The Classical Monetary Model

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Assumptions

- Perfect competition in goods and labor markets
- Flexible prices and wages
- Representative household
- Money in the utility function
- No capital accumulation
- No fiscal sector
- Closed economy

Households

- *Preferences*

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

- *Budget constraint*

$$P_t C_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + W_t N_t + D_t - T_t$$

with solvency constraint:

$$\lim_{T\to\infty} E_t \{\Lambda_{t,T}(\mathcal{A}_T / P_T)\} \geq 0$$

where $Q_t \equiv \exp\{-i_t\}$ and $\mathcal{A}_t \equiv B_t + M_t$.

Households

- *Optimality Conditions*

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

$$Q_t = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\}$$

$$\frac{U_{m,t}}{U_{c,t}} = 1 - Q_t$$

- Interpretation: $1 - Q_t = 1 - \exp\{-i_t\} \simeq i_t$

$\Rightarrow$ opportunity cost of holding money

Households

- *Assumption*:

$$U\left(C_t, \frac{M_t}{P_t}, N_t\right) = \begin{cases} \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} + \chi \frac{(M_t/P_t)^{1-\sigma}-1}{1-\sigma} \text{for } \sigma \neq 1 \\ \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} + \chi \log \frac{M_t}{P_t} \quad \text{for } \sigma = 1 \end{cases}$$

- *Remark*: *separable* real balances assumed
- Implied optimality conditions

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi}$$

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) \right\}$$

$$\frac{M_t}{P_t} = \chi^{\frac{1}{\sigma}} C_t \left(1 - \exp\{-i_t\}\right)^{-\frac{1}{\sigma}}$$

Households

- Implied optimality conditions

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi}$$

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right\}$$

$$\frac{M_t}{P_t} = \chi^{\frac{1}{\sigma}} C_t \, (1 - \exp\{-i_t\})^{-\frac{1}{\sigma}}$$

- Log-linear versions:

$$w_t - p_t = \sigma c_t + \varphi n_t$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho)$$

$$m_t - p_t = c_t - \eta i_t$$

where $\pi_t \equiv p_t - p_{t-1}$ and $\beta \equiv \exp\{-\rho\}$

Firms

- *Technology*

$$Y_t = A_t N_t^{1-\alpha} \tag{1}$$

where $a_t \equiv \log A_t$ follows an exogenous process

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

- *Profit maximization:*

$$\max \; P_t Y_t - W_t N_t$$

subject to (1), taking the price and wage as given

- *Optimality condition:*

$$\frac{W_t}{P_t} = (1-\alpha) A_t N_t^{-\alpha}$$

- *Log linear version*

$$w_t - p_t = a_t - \alpha n_t + \log(1-\alpha)$$

Equilibrium

- *Goods market clearing*

$$y_t = c_t$$

- *Labor market clearing*

$$\sigma c_t + \varphi n_t = w_t - p_t = a_t - \alpha n_t + \log(1 - \alpha)$$

- *Asset market clearing*

$$B_t = 0$$

$$M_t = M_t^S$$

- *Aggregate output:*

$$y_t = a_t + (1 - \alpha) n_t$$

Equilibrium

- Equilibrium values for real variables

$$n_t = \psi_{na} a_t + \psi_n$$

$$y_t = \psi_{ya} a_t + \psi_y$$

$$r_t = \rho - \sigma \psi_{ya} (1 - \rho_a) a_t$$

$$\omega_t \equiv w_t - p_t = \psi_{\omega a} a_t + \psi_\omega$$

where $\psi_{na} \equiv \frac{1-\sigma}{\sigma(1-\alpha)+\varphi+\alpha}$; $\psi_n \equiv \frac{\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$; $\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$;
$\psi_y \equiv (1-\alpha)\psi_n$; $\psi_{\omega a} \equiv \frac{\sigma+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$; $\psi_\omega \equiv \frac{(\sigma(1-\alpha)+\varphi)\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$

- *Neutrality*: real variables *independent of monetary policy*
- Monetary policy needed to determine *nominal* variables

Price Level Determination

- **Example I: A Simple Interest Rate Rule**

$$i_t = \rho + \pi + \phi_\pi(\pi_t - \pi) + v_t$$

where $\phi_\pi \geq 0$. Combined with definition of real rate:

$$\phi_\pi \widehat{\pi}_t = E_t\{\widehat{\pi}_{t+1}\} + \widehat{r}_t - v_t$$

Case I: $\phi_\pi > 1$

$$\begin{aligned} \widehat{\pi}_t &= \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} E_t\{\widehat{r}_{t+k} - v_{t+k}\} \\ &= -\frac{\sigma(1-\rho_a)\psi_{ya}}{\phi_\pi - \rho_a}\, a_t - \frac{1}{\phi_\pi - \rho_v}\, v_t \end{aligned}$$

$\Longrightarrow$ *nominal determinacy*

Price Level Determination

Case II: $\phi_\pi < 1$

$$\widehat{\pi}_t = \phi_\pi \widehat{\pi}_{t-1} - \widehat{r}_{t-1} + v_{t-1} + \xi_t$$

for any $\{\xi_t\}$ sequence with $E_t\{\xi_{t+1}\} = 0$ for all t

$\Rightarrow$ *nominal indeterminacy*

$\Rightarrow$ illustration of "Taylor principle" requirement

Price Level Determination

- *Responses to a monetary policy shock ($\phi_\pi > 1$ case):*

$$\frac{\partial \pi_t}{\partial \varepsilon_t^v} = -\frac{1}{\phi_\pi - \rho_v} < 0$$

$$\frac{\partial i_t}{\partial \varepsilon_t^v} = -\frac{\rho_v}{\phi_\pi - \rho_v} < 0$$

$$\frac{\partial m_t}{\partial \varepsilon_t^v} = \frac{\eta \rho_v - 1}{\phi_\pi - \rho_v} \lesseqgtr 0$$

$$\frac{\partial y_t}{\partial \varepsilon_t^v} = 0$$

- *Discussion*: liquidity effect and price response

Price Level Determination

- **Example II: An Exogenous Path for the Money Supply $\{m_t\}$**

Combining money demand and the definition of the real rate:

$$p_t = \left(\frac{\eta}{1+\eta}\right) E_t\{p_{t+1}\} + \left(\frac{1}{1+\eta}\right) m_t + u_t$$

where $u_t \equiv (1+\eta)^{-1}(\eta r_t - y_t)$. Solving forward:

$$p_t = \frac{1}{1+\eta} \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k E_t\{m_{t+k}\} + \overline{u}_t$$

where $\overline{u}_t \equiv \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k E_t\{u_{t+k}\}$

$\Rightarrow$ *price level determinacy*

Price Level Determination

In terms of money growth rates:

$$p_t = m_t + \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k E_t \{\Delta m_{t+k}\} + \overline{u}_t$$

Nominal interest rate:

$$\begin{aligned} i_t &= \eta^{-1} (y_t - (m_t - p_t)) \\ &= \eta^{-1} \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k E_t \{\Delta m_{t+k}\} + \underline{u}_t \end{aligned}$$

where $\underline{u}_t \equiv \eta^{-1}(\overline{u}_t + y_t)$ is independent of monetary policy.

Price Level Determination

Assumption

$$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m$$

$$r_t = y_t = 0$$

Price response:

$$p_t = m_t + \frac{\eta \rho_m}{1 + \eta(1 - \rho_m)} \Delta m_t$$

$\Rightarrow$ *large price response*

Nominal interest rate response:

$$i_t = \frac{\rho_m}{1 + \eta(1 - \rho_m)} \Delta m_t$$

$\Rightarrow$ *no liquidity effect*

The Case of Non-Separable Real Balances

- Labor supply affected by monetary policy $\Rightarrow$ non-neutrality
- Example:

$$U(X_t, N_t) = \frac{X_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

where

$$\begin{aligned} X_t &\equiv \left[(1-\vartheta) C_t^{1-\nu} + \vartheta \left(\frac{M_t}{P_t} \right)^{1-\nu} \right]^{\frac{1}{1-\nu}} \quad \text{for } \nu \neq 1 \\ &\equiv C_t^{1-\vartheta} \left(\frac{M_t}{P_t} \right)^{\vartheta} \quad \text{for } \nu = 1 \end{aligned}$$

The Case of Non-Separable Real Balances

- Optimality conditions:

$$\frac{W_t}{P_t} = N_t^{\varphi} X_t^{\sigma-\nu} C_t^{\nu} (1-\vartheta)^{-1}$$

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\nu} \left(\frac{X_{t+1}}{X_t} \right)^{\nu-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right\}$$

$$\frac{M_t}{P_t} = C_t \left(1 - \exp\{-i_t\}\right)^{-\frac{1}{\nu}} \left(\frac{\vartheta}{1-\vartheta} \right)^{\frac{1}{\nu}}$$

- Log-linearized money demand equation:

$$m_t - p_t = c_t - \eta i_t$$

where $\eta \equiv 1/[\nu(\exp\{i\} - 1)]$

The Case of Non-Separable Real Balances

- Log-linearized labor supply equation:

$$\begin{aligned} w_t - p_t &= \sigma c_t + \varphi n_t + (\nu - \sigma)(c_t - x_t) \\ &= \sigma c_t + \varphi n_t + \chi(\nu - \sigma)\left(c_t - (m_t - p_t)\right) \\ &= \sigma c_t + \varphi n_t + \eta\chi(\nu - \sigma) i_t \end{aligned}$$

where $\chi \equiv \frac{k_m(1-\beta)}{1+k_m(1-\beta)} \in [0,1)$ with $k_m \equiv \frac{M/P}{C} = \left(\frac{\vartheta}{(1-\beta)(1-\vartheta)}\right)^{\frac{1}{\nu}}$

Equivalently,

$$w_t - p_t = \sigma c_t + \varphi n_t + \varpi i_t$$

where $\varpi \equiv \frac{k_m \beta (1-\frac{\sigma}{\nu})}{1+k_m(1-\beta)}$

The Case of Non-Separable Real Balances

- Labor market clearing:

$$\sigma c_t + \varphi n_t + \varpi i_t = a_t - \alpha n_t + \log(1-\alpha)$$

which combined with aggregate production function:

$$y_t = \psi_{ya} a_t + \psi_{yi} i_t$$

where $\psi_{yi} \equiv -\frac{\varpi(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$ and $\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$

$\Rightarrow$ long run non-superneutrality

- *Assessment of size of short-run non-neutralities*

  Calibration: $\beta = 0.99$; $\sigma = 1$; $\varphi = 5$; $\alpha = 1/4$; $\nu = 1/\eta i$ "large"

$$\Rightarrow \varpi \simeq \frac{k_m \beta}{1 + k_m(1-\beta)} > 0 \quad ; \quad \psi_{yi} \simeq -\frac{k_m}{8} < 0$$

Monetary base inverse velocity: $k_m \simeq 0.3 \quad \Rightarrow \psi_{yi} \simeq -0.04$

M2 inverse velocity: $k_m \simeq 3 \quad \Rightarrow \psi_{yi} \simeq -0.4$

$\Rightarrow$ small output effects of monetary policy

Optimal Monetary Policy

- *Social Planner's problem*

$$\max U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

subject to

$$C_t = A_t N_t^{1-\alpha}$$

Optimality conditions:

$$-\frac{U_{n,t}}{U_{c,t}} = (1-\alpha)A_t N_t^{-\alpha}$$

$$U_{m,t} = 0$$

Optimal policy (Friedman rule): $\quad i_t = 0$ for all t

Intuition

Implied average inflation: $\quad \pi = -\rho < 0$

- *Implementation*

$$i_t = \phi(r_{t-1} + \pi_t)$$

for some $\phi > 1$. Combined with the definition of the real rate:

$$E_t\{i_{t+1}\} = \phi i_t$$

whose only stationary solution is $i_t = 0$ for all t.

Implied equilibrium inflation:

$$\pi_t = -r_{t-1}$$