

The Classical Monetary Model

Jordi Galí

CREI, UPF and Barcelona GSE

January 2019

Assumptions

- Perfect competition in goods and labor markets
- Flexible prices and wages
- Representative household
- Money in the utility function
- No capital accumulation
- No fiscal sector
- Closed economy

- *Preferences*

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t, L_t; Z_t)$$

where $L_t \equiv M_t / P_t$

- *Budget constraint*

$$P_t C_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + W_t N_t + D_t - T_t$$

with solvency constraint:

$$\lim_{T \rightarrow \infty} E_t \{ \Lambda_{t,T} (\mathcal{A}_T / P_T) \} \geq 0$$

where $Q_t \equiv \exp\{-i_t\}$ and $\mathcal{A}_t \equiv B_t + M_t$.

- *Optimality Conditions*

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

$$Q_t = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\}$$

$$\frac{U_{l,t}}{U_{c,t}} = 1 - Q_t$$

- Interpretation: $1 - Q_t = 1 - \exp\{-i_t\} \simeq i_t$

⇒ opportunity cost of holding money

- *Assumption:*

$$U(C_t, N_t, L_t) = \begin{cases} \left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} + \chi \frac{L_t^{1-\sigma}-1}{1-\sigma} \right) Z_t & \text{for } \sigma \neq 1 \\ \left(\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} + \chi \log L_t \right) Z_t & \text{for } \sigma = 1 \end{cases}$$

where $z_t \equiv \log Z_t$ follows the exogenous process $z_t = \rho_z z_{t-1} + \varepsilon_t^z$

- *Remark:* separable real balances assumed
- Implied optimality conditions

$$\frac{W_t}{P_t} = C_t^\sigma N_t^\varphi$$

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Z_{t+1}}{Z_t} \right) \left(\frac{P_t}{P_{t+1}} \right) \right\}$$

$$L_t = \chi^{\frac{1}{\sigma}} C_t (1 - \exp\{-i_t\})^{-\frac{1}{\sigma}}$$

- Log-linearized versions (around zero growth steady state):

$$w_t - p_t = \sigma c_t + \varphi n_t$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + \frac{1}{\sigma}(1 - \rho_z)z_t$$

$$l_t = c_t - \eta i_t + \zeta$$

where $\pi_t \equiv p_t - p_{t-1}$, $\beta \equiv \exp\{-\rho\}$, and $\eta \equiv \frac{1}{\sigma(\exp\{i\}-1)}$.

- *Technology*

$$Y_t = A_t N_t^{1-\alpha} \quad (1)$$

where $a_t \equiv \log A_t$ follows an exogenous process

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

- *Profit maximization:*

$$\max P_t Y_t - W_t N_t$$

subject to (1), taking the price and wage as given

- *Optimality condition:*

$$\frac{W_t}{P_t} = (1 - \alpha) A_t N_t^{-\alpha}$$

- *Log linear version*

$$w_t - p_t = a_t - \alpha n_t + \log(1 - \alpha)$$

- Government budget constraint

$$P_t G_t + B_{t-1}^G = T_t + Q_t B_t^G + \Delta M_t^S$$

- Fiscal policy: rule determining $\{G_t, B_t^G, T_t\}$
- Monetary policy: rule determining $\{M_t^S, i_t\}$

Equilibrium: Market Clearing Conditions

- *Goods market clearing*

$$Y_t = C_t + G_t$$

- *Labor market clearing*

$$(1 - \alpha)A_t N_t^{-\alpha} = \frac{W_t}{P_t} = C_t^\sigma N_t^\varphi$$

- *Asset market clearing*

$$B_t = B_t^G$$

$$M_t = M_t^S$$

- Equilibrium values for real variables (assuming $G_t = 0$)

$$n_t = \psi_{na} a_t + \psi_n$$

$$y_t = \psi_{ya} a_t + \psi_y$$

$$r_t \equiv i_t - E_t\{\pi_{t+1}\} = \rho - \sigma\psi_{ya}(1 - \rho_a)a_t + (1 - \rho_z)z_t$$

$$\omega_t \equiv w_t - p_t = \psi_{\omega a} a_t + \psi_\omega$$

where $\psi_{na} \equiv \frac{1-\sigma}{\sigma(1-\alpha)+\varphi+\alpha}$; $\psi_n \equiv \frac{\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$; $\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$;
 $\psi_y \equiv \frac{(1-\alpha)\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$; $\psi_{\omega a} \equiv \frac{\sigma+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$; $\psi_\omega \equiv -\frac{\alpha\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$

- *Two neutrality results:*

(i) non-fiscal variables independent of $\{B_t^G, T_t\}$ (Ricardian equivalence)

(ii) real variables *independent of monetary policy*

- Monetary policy needed to determine *nominal* variables

- **Example I: A Simple Interest Rate Rule**

$$i_t = \rho + \pi + \phi_\pi(\pi_t - \pi) + v_t$$

where $\phi_\pi \geq 0$. Combined with definition of real rate:

$$\phi_\pi \hat{\pi}_t = E_t\{\hat{\pi}_{t+1}\} + \hat{r}_t - v_t$$

Case I: $\phi_\pi > 1$

$$\begin{aligned}\hat{\pi}_t &= \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} E_t\{\hat{r}_{t+k} - v_{t+k}\} \\ &= -\frac{\sigma\psi_{ya}(1-\rho_a)}{\phi_\pi - \rho_a} a_t + \frac{1-\rho_z}{\phi_\pi - \rho_z} z_t - \frac{1}{\phi_\pi - \rho_v} v_t\end{aligned}$$

\implies *nominal determinacy*

Price Level Determination

Case II: $\phi_\pi < 1$

$$\hat{\pi}_t = \phi_\pi \hat{\pi}_{t-1} - \hat{r}_{t-1} + v_{t-1} + \zeta_t$$

for any $\{\zeta_t\}$ sequence with $E_t\{\zeta_{t+1}\} = 0$ for all t

\Rightarrow *nominal indeterminacy*

\Rightarrow illustration of "Taylor principle" requirement

Price Level Determination

- *Responses to a monetary policy shock ($\phi_\pi > 1$ case):*

$$\frac{\partial \pi_t}{\partial \varepsilon_t^v} = -\frac{1}{\phi_\pi - \rho_v} < 0$$

$$\frac{\partial i_t}{\partial \varepsilon_t^v} = -\frac{\rho_v}{\phi_\pi - \rho_v} < 0$$

$$\frac{\partial m_t}{\partial \varepsilon_t^v} = \frac{\eta \rho_v - 1}{\phi_\pi - \rho_v} \leq 0$$

$$\frac{\partial y_t}{\partial \varepsilon_t^v} = \frac{\partial r_t}{\partial \varepsilon_t^v} = 0$$

- *Discussion:* liquidity effect and other comovements.

- **Example II: An Exogenous Path for the Money Supply** $\{m_t\}$

Combining money demand and the definition of the real rate:

$$p_t = \left(\frac{\eta}{1 + \eta} \right) E_t \{ p_{t+1} \} + \left(\frac{1}{1 + \eta} \right) m_t + u_t$$

where $u_t \equiv (1 + \eta)^{-1}(\eta r_t - y_t)$. Solving forward:

$$p_t = \frac{1}{1 + \eta} \sum_{k=0}^{\infty} \left(\frac{\eta}{1 + \eta} \right)^k E_t \{ m_{t+k} \} + \bar{u}_t$$

where $\bar{u}_t \equiv \sum_{k=0}^{\infty} \left(\frac{\eta}{1 + \eta} \right)^k E_t \{ u_{t+k} \}$

\Rightarrow *price level determinacy*

Price Level Determination

In terms of money growth rates:

$$p_t = m_t + \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k E_t \{ \Delta m_{t+k} \} + \bar{u}_t$$

Nominal interest rate:

$$\begin{aligned} i_t &= \eta^{-1} (y_t - (m_t - p_t)) \\ &= \eta^{-1} \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k E_t \{ \Delta m_{t+k} \} + \underline{u}_t \end{aligned}$$

where $\underline{u}_t \equiv \eta^{-1}(\bar{u}_t + y_t)$ is independent of monetary policy.

Price Level Determination

Assumption

$$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m$$

$$r_t = y_t = 0$$

Price response:

$$p_t = m_t + \frac{\eta \rho_m}{1 + \eta(1 - \rho_m)} \Delta m_t$$

\Rightarrow *large price response*

Nominal interest rate response:

$$i_t = \frac{\rho_m}{1 + \eta(1 - \rho_m)} \Delta m_t$$

\Rightarrow *no liquidity effect*

The Case of Non-Separable Real Balances

- Labor supply affected by monetary policy \Rightarrow non-neutrality
- Example:

$$U(X_t, N_t) = \frac{X_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

where

$$\begin{aligned} X_t &\equiv [(1-\vartheta)C_t^{1-\nu} + \vartheta L_t^{1-\nu}]^{\frac{1}{1-\nu}} && \text{for } \nu \neq 1 \\ &\equiv C_t^{1-\vartheta} L_t^{\vartheta} && \text{for } \nu = 1 \end{aligned}$$

implying:

$$U_{c,t} = X_t^{\nu-\sigma} C_t^{-\nu} (1-\vartheta)$$

$$U_{l,t} = X_t^{\nu-\sigma} L_t^{-\nu} \vartheta$$

$$U_{n,t} = N_t^{\varphi}$$

Remark: $U_{cl,t} > 0 \Leftrightarrow \nu > \sigma$

The Case of Non-Separable Real Balances

- Optimality conditions:

$$\frac{W_t}{P_t} = N_t^\varphi X_t^{\sigma-\nu} C_t^\nu (1-\vartheta)^{-1}$$

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\nu} \left(\frac{X_{t+1}}{X_t} \right)^{\nu-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right\}$$

$$L_t = C_t (1 - \exp\{-i_t\})^{-\frac{1}{\nu}} \left(\frac{\vartheta}{1-\vartheta} \right)^{\frac{1}{\nu}}$$

- Log-linearized money demand equation:

$$l_t = c_t - \eta i_t + \zeta$$

where $\eta \equiv 1/[\nu(\exp\{i\} - 1)]$

The Case of Non-Separable Real Balances

- Log-linearized labor supply equation (ignoring constants):

$$\begin{aligned}w_t - p_t &= \sigma c_t + \varphi n_t - (\nu - \sigma)(x_t - c_t) \\ &= \sigma c_t + \varphi n_t - \chi(\nu - \sigma)(l_t - c_t) \\ &= \sigma c_t + \varphi n_t + \eta\chi(\nu - \sigma)i_t\end{aligned}$$

where $\chi \equiv \frac{\theta k_m^{1-\nu}}{1-\theta+\theta k_m^{1-\nu}}$ with $k_m \equiv \frac{L}{C}$. In a zero inflation steady state,

$$k_m \equiv \frac{L}{C} = \left(\frac{\vartheta}{(1-\beta)(1-\vartheta)} \right)^{\frac{1}{\nu}}, \text{ implying } \chi = \frac{k_m(1-\beta)}{1+k_m(1-\beta)}. \text{ In addition,}$$
$$\eta = \frac{\beta}{\nu(1-\beta)}.$$

Accordingly,

$$w_t - p_t = \sigma c_t + \varphi n_t + \omega i_t$$

$$\text{where } \omega \equiv \eta\chi(\nu - \sigma) = \frac{k_m\beta(1-\frac{\sigma}{\nu})}{1+k_m(1-\beta)} \leq 0.$$

The Case of Non-Separable Real Balances

- Labor market clearing:

which combined with aggregate production function:

$$y_t = \psi_{ya} a_t + \psi_{yi} i_t$$

where $\psi_{yi} \equiv -\frac{\omega(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$ and $\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$

\Rightarrow long run non-superneutrality

- *Assessment of size of short-run non-neutralities*

Calibration: $\beta = 0.99$; $\sigma = 1$; $\varphi = 5$; $\alpha = 1/4$

$$v = \frac{\beta}{\eta(1-\beta)} > \sigma \Rightarrow \psi_{yi} < 0$$

$$\Rightarrow \omega \simeq k_m > 0 \quad ; \quad \psi_{yi} \simeq -\frac{k_m}{8} < 0$$

Monetary base inverse velocity: $k_m \simeq 0.3 \quad \Rightarrow \psi_{yi} \simeq -0.04$

M2 inverse velocity: $k_m \simeq 3 \quad \Rightarrow \psi_{yi} \simeq -0.4$

\Rightarrow small output effects of monetary policy

Optimal Monetary Policy

- *Social Planner's problem*

$$\max U(C_t, N_t, L_t; Z_t)$$

subject to

$$C_t = A_t N_t^{1-\alpha}$$

Optimality conditions:

$$-\frac{U_{n,t}}{U_{c,t}} = (1 - \alpha) A_t N_t^{-\alpha}$$

$$U_{l,t} = 0$$

Optimal policy (Friedman rule): $i_t = 0$ for all t

Intuition

Implied average inflation: $\pi = -\rho < 0$

- *Implementation*

$$i_t = \phi(r_{t-1} + \pi_t)$$

for some $\phi > 1$. Combined with the definition of the real rate:

$$E_t\{i_{t+1}\} = \phi i_t$$

whose only stationary solution is $i_t = 0$ for all t .

Implied equilibrium inflation:

$$\pi_t = -r_{t-1}$$