Monetary Policy Design in the Basic New Keynesian Model

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**New Keynesian Phillips Curve**

\[ \pi_t = \beta E_t \{\pi_{t+1}\} + \kappa_p \tilde{y}_t \]

**Dynamic IS Equation**

\[ \tilde{y}_t = -\frac{1}{\sigma} (i_t - E_t \{\pi_{t+1}\} - r^n_t) + E_t \{\tilde{y}_{t+1}\} \]

where

\[ r^n_t = \rho - \frac{\sigma(1 + \varphi)(1 - \rho_a)}{\sigma + \varphi} a_t + (1 - \rho_z)z_t \]
Monetary Policy Design: The Case of an Efficient Natural Equilibrium

- Assumption:
  \[ y_t^n = y_t^e \]

- Optimal Policy
  \[ \tilde{y}_t = 0 \quad ; \quad \pi_t = 0 \]

- Implementation
  \[ i_t = r_t^n + \phi_\pi \pi_t \]
  where \( \phi_\pi > 1 \) (determinacy condition)
Evaluation of Alternative Policies

Welfare losses (second order approx.)

\[ \mathbb{W} \equiv -E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{U_t - U^n_t}{U_C C} \right) = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \tilde{y}_t^2 + \frac{\epsilon}{\lambda} \pi_t^2 \right] \]

Average unconditional welfare losses:

\[ \mathbb{I}L = \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \text{var}(\tilde{y}_t) + \frac{\epsilon}{\lambda} \text{var}(\pi_t) \]

Example:

\[ i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t \]
### Table 4.1
Evaluation of Simple Rules: Taylor Rule

<table>
<thead>
<tr>
<th></th>
<th>Technology</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_\pi)</td>
<td>1.5 1.5 5 1.5</td>
<td>1.5 1.5 5 1.5</td>
</tr>
<tr>
<td>(\phi_y)</td>
<td>0.125 0 0 1</td>
<td>0.125 0 0 1</td>
</tr>
<tr>
<td>(\sigma(y))</td>
<td>1.85 2.07 2.25 1.06</td>
<td>0.59 0.68 0.28 0.31</td>
</tr>
<tr>
<td>(\sigma(\tilde{y}))</td>
<td>0.44 0.21 0.03 1.23</td>
<td>0.59 0.68 0.28 0.31</td>
</tr>
<tr>
<td>(\sigma(\pi))</td>
<td>0.69 0.34 0.05 1.94</td>
<td>0.20 0.23 0.09 0.10</td>
</tr>
<tr>
<td>(L)</td>
<td>1.02 0.25 0.006 7.98</td>
<td>0.10 0.13 0.02 0.02</td>
</tr>
</tbody>
</table>
The Taylor Rule (Taylor 1993)

\[ i_t = 4 + 1.5(\pi_t - 2) + 0.5 y_t \]

Source: Taylor 1999
Source: Taylor 1999
Clarida, Galí and Gertler (QJE 2000)

\[ i_t = \rho i_{t-1} + (1 - \rho) \left[ r + \pi^* + \beta E_t \{ \pi_{t+1} - \pi^* \} + \gamma E_t \{ y_{t+1} - y_{t+1}^* \} \right] \]

<table>
<thead>
<tr>
<th>TABLE II</th>
<th>Baseline Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \pi^* )</td>
</tr>
<tr>
<td>Pre-Volcker</td>
<td>4.24</td>
</tr>
<tr>
<td></td>
<td>(1.09)</td>
</tr>
<tr>
<td>Volcker-Greenspan</td>
<td>3.58</td>
</tr>
<tr>
<td></td>
<td>(0.50)</td>
</tr>
</tbody>
</table>

Standard errors are reported in parentheses. The set of instruments includes four lags of inflation: output gap, the federal funds rate, the short-long spread, and commodity price inflation.
Orphanides (JME 2003)
Fig. 5. Then and now: Taylor rule with final and real-time data.
Monetary Policy Design: The Case of an Inefficient Natural Equilibrium

- **Assumption:** time-varying $y_t^n - y_t^e$
- **The New Keynesian Phillips Curve**
  \[ \pi_t = \beta E_t\{\pi_{t+1}\} + \kappa x_t + u_t \]
  where $x_t \equiv y_t - y_t^e$ and $u_t \equiv \kappa(y_t^e - y_t^n)$
- **Dynamic IS Equation**
  \[ x_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^e) + E_t\{x_{t+1}\} \]
  where $r_t^e \equiv \rho + \sigma E_t\{\Delta y_{t+1}^e\} + (1 - \rho_z)z_t = r_t^n$
The Optimal Monetary Policy Problem

\[
\min E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \vartheta x_t^2)
\]

subject to:

\[
\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa x_t + u_t
\]

where \(\{u_t\}\) evolves exogenously according to

\[
u_t = \rho_u u_{t-1} + \varepsilon_t
\]

In addition:

\[
x_t = -\frac{1}{\sigma}(i_t - E_t \{\pi_{t+1}\} - r_t^e) + E_t \{x_{t+1}\}
\]

**Note:** Utility based criterion requires \(\vartheta = \frac{\kappa}{\varepsilon}\)
Each period CB chooses \((x_t, \pi_t)\) to minimize

\[
\pi_t^2 + \vartheta x_t^2
\]

subject to

\[
\pi_t = \kappa x_t + v_t
\]

where \(v_t \equiv \beta E_t\{\pi_{t+1}\} + u_t\) is taken as given.

**Optimality condition:**

\[
x_t = -\frac{\kappa}{\vartheta} \pi_t
\]

**Equilibrium**

\[
\pi_t = \frac{\vartheta}{\kappa^2 + \vartheta(1 - \beta \rho_u)} u_t ; \quad x_t = -\frac{\kappa}{\kappa^2 + \vartheta(1 - \beta \rho_u)} u_t
\]

\[
i_t = r_t^e + \frac{\vartheta \rho_u + \sigma \kappa (1 - \rho_u)}{\kappa^2 + \vartheta(1 - \beta \rho_u)} u_t
\]
Implementation:

\[ i_t = r^e_t + \frac{\vartheta \rho_u + \sigma \kappa (1 - \rho_u)}{\kappa^2 + \vartheta (1 - \beta \rho_u)} u_t + \phi_\pi \left( \pi_t - \frac{\vartheta}{\kappa^2 + \vartheta (1 - \beta \rho_u)} u_t \right) \]

\[ = r^e_t + \Theta u_t + \phi_\pi \pi_t \]

where \( \Theta \equiv \frac{\sigma \kappa (1 - \rho_u) - \vartheta (\phi_\pi - \rho_u)}{\kappa^2 + \vartheta (1 - \beta \rho_u)} \) and \( \phi_\pi > 1 \).
Figure 5.1
Discretion vs. Commitment: Responses to a Transitory Cost-Push Shock

Output gap vs. time:
- Discretion line (blue)
- Commitment line (red)

Inflation vs. time:
- Discretion line (blue)
- Commitment line (red)

Price level vs. time:
- Discretion line (blue)
- Commitment line (red)

Cost-push shock vs. time:
- Discretion line (blue)
- Commitment line (red)
Figure 5.2
Discretion vs. Commitment: Responses to a Persistent Cost-Push Shock
State-contingent policy \( \{x_t, \pi_t\}_{t=0}^{\infty} \) that minimizes

\[
E_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \vartheta x_t^2)
\]

subject to the sequence of constraints:

\[
\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa x_t + u_t
\]

Lagrangean:

\[
\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[ \frac{1}{2} (\pi_t^2 + \vartheta x_t^2) + \xi_t (\pi_t - \kappa x_t - \beta \pi_{t+1}) \right]
\]

Optimality conditions:

\[
\vartheta x_t - \kappa \xi_t = 0
\]

\[
\pi_t + \xi_t - \xi_{t-1} = 0
\]

for \( t = 0, 1, 2, \ldots \) with \( \xi_{-1} = 0 \).
Eliminating multipliers:

\[
x_0 = -\frac{\kappa}{\vartheta} \pi_0
\]

\[
x_t = x_{t-1} - \frac{\kappa}{\vartheta} \pi_t
\]

for \( t = 1, 2, 3, \ldots \)

Alternative representation:

\[
x_t = -\frac{\kappa}{\vartheta} \hat{p}_t
\]

for \( t = 0, 1, 2, \ldots \) where \( \hat{p}_t \equiv p_t - p_{-1} \)
Equilibrium

\[ \hat{\rho}_t = \gamma \hat{\rho}_{t-1} + \gamma \beta E_t \{ \hat{\rho}_{t+1} \} + \gamma u_t \]

for \( t = 0, 1, 2, \ldots \) where \( \gamma \equiv \frac{\vartheta}{\vartheta(1+\beta)+\kappa^2} \)

Stationary solution:

\[ \hat{\rho}_t = \delta \hat{\rho}_{t-1} + \frac{\delta}{1 - \delta \beta \rho_u} \ u_t \]

for \( t = 0, 1, 2, \ldots \) where \( \delta \equiv \frac{1-\sqrt{1-4\beta \gamma^2}}{2\gamma \beta} \in (0, 1) \).

\[ \rightarrow \text{price level targeting!} \]

\[ x_t = \delta x_{t-1} - \frac{\kappa \delta}{\vartheta(1 - \delta \beta \rho_u)} \ u_t \]

for \( t = 1, 2, 3, \ldots, \) and

\[ x_0 = \frac{-\kappa \delta}{\vartheta(1 - \delta \beta \rho_u)} \ u_0 \]
Discussion: Gains from Commitment

\[ \pi_t = \kappa x_t + \kappa \sum_{k=1}^{\infty} \beta^k E_t \{ x_{t+k} \} + \frac{1}{1 - \beta \rho_u} u_t \]

Illustration: Optimal Monetary Policy under the Zero Lower Bound ("Forward Guidance")
Figure 5.3
Discretion vs. Commitment in the Presence of a ZLB