Monetary Policy Design
in the Basic New Keynesian Model

Jordi Galí

CREI, UPF and Barcelona GSE

January 2019
The Basic New Keynesian Model: Non-Policy Block

- **New Keynesian Phillips Curve**

  \[ \pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \tilde{y}_t \]

- **Dynamic IS Equation**

  \[ \tilde{y}_t = -\frac{1}{\sigma} (i_t - E_t\{\pi_{t+1}\} - r^n_t) + E_t\{\tilde{y}_{t+1}\} \]

  where

  \[ r^n_t = \rho - \sigma \psi_{ya} (1 - \rho_a) a_t + (1 - \rho_z) z_t \]
Monetary Policy Design: The Case of an Efficient Natural Equilibrium

- Assumption:
  \[ y_t^n = y_t^e \]

- Optimal Policy
  \[ \tilde{y}_t = 0 \quad ; \quad \pi_t = 0 \]

- Implementation
  \[ i_t = r_t^n + \phi_{\pi} \pi_t \]

where \( \phi_{\pi} > 1 \) (determinacy condition)
Monetary Policy Design: Simple Rules

- Evaluation of Alternative Policies

Welfare losses (second order approx.)

\[
W \equiv - E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{U_t - U_t^n}{U_C C} \right) = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \left( \sigma + \frac{\phi + \alpha}{1 - \alpha} \right) \tilde{y}_t^2 + \frac{\epsilon}{\lambda} \pi_t^2 \right]
\]

Average unconditional welfare losses:

\[
\mathbb{I}L = \left( \sigma + \frac{\phi + \alpha}{1 - \alpha} \right) \text{var}(\tilde{y}_t) + \frac{\epsilon}{\lambda} \text{var}(\pi_t)
\]

- Example:

\[
i_t = \rho + \phi_\pi \pi_t + \phi_\gamma \tilde{y}_t
\]
<table>
<thead>
<tr>
<th></th>
<th>Technology</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_\pi$</td>
<td>1.5 1.5 5 1.5</td>
<td>1.5 1.5 5 1.5</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>0.125 0 0 1</td>
<td>0.125 0 0 1</td>
</tr>
<tr>
<td>$\sigma(y)$</td>
<td>1.85 2.07 2.25 1.06</td>
<td>0.59 0.68 0.28 0.31</td>
</tr>
<tr>
<td>$\sigma(\hat{y})$</td>
<td>0.44 0.21 0.03 1.23</td>
<td>0.59 0.68 0.28 0.31</td>
</tr>
<tr>
<td>$\sigma(\pi)$</td>
<td>0.69 0.34 0.05 1.94</td>
<td>0.20 0.23 0.09 0.10</td>
</tr>
<tr>
<td>$\mathbb{L}$</td>
<td>1.02 0.25 0.006 7.98</td>
<td>0.10 0.13 0.02 0.02</td>
</tr>
</tbody>
</table>
Assumption: time-varying $y_t^n - y_t^e$

The New Keynesian Phillips Curve

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa x_t + u_t$$

where $x_t \equiv y_t - y_t^e$ and $u_t \equiv \kappa(y_t^e - y_t^n)$

Dynamic IS Equation

$$x_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^e) + E_t\{x_{t+1}\}$$

where $r_t^e \equiv \rho + \sigma E_t\{\Delta y_{t+1}^e\} + (1 - \rho_z)z_t = r_t^n$
The Optimal Monetary Policy Problem

\[
\min E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \vartheta x_t^2)
\]

subject to:

\[
\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa x_t + u_t
\]

where \{u_t\} evolves exogenously according to

\[
u_t = \rho u_{t-1} + \epsilon_t
\]

In addition:

\[
x_t = -\frac{1}{\sigma} (i_t - E_t \{\pi_{t+1}\} - r^e_t) + E_t \{x_{t+1}\}
\]

Note: utility based criterion requires \(\vartheta = \frac{\kappa}{\epsilon}\)
Each period CB chooses \((x_t, \pi_t)\) to minimize

\[ \pi_t^2 + \vartheta x_t^2 \]

subject to

\[ \pi_t = \kappa x_t + v_t \]

where \(v_t \equiv \beta E_t\{\pi_{t+1}\} + u_t\) is taken as given.

**Optimality condition:**

\[ x_t = -\frac{\kappa}{\vartheta} \pi_t \]

**Equilibrium**

\[ \pi_t = \frac{\vartheta}{\kappa^2 + \vartheta(1 - \beta \rho_u)} u_t \quad ; \quad x_t = -\frac{\kappa}{\kappa^2 + \vartheta(1 - \beta \rho_u)} u_t \]

\[ i_t = r^e_t + \frac{\vartheta \rho_u + \sigma \kappa (1 - \rho_u)}{\kappa^2 + \vartheta(1 - \beta \rho_u)} u_t \]
Figure 5.1
Discretion vs. Commitment: Responses to a Transitory Cost-Push Shock
Figure 5.2
Discretion vs. Commitment: Responses to a Persistent Cost-Push Shock
Optimal Monetary Policy under Discretion

Implementation:

\[ i_t = r_t^e + \frac{\vartheta \rho_u + \sigma \kappa (1 - \rho_u)}{\kappa^2 + \vartheta (1 - \beta \rho_u)} u_t + \phi_\pi \left( \pi_t - \frac{\vartheta}{\kappa^2 + \vartheta (1 - \beta \rho_u)} u_t \right) \]

\[ = r_t^e + \Theta u_t + \phi_\pi \pi_t \]

where \( \Theta \equiv \frac{\sigma \kappa (1 - \rho_u) - \vartheta (\phi_\pi - \rho_u)}{\kappa^2 + \vartheta (1 - \beta \rho_u)} \) and \( \phi_\pi > 1. \)
State-contingent policy \( \{x_t, \pi_t\}_{t=0}^{\infty} \) that minimizes

\[
E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \vartheta x_t^2)
\]

subject to the sequence of constraints:

\[
\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa x_t + u_t
\]

Lagrangean:

\[
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{1}{2} (\pi_t^2 + \vartheta x_t^2) + \xi_t (\pi_t - \kappa x_t - \beta \pi_{t+1}) \right]
\]

Optimality conditions:

\[
\vartheta x_t - \kappa \xi_t = 0
\]

\[
\pi_t + \xi_t - \xi_{t-1} = 0
\]

for \( t = 0, 1, 2, \ldots \) with \( \xi_{-1} = 0 \),
Optimal Monetary Policy under Commitment

Eliminating multipliers:

\[ x_0 = -\frac{\kappa}{\vartheta} \pi_0 \]

\[ x_t = x_{t-1} - \frac{\kappa}{\vartheta} \pi_t \]

for \( t = 1, 2, 3, \ldots \)

Alternative representation:

\[ x_t = -\frac{\kappa}{\vartheta} \hat{p}_t \]

for \( t = 0, 1, 2, \ldots \) where \( \hat{p}_t \equiv p_t - p_{-1} \)
Equilibrium

\[ \hat{p}_t = \gamma \hat{p}_{t-1} + \gamma \beta E_t \{ \hat{p}_{t+1} \} + \gamma u_t \]
for \( t = 0, 1, 2, \ldots \) where \( \gamma \equiv \frac{\vartheta}{\vartheta(1+\beta)+\kappa^2} \),

Stationary solution:

\[ \hat{p}_t = \delta \hat{p}_{t-1} + \frac{\delta}{1 - \delta \beta \rho_u} u_t \]
for \( t = 0, 1, 2, \ldots \) where \( \delta \equiv \frac{1 - \sqrt{1 - 4 \beta \gamma^2}}{2 \gamma \beta} \in (0, 1) \).

\[ \rightarrow \text{price level targeting} ! \]

\[ x_t = \delta x_{t-1} - \frac{\kappa \delta}{\vartheta(1 - \delta \beta \rho_u)} u_t \]
for \( t = 1, 2, 3, \ldots \), and

\[ x_0 = - \frac{\kappa \delta}{\vartheta(1 - \delta \beta \rho_u)} u_0 \]
Figure 5.1
Discretion vs. Commitment: Responses to a Transitory Cost-Push Shock
Figure 5.2
Discretion vs. Commitment: Responses to a Persistent Cost-Push Shock
Optimal Monetary Policy under Commitment

Discussion: Gains from Commitment

\[ \pi_t = \kappa x_t + \kappa \sum_{k=1}^{\infty} \beta^k E_t \{ x_{t+k} \} + \frac{1}{1 - \beta \rho_u} u_t \]

Illustration: Optimal Monetary Policy under the Zero Lower Bound ("Forward Guidance")
Figure 5.3
Discretion vs. Commitment in the Presence of a ZLB
The Taylor Rule (Taylor 1993)

\[ i_t = 4 + 1.5 (\pi_t - 2) + 0.5 y_t \]

Source: Taylor 1999
Source: Taylor 1999
Clarida, Galí and Gertler (QJE 2000)

\[ i_t = \rho i_{t-1} + (1 - \rho) [r + \pi^* + \beta E_t \{\pi_{t+1} - \pi^*\} + \gamma E_t \{y_{t+1} - y_{t+1}^*\} ] \]

**TABLE II**

**Baseline Estimates**

<table>
<thead>
<tr>
<th></th>
<th>(\pi^*)</th>
<th>(\beta)</th>
<th>(\gamma)</th>
<th>(\rho)</th>
<th>(p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Volcker</td>
<td>4.24</td>
<td>0.83</td>
<td>0.27</td>
<td>0.68</td>
<td>0.834</td>
</tr>
<tr>
<td></td>
<td>(1.09)</td>
<td>(0.07)</td>
<td>(0.08)</td>
<td>(0.05)</td>
<td></td>
</tr>
<tr>
<td>Volcker-Greenspan</td>
<td>3.58</td>
<td>2.15</td>
<td>0.93</td>
<td>0.79</td>
<td>0.316</td>
</tr>
<tr>
<td></td>
<td>(0.50)</td>
<td>(0.40)</td>
<td>(0.42)</td>
<td>(0.04)</td>
<td></td>
</tr>
</tbody>
</table>

Standard errors are reported in parentheses. The set of instruments includes four lags of inflation: output gap, the federal funds rate, the short-long spread, and commodity price inflation.
Orphanides (JME 2003)
Fig. 5. Then and now: Taylor rule with final and real-time data.
Table 1
Estimated policy rules

<table>
<thead>
<tr>
<th></th>
<th>$\theta_0$</th>
<th>$\theta_1$</th>
<th>$\theta_\pi$</th>
<th>$\theta_{\Delta y}$</th>
<th>$\theta_y$</th>
<th>see</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Greenbook forecasts</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1969:1–1997:4</td>
<td>-0.42</td>
<td>0.88</td>
<td>0.44</td>
<td>0.27</td>
<td>0.14</td>
<td>1.04</td>
</tr>
<tr>
<td></td>
<td>(0.29)</td>
<td>(0.04)</td>
<td>(0.10)</td>
<td>(0.10)</td>
<td>(0.03)</td>
<td></td>
</tr>
<tr>
<td>1969:1–1979:2</td>
<td>0.53</td>
<td>0.75</td>
<td>0.44</td>
<td>0.14</td>
<td>0.19</td>
<td>0.95</td>
</tr>
<tr>
<td></td>
<td>(0.92)</td>
<td>(0.14)</td>
<td>(0.12)</td>
<td>(0.15)</td>
<td>(0.04)</td>
<td></td>
</tr>
<tr>
<td>1982:3–1997:4</td>
<td>-0.33</td>
<td>0.81</td>
<td>0.52</td>
<td>0.51</td>
<td>0.10</td>
<td>0.56</td>
</tr>
<tr>
<td></td>
<td>(0.32)</td>
<td>(0.06)</td>
<td>(0.13)</td>
<td>(0.17)</td>
<td>(0.03)</td>
<td></td>
</tr>
<tr>
<td><strong>Survey forecasts</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1969:1–2002:4</td>
<td>-0.51</td>
<td>0.84</td>
<td>0.55</td>
<td>0.36</td>
<td>0.17</td>
<td>0.96</td>
</tr>
<tr>
<td></td>
<td>(0.22)</td>
<td>(0.05)</td>
<td>(0.12)</td>
<td>(0.13)</td>
<td>(0.03)</td>
<td></td>
</tr>
<tr>
<td>1969:1–1979:2</td>
<td>0.74</td>
<td>0.91</td>
<td>0.25</td>
<td>0.32</td>
<td>0.21</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>(1.28)</td>
<td>(0.29)</td>
<td>(0.16)</td>
<td>(0.35)</td>
<td>(0.05)</td>
<td></td>
</tr>
<tr>
<td>1982:3–2002:4</td>
<td>-0.66</td>
<td>0.83</td>
<td>0.58</td>
<td>0.53</td>
<td>0.16</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>(0.21)</td>
<td>(0.05)</td>
<td>(0.09)</td>
<td>(0.11)</td>
<td>(0.03)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Least-squares estimates of

$$i_t = \theta_0 + \theta_i i_{t-1} + \theta_\pi \pi_{t+3}^a + \theta_{\Delta y} \Delta^a y_{t+3} + \theta_y y_{t-1},$$

where $\pi_{t+3}^a = p_{t+3} - p_{t-1}, y_{t-1} = q_{t-1} - q_{t-1}^*$ and $\Delta^a y_{t+3} = y_{t+3} - y_{t-1} = \Delta^a q_{t+3} - \Delta^a q_{t+3}^*$. All variables dated $t$ and later reflect real-time forecasts formed during quarter $t$. HAC standard errors in parentheses.

Source: Orphanides (JME 2003 b)