Monetary Policy Design in the Basic New Keynesian Model

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The Basic New Keynesian Model: Non-Policy Block

- *New Keynesian Phillips Curve*

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa_p \widetilde{y}_t$$

- *Dynamic IS Equation*

$$\widetilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n) + E_t\{\widetilde{y}_{t+1}\}$$

where

$$r_t^n = \rho - \frac{\sigma(1+\varphi)(1-\rho_a)}{\sigma+\varphi} a_t + (1-\rho_z) z_t$$

Monetary Policy Design: The Case of an Efficient Natural Equilibrium

- Assumption:

$$y_t^n = y_t^e$$

- Optimal Policy

$$\widetilde{y}_t = 0 \quad ; \quad \pi_t = 0$$

- Implementation

$$i_t = r_t^n + \phi_\pi \pi_t$$

where $\phi_\pi > 1$ (determinacy condition)

Monetary Policy Design: Simple Rules

- Evaluation of Alternative Policies

  Welfare losses (second order approx.)

$$\mathbb{W} \equiv -E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{U_t - U_t^n}{U_c C} \right) = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \widetilde{y}_t^2 + \frac{\epsilon}{\lambda} \pi_t^2 \right]$$

  Average unconditional welfare losses:

$$\mathbb{L} = \left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) var(\widetilde{y}_t) + \frac{\epsilon}{\lambda} \, var(\pi_t)$$

- Example:

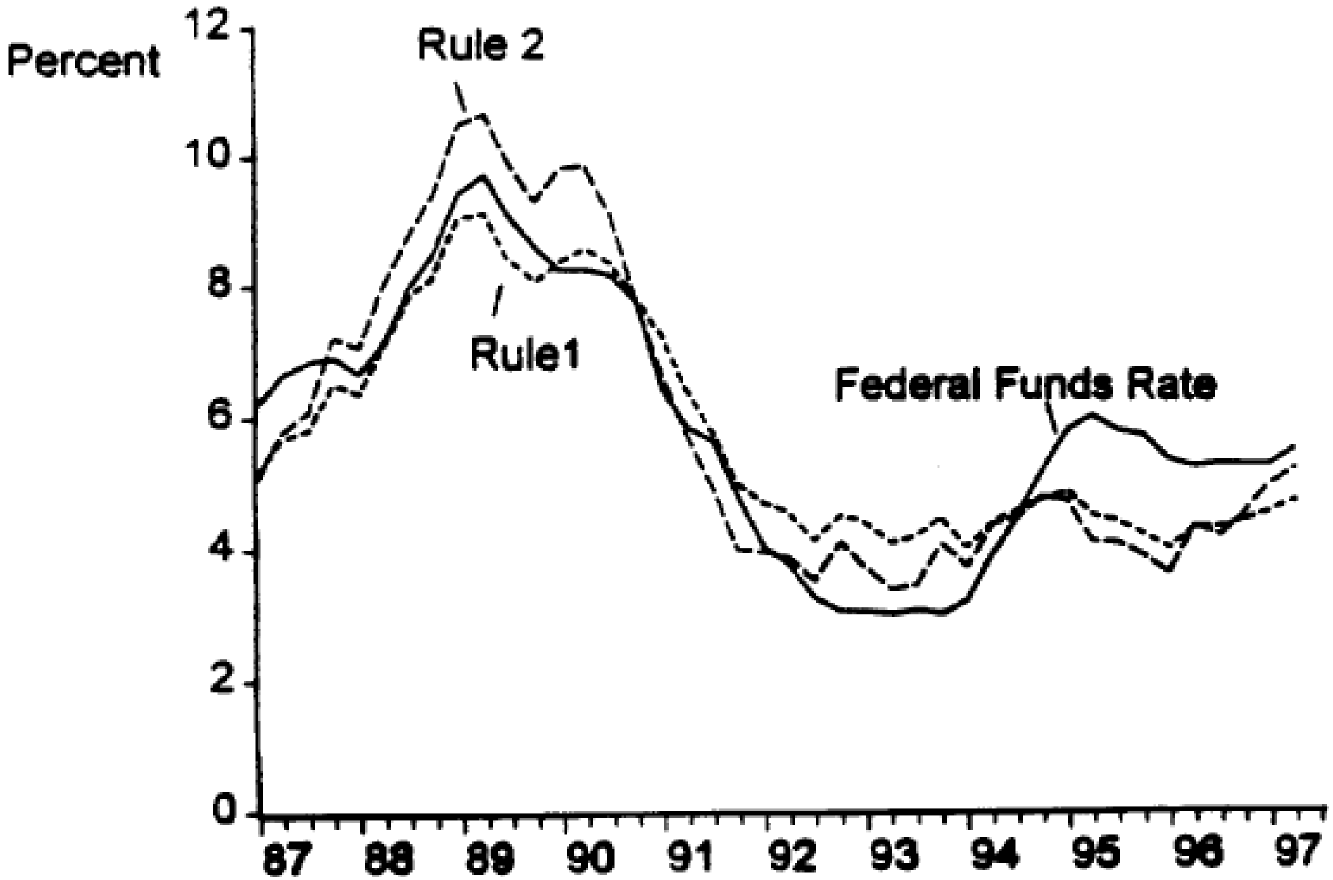
$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t$$

Table 4.1
Evaluation of Simple Rules: Taylor Rule

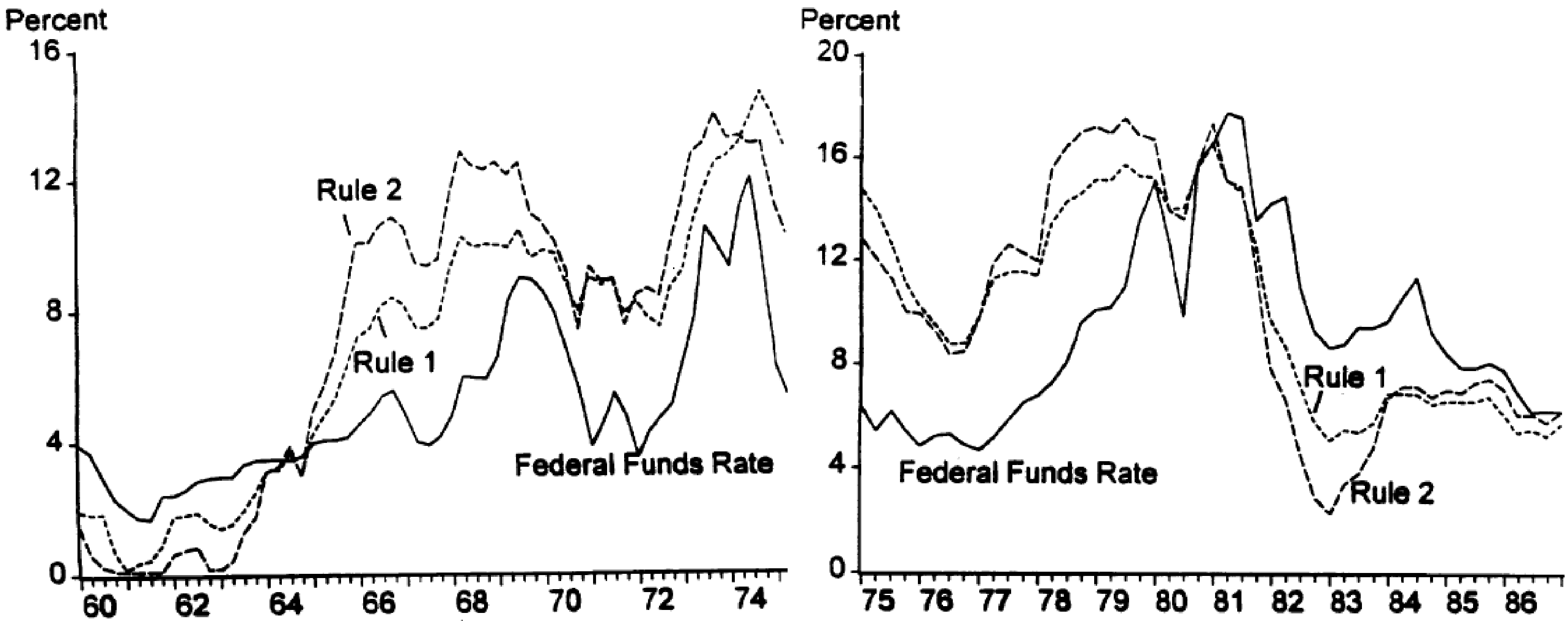
	Technology				*Demand*			
ϕ_π	1.5	1.5	5	1.5	1.5	1.5	5	1.5
ϕ_y	0.125	0	0	1	0.125	0	0	1
$\sigma(y)$	1.85	2.07	2.25	1.06	0.59	0.68	0.28	0.31
$\sigma(\widetilde{y})$	0.44	0.21	0.03	1.23	0.59	0.68	0.28	0.31
$\sigma(\pi)$	0.69	0.34	0.05	1.94	0.20	0.23	0.09	0.10
$\mathbb{L}$	1.02	0.25	0.006	7.98	0.10	0.13	0.02	0.02

The Taylor Rule (Taylor 1993)

$$i_t = 4 + 1.5(\pi_t - 2) + 0.5\, y_t$$



Source: Taylor 1999



Source: Taylor 1999

Clarida, Galí and Gertler (QJE 2000)

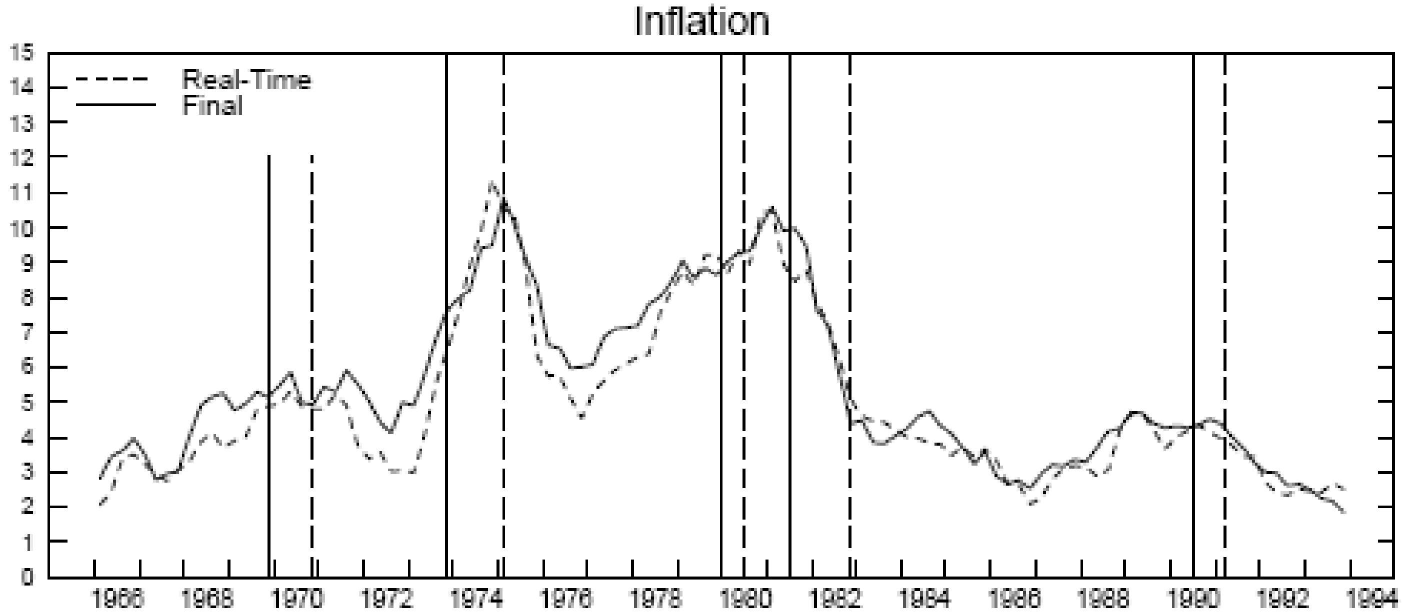
$$i_t = \rho i_{t-1} + (1-\rho)[r + \pi^* + \beta E_t\{\pi_{t+1} - \pi^*\} + \gamma E_t\{y_{t+1} - y^*_{t+1}\}]$$

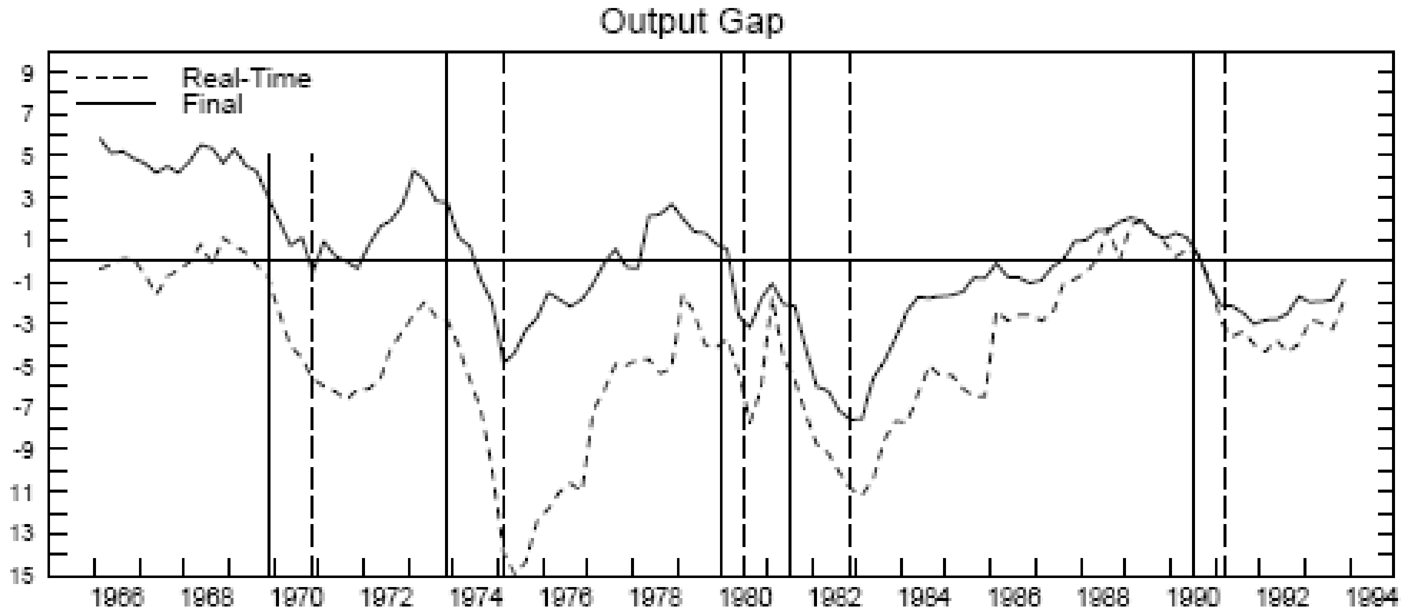
TABLE II
BASELINE ESTIMATES

	π^*	β	γ	ρ	p
Pre-Volcker	4.24	0.83	0.27	0.68	0.834
	(1.09)	(0.07)	(0.08)	(0.05)	
Volcker-Greenspan	3.58	2.15	0.93	0.79	0.316
	(0.50)	(0.40)	(0.42)	(0.04)	

Standard errors are reported in parentheses. The set of instruments includes four lags of inflation: output gap, the federal funds rate, the short-long spread, and commodity price inflation.

Orphanides (JME 2003)





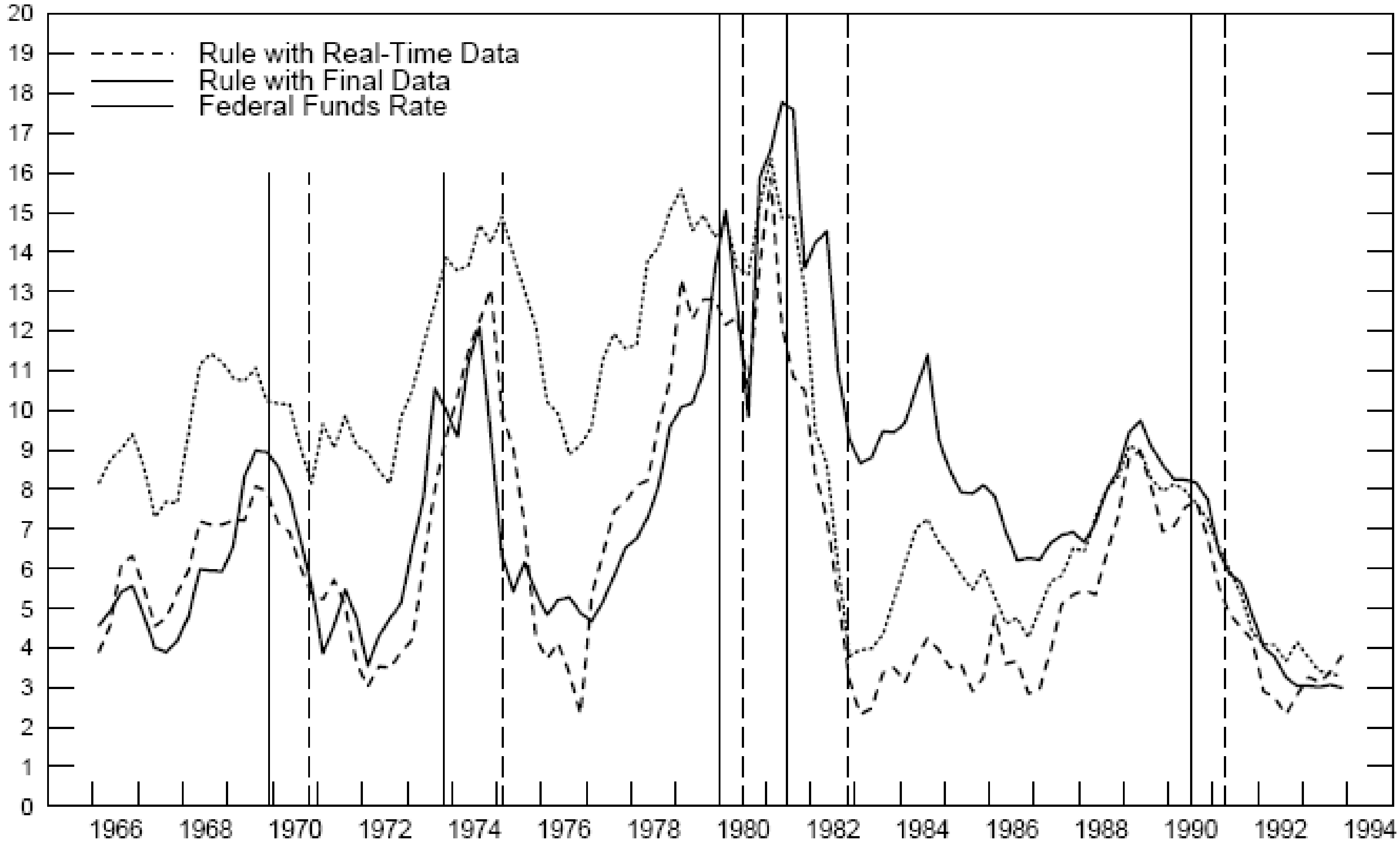


Fig. 5. Then and now: Taylor rule with final and real-time data.

Monetary Policy Design: The Case of an Inefficient Natural Equilibrium

- *Assumption*: time-varying $y_t^n - y_t^e$
- *The New Keynesian Phillips Curve*

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa x_t + u_t$$

where $x_t \equiv y_t - y_t^e$ and $u_t \equiv \kappa(y_t^e - y_t^n)$

- *Dynamic IS Equation*

$$x_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^e) + E_t\{x_{t+1}\}$$

where $r_t^e \equiv \rho + \sigma E_t\{\Delta y_{t+1}^e\} + (1-\rho_z)z_t = r_t^n$

The Optimal Monetary Policy Problem

$$\min \; E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta x_t^2\right)$$

subject to:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa x_t + u_t$$

where $\{u_t\}$ evolves exogenously according to

$$u_t = \rho_u u_{t-1} + \varepsilon_t$$

In addition:

$$x_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^e) + E_t\{x_{t+1}\}$$

Note: utility based criterion requires $\vartheta = \frac{\kappa}{\epsilon}$

Optimal Monetary Policy under Discretion

Each period CB chooses (x_t, π_t) to minimize

$$\pi_t^2 + \vartheta x_t^2$$

subject to

$$\pi_t = \kappa x_t + v_t$$

where $v_t \equiv \beta E_t\{\pi_{t+1}\} + u_t$ is taken as given.

Optimality condition:

$$x_t = -\frac{\kappa}{\vartheta}\pi_t$$

Equilibrium

$$\pi_t = \frac{\vartheta}{\kappa^2 + \vartheta(1 - \beta\rho_u)}\, u_t \qquad ; \qquad x_t = -\frac{\kappa}{\kappa^2 + \vartheta(1 - \beta\rho_u)}\, u_t$$

$$i_t = r_t^e + \frac{\vartheta\rho_u + \sigma\kappa(1 - \rho_u)}{\kappa^2 + \vartheta(1 - \beta\rho_u)} u_t$$

Optimal Monetary Policy under Discretion

Implementation:

$$
\begin{aligned}
i_t &= r_t^e + \frac{\vartheta\rho_u + \sigma\kappa(1-\rho_u)}{\kappa^2 + \vartheta(1-\beta\rho_u)} u_t + \phi_\pi \left(\pi_t - \frac{\vartheta}{\kappa^2 + \vartheta(1-\beta\rho_u)} \, u_t \right) \\
&= r_t^e + \Theta u_t + \phi_\pi \pi_t
\end{aligned}
$$

where $\Theta \equiv \frac{\sigma\kappa(1-\rho_u) - \vartheta(\phi_\pi - \rho_u)}{\kappa^2 + \vartheta(1-\beta\rho_u)}$ and $\phi_\pi > 1$.

Figure 5.1

Discretion vs. Commitment: Responses to a Transitory Cost-Push Shock

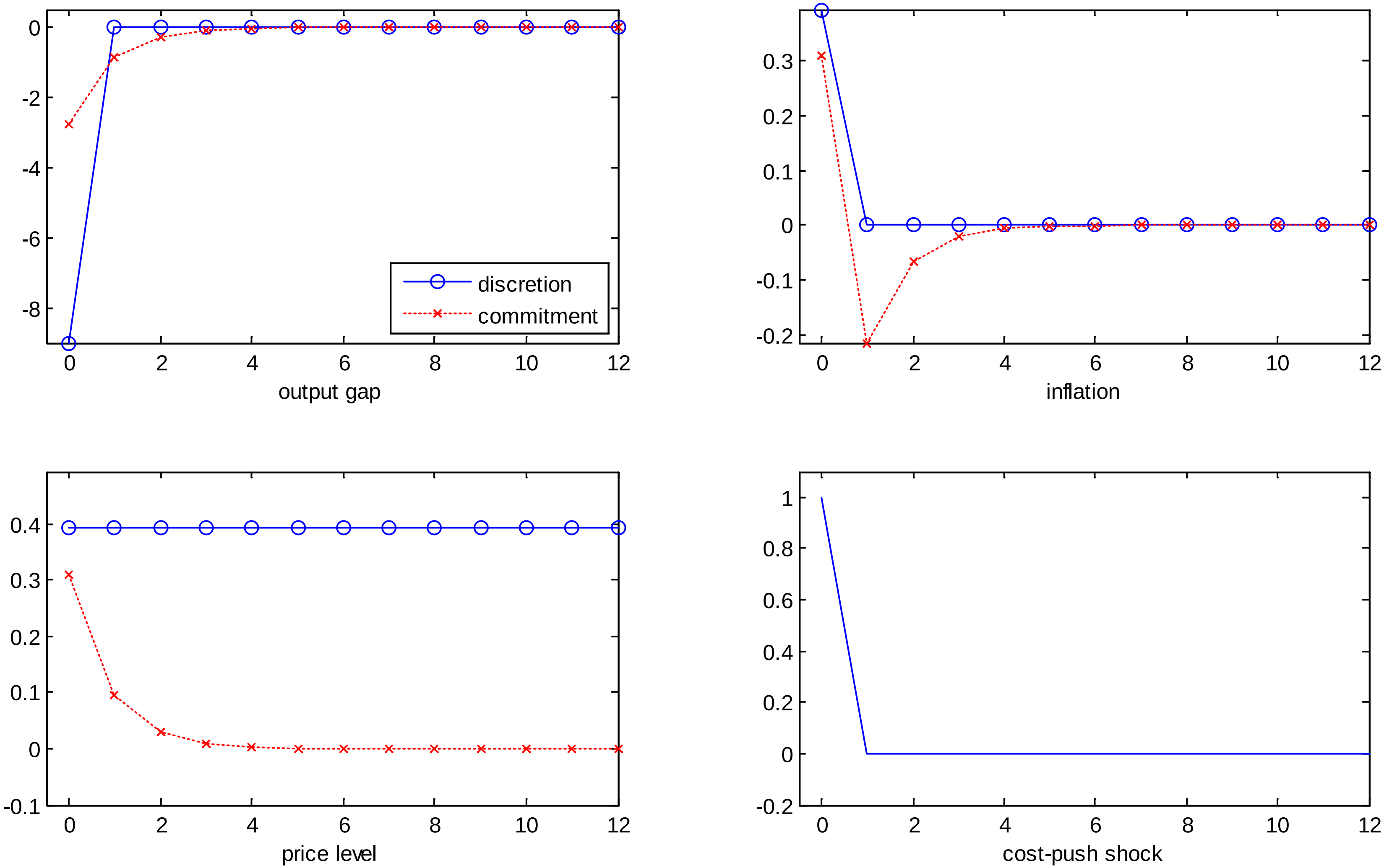
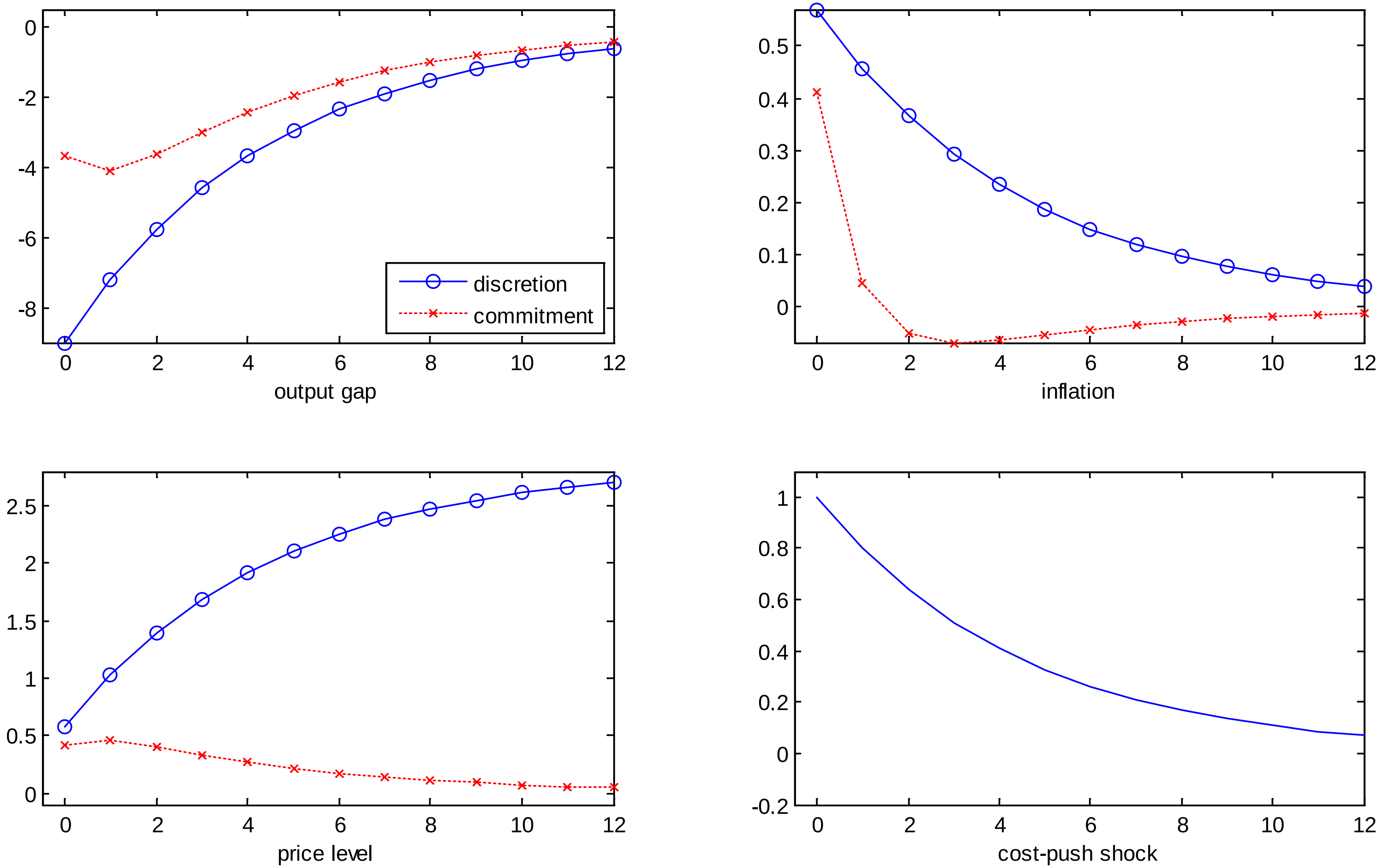


Figure 5.2

Discretion vs. Commitment: Responses to a Persistent Cost-Push Shock



Optimal Monetary Policy under Commitment

State-contingent policy $\{x_t, \pi_t\}_{t=0}^{\infty}$ that minimizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \vartheta x_t^2\right)$$

subject to the sequence of constraints:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa x_t + u_t$$

Lagrangean:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{2}\left(\pi_t^2 + \vartheta x_t^2\right) + \xi_t(\pi_t - \kappa x_t - \beta \pi_{t+1})\right]$$

Optimality conditions:

$$\vartheta x_t - \kappa \xi_t = 0$$

$$\pi_t + \xi_t - \xi_{t-1} = 0$$

for $t = 0, 1, 2, \ldots$with $\xi_{-1} = 0$,

Optimal Monetary Policy under Commitment

Eliminating multipliers:

$$x_0 = -\frac{\kappa}{\vartheta}\pi_0$$

$$x_t = x_{t-1} - \frac{\kappa}{\vartheta}\pi_t$$

for $t = 1, 2, 3, ...$

Alternative representation:

$$x_t = -\frac{\kappa}{\vartheta}\widehat{p}_t$$

for $t = 0, 1, 2, ...$where $\widehat{p}_t \equiv p_t - p_{-1}$

Optimal Monetary Policy under Commitment

Equilibrium

$$\widehat{p}_t = \gamma \widehat{p}_{t-1} + \gamma \beta E_t\{\widehat{p}_{t+1}\} + \gamma u_t$$

for $t = 0, 1, 2, ...$where $\gamma \equiv \frac{\vartheta}{\vartheta(1+\beta)+\kappa^2}$

Stationary solution:

$$\widehat{p}_t = \delta \widehat{p}_{t-1} + \frac{\delta}{1 - \delta\beta\rho_u} \; u_t$$

for $t = 0, 1, 2, ...$where $\delta \equiv \frac{1-\sqrt{1-4\beta\gamma^2}}{2\gamma\beta} \in (0,1)$.

$\longrightarrow$ *price level targeting !*

$$x_t = \delta x_{t-1} - \frac{\kappa\delta}{\vartheta(1 - \delta\beta\rho_u)} \; u_t$$

for $t = 1, 2, 3, ...$, and

$$x_0 = -\frac{\kappa\delta}{\vartheta(1 - \delta\beta\rho_u)} \; u_0$$

Optimal Monetary Policy under Commitment

Discussion: Gains from Commitment

$$\pi_t = \kappa x_t + \kappa \sum_{k=1}^{\infty} \beta^k E_t\{x_{t+k}\} + \frac{1}{1-\beta\rho_u}\, u_t$$

Illustration: *Optimal Monetary Policy under the Zero Lower Bound ("Forward Guidance")*

Figure 5.3
Discretion vs. Commitment in the Presence of a ZLB