The New Keynesian Model with Sticky Wages and Prices

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Alternative Labor Market Specifications

- Competitive labor markets

$$w_t - p_t = mrs_t$$

where $mrs_t = \sigma c_t + \varphi n_t$.

- General labor market imperfections

$$w_t - p_t = \mu_t^w + mrs_t$$

where μ_t^w: (log) wage markup.

Example: monopolistic union with isoelastic labor demand:

$$\mu_t^w = \log \frac{\epsilon_w}{\epsilon_w - 1} \equiv \mu^w$$

Sticky Wages: Implications for Inflation Dynamics

Recall

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} - \lambda_p(\mu_t^p - \mu^p)$$

Assuming constant returns (for simplicity)

$$\begin{aligned}
\mu_t^p &= p_t - (w_t - a_t) \\
&= a_t - \omega_t \\
&= a_t - (\mu_t^w + \sigma c_t + \varphi n_t) \\
&= (1+\varphi)a_t - (\sigma + \varphi)y_t - \mu_t^w
\end{aligned}$$

In deviations from *natural* levels (assuming constant natural markups):

$$\mu_t^p - \mu^p = -(\sigma + \varphi)\widetilde{y}_t - (\mu_t^w - \mu^w)$$

Implied New Keynesian Phillips Curve:

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} + \kappa_p \widetilde{y}_t + \lambda_p \widehat{\mu}_t^w$$

$\Longrightarrow$ tradeoff between inflation and output gap stabilization

Question: What determines the evolution of the wage markup?

An Economy With Sticky Wages and Prices (EHL (2000))

- **Firms**

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

where $N_t(i) \equiv \left(\int_0^1 N_t(i,j)^{1-\frac{1}{\epsilon_w}} dj \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}$ and $a_t \equiv \log A_t \sim AR(1)$

Cost minimization:

$$N_t(i,j) = \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t(i)$$

where $W_t \equiv \left(\int_0^1 W_t(j)^{1-\epsilon_w} dj \right)^{\frac{1}{1-\epsilon_w}}$

Implication:

$$\int_0^1 W_t(j) N_t(i,j) dj = W_t N_t(i)$$

Price setting (as in basic NK model):

Fraction of firms adjusting price each period: $1-\theta_p$

Firm's problem:

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta_p^k E_t \left\{ \Lambda_{t,t+k}(1/P_{t+k}) \left(P_t^* Y_{t+k|t} - \mathcal{C}_{t+k}(Y_{t+k|t}) \right) \right\}$$

subject to:

$$\mathcal{C}_{t+k}(Y_{t+k|t}) = W_{t+k} \left(Y_{t+k|t}/A_{t+k} \right)^{\frac{1}{1-\alpha}}$$
$$Y_{t+k|t} = \left(P_t^*/P_{t+k} \right)^{-\epsilon_p} C_{t+k}$$

Implied price setting rule (log-linearized):

$$p_t^* = \mu^p + (1-\beta\theta_p) \sum_{k=0}^{\infty} (\beta\theta_p)^k E_t\{\psi_{t+k|t}\}$$

where $\psi_{t+k|t} \equiv \log \Psi_{t+k|t}$ and $\mu^p \equiv \log \frac{\epsilon_p}{\epsilon_p - 1}$

Price dynamics:

$$p_t = \theta_p p_{t-1} + (1-\theta_p) p_t^*$$

Price inflation equation:

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} - \lambda_p(\mu_t^p - \mu^p)$$

where $\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$.

• **Households**

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, \{\mathcal{N}_t(j)\}; Z_t)$$

subject to:

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + \int_0^1 W_t(j) \mathcal{N}_t(j) dj + D_t$$

where $C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$ and

$$U(C_t, \{\mathcal{N}_t(j)\}; Z_t) = \begin{cases} \left(\frac{C_t^{1-\sigma}-1}{1-\sigma} - \int_0^1 \frac{\mathcal{N}_t(j)^{1+\varphi}}{1+\varphi} dj \right) Z_t & \text{for } \sigma \neq 1 \\ \left(\log C_t - \int_0^1 \frac{\mathcal{N}_t(j)^{1+\varphi}}{1+\varphi} dj \right) Z_t & \text{for } \sigma = 1 \end{cases}$$

with $z_t \equiv \log Z_t \sim AR(1)$.

Optimality conditions

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon_p} C_t$$

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \left(\frac{Z_{t+1}}{Z_t} \right) \left(\frac{P_t}{P_{t+1}} \right) \right\}$$

or, in log-linearized form:

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}^p\} - \rho) + \frac{1}{\sigma}(1 - \rho_z) z_t$$

- **Wage Setting**

Fraction of occupations/unions adjusting nominal wage: $1 - \theta_w$

Optimal wage setting:

$$\max_{W_t^*} E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left(C_{t+k}^{-\sigma} \frac{W_t^*}{P_{t+k}} N_{t+k|t} - \frac{N_{t+k|t}^{1+\varphi}}{1+\varphi} \right) Z_{t+k}$$

subject to

$$N_{t+k|t} = \left(\frac{W_t^*}{W_{t+k}} \right)^{-\epsilon_w} \left(\int_0^1 N_{t+k}(i) di \right)$$

Optimality condition:

$$\sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ N_{t+k|t} Z_{t+k} C_{t+k}^{-\sigma} \left(\frac{W_t^*}{P_{t+k}} - \mathcal{M}_w MRS_{t+k|t} \right) \right\} = 0$$

where $MRS_{t+k|t} \equiv C_{t+k}^{\sigma} N_{t+k|t}^{\varphi}$ and $\mathcal{M}_w \equiv \frac{\epsilon_w}{\epsilon_w - 1}$.

Log-linearized version:

$$w_t^* = \mu^w + (1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \{ mrs_{t+k|t} + p_{t+k} \}$$

Equivalently:

$$w_t^* = (1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k E_t \left\{ w_{t+k} - (1 + \epsilon_w \varphi)^{-1} \widehat{\mu}_{t+k}^w \right\}$$

where $\mu_t^w \equiv (w_t - p_t) - mrs_t$ and $mrs_t = \sigma c_t + \varphi n_t$.

Aggregate wage dynamics

$$w_t = \theta_w w_{t-1} + (1 - \theta_w) w_t^*$$

Wage inflation equation

$$\pi_t^w = \beta E_t \{\pi_{t+1}^w\} - \lambda_w (\mu_t^w - \mu^w)$$

where $\lambda_w \equiv \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w \ (1+\varphi\epsilon_w)}$

• Equilibrium

Goods market clearing:

$$Y_t(i) = C_t(i) \text{ all } i \in [0,1] \Rightarrow Y_t = C_t$$

where $Y_t \equiv \left(\int_0^1 Y_t(i)^{1-\frac{1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$.

Aggregate employment

$$\begin{aligned} N_t &\equiv \int_0^1 \int_0^1 N_t(i,j) dj di = \int_0^1 N_t(i) \int_0^1 \frac{N_t(i,j)}{N_t(i)} dj di = \Delta_{w,t} \int_0^1 N_t(i) di \\ &= \Delta_{w,t} \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{Y_t(i)}{Y_t} \right)^{\frac{1}{1-\alpha}} di = \Delta_{w,t} \Delta_{p,t} \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \end{aligned}$$

where $\Delta_{w,t} \equiv \int_0^1 \left(\frac{W_t(j)}{W_t} \right)^{-\epsilon_w} dj$ and $\Delta_{p,t} \equiv \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{\frac{-\epsilon_p}{1-\alpha}} di$.

Up to a first order approximation:

$$(1-\alpha) n_t = y_t - a_t$$

Wage gap:

$$\widetilde{\omega}_t \equiv \omega_t - \omega_t^n$$

where $\omega_t \equiv w_t - p_t$ and where ω_t^n is the *natural real wage*:

$$\begin{aligned} \omega_t^n &= \log(1-\alpha) + (a_t - \alpha n_t^n) - \mu^p \\ &= \log(1-\alpha) + \psi_{wa} a_t - \mu^p \end{aligned}$$

where $\psi_{wa} \equiv \frac{1-\alpha\psi_{ya}}{1-\alpha} > 0$ and $\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$.

Price markup gap:

$$\begin{aligned} \mu_t^p &= \log(1-\alpha) + (a_t - \alpha n_t) - \omega_t \\ \mu^p &= \log(1-\alpha) + (a_t - \alpha n_t^n) - \omega_t^n \\ &\Rightarrow \widehat{\mu}_t^p = -\frac{\alpha}{1-\alpha}\widetilde{y}_t - \widetilde{\omega}_t \end{aligned}$$

Hence:

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} + \varkappa_p \widetilde{y}_t + \lambda_p \widetilde{\omega}_t$$

where $\varkappa_p \equiv \frac{\alpha\lambda_p}{1-\alpha}$.

Wage markup gap:

$$\begin{aligned}\mu_t^w &= \omega_t - (\sigma y_t + \varphi n_t) \\ \mu^w &= \omega_t^n - (\sigma y_t^n + \varphi n_t^n) \\ &\Rightarrow \widehat{\mu}_t^w = \widetilde{\omega}_t - \left(\sigma + \frac{\varphi}{1-\alpha}\right)\widetilde{y}_t\end{aligned}$$

Hence:

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} + \varkappa_w \widetilde{y}_t - \lambda_w \widetilde{\omega}_t$$

where $\varkappa_w \equiv \lambda_w \left(\sigma + \frac{\varphi}{1-\alpha}\right)$.

In addition:

$$\widetilde{\omega}_t \equiv \widetilde{\omega}_{t-1} + \pi_t^w - \pi_t^p - \Delta\omega_t^n$$

Dynamic IS equation

$$\widetilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}^p\} - r_t^n) + E_t\{\widetilde{y}_{t+1}\}$$

where $r_t^n \equiv \rho - \sigma(1-\rho_a)\psi_{ya} a_t + (1-\rho_z) z_t$

Interest Rate Rule:

$$i_t = \rho + \phi_p \pi_t^p + \phi_w \pi_t^w + \phi_y \widehat{y}_t + v_t$$

Dynamical system:

$$\mathbf{A}_0^w \mathbf{x}_t = \mathbf{A}_\mathbf{1}^w E_t\{\mathbf{x}_{t+1}\} + \mathbf{B}_0^w \mathbf{u}_t$$

where $\mathbf{x}_t \equiv [\widetilde{y}_t,\ \pi_t^p,\ \pi_t^w,\ \widetilde{\omega}_{t-1}]'$, $\mathbf{u}_t \equiv [\widehat{r}_t^n - v_t - \phi_y \widehat{y}_t^n,\ \Delta\omega_t^n]'$,

$$\mathbf{A}_0^w \equiv \begin{bmatrix} \sigma + \phi_y & \phi_p & \phi_w & 0 \\ -\varkappa_p & 1 & 0 & 0 \\ -\varkappa_w & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 \end{bmatrix}$$

$$\mathbf{A}_\mathbf{1}^w \equiv \begin{bmatrix} \sigma & 1 & 0 & 0 \\ 0 & \beta & 0 & \lambda_p \\ 0 & 0 & \beta & -\lambda_w \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad ; \quad \mathbf{B}_0^w \equiv \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

Conditions for uniqueness of the equilibrium[1]

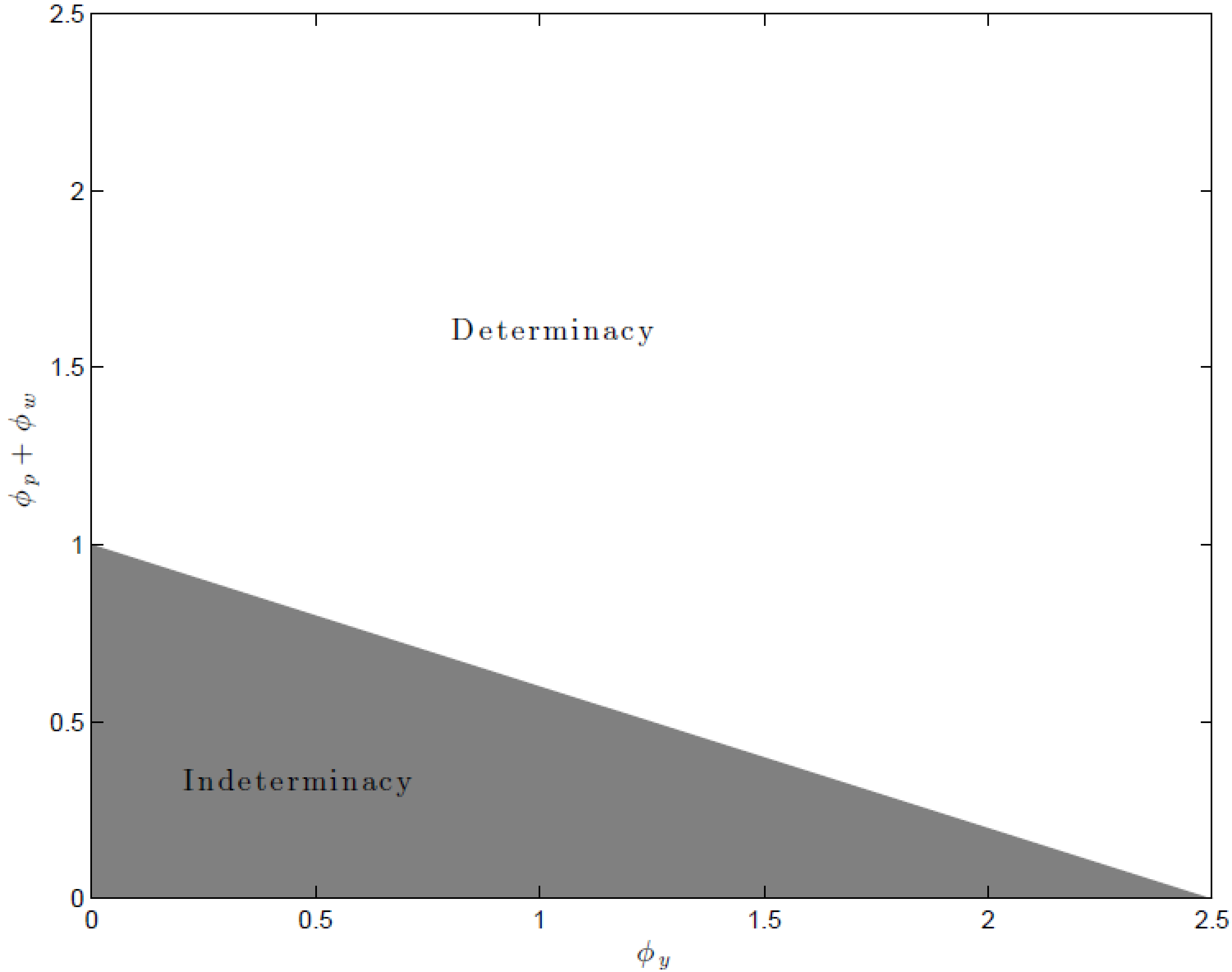
$$\phi_p + \phi_w + \phi_y \left(\frac{1-\beta}{\sigma + \frac{\alpha+\varphi}{1-\alpha}} \right) \left(\frac{1}{\lambda_p} + \frac{1}{\lambda_w} \right) > 1$$

Particular case ($\phi_y = 0$):

$$\phi_p + \phi_w > 1$$

[1] Flaschel-Franke (2008), Blasselle-Poissonier (2013)

Figure 6.1 Determinacy and Indeterminacy Regions



Dynamic Responses to a Monetary Policy Shock

Interest rate rule: $\phi_p = 1.5$, $\phi_y = \phi_w = 0$, $\rho_v = 0.5$

New parameter: $\epsilon_w = 4.5$

Three calibrations:

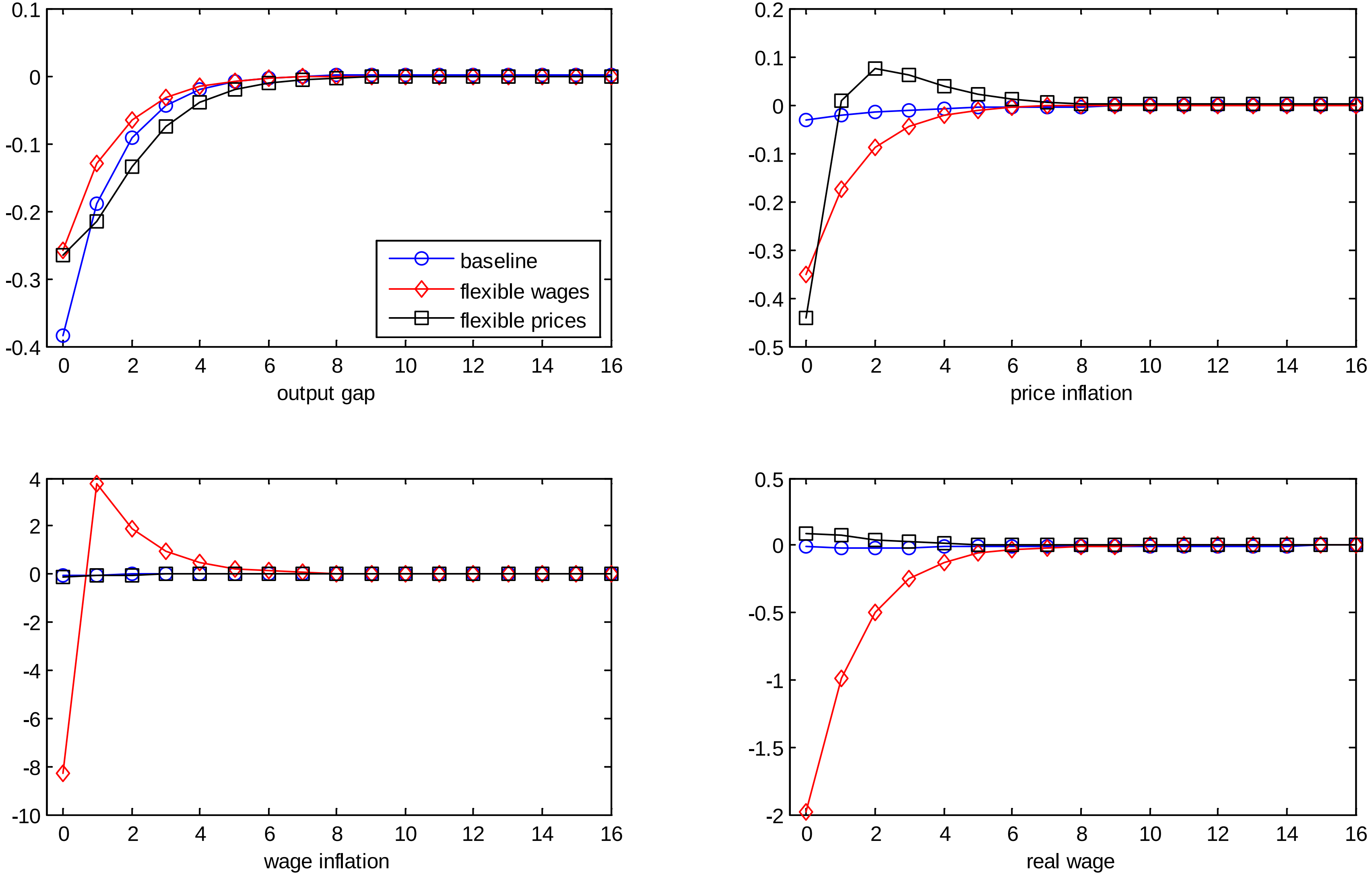
Baseline: $\theta_p = 3/4$, $\theta_w = 3/4$

Flexible wages: $\theta_p = 3/4$, $\theta_w = 0$

Flexible price: $\theta_p = 0$, $\theta_w = 3/4$

Simulations

Figure 6.2 Dynamic Responses to a Monetary Policy Shock



Monetary Policy Design: The Social Planner's Problem

$$\max U(C_t, \{\mathcal{N}_t(j)\}; Z_t)$$

subject to:

$$C_t(i) = A_t N_t(i)^{1-\alpha}$$

$$\mathcal{N}_t(j) = \int_0^1 N_t(i,j)di$$

where $C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon_p}} di\right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$ and $N_t(i) \equiv \left(\int_0^1 N_t(i,j)^{1-\frac{1}{\epsilon_w}} dj\right)^{\frac{\epsilon_w}{\epsilon_w - 1}}$.

Optimality conditions:

$$C_t(i) = C_t, \text{ all } i \in [0,1]$$

$$N_t(i,j) = \mathcal{N}_t(j) = N_t(i) = N_t, \text{ all } i,j \in [0,1]$$

$$-\frac{U_{n,t}}{U_{c,t}} = MPN_t$$

where $MPN_t = (1-\alpha)A_t N_t^{-\alpha}$

Efficiency of the Natural Equilibrium

In the descentralized economy with flexible prices and wages:

$$P_t = \mathcal{M}_p \frac{(1-\tau)W_t}{MPN_t}$$

$$\frac{W_t}{P_t} = -\frac{U_{n,t}}{U_{c,t}}\ \mathcal{M}_w$$

for all goods and occupations, where $\mathcal{M}_p \equiv \frac{\epsilon_p}{\epsilon_p - 1}$ and $\mathcal{M}_w \equiv \frac{\epsilon_w}{\epsilon_w - 1}$. Thus,

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{1}{\mathcal{M}(1-\tau)} MPN_t$$

where $\mathcal{M} \equiv \mathcal{M}_p \mathcal{M}_w$.

Condition for efficiency of the natural equilibrium: $\mathcal{M}(1-\tau) = 1$

Remark: natural equilibrium generally not attainable with sticky prices and wages (proof)

Optimal Monetary Policy Problem

$$\min \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left(\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \widetilde{y}_t^2 + \frac{\epsilon_p}{\lambda_p} (\pi_t^p)^2 + \frac{\epsilon_w (1 - \alpha)}{\lambda_w} (\pi_t^w)^2 \right)$$

subject to:

$$\pi_t^p = \beta E_t \{ \pi_{t+1}^p \} + \varkappa_p \widetilde{y}_t + \lambda_p \widetilde{\omega}_t$$

$$\pi_t^w = \beta E_t \{ \pi_{t+1}^w \} + \varkappa_w \widetilde{y}_t - \lambda_w \widetilde{\omega}_t$$

$$\widetilde{\omega}_{t-1} \equiv \widetilde{\omega}_t - \pi_t^w + \pi_t^p + \Delta \omega_t^n$$

Optimality conditions:

$$\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) \widetilde{y}_t + \varkappa_p \zeta_{1,t} + \varkappa_w \zeta_{2,t} = 0$$

$$\frac{\epsilon_p}{\lambda_p} \, \pi_t^p - \Delta\zeta_{1,t} + \zeta_{3,t} = 0$$

$$\frac{\epsilon_w (1 - \alpha)}{\lambda_w} \, \pi_t^w - \Delta\zeta_{2,t} - \zeta_{3,t} = 0$$

$$\lambda_p \zeta_{1,t} - \lambda_w \zeta_{2,t} + \zeta_{3,t} - \beta E_t\{\zeta_{3,t+1}\} = 0$$

for $t = 0, 1, 2, ...$given $\zeta_{1,-1} = \zeta_{2,-1} = 0$ and given $\widetilde{\omega}_{-1}$.

Equilibrium under the optimal policy:

$$\mathbf{A}_0^* \mathbf{x}_t = \mathbf{A}_\mathbf{1}^* E_t\{\mathbf{x}_{t+1}\} + \mathbf{B}_0^* \Delta a_t$$

where $\mathbf{x}_t \equiv [\widetilde{y}_t, \ \pi_t^p, \ \pi_t^w, \widetilde{\omega}_{t-1}, \zeta_{1,t-1}, \ \zeta_{2,t-1}, \ \zeta_{3,t}]'$.

Optimal Policy in Response to Demand Shocks

$$\mathbf{A}_0^* \mathbf{x}_t = \mathbf{A}_\mathbf{1}^* E_t\{\mathbf{x}_{t+1}\}$$

Under the assumption of $\widetilde{\omega}_{t-1} = 0$,

$$\mathbf{x}_t = 0$$

for all t

Implementation:

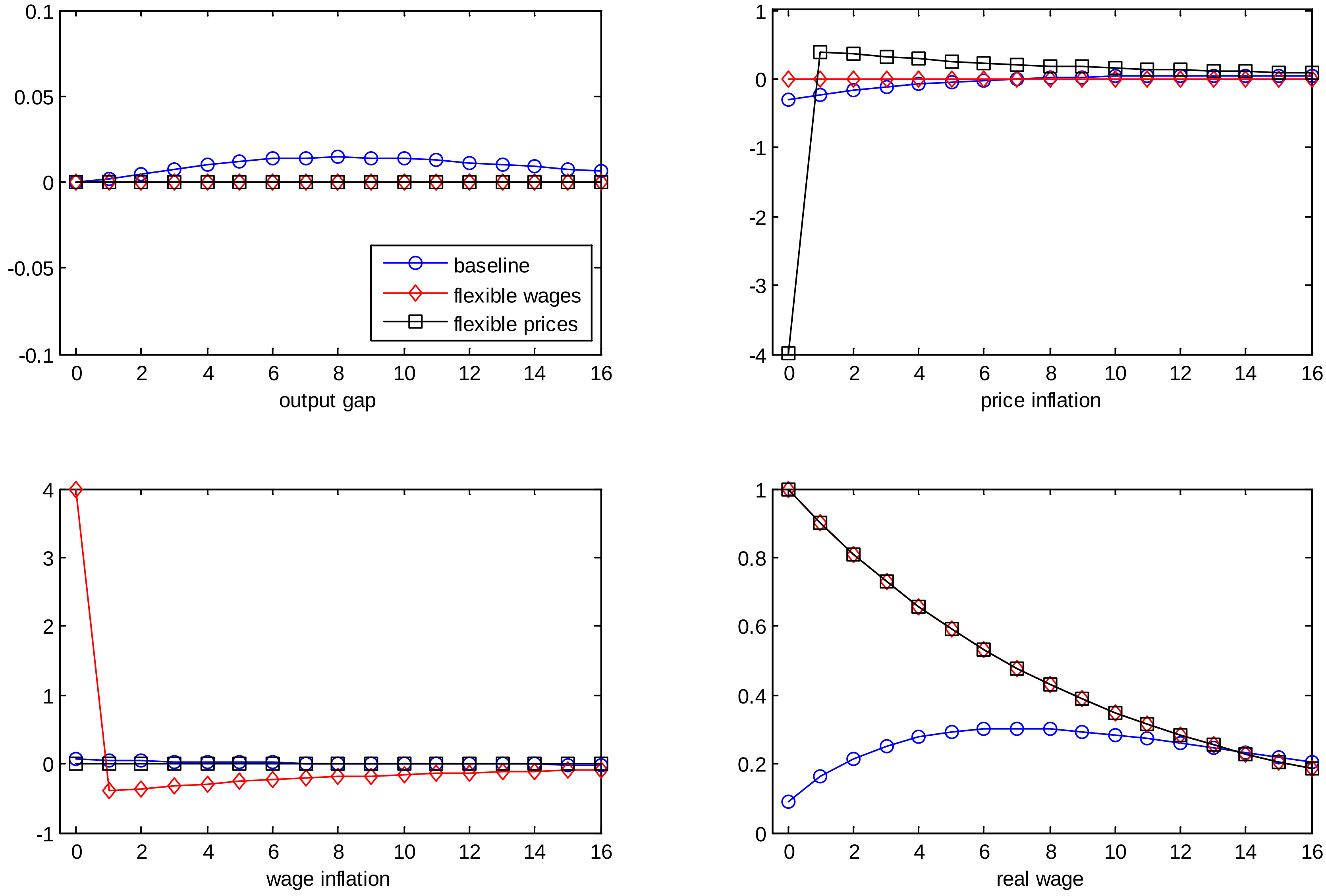
$$i_t = r_t^n + \phi_p \pi_t^p$$

where $r_t^n = \rho + (1 - \rho_z) z_t$, and $\phi_p > 1$

Optimal Policy in Response to Technology Shocks

Figure 6.3

Figure 6.3 Dynamic Responses to a Technology Shock under the Optimal Monetary Policy



A New Keynesian Phillips Curve for Composite Inflation

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

where $\kappa \equiv \frac{\lambda_w \lambda_p}{\lambda_p + \lambda_w} \frac{\alpha}{1-\alpha}$

$$\pi_t \equiv \frac{\lambda_w}{\lambda_p + \lambda_w} \, \pi_t^p + \frac{\lambda_p}{\lambda_p + \lambda_w} \, \pi_t^w$$

$$\Rightarrow \quad \text{no tradeoff}$$

A Special Case

$$\varkappa_p = \varkappa_w \equiv \varkappa$$
$$\epsilon_p = \epsilon_w(1-\alpha) \equiv \epsilon.$$

Implied optimality conditions:

$$\pi_t = -\frac{1}{\epsilon}\, \Delta\widetilde{y}_t \tag{1}$$

for $t = 1, 2, 3, ...$ as well as

$$\pi_0 = -\frac{1}{\epsilon}\, \widetilde{y}_0 \tag{2}$$

Optimal policy:

$$\pi_t = \widetilde{y}_t = 0$$

for all t

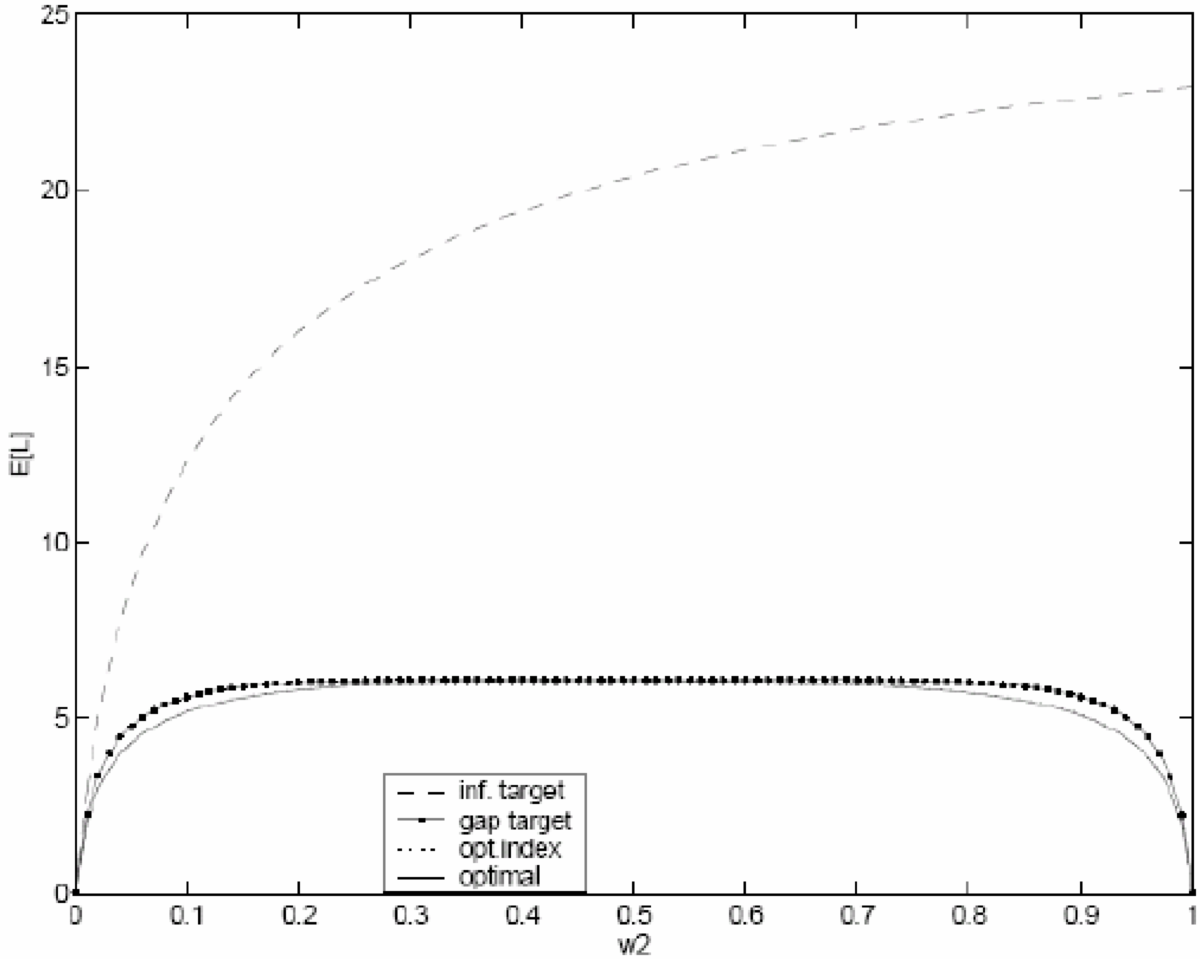


Figure 6.4: Welfare losses under alternative policies with sticky wages and prices.

Woodford (2003)

Evaluation of Simple Interest Rate Rules

Strict Targeting Rules:

$$\pi_t^i = 0$$

Flexible Targeting Rules:

$$i_t = 0.01 + 1.5\pi_t^i$$

Table 6.1

Table 6.1 Evaluation of Simple Rules

	Optimal	*Strict Targeting*			*Flexible Targeting*		
		Price	Wage	Composite	Price	Wage	Composite
Technology shocks							
$\sigma(\pi^p)$	0.11	0	0.13	0.12	0.29	0.24	0.24
$\sigma(\pi^w)$	0.03	0.26	0	0.02	0.23	0.16	0.16
$\sigma(\widetilde{y})$	0.04	3.38	0.20	0	0.84	1.18	1.11
$\mathbb{L}$	0.0330	0.78	0.039	0.0337	0.47	0.305	0.307
Demand shocks							
$\sigma(\pi^p)$	0	0	0	0	0.02	0.04	0.03
$\sigma(\pi^w)$	0	0	0	0	0.05	0.06	0.06
$\sigma(\widetilde{y})$	0	0	0	0	1.08	1.05	1.06
$\mathbb{L}$	0	0	0	0	0.061	0.067	0.066