Advanced Macroeconomics II

The Classical Monetary Model

Jordi Galí
Universitat Pompeu Fabra
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Assumptions

- Perfect competition
- Flexible prices and wages

Households

- Preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t, \frac{M_t}{P_t}\right)$$

where $\beta \equiv \frac{1}{1+\rho} \in [0,1]$, and $U_c > 0$, $U_m > 0$, $U_n < 0$, $U_{cc} \leq 0$, $U_{mm} \leq 0$, i $U_{nn} \leq 0$

- Budget constraint

$$P_t C_t + B_t + M_t = W_t N_t + (1 + i_{t-1}) B_{t-1} + M_{t-1} + D_t$$

per a $t = 0, 1, 2, ...$

- Optimality conditions

  - *intratemporal*

$$\frac{W_t}{P_t} = -\frac{U_{n,t}}{U_{c,t}}$$

  - *intertemporal*

$$U_{c,t} = \beta(1 + i_t)E_t\{U_{c,t+1}(P_t/P_{t+1})\}$$

$$U_{c,t} = U_{m,t} + \beta E_t\{U_{c,t+1}(P_t/P_{t+1})\}$$

Combining the last two conditions:

$$\frac{U_{m,t}}{U_{c,t}} = \frac{i_t}{1 + i_t}$$

- *Example*

$$U(C_t, N_t) = \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} - \frac{(M_t/P_t)^{1-v} - 1}{1-v}$$

Implied optimality conditions:

$$W_t/P_t = C_t^{\sigma} N_t^{\varphi}$$

$$1 = \beta(1+i_t)E_t\left\{(C_{t+1}/C_t)^{-\sigma}(P_t/P_{t+1})\right\}$$

$$M_t/P_t = C_t^{\sigma/v}\left(1+\frac{1}{i_t}\right)^{1/\nu}$$

Log-linear version (ignoring constants):

$$w_t - p_t = \sigma c_t + \varphi n_t$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho)$$

$$m_t - p_t = \frac{\sigma}{\nu}c_t - \eta i_t$$

Standard assumption: $\sigma = \nu$

Firms

- Technology

$$Y_t = A_t N_t^{1-\alpha} \tag{1}$$

where $a_t \equiv \log A_t$ evolves according to

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

with $\rho_a \in [0, 1)$.

- Firm's problem

$$\max P_t Y_t - W_t N_t$$

subject to (1).

- Optimality conditions

$$W_t/P_t = (1-\alpha) A_t N_t^{-\alpha}$$

- Log-linear version:

$$w_t - p_t = a_t - \alpha n_t + \log(1-\alpha)$$

Equilibriium

- Goods market

$$y_t = c_t$$

- Labor market

$$\sigma c_t + \varphi n_t = w_t - p_t = a_t - \alpha n_t + \log(1 - \alpha)$$

- Asset market

$$b_t = 0$$

$$r_t \equiv i_t - E_t\{\pi_{t+1}\} = \rho + \sigma E_t\{\Delta y_{t+1}\}$$

- Aggregate production function

$$y_t = a_t + (1 - \alpha) n_t$$

- *Equilibrium values for real variables* (ignoring constants):

$$n_t = \frac{1-\sigma}{\sigma(1-\alpha)+\varphi+\alpha}\, a_t \quad ; \quad y_t = c_t = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}\, a_t$$

$$w_t - p_t = \frac{\sigma+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}\, a_t \quad ; \quad r_t = \rho - \frac{\sigma(1+\varphi)(1-\rho_a)}{\sigma(1-\alpha)+\varphi+\alpha}\, a_t$$

$\Longrightarrow$ *neutrality*: real variables independent of monetary policy

$\Longrightarrow$ *monetary policy*: determination of price level and other *nominal* variables

Next: examples of alternative monetary policy rules

A Simple Interest Rate Rule

$$i_t = \rho + \phi_\pi \pi_t + v_t$$

where $\{v_t\} \sim AR(1)$ is an exogenous monetary policy shock:

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

Fisher equation:

$$i_t = r_t + E_t\{\pi_{t+1}\}$$

Combining the two equations and defining $\widehat{r}_t \equiv r_t - \rho$:

$$\phi_\pi \pi_t = E_t\{\pi_{t+1}\} + \widehat{r}_t - v_t$$

Under the assumption $\phi_\pi > 1$, the solution is:

$$\begin{aligned} \pi_t &= \frac{1}{\phi_\pi} \sum_{k=0}^{\infty} \left(\frac{1}{\phi_\pi} \right)^k E_t\{\widehat{r}_{t+k} - v_{t+k}\} \\ &= -\frac{\psi_r}{\phi_\pi - \rho_a}\, a_t - \frac{1}{\phi_\pi - \rho_v}\, v_t \end{aligned}$$

Response to a monetary policy shock:

$$y_t = 0$$

$$\pi_t = -\frac{1}{\phi_\pi - \rho_v}\, v_t$$

$$i_t = \rho - \frac{\rho_v}{\phi_\pi - \rho_v}\, v_t$$

$$\begin{aligned} \Delta m_t &= \pi_t - \eta \Delta i_t \\ &= -\frac{1 - \eta\rho_v}{\phi_\pi - \rho_v}\, v_t - \frac{\eta\rho_v}{\phi_\pi - \rho_v}\, v_{t-1} \end{aligned}$$

$$\Rightarrow cov(\pi_t, i_t) > 0$$
$$\Rightarrow cov(y_t, i_t) = 0$$
$$\Rightarrow cov(\Delta m_t, i_t) \gtrless 0$$

Exogenous money supply $\{m_t\}$

Money demand:

$$m_t - p_t = y_t - \eta i_t$$

Combined with Fisher equation

$$p_t = \left(\frac{\eta}{1+\eta}\right) E_t\{p_{t+1}\} + \left(\frac{1}{1+\eta}\right) m_t + u_t$$

where $u_t \equiv (1+\eta)^{-1}(\eta r_t - y_t)$ is independent from $\{m_t\}$.

Equilibrium price level:

$$p_t = m_t + \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k E_t\left\{\Delta m_{t+k}\right\} + \overline{u}_t$$

where $\overline{u}_t \equiv \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k E_t\{u_{t+k}\} = -\Phi_0 a_t$.

Nominal interest rate:

$$
\begin{aligned}
i_t &= \eta^{-1}(y_t - (m_t - p_t)) \\
&= \eta^{-1} \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k E_t \{ \Delta m_{t+k} \} + \underline{u}_t
\end{aligned}
$$

where $\underline{u}_t \equiv \eta^{-1}(y_t + \overline{u}_t) = -\Phi_1 a_t$.

Example

$$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m$$

Assuming no real shocks, for simplicity ($a_t = y_t = r_t = 0$).

Price level:

$$p_t = m_t + \frac{\eta \rho_m}{1 + \eta(1 - \rho_m)} \Delta m_t$$

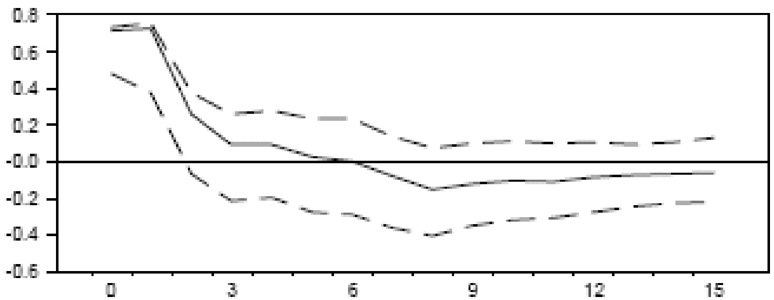
$\Longrightarrow$ *strong sensitivity to monetary policy shocks*

Nominal interest rate:

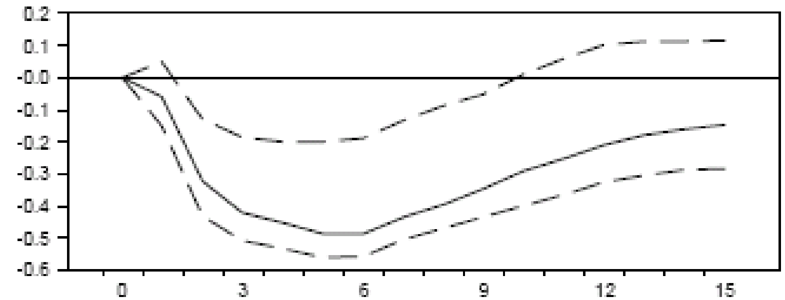
$$i_t = \frac{\rho_m}{1 + \eta(1 - \rho_m)} \Delta m_t$$

$\Longrightarrow$ *no liquidity effect*

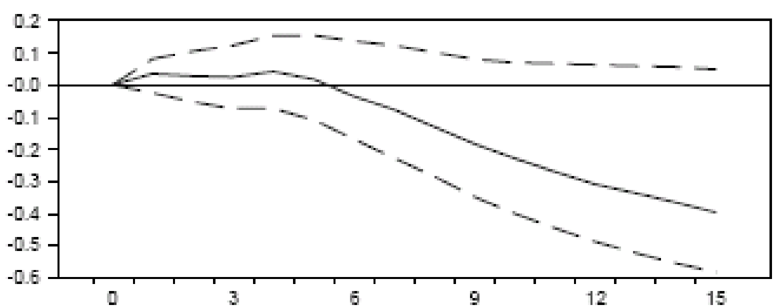
Estimated Dynamic Response to a Monetary Policy Shock



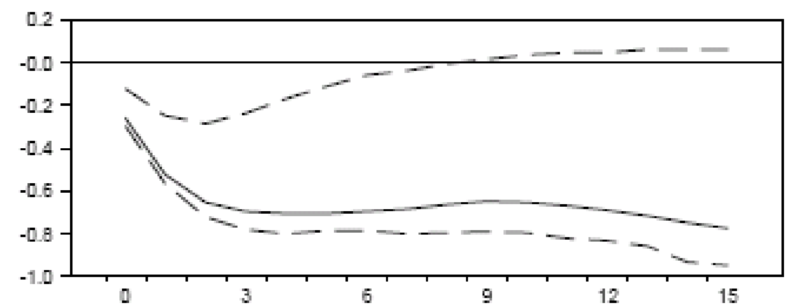
Federal funds rate



GDP



GDP deflator



M2

Source: Christiano, Eichenbaum and Evans (1999)

Money Growth, Inflation and the Nominal Rate: The Long Run

Using money demand equation

$$\pi_t = \Delta m_t - \Delta y_t + \eta \Delta i_t$$

Steady state:

$$\pi = \Delta m - \mathfrak{g}$$

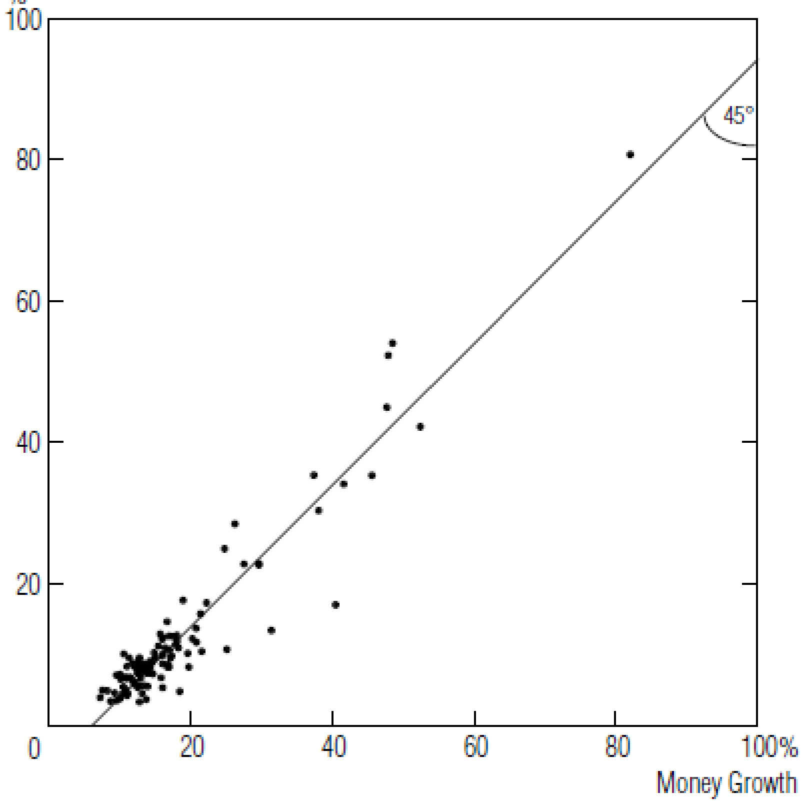
independently of monetary rule in place.

Empirical evidence

Chart 1

Money Growth and Inflation: A High, Positive Correlation

Average Annual Rates of Growth in M2 and in Consumer Prices During 1960–90 in 110 Countries



Source: International Monetary Fund

Table 1

Correlation Coefficients for Money Growth and Inflation*

Based on Data From 1960 to 1990

Sample	Coefficient for Each Definition of Money		
	M0	M1	M2
All 110 Countries	.925	.958	.950
Subsamples			
21 OECD Countries	.894	.940	.958
14 Latin American Countries	.973	.992	.993

*Inflation is defined as changes in a measure of consumer prices.
Source of basic data: International Monetary Fund

Source: McCandless and Weber (1995)

Optimal Monetary Policy

- Social planner's problem

$$\max U\left(C_t, N_t, \frac{M_t}{P_t}\right)$$

subject to

$$C_t = A_t N_t^{1-\alpha}$$

Optimality conditions

$$-\frac{U_{n,t}}{U_{c,t}} = (1-\alpha) A_t N_t^{-\alpha}$$

$$U_{m,t} = 0$$

- Optimal Policy (Friedman rule)

$$i_t = 0$$

for all t.