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On payoff heterogeneity in games with strategic complementarities

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Recent papers involving binary choices have argued that increasing heterogeneity decreases positive feedback. We show that no such result holds in models where all agents make interior choices. The results in the binary choice case arise for two reasons. First, if we increase heterogeneity without limit but impose a bounded choice set, then almost all players eventually become completely unresponsive, preferring some corner so strongly that they do not react to any feasible change in the behavior of others. Second, discrete choice permits the construction of strong, but fragile, positive feedbacks through composition effects.

1. Introduction

Many of the so-called 'new' theories in economics analyse the role of positive feedback. This is true of 'new' development and growth theories as well as 'new' trade theories. Positive feedback is also important in current thinking about financial crises, and in recent business cycle theories based on self-fulfilling prophecies. Positive feedback has aroused interest in these contexts since it may lead to multiple equilibria and since it may amplify the effect of exogenous variables on equilibrium outcomes.

Theories involving positive feedback are often developed using models in which all individuals have identical payoff functions.[1] This specification is usually regarded as an innocuous simplification. However, two recent papers claim that the lack of payoff heterogeneity is actually crucial for amplification and multiple equilibria. Schmutzler (1998) and Herrendorf *et al.* (2000) construct binary choice games in which payoff heterogeneity dampens the effect of changes in exogenous variables on equilibrium outcomes, and makes multiple equilibria less likely, respectively. Their conclusions are driven by the fact that payoff heterogeneity diminishes the strength of positive feedback in their models.

[1] We focus on uncorrelated heterogeneity of the payoff function in full information games. Modelling heterogeneity as correlated shocks across individuals gives rise to some additional subtle effects, not discussed here, which are particularly relevant in the case of incomplete information. See Morris and Shin (1998, 2003) and Chamley (1999).

Our objective here is to better understand whether and how heterogeneity of payoffs might affect positive feedback, and thus amplification and multiplicity. We begin by showing that if all players make interior choices, then the effect of payoff heterogeneity on feedback is ambiguous. Next, we consider a model with a bounded choice set in which greater heterogeneity tends to increase the number of players choosing corners. In this second model, increasing heterogeneity tends to diminish feedback, because more and more players become completely unreactive, choosing one given corner regardless of the actions of others. Finally, if we restrict the decision further, to a binary choice set, then increasing heterogeneity places a tighter upper bound on the amount of feedback. We conclude that the results of Schmutzler (1998) and Herrendorf *et al.* (2000) are mainly relevant for the binary choice case, on which they focus.

2. A game with strategic complementarities and heterogeneity

We ask how payoff heterogeneity affects feedback in a framework based on the simple static game of Cooper and John (1988). These authors showed that the key ingredient for models of amplification and multiplicity is strategic complementarities. While Schmutzler (1998) and Herrendorf *et al.* (2000) construct dynamic models, the relevant properties of their dynamics come from an underlying static game equivalent to the one analysed here, so little is lost by studying the static game itself.

There is a continuum of players of measure one. An individual's payoff depends on her own action $x \in X$ and on the actions of others. For simplicity, we focus on the case where only the mean action $\bar{x}$ of others matters. Note that $\bar{x}$ lies in $[x_*, x^*]$, where $x_* \equiv \inf X$, and $x^* \equiv \sup X$. Payoffs also depend on an individual characteristic z. The distribution of z across players is called $F(z)$, and the density, where it exists, is $f(z)$. We consider equilibria of the game in which individuals simultaneously choose x to maximize the payoff $V(x, \bar{x}, z)$.

We impose the following conditions on the utility function

$$V_{xx} < 0, \quad V_{xz} < -b < 0, \quad \text{and} \quad V_{x\bar{x}} > 0 \tag{1}$$

We assume that V and these derivatives are bounded and continuous. The derivative V_{xz} shows how strongly the marginal utility of x depends on the characteristic z; we bound it away from zero to ensure that differences in z matter for choice, since otherwise heterogeneity in z would be of no interest.[2] The main assumption here is that $V_{x\bar{x}} > 0$: a rise in the mean choice $\bar{x}$ increases every player's marginal utility of x. This assumption is necessary but not sufficient for strategic complementarities, as we define them.

[2] We focus throughout on heterogeneity that affects marginal payoffs and can thus affect behavior. Heterogeneity that changes only the level of payoffs has no fundamental effects.

Now suppose all players believe that the average choice will be $\bar{x}$. The individual best response y is given by

$$y = g(\bar{x}, z) = \arg\max_{x \in X} V(x, \bar{x}, z) \tag{2}$$

The assumption $V_{x\bar{x}} > 0$ guarantees that g is weakly increasing in $\bar{x}$. If, for a player with characteristic z, the function g is also strictly increasing at some $\bar{x}$ in $[x_*, x^*]$, then we will say that this agent's behavior exhibits 'strategic complementarities'. This definition differs from that of Cooper and John (1988), who define strategic complementarities as $V_{x\bar{x}} > 0$. But we allow for corner solutions, which makes it possible that an agent with $V_{x\bar{x}} > 0$ may nonetheless choose the same corner for all $\bar{x} \in [x_*, x^*]$. Such a nonreactive player, by our definition, does not exhibit strategic complementarities.

The actual average choice $\bar{y}$ made in response to a conjectured average choice $\bar{x}$, which we will call the 'aggregate best response function', is thus

$$\bar{y} = G(\bar{x}) \equiv \int_{-\infty}^{\infty} g(\bar{x}, z)\, dF(z) \tag{3}$$

This function maps the set $[x_*, x^*]$ into itself. The points where the aggregate best response function crosses the 45° line are the equilibria of the game. If X is bounded, then at least one equilibrium exists (given our assumptions on V), either at a corner, or at point where G crosses the 45° line continuously, or at a point where G jumps across the 45° line (which may occur if there are gaps in X).

The slope of the aggregate best response function shows how players' behavior, on average, responds to changes in average behavior, so it is natural to make the slope of G our measure of 'feedback'. What interests us is not only the feedback at a given point, but also the width of the intervals over which some level of feedback applies. Greater feedback in an interval around a stable equilibrium means that the equilibrium adjusts more in response to an infinitesimal exogenous shock (a larger 'multiplier', in the sense of Cooper and John 1988). A large multiplier to a non-infinitesimal shock requires also that the interval of high feedback be sufficiently wide, as seen in Fig. 1. A wide interval of strong positive feedback also matters for multiplicity: if there are two equilibria separated by distance r, then the mean feedback on the interval between them is one. Also, if the average feedback on an interval of width r containing an equilibrium is $\phi > 1$, then there exist multiple equilibria separated by at least ϕr; see Fig. 2. Thus by asking how heterogeneity affects feedback we address both the size of the multiplier, as in Schmutzler (1998), and the likelihood and economic significance of multiplicity, as in Herrendorf *et al.* (2000).

But what do we mean by 'heterogeneity'? There is no universally accepted criterion, so we study two simple and reasonable definitions. First, we can partially order distributions by saying that one is 'more heterogeneous' than another if

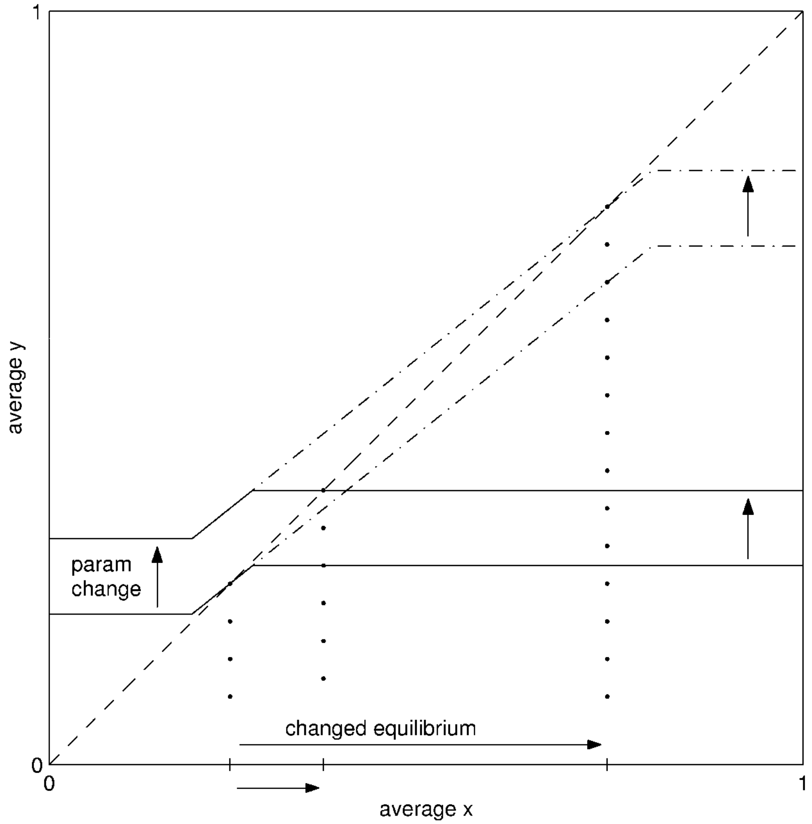

Fig. 1. Response to non-infinitesimal parameter change.

it is a mean-preserving spread of the other. But when we compare a uniform distribution on $[0, 1]$ with a pair of equal point masses at 0 and 1, we see some disadvantages of this definition. The pair of point masses is a mean-preserving spread of the uniform distribution, but it might also be reasonable to call it less heterogeneous, since every individual is exactly identical to half the population. Thus we also investigate a second definition, in which heterogeneity means lack of homogeneity, that is, the absence of any very uniform subpopulation. For this purpose, it suffices to define 'homogeneity' as the supremum of the density: $A \equiv \sup_z f(z)$, so that a population is heterogeneous when f is fairly flat, and homogeneous if f has spikes; homogeneity is infinite if there is any point mass.

3. The effect of heterogeneity on feedback is ambiguous

Using this minimalist model, we quickly see that feedback need not diminish in the face of heterogeneity. For starters, suppose we consider only interior solutions.

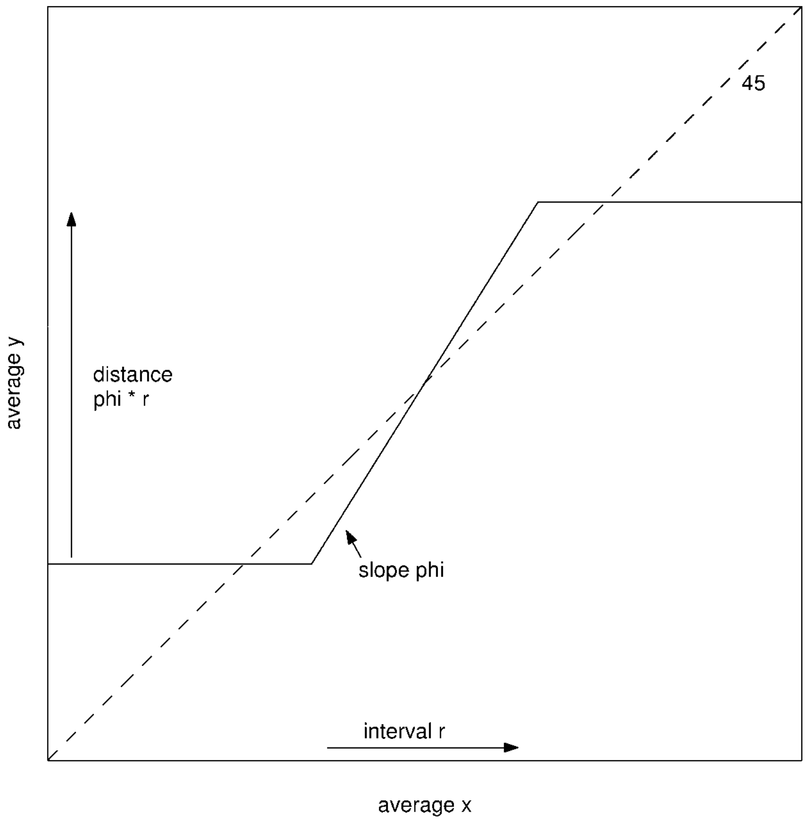

Fig. 2. Distance between equilibria.

Then we can write the slope of the aggregate best response function as an integral over the marginal changes in individual behavior

$$\frac{\partial \overline{y}}{\partial \overline{x}} = G_{\overline{x}}(\overline{x}) \equiv \int_{-\infty}^{\infty} g_{\overline{x}}(\overline{x}, z)\, dF(z) \tag{5}$$

Therefore, a mean-preserving spread of F has an ambiguous effect on feedback:

Proposition 1 Consider a choice space X, a utility function V satisfying (1), and a distribution F, such that all players choose an action in the interior of X at any $\overline{x} \in [x_*, x^*]$. Then the feedback at each $\overline{x} \in [x_*, x^*]$ increases (decreases) in response to a mean-preserving spread of F if $g_{\overline{x}}(\overline{x}, z)$ is convex (concave) in z over the support of the spread.[3]

[3] By the 'support of the spread', we mean the set of points z at which the new distribution function differs from the old one.

Proof Our assumptions on V, together with the assumption of interior choice, imply that $g_{\bar{x}}(\bar{x}, z)$ is well-defined. The effect of a mean preserving spread on the integral above follows directly from Rothschild and Stiglitz (1970).

There are no economic reasons to restrict the second z-derivative of the policy function slope $g_{\bar{x}}(\bar{x}, z)$; often $g_{\bar{x}}(\bar{x}, z)$ will be neither convex nor concave in z overall. Also, mean preserving spreads are not the only reasonable definition of heterogeneity. For still greater ambiguity, note that the point(s) where G crosses the 45° line will usually move if the F changes, so that even knowing how feedback changes at all points does not tell us how it changes at the equilibrium point(s), where it matters most. The bottom line is that with interior choice, there is no reason at all to presume that heterogeneity decreases feedback. Moreover, this result does not really require that all players make interior choices. What is needed is that spreading out the distribution does not increase the number of players choosing corners—because that can indeed diminish feedback, as we show next.

4. Limiting effects of heterogeneity when choice is bounded

Since interior solutions yield no clear relation between heterogeneity and feedback, we next consider corner solutions. So suppose the choice space is bounded: without loss of generality,[4] say $X \equiv [0, 1]$. Then by introducing enough heterogeneity in preferences for x so that most people choose some corner, regardless of the actions of the rest, we can eliminate strategic complementarities.

Proposition 2 Consider the bounded choice set $X = [0, 1]$, and a utility function V satisfying (1). If we consider a sequence of distributions $\{F_j\}_{j=1}^{\infty}$ such that $\lim_{j\to\infty}\sup_z F_j'(z) = 0$, then the aggregate best response functions $G_j(\bar{x})$ satisfy $\lim_{j\to\infty} G_j'(\bar{x}) = 0$ at all $\bar{x} \in [0, 1]$.

Proof Given our assumptions on V, we can define thresholds $Z_0(\bar{x})$ and $Z_1(\bar{x})$ such that $V_x(0, \bar{x}, z) < (>)\, 0$ iff $z > (<)\, Z_0(\bar{x})$, and $V_x(1, \bar{x}, z) < (>)\, 0$ iff $z > (<)\, Z_1(\bar{x})$. Then a player's behavior exhibits strategic complementarities iff $z \in [Z_1(0), Z_0(1)]$; for z outside this interval, $g_{\bar{x}}(\bar{x}, z) = 0$ for all $\bar{x} \in [0, 1]$. But as homogeneity A approaches zero, the mass in the interval $[Z_1(0), Z_0(1)]$ goes to zero. Thus the integral $G'(\bar{x})$ goes to zero at all $\bar{x} \in [0, 1]$.

Intuitively, if we bound V_{xz} away from zero so that z always matters for choice, then as we spread out the distribution of z there are less and less players who react to the choices of others. Agents with z less than $Z_1(0)$ prefer to choose $x = 1$ regardless of what others do. Likewise, agents with sufficiently large z prefer $x = 0$ at all possible $\bar{x} \in [0, 1]$. In the limit as we spread out z, everyone chooses corners, there are no strategic complementarities, and feedback $G'(\bar{x})$ is zero

[4] More precisely, if all individuals have the same bounds on their choice space, calling these 0 and 1 is a normalization. But it is not without loss of generality to assume that all individuals face the same bounds.

at every $\bar{x} \in [0,1]$. Thus equilibrium is unique and the multiplier on any (sufficiently small) shock is exactly zero.

This limiting result reiterates what we already know from Cooper and John (1988): strategic complementarities are needed for positive feedback. But does it have any relevance in contemporary macroeconomic applications? If we are willing to suppose that many players do not react at all to others' choices, it is no surprise to conclude that feedback is limited. But to derive such inertia from heterogeneity we must assume some bounds on the choice space, while imposing no bounds at all on the admissible degree of heterogeneity. In practice, for any model, the question is quantitative: are limits on heterogeneity more or less binding than limits on choice?

5. Composition effects when choice is binary

Schmutzler (1998) and Herrendorf *et al.* (2000) assume a binary choice set, which is a special case of bounded choice, and a version of Proposition 2 applies. But binary choice also permits stronger conclusions. To see how, we now define the choice set as $X = \{0,1\}$, so that the average choice $\bar{x}$ represents the measure m of individuals playing 1.

When everyone believes that measure m of individuals will choose 1, a player with characteristic z will choose 1 if

$$V(1, m, z) - V(0, m, z) \equiv U(m, z) \geqslant 0 \tag{6}$$

Our assumptions on V ensure that $U_m(m, z) > 0$ and $U_z(m, z) < -b < 0$: playing 1 is more advantageous if many others play 1 or if z is smaller. For each m, we can find a threshold $Z(m)$ such that people with z below (above) the threshold strictly prefer to play 1 (0). The threshold function is implicitly defined by $U(m, Z(m)) = 0$.

When players' behavior exhibits a distinct change across a threshold, strong feedbacks will occur if their threshold points are tightly clustered. In particular, if many agents have a characteristic near $z = Z(m)$, then average choice varies sharply around m. Note that the aggregate best response function now represents the measure n who choose 1 if everyone expects measure m to play 1; the definition (3) of the aggregate best response function G simplifies to

$$n = H(m) \equiv F(Z(m)) \tag{7}$$

Differentiating, we can decompose the feedback at any point m into two factors, one relating to strategic complementarities and the other to heterogeneity

$$\frac{\partial n}{\partial m} = H'(m) = f(Z(m))Z'(m) \tag{8}$$

The factor $Z'(m) = -U_m/U_z$ is positive. A larger $Z'(m)$ represents stronger strategic complementarities, since it means that a given change in m causes players to change their choice from $x = 0$ to $x = 1$ over a wider range of characteristics z. The factor $f(Z(m))$, the density of individuals with threshold $Z(m)$, captures what we call 'composition effects': even weak strategic complementarities can generate strong positive feedback, if there is a dense concentration of players who change their behaviour near m.[5] In fact, a point mass at $z = Z(m)$ implies infinite feedback at m. But this means high homogeneity, by our second definition: there exists a group of very similar people.

More formally, if the mean feedback on an interval of width r is ϕ, then the maximum density must be at least $A(\phi, r)$, which is an increasing function of ϕ and r. To distinguish this result from Proposition 2, here we hold fixed the mass of players exhibiting strategic complementarities, $\sigma = F(Z(1)) - F(Z(0))$. We thus compute homogeneity only over the set $[Z(0), Z(1)]$; the minimum homogeneity is now $A \equiv \sigma/(Z(1) - Z(0))$, given by a uniform distribution on $[Z(0), Z(1)]$. Now consider all subintervals of $[0, 1]$ with width r. The average feedback on such an interval cannot exceed $\phi^r \equiv \sigma/r$, which obtains if all those exhibiting strategic complementarities have their threshold points in the interval. Also, if mass σ is spread uniformly on $[Z(0), Z(1)]$, then on some subinterval of $[0, 1]$ of width r the following level of feedback is achieved

$$\phi_r = \max_{m \in [0,\, 1-r]} \left(\frac{\sigma}{r}\right) \frac{Z(m+r) - Z(m)}{Z(1) - Z(0)} \tag{9}$$

With these definitions, we have

Proposition 3 Consider the binary choice set $X = \{0,1\}$, a utility function V satisfying (1), and distributions F in which the mass of agents with strategic complementarities is σ. Then there is a minimum homogeneity $A(\phi, r)$ required to construct average feedback ϕ on an interval of width r. $A(\phi, r)$ is strictly increasing in r on [0, 1] and in ϕ on $[\phi_r, \phi^r]$.

Proof Fix the mass σ in $[Z(0), Z(1)]$. Choose intervals $I \equiv [Z(m_0), Z(m_0 + r)]$ and $J \equiv [Z(m_0 - \delta), Z(m_0 + r + \varepsilon)]$ so that $I \subseteq J \subseteq [Z(0), Z(1)]$. Consider the density f which places mass σ uniformly on J, and is zero elsewhere; this implies homogeneity $\sigma/(r + \varepsilon + \delta)$, and feedback on I equal to $(Z(m_0+r)-Z(m_0))/r$ times homogeneity. Since Z is continuous in m, there are densities of this shape that attain any feedback between ϕ_r and ϕ^r. Hence there is a minimum homogeneity $A(\phi, r)$ required to construct feedback $\phi \in [\phi_r, \phi^r]$ on an interval of width r. By the envelope theorem, $A(\phi, r)$ is strictly increasing in ϕ and r.

[5] Moreover, a model where high or even infinite feedback arises from the superposition of threshold strategies exhibits a 'fallacy of composition' in the sense of Caballero (1992): the high feedback comes from assuming that aggregate behavior has the same discontinuous form as individual behavior, in a context where there is no economic reason to suppose this.

Hence if we restrict ourselves to more heterogeneous distributions, a given level of feedback is only possible over smaller intervals, and less feedback is possible on intervals of a given width. Thus exogenous shocks are less likely to have a big effect, and the most widely separated equilibria (if multiple) cannot be so far apart (existence of equilibria separated by r implies at least homogeneity $A(1, r)$).

On the other hand, this result only puts bounds on the feedback occurring under any given heterogeneity; it does not imply a monotonic relationship between heterogeneity and feedback for all possible changes of F. In fact, any change in F (fixing σ) must lower feedback at some points and raise it at others; the effect at the equilibrium point(s) is ambiguous in general. Furthermore, while equilibrium must be unique if we eliminate the agents exhibiting strategic complementarities, composition effects alone do not suffice for uniqueness. Fixing σ, homogeneity A is minimized by going to a uniform distribution on $[Z(0), Z(1)]$. Nonetheless, since $Z'(m)$ is arbitrary, feedback may still be strong, and there may still be multiplicity.

6. Examples

We now illustrate our results in a version of Matsuyama's (1991) industrialization model, which Herrendorf *et al.* (2000) used as an example of the fragility of multiplicity. While these papers extended the model to a dynamic context, here it suffices to study the static framework from which Matsuyama's paper began.

6.1 A binary choice example

Suppose there are two sectors: agriculture, in which the wage is one for all individuals, and manufacturing, where the wage per unit of effective labor, as a function of manufacturing employment m, is $w = W(m)$. The assumption $W'(m) > 0$ implies strategic complementarities (i.e. increasing returns to scale) to participation in manufacturing. The number of units of effective labor that an individual can supply to manufacturing is $1/z$, where z varies in the population.

An individual wishes to work in manufacturing if $w/z \geqslant 1$, so the number who prefer manufacturing is $n = F(w)$, where F is the distribution function of z. Thus if m individuals are expected to work in manufacturing, the number who prefer manufacturing is $n = H(m) \equiv F(W(m))$, which defines the aggregate best response function. Equilibria are fixed points of $H(m)$.

Figure 3 illustrates the model under homogeneity and heterogeneity. Under the initial distribution, we assume that many agents are concentrated around a certain z_0, so that around $m = Z^{-1}(z_0)$, most agents switch from agriculture to manufacturing, yielding multiple equilibria. If we spread out the distribution of z, there is less of a jump in the aggregate best response function, implying uniqueness.

6.2 A continuous example

The Matsuyama model can easily be altered to demonstrate the ambiguous relation between heterogeneity and multiplicity in a continuous choice space. Suppose the

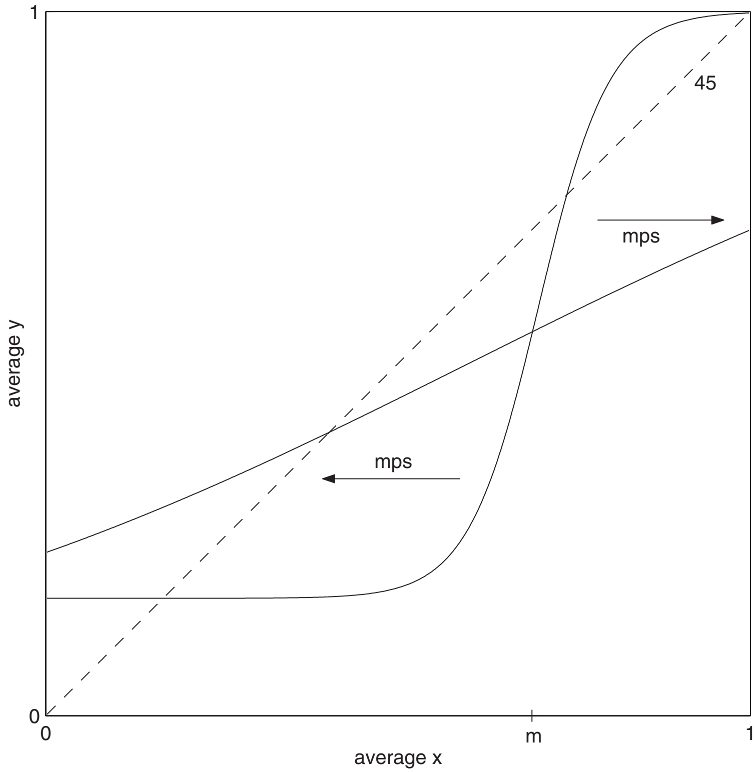

Fig. 3. Mean-preserving spread under binary choice.

wage in agriculture is one, and that each individual can produce manufacturing output $q = zx^{\alpha}Q(\overline{x})$. Assume z varies in the population with density f; x, and $\overline{x}$ are the individual and aggregate labor inputs to manufacturing, respectively; $0 < \alpha < 1$, and $Q'(\overline{x}) > 0$. Assume for simplicity that the disutility of labor is sufficiently low so that all individuals choose positive labor input in both sectors. Then individual labor input to manufacturing is

$$y = g(\overline{x}, z) = (\alpha\, zQ(\overline{x}))^{1/(1-\alpha)} \tag{11}$$

Notice that in this example, both the individual best response function and its slope $g_{\overline{x}}(\overline{x}, z)$ will be convex functions of z. The aggregate best response function is

$$\overline{y} = G(\overline{x}) = (\alpha\, Q(\overline{x}))^{1/(1-\alpha)} \int_0^{\infty} z^{1/(1-\alpha)} f(z)\, dz \tag{12}$$

We see that both the aggregate best response function and its slope will be raised at all points $\overline{x}$ by a mean-preserving spread of z. An example is shown in Fig. 4.

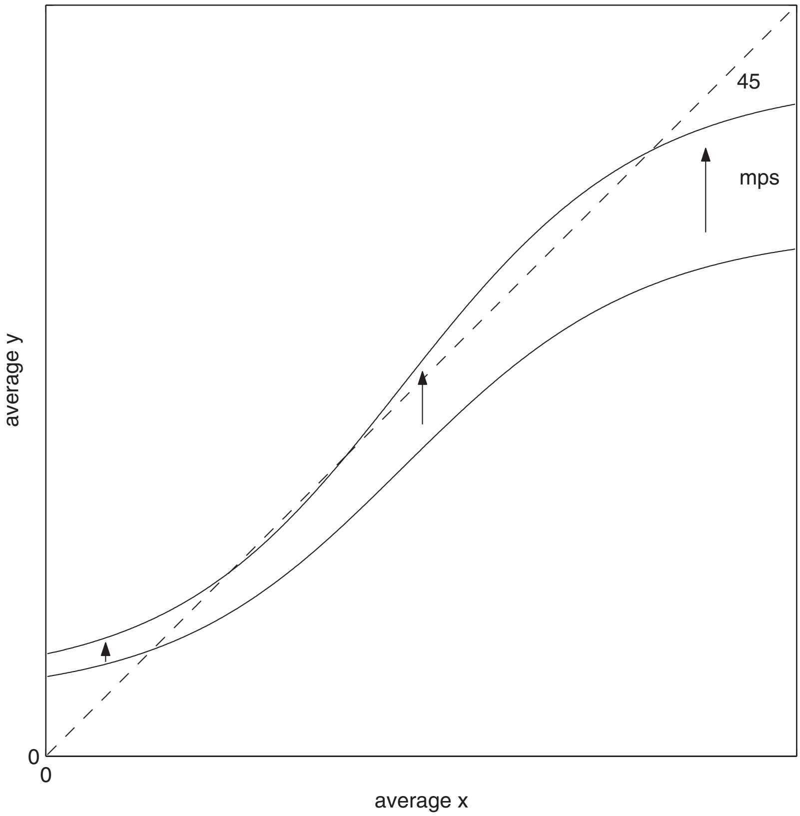

Fig. 4. Mean-preserving spread under continuous choice.

This model has increasing returns to labor in manufacturing, like Matsuyama's model, but does not impose binary choice. It can display multiple industrialization equilibria, but these need not be eliminated by composition effects. In the example of Fig. 4, a mean-preserving spread of the distribution of z gives rise to multiple equilibria. While we could have changed the parameters to obtain a different result, our point is that multiplicity is not generally vulnerable to heterogeneity in this context.

7. Conclusions

It has been proposed that we should be skeptical of economic arguments based on positive feedback, because, in some cases, feedback is weakened or eliminated by payoff heterogeneity. This paper has characterized the set of models in which this concern is justified. As long as we consider interior solutions, there is no relationship whatsoever between heterogeneity and feedback. But if we are willing to impose unbounded amounts of heterogeneity in payoffs, while fixing and bounding the choice set, then strategic complementarities are eventually eliminated, and hence feedback as well. Also, in the binary choice case, stronger feedback can be constructed when thresholds overlap than when agents are heterogeneous.

Positive feedbacks arising from interior choice of prices or quantities under imperfect competition, like the multipliers in Blanchard and Kiyotaki (1985), should be robust. But in the same paper, the multiplicity of equilibrium generated by menu costs will disappear if the payoff to adjusting prices is sufficiently heterogeneous—which is a point related to that of Caballero (1992). Positive feedback in the increasing-returns search model of Diamond (1982) will also be robust, as it comes from an interior choice of search intensity.[6] However, the feedbacks from the (binary) market entry decision in Pagano (1989) will eventually be eliminated as heterogeneity in entry costs increases. Speculative attack decisions, though not binary, may be best regarded as bounded (between selling none, and selling all, of one's currency holdings). Thus the interval of multiplicity in Obstfeld (1996) should shrink as heterogeneity increases.[7] But always, the question is whether bounds on choice matter more or less than bounds on heterogeneity. Are agents likely to differ so much in their currency demands that many willingly maintain their currency holdings even when they know for sure that many others are successfully attacking? Heterogeneity may ensure uniqueness of equilibrium when the answer is affirmative. But the answer depends greatly on parameters, so the mechanism of Schmutzler (1988) and Herrendorf *et al.* (2000) is likely often to be quantitatively insignificant.

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[7] Proposition 3 of Costain (2003) states a result of this sort.

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