CREI Lectures in Macroeconomics:
The Economics of Sovereign Debt and Default
Part I

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November 2-4, 2016
Introduction

Sovereign External Debt: 1800-2012
Percent of Countries in Default or Restructuring

Source: Reinhart and Rogoff
Introduction

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Eaton and Gersovitz (1981)

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Eaton and Gersovitz (1981)
Started PhD AG (2006)

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Introduction
Motivation
What are we trying to understand

1. Default
   ▶ Traditional focus of the literature
   ▶ Fairly rare
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2. Large spikes in spreads or “loss of access”
   ▶ Much more frequent than default
   ▶ May be more important to understand
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3. Equilibrium debt dynamics and maturity choice
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3. Equilibrium debt dynamics and maturity choice

4. Role for third-party policies or institutions
Motivating Facts

Major Defaults

▶ Sturzenegger and Zettlemeyer list 29 countries that defaulted or restructured between 1980 and 1983

▶ Major defaults in late 1990s/early 2000s
  ▶ Russia - 1998
  ▶ Ecuador - 1999
  ▶ Argentina - 2001
  ▶ Uruguay - 2003

▶ More recent examples
  ▶ Ecuador - 2008
  ▶ Greece - 2012
  ▶ Argentina - 2014
  ▶ Venezuela - ?
Motivating Facts

Serial Defaulters

---

External sovereign defaults since 1800

Selected countries (number of defaults)

July 31st 2014

- Ecuador (10)
- Venezuela (10)
- Uruguay (9)
- Costa Rica (9)
- Brazil (9)
- Chile (9)
- Argentina (8)
- Peru (8)
- Mexico (8)
- Turkey (8)
- Greece (7)
- Dominican Rep. (7)
- Nicaragua (7)
- Paraguay (7)
- Guatemala (7)
- Austria (7)
- Colombia (7)
- Spain (6)
- Nigeria* (5)
- Russia (5)
- Bolivia (5)
- Ghana* (5)
- Tunisia (5)
- El Salvador (5)
- Germany (4)
- Portugal (4)

Source: Carmen Reinhart and Kenneth Rogoff

*Data from 1960

Economist.com/graphicdetail
Motivating Facts
Why Default?

- Low output
  - Tomz and Wright document 62% of defaults start when output is below trend
  - Average deviation of output is only -1.6%
  - Correlation of output and default status is only -0.08
Motivating Facts
Why Default?

- External Fundamentals
  - Latin American Debt Crisis of 1980s
  - Global Financial Crises/Risk Premia
Motivating Facts

Why Default?

- External Fundamentals
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  - Global Financial Crises/Risk Premia

- Self-fulfilling Runs
  - Mexico 1994/95
  - Europe 2012 ("Whatever it takes...")
Motivating Facts
Why Default?

- Political Shocks
  - Ecuador default in 2008
    - Oil prices high
    - President argued foreign debt was “illegitimate” and bondholders “monsters”
    - Contrast with repayment in 2015 when oil prices were low
  - Greece near default in 2015
    - Syriza elected in January 2015
    - Referendum in July 2015 rejects Troika’s proposed bailout terms
    - Agreement reached a week later averting default
Motivating Facts

Spreads: Italy

<table>
<thead>
<tr>
<th>Year</th>
<th>Spread</th>
<th>Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005q1</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>2008q1</td>
<td>4.0</td>
<td>-0.04</td>
</tr>
<tr>
<td>2011q1</td>
<td>5.0</td>
<td>0.00</td>
</tr>
<tr>
<td>2014q1</td>
<td>6.0</td>
<td>0.02</td>
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</table>

Graph showing trends in spreads and growth.
Motivating Facts

Spreads: Mexico

<table>
<thead>
<tr>
<th>Year</th>
<th>Spread</th>
<th>Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995q1</td>
<td>-0.06</td>
<td></td>
</tr>
<tr>
<td>2000q1</td>
<td>-0.04</td>
<td></td>
</tr>
<tr>
<td>2005q1</td>
<td>-0.02</td>
<td></td>
</tr>
<tr>
<td>2010q1</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>2015q1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Motivating Facts

Spreads and Growth: Emerging Markets

Crisis: Contemporaneous with $\Delta EMBI > 158 bp$
Median Growth: $-0.4$ and $1.1$, resp
Motivating Facts

Spreads and Deleveraging
Motivating Facts

Maturity Choice

- Issuances shorten in crises
- Yield curve flattens or inverts
  - Keep in mind: Secondary market yield curve is not marginal yield
Motivating Facts
Maturity Choice (Spain)

Average Interest Rate
(3–Months MA)

Short  Medium  Long
Motivating Facts
Maturity Choice (Spain)

Monthly Issuances (Billion Euros)
By Maturity (5 Month MA)
Motivating Facts
Marginal vs. Average Yields

Marginal minus Average Interest Rate
Spanish Treasury Auctions (Basis Points)
Taking Stock

- Defaults and spikes in spreads occur regularly
  - But only mildly correlated with output
  - Plausibly some role for self-fulfilling beliefs
  - Political risk important

- Some evidence that high spreads associated with deleveraging

- Maturity choice shifts during crises
Road Map

1. Discuss general framework
Road Map

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2. Analyze one-period bond economy
   ▶ Efficiency and uniqueness
   ▶ Debt dynamics

3. Long-term bonds
   ▶ Inefficiency
   ▶ Debt dynamics
   ▶ Multiplicity

4. Maturity choice
   ▶ With and without rollover risk
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Framework

Nests Key Variations:

- Complete Markets (Arrow-Debreu, Thomas-Worral, Kehoe-Levine)

- Eaton-Gersovitz and descendants (Arellano, Aguiar-Gopinath, Chatterjee-Eygingur, Hatchondo-Martinez, etc.)

- Cole-Kehoe and descendants (e.g. Aguiar-Chatterjee-Cole-Stangebye)

- As well as the models Manuel and I have used in various papers
  - Aguiar-Amador
  - Aguiar-Amador-Gopinath-Farhi
  - Aguiar-Amador-Hopenhayn-Werning
Framework
Basic Environment

- Study a small open economy (SOE) – pins down world risk-free rate
- A single, freely traded good – numeraire
- Benchmark: Time is discrete
  - Some extensions will be easier to discuss in continuous time
Framework

Notation: Exogenous States

- Denote the exogenous state at time $t$ by $s_t$
  - Endowment
  - Punishments
  - Sunspots

- $s^t = \{s_0, s_1, \ldots, s_t\}$

- Date zero probability of history $s^t$: $\pi(s^t)$
Framework

Government

- A single decision maker: Government or Sovereign
- Not necessarily benevolent
- Benchmark preferences:

\[
U(c) = \sum_{t=0}^{\infty} \beta^t \sum_{s^t} \pi(s^t)u(c(s^t))
\]

\[
= E \sum_{t=0}^{\infty} \beta^t u(c_t)
\]
Framework

Lenders

- Atomistic – competitive asset markets
- Discount at $R^{-1} = (1 + r)^{-1}$
- Risk Neutral
  - Have explored extensions with risk-averse lenders
- Have full commitment
Framework

Asset Markets

1. Complete Markets
Framework

Asset Markets

1. Complete Markets

2. One-period non-contingent bond
   - Discount bond: Pays one in all states next period
Framework

Asset Markets

1. Complete Markets

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   - Discount bond: Pays one in all states next period

3. Random maturity bond
   - Poisson process for maturity: $\lambda$
     - Independent across units: LLN implies fraction $\lambda$ matures each period
   - Non-maturing bonds are identical – “perpetual youth” property
   - Special cases:
     - $\lambda = 1$: One-period bonds
     - $\lambda = 0$: Perpetuities
4. Arbitrary portfolio of non-contingent bonds
   ▶ Random maturity or time dependent
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5. Nominal bonds
   ▶ Mention only in passing
   ▶ Interesting to the extent punishment for “default” differs
   ▶ Adds some additional state contingency but brings in additional commitment issues
Framework

Endowment

- Endowment $y_t = y(s^t)$

- Endowment fluctuations dominate discussion of sovereign default models
Framework

Default Payoffs

- How to support repayment is crucial in this class of models
  - Eaton-Gersovitz: Default triggers financial autarky
  - Bulow-Rogoff: Reputation “not enough”
  - Quantitative models: Combination of temporary autarky and direct punishments (endowment loss)
Framework
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- Short-hand for this is value of default: $V^D(s)$
  - Incorporates stochastic punishments (endowment loss, political consequences, etc.)
  - We will treat as a primitive of environment
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  - Incorporates stochastic punishments (endowment loss, political consequences, etc.)
  - We will treat as a primitive of environment
  - Our main source of risk
Framework
Timing

- Timing of actions within a period important
  - Does an auction occur before or after default decision?
  - Does choice of amount of debt occur before or after auction begins?
  - Can there be more than one auction per period?
Framework

Timing

- Canonical “Eaton-Gersovitz” Timing
  1. Exogenous states realized (endowment, default cost, sunspot)
  2. Government decides (commits) to repay or default that period
  3. If repay, decides (commits) how much (face value) new debt to auction that period
  4. Auction occurs
  5. Repayment and consumption
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- Will introduce “Cole-Kehoe” timing later
Framework

Taking Stock

- What our framework captures:
  - Uninsurable risk and default
  - Limited commitment to repayment and fiscal plans more generally
  - Multiplicity and self-fulfilling crises

- Some things we are missing:
  - Richer post-default environments (renegotiation, hold outs, haircuts, etc)
  - Information frictions (other than default payoff)
  - Richer political economy frictions (other than default payoffs)
  - Richer domestic economic environment (private agents, externalities)
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Our Approach
Planning Problems

- To the extent possible, analyze planning problems
- Establish equivalence between competitive equilibrium and a dynamic contract
  - Representative lender as Principal
  - Government as Agent
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Planning Problems

- To the extent possible, analyze planning problems
- Establish equivalence between competitive equilibrium and a dynamic contract
  - Representative lender as Principal
  - Government as Agent
- Useful to highlight in what sense efficiency holds or fails in competitive equilibria
- Requires “flipping” between primal and dual problems
- Approach taken in Aguiar-Amador-Hopenhayn-Werning
Two Planning Problems

1. Complete Markets with Limited Commitment
   ▶ Implication of limited commitment: saving
   ▶ Motivation for saving: Improve insurance
Two Planning Problems

1. Complete Markets with Limited Commitment
   - Implication of limited commitment: saving
   - Motivation for saving: Improve insurance

2. One-period non-contingent bonds
   - Modified welfare theorem for the competitive equilibrium
   - Where inefficiencies arise relative to CM benchmark
   - What incompleteness does to equilibrium allocation relative to CM
   - Incentives to save
Complete Markets Benchmark

Primal Problem

- Government begins with some initial debt $b$
- Trades contingent assets with risk-neutral lenders
- Cannot commit to contracts
  - If reneges, receives $V^D(s)$ in state $s$
  - Gains from trade: Cheaper to provide $V^D(s)$ within relationship
Complete Markets Benchmark
Primal Problem

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- Appeal to Welfare Theorems and solve a planning problem
Pareto Frontier
Complete Markets Benchmark

Pareto Planning Problem

\[ B(s_0, \nu) = \max_c \sum_{t=0}^{\infty} R^{-t} \sum_{s^t} \pi(s^t) \left( y(s^t) - c(s^t) \right) \]
Complete Markets Benchmark

Pareto Planning Problem

\[ B(s_0, v) = \max_c \sum_{t=0}^{\infty} R^{-t} \sum_{s^t} \pi(s^t) (y(s^t) - c(s^t)) \]

subject to:

\[ v \leq U(c) \]
Complete Markets Benchmark

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\]

\[
V^D(s_t) \leq \sum_{k=0}^{\infty} \beta^k \sum_{s^{t+k}} \pi(s^{t+k} | s^t) u(c(s^{t+k})) \text{ for all } t, s^t
\]
Complete Markets Benchmark

- Let $\mu_0$ be multiplier on promised utility
- Let $\mu_0 \beta^t \pi(s^t) \lambda(s^t)$ be multiplier on participation constraint
Complete Markets Benchmark

- Let $\mu_0$ be multiplier on promised utility
- Let $\mu_0 \beta^t \pi(s^t) \lambda(s^t)$ be multiplier on participation constraint
- FOC:

$$0 = -R^{-t} \pi(s^t) + \mu_0 \beta^t \pi(s^t) u'(c(s^t)) + \mu_0 \beta^t \pi(s^t) u'(c(s^t)) \sum_{s^{t-k} \in s^t} \lambda(s^{t-k})$$
Complete Markets Benchmark

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+ \mu_0 \beta^t \pi(s^t) u'(c(s^t)) \sum_{s^{t-k} \in s^t} \lambda(s^{t-k})
$$

- Rearranging:

$$
\frac{1}{\mu_0} = R^t \beta^t u'(c(s^t)) \left(1 + \sum_{s^{t-k} \in s^t} \lambda(s^{t-k})\right)
$$
Complete Markets Benchmark

Backloading

- Suppose $\beta R = 1$:

$$\frac{1}{\mu_0} = u'(c(s^t)) \left( 1 + \sum_{s^{t-k} \in s^t} \lambda(s^{t-k}) \right)$$
Complete Markets Benchmark

Backloading

- Suppose $\beta R = 1$:

$$\frac{1}{\mu_0} = u'(c(s^t)) \left( 1 + \sum_{s^{t-k} \in s^t} \lambda(s^{t-k}) \right)$$

- $\lambda(s^t) \geq 0$

- $\sum \lambda(s^{t-k})$ converges $\Rightarrow \lim \lambda(s^t) \to 0$.

- $c(s^t)$ weakly increases over time and converges to a constant

- Full risk sharing after first realization of $\bar{V}^D = \max_{s \in S} V^D(s)$
Move up whenever \( V^D(s_t) > \max_{j<t} V^D(s_j) \)
Complete Markets Benchmark

Key Implications

- Risk-neutral foreign lenders insuring a risk-averse government
- Limited commitment is the only friction in the model
- Promising consumption in the future relaxes participation constraints along the path
- Extra return to saving: Improves insurance
- No “default” in this environment given complete markets
  - Never exercise outside option $V^D(s)$
Eaton-Gersovitz
Incomplete Markets Planning Problem

- One period non-contingent bond
- Canonical Eaton-Gersovitz (Arellano, Aguiar-Gopinath, etc.) model
Eaton-Gersovitz
Incomplete Markets Planning Problem

- One period non-contingent bond
- Canonical Eaton-Gersovitz (Arellano, Aguiar-Gopinath, etc.) model
- Recast competitive equilibrium as a constrained planning problem
- Highlight how it contrasts with complete markets planning problem
- Shed light on aspects of the equilibrium
Eaton-Gersovitz
Recursive Competitive Equilibrium

- One period discount bond: $b$
Eaton-Gersovitz
Recursive Competitive Equilibrium

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- Exogenous state vector $s \in S$
Eaton-Gersovitz
Recursive Competitive Equilibrium

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- Exogenous state vector $s \in S$
- Equilibrium objects:
  - Price schedule: $q(s, b, b')$
  - Value of repayment: $V^R(s, b)$
  - Default if $V^R(s, b) < V^D(s)$
Eaton-Gersovitz
Recursive Competitive Equilibrium

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  - Value of repayment: $V^R(s, b)$
  - Default if $V^R(s, b) < V^D(s)$
- EG timing:
  1. $s$
  2. Default or Repay Decision
  3. Choose $b'$
  4. Auction
Eaton-Gersovitz
Recursive Competitive Equilibrium

- Lender’s break-even condition:

\[
q(s, b, b') = \begin{cases} 
R^{-1} & \text{if } b' \leq 0 \\
R^{-1} \sum_{s' \in S} \pi(s'|s) \mathbb{1}_{\{V^R(s, b') \geq V^D(s')\}} & \text{otherwise}
\end{cases}
\]

- First row: Risk-free rate if NFA > 0
- Second row: Repayment only if optimal for government
Lender’s break-even condition:

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\end{cases}
\]

- First row: Risk-free rate if NFA > 0
- Second row: Repayment only if optimal for government
- Inherited debt $b$ irrelevant: $q(s, b, b') \rightarrow q(s, b')$
Government’s problem if Repay:

\[ V^R(s, b) = \sup_{c,b'} u(c) + \beta \sum_{s' \in S} \pi(s'|s) \max \langle V^R(s', b'), V^D(s') \rangle \]

subject to:

\[ c \leq y(s) - b + q(s, b') b' \]
Definition 1

An **equilibrium** consists of functions $V^R : S \times \mathbb{R} \rightarrow \mathbb{R}$ and $q : S \times \mathbb{R} \rightarrow [0, R^{-1}]$ such that:

(i) Given $q$, $V^R$ solves government’s problem

(ii) Given $V^R$, $q$ satisfies lenders’ break-even condition and NPC
Show that the competitive equilibrium is solution to a “planning” problem

Highlight how incompleteness changes the complete-markets planning problem
Show that the competitive equilibrium is solution to a “planning” problem

Highlight how incompleteness changes the complete-markets planning problem

Additionally:
- Show equilibrium is fixed point of a contraction operator
- Existence, uniqueness, and a fast method of computation
Road Map:
1. Introduce an operator of a planning problem
Road Map:

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2. Argue operator is a contraction
Eaton-Gersovitz
A Dual Problem

- Road Map:
  1. Introduce an operator of a planning problem
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  3. Show that the inverse of $V^R$ is a fixed point of the operator
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1. Introduce an operator of a planning problem
2. Argue operator is a contraction
3. Show that the inverse of $V^R$ is a fixed point of the operator
4. Result is that equilibrium is solution to a planning problem
Eaton-Gersovitz
A Dual Problem

- Road Map:
  1. Introduce an operator of a planning problem
  2. Argue operator is a contraction
  3. Show that the inverse of $V^R$ is a fixed point of the operator
  4. Result is that equilibrium is solution to a planning problem
  5. Discuss similarities and differences with complete markets planning problem
Start with an equilibrium pair \( \{q, V^R\} \) and define the inverse of \( V^R \) as \( B \):

\[
B(s, V^R(s, b)) = b
\]

for any \( b \leq \bar{b}(s) \)
Inverse Value Function

- Start with an equilibrium pair \( \{ q, V^R \} \) and define the inverse of \( V^R \) as \( B \):

\[
B(s, V^R(s, b)) = b
\]

for any \( b \leq \bar{b}(s) \)

- Given monotonicity, can move between \( V^R \) and its inverse
Inverse Value Function

$V^D(s)$

$\Omega(s, v)$

$B(s, v)$

$v$

$\overline{V}$

$b$

$\overline{b}$
Will argue $B$ is fixed point of operator $T$:

$$[TB](s, v) = \max_{c, \nu(s'), b'} y(s) - c$$

$$+ R^{-1} \max \langle 0, b' \rangle \sum_{s' \in S} \pi(s'|s) \mathbb{1}_{\{\nu(s') \geq V_D(s')\}}$$

$$+ R^{-1} \min \langle 0, b' \rangle$$

subject to:
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subject to:

$$\nu \leq u(c) + \beta \sum_{s' \in S} \pi(s'|s) \max \langle \nu(s'), V^D(s') \rangle$$
Will argue $B$ is fixed point of operator $T$:

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$$+ R^{-1} \min \langle 0, b' \rangle$$

subject to:

$$v \leq u(c) + \beta \sum_{s' \in S} \pi(s'|s) \max \langle v(s'), V^D(s') \rangle$$

$$b' \leq B(s', v(s')) \text{ for } s' \in S \text{ such that } v(s') \geq V^D(s')$$
Eaton-Gersovitz
A Dual Problem

- Blackwell’s Sufficient Conditions: Monotonicity
  - \( B \) shows up on the right-hand side only in an inequality constraint
  - Objective must be weakly increasing in \( B \)
Eaton-Gersovitz
A Dual Problem

- Blackwell’s Sufficient Conditions: Discounting
  - \( B + a \) for \( a > 0 \):
    \[
    b' \leq B(s', v(s')) + a
    \]
  - Rewrite choice as \( \hat{b} \equiv b' - a \):
    \[
    [T(B + a)](s, v) \leq \max_{c, v(s'), \hat{b}} y(s) - c
    + R^{-1} \max\langle 0, \hat{b} \rangle \sum_{s' \in S} \pi(s'|s) \mathbb{1}_{\{v(s') \geq V^D(s')\}}
    + R^{-1} \min\langle 0, \hat{b} \rangle + R^{-1} a
    \]
    subject to \( \hat{b} \leq B(s', v(s')) \)
  - Identical problem with an added constant:
    \( \Rightarrow T(B + a) = TB + R^{-1}a \)
Eaton-Gersovitz
A Dual Problem

- Contraction Mapping Theorem gives us existence and uniqueness of a fixed point
  - Alternative to Auclert-Rognlie
Eaton-Gersovitz
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- Contraction Mapping Theorem gives us existence and uniqueness of a fixed point
  - Alternative to Auclert-Rognlie
  - Contrast with Passadore-Xandri
Eaton-Gersovitz
A Dual Problem

- Contraction Mapping Theorem gives us existence and uniqueness of a fixed point
  - Alternative to Auclert-Rognlie
  - Contrast with Passadore-Xandri

- Next key step is to show that the equilibrium can be mapped into this planning problem
Eaton-Gersovitz
A Dual Problem

\[ [TB](s, \nu) = \max_{c, v(s'), b'} y(s) - c \]
\[ + \max\langle 0, b'\rangle R^{-1} \sum_{s' \in S} \pi(s'|s) \mathbb{1}\{\nu(s') \geq V^D(s')\} \]
\[ + \min\langle 0, b'\rangle R^{-1} \]
subject to:
\[ \nu \leq u(c) + \beta \sum_{s' \in S} \pi(s'|s) \max\langle \nu(s'), V^D(s')\rangle \]
\[ b' \leq B(s', \nu(s')) \text{ for } s' \in S \text{ such that } \nu(s') \geq V^D(s') \]

▶ Show the inverse of \( V^R \) if fixed point
Eaton-Gersovitz
A Dual Problem

\[ [TB](s, \nu) = \max_{c, \nu(s'), b'} \ y(s) - c \]

\[ + \max \langle 0, b' \rangle R^{-1} \sum_{s' \in S} \pi(s'|s) \mathbb{1}_{\{\nu(s') \geq V_D(s')\}} \]

\[ + \min \langle 0, b' \rangle R^{-1} \]

subject to:

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\[ b' \leq B(s', \nu(s')) \text{ for } s' \in S \text{ such that } \nu(s') \geq V_D(s') \]

- Note that objective is weakly increasing in \( b' \) and strictly if \( b' > 0 \)
Eaton-Gersovitz

A Dual Problem

$$[TB](s, v) = \max_{c, v(s'), b'} y(s) - c$$

$$+ \max \langle 0, b' \rangle R^{-1} \sum_{s' \in S} \pi(s'|s) 1\{v(s') \geq V^D(s')\}$$

$$+ \min \langle 0, b' \rangle R^{-1}$$

subject to:

$$v \leq u(c) + \beta \sum_{s' \in S} \pi(s'|s) \max \langle v(s'), V^D(s') \rangle$$

$$b' = B(s', v(s'))$$ for $$s' \in S$$ such that $$v(s') \geq V^D(s')$$

- Note that objective is weakly increasing in $$b'$$ and strictly if $$b' > 0$$
Eaton-Gersovitz

A Dual Problem

\[
[TB](s, v) = \max_{c, v(s'), b'} y(s) - c \\
+ \max \langle 0, b' \rangle R^{-1} \sum_{s' \in S} \pi(s'|s) \mathbb{1}_{\{v(s') \geq V^D(s')\}} \\
+ \min \langle 0, b' \rangle R^{-1}
\]

subject to:

\[
v \leq u(c) + \beta \sum_{s' \in S} \pi(s'|s) \max \langle v(s'), V^D(s') \rangle
\]

\[
b' = B(s', v(s')) \text{ for } s' \in S \text{ such that } v(s') \geq V^D(s')
\]

- By definition of \( B \): \( b' = B(s', v(s')) \Rightarrow V^R(s', b') = v(s') \)
Eaton-Gersovitz
A Dual Problem

\[ [TB](s, v) = \max_{c,v(s'),b'} y(s) - c \]

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subject to:

\[ v \leq u(c) + \beta \sum_{s' \in S} \pi(s'|s) \max \langle v(s'), V^D(s') \rangle \]

\[ V^R(s', b') = v(s') \text{ for } s' \in S \text{ such that } v(s') \geq V^D(s') \]

By definition of \( B \): \( b' = B(s', v(s')) \Rightarrow V^R(s', b') = v(s') \)
Eaton-Gersovitz

A Dual Problem

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[TB](s, \nu) = \max_{c, \nu(s'), b'} y(s) - c
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+ \min \langle 0, b' \rangle R^{-1}
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subject to:
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\nu \leq u(c) + \beta \sum_{s' \in S} \pi(s'|s) \max \langle \nu(s'), V^D(s') \rangle
\]
\[
V^R(s', b') = \nu(s') \text{ for } s' \in S \text{ such that } \nu(s') \geq V^D(s')
\]

- To substitute out \( \nu(s') \) need to rule out:
  \[
  V^R(s', b') \geq V^D(s') > \nu(s')
  \]
Eaton-Gersovitz
A Dual Problem

\[
[TB](s, v) = \max_{c,v(s'),b'} y(s) - c \\
+ \max \langle 0, b' \rangle R^{-1} \sum_{s' \in S} \pi(s'|s) 1 \{v(s') \geq V^D(s')\} \\
+ \min \langle 0, b' \rangle R^{-1}
\]

subject to:

\[
v \leq u(c) + \beta \sum_{s' \in S} \pi(s'|s) \max \langle v(s'), V^D(s') \rangle
\]

\[
V^R(s', b') = v(s') \text{ for } s' \in S \text{ such that } v(s') \geq V^D(s')
\]

- But \(V^R(s', b') \geq V^D(s') > v(s')\) is never optimal
Eaton-Gersovitz
A Dual Problem

\[
[TB](s, \nu) = \max_{c, b'} y(s) - c \\
+ \max \langle 0, b' \rangle R^{-1} \sum_{s' \in S} \pi(s'|s) \mathbb{1}\{VR(s',b') \geq VD(s')\} \\
+ \min \langle 0, b' \rangle R^{-1}
\]

subject to:

\[
\nu \leq u(c) + \beta \sum_{s' \in S} \pi(s'|s) \max \langle VR(s',b'), VD(s') \rangle
\]

- Substitute out \( \nu(s') \) using \( VR(s',b') = \nu(s') \) constraint
Eaton-Gersovitz
A Dual Problem

\[ [TB](s, v) = \max_{c, b'} \left( y(s) - c \right) \]
\[ + \max \langle 0, b' \rangle R^{-1} \sum_{s' \in S} \pi(s' \mid s) \mathbb{I}\{V_R(s', b') \geq V_D(s')\} \]
\[ + \min \langle 0, b' \rangle R^{-1} \]
\[ \text{subject to:} \]
\[ v \leq u(c) + \beta \sum_{s' \in S} \pi(s' \mid s) \max \langle V^R(s', b'), V^D(s') \rangle \]

▶ Lender’s break-even condition:

\[ q(s, b') = \begin{cases} 
R^{-1} & \text{if } b' \leq 0 \\
R^{-1} \sum_{s'} \pi(s' \mid s) \mathbb{I}\{V_R(s', b') \geq V_D(s')\} & \text{if } b' \geq 0
\end{cases} \]
Eaton-Gersovitz
A Dual Problem

\[ [TB](s, v) = \max_{c, b'} y(s) - c + q(s, b')b' \]

subject to:
\[ v \leq u(c) + \beta \sum_{s' \in S} \pi(s'|s) \max \langle V^R(s', b'), V^D(s') \rangle \]

- Lender’s break-even condition:
\[ q(s, b') = \begin{cases} R^{-1} & \text{if } b' \leq 0 \\ R^{-1} \sum_{s'} \pi(s'|s) \mathbb{I}\{ V^R(s', b') \geq V^D(s') \} & \text{if } b' \geq 0 \end{cases} \]
This is the dual of the government’s problem:

\[ b = \max_{c,b'} y(s) - c + q(s, b') b' \]

subject to

\[ v \leq u(c) + \beta \sum_{s' \in S} \pi(s'|s) \max \langle V^R(s', b'), V^D(s') \rangle \]
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Hence \( B(s, v) = b = TB(s, v) \)
Eaton-Gersovitz

A Dual Problem

- This is the dual of the government’s problem:

\[
b = \max_{c,b'} y(s) - c + q(s, b') b'
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subject to

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v \leq u(c) + \beta \sum_{s' \in S} \pi(s'|s) \max \langle V^R(s', b'), V^D(s') \rangle
\]

- Hence \( B(s, v) = b = TB(s, v) \)

- Thus an equilibrium pair \( \{q, V^R\} \) generates an inverse value that is a fixed point of our operator

- From the Contraction Mapping Theorem existence and uniqueness follows
The Eaton-Gersovitz Planning Problem
Two Key Steps

- Choosing continuation values resembles complete markets planning problem
- How can $v(s')$ be a state-by-state choice in an incomplete markets environment?
The Eaton-Gersovitz Planning Problem
Two Key Steps

- Choosing continuation values resembles complete markets planning problem
- How can $\nu(s')$ be a state-by-state choice in an incomplete markets environment?
- Role of the constraint:

\[ b' = B(s', \nu(s')) \text{ for all } s' \text{ such that } \nu(s') \geq V^D(s') \]

- Restricts freedom to allocate utility across states
The Eaton-Gersovitz Planning Problem

Two Key Steps

- How to replace $V^R(s', b')$, an equilibrium object, with a choice $v(s')$?
The Eaton-Gersovitz Planning Problem

Two Key Steps

- How to replace $V^R(s', b')$, an equilibrium object, with a choice $v(s')$?
- How is planning problem independent of $q(s, b')$?
The Eaton-Gersovitz Planning Problem

Two Key Steps

- How to replace $V^R(s', b')$, an equilibrium object, with a choice $v(s')$?

- How is planning problem independent of $q(s, b')$?

- Both are related:
  - $q(s, b')$ uniquely pinned down by $V^R(s', b')$ – really only one equilibrium object
  - Constraint $b' = B(s', v(s'))$ ensures that $v(s') = V^R(s', b')$ as long as $B$ is the inverse of the equilibrium value
The EG Planning Problem

Frictions

- Planning problem suggests cannot find a better allocation that satisfies limited commitment to repay and incompleteness of markets
The EG Planning Problem
Frictions

- Planning problem suggests cannot find a better allocation that satisfies limited commitment to repay and incompleteness of markets

- Two (related) frictions:
  1. Incomplete Markets
     - Cannot insure fluctuations in $y(s)$
The EG Planning Problem

Incomplete Markets

\[ B(s, v) = \max_{c, v(s'), b'} y(s) - c + R^{-1} b' \sum_{s' \in S} \pi(s'|s) \mathbb{1}_{\{v(s') \geq V^D(s')\}} \]

subject to:

\[ v \leq u(c) + \beta \sum_{s' \in S} \pi(s'|s) \max\langle v(s'), V^D(s') \rangle \]

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- Two (related) frictions:
  1. Incomplete Markets
     - Cannot insure fluctuations in $y(s)$
  2. Deadweight Costs of Default
The EG Planning Problem

Costs of Default

\[ B(s, \nu) = \max_{c, \nu(s'), b'} \nu(s) - c + R^{-1} b' \sum_{s' \in S} \pi(s'|s) \mathbb{1}_{\{\nu(s') \geq V^D(s')\}} \]

subject to:

\[ \nu \leq u(c) + \beta \sum_{s' \in S} \pi(s'|s) \max\langle \nu(s'), V^D(s') \rangle \]

\[ b' \leq B(s', \nu(s')) \text{ for } s' \in S \text{ such that } \nu(s') \geq V^D(s') \]
The EG Planning Problem

Frictions

- Planning problem suggests cannot find a better allocation that satisfies limited commitment to repay and incompleteness of markets

- Two (related) frictions:
  1. Incomplete Markets
     - Cannot insure fluctuations in $y(s)$
  2. Deadweight Costs of Default
     - Moving from $v(s') = V^D(s') - \epsilon$ to $v(s') = V^D(s')$
     - Second-order costs
     - First-order gain: $b'\pi(s'|s)$
     - Cannot avoid due to IM restriction on $v(s')$
The Eaton-Gersovitz Planning Problem

Inefficiency

- Both frictions provide an incentive to save
- Lack of insurance generates precautionary saving
  - Close parallel to CM benchmark: More wealth implies better insurance
The Eaton-Gersovitz Planning Problem

Inefficiency

- Both frictions provide an incentive to save
- Lack of insurance generates precautionary saving
  - Close parallel to CM benchmark: More wealth implies better insurance
- Deadweight loss of default also generates saving
  - But how is this internalized in equilibrium when...
    - Prices are actuarially fair
    - Government chooses default because it is optimal
Taking Stock and Next Steps

- One-period bond model is solution to planning problem
- Equilibrium unique
  - Not true with long-term bonds
    - Will show examples in next lecture
- Has some nice efficiency properties
  - This will be important when we discuss maturity choice
- All of these issues will be related to the incentives to save