1 Endogenous Technical Change

Endogenous growth theory provides the tools for understanding sustained productivity growth due to technical change. Productivity increases through innovation which is motivated by the prospect of the monopoly rents it generates. These models will be a starting point to study why productivity differs across countries.

1.1 Benchmark model (Romer, 1990)

We study a simplified version of Romer (1990) with no physical capital. See Gancia and Zilibotti (2005). Barro and Sala-I-Martin (2003) also provide an excellent textbook treatment.

Horizontal innovation = introduction of new product variety that does not displace existing varieties. More varieties, in turn translates into higher productivity.

Households

L infinitely lived agents, inelastic labor supply. Consumption path is set to maximize utility:

$$\begin{aligned} \max U &= \int_0^\infty e^{-\rho t} \frac{c_t^{1-\theta} - 1}{1-\theta} dt \\ st &: \dot{b}_t = r_t b_t + w_t - c_t \\ & \quad \text{No-Ponzi condition} \end{aligned}$$

where b is bond holdings, w the wage and r the interest rate. Current value Hamiltonian:

$$H = \frac{c_t^{1-\theta} - 1}{1-\theta} + \mu_t \left[r_t b_t + w_t - c_t \right]$$

FOCs:

$$\begin{aligned} H_c &= 0 \rightarrow c_t^{-\theta} = \mu_t \quad \text{log-differentiate} \quad \frac{\dot{c}_t}{c_t} = -\frac{1}{\theta} \frac{\dot{\mu}_t}{\mu_t} \\ H_b &= -\dot{\mu}_t + \rho \mu_t \rightarrow \frac{\dot{\mu}_t}{\mu_t} = -\left(r_r - \rho \right) \\ & \quad \text{Transversality condition} \end{aligned}$$

Solving yields a standard Euler equation for consumption growth:

$$\frac{\dot{c}_t}{c_t} = \frac{r_t - \rho}{\theta}$$

The production side has two sectors: a competitive sector producing a homogenous final good, and a non-competitive sector producing differentiated intermediate goods.

Final Good Sector (competitive) - FGS

Employs labor and intermediate goods as inputs. Production function:

$$Y_t = L_{y,t}^{1-\alpha} \int_0^{A_t} x_{j,t}^{\alpha} \, dj,$$

where x_j is the quantity of the intermediate good j, A_t is the measure of intermediate goods available at t, L_y is labor and $\alpha \in (0, 1)$.

Note: different inputs are imperfect substitutes and enter symmetrically the production function. No intermediate good is intrinsically better or worse than any other. Also, if $x_{j,t} = K/A_t$ productivity grows with A_t.

Demand for labor and intermediates are found from the profit maximization program of the representative firm:

$$\max \pi_{Y,t} = L_{y,t}^{1-\alpha} \int_0^{A_t} {x_{j,t}}^{\alpha} dj - w_t L_{y,t} - \int_0^{A_t} p_{j,t} x_{j,t} dj.$$

where p_j = price of variety j and the price of Y is one. FOCs:

$$\begin{aligned} \frac{\partial \pi_{Y,t}}{\partial x_{j,t}} &= 0 \to p_{j,t} = \alpha L_{y,t}^{1-\alpha} \left(x_{j,t}\right)^{\alpha-1} \qquad \forall j \\ \frac{\partial \pi_{Y,t}}{\partial L_{y,t}} &= 0 \to w_{y,t} = (1-\alpha) L_{y,t}^{-\alpha} \int_0^{A_t} {x_{j,t}}^{\alpha} dj \end{aligned}$$

Note: demand for $x_{j,t}$ has a constant price elasticity of $\epsilon = \frac{1}{\alpha - 1}$.

Intermediate Good Sector (monopolistic) - IGS

Each existing firm holds a patent to produce a single variety j. Technology: one unit of intermediate good requires one unit of final good. Prices are set to maximize profits subject to (isoelastic) demand for $x_{j,t}$. Monopoly pricing:

$$p_{j,t}\left(1-\frac{1}{|\epsilon|}\right)=\text{MC}\rightarrow p_{j,t}=p_t=\frac{1}{\alpha}$$

where ϵ is the price elasticity of demand. Substituting into demand, we find the quantity:

$$x_{j,t}=x_t=\alpha^{\frac{2}{1-\alpha}}L_{y,t}$$

Hence, profit is:

$$\pi_{j,t}=\pi_t=(p-1)x_t=\frac{1-\alpha}{\alpha}\alpha^{\frac{2}{1-\alpha}}L_{y,t}.$$

Substitution of x_t in the demand for labor yields the wage:

$$w_t=(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}A_t.$$

Innovation - R&D

Designing a new variety requires a sunk cost of $1/\delta A_t$ units of labor. Thus, the cost of innovation is:

$$\frac{w_t}{\delta A_t}$$

Note the knowledge spillover: productivity in R&D increases with the stock of "knowledge", A_t (i.e., the number of known varieties). The idea is that researchers benefit from past discoveries, obtaining inspiration for new designs ("standing on the shoulders of giants"). The law of motion of A_t follows:

$$\dot{A}_t=\delta A_t L_{RD,t}$$

The rate of technological change is a linear function of total employment in R&D.

Balanced Growth Equilibrium

Guess-and-verify the existence of a balanced growth (BG) equilibrium where c_t, Y_t, w_t and A_t grow at the constant rate, γ. Note that in BG, production and the profits of intermediate firms are constant over time and across industries, $x_t=x$ and $\pi_t=\pi$. By the Euler equation, the interest rate is also constant in BG.

Free entry

For an interior solution with positive growth, the present discounted value (PDV) of profits from innovation has to be equal to the sunk cost of entry:

$$\text{value of innovation } = \frac{\pi}{r} = \frac{w_t}{\delta A_t} = \text{ cost of innovation}$$

Note: both sides are constant in BG due to the knowledge externality. Without it, the cost of innovation would grow over time and technical progress would stop like in the neoclassical model.

Note the role of patents: in the absence of intellectual property rights, free-riding would prevent any innovative activity. If firms could copy, competition would drive ex-post rents to zero. Then, no firms would have an incentive, ex-ante, to pay a sunk cost to design a new input.

The growth rate can be found solving the system:

$$\begin{aligned} \text{Free entry} &: \frac{\pi}{r} = \frac{w_t}{\delta A_t} \\ \text{Euler equation} &: \gamma = \frac{r-\rho}{\theta} \\ \text{Full employment} &: L_y = L - \frac{\gamma}{\delta} \end{aligned}$$

Using π and w_t, and solving:

$$\gamma = \frac{\delta \alpha L - \rho}{\alpha + \theta}$$

Note that an interior solution exists if and only if $\alpha\delta L > \rho$. The growth rate is increasing in the productivity of the research sector (δ), the size of the labor force (L) and the intertemporal elasticity of substitution of consumption ($1/\theta$), while it is decreasing in the elasticity of final output to labor, $(1-\alpha)$, and the discount rate.

The decentralized equilibrium is inefficient (and growth sub-optimally low) for two reasons:

1. Monopoly pricing, higher than marginal cost → underproduction of each variety of intermediates.

2. Ideas produce externalities: innovating firms compare the private cost of innova-

tion, $w_t/(\delta A_t)$, with the present discounted value of profits, π/r and ignore the spillover on the future productivity of innovation.

1.2 "Lab-Equipment" version (Romer & Rivera Batiz, 1991)

In the "lab-equipment" model research uses final output instead of labor as a productive input. One innovation requires μ units of Y:

$$\begin{aligned} \text{Free entry} &: \frac{\pi}{r} = \mu \\ \text{Euler equation} &: \gamma = \frac{r-\rho}{\theta} \\ \text{Full employment} &: L_y = L \end{aligned}$$

Together with π and w_t, this yields:

$$\gamma = \frac{1}{\theta}\left[(1-\alpha)\,\alpha^{\frac{1+\alpha}{1-\alpha}}\frac{L}{\mu} - \rho\right]$$

Also, $\dot{A}_t = Y_x/\mu$, where $Y_x =$ units of final output devoted to R&D (hence, consumption is $C = Y - Ax - Y_x$). Note that there is no research spillover. Sustained growth is attained by allocating a constant share of production to finance the research activity.

1.3 Vertical Innovation (Aghion & Howitt, 1992)

Vertical innovation = higher **quality** variety that **replace** an existing variety. Key new feature: innovation generates obsolescence of previous innovations (creative destruction).

Preferences: agents are risk neutral ($\theta = 0$) $\rightarrow$ the Euler equation requires: $r = \rho$.

Technology:

$$\begin{aligned} \text{FGS} &: Y(i) = A(i)\,x(i)^{\alpha} \\ \text{IGS} &: x(i) = L_x \\ \text{R\&D} &: \Pr(innov) = \lambda L_{RD} \rightarrow A(i) = \gamma A(i-1) \end{aligned}$$

Where $i \in \mathbb{N}^+$ indexes the latest generation of innovation. Focus on BG where L_{RD} is constant. Solution:

$$\begin{aligned} \text{FGS (competitive)} &: \max \pi_Y \rightarrow p(i) = \alpha A(i)\, x(i)^{\alpha-1} \\ \text{IGS (monopoly)} &: p(i) = \frac{w(i)}{\alpha} \rightarrow \pi(i) = \left(\frac{1}{\alpha} - 1\right) w(i)\, x(i) \end{aligned}$$

Define $V(i+1)$ the PDV of the next innovation. Arbitrage equation for R&D:

$$MB = \lambda V(i+1) = \frac{\lambda \pi(i+1)}{r + \lambda L_{RD}} = w(i) = MC$$

Substitute $w(i) = \alpha p(i) = \alpha^2 A(i)\, x(i)^{\alpha-1}$, $\pi(i+1) = \left(\frac{1}{\alpha} - 1\right) w(i+1)\, x(i+1)$ and $x(i) = L - L_{RD}$:

$$\frac{\lambda \left(\frac{1}{\alpha} - 1\right) A(i+1)(L - L_{RD})}{r + \lambda L_{RD}} = A(i)$$

Using $A(i+1) = \gamma A(i)$:

$$\frac{\frac{1-\alpha}{\alpha}\gamma L - \frac{r}{\lambda}}{1 + \frac{1-\alpha}{\alpha}\gamma} = L_{RD}$$

Investment in R&D is increasing in γ, λ, L and decreasing in α and r. Growth, of course, is proportional to L_{RD}.

1.4 Convergence and endogenous growth (Barro & Sala-I-Martin, 1997, Howitt, 2000, Nelson & Phepls, 1966)

Endogenous growth models usually do not predict convergence, while conditional convergence is a common feature in developing countries. Yet, these models can easily be modified to account for it. Consider the lab equipment version of the model with horizontal innovation and assume that innovation (imitation?) is easier when a country is far from the technology frontier:

$$\text{Cost of innovation} = \mu \left(\frac{A}{A^*}\right)^{\xi}$$

where A^* = world technology frontier. Assume that A^* grows at the growth rate γ^*, determined by a set of advanced countries. The economy will converge to a BG equilibrium where A also grows at the rate γ^*. In this BG equilibrium the cost of innovation is constant and the analysis is as before. Also, r has to be equalized across countries, i.e., $r = r^*$. This implies:

$$\frac{\pi^*}{\mu^*} = r^* = r = \frac{\pi}{\mu}\left(\frac{A}{A^*}\right)^{-\xi}$$

Thus, in BG the equilibrium distance to frontier, $\frac{A}{A^*}$, is constant and pin down by:

$$\frac{A}{A^*} = \left(\frac{\mu^*}{\pi^*}\frac{\pi}{\mu}\right)^{1/\xi}$$

More "productive" economies - in terms of the exogenous parameters - will have a higher $\frac{\pi}{\mu}$ and will get closer to the world technology frontier. In other words, the model predicts **conditional** convergence.