of volatility and persistence of unemployment, conditioned on monetary policy and technology shocks, for alternative configurations of values for the degree of wage stickiness θ_w and the autoregressive coefficient of the shock process ρ_i $(i = v, a)$. I choose the standard deviation of the unemployment rate relative to (log) output as a measure or unemployment volatility, for that measure is invariant to the the assumed volatility of each shock.[16] The first order autocorrelation of the unemployment rate is reported as a measure of persistence. As a reference, note that the corresponding empirical values for the previous two statistics in the U.S. economy over the period 1964:I-2007:IV are 0.99 and 0.98, respectively.[17] The question I pose here can be stated as follows: To what extent does the standard New Keynesian model with staggered wage and price setting have the potential to generate fluctuations in the unemployment rate as volatile (relative to output) and persistent as observed in the data, even under the tightrope of a constant natural rate?

The statistics reported in Table 2 suggest that the answer to the previous question is a qualified yes. I start by discussing the second moments generated by monetary/demand shocks, shown in the top panel of the Table. First, we see that unemployment volatility relative to output increases with the degree of nominal wage rigidities θ_w. This should not be surprising since such rigidities are the only source of unemployment variations in the model Note also that the same statistic is decreasing in the persistence of the mon-

[16]Note that one can always match an arbitrary (absolute) standard deviation of the unemployment rate by adjusting the volatility of the exogenous shock, the latter being a parameter whose calibration would always be controversial.

[17]I use the standard deviation of HP-filtered (log) GDP (with a smoothing parameter of 1600) as a reference when computing the relative volatility of the unemployment rate (the latter variable is not detrended).

etary shock, and more so when wages are less rigid.[18] For the range of values for θ_w and ρ_v considered, the relative volatility of unemployment conditional on monetary shocks is larger than observed in the data, except when the shock is highly persistent.and wages highly flexible. Turning to the implied persistence of the unemployment rate, we see how the autocorrelation of the latter is increasing in the persistence of the shock and, more interestingly, in the degree of nominal wage rigidities. Note, however, that the conditional autocorrelation always remains below the autocorrelation of the shock process itself, and substantially so when wages are relatively flexible. Yet, under the estimated value for the wage rigidity parameter ($\theta_w = 0.8$) the unemployment rate essentially inherits the persistence of the shock itself.

Turning to the effects of technology shocks, we see that the latter also imply a relative volatility of unemployment that is increasing in the degree of wage rigidities and decreasing in the persistence of the shock, the same pattern that we detected under monetary shocks. With technology shocks, however, the sensitivity to variations in both parameters appears considerably enhanced. In particular, when technology shocks are highly persistent, the relative volatility of unemployment is substantially lower than that generated by monetary shocks. This may be viewed as good news, for it implies that the simultaneous operation of the two shocks will tend to generate a measure of relative volatility close to the unconditional one observed in the

[18] Even though the standard deviations of both unemployment and (log) output increase as we raise the persistence of the shock process, that of unemployment increases less than proportionally. Since, by construction, the relative volatility of (log) employment and (log) output is constant under monetary policy shocks, the smaller increase in the standard deviation of unemployment must be due to the response of participation. A detailed analysis of the latter is left for future research.

data. On the other hand, the Table shows that the pattern of conditional autocorrelations is very similar to that obtained for monetary shocks.

The simple exercise summarized in Table 2 should not be viewed as a substitute for a more thorough analysis of the properties of unemployment in the standard New Keynesian model. Such an analysis, which is beyond the scope of the present paper, should include a comparison of the predicted and estimated joint responses of unemployment, its two "components" (i.e. employment and participation), and other key macro variables to a variety of shocks, thus going beyond the simple summary statistics of Table 2. That caveat notwithstanding, the simulation findings reported in Table 2 yield at least one important lessons: they clearly suggest that realistic nominal wage rigidities may, in themselves, be a significant source of unemployment fluctuations, of size and persistence comparable to those found in postwar U.S. data. More research would thus seem to be warranted to determine the extent to which frictions other than nominal wage rigidities (e.g. real wage rigidities or search frictions), as well as factors accounting for possible variations in the natural rate, play a role as a source of unemployment variations. That analysis should also provide some guidance regarding the importance of incorporating these different factors in current monetary DSGE models, if we want the latter to provide a good account of unemployment.

5 Concluding comments

In his seminal 1958 paper, A.W. Phillips uncovered a tight inverse relation between unemployment and wage inflation in the U.K.. That relation was largely abandoned on both theoretical and empirical grounds. From a theoretical viewpoint, it was not clear why the rate of change of the nominal wage (as opposed to the level of the real wage) should be related to unemployment. From an empirical viewpoint, economists' attention shifted to the relation between price inflation and unemployment, but hopes of establishing a stable relationship between those variables faded with the stagflation of the 1970s.

The present paper has made two main contributions. First, it provides some theoretical foundations to a Phillips-like relation between wage inflation and unemployment. It does so not by developing a new model but, instead, by showing that such a relation underlies a standard New Keynesian framework with staggered wage setting, even though versions of the latter found in the literature do not explicitly incorporate or even discuss unemployment. Secondly, the implied structural wage equation is shown to account reasonably well for the comovement of wage inflation and the unemployment rate in the U.S. economy, even under the strong assumption of a constant natural rate of unemployment. In particular, that equation can explain the strong negative comovement between wage inflation and unemployment observed during the past two decades of price stability. Finally, simulations of a calibrated version of a standard New Keynesian model suggest that staggered nominal wage setting may, in itself, be an significant source of observed unemployment fluctuations, which may complement the search and matching frictions emphasized in the recent literature.

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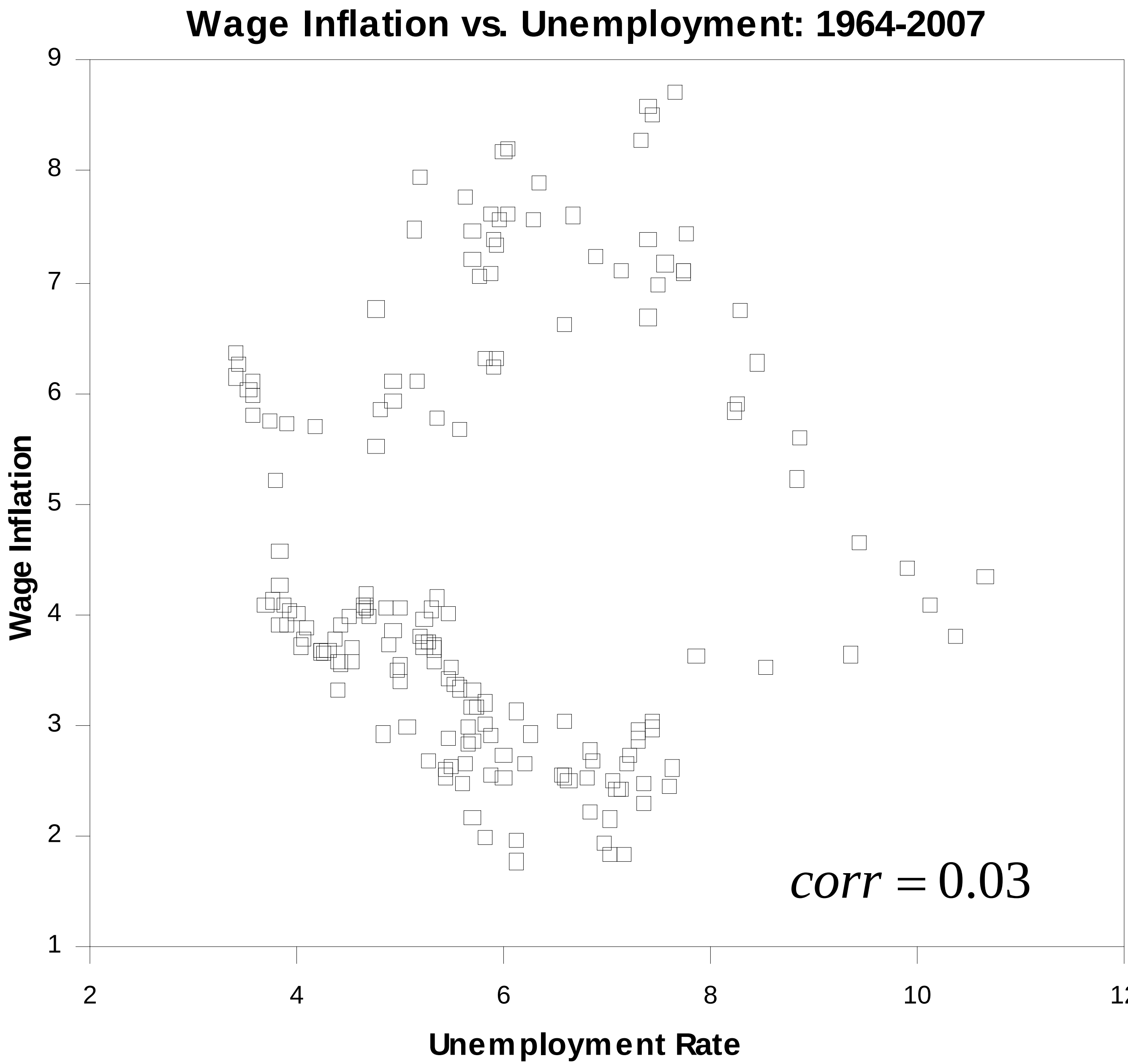
Table 1. Baseline Calibration

Parameter	*Description*	*Value*
φ	Curvature of labor disutility	5
ϵ_w	Elasticity of substitution among labor types	4.52
ϵ_p	Elasticity of substitution among goods	6
θ_p	Calvo index of price rigidities	2/3
α	Index of decrasing returns to labor	1/3
ϕ_p	Inflation coefficient in policy rule	1.5
ϕ_y	Output coefficient in policy rule	0.125
β	Discount factor	0.99

Table 2. Properties of Unemployment in the NK Model

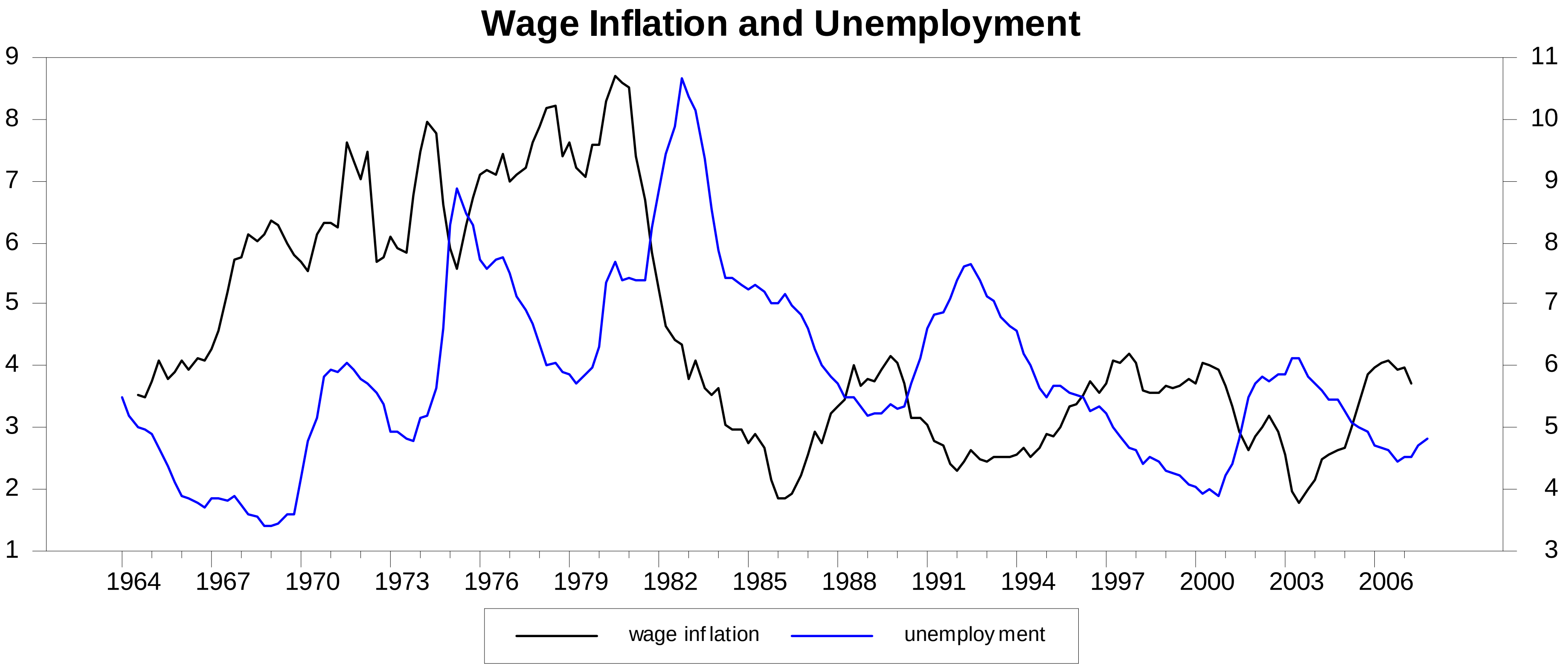
	Monetary Shocks					
	Relative Volatility			Persistence		
	$\theta_w = 0.1$	$\theta_w = 0.5$	$\theta_w = 0.8$	$\theta_w = 0.1$	$\theta_w = 0.5$	$\theta_w = 0.8$
$\rho_v = 0.0$	1.43	1.67	1.71	-0.16	-0.03	-0.01
$\rho_v = 0.5$	1.24	1.64	1.70	0.21	0.42	0.48
$\rho_v = 0.9$	0.81	1.50	1.68	0.45	0.76	0.87
	Technology Shocks					
	Relative Volatility			Persistence		
	$\theta_w = 0.1$	$\theta_w = 0.5$	$\theta_w = 0.8$	$\theta_w = 0.1$	$\theta_w = 0.5$	$\theta_w = 0.8$
$\rho_a = 0.0$	2.93	5.38	6.46	-0.15	-0.03	0.01
$\rho_a = 0.5$	0.72	1.77	2.68	0.22	0.43	0.52
$\rho_a = 0.9$	0.04	0.13	0.24	0.41	0.67	0.87

Figure 1



Wage Inflation vs. Unemployment: 1964-2007
Wage Inflation
Unemployment Rate
1
2
3
4
5
6
7
8
9
2
4
6
8
10
12
$corr = 0.03$

Figure 2



Wage Inflation and Unemployment
9
8
7
6
5
4
3
2
1
11
10
9
8
7
6
5
4
3
1964
1967
1970
1973
1976
1979
1982
1985
1988
1991
1994
1997
2000
2003
2006
wage inflation
unemployment

Figure 3

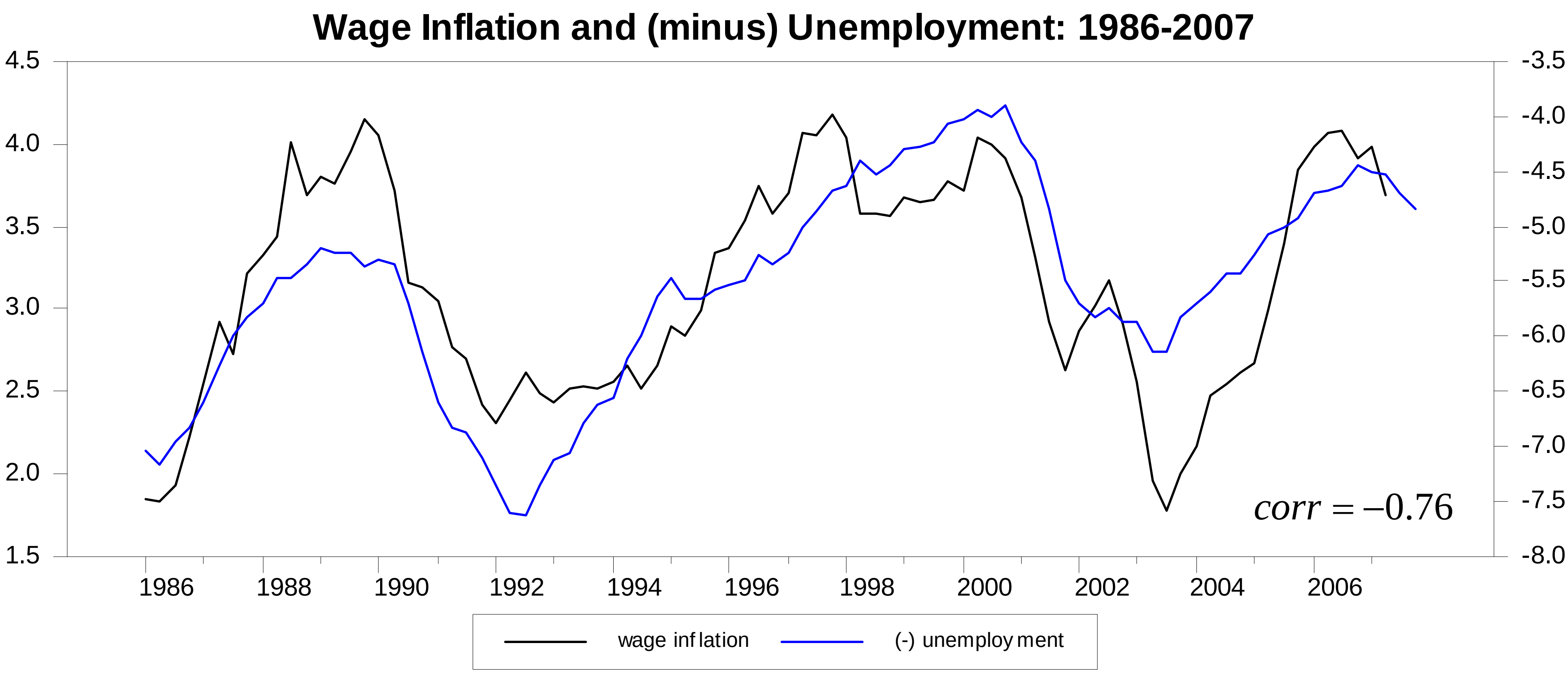

Wage Inflation and (minus) Unemployment: 1986-2007
4.5
4.0
3.5
3.0
2.5
2.0
1.5
-3.5
-4.0
-4.5
-5.0
-5.5
-6.0
-6.5
-7.0
-7.5
-8.0
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
$corr = -0.76$
wage inflation
(-) unemployment

Figure 4

Wage Inflation and Unemployment over Time

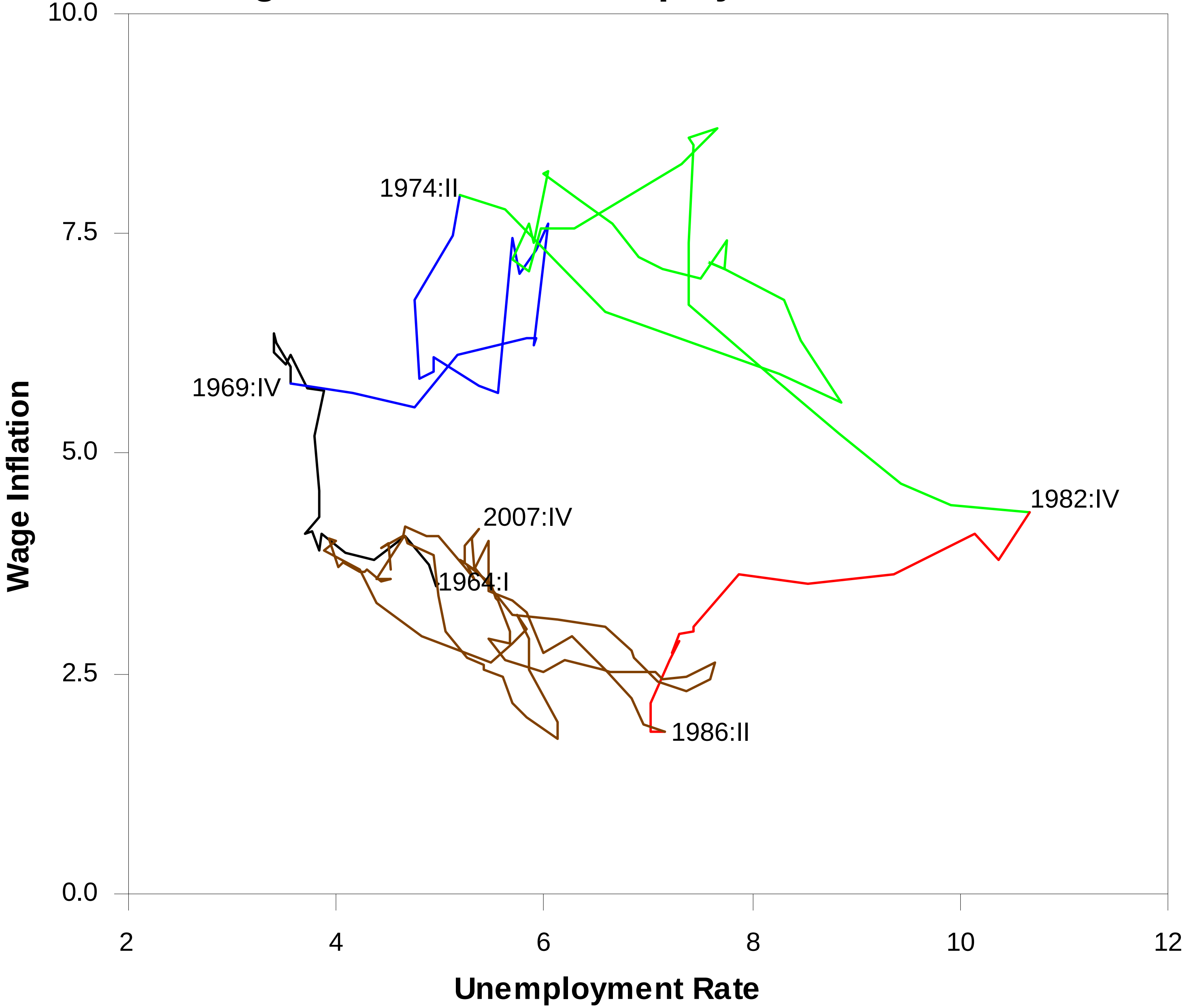
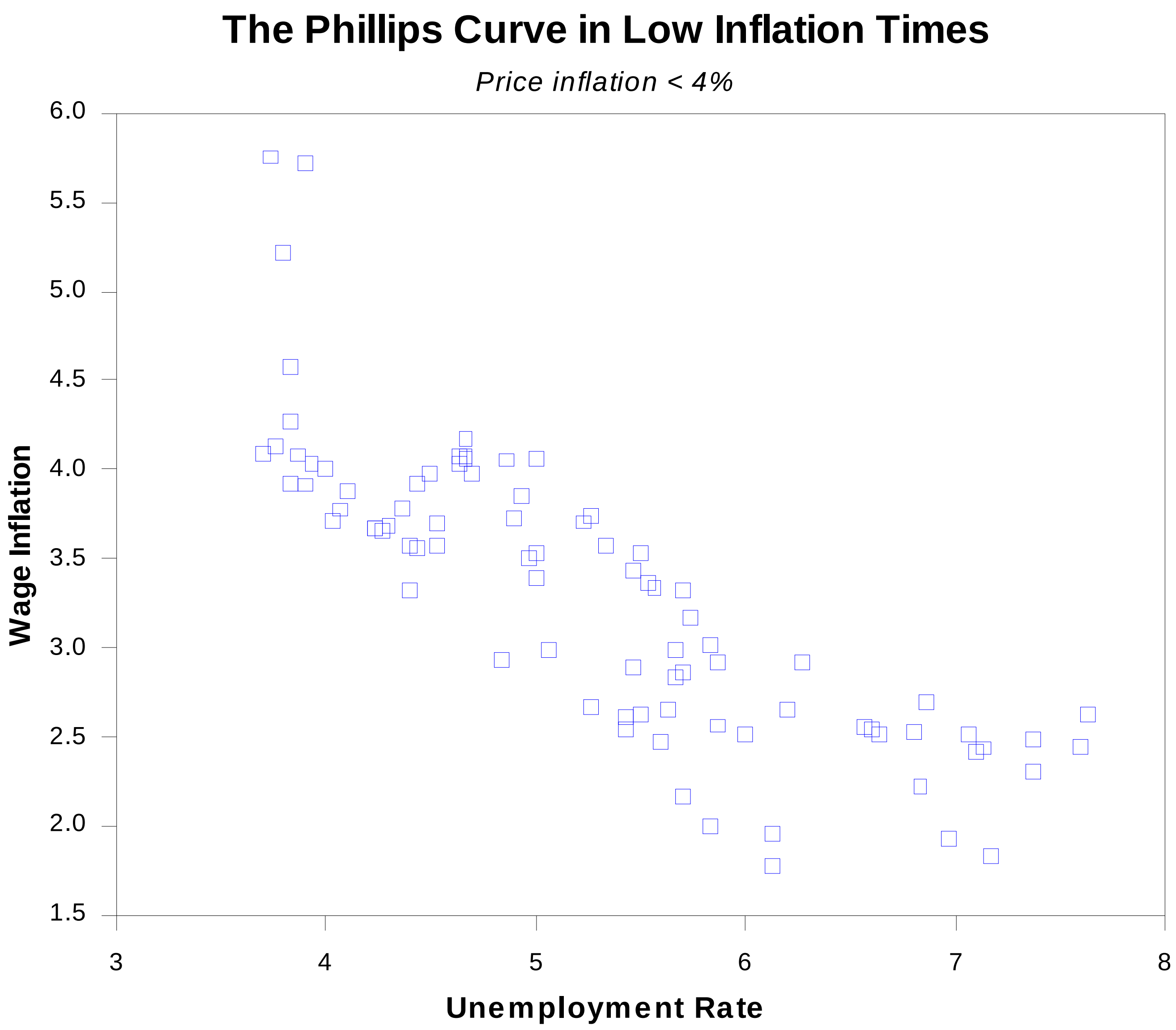
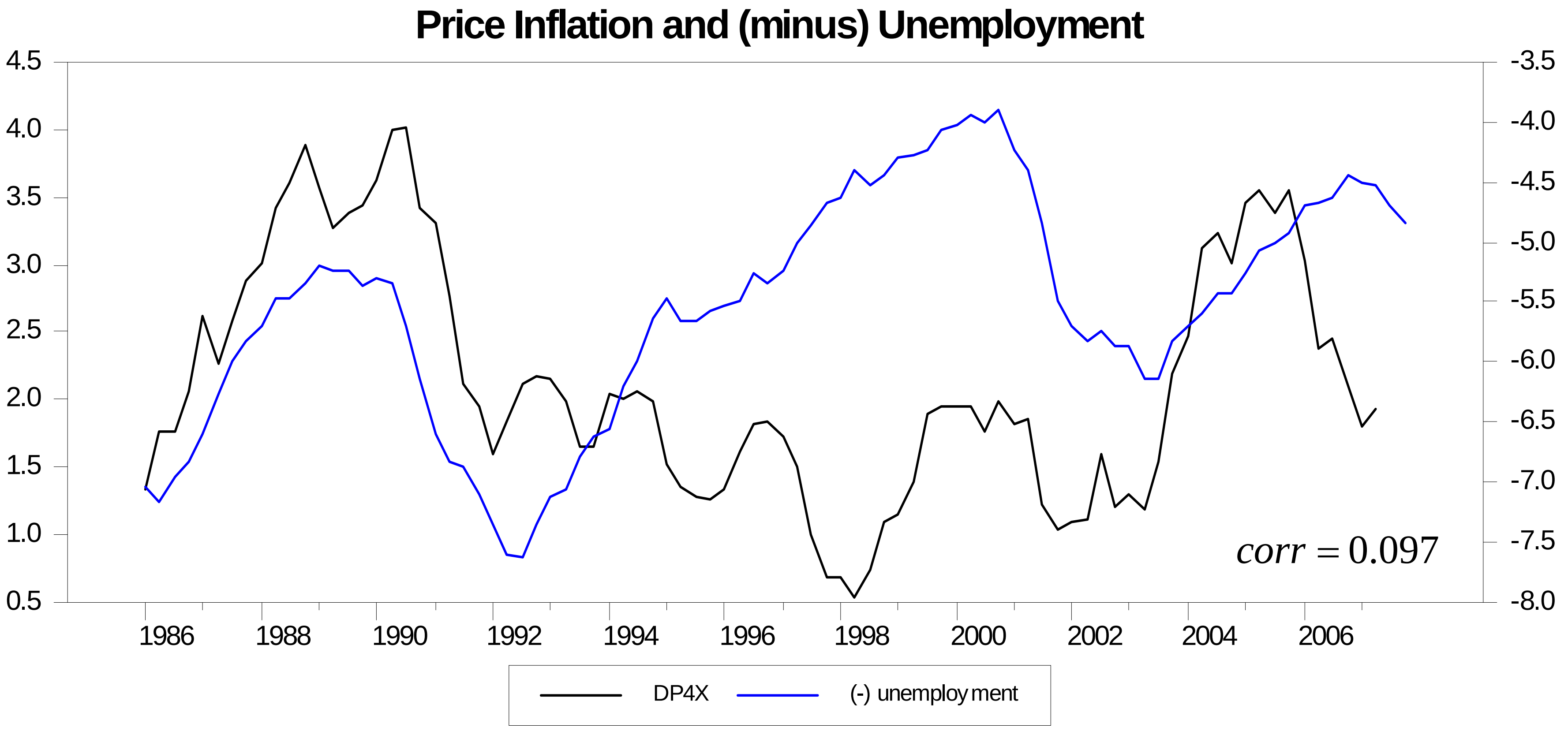


Figure 5



The Phillips Curve in Low Inflation Times
Price inflation < 4%
Wage Inflation
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
3
4
5
6
7
8
Unemployment Rate

Figure 6



Price Inflation and (minus) Unemployment
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
-3.5
-4.0
-4.5
-5.0
-5.5
-6.0
-6.5
-7.0
-7.5
-8.0
1986
1988
1990
1992
1994
1996
1998
2000
2002
2004
2006
$corr = 0.097$
DP4X
(-) unemploy ment

Figure 7

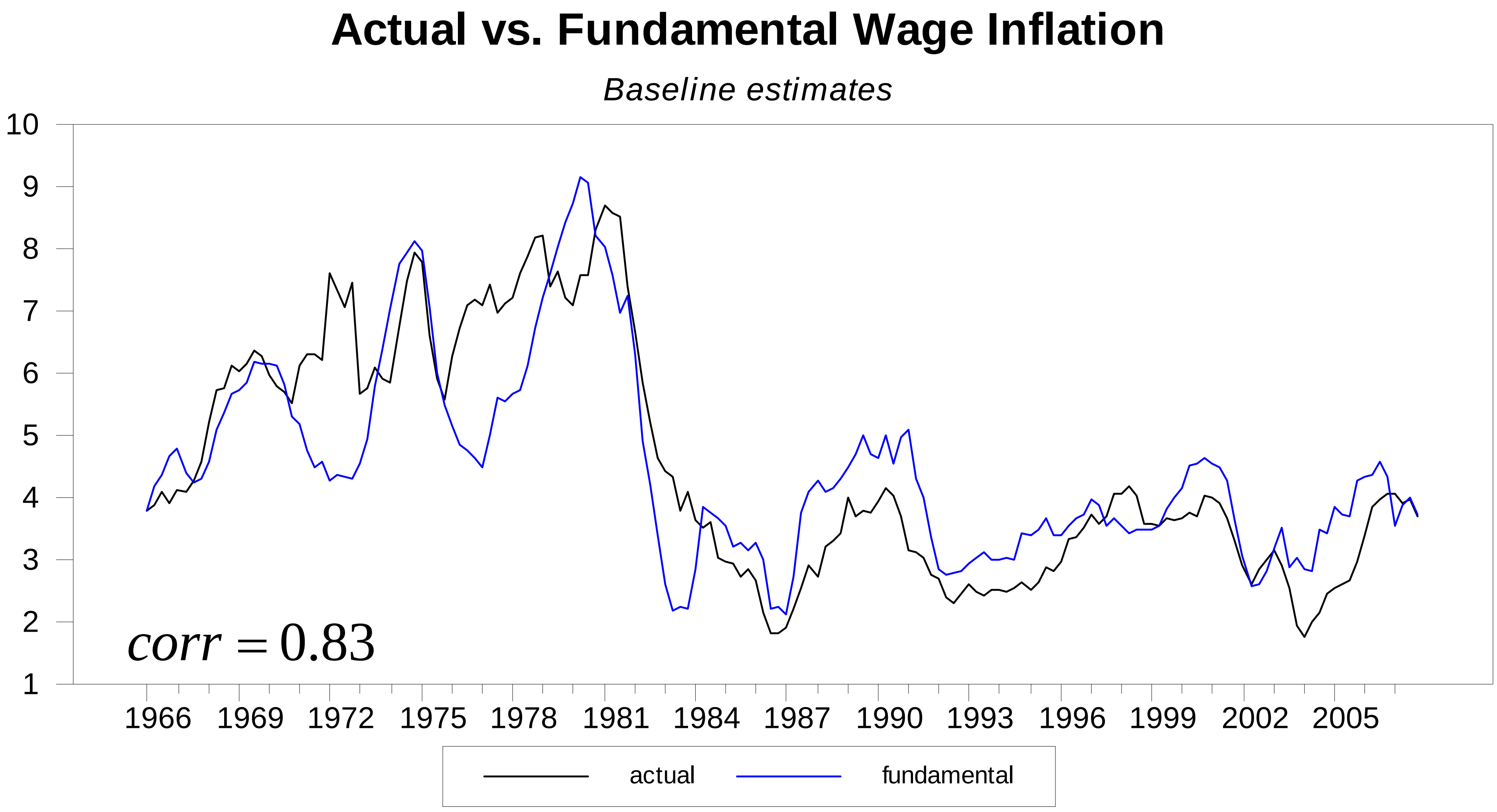

Actual vs. Fundamental Wage Inflation
Baseline estimates
10
9
8
7
6
5
4
3
2
1
corr = 0.83
1966
1969
1972
1975
1978
1981
1984
1987
1990
1993
1996
1999
2002
2005
actual
fundamental

Figure 8

Conditional Estimates of Calvo Parameter

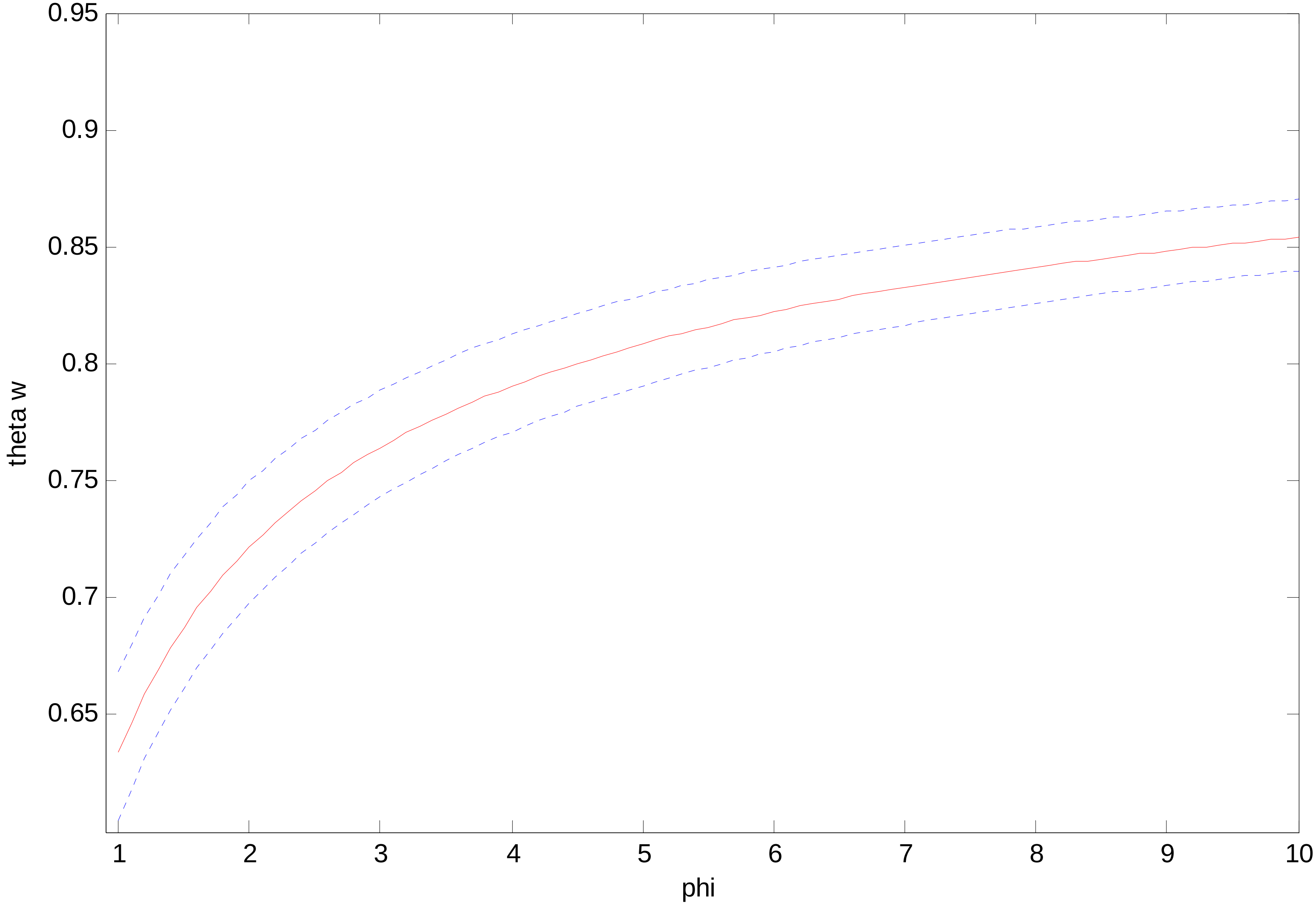
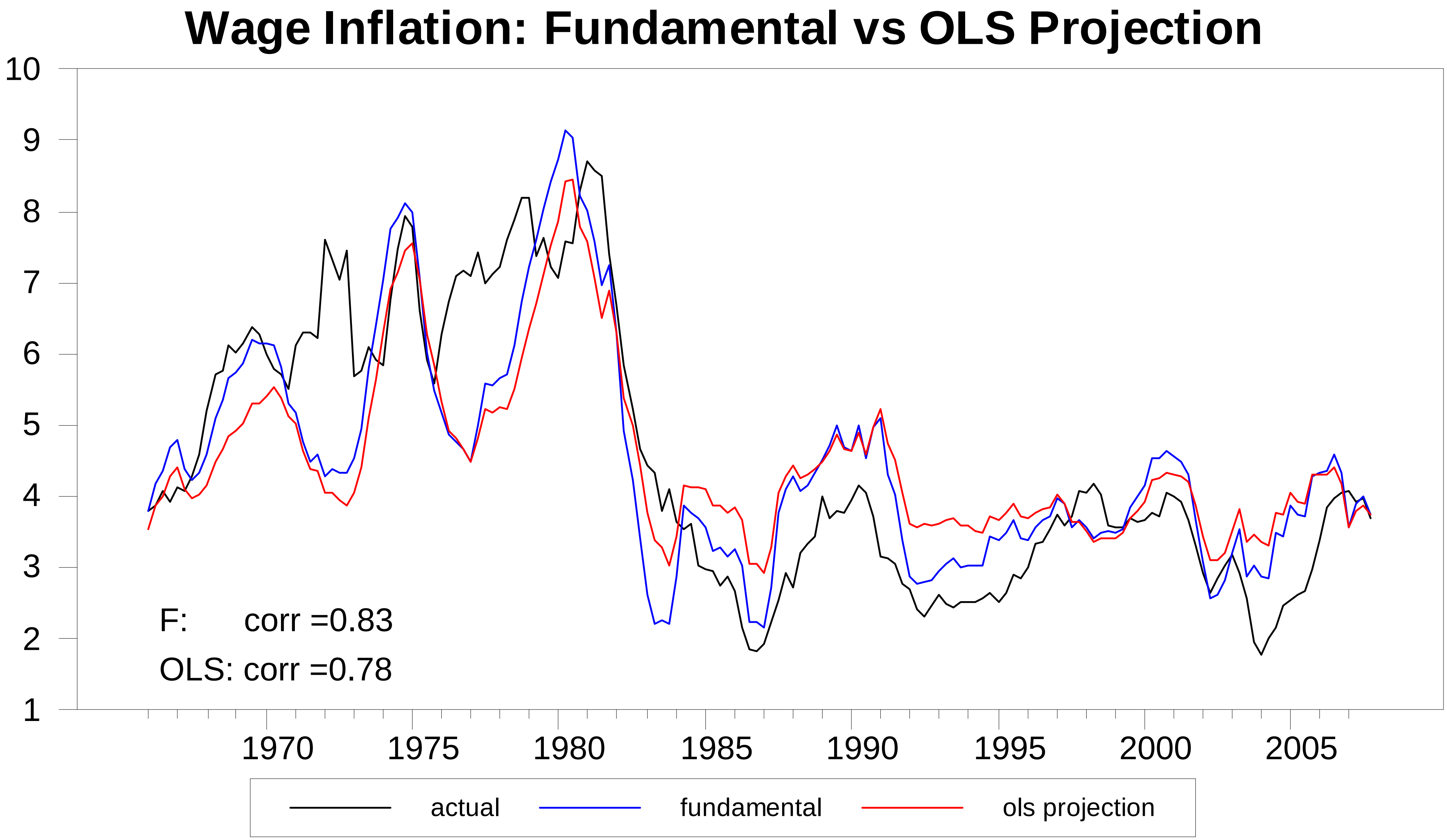


Figure 9



Wage Inflation: Fundamental vs OLS Projection
10
9
8
7
6
5
4
3
2
1
1970
1975
1980
1985
1990
1995
2000
2005
F: corr =0.83
OLS: corr =0.78
actual
fundamental
ols projection