The Basic New Keynesian Model

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Introduction

Evidence on Monetary Policy, Output, and Prices:

- Short run effects of monetary policy shocks

  (i) persistent effects on real variables
  (ii) slow adjustment of aggregate price level
  (iii) liquidity effect

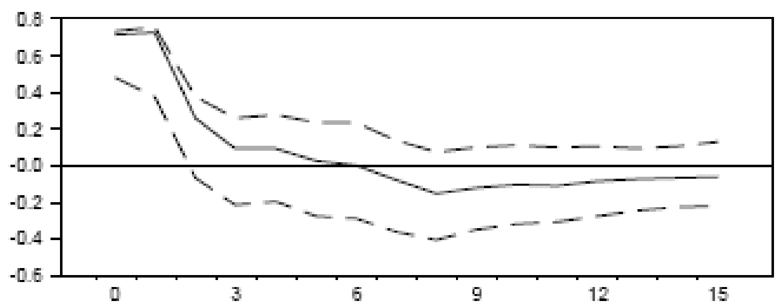
- Micro evidence on price and wage-setting behavior: significant rigidities

Failure of Classical Monetary Models

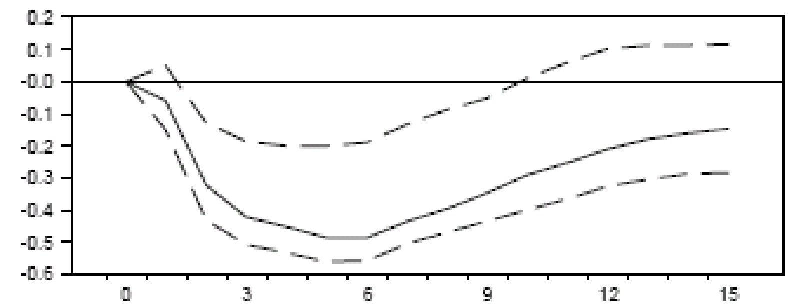
Key ingredients of the NK Model

- monopolistic competition
- nominal rigidities

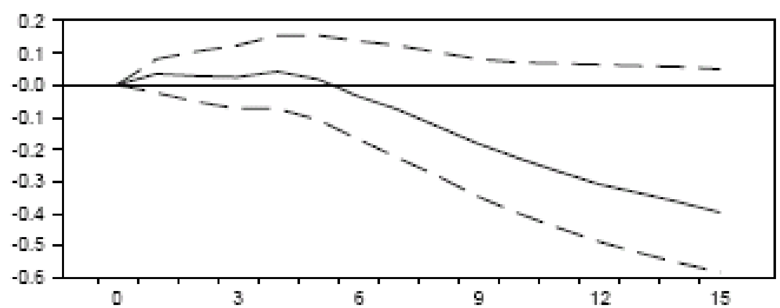
Figure 1. Estimated Dynamic Response to a Monetary Policy Shock



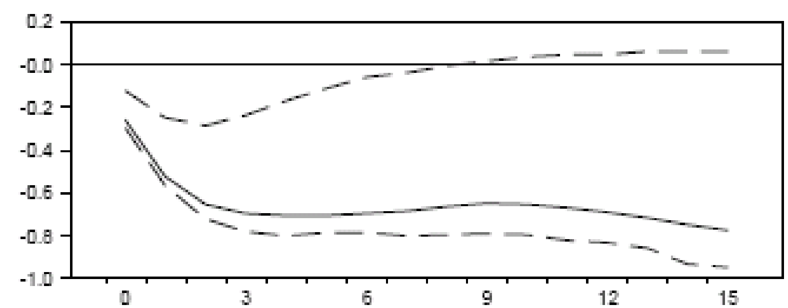
Federal funds rate



GDP



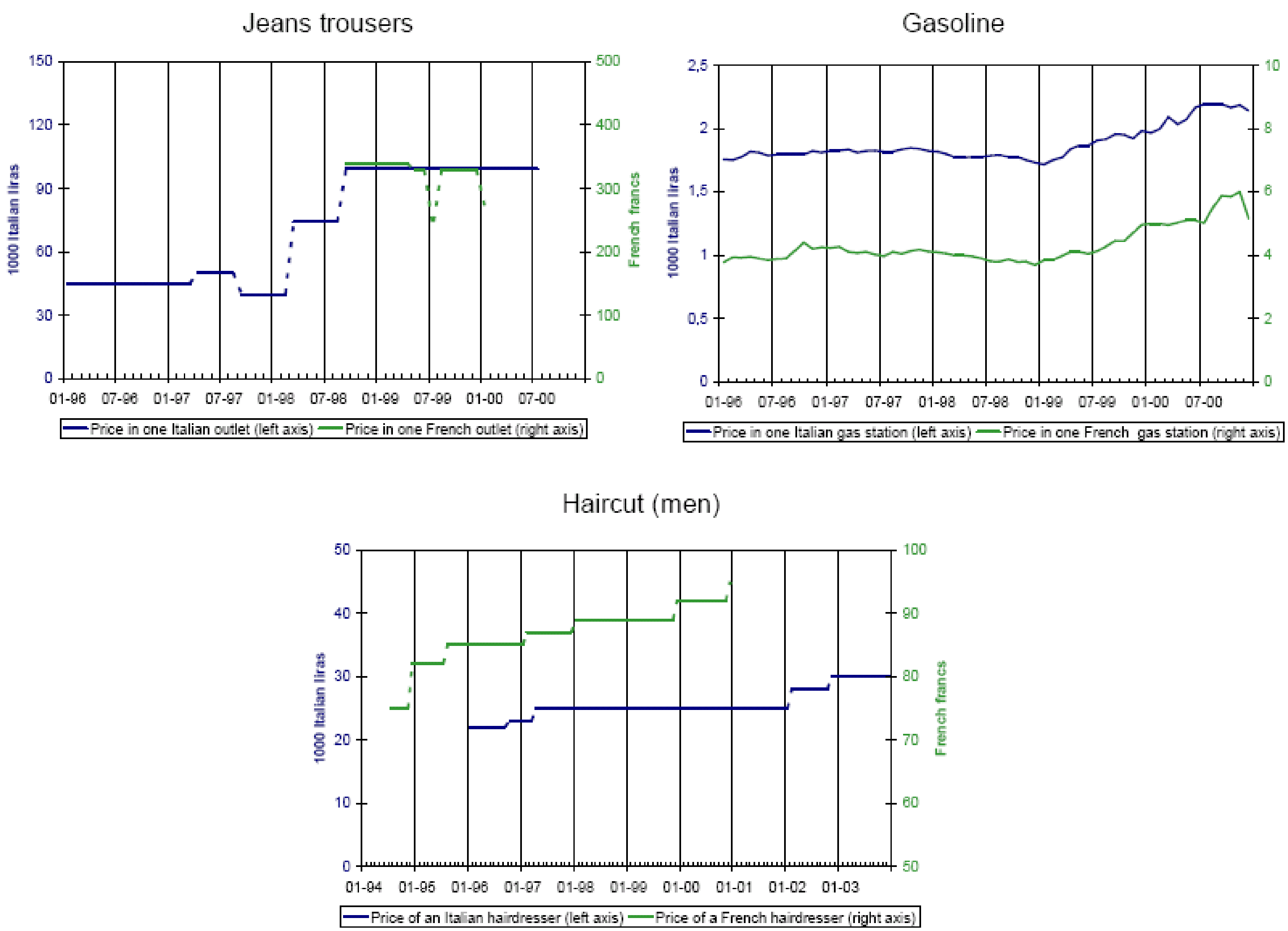
GDP deflator



M2

Source: Christiano, Eichenbaum and Evans (1999)

Figure 1 - Examples of individual price trajectories (French and Italian CPI data)



Note : Actual examples of trajectories, extracted from the French and Italian CPI databases. The databases are described in Baudry *et al.* (2004) and Veronese *et al.* (2005). Prices are in levels, denominated in French Francs and Italian Lira respectively. The dotted lines indicate events of price changes.

Source: Dhyne et al. (JEP, 2006)

Table 1. Measures of price stickiness in the euro area and the US (% per month unless otherwise stated).

	Statistics	Euro area	US
CPI*	Frequency	15.1	24.8
	Average duration (*months*)	13.0	6.7
	Median duration (*months*)	10.6	4.6
PPI†	Frequency	20.0	n.a
Surveys‡	Frequency	15.9	20.8
	Average duration (*months*)	10.8	8.3
NKPC§	Average durations (*months*)	13.5–19.2	7.2–8.4
Internet prices¶	Frequency	79.2	64.3

Source: Álvarez et al. (JEEA, 2006)

The Basic New Keynesian Model: Key Blocks

- *Assumptions:*

  - monopolistic competition in the goods market
  - staggered price setting
  - perfectly competitive labor market

- *New Keynesian Phillips Curve*

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

where $\pi_t \equiv p_t - p_{t-1}$ and $\widetilde{y}_t \equiv y_t - y_t^n$

- *Dynamic IS Equation*

$$\widetilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n) + E_t\{\widetilde{y}_{t+1}\}$$

- *Monetary Policy Rule*

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t + v_t$$

where $\widehat{y}_t \equiv y_t - y$

Households

- *Preferences*

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t, L_t; Z_t\right)$$

where $L_t \equiv M_t / P_t$

- *Budget constraint*

$$P_t C_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + W_t N_t + D_t - T_t$$

with solvency constraint:

$$\lim_{T \to \infty} E_t \left\{ \Lambda_{t,T} (\mathcal{A}_T / P_T) \right\} \geq 0$$

where $Q_t \equiv \exp\{-i_t\}$ and $\mathcal{A}_t \equiv B_t + M_t$.

Households

- *Optimality Conditions*

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

$$Q_t = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\}$$

$$\frac{U_{l,t}}{U_{c,t}} = 1 - Q_t$$

- Interpretation: $1 - Q_t = 1 - \exp\{-i_t\} \simeq i_t$

$\Rightarrow$ opportunity cost of holding money

Households

- *Assumption*:

$$U(C_t, N_t, L_t) = \begin{cases} \left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} + \chi \frac{L_t^{1-\sigma}-1}{1-\sigma} \right) Z_t & \text{for } \sigma \neq 1 \\ \left(\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} + \chi \log L_t \right) Z_t & \text{for } \sigma = 1 \end{cases}$$

- *Remark*: *separable* real balances assumed
- Implied optimality conditions

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi}$$

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Z_{t+1}}{Z_t} \right) \left(\frac{P_t}{P_{t+1}} \right) \right\}$$

$$L_t = \chi^{\frac{1}{\sigma}} C_t \, (1 - \exp\{-i_t\})^{-\frac{1}{\sigma}}$$

Households

- Log-linearized versions (around zero growth steady state):

$$w_t - p_t = \sigma c_t + \varphi n_t$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + \frac{1}{\sigma}(1 - \rho_z) z_t$$

$$l_t = c_t - \eta i_t + \varsigma$$

where $\pi_t \equiv p_t - p_{t-1}$, $\beta \equiv \exp\{-\rho\}$, and $z_t \equiv \log Z_t$ is assumed to follow the exogenous process

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z$$

Firms: Final Goods

- Representative, perfectly competitive firm
- Technology

$$Y_t = \left[\int_0^1 X_t(i)^{1-\frac{1}{\epsilon}} di\right]^{\frac{\epsilon}{\epsilon-1}} \tag{1}$$

- Profit maximization

$$\max_{\{X_t(i)\}} P_t Y_t - \int_0^1 P_t(i) X_t(i) di$$

  subject to (1).
- Optimality conditions:

$$X_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_t$$

- Implication:

$$P_t = \left[\int_0^1 P_t(i)^{1-\epsilon} di\right]^{\frac{1}{1-\epsilon}}$$

Firms: Intermediate Goods

- Continuum of firms, indexed by $i \in [0,1]$
- Each firm produces a differentiated intermediate good
- Identical technology

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

where $a_t \equiv \log A_t = \rho_a a_{t-1} + \varepsilon_t^a$

- Isoelastic demand $\Rightarrow$ constant desired markup $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$ under flexible prices
- Probability of being able to reset price in any given period: $1-\theta$, independent across firms (Calvo (1983))
- $\theta \in [0,1]$: index of price stickiness
- Implied average price duration $\frac{1}{1-\theta}$

The New Keynesian Phillips Curve

- *Price level dynamics*

$$P_t = \left[\theta (P_{t-1})^{1-\epsilon} + (1-\theta)(P_t^*)^{1-\epsilon}\right]^{\frac{1}{1-\epsilon}}$$

Log-linearization (around zero inflation):

$$p_t = \theta p_{t-1} + (1-\theta) p_t^*$$

- *Optimal Price Setting*

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} (1/P_{t+k}) \left(P_t^* Y_{t+k|t} - \mathcal{C}_{t+k}(Y_{t+k|t}) \right) \right\}$$

subject to

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} Y_{t+k}$$

for $k = 0, 1, 2, \ldots$where $\Lambda_{t,t+k} \equiv \beta^k U_{c,t+k} / U_{c,t}$

- *Optimal Price Setting* (cont.)

Optimality condition:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} Y_{t+k|t} (1/P_{t+k}) \left(P_t^* - \mathcal{M} \Psi_{t+k|t} \right) \right\} = 0$$

Log-linearized version:

$$p_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t \{ \psi_{t+k|t} \}$$

Remark: if $\alpha = 0$, then $\psi_{t|t-k} = \psi_t$ for $k = 0, 1, 2, ...$

The New Keynesian Phillips Curve

- *Inflation and the Markup Gap*

$$\pi_t = \beta E_t\{\pi_{t+1}\} - \lambda(\mu_t - \mu)$$

where $\mu \equiv \log \mathcal{M}$, $\mu_t \equiv p_t - \psi_t$, $\psi_t \equiv w_t - (a_t - \alpha n_t + \log(1-\alpha))$ and $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$

- *Aggregate Employment*

$$\begin{aligned} N_t &= \int_0^1 N_t(i) di \\ &= (Y_t/A_t)^{\frac{1}{1-\alpha}} \; \Delta_{p,t} \end{aligned}$$

where $\Delta_{p,t} \equiv \int_0^1 (P_t(i)/P_t)^{-\frac{\epsilon}{1-\alpha}} di$. Up to first order:

$$(1-\alpha)n_t = y_t - a_t$$

- *Labor Supply*

$$w_t - p_t = \sigma c_t + \varphi n_t$$

- *Goods Market Clearing*

$$y_t = c_t$$

- *Average Markup and the Output Gap*

$$\mu_t = -\left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) y_t + \left(\frac{1+\varphi}{1-\alpha}\right) a_t + \log(1-\alpha)$$

Under flexible prices:

$$\mu = -\left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) y_t^n + \left(\frac{1+\varphi}{1-\alpha}\right) a_t + \log(1-\alpha)$$

Combining both:

$$\mu_t - \mu = -\left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) \widetilde{y}_t$$

- *The New Keynesian Phillips Curve*

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

where $\kappa \equiv \lambda \left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)$

- Properties of the NKPC::

(i) Forward-looking

$$\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k E_t\{\widetilde{y}_{t+k}\}$$

$\Rightarrow$ no role for past inflation

(ii) No tradeoff between output gap and inflation stabilization
$\Rightarrow$ "the Divine Coincidence"

(iii) Model-based vs. traditional output gap

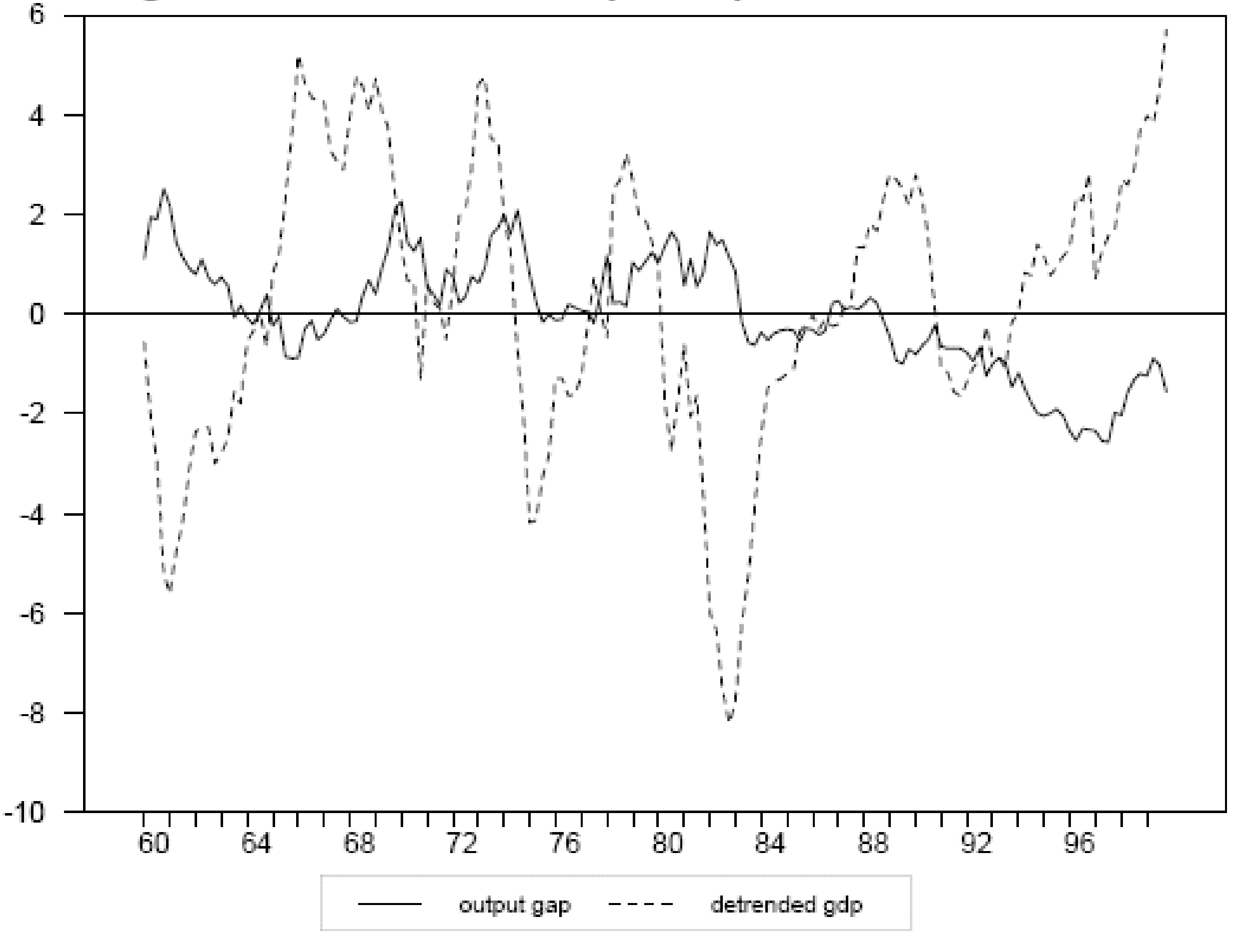
$$\widehat{y}_t = y_t - f(t)$$

$$\widetilde{y}_t \equiv y_t - y_t^n$$

$\Rightarrow$ may distort empirical assessments

Empirical evidence

Figure 2. Model-Based Output Gap vs. Detrended GDP



Source: Galí (2003)

Table 1
Structural estimates

	Parameters				Test
	θ	β	λ	D	J
Euro Area					
$\mu = 1.1$, $\alpha = 0.175$					
(1)	0.777 (0.021)	0.843 (0.046)	0.099 (0.025)	4.5 (0.09)	8.843 (0.452)
(2)	0.834 (0.032)	0.915 (0.040)	0.047 (0.022)	6.0 (0.19)	8.214 (0.513)
United Sates					
$\mu = 1.1$, $\alpha = 0.270$					
(1)	0.603 (0.051)	0.872 (0.041)	0.311 (0.106)	2.5 (0.13)	7.022 (0.534)
(2)	0.698 (0.058)	0.923 (0.029)	0.154 (0.070)	3.3 (0.19)	5.760 (0.674)

Note: Parameter α is calibrated so that $(1 - \alpha)$ equals the average labor income share times the chosen markup (μ). The average labor income shares are taken to be equal to $\frac{2}{3}$ for the US and $\frac{3}{4}$ for the Euro Area. Sample Period: 1970–1998. Column D reports the implied average price duration. J is the Hansen test statistic for the overidentifying restrictions (p-value in brackets). Instruments for Euro area estimation: inflation $t-1$ to $t-5$, output gap, labor income share and wage inflation: $t-1$ to $t-2$. Instruments for US estimation: the same excepts inflation from $t-1$ to $t-4$.

Source: Galí, Gertler, López-Salido (EER 03)

The Dynamic IS Equation

- Euler equation + goods market clearing

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + \frac{1}{\sigma}(1-\rho_z)z_t$$

Combined with $\widetilde{y}_t \equiv y_t - y_t^n$

$$\widetilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n) + E_t\{\widetilde{y}_{t+1}\}$$

where

$$\begin{aligned} r_t^n &\equiv \rho + \sigma E_t\{\Delta y_{t+1}^n\} + (1-\rho_z)z_t \\ &= \rho - \sigma\psi_{ya}(1-\rho_a)a_t + (1-\rho_z)z_t \end{aligned}$$

where $\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$

Monetary Policy

- Interest rate rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t + v_t$$

- The role of monetary aggregates: implementation

- money market clearing:

$$m_t - p_t = y_t - \eta i_t$$

- implied money supply growth:

$$\Delta m_t = \pi_t + \Delta y_t - \eta \Delta i_t$$

The Basic New Keynesian Model: Key Blocks

- *New Keynesian Phillips Curve*

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

- *Dynamic IS Equation*

$$\widetilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n) + E_t\{\widetilde{y}_{t+1}\}$$

where

$$r_t^n = \rho - \sigma\psi_{ya}(1-\rho_a)a_t + (1-\rho_z)z_t$$

- *Monetary Policy Rule*

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t + v_t$$

where $\widehat{y}_t \equiv y_t - y$

Equilibrium Dynamics

- *System of difference equations:*

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_T \; u_t$$

where

$$\begin{aligned} u_t &\equiv \widehat{r}_t^n - \phi_y \widehat{y}_t^n - v_t \\ &= -\psi_{ya}[\sigma(1-\rho_a) + \phi_y]a_t + (1-\rho_z)z_t - v_t \end{aligned}$$

$$\mathbf{A}_T \equiv \Omega \begin{bmatrix} \sigma & 1-\beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix} \quad ; \quad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

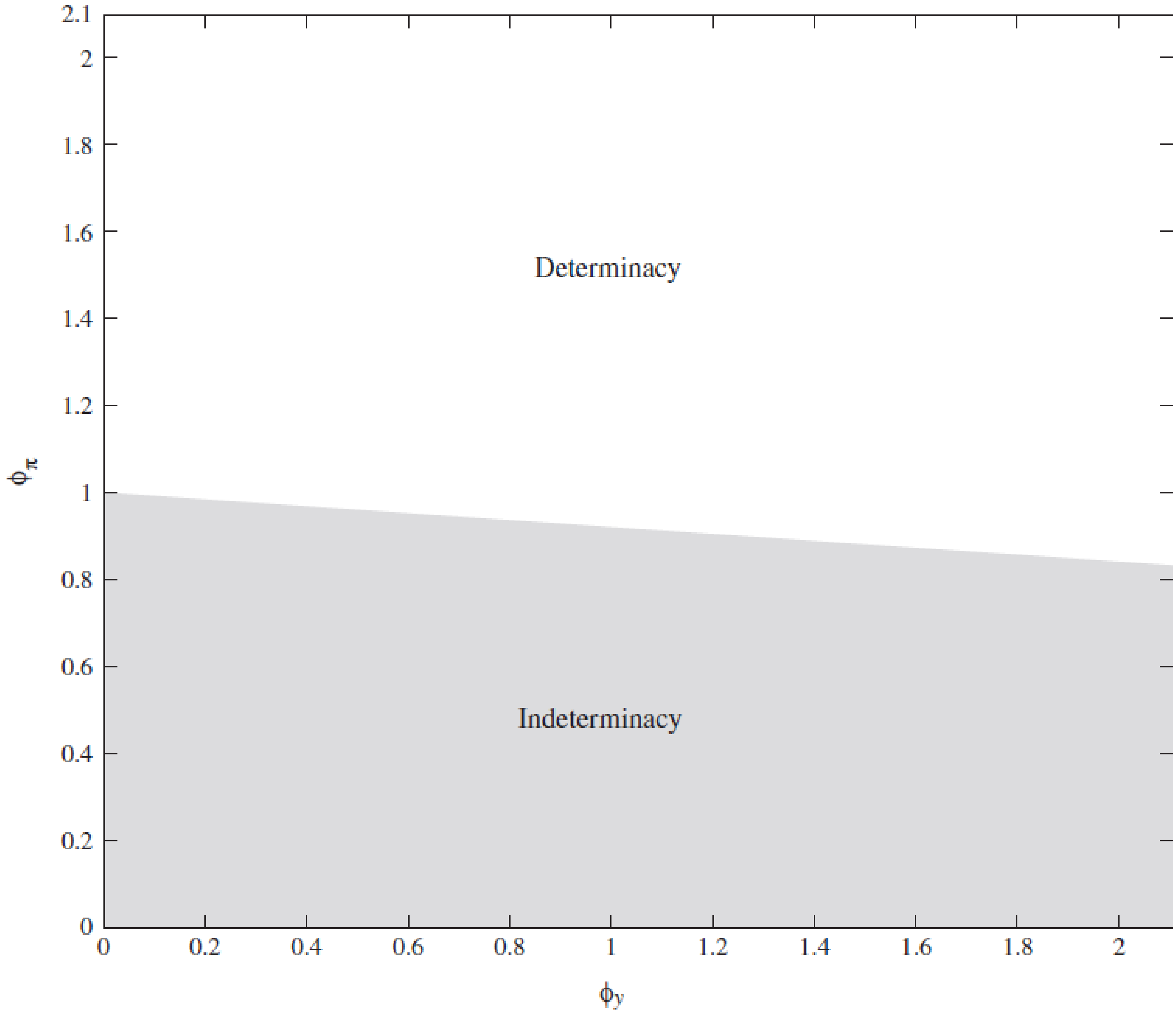
with $\Omega \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi}$

- Uniqueness condition (Bullard and Mitra):

$$\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$$

- Analytical solution (method of undetermined coefficients)

Equilibrium uniqueness under the simple interest rate rule



Equilibrium Dynamics

- Calibration (Galí (2015))

  Households: $\sigma = 1$; $\varphi = 5$; $\beta = 0.99$; $\epsilon = 9$; $\eta = 4$; $\rho_z = 0.5$

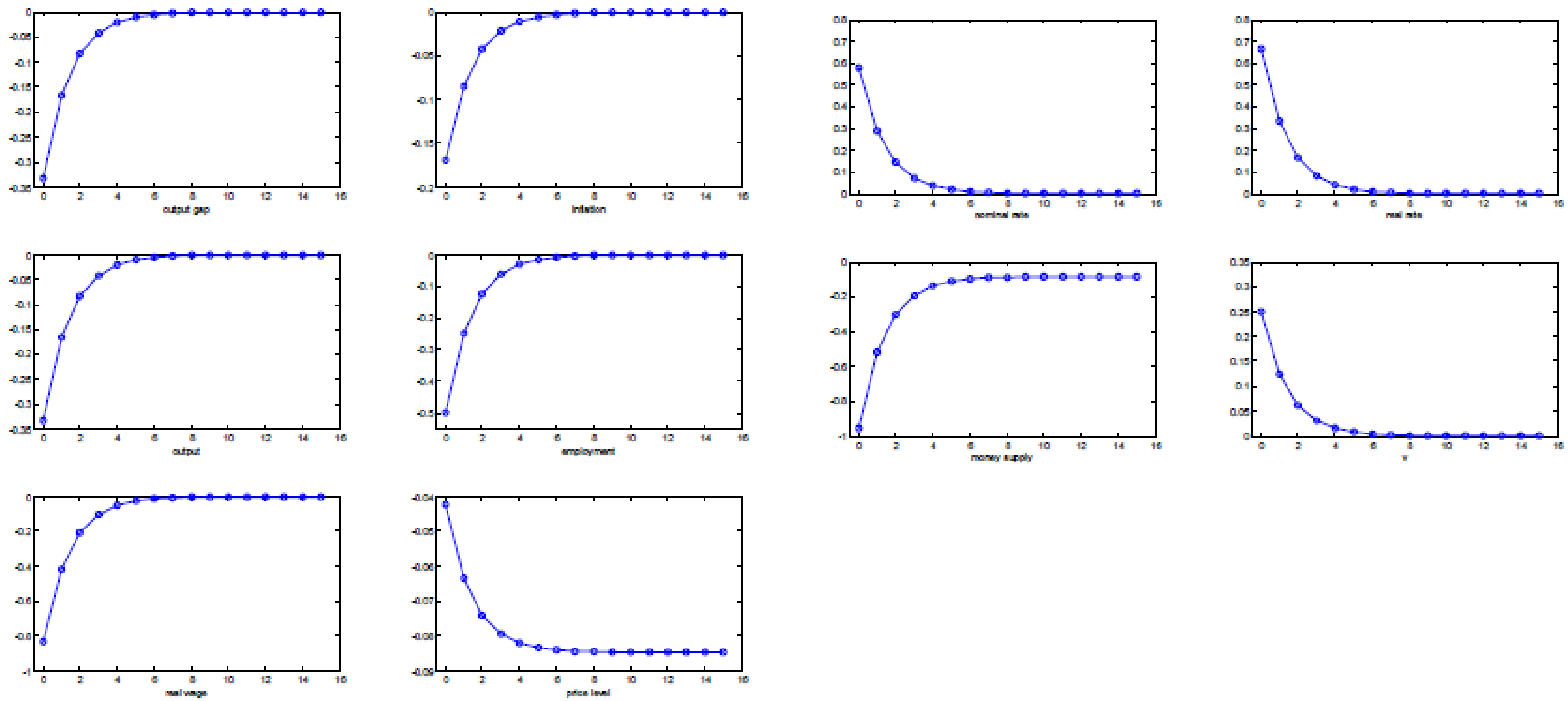
  Firms: $\alpha = 1/4$; $\theta = 3/4$; $\rho_a = 0.9$

  Policy rules: $\phi_\pi = 1.5$, $\phi_y = 0.125$; $\rho_v = 0.5$

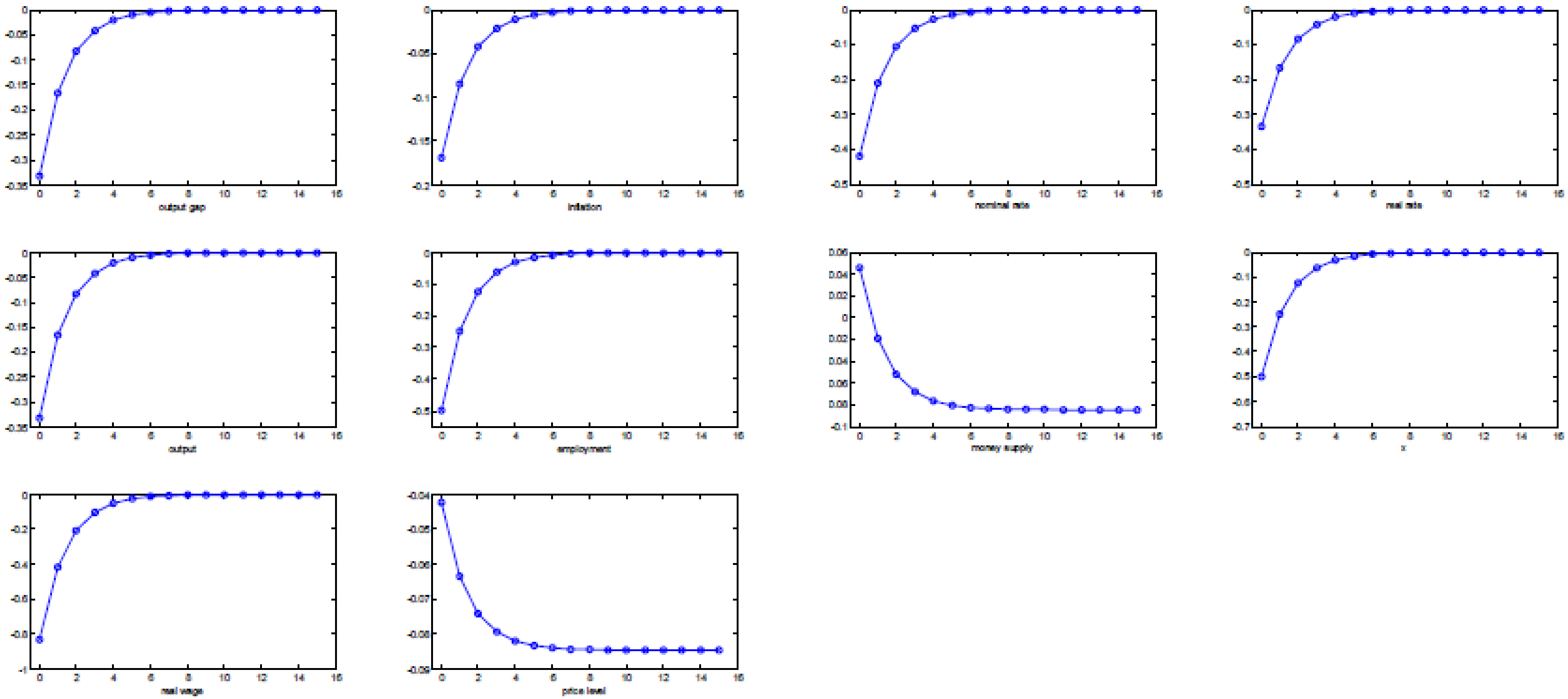
- Dynamic Responses to Exogenous Shocks

  (i) Monetary policy
  (ii) Discount rate
  (iii) Technology

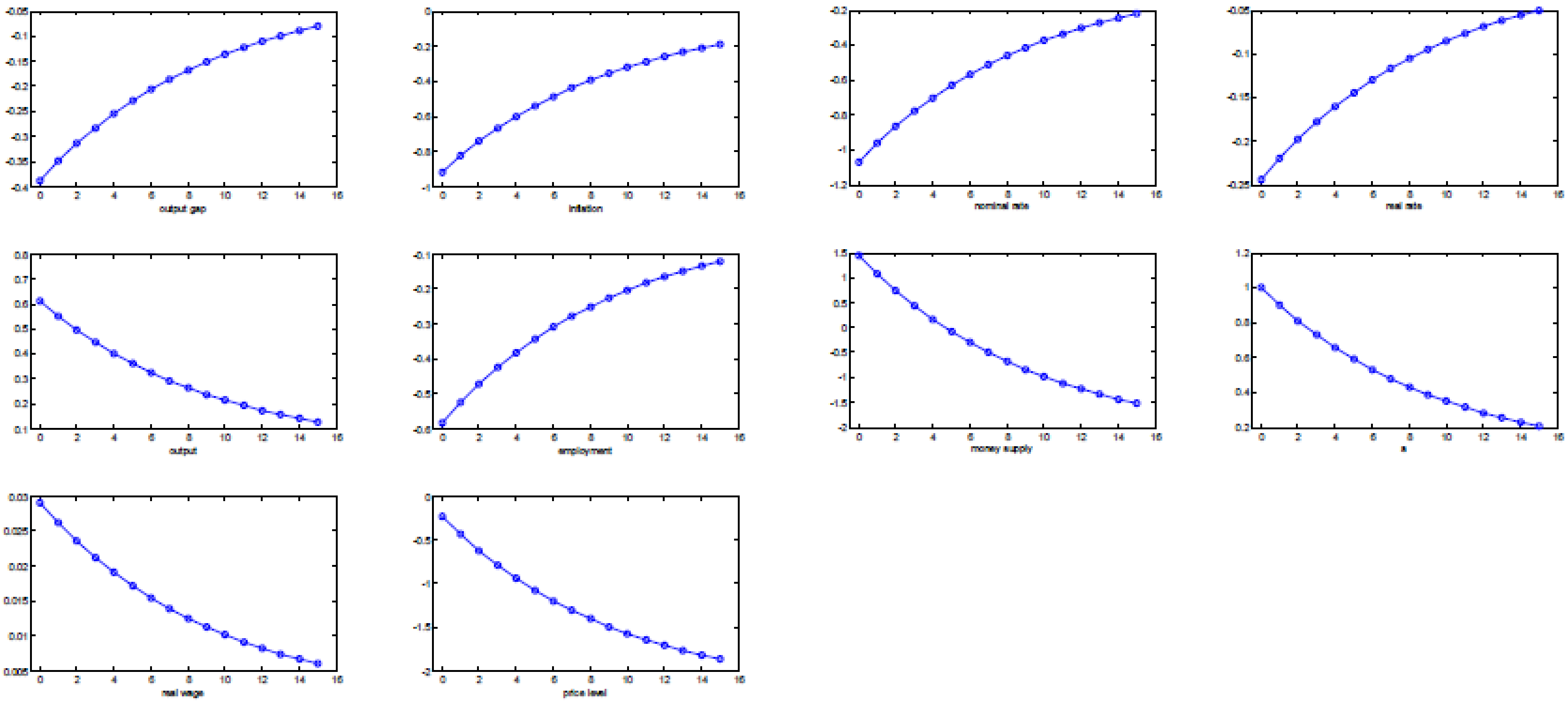
Dynamic responses to a monetary policy shock: Interest rate rule



Dynamic responses to a discount rate shock: Interest rate rule



Dynamic responses to a technology shock: Interest rate rule



Estimated Effects of Technology Shocks

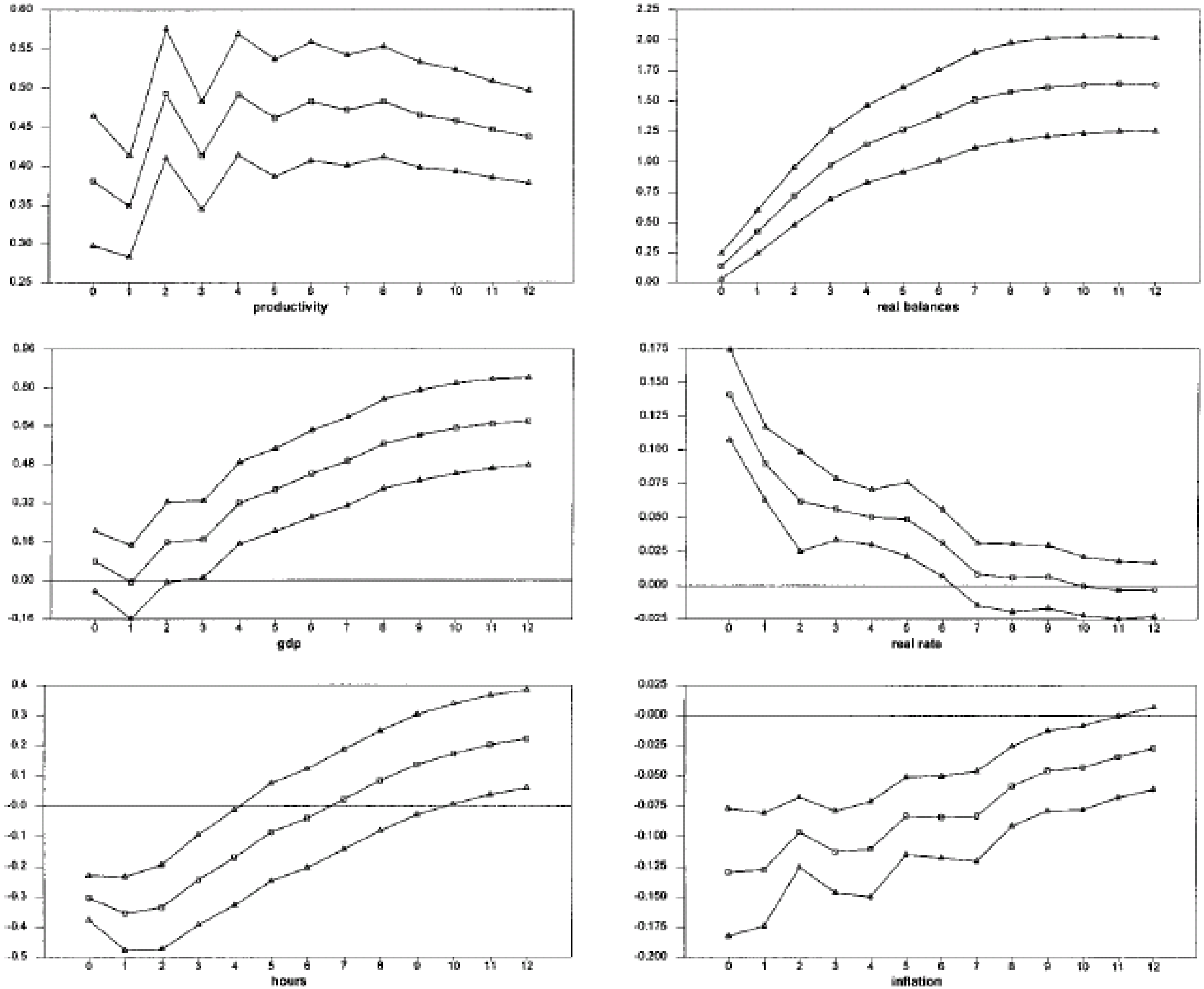
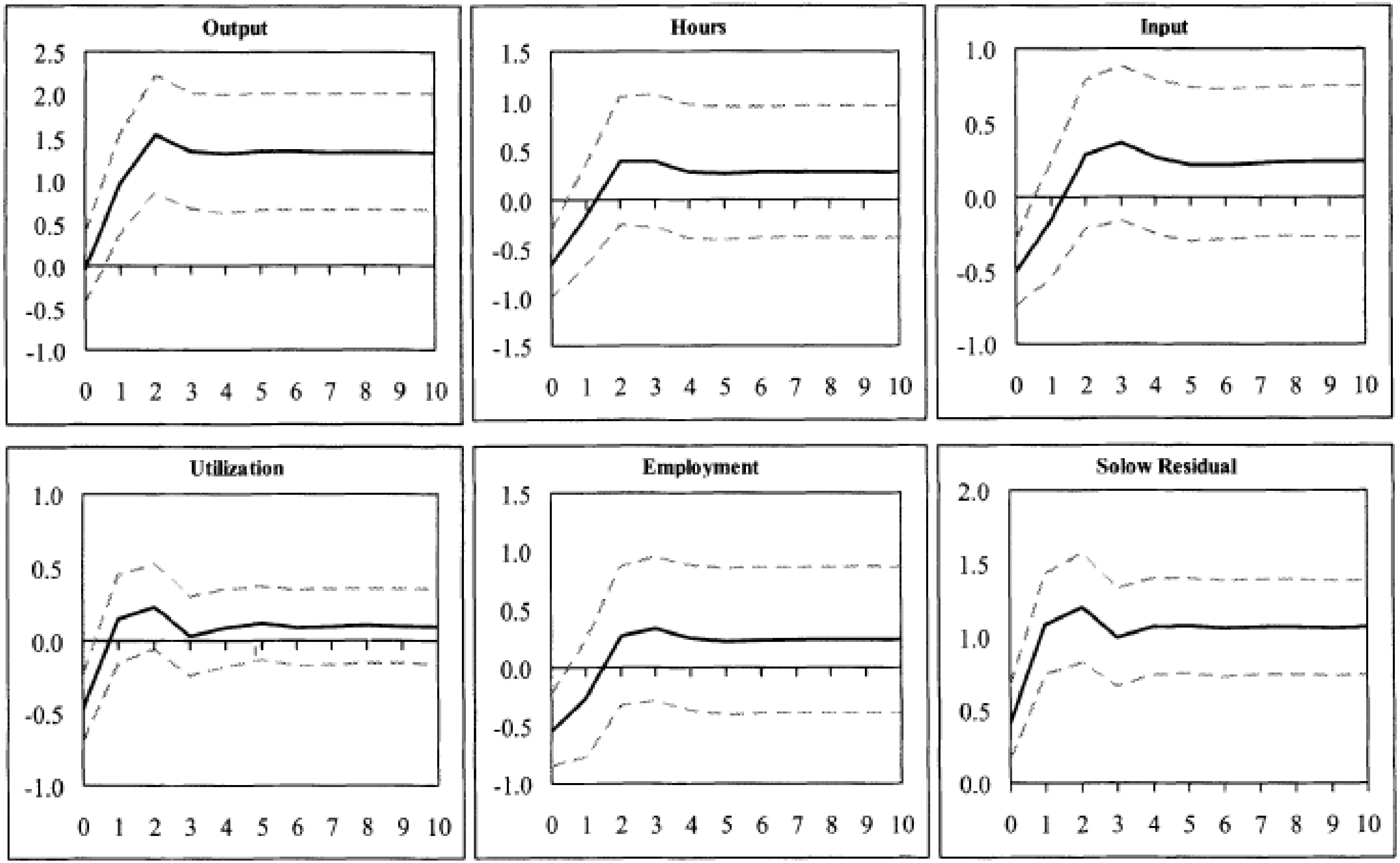


FIGURE 4. ESTIMATED IMPULSE RESPONSES FROM A FIVE-VARIABLE MODEL: U.S. DATA, FIRST-DIFFERENCED HOURS (POINT ESTIMATES AND ±2 STANDARD ERROR CONFIDENCE INTERVALS)

Source: Galí (1999)

Estimated Effects of Technology Shocks



Source: Basu, Fernald and Kimball (2006)