Advanced Macroeconomics II

Real Business Cycle Models

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Assumptions

- Optimization by consumers and firms
- Perfect competition
- General equilibrium
- Absence of a monetary sector or nominal variables

Outline:

- Basic RCB model without capital
- RBC model with capital accumulation
- Fiscal policy

Basic RBC Model without Capital

Households

- Preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

where $\beta \equiv \frac{1}{1+\rho} \in [0, 1]$, $U_c > 0$, $U_n < 0$, $U_{cc} \leq 0$, and $U_{nn} \leq 0$

- Budget constraint

$$C_t + B_t = W_t N_t + (1 + r_{t-1}) B_{t-1} + D_t$$

- Optimality conditions

  - *intratemporal*

$$W_t = -\frac{U_{n,t}}{U_{c,t}} \equiv MRS_t$$

  - *intertemporal*

$$U_{c,t} = \beta(1 + r_t) E_t\{U_{c,t+1}\}$$

Example:

$$\begin{aligned} U(C_t, N_t) &= \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \quad \text{if } \sigma \neq 1 \\ &= \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \quad \text{if } \sigma = 1 \end{aligned}$$

Optimality conditions:

$$W_t = C_t^{\sigma} N_t^{\varphi}$$

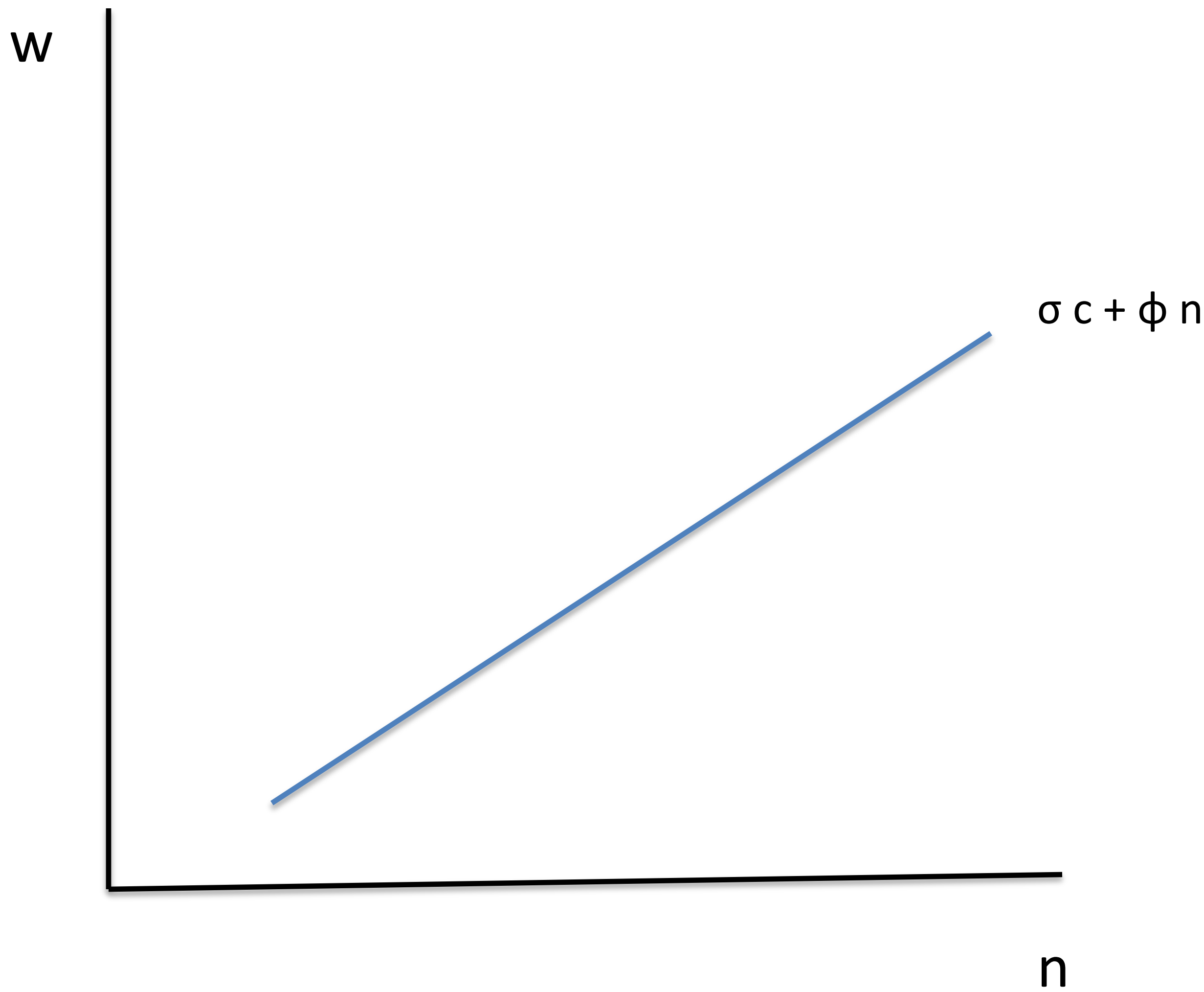
$$1 = \beta(1 + r_t) E_t \left\{ (C_{t+1}/C_t)^{-\sigma} \right\}$$

Log-linear version:

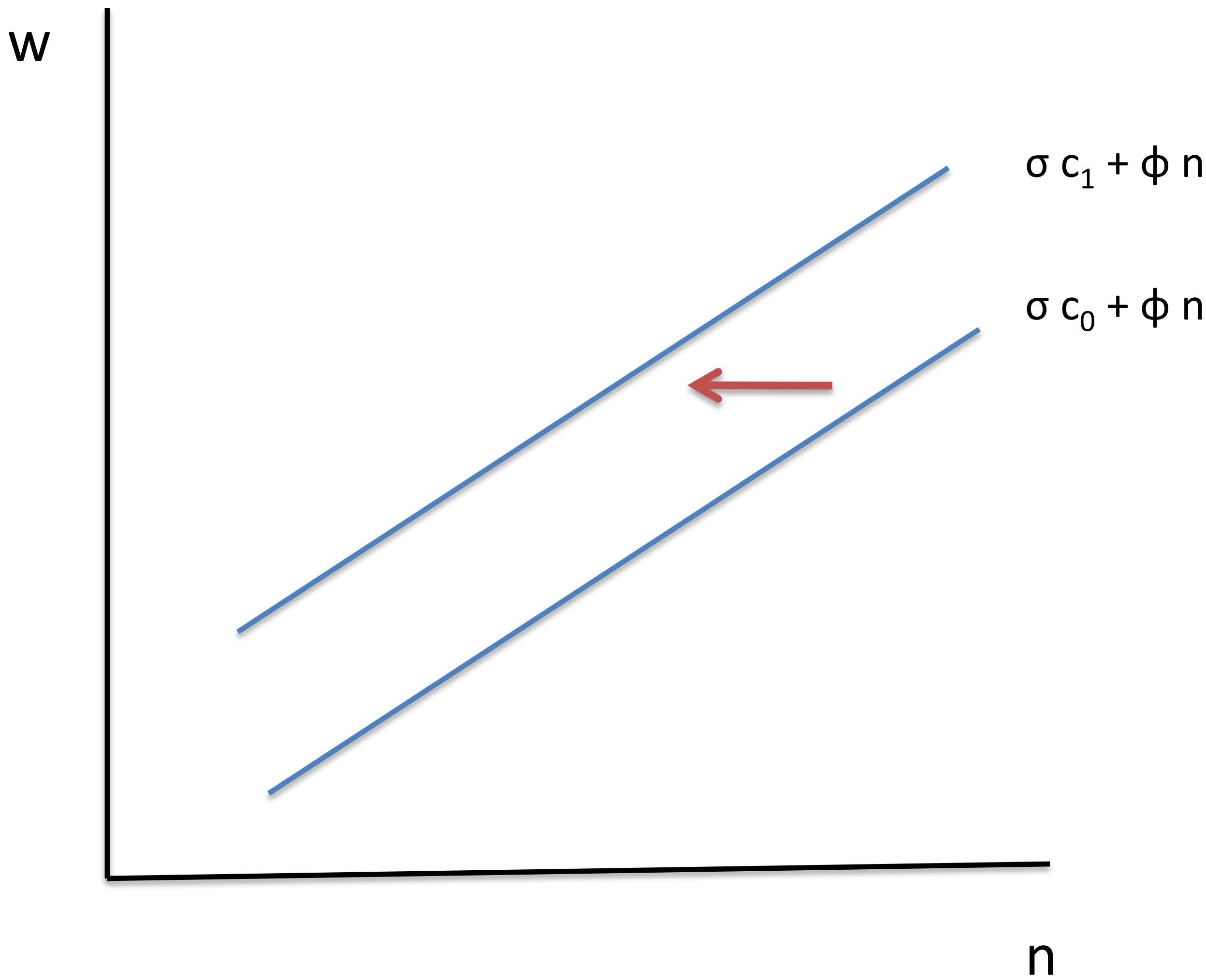
$$w_t = \sigma c_t + \varphi n_t$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(r_t - \rho)$$

Labor Supply



Labor Supply



Firms

- Technology

$$Y_t = A_t F(N_t) \tag{1}$$

where $F_n > 0$, $F_{nn} \leq 0$ and $A_t \equiv \exp\{a_t\}$ is a technology parameter that evolves according to the $AR(1)$ process:

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

where $\rho_a \in [0, 1)$, and $\{\varepsilon_t^a\}$ is white noise.

- Firm's problem

$$\max Y_t - W_t N_t$$

subject to (1).

- Optimality condition

$$W_t = A_t F_{n,t} \equiv MPN_t$$

Example

$$Y_t = A_t N_t^{1-\alpha}$$

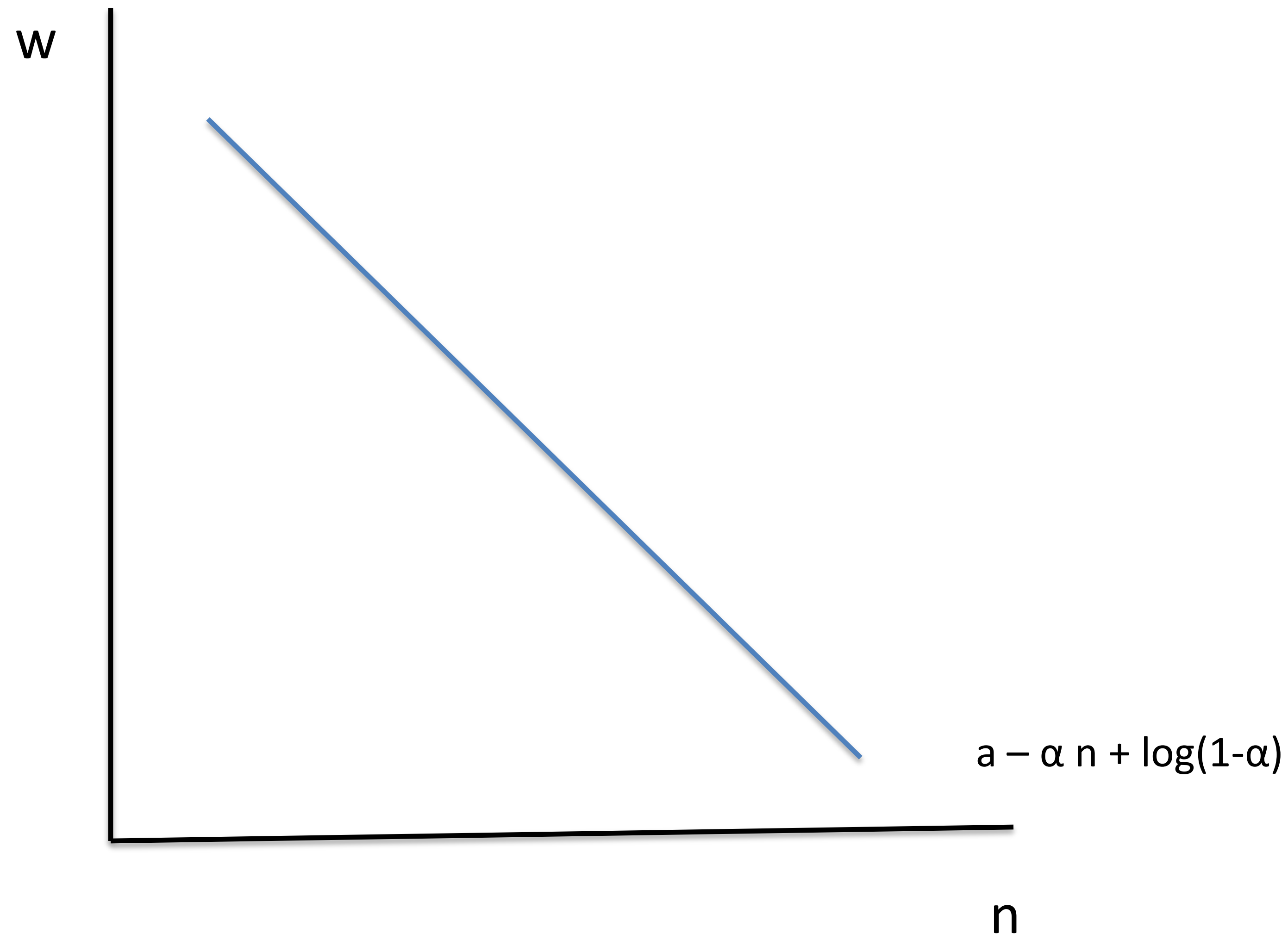
Optimality condition

$$\begin{aligned} W_t &= (1-\alpha) A_t N_t^{-\alpha} \\ &= (1-\alpha)(Y_t / N_t) \end{aligned}$$

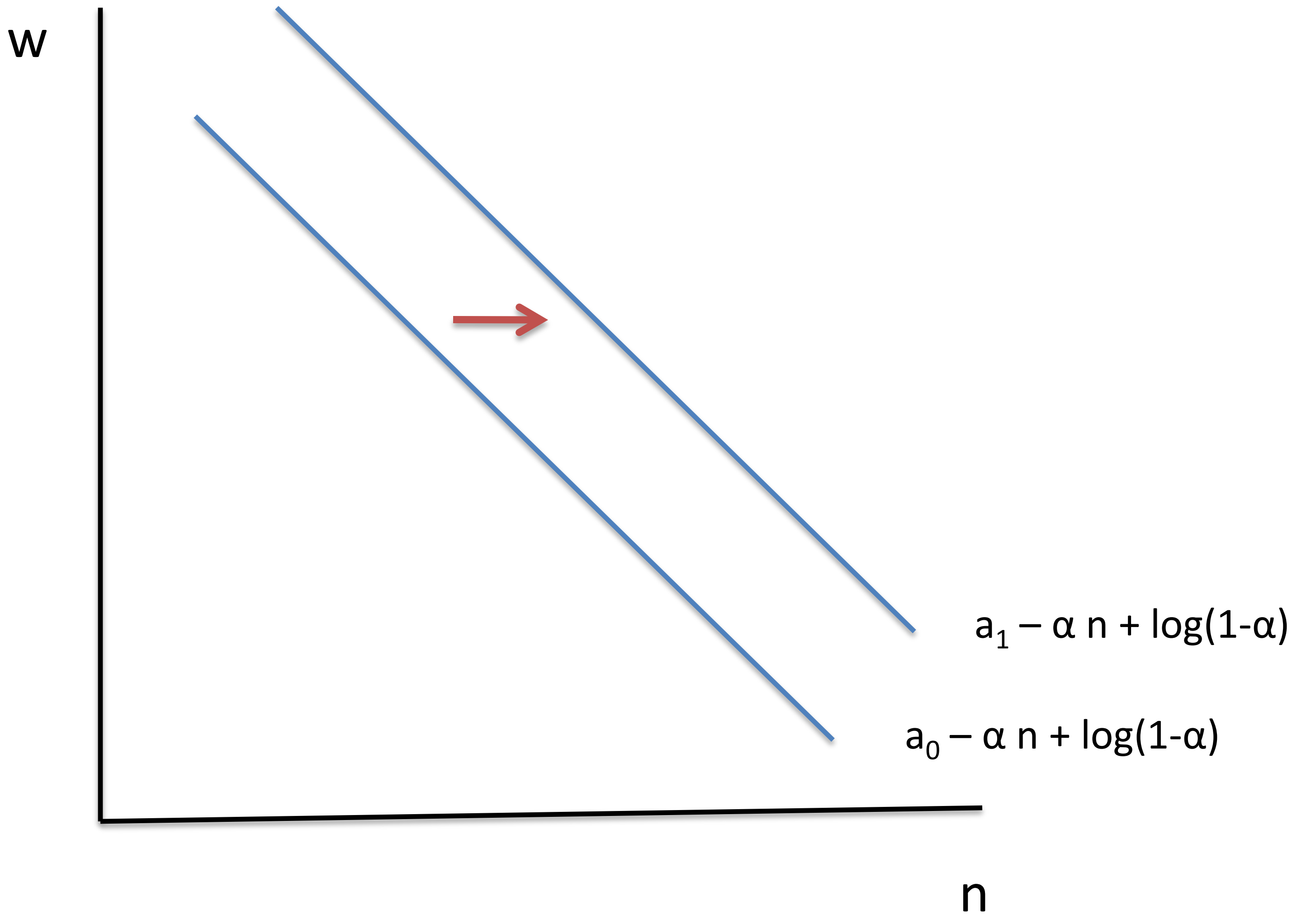
Log-linear version:

$$w_t = a_t - \alpha n_t + \log(1-\alpha)$$

Labor Demand



Labor Demand



Equilibrium

- Goods markets

$$Y_t = C_t$$

- Labor market

$$C_t^{\sigma} N_t^{\varphi} = W_t = (1 - \alpha) A_t N_t^{-\alpha}$$

- Asset market

$$B_t = 0$$

$$1 = \beta(1 + r_t) E_t \left\{ (C_{t+1}/C_t)^{-\sigma} \right\}$$

all for all t

- *Equilibrium values (in logs and ignoring constant terms)*:

$$n_t = \frac{1-\sigma}{\sigma(1-\alpha)+\varphi+\alpha}\ a_t$$

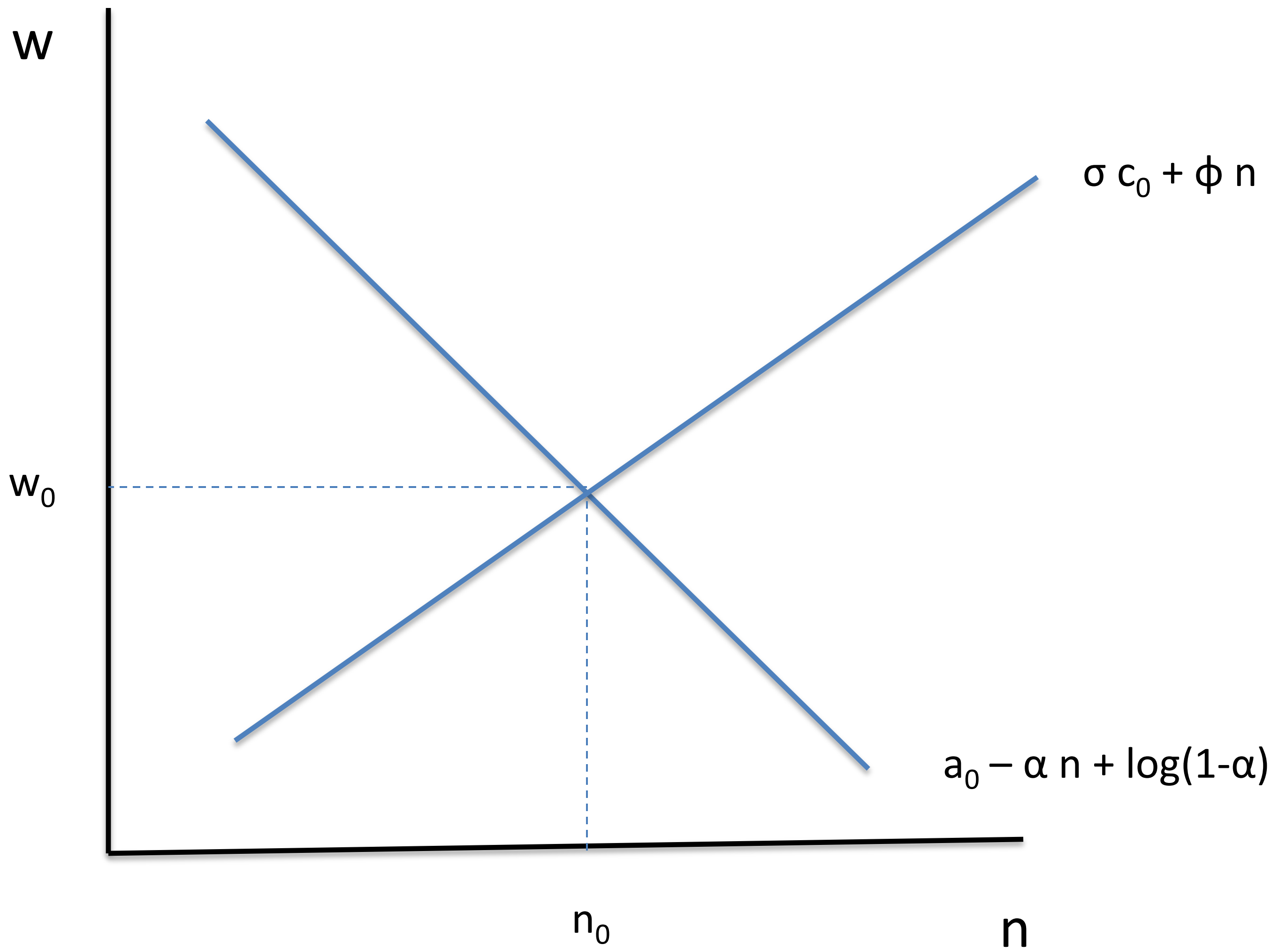
$$y_t = \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}\ a_t$$

$$w_t = \frac{\sigma+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}\ a_t$$

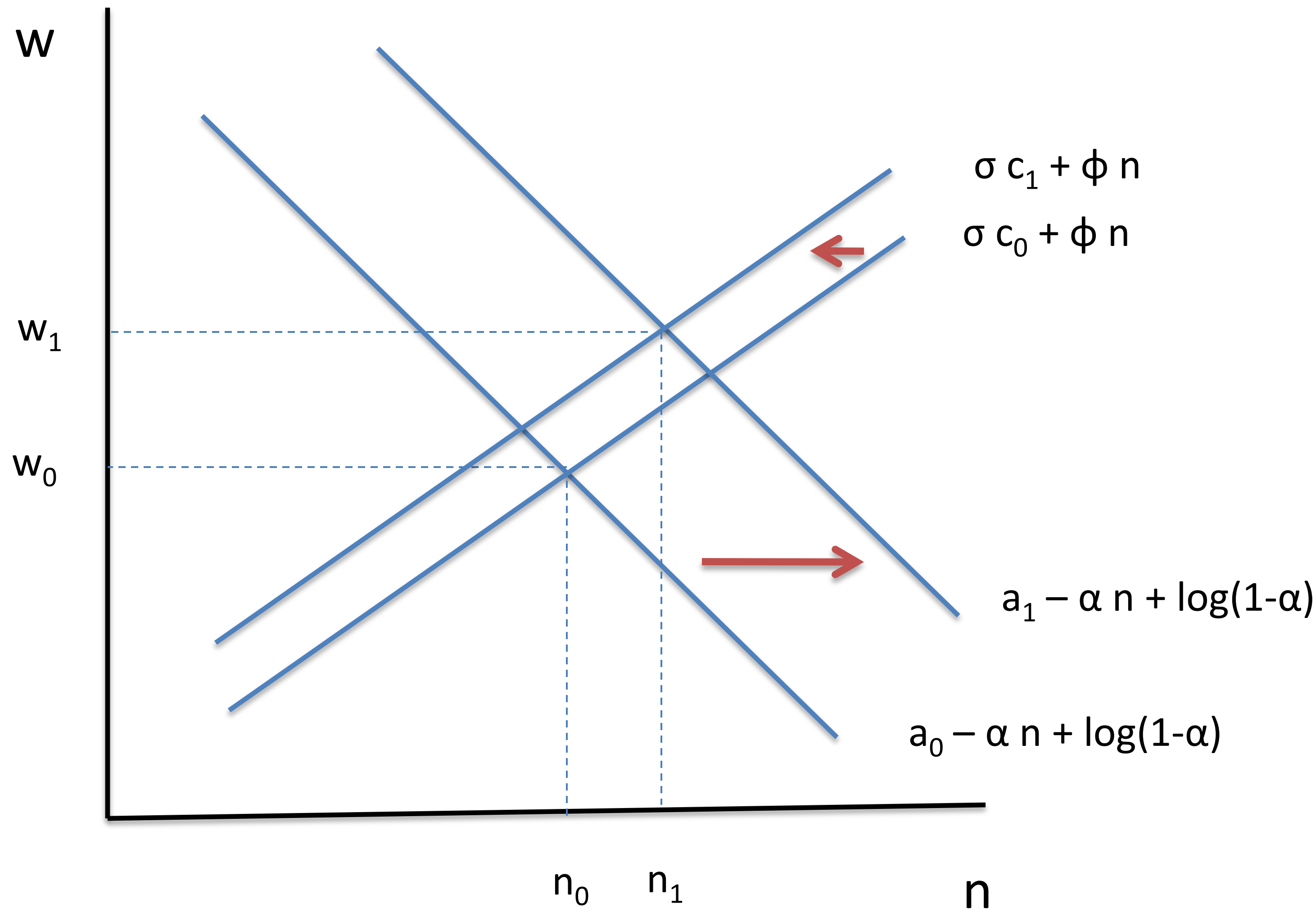
$$r_t = \rho - \frac{\sigma(1+\varphi)(1-\rho_a)}{\sigma(1-\alpha)+\varphi+\alpha}\ a_t$$

- Predictions vs Evidence

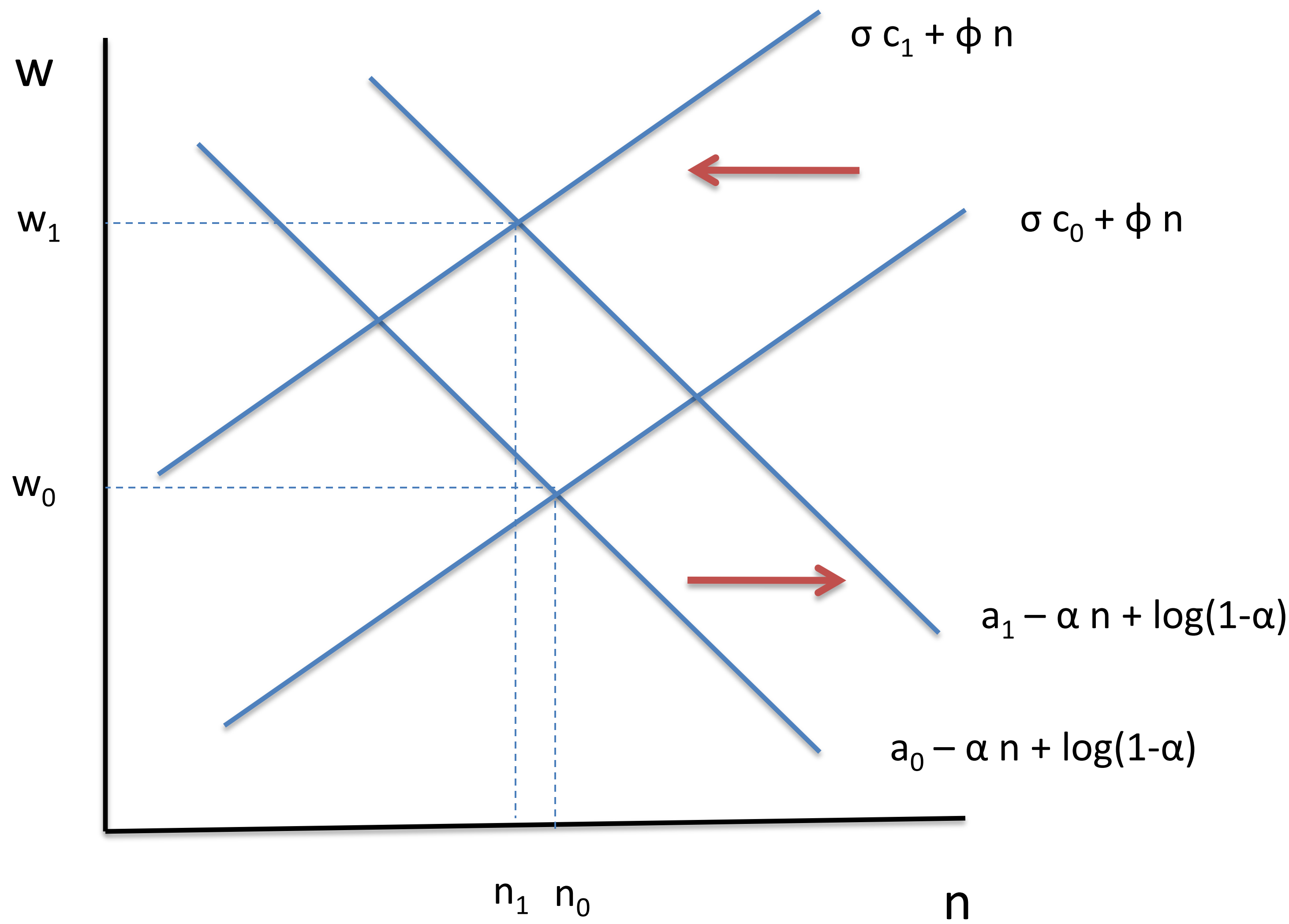
Labor Market Equilibrium



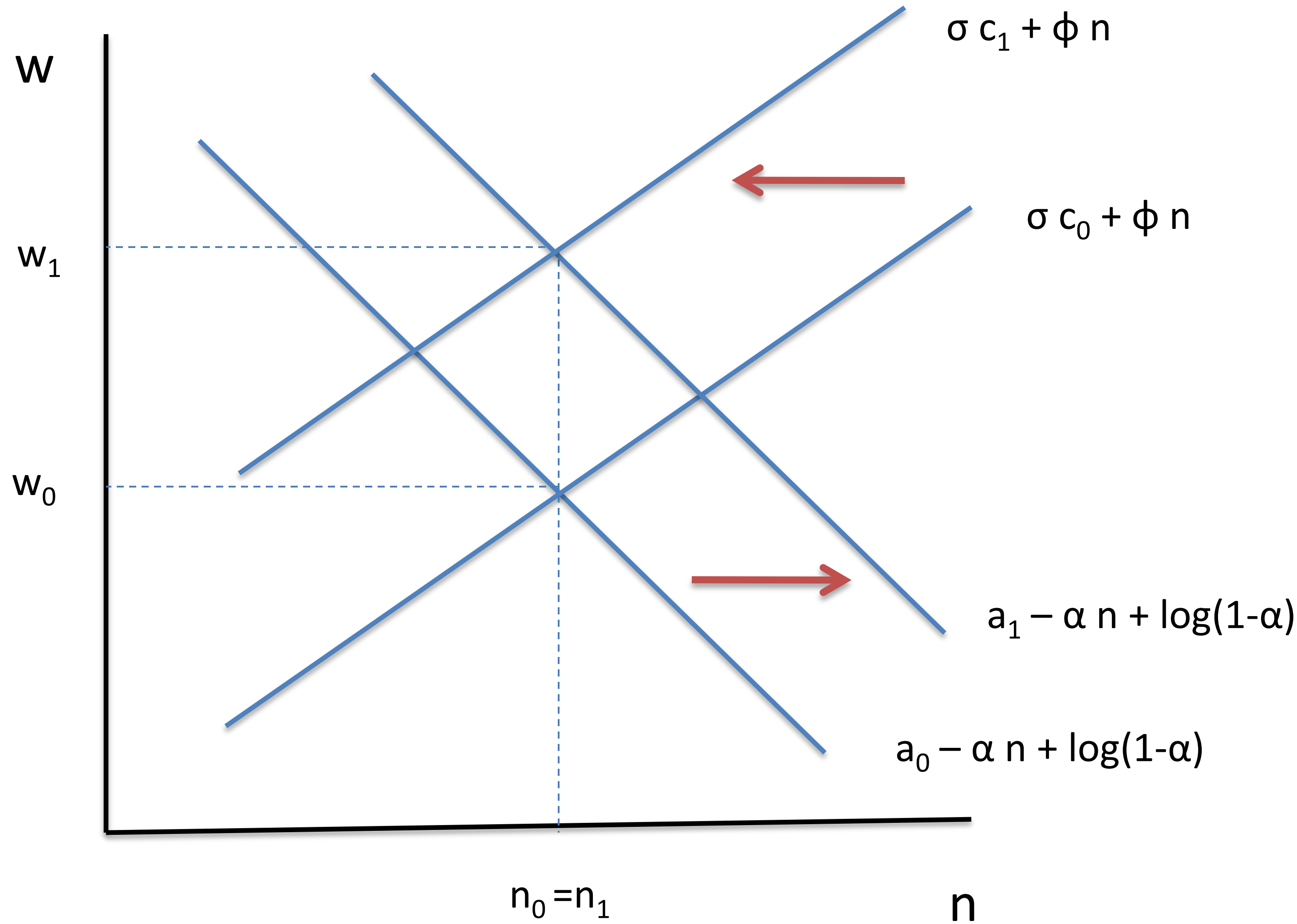
Effects of a Technology Shock ($\sigma < 1$)



Effects of a Technology Shock ($\sigma > 1$)



Effects of a Technology Shock ($\sigma = 1$)



Efficient Allocation: The Social Planner's Problem

$$\max U(C_t, N_t)$$

subject to

$$C_t = A_t F(N_t)$$

Optimality condition:

$$-\frac{U_{n,t}}{U_{c,t}} = A_t F_{n,t}$$

Example:

$$C_t^{\sigma} N_t^{\varphi} = (1-\alpha) A_t N_t^{-\alpha}$$

$\Longrightarrow$ Equivalence with competitive equilibrium allocation
$\Longrightarrow$ Any observed fluctuations are optimal
$\Longrightarrow$ Stabilization policies are not justified

The Basic RBC Model with Capital

Households

- Preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

where $\beta \equiv \frac{1}{1+\rho} \in [0,1]$, $U_c > 0$, $U_n < 0$, $U_{cc} \leq 0$, and $U_{nn} \leq 0$

- Budget constraint and capital accumulation equation

$$C_t + I_t + B_t = W_t N_t + R_t^k K_t + (1 + r_{t-1}) B_{t-1} + D_t$$

$$K_{t+1} = (1 - \delta) K_t + I_t$$

- Optimality conditions

- *intratemporal*

$$W_t = -\frac{U_{n,t}}{U_{c,t}} \equiv MRS_t$$

- *intertemporal*

$$U_{c,t} = \beta(1 + r_t)E_t\{U_{c,t+1}\}$$
$$U_{c,t} = \beta E_t\{U_{c,t+1}(1 - \delta + R^k_{t+1})\}$$

Example:

$$U(C_t, N_t) = \frac{C_t^{1-\sigma} - 1}{1 - \sigma} - \frac{N_t^{1+\varphi}}{1 + \varphi}$$

Optimality conditions*:*

$$W_t = C_t^{\sigma} N_t^{\varphi}$$
$$1 = \beta(1 + r_t)E_t\left\{(C_{t+1}/C_t)^{-\sigma}\right\}$$
$$1 = \beta E_t\left\{(C_{t+1}/C_t)^{-\sigma}(1 - \delta + R^k_{t+1})\right\}$$

Firms

- Technology

$$Y_t = A_t F(K_t, N_t) \tag{2}$$

where $F_k > 0$, $F_n > 0$, $F_{kk} \leq 0$, and $F_{nn} \leq 0$. Defining $a_t \equiv \log A_t$, we assume

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

where $\rho_a \in [0, 1)$, and $\{\varepsilon_t^a\}$ is white noise.

- Firm's problem

$$\max Y_t - W_t N_t - R_t^k K_t$$

subject to (2).

- Optimality conditions

$$\begin{aligned} W_t &= A_t F_n(K_t, N_t) \equiv MPN_t \\ R_t^k &= A_t F_k(K_t, N_t) \equiv MPK_t \end{aligned}$$

Example (Cobb-Douglas)

$$Y_t = A_t K_t^{\alpha} N_t^{1-\alpha}$$

Optimality conditions:

$$R_t^k = \alpha A_t \left(K_t / N_t\right)^{-(1-\alpha)}$$

$$W_t = (1-\alpha) A_t \left(K_t / N_t\right)^{\alpha}$$

Equilibrium

- Goods market

$$Y_t = C_t + I_t \tag{3}$$

$$K_{t+1} = (1-\delta)K_t + A_t K_t^{\alpha} N_t^{1-\alpha} - C_t \tag{4}$$

- Labor market

$$C_t^{\sigma} N_t^{\varphi} = W_t = (1-\alpha)A_t \left(K_t/N_t\right)^{\alpha} \tag{5}$$

- Asset market

$$B_t = 0 \tag{6}$$

$$1 = \beta E_t \left\{ \left(C_{t+1}/C_t\right)^{-\sigma} \left(1 - \delta + \alpha A_{t+1} \left(K_{t+1}/N_{t+1}\right)^{-(1-\alpha)}\right) \right\} \tag{7}$$

$$1 = \beta(1+r_t)E_t \left\{ \left(C_{t+1}/C_t\right)^{-\sigma} \right\} \tag{8}$$

An Example with an Exact Solution

- Long and Plosser, JPE 1983
- Complete depreciation ($\delta = 1$) + logarithmic utility ($\sigma = 1$).
- Equilibrium conditions

$$(1-\alpha)(Y_t/N_t) = C_t N_t^{\varphi}$$
$$1 = \alpha\beta E_t \left\{ (C_t/C_{t+1}) \left(Y_{t+1}/K_{t+1}\right) \right\}$$
$$K_{t+1} + C_t = Y_t$$

- Conjecture:

$$\begin{aligned} K_{t+1} &= \lambda Y_t \\ C_t &= (1-\lambda) Y_t \end{aligned}$$

Implications:

$$\lambda = \alpha\beta$$

$$N_t = ((1-\alpha)(Y_t/C_t))^{\frac{1}{1+\varphi}} = \left(\frac{1-\alpha}{1-\alpha\beta} \right)^{\frac{1}{1+\varphi}} \equiv N$$

Equilibrium dynamics (in logs)

$$y_t = \alpha k_t + a_t + const.$$

$$\begin{aligned} c_t &= y_t + const. \\ &= \alpha k_t + a_t + const. \end{aligned}$$

$$\begin{aligned} k_{t+1} &= y_t + const. \\ &= \alpha k_t + a_t + const. \end{aligned}$$

Discussion:

- dynamic effects of a technology shock
- "intrinsic" persistence
- limitations: constant employment, uniform volatility,...

General Case

Step 1: determination of steady state

Step 2: approximate equilibrium conditions around the steady state ("log-linearization")

Step 3: calibration

Step 4: simulation of calibrated model

Determination of Steady State

Steady state: equilibrium $A_t = A$, $C_t = C$, $K_t = K$, $N_t = N$,...

Evaluating (7) at the steady state:

$$K/N = \left(\frac{\alpha A}{\rho + \delta}\right)^{\frac{1}{1-\alpha}} \tag{9}$$

Evaluating (4) at the steady state (dividing by N),

$$C/N = A^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{\rho + \delta}\right)^{\frac{\alpha}{1-\alpha}} \left(1 - \frac{\delta\alpha}{\rho + \delta}\right) \tag{10}$$

Evaluating (5) at the steady state:

$$\begin{aligned} N^{\sigma+\varphi} &= (1-\alpha)(C/N)^{-\sigma} A \left(K/N\right)^{\alpha} \\ &= (1-\alpha) A^{\frac{1-\sigma}{1-\alpha}} \left(\frac{\alpha}{\rho + \delta}\right)^{\frac{\alpha(1-\sigma)}{1-\alpha}} \left(1 - \frac{\delta\alpha}{\rho + \delta}\right)^{-\sigma} \end{aligned}$$

Given N, use (9) and (10) to determine K and C, etc.

Exercise: What determines long run labor productivity Y/N?

Log-linearization of Equilibrium Conditions around Steady State

- Goods market

$$\alpha\beta\widehat{k}_{t+1} = \alpha\widehat{k}_t + (1-(1-\delta)\beta)((1-\alpha)\,\widehat{n}_t + a_t) - (1-\beta+\beta\delta(1-\alpha))\widehat{c}_t$$

- Labor market

$$\sigma\widehat{c}_t + (\alpha+\varphi)\widehat{n}_t = a_t + \alpha\widehat{k}_t$$

- Capital rental market

$$\sigma\widehat{c}_t = \sigma E_t\{\widehat{c}_{t+1}\} + (1-(1-\delta)\beta)\left((1-\alpha)(\widehat{k}_{t+1} - E_t\{\widehat{n}_{t+1}\}) - \rho_a a_t\right)$$

where $\widehat{x}_t \equiv \log(X_t/X)$. More compactly:

$$\begin{bmatrix} \widehat{c}_t \\ \widehat{n}_t \\ \widehat{k}_t \end{bmatrix} = \widetilde{\mathbf{A}} \begin{bmatrix} E_t\{\widehat{c}_{t+1}\} \\ E_t\{\widehat{n}_{t+1}\} \\ \widehat{k}_{t+1} \end{bmatrix} + \widetilde{\mathbf{B}}\; a_t$$

Technical Note on the Solution to Dynamical Systems

- Dynamical System

$$\mathbf{y}_t = \mathbf{A} E_t\{\mathbf{y}_{t+1}\} + \mathbf{B}\mathbf{z}_t$$

$$\mathbf{z}_t = \mathbf{R}\mathbf{z}_{t-1} + \boldsymbol{\varepsilon}_t$$

$\mathbf{y}_t = [\mathbf{x}_t',\ \mathbf{k}_t']'$: vector $(n \times 1)$ of endogenous variables
$\mathbf{x}_t$: vector $(n_x \times 1)$ of non-predetermined endogenous variables
$\mathbf{k}_t$: vector $(n_k \times 1)$ of predetermined endogenous variables
$\mathbf{z}_t$: vector $(n_z \times 1)$ of exogenous variables
$\boldsymbol{\varepsilon}_t$: vector $(n_z \times 1)$ following a white noise process

- Solution ("state-space representation"):

$$\begin{aligned} \mathbf{s}_t &= \mathbf{C}\mathbf{s}_{t-1} + \mathbf{D}\boldsymbol{\varepsilon}_t \\ \mathbf{x}_t &= \mathbf{M}\mathbf{s}_t \end{aligned}$$

where $\mathbf{s}_t = [\mathbf{k}_t',\ \mathbf{z}_t']'$ is the vector of state variables.

In our RBC example:

$$\begin{aligned}\widehat{k}_t &= \psi_{kk}\widehat{k}_{t-1} + \psi_{ka}a_{t-1} \\ a_t &= \rho_a a_{t-1} + \varepsilon_t^a\end{aligned}$$

and for any other endogenous variable $\widehat{x}_t$:

$$\widehat{x}_t = \psi_{xk}\widehat{k}_t + \psi_{xa}a_t$$

Calibration

$[\beta\ ]$: $\qquad \beta R = 1$
average return S&P500 = 6.5% $\Longrightarrow \beta = (1 + (0.065/4))^{-1} \simeq 0.985$

$[\delta\ ]$: $\quad 0.10/4 = 0.025.$

$[\alpha\ ]$: $\quad W = (1-\alpha)(Y/N) \quad \Longrightarrow \quad \alpha = 1 - S_{n,t} \;\; S_{n,t} \equiv \frac{W_t N_t}{Y_t} \simeq 2/3$
$\qquad S_{n,t} \equiv \frac{W_t N_t}{Y_t} \simeq 2/3 \Longrightarrow \alpha = 1/3$

$[\sigma]$: $\quad (1-\alpha)\frac{Y_t}{N_t} = C_t^{\sigma} N_t^{\varphi}$... balanced growth requirement $\Longrightarrow \sigma = 1$

$[\varphi\ ]$: $\quad w_t = \sigma c_t + \varphi n_t ... \Longrightarrow ...\; n_t = \varphi^{-1} w_t - \sigma \varphi^{-1} c_t$
$\qquad \Longrightarrow \varphi^{-1}$: labor supply elasticity $\simeq 4$ (controversial)
Aside: King-Rebelo specification: $U(C, L) = \frac{C_t^{1-\sigma}-1}{1-\sigma} + \theta \frac{L_t^{1-\eta}-1}{1-\eta}$ with restriction $N_t + L_t = 1$
Implied labor supply elasticity: $\frac{L}{\eta N} = \frac{0.8}{(1)(0.2)} = 4$

$[\rho_a, \sigma_a^2\ ]$: $\qquad a_t = y_t - \alpha k_t - (1-\alpha) n_t$
Estimated AR(1) process for $\{a_t\}$: $\rho_a = 0.979, \sigma_a^2 = (0.007)^2$

Predictions vs Empirical Evidence (KR, Tables 1 and 3)

- Volatility:

- the model accounts for 70 percent of observed output volatility
- can explain relative volatility of consumption and investment
- consumption and hours too little volatile relative to output

- Persistence:

- accounts for high positive autocorrelation

- Cyclical patterns

- accounts for procyclicality of consumption, investment and hours.
- main limitation: predicts too high procyclicality of interest rate and wages

- Simulations (KR Figure 7)

- correlation simulated and actual output $\simeq 0.8$
- weaker comovement for labor market variables

Table 1
Business Cycle Statistics for the U.S. Economy

	Standard Deviation	Relative Standard Deviation	First Order Auto-correlation	Contemporaneous Correlation with Output
Y	1.81	1.00	0.84	1.00
C	1.35	0.74	0.80	0.88
I	5.30	2.93	0.87	0.80
N	1.79	0.99	0.88	0.88
Y/N	1.02	0.56	0.74	0.55
w	0.68	0.38	0.66	0.12
r	0.30	0.16	0.60	-0.35
A	0.98	0.54	0.74	0.78

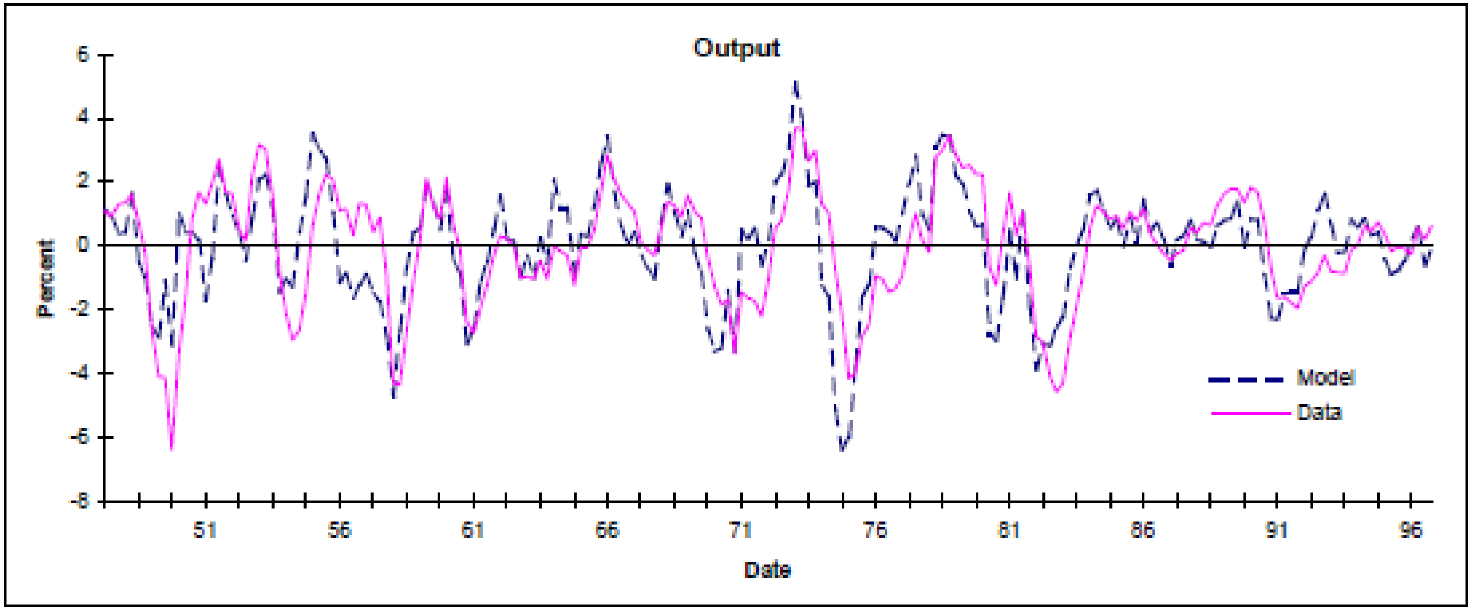
Source: King and Rebelo (1999)

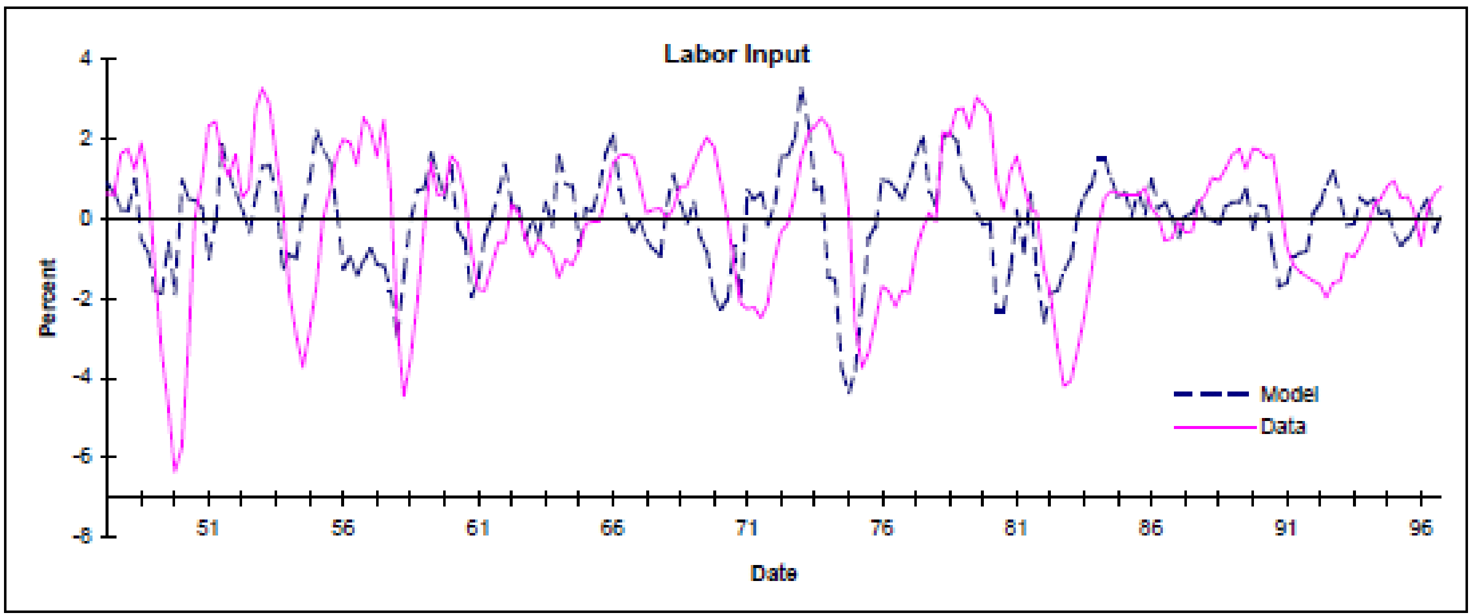
Table 3
Business Cycle Statistics for Basic RBC Model[35]

	Standard Deviation	Relative Standard Deviation	First Order Auto-correlation	Contemporaneous Correlation with Output
Y	1.39	1.00	0.72	1.00
C	0.61	0.44	0.79	0.94
I	4.09	2.95	0.71	0.99
N	0.67	0.48	0.71	0.97
Y/N	0.75	0.54	0.76	0.98
w	0.75	0.54	0.76	0.98
r	0.05	0.04	0.71	0.95
A	0.94	0.68	0.72	1.00

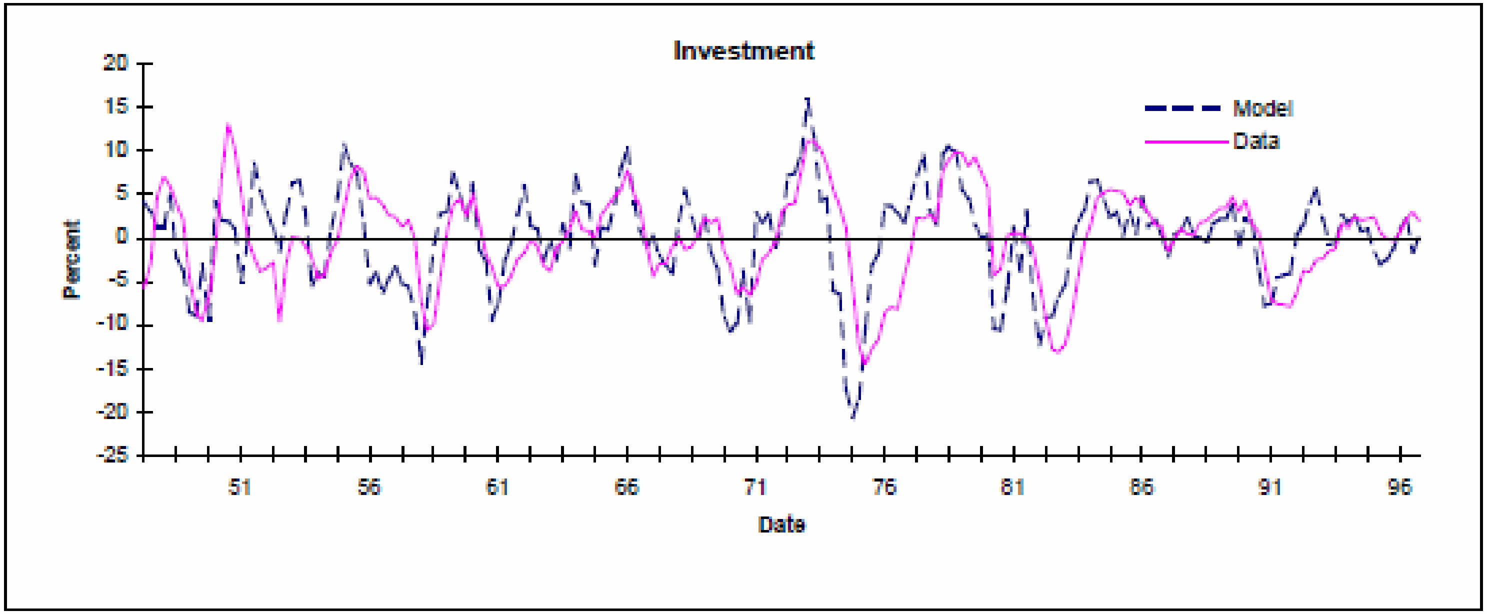
Note: All variables have been logged (with the exception of the real interest rate)and detrended with the HP filter.

Source: King and Rebelo (1999)

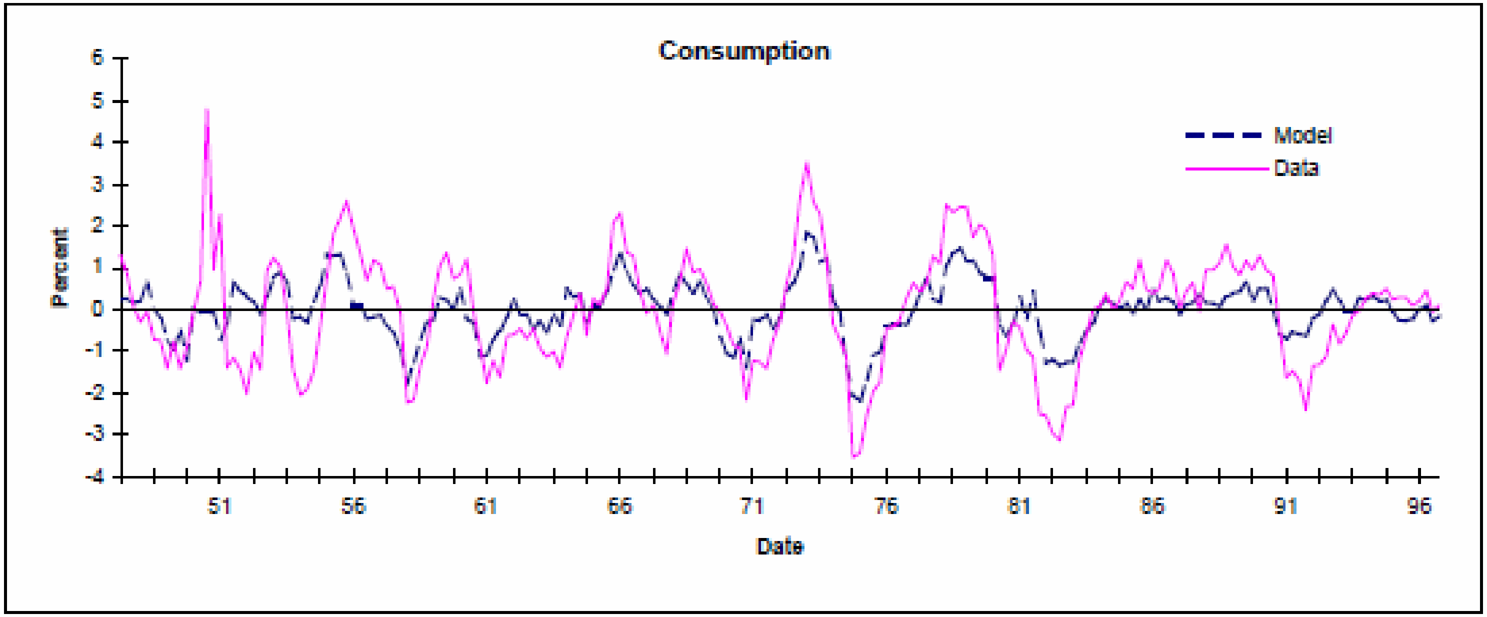




Source: King and Rebelo (1999)



Investment
Model
Data
Percent
20
15
10
5
0
-5
-10
-15
-20
-25
51
56
61
66
71
76
81
86
91
96
Date



Consumption
Model
Data
Percent
6
5
4
3
2
1
0
-1
-2
-3
-4
51
56
61
66
71
76
81
86
91
96
Date

Significance of Findings

- Role of technology: end of growth vs fluctuations dychotomy.
- Fluctuations are not necessarily inefficient, given optimality of equilibrium allocation $\Rightarrow$ stabilization policies may be counterproductive

Exercise: Social planner's problem:

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t)$$

subject to

$$K_{t+1} = (1-\delta)K_t + A_t F(K_t, N_t) - C_t$$

Derive optimality conditions and check equivalence with decentralized equilibrium

Criticisms

- No role for monetary policy
- No involuntary unemployment
- What is a negative technology shock?
- Shortcomings of Solow residual as a measure of technology
- Evidence on the effects of technology shocks (Galí (AER, 1999), Basu et al. (AER, 2006))

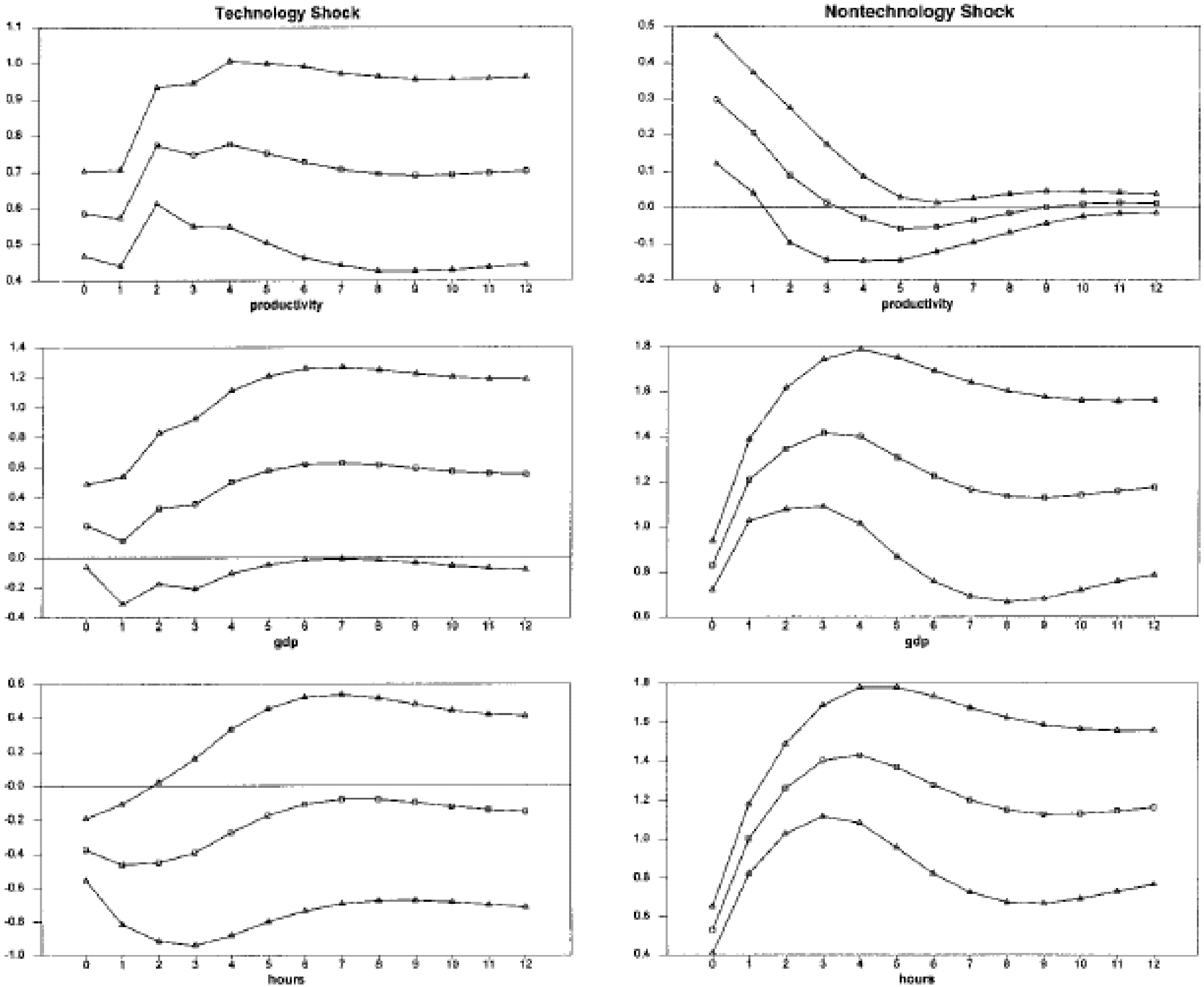
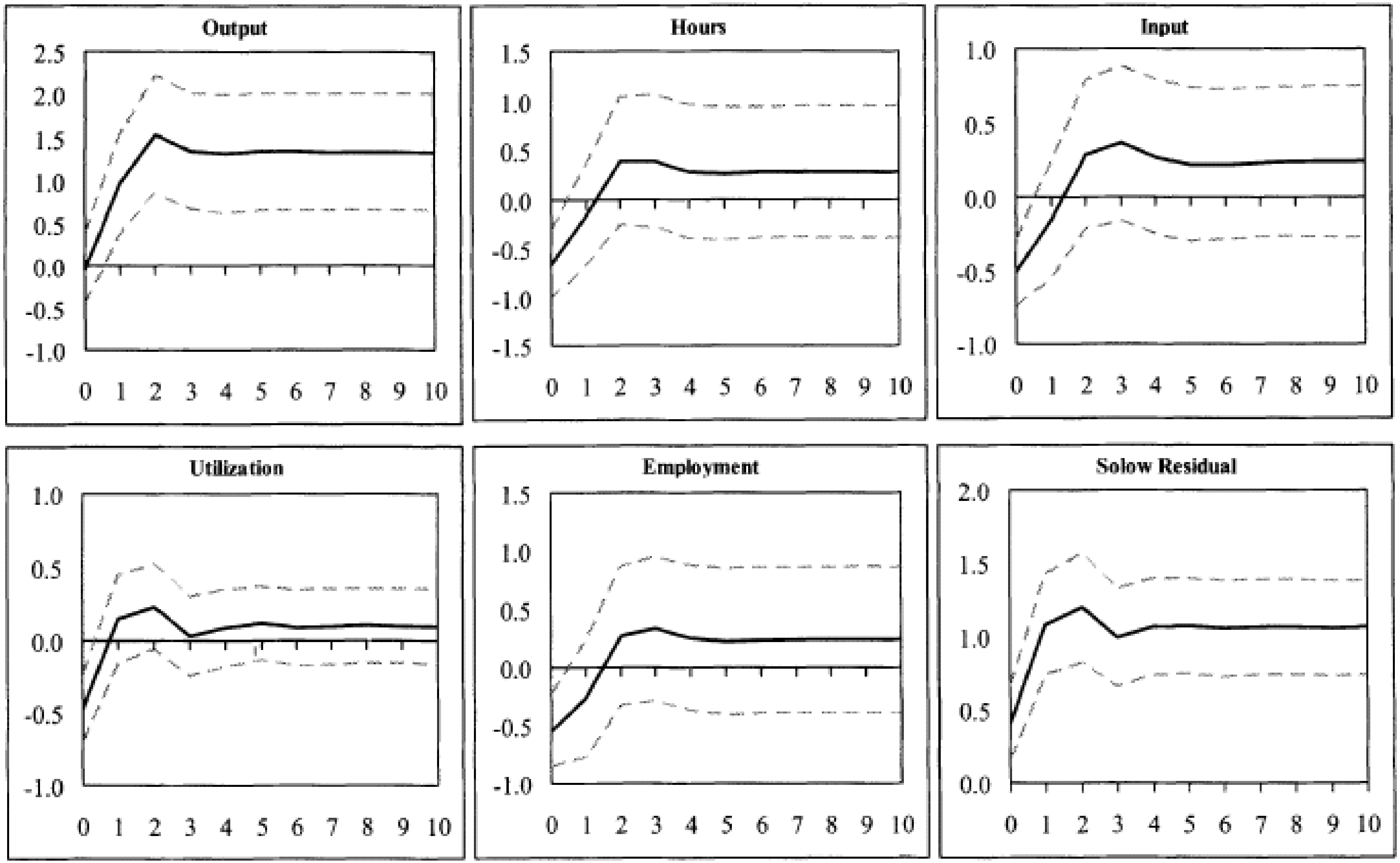


FIGURE 2. ESTIMATED IMPULSE RESPONSES FROM A BIVARIATE MODEL: U.S. DATA, FIRST-DIFFERENCED HOURS (POINT ESTIMATES AND ±2 STANDARD ERROR CONFIDENCE INTERVALS)

Source: Galí (1999)



Source: Basu, Fernald and Kimball (2006)