The New Keynesian Model with Sticky Wages and Prices

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Modelling the Labor Market

- Competitive labor markets

$$w_t - p_t = mrs_t$$

where $mrs_t = \sigma c_t + \varphi n_t$

- General labor market imperfections

$$w_t - p_t = \mu_t^w + mrs_t$$

where μ_t^w: (log) wage markup.

Example: monopolistic union with isoelastic labor demand:

$$\mu_t^w = \log \frac{\epsilon_w}{\epsilon_w - 1} \equiv \mu^w$$

Sticky Wages and Inflation Dynamics

Recall

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} - \lambda_p(\mu_t^p - \mu^p)$$

Assuming constant returns (for simplicity)

$$\begin{aligned} \mu_t^p &= p_t - (w_t - a_t) \\ &= a_t - \omega_t \\ &= a_t - (\mu_t^w + \sigma c_t + \varphi n_t) \\ &= (1+\varphi)a_t - (\sigma+\varphi)y_t - \mu_t^w \end{aligned}$$

In deviations from *natural* levels (assuming constant natural markups):

$$\mu_t^p - \mu^p = -(\sigma+\varphi)\widetilde{y}_t - (\mu_t^w - \mu^w)$$

Implied New Keynesian Phillips Curve:

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} + \kappa_p \widetilde{y}_t + \lambda_p \widehat{\mu}_t^w$$

$\Longrightarrow$ tradeoff between inflation and output gap stabilization

Question: *What determines the evolution of the wage markup?*

A Model with Sticky Wages and Prices: Assumptions

- Original reference: Erceg-Henderson-Levin (JME 2000)
- Price setting: as in basic NK model
- Differentiated labor services ("occupations") represented by unions

  $\Rightarrow$ market power when setting wages

- Staggered nominal wage setting
- Cashless limit (focus on interest rate rules)

Households

- Representative household's problem

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, \{\mathcal{N}_t(j)\}; Z_t)$$

subject to

$$P_t C_t + Q_t B_t \leq B_{t-1} + \int_0^1 W_t(j)\mathcal{N}_t(j)dj + D_t$$

where

$$U(C_t, \{\mathcal{N}_t(j)\}; Z_t) = \begin{cases} \left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \int_0^1 \frac{\mathcal{N}_t(j)^{1+\varphi}}{1+\varphi}dj\right) Z_t & \text{for } \sigma \neq 1 \\ \left(\log C_t - \int_0^1 \frac{\mathcal{N}_t(j)^{1+\varphi}}{1+\varphi}dj\right) Z_t & \text{for } \sigma = 1 \end{cases}$$

with $z_t \equiv \log Z_t \sim AR(1)$ and $\{\mathcal{N}_t(j)\}$ *taken as given.*

Households

- *Optimality condition*

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Z_{t+1}}{Z_t} \right) \left(\frac{P_t}{P_{t+1}} \right) \right\}$$

or, in log-linearized form:

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi^p_{t+1}\} - \rho) + \frac{1}{\sigma}(1 - \rho_z)z_t$$

Wage Setting

- Fraction of occupations/unions adjusting nominal wage: $1 - \theta_w$
- Optimal wage setting

$$\max_{W_t^*} E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(C_{t+k}^{-\sigma} \frac{W_t^*}{P_{t+k}} N_{t+k|t} - \frac{N_{t+k|t}^{1+\varphi}}{1+\varphi} \right) Z_{t+k}$$

subject to labor demand schedule

$$N_{t+k|t} = \left(\frac{W_t^*}{W_{t+k}} \right)^{-\epsilon_w} \left(\int_0^1 N_{t+k}(i) di \right)$$

Wage Setting

- Optimality condition:

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ N_{t+k|t} Z_{t+k} C_{t+k}^{-\sigma} \left(\frac{W_t^*}{P_{t+k}} - \mathcal{M}_w MRS_{t+k|t} \right) \right\} = 0$$

where $MRS_{t+k|t} \equiv C_{t+k}^{\sigma} N_{t+k|t}^{\varphi}$ and $\mathcal{M}_w \equiv \frac{\epsilon_w}{\epsilon_w - 1}$.

- Log-linearized version:

$$w_t^* = \mu^w + (1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \{ mrs_{t+k|t} + p_{t+k} \}$$

- Equivalently:

$$w_t^* = (1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ w_{t+k} - (1 + \epsilon_w \varphi)^{-1} \widehat{\mu}_{t+k}^{w} \right\}$$

where $\mu_t^w \equiv (w_t - p_t) - mrs_t$ and $mrs_t = \sigma c_t + \varphi n_t$.

Wage Setting

- *Aggregate wage dynamics*

$$w_t = \theta_w w_{t-1} + (1 - \theta_w) w_t^*$$

- *Wage inflation equation*

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} - \lambda_w(\mu_t^w - \mu^w)$$

where $\lambda_w \equiv \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w \ (1+\varphi\epsilon_w)}$

Firms

- Final goods: same as in the basic NK model
- Intermediate goods: technology given by

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

where $N_t(i) \equiv \left(\int_0^1 N_t(i,j)^{1-\frac{1}{\epsilon_w}} dj \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}$ and $a_t \equiv \log A_t \sim AR(1)$

Cost minimization:

$$N_t(i,j) = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t(i)$$

where $W_t \equiv \left(\int_0^1 W_t(j)^{1-\epsilon_w} dj \right)^{\frac{1}{1-\epsilon_w}}$

Implication:

$$\int_0^1 W_t(j) N_t(i,j) dj = W_t N_t(i)$$

Firms

- Price setting: same as in the basic NK model

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} - \lambda_p(\mu_t^p - \mu^p)$$

where $\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p}\frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$

Equilibrium

- *Goods market clearing:*

$$Y_t(i) = C_t(i) \text{ all } i \in [0,1] \Rightarrow Y_t = C_t$$

where $Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$

- *Aggregate employment and output*

$$\begin{aligned} N_t &\equiv \int_0^1 \int_0^1 N_t(i,j) dj di = \Delta_{w,t} \int_0^1 N_t(i) di \\ &= \Delta_{w,t} \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{Y_t(i)}{Y_t} \right)^{\frac{1}{1-\alpha}} di = \Delta_{w,t} \Delta_{p,t} \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \end{aligned}$$

where $\Delta_{w,t} \equiv \int_0^1 \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} dj$ and $\Delta_{p,t} \equiv \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{\frac{-\epsilon_p}{1-\alpha}} di$.

Up to a first order approximation:

$$(1-\alpha) n_t = y_t - a_t$$

Equilibrium

- *The Wage Gap*

$$\widetilde{\omega}_t \equiv \omega_t - \omega_t^n$$

where $\omega_t \equiv w_t - p_t$ and ω_t^n is the *natural real wage*:

$$\omega_t^n = \psi_\omega + \psi_{\omega a} a_t - \mu^p$$

where $\psi_{\omega a} \equiv \frac{\sigma+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$ and $\psi_\omega \equiv -\frac{\alpha \log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$.

Identity:

$$\widetilde{\omega}_t \equiv \widetilde{\omega}_{t-1} + \pi_t^w - \pi_t^p - \Delta\omega_t^n$$

Equilibrium

- *Price markup gap*

$$
\begin{aligned}
\mu_t^p &= \log(1-\alpha) + (a_t - \alpha n_t) - \omega_t \\
\mu^p &= \log(1-\alpha) + (a_t - \alpha n_t^n) - \omega_t^n \\
&\Rightarrow \ \widehat{\mu}_t^p = -\frac{\alpha}{1-\alpha}\widetilde{y}_t - \widetilde{\omega}_t
\end{aligned}
$$

- Implied price inflation equation:

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} + \varkappa_p \widetilde{y}_t + \lambda_p \widetilde{\omega}_t$$

where $\varkappa_p \equiv \frac{\alpha\lambda_p}{1-\alpha}$.

Equilibrium

- *Wage markup gap:*

$$\mu_t^w = \omega_t - (\sigma y_t + \varphi n_t)$$
$$\mu^w = \omega_t^n - (\sigma y_t^n + \varphi n_t^n)$$
$$\Rightarrow \widehat{\mu}_t^w = \widetilde{\omega}_t - \left(\sigma + \frac{\varphi}{1-\alpha}\right)\widetilde{y}_t$$

- Implied wage inflation equation:

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} + \varkappa_w \widetilde{y}_t - \lambda_w \widetilde{\omega}_t$$

where $\varkappa_w \equiv \lambda_w \left(\sigma + \frac{\varphi}{1-\alpha}\right)$

Equilibrium

- *Dynamic IS equation*

$$\widetilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi^p_{t+1}\} - r^n_t) + E_t\{\widetilde{y}_{t+1}\}$$

where $r^n_t \equiv \rho - \sigma\psi_{ya}(1-\rho_a)a_t + (1-\rho_z)z_t$

- *Interest Rate Rule:*

$$i_t = \rho + \phi_p \pi^p_t + \phi_w \pi^w_t + \phi_y \widehat{y}_t + v_t$$

Equilibrium

- *Dynamical system*:

$$\mathbf{A}_0^w \mathbf{x}_t = \mathbf{A}_1^w E_t\{\mathbf{x}_{t+1}\} + \mathbf{B}_0^w \mathbf{u}_t$$

where $\mathbf{x}_t \equiv [\widetilde{y}_t, \, \pi_t^p, \, \pi_t^w, \, \widetilde{\omega}_{t-1}]'$, $\mathbf{u}_t \equiv [\widehat{r}_t^n - v_t - \phi_y \widehat{y}_t^n, \, \Delta\omega_t^n]'$,

$$\mathbf{A}_0^w \equiv \begin{bmatrix} \sigma + \phi_y & \phi_p & \phi_w & 0 \\ -\varkappa_p & 1 & 0 & 0 \\ -\varkappa_w & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}$$

$$\mathbf{A}_1^w \equiv \begin{bmatrix} \sigma & 1 & 0 & 0 \\ 0 & \beta & 0 & \lambda_p \\ 0 & 0 & \beta & -\lambda_w \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ; \quad \mathbf{B}_0^w \equiv \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Equilibrium

- *Local uniqueness condition*[1]

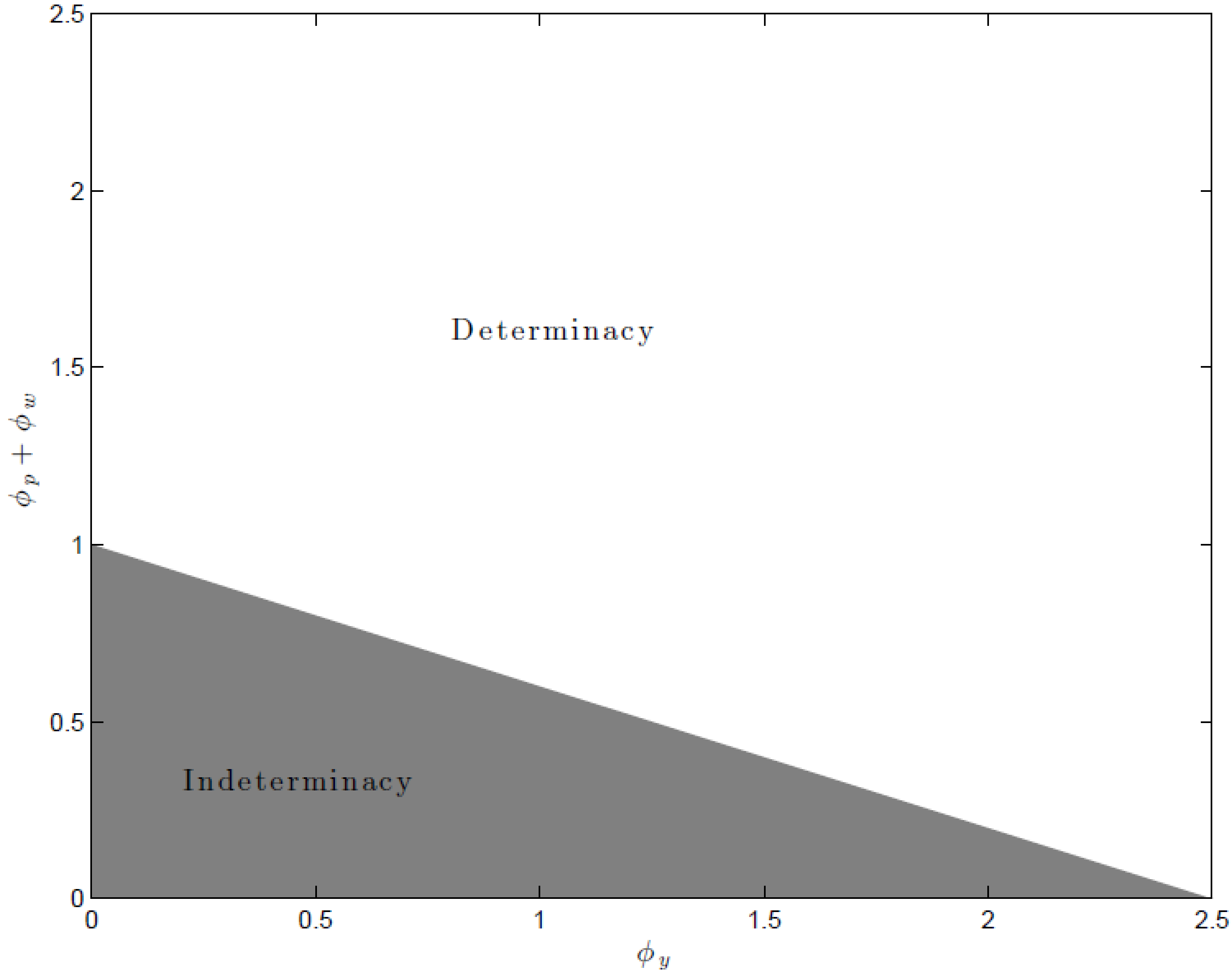
$$\phi_p + \phi_w + \phi_y \left(\frac{1-\beta}{\sigma + \frac{\alpha+\varphi}{1-\alpha}} \right) \left(\frac{1}{\lambda_p} + \frac{1}{\lambda_w} \right) > 1$$

Particular case ($\phi_y = 0$):

$$\phi_p + \phi_w > 1$$

[1] Flaschel-Franke (2008), Blasselle-Poissonier (2013)

Figure 6.1 Determinacy and Indeterminacy Regions



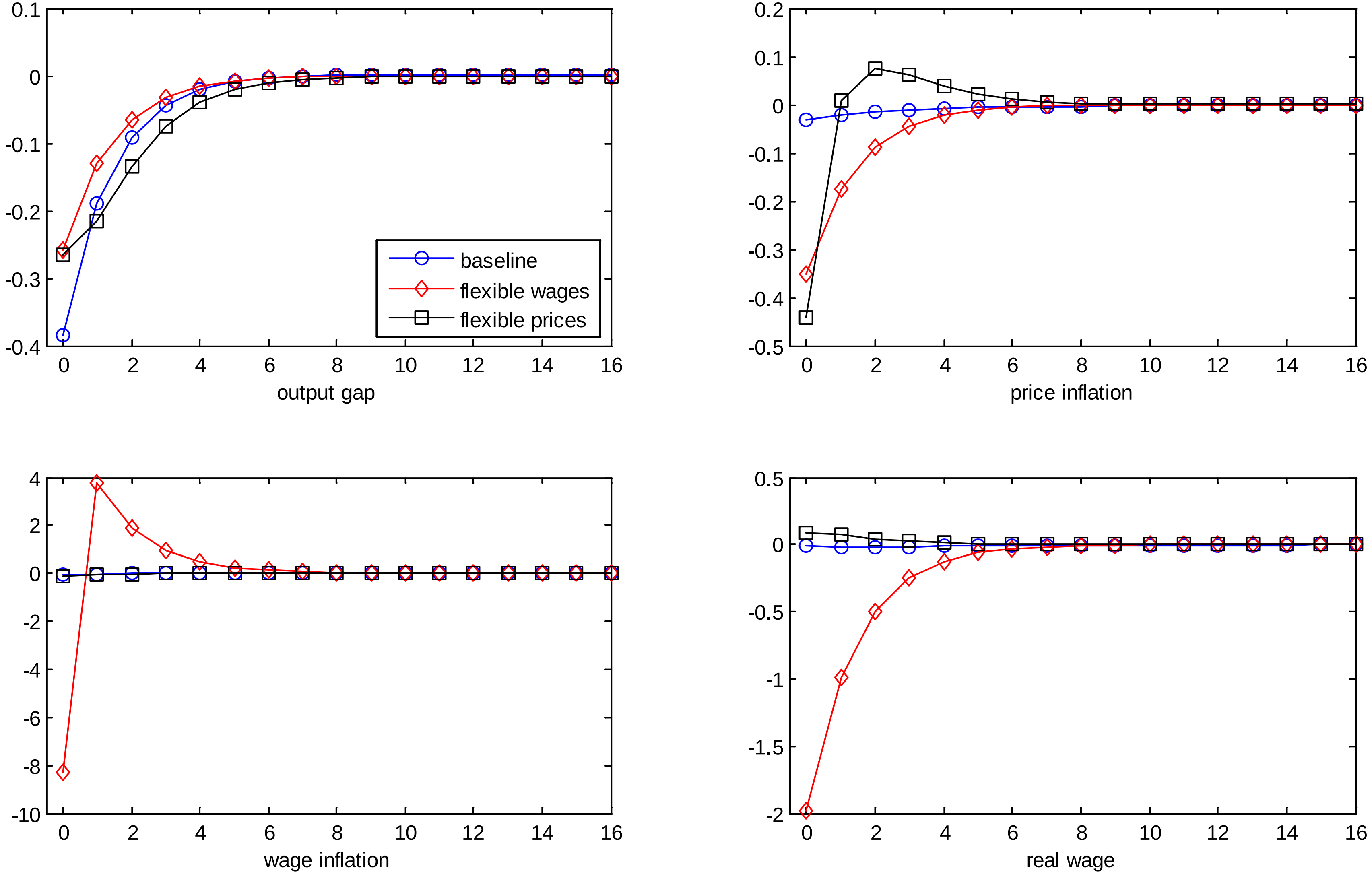
Calibration and Simulations

- *Interest rate rule*: $\phi_p = 1.5$, $\phi_y = \phi_w = 0$, $\rho_v = 0.5$
- *New parameter*: $\epsilon_w = 4.5$
- *Three calibrations of price and wage rigidities*:

  (i) Baseline: $\theta_p = 3/4$, $\theta_w = 3/4$
  (ii) Flexible wages: $\theta_p = 3/4$, $\theta_w = 0$
  (iii) Flexible prices: $\theta_p = 0$, $\theta_w = 3/4$

- Remaining parameters as in baseline model
- *Simulations: Dynamic responses to monetary policy shock*

Figure 6.2 Dynamic Responses to a Monetary Policy Shock



The Social Planner's Problem

$$\max U(C_t, \{\mathcal{N}_t(j)\}; Z_t)$$

subject to:

$$C_t = A_t \left[\int_0^1 \left(\int_0^1 N_t(i,j)^{1-\frac{1}{\epsilon_w}} dj \right)^{\frac{\epsilon_w(1-\alpha)}{\epsilon_w - 1}\left(1-\frac{1}{\epsilon_p}\right)} di \right]^{\frac{\epsilon_p}{\epsilon_p - 1}}$$

$$\mathcal{N}_t(j) = \int_0^1 N_t(i,j) di$$

- *Optimality conditions*:

$$C_t(i) = C_t, \ \text{all } i \in [0,1]$$

$$N_t(i,j) = \mathcal{N}_t(j) = N_t(i) = N_t, \ \text{all } i,j \in [0,1]$$

$$-\frac{U_{n,t}}{U_{c,t}} = MPN_t$$

where $MPN_t = (1-\alpha) A_t N_t^{-\alpha}$

Efficiency of Natural Equilibrium

- In the descentralized economy with flexible prices and wages:

$$P_t = \mathcal{M}_p \frac{(1-\tau)W_t}{MPN_t}$$

$$\frac{W_t}{P_t} = -\frac{U_{n,t}}{U_{c,t}} \mathcal{M}_w$$

for all goods and occupations, where $\mathcal{M}_p \equiv \frac{\epsilon_p}{\epsilon_p - 1}$ and $\mathcal{M}_w \equiv \frac{\epsilon_w}{\epsilon_w - 1}$. Letting $\mathcal{M} \equiv \mathcal{M}_p \mathcal{M}_w$

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{1}{\mathcal{M}(1-\tau)} MPN_t$$

- Condition for efficiency of the natural equilibrium: $\mathcal{M}(1-\tau) = 1$
- *Remark*: natural equilibrium generally not attainable with sticky prices and wages (proof)

Optimal Monetary Policy

$$\min \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \widetilde{y}_t^2 + \frac{\epsilon_p}{\lambda_p} (\pi_t^p)^2 + \frac{\epsilon_w (1 - \alpha)}{\lambda_w} (\pi_t^w)^2 \right)$$

subject to:

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} + \varkappa_p \widetilde{y}_t + \lambda_p \widetilde{\omega}_t$$

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} + \varkappa_w \widetilde{y}_t - \lambda_w \widetilde{\omega}_t$$

$$\widetilde{\omega}_{t-1} \equiv \widetilde{\omega}_t - \pi_t^w + \pi_t^p + \Delta\omega_t^n$$

Optimal Monetary Policy

- *Optimality conditions*:

$$\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right)\widetilde{y}_t + \varkappa_p \zeta_{1,t} + \varkappa_w \zeta_{2,t} = 0$$

$$\frac{\epsilon_p}{\lambda_p}\ \pi_t^p - \Delta\zeta_{1,t} + \zeta_{3,t} = 0$$

$$\frac{\epsilon_w(1-\alpha)}{\lambda_w}\ \pi_t^w - \Delta\zeta_{2,t} - \zeta_{3,t} = 0$$

$$\lambda_p \zeta_{1,t} - \lambda_w \zeta_{2,t} + \zeta_{3,t} - \beta E_t\{\zeta_{3,t+1}\} = 0$$

for $t = 0, 1, 2, ...$given $\zeta_{1,-1} = \zeta_{2,-1} = 0$ and given $\widetilde{\omega}_{-1}$.

- *Equilibrium under the optimal policy*:

$$\mathbf{A}_0^* \mathbf{x}_t = \mathbf{A}_1^* E_t\{\mathbf{x}_{t+1}\} + \mathbf{B}_0^* \Delta a_t$$

where $\mathbf{x}_t \equiv [\widetilde{y}_t,\ \pi_t^p,\ \pi_t^w,\ \widetilde{\omega}_{t-1},\ \zeta_{1,t-1},\ \zeta_{2,t-1},\ \zeta_{3,t}]'$

Optimal Monetary Policy

- *Optimal Policy in Response to Demand Shocks*

$$\mathbf{A}_0^* \mathbf{x}_t = \mathbf{A}_\mathbf{1}^* E_t\{\mathbf{x}_{t+1}\}$$

Under the assumption of $\widetilde{\omega}_{t-1} = 0$,

$$\mathbf{x}_t = 0$$

for all t

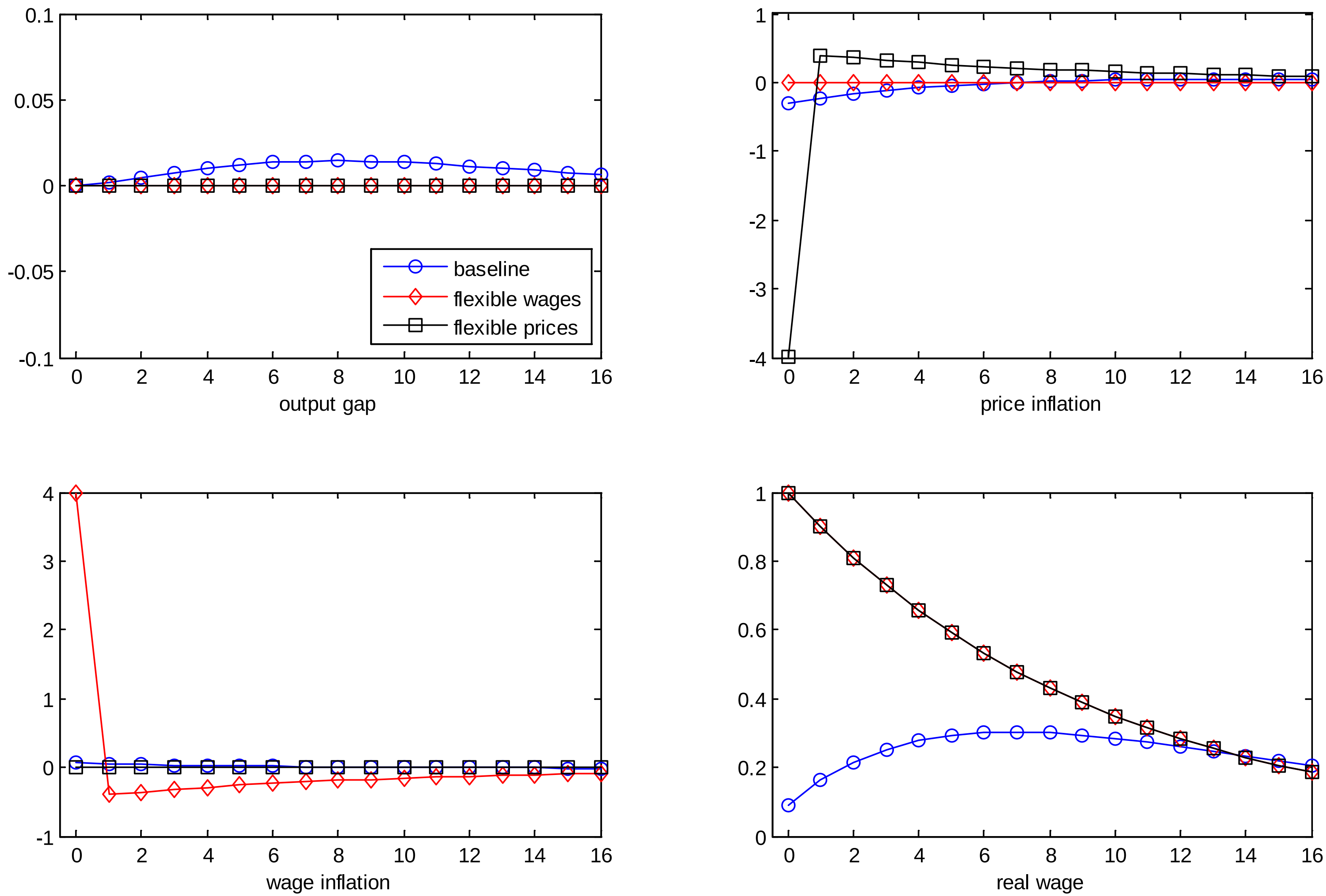
Implementation:

$$i_t = r_t^n + \phi_p \pi_t^p$$

where $r_t^n = \rho + (1 - \rho_z) z_t$, and $\phi_p > 1$

- *Optimal Policy in Response to Technology Shocks*

Figure 6.3 Dynamic Responses to a Technology Shock under the Optimal Monetary Policy



Monetary Policy Design: A Divine Coincidence Result

Combining the wage and price inflation equations:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

where $\kappa \equiv \frac{\lambda_w \lambda_p}{\lambda_p + \lambda_w} \frac{\alpha}{1-\alpha}$ and

$$\pi_t \equiv \frac{\lambda_w}{\lambda_p + \lambda_w} \pi_t^p + \frac{\lambda_p}{\lambda_p + \lambda_w} \pi_t^w$$

$\Rightarrow$ no tradeoff

Monetary Policy Design: A Special Case

- Assumptions

$$\varkappa_p = \varkappa_w \equiv \varkappa$$

$$\epsilon_p = \epsilon_w (1 - \alpha) \equiv \epsilon.$$

- Implied optimality conditions:

$$\pi_t = -\frac{1}{\epsilon} \, \Delta \widetilde{y}_t$$

for $t = 1, 2, 3, \ldots$ as well as

$$\pi_0 = -\frac{1}{\epsilon} \, \widetilde{y}_0$$

- *Optimal policy*:

$$\pi_t = \widetilde{y}_t = 0$$

for all t

Monetary Policy Design: Evaluation of Simple Interest Rate Rules

- *Strict Targeting Rules:*

$$\pi_t^i = 0$$

- *Flexible Targeting Rules:*

$$i_t = 0.01 + 1.5\pi_t^i$$

- *Evaluation*

Table 6.1 Evaluation of Simple Rules

	Optimal	*Strict Targeting*			*Flexible Targeting*		
		Price	Wage	Composite	Price	Wage	Composite
Technology shocks							
$\sigma(\pi^p)$	0.11	0	0.13	0.12	0.29	0.24	0.24
$\sigma(\pi^w)$	0.03	0.26	0	0.02	0.23	0.16	0.16
$\sigma(\widetilde{y})$	0.04	3.38	0.20	0	0.84	1.18	1.11
$\mathbb{L}$	0.0330	0.78	0.039	0.0337	0.47	0.305	0.307
Demand shocks							
$\sigma(\pi^p)$	0	0	0	0	0.02	0.04	0.03
$\sigma(\pi^w)$	0	0	0	0	0.05	0.06	0.06
$\sigma(\widetilde{y})$	0	0	0	0	1.08	1.05	1.06
$\mathbb{L}$	0	0	0	0	0.061	0.067	0.066

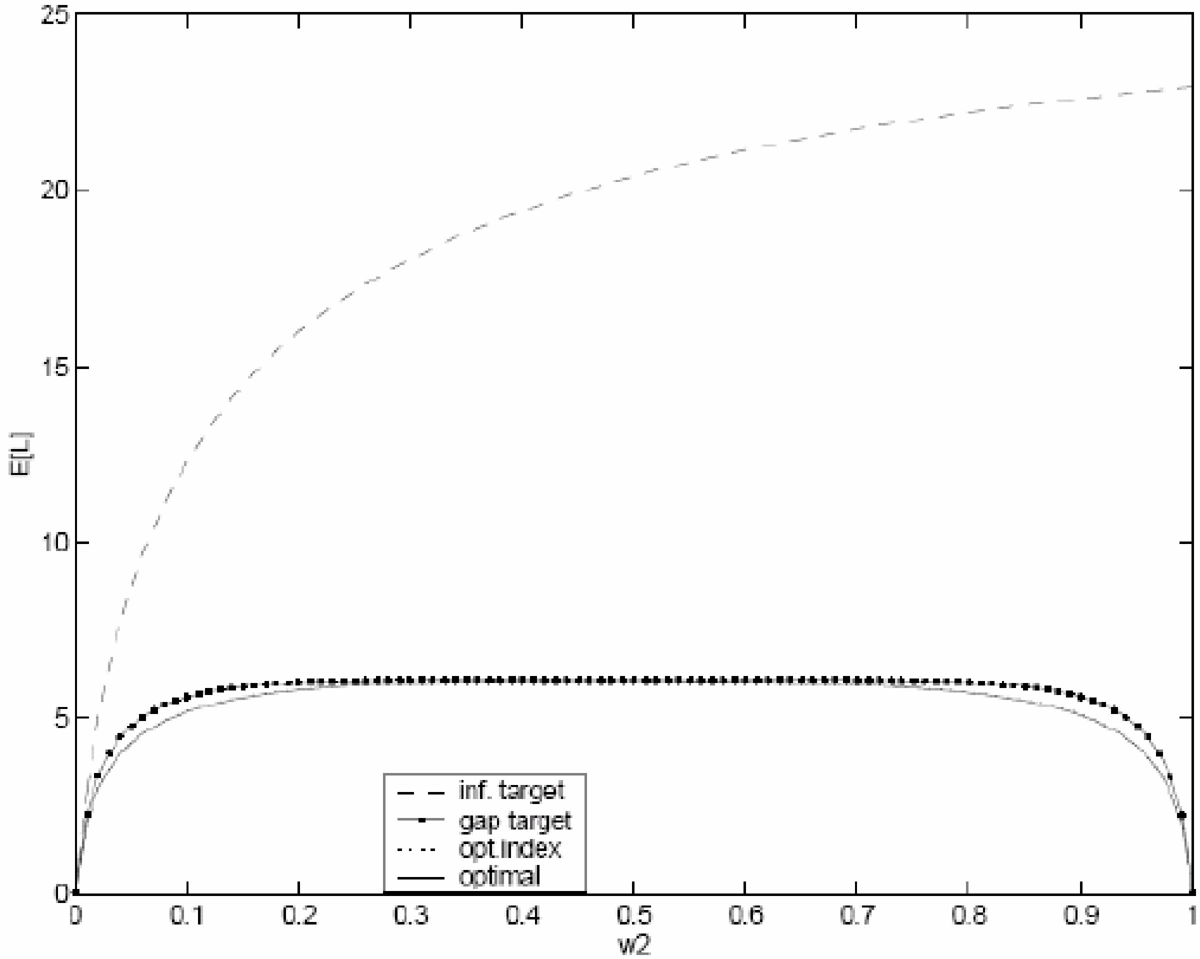


Figure 6.4: Welfare losses under alternative policies with sticky wages and prices.

Woodford (2003)

Households: An Alternative Formulation

- Representative household with a continuum of members, indexed by $(j, s) \in [0,1] \times [0,1]$
- Continuum of differentiated labor services, indexed by $j \in [0,1]$
- Indivisible labor
- Disutility from working: χs^{φ}, for $s \in [0,1]$, where $\varphi \geq 0$
- Full consumption risk sharing within the household
- Household utility: $E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, \{\mathcal{N}_t(j)\}; Z_t)$

$$U(C_t, \{\mathcal{N}_t(j)\}; Z_t) \equiv \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \chi_t \int_0^1 \int_0^{\mathcal{N}_t(j)} s^{\varphi} ds dj \right) Z_t$$

$$= \left(\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \chi_t \int_0^1 \frac{\mathcal{N}_t(j)^{1+\varphi}}{1+\varphi} dj \right) Z_t$$

where χ_t is a labor disutility shifter

Unemployment

Participation

- Participation condition for an individual (j, s):

$$\frac{W_t(j)}{P_t} \geq \chi_t C_t^{\sigma} s^{\varphi}$$

- Marginal participant, $L_t(j)$, defined by:

$$\frac{W_t(j)}{P_t} = \chi_t C_t^{\sigma} L_t(j)^{\varphi}$$

- Taking logs and integrating over i,

$$w_t - p_t = \sigma c_t + \varphi l_t + \xi_t$$

where $w_t \simeq \int_0^1 w_t(j)dj$ and $l_t \equiv \int_0^1 l_t(j)dj$ is the (log) labor force.

Introducing Unemployment

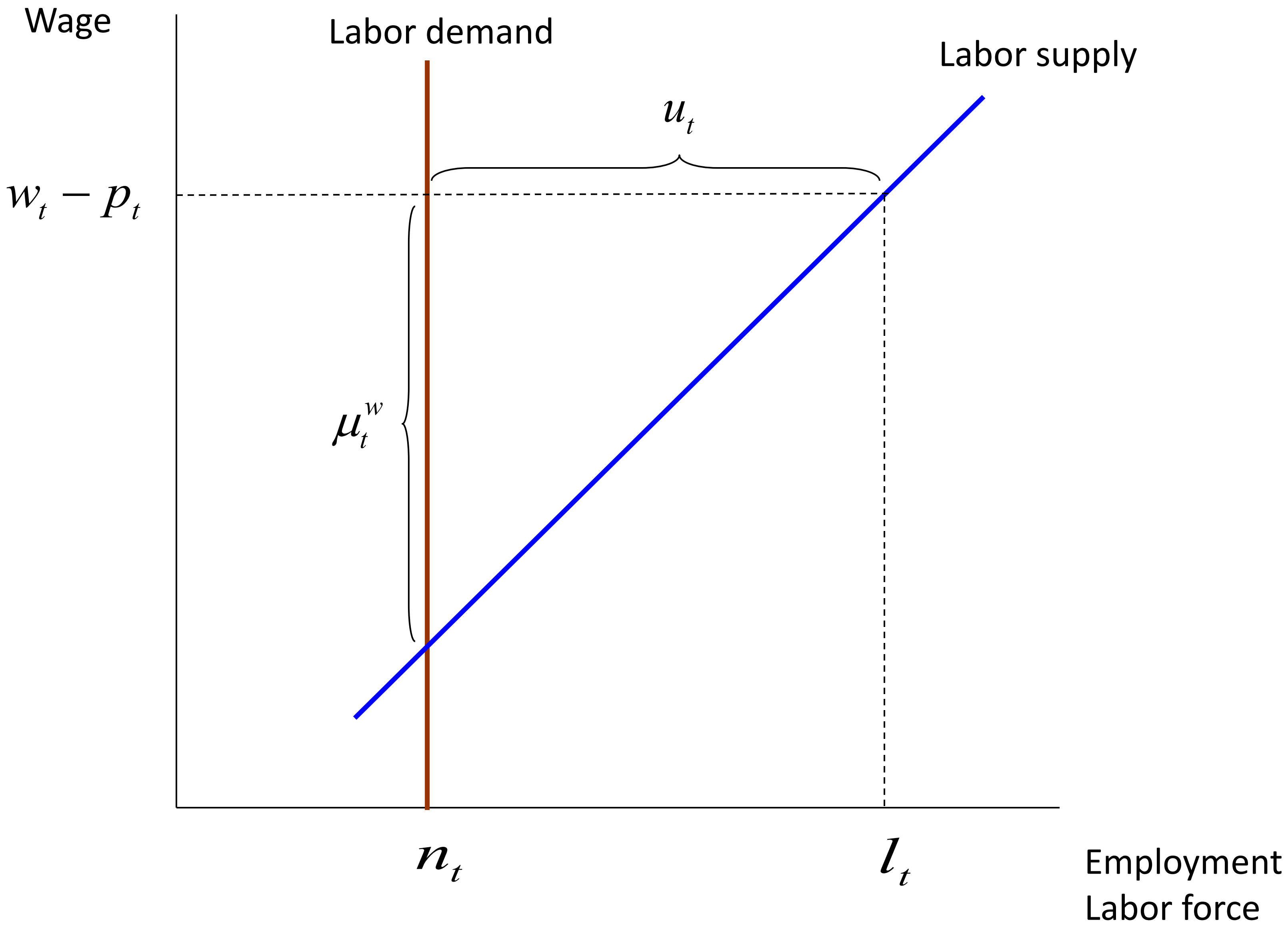
- Unemployment rate

$$u_t \equiv l_t - n_t$$

- Average wage markup and unemployment

$$\begin{aligned} \mu_t^w &= (w_t - p_t) - (\sigma c_t + \varphi n_t + \xi_t) \\ &= \varphi u_t \end{aligned}$$

Figure 7.1 The Wage Markup and the Unemployment Rate



Introducing Unemployment

- Unemployment rate

$$u_t \equiv l_t - n_t$$

- Average wage markup and unemployment

$$\begin{aligned} \mu_t^w &= (w_t - p_t) - (\sigma c_t + \varphi n_t + \xi_t) \\ &= \varphi u_t \end{aligned}$$

- Under flexible wages:

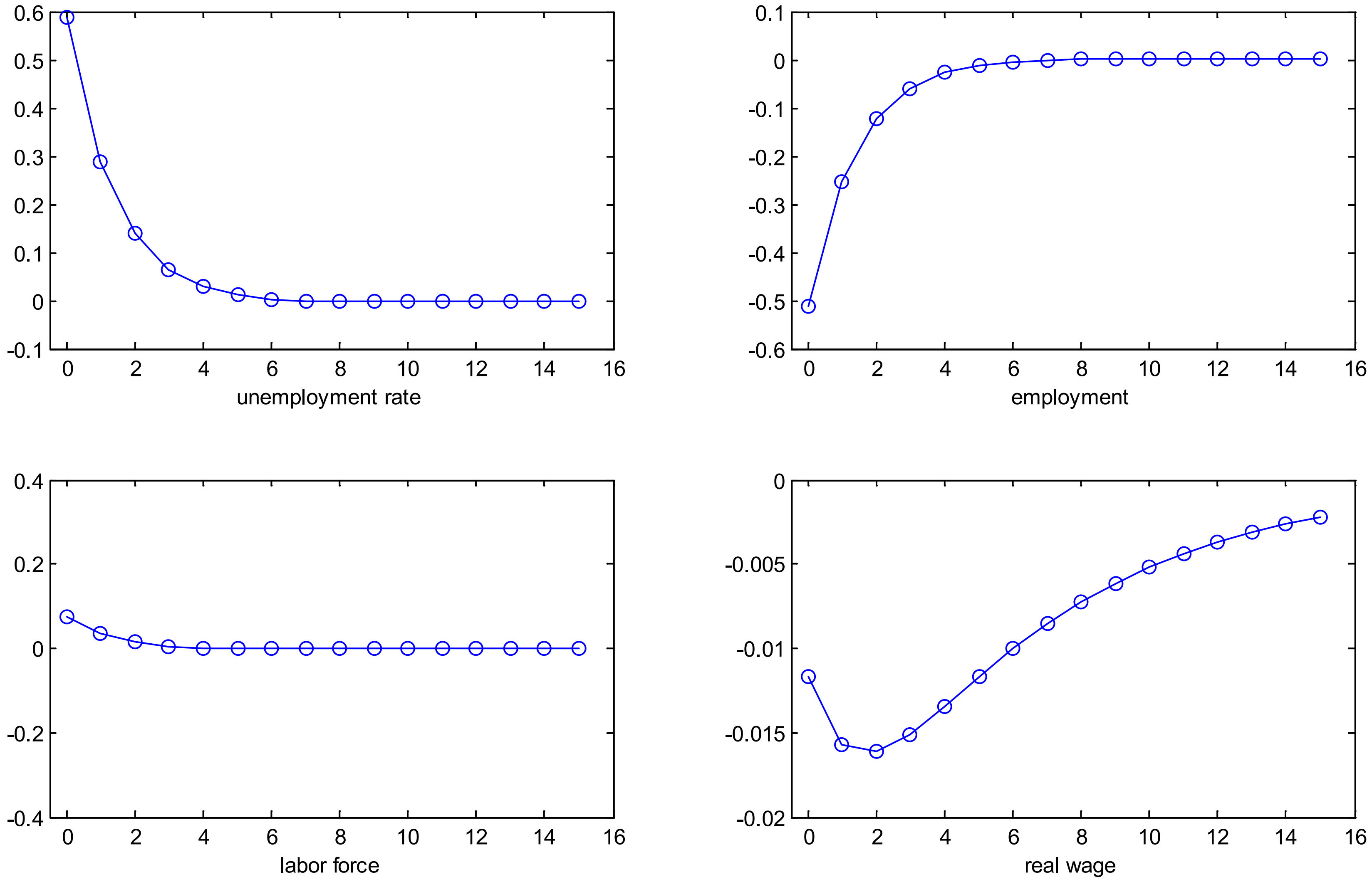
$$\mu^w = \varphi u^n$$

$$\Rightarrow u^n\text{: } \textit{natural} \text{ rate of unemployment}$$

- Combined with wage inflation equation:

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} - \lambda_w \varphi (u_t - u^n)$$

Figure 7.2 Response of Labor Market Variables to a Monetary Policy Shock



Extensions (I): Indexation

- Indexation rule

$$w_{t+k|t} = w_{t+k-1|t} + \gamma \pi^p_{t-1}$$

- Implied wage inflation equation

$$\begin{aligned} \widetilde{\pi}^w_t &= \beta E_t\{\widetilde{\pi}^w_{t+1}\} - \lambda_w(\mu^w_t - \mu^w) \\ &= \beta E_t\{\widetilde{\pi}^w_{t+1}\} - \lambda_w \varphi(u_t - u^n) \end{aligned}$$

where $\widetilde{\pi}^w_t \equiv \pi^w_t - \gamma \pi^p_{t-1}$

- Solving forward and assuming $\{u_t\} \sim AR(1)$,

$$\pi^w_t = \gamma \pi^p_{t-1} - \frac{\lambda_w \varphi}{1 - \beta \rho_u} u_t$$

- Disutility from working: $\chi \Theta_t s^{\varphi}$, where $\Theta_t \equiv Z_t / \overline{C}_t$ and $Z_t = Z_{t-1}^{1-\upsilon} \overline{C}_t^{\upsilon}$. Log utility of consumption (consistent with BGP).
- Participation condition for an individual (j, s):

$$\frac{W_t(j)}{P_t} \geq \chi \Theta_t C_t s^{\varphi}$$

- Marginal participant, $L_t(j)$, defined by:

$$\frac{W_t(j)}{P_t} = \chi \Theta_t C_t L_t(j)^{\varphi}$$

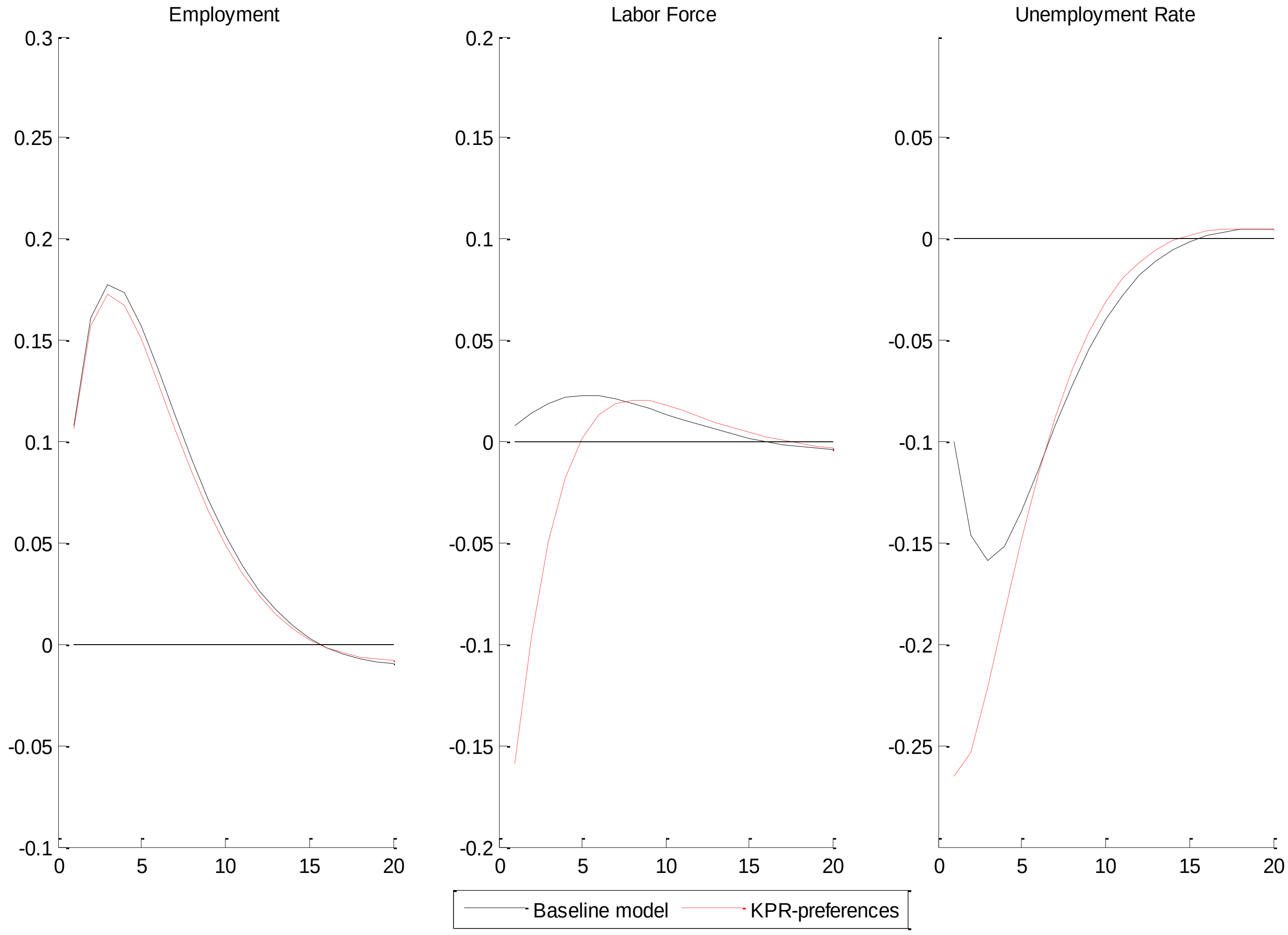
- Taking logs, symmetric equilibrium, and integrating over j,

$$w_t - p_t = z_t + \varphi l_t + \xi$$

where $w_t \simeq \int_0^1 w_t(j) dj$ and $l_t \equiv \int_0^1 l_t(j) dj$ is the (log) labor force.

- Limiting cases: KPR ($\upsilon = 1$) and GHH ($\upsilon = 0$).

Figure 6. Monetary Policy Shocks and the Role of Wealth Effects



Extensions (III): Wage Markup vs Labor Supply Shocks

- Wage inflation equation (standard)

$$\begin{aligned}\pi_t^w &= \beta E_t\{\pi_{t+1}^w\} - \lambda_w(\mu_{w,t} - \mu_{w,t}^n) \\ &= \beta E_t\{\pi_{t+1}^w\} - \lambda_w(\omega_t - \sigma c_t - \varphi n_t) - \lambda_w \xi_t + \lambda_w \mu_{w,t}^n\end{aligned}$$

- Wage inflation equation (alternative)

$$\begin{aligned}\pi_t^w &= \beta E_t\{\pi_{t+1}^w\} - \lambda_w(\mu_{w,t} - \mu_{w,t}^n) \\ &= \beta E_t\{\pi_{t+1}^w\} - \lambda_w \varphi u_t + \lambda_w \mu_{w,t}^n\end{aligned}$$

with labor supply shock identified from

$$w_t - p_t = \sigma c_t + \varphi l_t + \xi_t$$

Figure 10. The Natural Rate of Unemployment

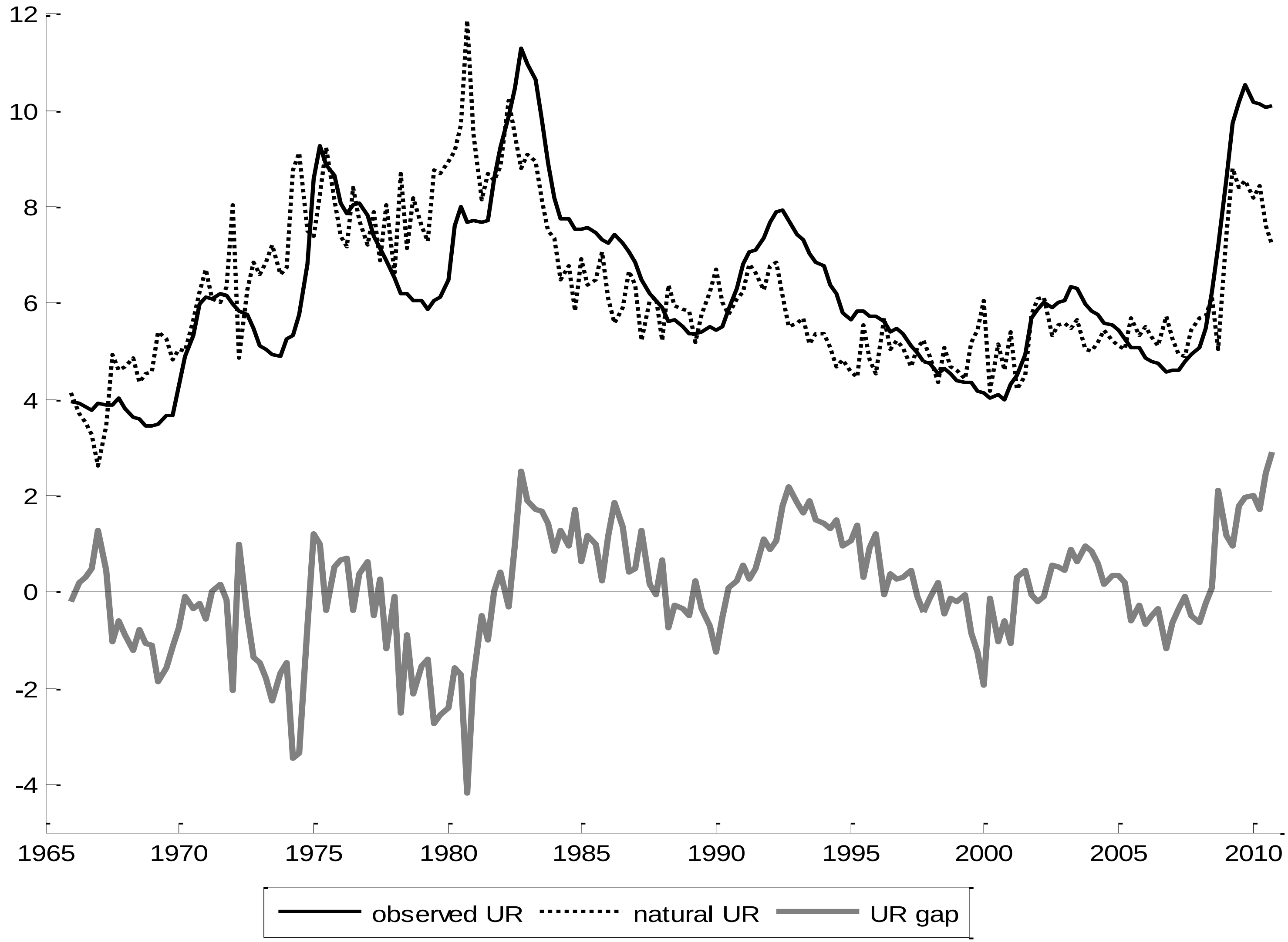


Figure 11. Sources of Unemployment Rate Fluctuations