Advanced Macroeconomics II

Monetary Models with Nominal Rigidities

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Motivation

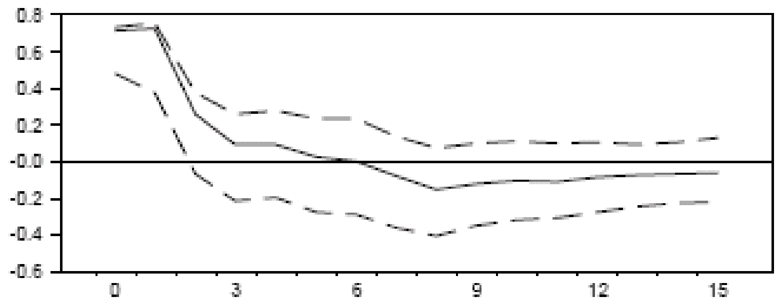
Empirical Evidence

- Macro evidence on the effects of monetary policy shocks

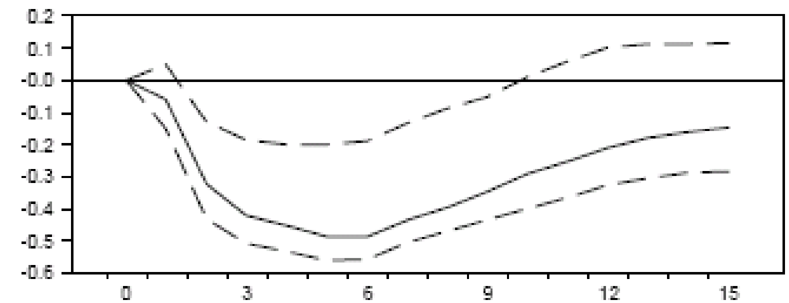
  (i) persistent real effects
  (ii) sluggish adjustment of aggregate price level
  (iii) liquidity effect

- Micro evidence on patterns of price setting

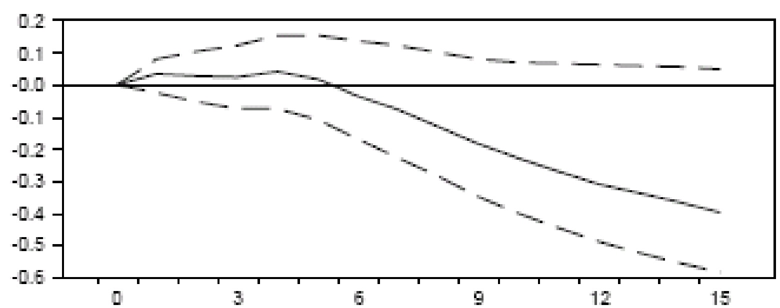
Figure 1. Estimated Dynamic Response to a Monetary Policy Shock



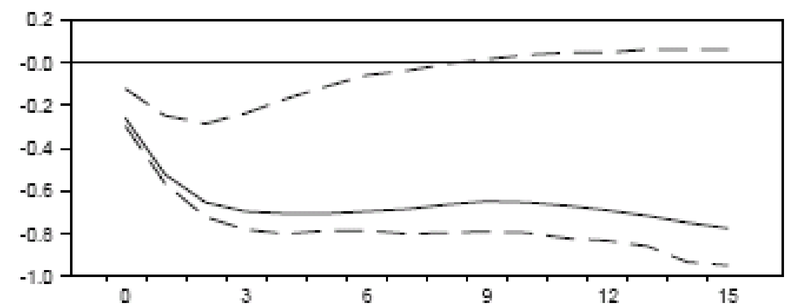
Federal funds rate



GDP



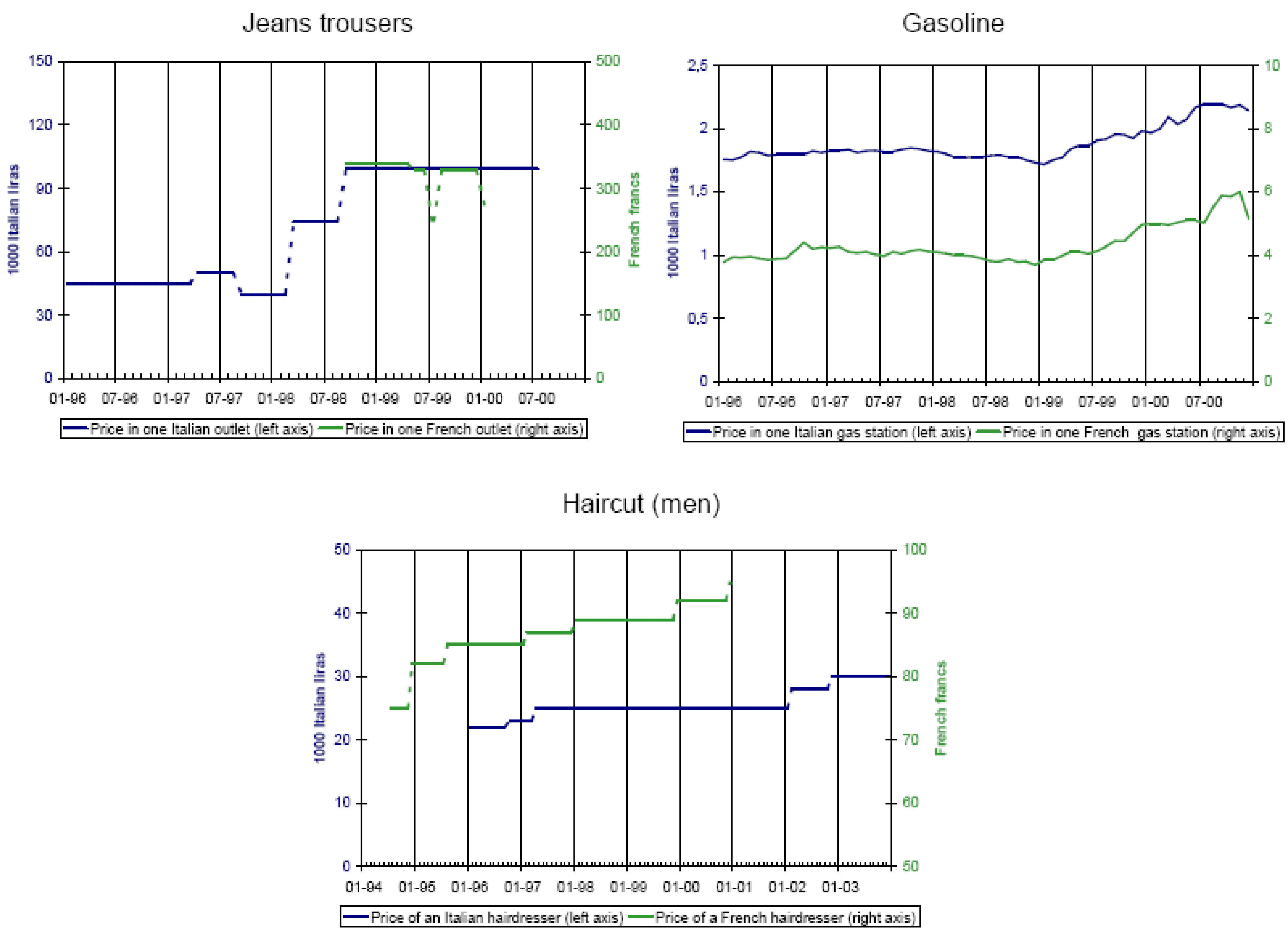
GDP deflator



M2

Source: Christiano, Eichenbaum and Evans (1999)

Figure 1 - Examples of individual price trajectories (French and Italian CPI data)



Note : Actual examples of trajectories, extracted from the French and Italian CPI databases. The databases are described in Baudry *et al.* (2004) and Veronese *et al.* (2005). Prices are in levels, denominated in French Francs and Italian Lira respectively. The dotted lines indicate events of price changes.

Source: Dhyne et al. (JEP, 2006)

TABLE 1. Measures of price stickiness in the euro area and the US (% per month unless otherwise stated).

	Statistics	Euro area	US
CPI*	Frequency	15.1	24.8
	Average duration (*months*)	13.0	6.7
	Median duration (*months*)	10.6	4.6
PPI†	Frequency	20.0	n.a
Surveys‡	Frequency	15.9	20.8
	Average duration (*months*)	10.8	8.3
NKPC§	Average durations (*months*)	13.5–19.2	7.2–8.4
Internet prices¶	Frequency	79.2	64.3

Source: Álvarez et al. (JEEA, 2006)

Models with Nominal Rigidities

- differentiated goods and monopolistic competition in goods market
- sticky prices
- competitive labor market, closed economy, no capital accumulation

Maintained assumptions

- households' optimality conditions

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho)$$
$$w_t - p_t = \sigma c_t + \varphi n_t$$
$$m_t - p_t = c_t - \eta i_t$$

- continuum of firms/differentiated goods, indexed by $i \in [0, 1]$
- technology: $Y_t(i) = A_t N_t(i)^{1-\alpha}$
- implied marginal cost:

$$\psi_t(i) = w_t - a_t + \alpha n_t(i) - \log(1 - \alpha)$$

Monopolistic Competition with Flexible Prices

Firm's problem

$$\max_{P_t(i)} \; P_t(i)Y_t(i) - \mathcal{C}_t(Y_t(i))$$

subject to demand schedule:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} C_t$$

Optimality condition:

$$P_t(i) = \mathcal{M}\Psi_t(i)$$

where $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$ ("optimal gross markup") and $\Psi_t(i) \equiv \mathcal{C}_t'(Y_t(i))$ ("marginal cost").
In logs:

$$\begin{aligned} p_t(i) &= \mu + \psi_t(i) \\ &= \mu + w_t - (a_t - \alpha n_t(i) + \log(1-\alpha)) \end{aligned}$$

where $\mu \equiv \log \mathcal{M}$ and $\psi_t(i) \equiv \log \Psi_t(i)$.

Equilibrium

- *Goods market*

$$y_t(i) = c_t(i), \; i \in [0,1] \quad \Longrightarrow \quad y_t = c_t$$

- *Aggregate demand*

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho)$$

- *Labor market*

$$n_t = \frac{1}{1-\alpha}(y_t - a_t)$$

$$w_t = p_t + \sigma c_t + \varphi n_t$$

- *Price setting*

$$p_t = \mu + w_t - (a_t - \alpha n_t + \log(1-\alpha))$$

- *Asset market*

$$b_t = 0$$

Equilibrium values:

$$n_t = \frac{1-\sigma}{\sigma(1-\alpha)+\varphi+\alpha} a_t + \frac{\log(1-\alpha)-\mu}{\sigma(1-\alpha)+\varphi+\alpha}$$

$$y_t = c_t = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} a_t + \frac{(1-\alpha)(\log(1-\alpha)-\mu)}{\sigma(1-\alpha)+\varphi+\alpha}$$

$$w_t - p_t = \frac{\sigma+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} a_t + \frac{(\sigma(1-\alpha)+\varphi)(\log(1-\alpha)-\mu)}{\sigma(1-\alpha)+\varphi+\alpha}$$

$$r_t \equiv i_t - E_t\{\pi_{t+1}\} = \rho - \frac{\sigma(1+\varphi)(1-\rho_a)}{\sigma(1-\alpha)+\varphi+\alpha}\, a_t$$

$\Longrightarrow$ role of market power (μ)
$\Longrightarrow$ inefficiencies
$\Longrightarrow$ monetary policy neutrality
$\Longrightarrow$ role of monetary policy and optimal monetary policy as in the classical model

Monopolistic Competition with Constant Prices

Assumptions:

- constant prices: $p_t = p = 0$ (normalization), $t = 0, 1, 2, ...$
- non-negative markup: $\mu_t = p - \psi_t \geq 0$, $t = 0, 1, 2, ...$

Equilibrium

$$y_t = c_t$$

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - \rho)$$

$$n_t = \frac{1}{1-\alpha}(y_t - a_t)$$

$$w_t = \sigma y_t + \varphi n_t$$

$$\mu_t = w_t - (a_t - \alpha n_t - \log(1-\alpha))$$

Monetary policy (I): Interest rate rule

$$i_t = \rho + \phi_y y_t + v_t$$

where

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

Equilibrium:

$$y_t = -\frac{1}{\sigma(1-\rho_v)+\phi_y}\, v_t$$

$$n_t = -\frac{1}{[\sigma(1-\rho_v)+\phi_y](1-\alpha)}\, v_t - \frac{1}{1-\alpha}\, a_t$$

$\Longrightarrow$ *non-neutrality* of monetary policy: two dimensions

$\Longrightarrow$ technology shocks lower employment, unless offset by policy response

Exercise: derive an interest rate rule that will replicate the flexible price equilibrium

Monetary policy (II): Exogenous money supply

Combining money demand and aggregate demand:

$$y_t = \frac{1}{1+\sigma\eta} \sum_{k=0}^{\infty} \left(\frac{\sigma\eta}{1+\sigma\eta} \right)^k E_t\{m_{t+k}\}$$

Example: $m_t = \rho_m m_{t-1} + \varepsilon_t^m$

$$y_t = \frac{1}{1+\sigma\eta(1-\rho_m)}\, m_t$$

$$n_t = \frac{1}{(1-\alpha)[1+\sigma\eta(1-\rho_m)]}\, m_t - \frac{1}{1-\alpha}\, a_t$$

$\Longrightarrow$ *non-neutrality* of money supply changes (anticipated and unanticipated)
$\Longrightarrow$ technology shocks lower employment, unless offset by policy response

The Basic New Keynesian Model

- *The New Keynesian Phillips Curve*

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

where $\widetilde{y}_t \equiv y_t - y_t^n$ ("output gap").

- *Dynamic IS equation*

$$\widetilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n) + E_t\{\widetilde{y}_{t+1}\}$$

- *Interest rate rule*

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t + v_t$$

The New Keynesian Phillips Curve

- *Assumption:* probability of price adjustment: $1-\theta$ (independent across firms)

  $\Rightarrow$ fraction of firms keeping price unchanged: θ
  $\Rightarrow \theta \in [0,1]$: índex of price rigidity

- *Price level evolution*

$$p_t = \theta p_{t-1} + (1-\theta) p_t^*$$

- *Optimal price setting*

$$p_t^* = \mu + (1-\beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t\{\psi_{t+k}\}$$

- *Combining both equations*

$$\pi_t = \beta E_t\{\pi_{t+1}\} - \lambda(\mu_t - \mu)$$

where $\mu_t \equiv p_t - \psi_t$ (average price markup) and $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$

- *Average markup* (assumption: $\alpha = 0$)

$$\mu_t = p_t - (w_t - a_t)$$

- *Labor market equilibrium*

$$w_t - p_t = \sigma c_t + \varphi n_t$$

$$n_t = y_t - a_t$$

- *Goods market equilibrium*

$$y_t = c_t$$

- *Average price markup and the output gap*

$$\mu_t = (1 + \varphi) a_t - (\sigma + \varphi) y_t$$

With flexible prices:

$$\mu = (1 + \varphi) a_t - (\sigma + \varphi) y_t^n$$

$$\Rightarrow \; y_t^n = -\frac{\mu}{\sigma + \varphi} + \frac{1 + \varphi}{\sigma + \varphi} a_t$$

Combining both equations:

$$\mu_t - \mu = -(\sigma + \varphi)\,\widetilde{y}_t$$

- *New Keynesian Phillips curve*

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

where $\kappa \equiv \lambda\,(\sigma + \varphi)$

- *Properties*

(i) "Forward-looking"

$$\pi_t = \kappa \sum_{k=0}^{\infty} \beta^k E_t\{\widetilde{y}_{t+k}\}$$

$\Rightarrow$ past inflation is irrelevant

(ii) No trade-off between stabilization of inflation and output gap.

$\Rightarrow$ "Divine coincidence" (Blanchard-Galí)

$\Rightarrow$ no costs of disinflations.

(iii) Two "output gap" concepts:

$$\widehat{y}_t = y_t - f(t)$$

$$\widetilde{y}_t \equiv y_t - y_t^n$$

$\Rightarrow$ complicates empirical evaluation (Galí-Gertler 1998).

Dynamic IS Equation

- Intertemporal optimality condition + good market equilibrium

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho)$$

Combined with $\widetilde{y}_t \equiv y_t - y_t^n$

$$\widetilde{y}_t = -\frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - r_t^n\right) + E_t\{\widetilde{y}_{t+1}\}$$

where

$$\begin{aligned} r_t^n &\equiv \rho + \sigma E_t\{\Delta y_{t+1}^n\} \\ &= \rho + \frac{\sigma(1+\varphi)}{\sigma+\varphi} E_t\{\Delta a_{t+1}\} \end{aligned}$$

Monetary Policy

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t + v_t$$

The Basic New Keynesian Model

- *The New Keynesian Phillips Curve*

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

where $\widetilde{y}_t \equiv y_t - y_t^n$ ("output gap").

- *Dynamic IS equation*

$$\widetilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n) + E_t\{\widetilde{y}_{t+1}\}$$

- *Interest Rate Rule*

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t + v_t$$

- *Exogenous variables*

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

Solving for the Equilibrium

- Assumptions (for simplicity):

  (i) $\{v_t\}$ and $\{a_t\}$ white noise ($\rho_a = \rho_v = 0$)

  (ii) $i_t = \rho + \phi_\pi \pi_t + v_t$

  (iii) $\sigma = 1$

  $$\Rightarrow \widehat{r}_t^n = -a_t$$
  $$\Rightarrow \widehat{y}_t^n = a_t$$

- Conjecture ("method of undetermined coefficients"):

$$\widetilde{y}_t = \psi_{ya} a_t + \psi_{yv} v_t$$

$$\pi_t = \psi_{\pi a} a_t + \psi_{\pi v} v_t$$

- Solution

$$\widetilde{y}_t = -\frac{1}{1+\kappa\phi_\pi}a_t - \frac{1}{1+\kappa\phi_\pi}v_t$$

$$\pi_t = -\frac{\kappa}{1+\kappa\phi_\pi}a_t - \frac{\kappa}{1+\kappa\phi_\pi}v_t$$

$$\widehat{y}_t = \widetilde{y}_t + \widehat{y}_t^n = \frac{\kappa\phi_\pi}{1+\kappa\phi_\pi}a_t - \frac{1}{1+\kappa\phi_\pi}v_t$$

$$\widehat{n}_t = \widehat{y}_t - a_t = -\frac{1}{1+\kappa\phi_\pi}a_t - \frac{1}{1+\kappa\phi_\pi}v_t$$

$$i_t = \rho - \frac{\kappa\phi_\pi}{1+\kappa\phi_\pi}a_t + \frac{1}{1+\kappa\phi_\pi}v_t$$

$$m_t = \frac{\kappa(\phi_\pi(1+\eta)-1)}{1+\kappa\phi_\pi}a_t - \frac{1+\kappa+\eta}{1+\kappa\phi_\pi}v_t + p_{t-1}$$

- Discussion

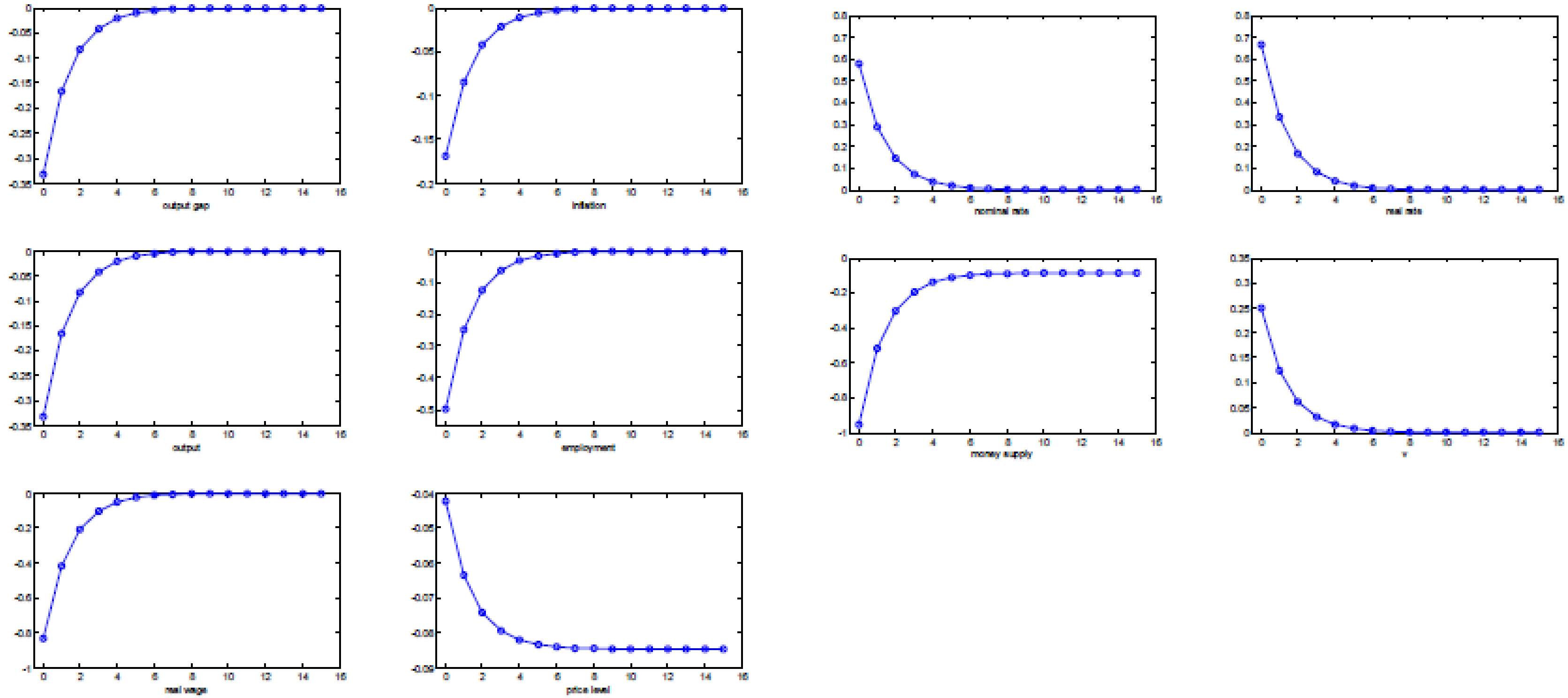
Simulations of Calibrated Model (Galí 2008/ rev2015)

- Calibration

  $\beta = 0.99,\ \sigma = 1,\ \varphi = 5,\ \epsilon = 9$
  $\alpha = 1/4$
  $\phi_\pi = 1.5,\ \phi_y = 0.5/4$
  $\theta = 3/4$
  $\eta = 4$
  $\rho_v = 0.5,\ \rho_a = 0.9$

- Effects of monetary policy shock
- Effects of technology shock

Dynamic responses to a monetary policy shock: Interest rate rule



Dynamic responses to a technology shock: Interest rate rule