

International Debt Deleveraging

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This draft: September 2016

First draft: November 2012

Abstract

This paper provides a framework to understand debt deleveraging in a group of financially integrated countries. During an episode of international deleveraging, world consumption demand is depressed and the world interest rate is low, reflecting a high propensity to save. If exchange rates are allowed to float, deleveraging countries can rely on depreciations to increase production and mitigate the fall in consumption associated with debt reduction. The key insight of the paper is that in a monetary union this channel of adjustment is shut off, because deleveraging countries cannot depreciate against the other countries in the monetary union, and therefore the fall in the demand for consumption and the downward pressure on the interest rate are amplified. As a result, deleveraging in a monetary union can generate a liquidity trap and an aggregate recession.

JEL Classification Numbers: E31, E44, E52, F32, F34, F41, G01, G15.

Keywords: Global Debt Deleveraging, Sudden Stops, Liquidity Trap, Monetary Union, Precautionary Savings, Debt Deflation.

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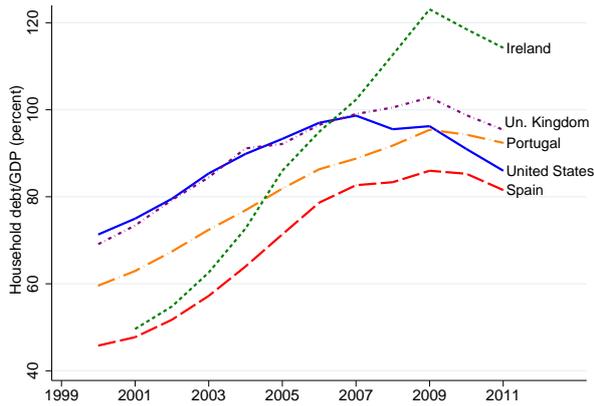
1 Introduction

Episodes of global debt deleveraging are rare, but when they occur they come with deep recessions and destabilize the international monetary system. In the Great Depression of the 1930s the world entered a period of global debt reduction and experienced the most severe recession in modern history. The cornerstone of the international monetary system, the Gold Standard, came under stress and was abandoned in 1936, when the remaining countries belonging to the Gold Block gave up their exchange rate pegs against gold. Almost 80 years later, history seems to be repeating itself. Following the 2007-2008 turmoil in financial markets several countries experienced sudden stops in capital inflows and embarked in a process of private debt deleveraging (Figure 1), accompanied by a deep economic downturn, the Great Recession. Once again, the status quo in the international monetary system has been challenged, and this time the survival of the euro area has been called into question. These events suggest that fixed exchange arrangements, such as monetary unions, are hard to maintain during times of global debt deleveraging. But more research is needed to understand exactly why this is the case.

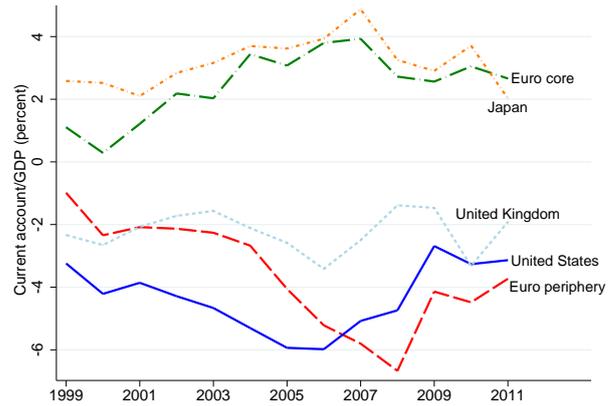
This paper provides a novel framework to study the adjustment triggered by an episode of debt deleveraging among financially integrated countries, and particularly the role played by the exchange rate regime. The model features a continuum of small open economies trading with each other. Each economy is inhabited by identical households which borrow and lend to smooth the impact of temporary, country-specific, productivity shocks on consumption, in the spirit of the Bewley (1977) closed economy model. Foreign borrowing and lending arise endogenously as households use the international credit markets to insure against country-specific productivity shocks. Crucially, each household is subject to an exogenous borrowing limit. I study the response of the world economy to a *deleveraging shock*, which consists in a permanent tightening of the borrowing limit. The model cannot be solved analytically, and I analyse its properties through simulations of a deleveraging event that captures some salient features of the euro area adjustment to the 2008 global financial crisis.

I start by considering a baseline economy in which the only frictions present are the borrowing limit and incomplete financial markets. The first result is that the process of debt reduction generates a fall in the world interest rate, which overshoots its long run value. The drop in the world interest rate is due to two different effects. On the one hand, the most indebted countries are hit by a sudden stop in capital inflows and are forced to increase savings, in order to reduce their debt and satisfy the new borrowing limit. On the other hand, the countries starting with a low stock of debt, as well as those starting with a positive stock of foreign assets, want to increase precautionary savings as a buffer against the risk of hitting the borrowing limit in the future. Both effects lower global consumption demand and generate a rise in the propensity to save. As a consequence, the world interest rate falls to guarantee that the rest of the world absorbs the forced savings of high-debt borrowing-constrained economies.

In the baseline model, deleveraging also affects the supply side of the economy. In fact, high-



(a) Household debt in percent of GDP



(b) Current account in percent of GDP

Figure 1: Motivating facts. Notes: the left panel illustrates the fall in private debt characterizing the US, the UK and the euro area periphery in the aftermath of the 2008 financial crisis. The right panel shows that deleveraging has been accompanied by improvements in the current account, especially in the case of euro area peripheral countries. It also illustrates the contemporaneous fall in the current account surplus of creditor countries, here captured by core euro area countries and Japan. Data are from Eurostat and the OECD.

debt countries respond to the deleveraging shock by increasing their production of tradable goods, so as to repay their external debt without cutting consumption too severely. The opposite occurs in the rest of the world, which experiences a contraction in the production of tradable goods. This process redistributes income from wealthy countries, characterized by a low propensity to consume, toward high-debt borrowing-constrained countries, featuring a high propensity to consume. Hence, the supply-side response to the deleveraging shock mitigates the fall in global consumption demand, and consequently the drop in the world interest rate.

Importantly, in order for the supply-side adjustment to take place, real wages need to fall in high-debt countries and rise in the rest of the world. A large body of evidence, however, suggests that nominal wages adjust slowly to shocks. In particular nominal wages do not fall much during deep recessions, in spite of sharp rises in unemployment.¹ To understand the implications of this friction, I then turn to a model in which nominal wages are partially rigid.

With nominal wage rigidities, monetary policy and the exchange rate regime affect the response of real variables to the deleveraging shock. I find that, when exchange rates are flexible and monetary policy stabilizes CPI inflation, the adjustment to deleveraging is essentially identical to the one occurring in the baseline model with flexible wages. In fact, under flexible exchange rates the fall in real wages in high-debt countries is attained with a nominal exchange rate depreciation. Conversely, countries in the rest of the world experience a nominal exchange rate appreciation which leads to an increase in real wages. But in a monetary union exchange rates between members are

¹In their empirical studies, Eichengreen and Sachs (1985) and Bernanke and Carey (1996) find that nominal wage rigidities contributed substantially to the fall in output during the Great Depression, in particular among countries belonging to the Gold Block. More recently, Schmitt-Grohé and Uribe (2016) have documented the importance of nominal wage rigidities in the context of the 2001 Argentine crisis and of the Great Recession in countries at the Eurozone periphery. Another strand of the literature shows the relevance of nominal wage rigidities using micro data. For example, Fehr and Goette (2005), Gottschalk (2005) and Barattieri et al. (2014) use worker-level data to show that changes in nominal wages, especially downward, happen infrequently. Fabiani et al. (2010) obtain similar results using firm-level data from several European countries.

fixed, and the adjustment in real wages cannot be achieved through movements in the nominal exchange rate. Indeed, when I consider a world in which all countries belong to a single monetary union, and in which monetary policy stabilizes average CPI inflation, I find that the production response to the deleveraging shock is essentially muted. Thus, in a monetary union households living in high-debt countries have to reduce their debt mainly by decreasing consumption. The deep fall in consumption demand coming from high-debt countries amplifies the increase in the propensity to save and the downward pressure on the interest rate. The result is that during deleveraging the drop in the world interest rate is much larger in a monetary union, compared to the economy with flexible exchange rates.

In the last part of the paper I focus on a monetary union, and study the impact of deleveraging on output and welfare. First, I show that plausible values of the deleveraging shock give rise to quantitatively relevant union-wide recessions. This happens because, following the deleveraging shock, monetary policy ends up being constrained by the zero lower bound on the nominal interest rate. Since the interest rate cannot fall enough to guarantee market clearing at the central bank's inflation target, firms decrease prices in order to eliminate excess supply. Given the sticky nominal wages, the fall in prices translates into a rise in real wages that reduces employment and production. Thus, during deleveraging the monetary union enters a liquidity trap, characterized by a deflationary recession. For instance, the benchmark deleveraging shock, which generates a fall in capital inflows toward high-debt countries similar to the one experienced in 2009 by peripheral euro area countries, produces over two years a cumulated fall in the output of the whole monetary union equal to 11 percent of steady state production. The recession hits high-debt countries particularly hard, but the economic downturn also spreads to the countries that are not financially constrained. I also show that the frictions associated with participation in a monetary union generate substantial welfare losses during deleveraging, especially in high-debt countries.

Finally, I discuss policy interventions that mitigate the recession during deleveraging in a monetary union. First, I show that a higher inflation target mitigates the fall in output during deleveraging. Indeed, when the nominal interest rate hits the zero bound the real interest rate is equal to the inverse of expected inflation, so that a higher inflation target implies a lower real interest rate, which stimulates consumption demand and production. Second, I consider the impact of transfers from creditor to debtor countries. Since debtor countries have a higher propensity to consume out of income than creditors, the transfers stimulate aggregate demand and limit the drop in output during deleveraging. I show that both policy interventions have a positive impact on aggregate welfare. However, in both cases the welfare gains are unevenly distributed across countries. In fact, while high-debt financially-constrained economies enjoy large welfare gains from both policy interventions, wealthy countries suffer welfare losses.

This paper is related to several strands of the literature. First, the paper is related to the literature on sudden stops in capital inflows. Some examples of this literature are Caballero and Krishnamurthy (2003), Cespedes et al. (2004), Christiano et al. (2004), Cook (2004), Devereux et al. (2006), Braggion et al. (2007), Gertler et al. (2007), Schmitt-Grohé and Uribe (2016), Fornaro

(2015) and Ottonello (2013). While all these papers study a single small open economy that takes the world interest rate as given, my paper contributes to this literature by considering a global economy in which the endogenous determination of the world interest rate is crucial.²

Second, the paper is about deleveraging and liquidity traps. Recently, Guerrieri and Lorenzoni (2011) and Eggertsson and Krugman (2012) have drawn a connection between deleveraging and drops in the interest rate in closed economies, while the focus of this paper is on the international dimension of a deleveraging episode. Deleveraging in open economies is also studied by Midrigan and Philippon (2011) and Benigno and Romei (2014). Midrigan and Philippon (2011) consider a monetary union featuring a cash-in-advance constraint in which credit can be used as a substitute for fiat money. In their model, the fall in consumption is generated by a decrease in the provision of private credit that tightens households' cash-in-advance constraints, while here the emphasis is on intertemporal debt and liquidity traps. Benigno and Romei (2014) study a global liquidity trap triggered by deleveraging in a two-country model. Their main focus is on the equilibrium reached under the cooperative optimal policy when exchange rates are flexible. Instead, here the focus is on the constraints on the macroeconomic adjustment to deleveraging imposed by participation in a monetary union. Moreover, compared to the standard two-country model studied by Benigno and Romei (2014), the model proposed by this paper captures the rise in precautionary savings triggered by the deleveraging shock,³ and allows for the study of the heterogeneous impact of deleveraging on output and welfare across countries member of the monetary union.⁴

The paper also relates to the literature studying precautionary savings in incomplete-market economies with idiosyncratic shocks. The literature includes the seminal works of Bewley (1977), Huggett (1993) and Aiyagari (1994), who consider closed economies in which consumers borrow and lend to self-insure against idiosyncratic income shocks. Guerrieri and Lorenzoni (2011) use a Bewley model to study the impact of deleveraging on the interest rate in a closed economy. My paper shares with their work the focus on precautionary savings. Starting from Clarida (1990), some authors have used multi-country models with idiosyncratic shocks and incomplete markets to study international capital flows. Examples are Castro (2005), Bai and Zhang (2010) and Chang et al. (2013). This is the first paper that employs a multi-country Bewley model to study the interactions between deleveraging, the exchange rate regime and liquidity traps.

From an empirical perspective, this paper is linked to the work of Lane and Milesi-Ferretti (2012), who look at the adjustment in the current account balances during the Great Recession. They find that the compression in the current account deficits was larger for those countries that

²The current events in the Eurozone have revived the literature on the macroeconomic management of monetary unions. Recent contributions build on the multi-country framework developed by Galí and Monacelli (2005). Examples are Werning and Farhi (2012), who look at the optimal management of fiscal policy in a monetary union, and Farhi et al. (2014), who derive a set of fiscal measures able to substitute for exchange rate flexibility inside a currency union. Instead, Benigno (2004) uses a two-country model to study monetary unions. These frameworks abstract from financial frictions, a key element in my analysis.

³Carroll et al. (2012) and Mody et al. (2012) show that precautionary savings increased in the aftermath of the 2008 financial crisis.

⁴Another difference is that in Benigno and Romei (2014) nominal rigidities prevent the adjustment in the terms of trade. Instead, here nominal rigidities impede the reallocation of production between tradable and non-tradable goods. These are two complementary adjustment mechanisms.

were relying more heavily on external financing before the crisis. Moreover, they find that most of the adjustment passed through a compression in domestic demand, contributing to the severity of the crisis in deficit countries. My model rationalizes these facts. This paper also speaks to the empirical findings of Mian et al. (2013) and Mian and Sufi (2014). These authors find that the fall in consumption and employment in the US during the 2008-2009 recession was stronger in those counties where the pre-crisis expansion in credit driven by the rise in house prices was more pronounced. This evidence is consistent with the results of my paper, if the monetary union version of the model is interpreted as a large country composed of many different regions.

The rest of the paper is structured as follows. Section 2 introduces the baseline model and briefly analyzes the steady state. Section 3 considers the adjustment following a deleveraging shock in the baseline model. Section 4 describes the response to a deleveraging shock in a model with nominal wage rigidities. Section 5 highlights the role of the zero lower bound in translating a deleveraging episode into a recession in a monetary union, and presents some policy experiments. Section 6 concludes.

2 Baseline model

I start by studying a baseline model in which the only frictions present are located in the financial markets. This simple model is useful to obtain intuition about some crucial channels of adjustment triggered by the deleveraging shock. It will also serve as a comparison benchmark for the, more realistic, model with nominal wage rigidities studied in Section 4.

Consider a world composed of a continuum of measure one of small open economies indexed by $i \in \{0, 1\}$. Each economy can be thought of as a country.⁵ Time is discrete and indexed by t . Each country is populated by a continuum of measure one of identical infinitely lived households and by a large number of firms. All economies produce two consumption goods: a homogeneous tradable good and a non-tradable good. Countries face idiosyncratic shocks in their production technologies, while the world economy has no aggregate uncertainty. Households borrow and lend on the international credit markets in order to smooth the impact of productivity shocks on consumption. There is an exogenous limit on how much each household can borrow. I start by analyzing the steady state of the model, in which the borrowing limit is held constant. The next section studies the transition after an unexpected shock that tightens the borrowing limit.

Households. Households derive utility from consumption $C_{i,t}$ and experience disutility from labor effort $L_{i,t}$. The expected lifetime utility of the representative household in a generic country i is

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_{i,t}^{1-\gamma} - 1}{1-\gamma} - \frac{L_{i,t}^{1+\psi}}{1+\psi} \right) \right], \quad (1)$$

with $\gamma \geq 1$ and $\psi \geq 0$. In this expression, $E_t[\cdot]$ is the expectation operator conditional on information available at time t and $0 < \beta < 1$ is the subjective discount factor. The period utility

⁵Another possibility is to think of an economy as a region inside a large country, for example a US state or county.

function is separable in consumption and labor effort, as it is commonly assumed in the literature on monetary economics.⁶ Consumption is a Cobb-Douglas aggregate of a tradable good $C_{i,t}^T$ and a non-tradable good $C_{i,t}^N$

$$C_{i,t} = (C_{i,t}^T)^\omega (C_{i,t}^N)^{1-\omega},$$

where $0 < \omega < 1$.

Each household can trade in one period risk-free bonds. Bonds are denominated in units of the tradable consumption good and pay the gross interest rate R_t . The interest rate is common across countries, and hence R_t can be interpreted as the *world interest rate*.

There are no trade frictions and the price of the tradable good is the same in every country. Normalizing the price of the traded good to 1, the household budget constraint expressed in units of the tradable good is

$$C_{i,t}^T + p_{i,t}^N C_{i,t}^N + \frac{B_{i,t+1}}{R_t} = w_{i,t} L_{i,t} + B_{i,t} + \Pi_{i,t}. \quad (2)$$

The left-hand side of this expression represents the household's expenditure. $p_{i,t}^N$ denotes the price of a unit of non-tradable good in terms of the tradable good in country i .⁷ Hence, the term $C_{i,t}^T + p_{i,t}^N C_{i,t}^N$ is the total expenditure of the household in consumption expressed in units of the tradable good. $B_{i,t+1}$ denotes the purchase of bonds made by the household at time t at price $1/R_t$. If $B_{i,t+1} < 0$ the household is a borrower.

The right-hand side captures the household's income. $w_{i,t} L_{i,t}$ is the household's labor income. Labor is immobile across countries and hence the wage $w_{i,t}$ is country-specific. $B_{i,t}$ is the gross return on investment in bonds made at time $t - 1$. Finally, $\Pi_{i,t}$ denotes the total profits received from firms. All domestic firms are wholly owned by domestic households and equity holdings within these firms are evenly divided among them.

There is a limit on how much each household is able to borrow. In particular, debt repayment cannot exceed the exogenous limit κ_t , so that the bond position has to satisfy⁸

$$B_{i,t+1} \geq -\kappa_t. \quad (3)$$

This constraint captures in a simple form a case in which a household cannot credibly commit in period t to repay more than κ_t units of the tradable good to its creditors in period $t + 1$.⁹

The household's optimization problem is to choose a sequence $\{C_{i,t}^T, C_{i,t}^N, L_{i,t}, B_{i,t+1}\}_{t \geq 0}$ to maximize the expected present discounted value of utility (1), subject to the budget constraint (2) and the borrowing limit (3), taking the initial bond holdings $B_{i,0}$, prices $\{R_t, p_{i,t}^N, w_{i,t}\}_{t \geq 0}$, and

⁶See, for example, Galí (2009).

⁷ $p_{i,t}^N$ is not necessarily equalized across countries because the non-traded good is, by definition, not traded internationally.

⁸Throughout the analysis I assume that the exogenous borrowing limit κ_t is tighter than the natural borrowing limit.

⁹In reality tight access to credit may manifest itself through high interest rates, rather than through a quantity restriction on borrowing. In appendix A I show that it is possible to recast the borrowing limit (3) in terms of positive spreads over the world interest rate without changing any of the results.

the path for the borrowing limit $\{\kappa_t\}_{t \geq 0}$ as given. The household's first-order conditions can be written as

$$p_{i,t}^N = \frac{1 - \omega}{\omega} \frac{C_{i,t}^T}{C_{i,t}^N} \quad (4)$$

$$L_{i,t}^\psi = w_{i,t} \lambda_{i,t} \quad (5)$$

$$\frac{\lambda_{i,t}}{R_t} = \beta E_t [\lambda_{i,t+1}] + \mu_{i,t} \quad (6)$$

$$B_{i,t+1} \geq -\kappa_t, \quad \text{with equality if } \mu_{i,t} > 0, \quad (7)$$

where $\lambda_{i,t} \equiv \omega C_{i,t}^{1-\gamma} / C_{i,t}^T$ denotes the marginal utility from consumption of the tradable good, while $\mu_{i,t}$ is the non-negative Lagrange multiplier associated with the borrowing limit. The optimality condition (4) equates the marginal rate of substitution of the two consumption goods, tradables and non-tradables, to their relative price. Equation (5) is the optimality condition for labor supply. Equation (6) is the Euler equation for bonds. When it binds, the borrowing constraint generates a wedge between the marginal utility from consuming in the present and the marginal utility from consuming next period, given by the shadow price of relaxing the borrowing constraint $\mu_{i,t}$. Finally, equation (7) is the complementary slackness condition associated with the borrowing limit.

Firms. Firms rent labor from households and produce both consumption goods, taking prices as given. A typical firm in the tradable sector in country i maximizes profits

$$\Pi_{i,t}^T = Y_{i,t}^T - w_{i,t} L_{i,t}^T,$$

where $Y_{i,t}^T$ is the output of tradable good and $L_{i,t}^T$ is the amount of labor employed by the firm. The production function is

$$Y_{i,t}^T = A_{i,t}^T (L_{i,t}^T)^{\alpha_T},$$

where $0 < \alpha_T < 1$.¹⁰ $A_{i,t}^T$ determines labor productivity in the tradable sector. Profit maximization implies

$$\alpha_T A_{i,t}^T (L_{i,t}^T)^{\alpha_T - 1} = w_{i,t}.$$

This expression says that at the optimum firms equalize the marginal profit from an increase in labor, the left-hand side of the expression, to the marginal cost, the right-hand side.

Similarly, firms in the non-tradable sector maximize profits

$$\Pi_{i,t}^N = p_{i,t}^N Y_{i,t}^N - w_{i,t} L_{i,t}^N,$$

where $Y_{i,t}^N$ is the output of non-tradable good and $L_{i,t}^N$ is the amount of labor employed in the non-tradable sector. Labor is perfectly mobile across sectors within a country and hence firms in both sectors pay the same wage $w_{i,t}$. The production function available to firms in the non-tradable

¹⁰To introduce constant returns-to-scale in production we can assume a production function of the form $Y_{i,t}^T = A_{i,t}^T (L_{i,t}^T)^{\alpha_T} K^{1-\alpha_T}$, where K is a fixed production factor owned by the firm, for example physical or organizational capital. The production function in the main text corresponds to the normalization $K = 1$.

sector is

$$Y_{i,t}^N = A_{i,t}^N (L_{i,t}^N)^{\alpha_N},$$

where $0 < \alpha_N < 1$. The term $A_{i,t}^N$ determines the productivity of firms in the non-tradable sector. The optimal choice of labor in the non-tradable sector implies

$$p_{i,t}^N \alpha_N A_{i,t}^N (L_{i,t}^N)^{\alpha_N - 1} = w_{i,t}.$$

Just as firms in the tradable sector, at the optimum firms in the non-tradable sector equalize the marginal benefit from increasing employment to its marginal cost.¹¹

Every period countries are hit by idiosyncratic shocks to their labor productivity. Specifically, both $A_{i,t}^T$ and $A_{i,t}^N$ are stochastic and follow Markov processes. These shocks are the source of idiosyncratic uncertainty that gives rise to cross-country financial flows in steady state.

Market clearing. Since households inside a country are identical, we can interpret equilibrium quantities as either household or country specific. For instance, the end-of-period net foreign asset position of country i is equal to the end-of-period holdings of bonds of the representative household divided by the world interest rate¹²

$$NFA_{i,t} = \frac{B_{i,t+1}}{R_t}.$$

Market clearing for the non-tradable consumption good requires that in every country consumption is equal to production, that is $C_{i,t}^N = Y_{i,t}^N$. Moreover, equilibrium on the labor market implies that in every country the labor supplied by the households is equal to the labor demanded by firms, $L_{i,t} = L_{i,t}^T + L_{i,t}^N$.

These two market clearing conditions, in conjunction with the budget constraint of the household, as well as with the equilibrium condition $\Pi_{i,t} = \Pi_{i,t}^T + \Pi_{i,t}^N$, give the market clearing condition for the tradable consumption good in country i

$$C_{i,t}^T = Y_{i,t}^T + B_{i,t} - \frac{B_{i,t+1}}{R_t}.$$

This expression can be rearranged to obtain the law of motion for the stock of net foreign assets owned by country i , i.e. the current account

$$NFA_{i,t} - NFA_{i,t-1} = CA_{i,t} = Y_{i,t}^T - C_{i,t}^T + B_{i,t} \left(1 - \frac{1}{R_{t-1}}\right).$$

As usual, the current account is given by the sum of net exports, $Y_{i,t}^T - C_{i,t}^T$, and net interest payments on the stock of net foreign assets owned by the country at the start of the period, $B_{i,t}(1 - 1/R_{t-1})$.

Finally, in every period the world consumption of the tradable good has to be equal to the

¹¹Throughout the paper I focus on equilibria in which production always occurs in both sectors. Given the functional forms assumed, it is indeed optimal for firms to always operate in both sectors.

¹²I follow the convention of netting interest payments out of the net foreign asset position.

world production, $\int_0^1 C_{i,t}^T di = \int_0^1 Y_{i,t}^T di$. This equilibrium condition implies that bonds are in zero net supply at the world level, $\int_0^1 B_{i,t+1} di = 0$.

2.1 Equilibrium

Given a sequence of the world interest rate $\{R_t\}_{t \geq 0}$ and of the borrowing limit $\{\kappa_t\}_{t \geq 0}$, define the period t optimal decisions of the household as $C_t^T(B, A^T, A^N)$, $C_t^N(B, A^T, A^N)$ and $L_t(B, A^T, A^N)$, the period t optimal labor demand decisions as $L_t^T(B, A^T, A^N)$ and $L_t^N(B, A^T, A^N)$, and the period t equilibrium prices $w_t(B, A^T, A^N)$ and $p_t^N(B, A^T, A^N)$, in a country with bond holdings $B_{it} = B$ and productivities $A_{i,t}^T = A^T$ and $A_{i,t}^N = A^N$. Notice that these decision rules fully determine the transition for bond holdings.

Define $\Psi_t(B, A^T, A^N)$ as the joint distribution of bond holdings and current productivities across countries. The optimal decision rules for bond holdings together with the process for productivities yield a transition probability for the country-specific states (B, A^T, A^N) . This transition probability can be used to compute the next period distribution $\Psi_{t+1}(B, A^T, A^N)$, given the current distribution $\Psi_t(B, A^T, A^N)$. We can now define an equilibrium.

Definition 1 *An equilibrium is a sequence of the world interest rate $\{R_t\}_{t \geq 0}$, a sequence of pricing functions $\{w_t(B, A^T, A^N), p_t^N(B, A^T, A^N)\}_{t \geq 0}$, a sequence of policy rules $\{C_t^T(B, A^T, A^N), C_t^N(B, A^T, A^N), L_t(B, A^T, A^N), L_t^T(B, A^T, A^N), L_t^N(B, A^T, A^N)\}_{t \geq 0}$, and a sequence of joint distributions for bond holdings and productivity $\{\Psi_t(B, A^T, A^N)\}_{t \geq 0}$, such that given the initial distribution $\Psi_0(B, A^T, A^N)$ and a sequence of the borrowing limit $\{\kappa_t\}_{t \geq 0}$*

- $C_t^T(B, A^T, A^N), C_t^N(B, A^T, A^N), L_t(B, A^T, A^N), L_t^T(B, A^T, A^N), L_t^N(B, A^T, A^N)$ satisfy households' and firms' optimality conditions.
- Markets for consumption and labor clear in every country

$$\frac{B_{t+1}(B, A^T, A^N)}{R_t} = A^T (L_t^T(B, A^T, A^N))^{\alpha^T} - C_t^T(B, A^T, A^N) + B$$

$$C_t^N(B, A^T, A^N) = A^N (L_t^N(B, A^T, A^N))^{\alpha^N}$$

$$L_t(B, A^T, A^N) = L_t^T(B, A^T, A^N) + L_t^N(B, A^T, A^N).$$

- $\Psi_t(B, A^T, A^N)$ is consistent with the decision rules.
- The market for bonds clears at the world level

$$\int B d\Psi_t(B, A^T, A^N) = 0.$$

2.2 Parameters

The model cannot be solved analytically and I analyze its properties using numerical simulations. I employ a global solution method in order to deal with the nonlinearities involved by a large shock

Table 1: Parameters

	Value	Source/Target
Risk aversion	$\gamma = 2$	Standard value
Discount factor	$\beta = 0.9938$	$R = 1.025$ (annual)
Frisch elasticity of labor supply	$1/\psi = 1/2.2$	Galí and Monacelli (2016)
Share of tradables in consumption	$\omega = 0.2$	Estimate for the euro area
Labor share in tradable sector	$\alpha_T = 0.65$	Estimate for the euro area
Labor share in non-tradable sector	$\alpha_N = 0.65$	Estimate for the euro area
Productivity process	$\sigma_A = 0.024, \rho = 0.92$	Estimate for the euro area
Initial borrowing limit	$\kappa = 4.56$	World debt/GDP = 21% (annual)

such as the deleveraging shock studied in the next section. Appendix B describes the numerical solution method.

One period corresponds to one quarter. The risk aversion is set to $\gamma = 2$, a standard value. The discount factor is set to $\beta = 0.9938$ in order to match an annualized real interest rate in the initial steady state of 2.5 percent. This is meant to capture the low interest rate environment characterizing the US and the euro area in the years preceding the start of the 2007 crisis. The inverse of the Frisch elasticity of labor supply ψ is set equal to 2.2, following Galí and Monacelli (2016).

The remaining parameters are chosen using data from the euro area.¹³ The euro area is an interesting case because it is a large currency union that developed significant imbalances across its members in the run-up to the global financial crisis, while these imbalances were reversed during the post-crisis years. The calibration strategy consists in choosing values for the parameters so that the steady state of the model matches some key aspects of euro area countries. Appendix E provides details on the construction of the series used in the calibration.

The share of tradable goods in consumption and the labor share in both sectors are chosen to match the corresponding statistics for the euro area. Hence, the share of tradable goods in consumption is set to $\omega = 0.2$, while the labor share in production in both sectors is set to $\alpha_T = \alpha_N = 0.65$. These are in the range of the values commonly assumed in the literature.

To save on state variables I assume that productivity is the same in both sectors, so that $A_{i,t}^T = A_{i,t}^N = A_{i,t}$.¹⁴ Productivity follows a log-normal AR(1) process $\log(A_{i,t}) = \rho \log(A_{i,t-1}) + \epsilon_{i,t}$. This process is approximated with the quadrature procedure of Tauchen and Hussey (1991) using 13 nodes.¹⁵ The first order autocorrelation ρ and the standard deviation of the productivity process σ_A are set respectively to 0.92 and to 0.024, to reproduce the average across euro area countries of the corresponding moments of de-trended labor productivity.

The existing literature offers little guidance on how to set κ , the borrowing limit in the initial steady state. One of the key variables determined by κ is the stock of gross world debt, that is

¹³In the calibration, the euro area is defined as the aggregate of Austria, Belgium, Finland, Germany, Greece, Ireland, Italy, Netherlands, Portugal and Spain.

¹⁴In a previous version of the paper I experimented with a version of the model in which productivity shocks are present only in the tradable sector. None of the key results of the paper is affected by this alternative assumption.

¹⁵I use the weighting function proposed by Flodén (2008), which delivers a better approximation to high-persistence AR(1) processes than the weighting function originally suggested by Tauchen and Hussey (1991).

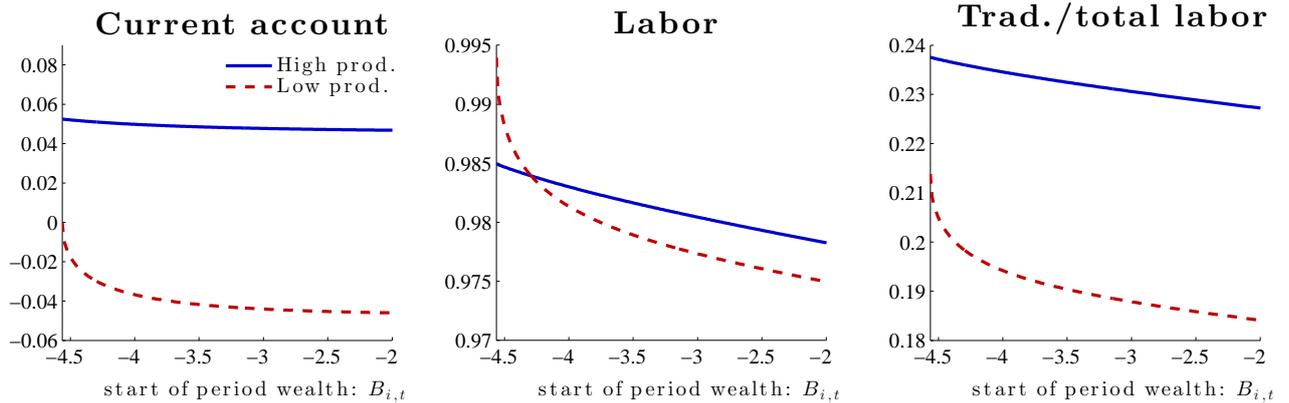


Figure 2: Policy functions in steady state. Notes: the high (low) productivity lines refer to economies hit by a productivity shock about two standard deviations above (below) the mean.

the sum of the net foreign asset positions of debtor countries.¹⁶ I set $\kappa = 4.56$ to match a world gross debt-to-annual GDP ratio of 21 percent.¹⁷ This target corresponds to the sum of the net external debt positions of the euro area debtor countries in 2008, expressed as a fraction of the euro area annual GDP.¹⁸ I choose 2008 as the benchmark year because later on I will use the sharp contraction in capital inflows experienced in 2009 by euro area debtor countries to parametrize the deleveraging shock.

2.3 Steady state

Before proceeding with the analysis of the deleveraging episode, this section briefly describes the steady state policy functions and the stationary distribution of the net foreign asset-to-GDP ratio.

Figure 2 displays the optimal choices for the current account, total labor and the fraction of labor allocated to the tradable sector as a function of $B_{i,t}$, the stock of wealth at the start of the period, for an economy hit by a good productivity shock, solid lines, and by a bad productivity shock, dashed lines. The left panel shows the current account. As it is standard in models in which the current account is used to smooth consumption over time, a country runs a current account surplus and accumulates foreign assets when productivity is high, while it runs a current account deficit and reduces its stock of foreign assets when productivity is low.¹⁹ Intuitively, fluctuations in

¹⁶Given that bonds are in zero net supply at the world level, the stock of gross world debt also corresponds to the sum of the net foreign asset positions of creditor countries.

¹⁷Throughout the paper, consistent with national accounts, I define GDP as the value of production at constant prices

$$GDP_{i,t} = Y_{i,t}^T + p^N Y_{i,t}^N,$$

where p^N is the unconditional mean of the relative price of non-tradable goods in the initial steady state, which is equal for every country. Naturally, world GDP is defined as $\int_0^1 GDP_{i,t} di$.

¹⁸Spain is, by far, the country that had the highest net foreign debt-to-euro area GDP ratio in 2008, equal to 9 percent. Other countries that in 2008 had sizable net foreign debt positions expressed as a fraction of euro area GDP are France, Greece, Ireland, Italy and Portugal. Austria and Finland both had a negative net foreign asset position in 2008, but their external debt was very small compared to euro area GDP. Taking 2007 as the base year would give a very similar target, precisely a world gross debt-to-GDP ratio of 21.4 percent. Data are from Lane and Milesi-Ferretti (2007).

¹⁹This effect generates a positive steady state correlation between the current account and GDP, while in the data the current account is typically countercyclical. There are several approaches that could correct this counterfactual

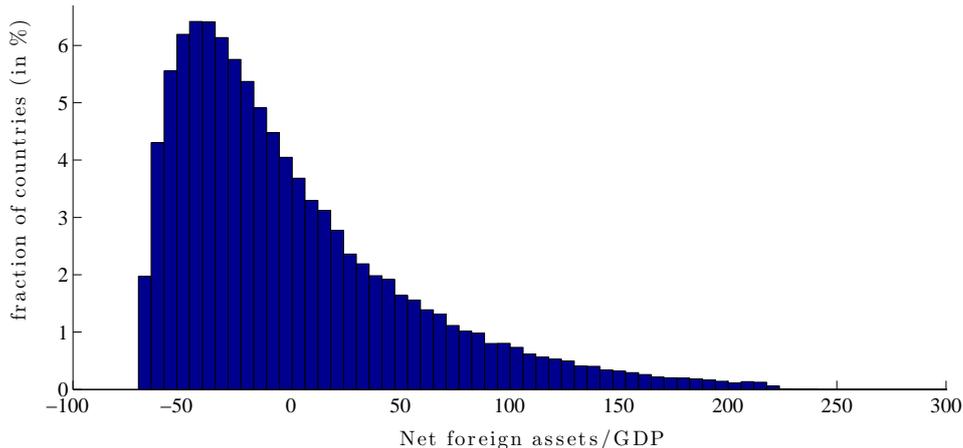


Figure 3: Steady state distribution of net foreign assets/GDP.

productivity generate fluctuations in wages and profits, and so in households' income. For instance, when productivity is low income is also low, and households borrow to mitigate the impact of the temporarily low income on consumption, giving rise to a current account deficit. Conversely, when productivity is high income is high, and households save generating a current account surplus. The borrowing limit, however, interferes with consumption smoothing because it restricts the amount of new debt that an already indebted household can take in response to a negative income shock. This feature of the economy explains why the deficit in the current account associated with a low realization of the productivity shock decreases as the start-of-period wealth falls. For instance, when $B_{i,t} = -\kappa$, households cannot increase their debt further and the change in net foreign assets following a low realization of the productivity shock is equal to zero.

The middle panel illustrates the optimal choice of labor. In general, equilibrium labor is higher when productivity is high, because when productivity is higher firms are able to pay higher wages and this induces households to supply more labor. But this pattern is reversed for low levels of wealth. This is due to the fact that highly indebted households cannot rely extensively on borrowing to smooth the impact of negative income shocks on consumption. Hence, at low levels of wealth, households mitigate the impact of negative productivity shocks on consumption by increasing their labor supply. As illustrated by the right panel, the share of labor allocated to the tradable sector follows a similar pattern. In particular, as the start of period wealth falls more labor is allocated to the tradable sector. Intuitively, credit frictions impact disproportionately production in the tradable sector, because they interfere with tradable consumption smoothing.

Figure 3 shows the steady state distribution of the net foreign asset-to-annual GDP ratio. The distribution is truncated and skewed toward the left. Both of these features are due to the implication of the model. One possibility would be to introduce endogenous capital accumulation. Modeling capital accumulation would make the framework more realistic, but at the cost of making it much more complicated to solve, and I leave this relevant extension for future work. Another possibility would be to introduce shocks to the supply of savings, for example in the form of shocks to the discount factor β . I explored this possibility and the introduction of saving shocks does not affect significantly the behavior of the economy during deleveraging. I chose to focus on productivity shocks because they are easier to quantify.

borrowing limit. In fact, while there is no limit to the positive stock of net foreign assets that a country can accumulate, the borrowing constraint imposes a bound on the negative net foreign asset position that a country can reach. In particular, the largest net foreign debt position-to-GDP ratio that a country can reach in the initial steady state is close to 70 percent.²⁰

3 Adjustment to a deleveraging shock

This section analyzes the response of the economy to a deleveraging shock, defined as a large tightening of the borrowing limit.²¹ I consider a world economy that starts from the steady state described in Section 2.3 and that, from period 0 on, transitions toward a new steady state characterized by a tighter borrowing limit $\bar{\kappa}$, where $\bar{\kappa} < \kappa$. The adjustment of the borrowing limit is gradual and follows the log-linear path

$$\log(\kappa_t) = \rho_\kappa \log(\kappa_{t-1}) + (1 - \rho_\kappa) \log(\bar{\kappa}),$$

for $t \geq 0$.²² The initial fall in the borrowing limit happening in $t = 0$ is not anticipated by agents, while from period 0 on agents correctly anticipate the path of κ_t .

Choosing values for the parameters $\bar{\kappa}$ and ρ_κ is a difficult task. Hence, I start to present the results for a benchmark parameterization, and later on provide some robustness analysis. I set the benchmark values of $\bar{\kappa}$ and ρ_κ to match the abrupt improvement in the current account experienced by Ireland, Greece, Portugal and Spain, the so-called GIPS countries, in the aftermath of the 2008 global financial crisis. I set the final borrowing limit to $\bar{\kappa} = 3.2$, so that the fraction of countries constrained by the new borrowing limit, that is those countries for which $B_{i,0} < -\bar{\kappa}$, accounts for 18.5% of world GDP in the initial steady state. This is in line with the fraction of euro area GDP accounted by GIPS countries in 2008, which is 18.4 percent. To set ρ_κ , the parameter which determines the speed of adjustment of the borrowing limit, I employ the following strategy. In 2009 the GIPS countries experienced an improvement in their current accounts collectively equal to 1 percent of 2008 euro area GDP.²³ Accordingly, I set $\rho_\kappa = 0.7$ so that after four quarters the tightening of the borrowing limit generates an amount of forced savings from high-debt countries equal to 1 percent of initial-steady-state world GDP.²⁴

²⁰For comparison, the net foreign debt-to-GDP ratio in Spain in 2008 was 75 percent.

²¹The model is silent about the causes behind the drop in the borrowing limit. For example, access to credit could be restricted because of a banking crisis. Or alternatively, a drop in house prices, perhaps due to the bursting of a bubble as in Martin and Ventura (2012), could reduce the value of collateral in the hands of households and lead to a reduction in their ability to borrow.

²²One reason to consider a gradual adjustment of the borrowing limit is the fact that the model features only debt contracts that last one period, that is one quarter. In reality, debt can take maturities that are longer than one quarter. Considering a gradual adjustment in the borrowing limit is a simple way of capturing the fact that long term debt allows agents to adjust gradually to the new, tighter, credit conditions.

²³Indeed, the sum of the current account deficits of GIPS countries expressed as a fraction of 2008 euro area GDP passed from 2.8 percent in 2008 to 1.8 percent in 2009. To compute these statistics I used data provided by Lane and Milesi-Ferretti (2007).

²⁴Formally, define forced savings between period 0 and period $t \geq 0$ as the reduction in world debt needed to satisfy the period t borrowing limit $\int_{-\kappa}^{-\kappa_t} (-\kappa_t - B_i) \Psi(B) di$, where the absence of time subscript denotes variables

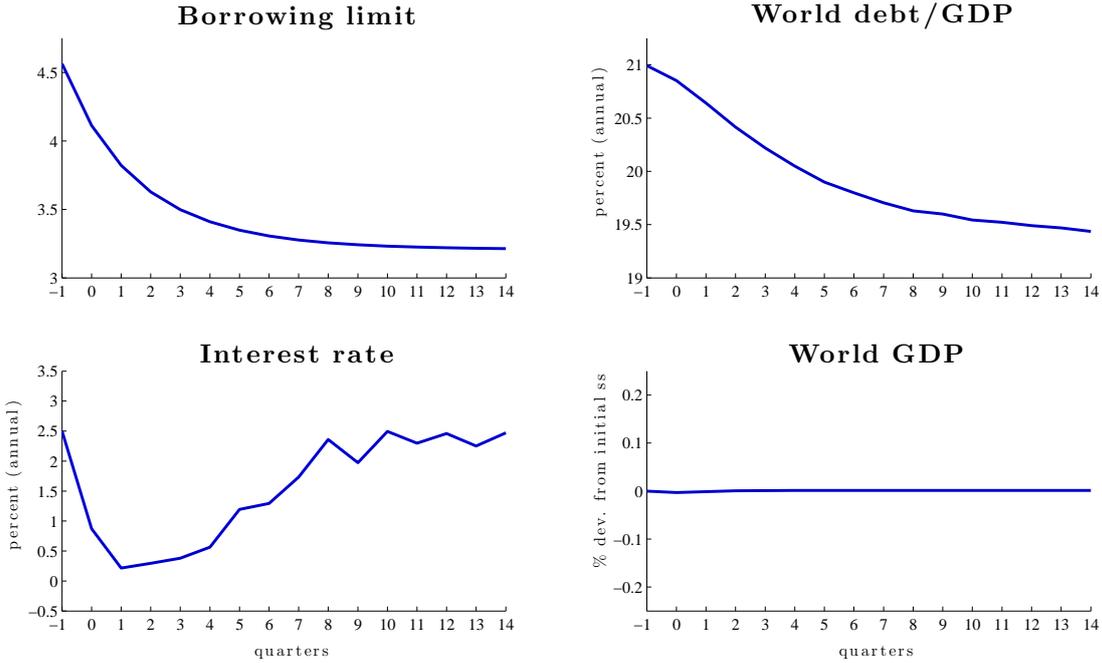


Figure 4: Response to deleveraging shock. Notes: world debt refers to world gross debt, defined as $\int_{-\kappa_t}^0 B_{i,t+1}/R_t di$. World GDP is defined as $\int_0^1 Y_{i,t}^T + p^N Y_{i,t}^N di$, where p^N denotes the unconditional mean of $p_{i,t}^N$ in the initial steady state.

3.1 Aggregate dynamics

Figure 4 displays the transitional dynamics of the world economy following the deleveraging shock. The figure shows the path for the exogenous borrowing limit, and the responses of the world gross debt-to-GDP ratio, the world interest rate and world GDP.

The tightening of the borrowing limit triggers a decrease in the foreign debt position of highly indebted countries. At the same time, surplus countries are forced to reduce their positive net foreign asset position, which is the counterpart of foreign debt in indebted countries. The result is a progressive compression of the net foreign asset distribution. As showed by the the top-right panel of Figure 4, the world debt-to-GDP ratio gradually falls toward its value in the final steady state, which is equal to 14.7 percent.

The world interest rate drops sharply in response to the deleveraging shock and overshoots its value in the new steady state.²⁵ The fall in the interest rate signals an increase in the desire to save, or equivalently a fall in the desire to consume. This is due to two distinct effects. First, countries that start with a high level of foreign debt, more precisely countries that start with a

²⁵referring to the initial steady state. I set $\rho_\kappa = 0.7$ so that

$$\frac{\int_{-\kappa}^{-\kappa_4} (-\kappa_4 - B_i) \Psi(B) di}{4GDP} = 0.01,$$

where steady state GDP is multiplied by 4 to convert it to its annual value. In words, the above expression means that the group of countries that have a debt position in the initial steady state higher than the period 4 borrowing limit is forced by the deleveraging shock to reduce debt by an amount equal to 1 percent of initial-steady-state world GDP.

²⁵The interest rate in the final steady state is lower compared to its value in the initial steady state, but quantitatively the difference is minuscule.

stock of bonds $B_{i,0} < -\bar{k}$, are forced to reduce their foreign debt position. This forced reduction in debt corresponds to a forced increase in savings that depresses the demand for consumption in high-debt countries. Second, even the countries that are sufficiently wealthy so that they are not directly affected by the tightening of the borrowing limit, the unconstrained countries, experience an increase in the propensity to save. In fact, unconstrained countries want to accumulate precautionary savings to self-insure against the risk of hitting the now-tighter borrowing limit in the future. These two effects point toward an increase in the propensity to save at the world level. In order to restore equilibrium on the bonds market the world interest rate has to fall, so as to induce the unconstrained countries to absorb the forced savings coming from high-debt, borrowing-constrained economies.

To illustrate these effects, I perform a partial equilibrium experiment. I consider a “disequilibrium” economy in which, in response to the deleveraging shock, the interest rate jumps immediately to its value in the final steady state, so that no overshooting occurs. Hence, along the transition to the final steady state of this “disequilibrium” economy the world bonds market might not clear. The left panel of Figure 5 displays the behavior of the world current account-to-GDP ratio. The figure shows that, in absence of the short-run drop in the interest rate, the world experiences several periods of large excess savings. The right panel of Figure 5 shows, again for the “disequilibrium” economy, the period 0 response of the current account to the deleveraging shock across the initial distribution of net foreign assets.²⁶ The shaded area denotes the countries that start the transition with $B_{i,0} < -\bar{k}$, and hence are forced to reduce their foreign debt by the tightening of the borrowing constraint. Naturally, these countries increase their current account surplus to deleverage. The key point, however, is that also those countries with $B_0 > -\bar{k}$, but with an amount of initial foreign debt sufficiently close to the borrowing limit, experience an improvement in their current account. This is the result of the increase in the desire to save for precautionary reasons triggered by the deleveraging shock. The rise in precautionary savings in response to the deleveraging shock is a distinctive feature of this framework, and is absent in more stylized two-country models such as the one studied by Benigno and Romei (2014).²⁷

Going back to the general equilibrium experiment, the bottom-right panel of Figure 4 shows that the deleveraging shock leaves world GDP essentially unchanged. However, the lack of aggregate movements in world output masks important country-level composition effects, to which we turn next.

²⁶To construct this figure, I first computed the response in period 0 to the deleveraging shock, assuming that the interest rate jumps immediately to its value in the final steady state, for every possible realization of the state variables $\{A_0, B_0\}$. Then I computed an aggregate response as a function of B_0 by taking the weighted average of the single country responses. The weights are given by the fraction of countries having a given realization of A_0 conditional on B_0 .

²⁷In this respect, the model is close to Guerrieri and Lorenzoni (2011), who study the response of precautionary savings to a deleveraging shock in a closed economy.

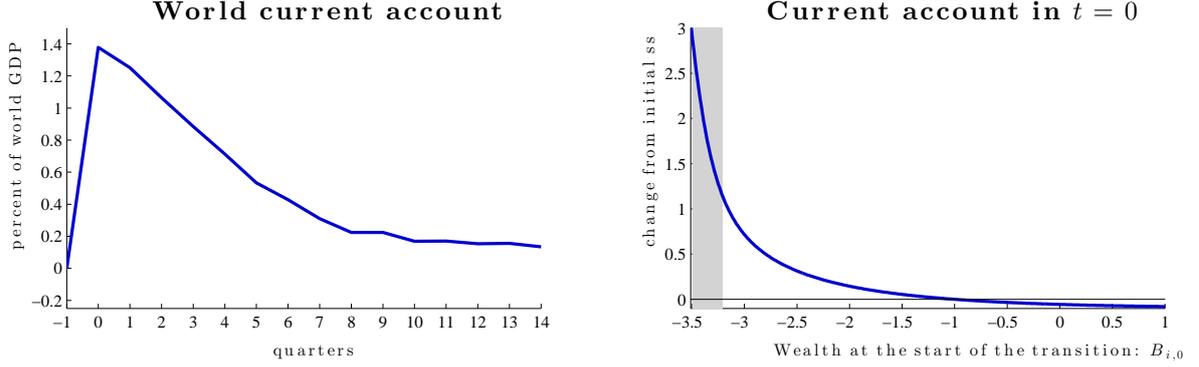


Figure 5: Current account response in partial equilibrium. Notes: the transitional dynamics are computed assuming that $R_t = \bar{R}$ for $t \geq 0$, where \bar{R} is the value of the world interest rate in the final steady state. World current account is defined as $\int_0^1 CA_{i,t} di$

3.2 Response across the net foreign asset distribution

Figure 6 illustrates how the response to the deleveraging shock varies across the initial distribution of net foreign assets. The figure shows the response of the current account-to-GDP ratio, GDP, consumption, real wage and the share of output and consumption accounted by the tradable sector for three countries. The three countries are at the 10th, 20th and 75th percentile of the initial net foreign asset distribution. In order to highlight the heterogeneity due to the initial net foreign asset position, for the three countries productivity is held constant to its mean value.

The country at the 10th percentile captures the typical behavior of a high-debt financially-constrained economy. As shown by the top-left panel, the tightening of the borrowing limit generates a sudden stop in capital inflows, giving rise to several periods of sustained current account surpluses. To understand the macroeconomic implications, it is useful to go back to the equation describing the current account

$$CA_{i,t} = Y_{i,t}^T - C_{i,t}^T + B_{i,t} \left(1 - \frac{1}{R_{t-1}} \right).$$

This expression makes clear that a country can improve its current account by increasing its output of the tradable good, by decreasing the consumption of the tradable good or through a combination of both. Figure 6 shows that the high-debt country adjusts both through the output and the consumption margins. In fact, both output and the share of output accounted by tradables rise, while the opposite occurs for consumption. Hence, in high-debt countries the sudden stop in capital inflows leads to an economic expansion, and to a shift of productive resources from the non-tradable to the tradable sector.

The countries at the 20th and 75th percentile are sufficiently wealthy so that they are not directly affected by the constraint, and their adjustment follows an opposite pattern compared to high-debt economies. Indeed, the decrease in the world interest rate induces unconstrained countries to reduce their stock of foreign assets by running current account deficits. This is achieved through a combination of lower production and higher consumption of tradable goods.

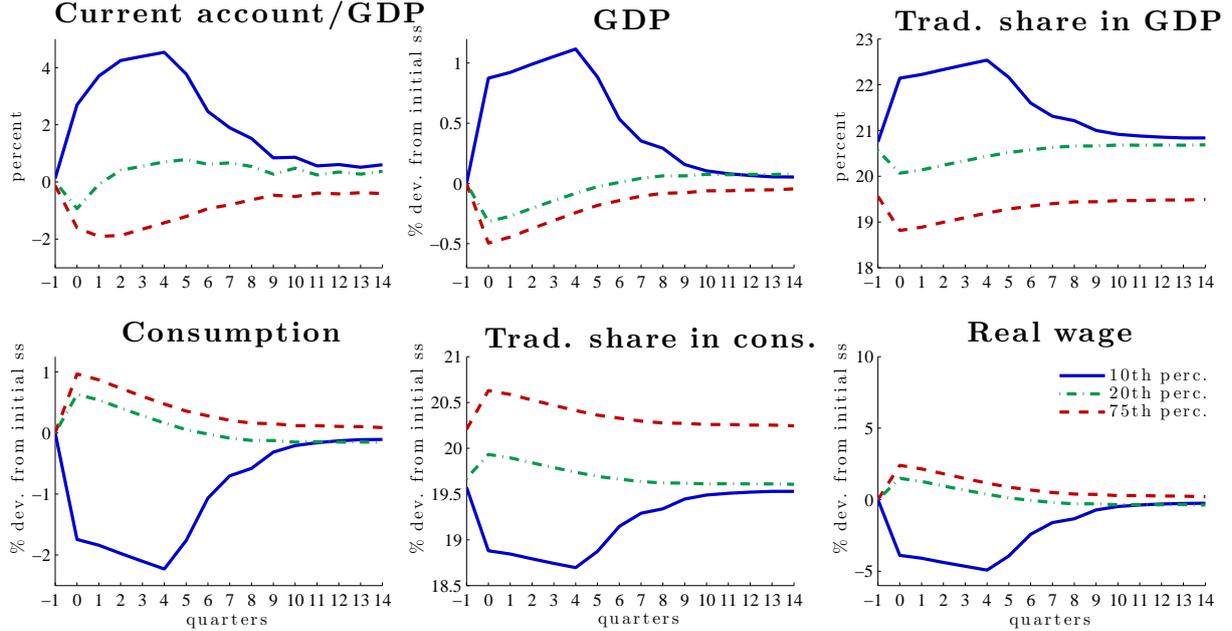


Figure 6: Response to deleveraging shock across the NFA distribution. Notes: GDP and consumption are the value of production and consumption at constant prices. Non-tradable goods are weighted using the unconditional mean of p^N in the initial steady state. The real wage is the wage in units of tradable goods.

Hence, following a deleveraging shock the baseline model displays a shift of production of tradable goods from wealthy unconstrained countries toward high-debt constrained ones. This change in the pattern of production plays a key role in the adjustment, because it redistributes income from countries which have a low propensity to consume, the unconstrained countries, toward countries that have a high propensity to consume, the borrowing constrained countries. In fact, the asymmetry in the response of production of tradable goods between borrowing constrained and unconstrained countries mitigates the rise in the world propensity to save caused by the deleveraging shock, thus limiting the fall in the world interest rate.²⁸

The real wage is the key price that has to adjust to allow production to respond to the deleveraging shock. This can be seen by rearranging the optimality condition for firms in the tradable sector to obtain

$$L_{i,t}^T = \left(\frac{\alpha_T A_{i,t}}{w_{i,t}} \right)^{\frac{1}{1-\alpha_T}}.$$

This expression implies that, given $A_{i,t}$, an increase in employment in the tradable sector in country i has to come with a decrease in the real wage $w_{i,t}$. In fact, as shown by the bottom-right panel of Figure 6, in the baseline economy the adjustment to the deleveraging shock entails a decrease in real wages in high-debt constrained economies and an increase in real wages in the rest of the world.

The adjustment in the real wage is due to two different effects. First, following the deleveraging

²⁸In Appendix C I study a simplified version of the model which allows for an analytic solution. This exercise provides further insights on the interaction between the endogenous response of production to the deleveraging shock and the behavior of the world interest rate.

shock households in high-debt countries increase their labor supply to boost labor income and to repay debts without cutting consumption too severely. Conversely, households in unconstrained countries contract their labor supply in response to the fall in the interest rate and the subsequent rise in consumption. Second, the fall in consumption in high-debt countries corresponds to a negative demand shock for the non-tradable sector, which leads to a fall in labor demand from firms producing non-tradable goods. The opposite occurs in wealthy unconstrained countries. Both of these effects point toward a fall in real wages in high-debt constrained economies, and a rise in the rest of the world.

The empirical evidence reviewed in the introduction suggests that nominal wages adjust sluggishly to shocks. In particular, a recurrent pattern in severe recessions is that nominal wages do not fall much, even in the face of large rises in unemployment. It is then difficult to imagine that the adjustment in real wages required by the deleveraging shock could come from an adjustment in nominal wages. But what are the macroeconomic implications of this friction? To answer this question we need to introduce a model with nominal wage rigidities.

4 A model with nominal rigidities

This section studies the adjustment to a deleveraging shock in presence of nominal wage rigidities. In the interest of space, here I provide an informal description of the model, while the details can be found in Appendix *D*.

The basic structure of the model is the same as the one of the baseline model of Section 2. There are two main differences. First, there is monopolistic competition on the labor market. In fact, as in Erceg et al. (2000), households supply differentiated labor services, which enter firms' production function with the elasticity of substitution ϵ . Second, nominal wages are negotiated by labor unions, which act in the interest of households. Specifically, each labor union sets the wage of a single type of labor service. Once wages are set, households stand ready to supply the quantity of labor demanded by firms. In spite of these differences, in absence of frictions in the wage-setting process the model is isomorphic to the baseline one.

I introduce frictions in the adjustment of nominal wages by assuming that labor unions update their information about the state of the economy infrequently. This implies that unions might set nominal wages based on outdated information, and so nominal wages might not respond immediately to unexpected shocks or changes in monetary policy. This friction creates a channel through which monetary policy can affect the real economy.

To implement this idea, I adopt a variant of the Mankiw and Reis (2002) model of imperfect information, in which in every period agents have a constant probability of updating their information set. More precisely, I assume that every period only a fraction $0 < \phi < 1$ of the unions observes the state variables describing the global economy, that is the cross-country distribution of net foreign assets and the path of the borrowing limit κ_t . Instead, wage setters update continuously their information about the country-level state variables, that is the stock of foreign assets held by

the country at the start of the period and the realization of the productivity shock. This setting captures an environment in which wage setters pay more attention to the idiosyncratic shocks that hit their country frequently, rather than to the rare shocks hitting the global economy. More broadly, this asymmetric information structure is meant to capture an environment in which there is enough wage flexibility to deal with normal business cycle fluctuations driven by the productivity shocks. Instead, wages fail to adjust immediately to large and rare shocks, such as the one-time previously unexpected drop in the borrowing limit considered in our deleveraging experiment.²⁹

It turns out that, given that the only aggregate shock considered is a one-time fully unanticipated shock to the borrowing limit κ_t , the equilibrium behavior of wage setters takes a very simple form. In fact, both in the initial and final steady states wage setters have perfect information about the state of the economy. Hence, in steady state the allocations correspond to the one of the baseline model with flexible wages discussed in Section 2. Instead, during the transition from the initial to the final steady state, in every period t a fraction $(1 - \phi)^t$ of wage setters has not yet received information about the global deleveraging shock. Uninformed unions act on the basis of outdated information, and set nominal wages according to the pricing rule of the initial steady state. More formally, during the transition the aggregate nominal wage $W_{i,t}$ follows the path

$$W_{i,t} = \left((1 - (1 - \phi)^t) (W_{i,t}^{in})^{1-\epsilon} + (1 - \phi)^t (W_{i,t}^{un})^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}},$$

where $W_{i,t}^{in}$ and $W_{i,t}^{un}$ denote respectively the nominal wage set by informed and uninformed unions. This equation implies that nominal wages adjust sluggishly to the deleveraging shock in the short run, but the economy approaches the full information benchmark as $t \rightarrow \infty$.

Because of the sluggish adjustment of nominal wages, monetary policy can affect the transitional dynamics triggered by the deleveraging shock. I consider two different monetary policy regimes. The main focus of the analysis is on a world in which every country belongs to a single monetary union. Under this regime every country $i \in \{0, 1\}$ shares the same currency, and there is a single central bank setting monetary policy. Consistent with the inflation objective of the European Central Bank, I focus attention on a central bank that targets the average CPI inflation across the member countries. More precisely, defining $\pi_{i,t}$ as CPI inflation, the objective of the central bank of the monetary union is to set $\int_0^1 \pi_{i,t} di = \bar{\pi}$. As a comparison, in this section I also consider a flexible exchange rate regime, in which every country has its own currency and runs monetary policy independently. To allow for a clean comparison with the monetary union, in every country i the central bank objective is to set $\pi_{i,t} = \bar{\pi}$. I will start by assuming that central banks always attain their inflation targets. Later on, in Section 5, I consider a case in which the central bank might fail to reach its inflation objective because of the zero lower bound on the nominal interest rate.

²⁹To be clear, this assumption is not made on the ground that wage rigidities are unimportant to explain normal business cycle fluctuations. Rather, the objective is to focus attention on the interactions between wage rigidities and the transitional dynamics triggered by a large global deleveraging shock, abstracting from the, already well-understood, role of wage rigidities in shaping the response of the economy to standard productivity shocks.

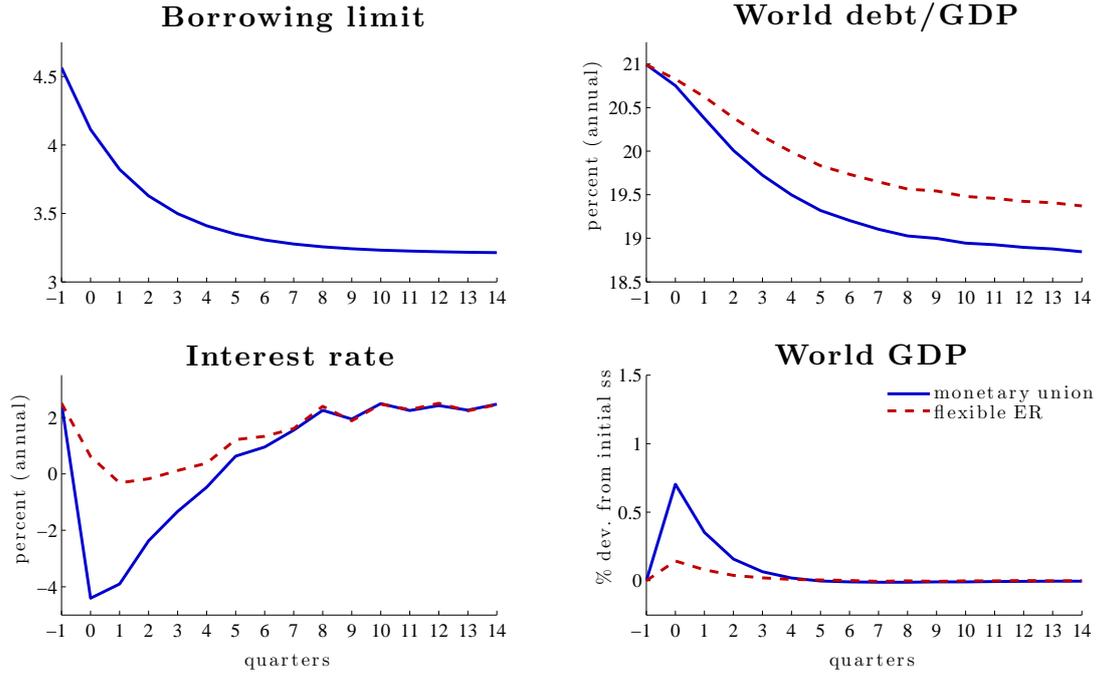


Figure 7: Response to deleveraging shock with nominal rigidities. Notes: world debt refers to world gross debt, defined as $\int_{-\kappa_t}^0 B_{i,t+1}/R_t di$. World GDP is defined as $\int_0^1 Y_{i,t}^T + p^N Y_{i,t}^N di$, where p^N denotes the unconditional mean of $p_{i,t}^N$ in the initial steady state.

For the model with nominal rigidities there are two additional parameters to be set, ϵ and ϕ . The elasticity of substitution across different labor types is set to $\epsilon = 4.3$, following Galí and Monacelli (2016). In the benchmark calibration I set ϕ , the probability that a wage setter updates its information set, to .2. This is in line with the average duration of wages usually assumed in models featuring a Calvo friction (Erceg et al., 2000; Galí and Monacelli, 2016). I later perform a robustness analysis along different assumptions on the value of ϕ .

Figure 7 presents the response of aggregate variables to the deleveraging shock. The solid lines refer to the monetary union, while the dashed lines refer to the world with flexible exchange rates. The first result is that the behavior of the flexible exchange rate economy is strikingly similar to the one of the flexible-wage economy studied in Section 2. Hence, as long as exchange rates are flexible and monetary policy stabilizes CPI inflation, frictions in the adjustment of nominal wages do not affect aggregate dynamics in any significant way.

The monetary union, instead, experiences a particularly large drop in the world interest rate. In fact, under a monetary union the fall in the interest rate is about three times larger than under flexible exchange rates. Hence, the combination of nominal wage rigidities and fixed exchange rates amplifies the fall in the interest rate following a deleveraging shock.

To gain intuition about this result, it is useful to look at the country-level responses, shown by Figure 8. The solid lines refer to a high-debt economy at the 10th percentile of the initial net foreign asset distribution, while the dashed lines are for a wealthy country at the 75th percentile.³⁰ Moreover, the unmarked lines refer to the monetary union, while cross marks indicate the world

³⁰For both economies, productivity is kept constant and equal to its mean value.

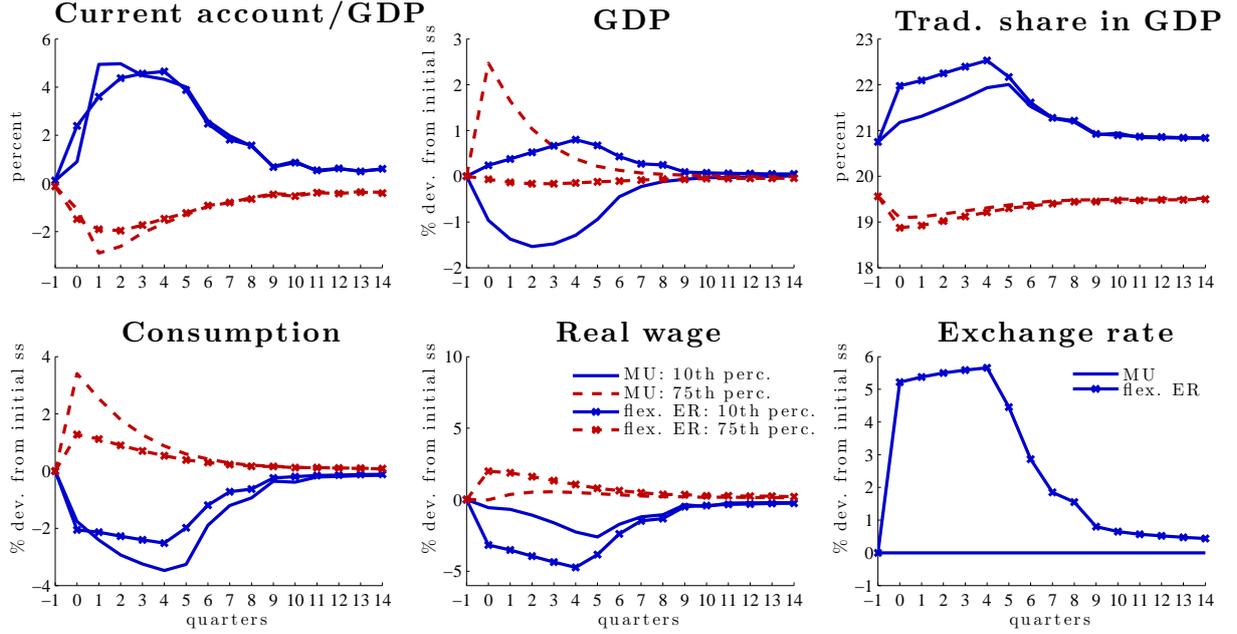


Figure 8: Response to deleveraging shock across the NFA distribution with nominal rigidities. Notes: GDP and consumption are the value of production and consumption at constant prices. Non-tradable goods are weighted using the unconditional mean of p^N in the initial steady state. The real wage is the wage in units of tradable goods. The exchange rate is defined as the units of the 10th percentile country's currency needed to buy one unit of the currency of the 75th percentile country.

with flexible exchange rates. Under both regimes, the high-debt country is forced to improve its current account by the tightening of the borrowing constraint. As in the flexible wage economy, under flexible exchange rates the high-debt country improves its current account by increasing the production of tradable goods, while the wealthy country contracts its production of tradables. Instead, in the monetary union this channel of adjustment is essentially shut off. In fact, in the monetary union the high-debt country experiences an economic contraction, and the shift of productive resources from the non-tradable to the tradable sector is much smaller than under flexible exchange rates. Similarly, in the monetary union tradable production in the wealthy unconstrained country does not fall as much as under flexible exchange rates.

The muted response of tradable production to the deleveraging shock can be traced to the fact that in the monetary union real wages fail to adjust, as shown by the bottom-middle panel of Figure 8. To understand how the exchange rate regime affects the behavior of real wages, consider that the real wage is defined as $W_{i,t}/P_{i,t}^T$, where $P_{i,t}^T$ is the price of the tradable good in terms of country i 's currency. Since the nominal wage is sticky, the adjustment has to come through movements in $P_{i,t}^T$. In particular, to mimic the adjustment under flexible wages, $P_{i,t}^T$ must rise in high-debt economies and fall in wealthy unconstrained countries. Now consider that the absence of trade frictions implies that the law of one price holds for the tradable good, so that

$$P_{i,t}^T = S_{i,t}^j P_{j,t}^T \text{ for any } i, j \in \{0, 1\}.$$

In this expression, $S_{i,t}^j$ is the exchange rate between country i and country j , defined as the units

of country i 's currency needed to purchase one unit of country j 's currency. This equation implies that, to replicate the adjustment under flexible wages, the currencies of high-debt countries need to depreciate against those of wealthy countries. This is precisely what happens under flexible exchange rates, as shown by the bottom-right panel of Figure 7.³¹ Instead, in a currency union by definition $S_{i,t}^j = 1$ for any i and j , so that the asymmetric adjustment in $P_{i,t}^T$ across financially constrained and unconstrained countries cannot occur. Hence, the combination of nominal wage rigidities and fixed exchange rates shuts down the response of real wages and of the output of tradable goods to the deleveraging shock.

The outcome of this lack of adjustment is that in the monetary union the improvement in the current account in high-debt countries comes mainly from a large drop in consumption, as displayed by the bottom-left panel of Figure 8. The fact that constrained countries have to adjust mainly through the consumption margin implies that, following the deleveraging shock, world demand for consumption falls more in the monetary union than under flexible exchange rates. In turn, the interest rate has to fall by more to induce unconstrained countries to increase consumption and pick up the slack left by borrowing constrained economies. Hence, the lack of exchange rate flexibility places the burden of adjustment on the interest rate. As we will see, this has important implications for the macroeconomic adjustment to deleveraging in a monetary union if interest rates are close to the zero lower bound.

Before turning to the zero lower bound, however, it is useful to understand why in a monetary union deleveraging generates a recession in high-debt countries. Taken together, the top-center and top-right panels of Figure 8 indicate that the recession in high-debt countries is driven by a fall in the production of non-tradable goods.³² To understand why this is the case, it is useful to recast the equilibrium on the market for non-tradables as the intersection of an aggregate demand and an aggregate supply schedule. The aggregate demand (AD) schedule can be obtained by rewriting equation (4) as

$$C_{i,t}^N = \frac{1 - \omega}{\omega} \frac{P_{i,t}^T}{P_{i,t}^N} C_{i,t}^T, \quad (\text{AD})$$

where $P_{i,t}^N$ denotes the price of the non-tradable good in terms of the domestic currency. Instead, the aggregate supply schedule can be obtained by rewriting the labor demand by firms in the non-tradable sector as

$$Y_{i,t}^N = A_{i,t}^{\frac{1}{1-\alpha_N}} \left(\alpha_N \frac{P_{i,t}^N}{W_{i,t}} \right)^{\frac{\alpha_N}{1-\alpha_N}}, \quad (\text{AS})$$

³¹To understand why under flexible exchange rates $P_{i,t}^T$ rises in high-debt countries, consider that, as explained in Appendix D, the CPI is

$$\left(\frac{P_{i,t}^T}{\omega} \right)^\omega \left(\frac{P_{i,t}^N}{1-\omega} \right)^{1-\omega}.$$

In high-debt countries the deleveraging shock generates a fall in the demand for non-tradable goods, and hence a fall in $P_{i,t}^N$. It follows that, in order to insulate the CPI from the shock, $P_{i,t}^T$ has to rise. The opposite occurs in wealthy unconstrained countries.

³²This is consistent with fact that in the data contractions in capital inflows tend to be accompanied by recessions localized in the sectors producing non-tradable goods (Benigno et al., 2015).

where I have used $Y_{i,t}^N = A_{i,t} \left(L_{i,t}^N \right)^{\alpha_N}$. In a monetary union, since tradable production does not react, the deleveraging shock generates a sharp fall in tradable consumption in high-debt countries. The AD equation shows that the fall in tradable consumption corresponds to a negative demand shock for non-tradable goods. This effect points toward lower production of non-traded goods. The fall in tradable consumption, however, also generates a rise in labor supply, and, with flexible wages, a fall in $W_{i,t}$. According to the AS equation, a lower $W_{i,t}$ reduces $P_{i,t}^N$, because the fall in labor costs leads firms to cut prices. This effect mitigates the drop in non-tradable production. With sticky wages, instead, $W_{i,t}$ cannot adjust. Still, if exchange rates are flexible, high-debt countries experience a rise in $P_{i,t}^T$ because of the exchange rate depreciation. From the AD equation, this induces an expenditure switching effect that sustains demand for non-tradables and mitigates the fall in $Y_{i,t}^N$. In a monetary union with wage rigidities both channels of adjustment are absent, which explains why high-debt countries experience a drop in GDP concentrated in the non-tradable sector.

Summarizing, the interaction between nominal wage rigidities and fixed exchange rates gives rise to a large drop in the world interest rate following the deleveraging shock, and a recession in the countries that end up being financially constrained. The next section shows how the recession can spread to unconstrained countries if the deleveraging shock pushes the union into a liquidity trap.

5 Deleveraging and liquidity trap in a monetary union

As we have seen, deleveraging in a monetary union entails a sharp drop in the real interest rate. This result suggests that, if expected inflation is low enough, deleveraging can push the nominal interest rate of the currency union all the way to its zero lower bound. At that point, the central bank is not able to stimulate the economy enough to hit its inflation target, and a liquidity trap occurs. The objective of this section is to understand under which conditions deleveraging across members of a monetary union gives rise to a liquidity trap, as well as the impact that the liquidity trap has on output and welfare.

To understand whether the deleveraging shock generates a liquidity trap, we have to take a stance on the central bank's inflation target, which determines agents' inflation expectations. In line with the price stability objective of the European Central Bank, I set the benchmark inflation target to 2 percent on a yearly basis, so that $\bar{\pi} = 1.02^{1/4}$. Now define \hat{R}_t^n as the gross nominal interest rate consistent with the central bank's inflation target. In this section the focus is on a currency union in which the central bank sets the interest rate according to $R_t^n = \max(\hat{R}_t^n, 1)$. In words, the central bank implements the inflation target as long as this does not imply a negative nominal rate, otherwise it sets the nominal interest rate to zero.

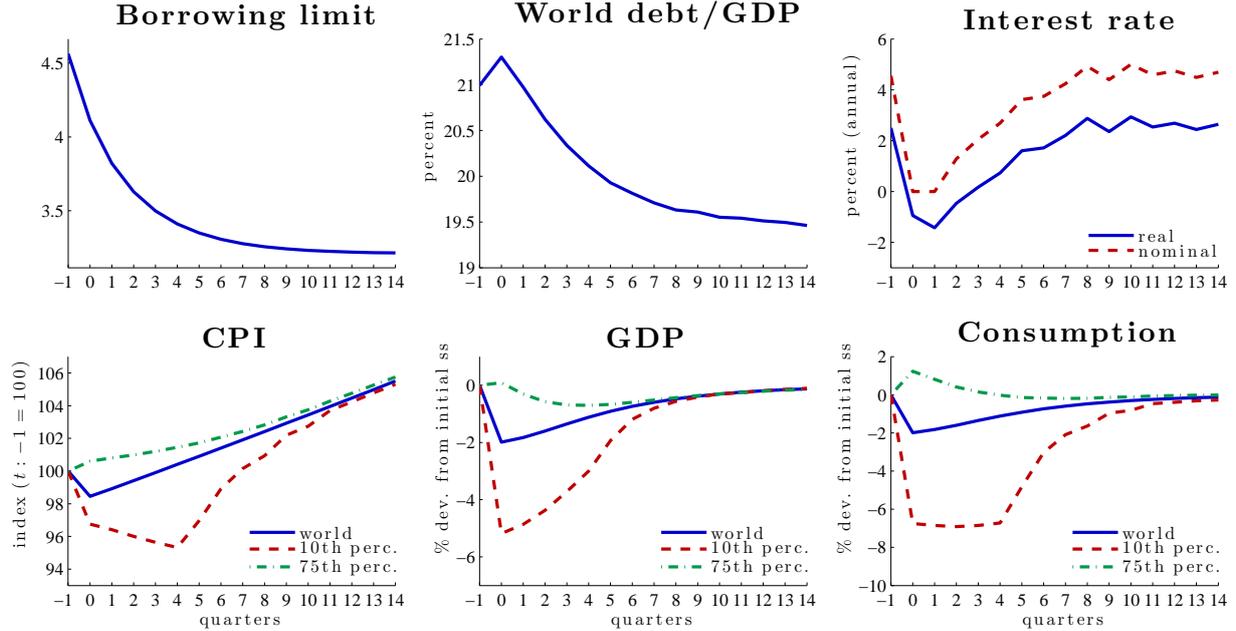


Figure 9: Response to deleveraging shock in a monetary union. Notes: world debt refers to world gross debt, defined as $\int_{-\kappa_t}^0 B_{i,t+1}/R_t di$. World GDP and consumption are respectively defined as $\int_0^1 Y_{i,t}^T + p^N Y_{i,t}^N di$ and $\int_0^1 C_{i,t}^T + p^N C_{i,t}^N di$, where p^N denotes the unconditional mean of $p_{i,t}^N$ in the initial steady state. CPI is the average consumer price index.

5.1 Dynamics during liquidity trap

Figure 9 shows the response of the monetary union to the deleveraging shock. As shown by the top-right panel, the nominal interest rate hits the zero lower bound during the first two periods of deleveraging. The binding zero lower bound indicates that it is not possible for the central bank to attain its inflation target. In fact, at the target inflation rate the goods market does not clear, since demand for consumption is too weak to absorb producers' desired output. Excess supply induces firms to cut prices, as illustrated by the bottom-left panel. The unexpected fall in prices leads to a rise in real wages, since nominal wages do not perfectly adjust to the deleveraging shock. In turn, higher wages reduce the profitability of employing labor, leading to a fall in production. Indeed, this is the mechanism through which the goods market equilibrium is restored.

The result is that deleveraging gives rise to a prolonged recession, lasting about two years, that affects all the countries belonging to the monetary union. As shown by the bottom-middle panel of Figure 9, on impact world output falls by almost 2 percentage points below its value in the initial steady state. Interestingly, the drop in GDP is sufficiently large so that initially the world gross debt-to-GDP ratio increases. Only starting from the second quarter the ratio of world debt-to-GDP starts declining.³³ There is, moreover, substantial heterogeneity in the output response across the members of the union. High-debt countries suffer a particularly severe recession. For instance, as shown in Figure 9, on impact GDP in the country at the 10th percentile of the initial net foreign asset distribution falls by almost 6 percent. Instead, wealthy countries experience a mild

³³This is consistent with the path of the private debt-to-GDP ratio observed during several deleveraging episodes. See McKinsey (2010, 2012).

Table 2: Cumulative output loss

	World	5th perc.	10th perc.	25th perc.	50th perc.	75th perc.
Benchmark	10.1	49.6	25.1	5.4	4.3	4.0
$\rho = 0.65$	23.0	69.6	44.4	17.6	16.3	15.9
$\rho = 0.75$	1.9	34.9	11.8	-2.1	-3.1	-3.4
$\phi = 0.1$	36.5	105.6	69.1	28.6	25.9	25.2
$\phi = 0.3$	3.3	25.9	9.8	0.6	0.0	-0.1

Notes: The cumulative output loss is computed as $\sum_{t=0}^T (GDP_{i,t}/GDP_i - 1) \cdot 100$, where GDP_i denotes GDP in the initial steady state. The output loss is computed over the two years following the deleveraging shock, so $T = 8$. For the country-level loss, productivity is assumed to be constant and equal to its mean value.

contraction, as it is the case for the country at the 75th percentile of the initial wealth distribution. The heterogeneous output response is due to the presence of non-traded goods. In fact, the fall in production of tradable goods is uniform across the monetary union, because production of the traded good depends on the demand from all the countries in the union. Instead, production of non-traded goods, which depends on local demand, falls more in high-debt countries, because these are the countries that experience the largest drop in consumption. In fact, as shown by the bottom-right panel of Figure 9, the heterogeneity in the consumption response is particularly large. While the 10th percentile country experiences a deep fall in consumption, consumption in the 75th percentile country rises slightly above its value in the initial steady state.

Table 2 provides further information on the impact on output of the deleveraging episode, by showing the cumulative output loss occurring in the two years after the deleveraging shock. The aggregate cumulative output loss amounts to a sizable 10 percent of GDP in the initial steady state.³⁴ These aggregate output losses are mainly driven by the deep recession experienced by high-debt countries. For instance, the losses in the countries at the 5th and 10th percentile of the initial net foreign asset distribution are respectively equal to 49.6 and 25.1 percent of their GDP in the initial steady state. This is one order of magnitude larger than the losses experienced by unconstrained countries. As an example, countries at the 25th, 50th and 75th percentile have cumulative losses respectively equal to 5.4, 4.3 and 4 percent of their initial GDP.

Table 2 also shows how the impact of deleveraging on output varies with two key parameters, ρ and ϕ . First, the output losses are greater when deleveraging is faster, i.e. the lower ρ .³⁵ For example, moving from the benchmark value of $\rho = 0.7$ to $\rho = 0.65$ more than doubles the output losses. Instead, when $\rho = 0.75$ the output loss associated with deleveraging is negligible. These results point toward the importance of the “surprise” aspect of the deleveraging shock in generating a large drop in output.³⁶

³⁴One might wonder how this number compares to the data from the euro area. In Appendix F I estimate the cumulative output loss experienced by the euro area during the first two years after the 2008 global financial crisis. Though it is difficult to provide a precise estimate, the procedure I use suggests that during the period 2008Q4-2010Q3 the euro area experienced a cumulative output loss equal to 41.7 percent of 2008Q3 GDP. Hence, according to the benchmark calibration the deleveraging shock explains about 1/4 of the output loss experienced by the euro area following the 2008 financial crisis.

³⁵Instead, holding constant the short-run path of κ_t , the results are largely unaffected by changes in the borrowing limit in the final steady state $\bar{\kappa}$.

³⁶To further investigate this point, I performed an experiment in which κ_t follows the same path as in the benchmark

Second, higher wage rigidities magnify the output losses due to deleveraging. For instance, when $\phi = 0.1$, so that every quarter a wage setter has a 10 percent probability of receiving information about the deleveraging shock, the aggregate cumulative output loss is almost four times larger than under the benchmark parametrization.³⁷ This is an interesting result, in light of the fact that in the standard New Keynesian model higher price or wage flexibility amplifies the drop in output associated with a liquidity trap (Werning, 2011). This can be explained with the presence of two opposing effects. In the New Keynesian model, higher wage flexibility increases the deflation associated with a liquidity trap. In turn, expectations of future deflation raise the real interest rate, which depresses output. This effect implies that more flexible wages are associated with a larger output drop. While here this effect is present, there is another effect that goes in the opposite direction. In fact, as discussed in Section 4, in a monetary union wage rigidities prevent the reallocation of production from wealthy to high-debt countries, deepening the drop in world demand for consumption generated by the deleveraging shock. This second effect, which turns out to dominate in the simulations, implies that more flexible wages mitigate the output drop during the liquidity trap.

Overall, this exercise shows that in a monetary union a reasonably calibrated deleveraging shock generates a significant aggregate recession. An important question, to which now I turn, is whether the recession generates significant welfare losses, and how these welfare losses are distributed across the different countries.

5.2 Welfare

In this section, I provide an estimate of the welfare losses associated with deleveraging in a monetary union. Specifically, I calculate the difference in welfare during the transition between the monetary union and the baseline model of Section 2. The baseline model is an interesting benchmark because, as shown in Appendix G, it corresponds to the uncooperative constrained-efficient allocation. In other words, the baseline model is isomorphic to an economy in which country-level policymakers, endowed with enough instruments to offset the distortions due to nominal rigidities, implement the uncooperative optimal policy.³⁸

More precisely, I compute the welfare losses associated with the monetary union as the proportional increase in consumption for all possible future histories that agents living in a monetary union must receive, in order to be indifferent between remaining in the monetary union and switching to the baseline frictionless economy. Since I am studying a single crisis event, following Gertler and Karadi (2011), I calculate the present value of the consumption-equivalent benefits and normalize

parametrization, except that the drop in the borrowing limit is announced to agents two periods in advance. Under this scenario, the fall in the interest rate is not large enough to make the zero lower bound bind. Moreover, the impact of deleveraging on aggregate output is negligible, while high-debt countries experience a mild contraction. Taking stock, these experiments suggest that to have a large impact on output the deleveraging shock must be perceived by agents as a low probability event.

³⁷Indeed, as discussed in Appendix F, with $\phi = 0.1$ the model gets close to reproduce the output loss experienced by the euro area in the aftermath of the 2008 financial crisis.

³⁸Of course, there might be gains from cooperation, and hence the allocation reached in the baseline model might not correspond to the cooperative constrained-efficient allocation.

Table 3: Welfare losses

	Mean	5th perc.	10th perc.	25th perc.	50th perc.	75th perc.
Benchmark	9.8	52.4	13.4	0.4	0.6	0.7
$\bar{\pi} = 1.04^{1/4}$	8.7	42.7	8.9	0.6	1.3	1.5
Transfer	7.9	40.0	7.4	-0.5	0.6	1.6

Notes: The welfare losses are computed as the proportional increase in consumption for all possible future histories that agents living in a monetary union must receive, in order to be indifferent between remaining in the monetary union and switching to the baseline frictionless economy. The consumption-equivalent benefits are expressed as percentage of consumption in the initial steady state.

them by consumption in the initial steady state.³⁹

As reported in Table 3, the mean welfare loss associated with deleveraging in the monetary union is equal to 9.8 percent of one-period consumption in the initial steady state. Perhaps unsurprisingly, the welfare losses are concentrated in high-debt economies. For instance, countries at the 5th and 10th percentile of the initial net foreign asset distribution suffer losses respectively equal to 52.4 and 13.4 percent of their consumption in the initial steady state. Instead, the losses experienced by unconstrained countries are very small. Interestingly, the welfare losses follow a U-shaped pattern with respect to the initial stock of net foreign assets held by a country. This is due to the fact that the countries at the extremes of the net foreign asset distribution are the ones that experience the largest adjustment in real wages in the baseline model. Hence, for these countries nominal wage rigidities represent a particularly severe friction in the adjustment to the deleveraging shock.

These results imply that the frictions associated with a monetary union, that is the inability to adjust exchange rates across member countries and to set the nominal interest rate below zero, generate substantial welfare losses. In the next sections I discuss some examples of policies that can mitigate the negative impact of deleveraging on welfare, in order to illustrate how the model can be used to evaluate policy interventions.

³⁹Formally, for every country i the welfare losses η_i are defined as

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{((1 + \eta_i)C_{i,t}^{mu})^{1-\gamma} - 1}{1-\gamma} - \frac{(L_{i,t}^{mu})^{1+\psi}}{1+\psi} \right) \right] = E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{((1 + \eta_i)C_{i,t}^{bas})^{1-\gamma} - 1}{1-\gamma} - \frac{(L_{i,t}^{bas})^{1+\psi}}{1+\psi} \right) \right],$$

where the superscripts *mu* and *bas* refer to the allocations respectively under the monetary union and the baseline frictionless model, while

$$L_{i,t}^{mu} \equiv \left((1 - (1 - \phi)^t) (L_{i,t}^{in})^{1+\psi} + (1 - \phi)^t (L_{i,t}^{un})^{1+\psi} \right)^{\frac{1}{1+\psi}},$$

where $L_{i,t}^{in}$ and $L_{i,t}^{un}$ denote the labor supply of members of respectively informed and uninformed unions. The normalized present value of the consumption equivalent benefits is then defined as

$$\frac{E_0 \left[\sum_{t=0}^{\infty} \beta^t \eta_i C_{i,t}^{mu} \right]}{C_i} \cdot 100,$$

where C_i denotes consumption in the initial steady state.

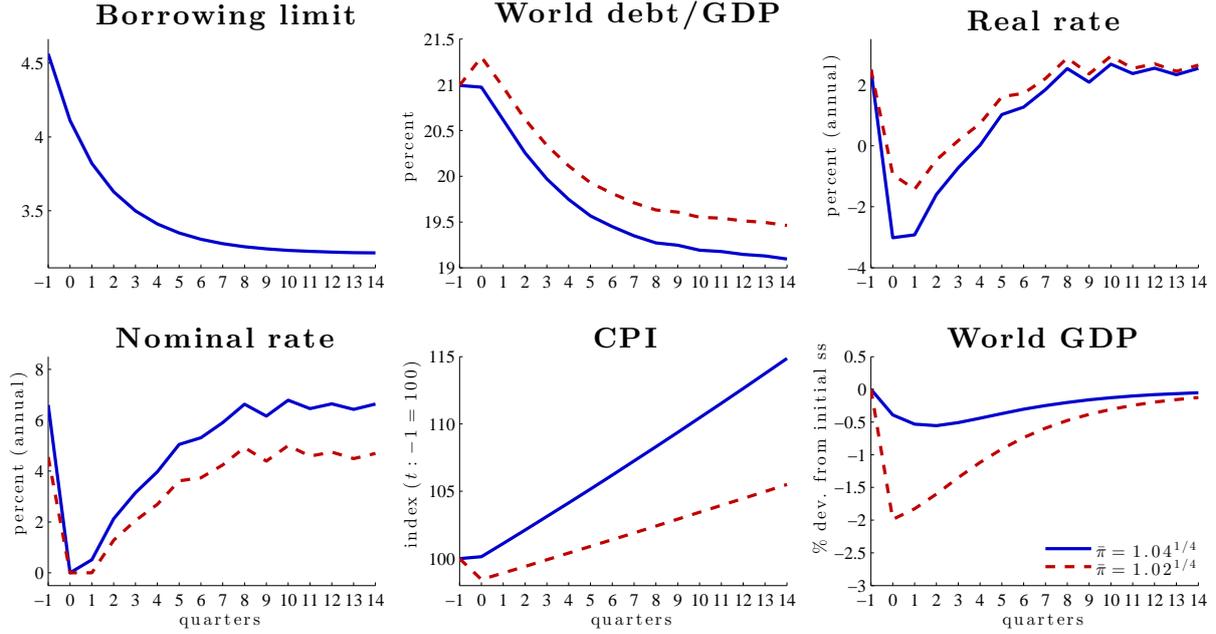


Figure 10: Higher inflation target. Notes: world debt refers to world gross debt, defined as $\int_{-\kappa_t}^0 B_{i,t+1}/R_t di$. World GDP and consumption are respectively defined as $\int_0^1 Y_{i,t}^T + p^N Y_{i,t}^N di$ and $\int_0^1 C_{i,t}^T + p^N C_{i,t}^N di$, where p^N denotes the unconditional mean of $p_{i,t}^N$ in the initial steady state. CPI is the average consumer price index.

5.3 Raising the inflation target

One policy that can mitigate the recession during debt deleveraging consists in adopting a higher inflation target. In fact, a higher inflation target relaxes the zero lower bound constraint, giving more room for monetary policy to lower the interest rate in response to the deleveraging shock. Figure 10 compares two monetary unions with different steady state inflation targets.⁴⁰ The solid lines refer to an economy with a high inflation target, of 4 percent per year, while the dashed lines refer to the baseline economy with an annual inflation target of 2 percent. Clearly, a higher inflation target reduces the drop in output associated with deleveraging. In fact, doubling the inflation target from 2 to 4 percent per year reduces the aggregate cumulative output losses in the two years following the deleveraging shock from 10.1 to 3.4 percent of GDP in the initial steady state. This happens because in the high inflation target scenario the central bank is able to cut the real rate by about 200 basis points more compared to the benchmark inflation target.

Table 3 shows the impact on welfare associated with a higher inflation target. On average, the impact on welfare from having a higher inflation target is positive, since the mean welfare loss is lower in the high inflation target economy than in the benchmark. However, not all countries benefit from a higher inflation target. In fact, while high-debt countries are better off in the high inflation target economy, the opposite is true for wealthy unconstrained countries. One possible

⁴⁰This section looks at two economies whose steady state inflation target is different. An alternative would be to consider a change in the inflation target in response to the tightening of the borrowing limit. However credibility issues are likely to prevent a central bank from changing the inflation target in the middle of a deleveraging episode. This point is discussed by Eggertsson (2008), who considers credibility issues faced by the FED during the Great Depression.

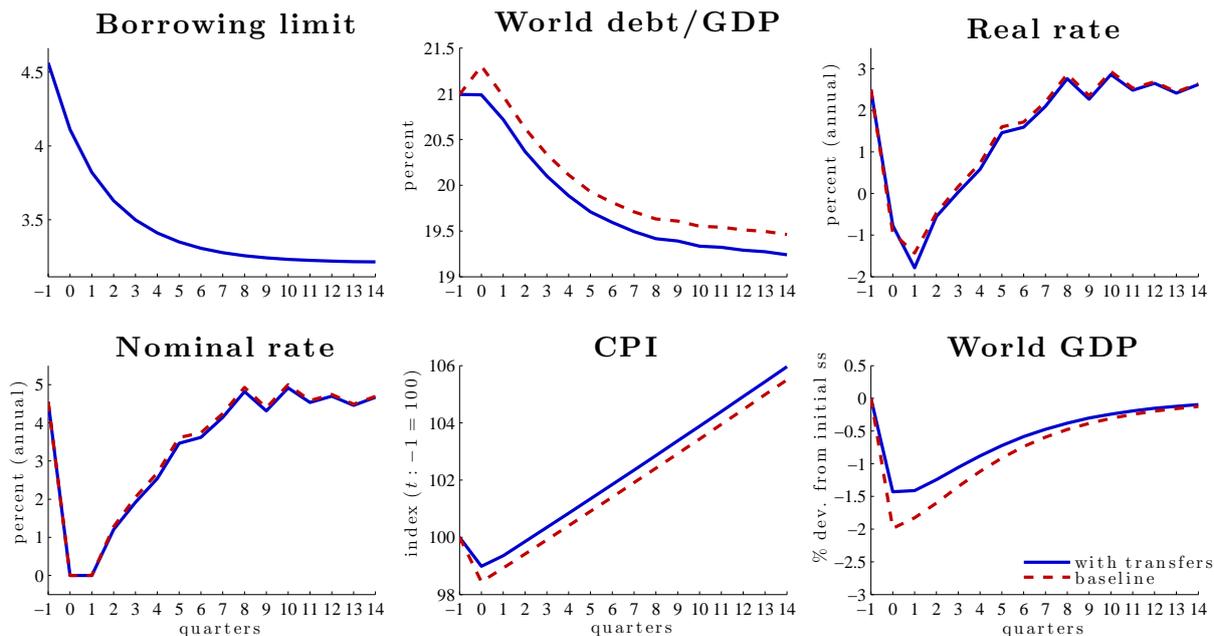


Figure 11: Response to deleveraging shock - transfers. Notes: world debt refers to world gross debt, defined as $\int_{-\kappa_t}^0 B_{i,t+1}/R_t di$. World GDP and consumption are respectively defined as $\int_0^1 Y_{i,t}^T + p^N Y_{i,t}^N di$ and $\int_0^1 C_{i,t}^T + p^N C_{i,t}^N di$, where p^N denotes the unconditional mean of $p_{i,t}^N$ in the initial steady state. CPI is the average consumer price index.

explanation for this fact is that a higher inflation target generates a larger drop in the real interest rate during the transition. This has a negative impact on wealthy countries, because it reduces the return that they enjoy on their wealth.

5.4 Transfers within members of the monetary union

One policy that has been much discussed in the context of the ongoing Eurozone crisis concerns transfers among members of a monetary union.⁴¹ In this section I perform a simple experiment to evaluate the macroeconomic impact of transfers from creditor to debtor countries. I consider a scenario in which at the start of period 0 every debtor country receives a transfer equal to 1 percent of its external debt. This transfer is financed by creditors countries, and each creditor country contributes with a sum equal to 1 percent of its stock of assets.⁴² Under this transfer scheme, collectively creditor countries transfer to debtor countries a sum equal to 0.2 percent of the annual GDP of the monetary union in the initial steady state.

The results are shown in Figure 11. The solid lines refer to the economy with transfers, while the dashed lines refer to the baseline economy. Figure 11 makes clear that transfers from creditor to debtors countries reduce deflation and the output contraction. This happens because debtor countries have a higher propensity to consume out of income than creditor countries. Hence, transfers toward debtor countries stimulate aggregate demand. In turn, the increase in aggregate

⁴¹See Werning and Farhi (2012) for an insightful analysis of optimal fiscal transfers inside monetary unions.

⁴²This transfer scheme captures a variety of policies, such as fiscal transfers inside a monetary union or debt relief policies. This experiment also captures some of the aspects of the public flows passing via the ECB which played a major role in cushioning the fall in foreign credit in the countries at the eurozone periphery in the first phase of the 2008/2009 recession, as shown by Lane and Milesi-Ferretti (2012).

demand has a positive impact on output, because the recession during the liquidity trap is due to weak aggregate demand. Quantitatively, even a relatively modest transfer such as the one considered in this section can have a significant impact on output. For instance, the transfer scheme considered in this experiment reduces the cumulative loss in output during the two years after the deleveraging shock from 10.1 to 7.8 percent of steady state output.

As reported in Table 3, the transfer scheme has on average a positive impact on welfare. The welfare gains are, however, unevenly distributed. In general debtor countries, captured in the table by the countries from the 5th to the 50th percentile of the initial debt distribution, gain from the transfer scheme. In fact, these countries enjoy both a direct welfare gain from the transfer, and an indirect gain coming from the boost in aggregate demand generated by the transfer. Creditor countries, captured in the table by the 75th percentile country, tend to experience a welfare loss. In fact, in these countries the benefits coming from higher aggregate demand are not sufficiently big to compensate the wealth loss implied by the transfer scheme.

This experiment suggests that transfers from creditor to debtor countries of a monetary union can play a role in mitigating the recession associated with an episode of debt deleveraging. These transfers, however, might entail welfare losses in wealthy countries, and hence be hard to implement from a political perspective.

6 Conclusion

I propose a multi-country model for understanding deleveraging among a group of financially integrated countries. The model highlights the channels through which participation in a monetary union impede a smooth adjustment to deleveraging. Deleveraging leads to a drop in the world interest rate, both because high-debt countries are forced to save more in order to reduce their debt and because the rest of the world experiences an increase in the desire to accumulate precautionary savings. In the absence of nominal rigidities, deleveraging also triggers a rise in production in high-debt countries. If wages are nominally rigid but nominal exchange rates are allowed to float, the rise in production involves a nominal depreciation in high-debt countries. In a monetary union, the combination of nominal wage rigidities and fixed exchange rates prevents any increase in production in indebted countries. This amplifies the fall in the world consumption demand and the drop in the world interest rate. Hence, monetary unions are prone to enter a liquidity trap during an episode of deleveraging. In a liquidity trap deleveraging generates a deflationary union-wide recession, hitting high-debt countries especially hard.

The analysis presented in this paper can be extended in a number of directions. First, the model could be used to investigate a richer menu of policies. In particular, the recent experience of the eurozone has sparked a lively debate on the role of fiscal policy inside monetary unions, and the model has the potential to shed light on this key policy issue. In addition, it would be interesting to consider collateral constraints in which asset prices play a role in determining access to credit. For instance, Fornaro (2015) and Ottonello (2013) study the interactions between

collateral constraints and exchange rate policy in small open economies. An open research question concerns the interactions between these types of constraints and the zero lower bound in a model of the world economy.

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Appendix

A A model with interest rate spreads

In the model presented in the main text all the agents, and countries, are subject to the same interest rate. However, in many cases tight credit conditions manifest themselves with high interest rates. In fact, often countries whose access to the international credit markets is restricted are charged a spread over the interest rate paid by unconstrained countries. This appendix shows how it is possible to reconcile this fact with the model without changing any of the results. In particular, in this appendix I present a model in which the borrowing limit is enforced through interest rate spreads and show that this model is isomorphic to the framework studied in the main text. The discussion draws on Uribe (2006).

For simplicity I focus on the economy without nominal rigidities described in section 2, but the results can be extended to the case of a monetary union with nominal wage rigidities. Suppose that the representative household in country i is charged the country-specific interest rate $R_{i,t}$, potentially different from the world interest rate R_t . Suppose also that there is no limit to how much the household can borrow at the interest rate $R_{i,t}$. The Euler equation then writes

$$\lambda_{i,t} = R_{i,t} \beta E_t [\lambda_{i,t+1}],$$

where $\lambda_{i,t}$ is the Lagrange multiplier on the budget constraint.

In the model in the main text, in which the household is constrained by the borrowing limit (3) and it is charged the world interest rate R_t , the Euler equation can instead be written as

$$\lambda_{i,t} = \frac{R_t}{1 - \frac{\mu_{i,t} R_t}{\lambda_{i,t}}} \beta E_t [\lambda_{i,t+1}],$$

where $\mu_{i,t}$ is the Lagrange multiplier on the borrowing constraint.

Notice that if the household is charged the interest rate

$$R_{i,t} = \frac{R_t}{1 - \frac{\mu_{i,t} R_t}{\lambda_{i,t}}},$$

the two Euler equations coincide. Moreover, we have that $R_{i,t} = R_t$ when $\mu_{i,t} = 0$, while $R_{i,t} > R_t$ when $\mu_{i,t} > 0$.⁴³ Intuitively, investors can make sure that an household respects the borrowing limit by charging a positive spread over the world interest rate anytime the household would violate the constraint if charged the world interest rate. In equilibrium, we would thus observe that high-debt constrained countries are charged a positive spread over the world interest rate.

To obtain a version of the model with interest rate spreads isomorphic to the model in the main text, we must make sure that the resource constraint of the household is not affected by

⁴³Using the fact that $\mu_{i,t} \geq 0$ and rearranging the Euler equation in the main text, it is easy to check that $\mu_{i,t} R_t / \lambda_{i,t} < 1$.

the interest rate spreads. Indeed, whenever the constraint is binding there is a financial rent given by the difference between the cost of funds for the investor and the interest rate that the borrower would like to pay. In the model in the main text this rent accrues to the borrower, since constrained borrowers are charged the world interest rate, that is the cost of funds for investors. We must then ensure that financial rents go fully to the borrower also in the version of the model with interest rate spreads. Following Uribe (2006), this can be done by assuming the existence of domestic financial intermediaries that borrow at the world interest rate R_t and lend to households at the interest rate $R_{i,t}$. Assuming that the profits of the domestic financial intermediaries are fully rebated to households in a lump sum fashion, we obtain that the economy with the borrowing constraint described in the main text is isomorphic to the economy with spreads described in this appendix.

B Numerical solution method

To solve the model numerically I employ the method proposed by Guerrieri and Lorenzoni (2011).

Computing the steady state of the model involves finding the interest rate that clears the bond market at the world level. The first step consists in deriving the optimal policy functions $C^T(B, A)$, $C^N(B, A)$ and $L(B, A)$ for a given interest rate R . To compute the optimal policy functions I discretize the endogenous state variable B using a grid with 600 points, and then iterate on the Euler equation and on the intratemporal optimality conditions using the endogenous gridpoints method of Carroll (2006). Using the optimal policies, it is possible to derive the inverse of the bond accumulation policy $g(B, A)$. This is used to update the conditional bond distribution $\Psi(B, A)$ according to the formula $\Psi_\tau(B, A) = \sum_A \Psi_{\tau-1}(g(B, \tilde{A}), \tilde{A}) P(A|\tilde{A})$ for all $B \leq -\kappa$, where τ is the τ -th iteration and $P(A|\tilde{A})$ is the probability that $A_{t+1} = A$ if $A_t = \tilde{A}$. The bond accumulation function is not invertible at $B = -\kappa$, but the formula above holds if $g(-\kappa, A)$ is defined as the largest B such that $B' = -\kappa$ is optimal. Once the bond distribution has converged to the stationary distribution, I check whether the market for bonds clears. If not, I update the guess for the interest rate.

To compute the transitional dynamics, I first derive the initial and final steady states. I then choose a T large enough so that the economy has approximately converged to the final steady state at $t = T$ (I use $T = 150$, increasing T does not affect the results reported). The next step consists in guessing a path for the interest rate. I then set the policy functions for consumption in period T equal to the ones in the final steady state and iterate backward on the Euler equation and on the intratemporal optimality conditions to find the sequence of optimal policies $\{C_t^T(B, A), C_t^N(B, A), L_t(B, A)\}$. Next, I use the optimal policies to compute the sequence of bond distributions $\Psi_t(B, A)$ going forward from $t = 0$ to $t = T$, starting with the distribution in the initial steady state. Finally, I compute the world demand for bonds in every period and update the path for the interest rate until the market clears in every period.

To compute the transitional dynamics with wage rigidities I follow a similar method. The only

difference is that in every period $t \geq 0$ uninformed wage setters behave according to their policy functions in the initial steady state. However, since in period $T = 150$ the measure of uninformed wage setters is essentially 0, we can still set the policy functions for consumption in period T equal to the ones in the final steady state.

C Analytic example

This appendix presents a simplified version of the baseline model, useful to sharpen intuition about the adjustment triggered by an episode of international deleveraging. To enhance the tractability of the model, here I study a perfect foresight economy with a stylized form of initial wealth heterogeneity, in which deleveraging takes place in a single period.

The equilibrium conditions are the same as in Section 2. I restate them here for convenience:

$$(C_{i,t}^T)^{\omega(1-\gamma)-1} (C_{i,t}^N)^{(1-\omega)(1-\gamma)} = R_t \left(\beta E_t \left[(C_{i,t+1}^T)^{\omega(1-\gamma)-1} (C_{i,t+1}^N)^{(1-\omega)(1-\gamma)} \right] + \frac{\mu_{i,t}}{\omega} \right) \quad (C.1)$$

$$B_{i,t+1} \geq -\kappa_t, \quad \text{with equality if } \mu_{i,t} > 0, \quad (C.2)$$

$$p_{i,t}^N = \frac{1 - \omega}{\omega} \frac{C_{i,t}^T}{C_{i,t}^N} \quad (C.3)$$

$$L_{i,t}^\psi = w_{i,t} (C_{i,t}^T)^{\omega(1-\gamma)-1} (C_{i,t}^N)^{(1-\omega)(1-\gamma)} \quad (C.4)$$

$$\alpha_T A_{i,t}^T (L_{i,t}^T)^{\alpha_T-1} = w_{i,t}. \quad (C.5)$$

$$p_{i,t}^N \alpha_N A_{i,t}^N (L_{i,t}^N)^{\alpha_N-1} = w_{i,t}. \quad (C.6)$$

$$C_{i,t}^N = A_{i,t}^N (L_{i,t}^N)^{\alpha_N} \quad (C.7)$$

$$C_{i,t}^T = A_{i,t}^T (L_{i,t}^T)^{\alpha_T} + B_{i,t} - \frac{B_{i,t+1}}{R_t} \quad (C.8)$$

$$L_{i,t} = L_{i,t}^T + L_{i,t}^N \quad (C.9)$$

$$\int_0^1 B_{i,t+1} di = 0. \quad (C.10)$$

I make some parametric assumptions to simplify the derivations. I thus assume that $\gamma = 1$, so that utility from consumption is logarithmic, and focus on the limit $\psi \rightarrow +\infty$, so that labor supply is infinitely inelastic. This last assumption and equation (C.4) imply that labor supply is constant and equal to 1. Moreover, to abstract from uncertainty and precautionary savings, I assume that in every country productivity in both sectors is constant and normalized to 1, so that $A_{i,t}^T = A_{i,t}^N = 1$ for all t .

For future reference, notice that under these assumptions the Euler equation (C.1) becomes

$$\frac{1}{C_{i,t}^T} = \frac{\beta R_t}{C_{i,t+1}^T} + R_t \mu_{i,t}. \quad (C.11)$$

In addition, using (C.3), (C.5) and (C.6) gives

$$L_{i,t}^N = \frac{\alpha_N}{\alpha_T} \frac{1-\omega}{\omega} (L_{i,t}^T)^{1-\alpha_T} C_{i,t}^T. \quad (\text{C.12})$$

Finally, the expression above can be combined with (C.9) to obtain

$$1 - L_{i,t}^T = \frac{\alpha_N}{\alpha_T} \frac{1-\omega}{\omega} (L_{i,t}^T)^{1-\alpha_T} C_{i,t}^T. \quad (\text{C.13})$$

The economy starts from a steady state in which half of the countries are creditors and half are debtors. In what follows I denote creditor and debtor countries respectively with subscripts c and d . In the initial steady state each creditor (debtor) country holds assets $B_0 > 0$ ($-B_0 < 0$). In period 0 there is a permanent drop in the borrowing limit that forces debtor countries to reduce their debt. In particular, starting from period 0 onward the borrowing limit is equal to $\bar{\kappa}$, with $\bar{\kappa} < B_0$. This implies that deleveraging takes place in a single period after which the economy reaches its final steady state. Hence, we can divide the analysis into a short run (period 0) and a long run (periods $t \geq 1$).

Initial and final steady states. In steady state consumption of both creditor and debtor countries is constant, and the borrowing constraint is not binding for creditors. Hence, by creditors' Euler equation (C.1), the steady state interest rate is $R = 1/\beta$. Moreover, in steady state each country rolls over its stock of foreign assets, so that by (C.8) steady state tradable consumption in a generic country i is

$$C_i^T = (L_i^T)^{\alpha_T} + B_i(1-\beta), \quad (\text{C.14})$$

where the absence of a time subscript denotes the value of a variable in steady state. In steady state the labor market clears in every country. Hence, combining (C.14) with (C.12) and (C.13) gives the steady state values of C_i^T , L_i^T and L_i^N as a function of B_i . For future reference, notice that (C.14) and (C.13) imply that $C_i^T = C^T(B_i)$ with $C^{T'}(\cdot) > 0$, and $L_i^T = L^T(B_i)$ with $L^{T'}(\cdot) < 0$. In words, in steady state tradable consumption (labor) is strictly increasing (decreasing) in foreign assets.

The economy starts from a steady state in which $B_c = B_0$ and $B_d = -B_0$. In period 1 the economy converges to the final steady state in which $B_c = \bar{\kappa}$ and $B_d = -\bar{\kappa}$. Recalling that $\bar{\kappa} < B_0$, the analysis above implies that comparing the initial steady state to the final one debtor countries increase their consumption of tradables and decrease the fraction of labor allocated to the tradable sector. This happens because in the post-deleveraging final steady state debtor countries have higher financial wealth compared to the initial one. The opposite is true for creditor countries. I now turn to the short-run adjustment triggered by a deleveraging shock.

Short-run response to deleveraging shock. I now derive the short run adjustment to the deleveraging shock. To gain intuition, it is useful to recast the equilibrium in terms of an aggregate demand/aggregate supply diagram. To derive an aggregate demand equation for tradable goods, start by noticing that in period 0 debtor countries are borrowing constrained, so their demand for

tradables is

$$C_{d,0}^T = (L_{d,0}^T)^{\alpha_T} + \frac{\bar{\kappa}}{R_0} - B_0. \quad (C.15)$$

Effectively, in period 0 debtor countries act as hand-to-mouth consumers, and consume all their production of tradables net of the amount needed to reduce external debt, so as to satisfy the new borrowing limit. Instead, creditors are on their Euler equation, and so their demand for consumption is

$$C_{c,0}^T = \frac{C^T(\bar{\kappa})}{\beta R_0}. \quad (C.16)$$

Importantly, in the short run creditors' demand for consumption does not depend directly on their production of tradable goods, since $C^T(\bar{\kappa})$ is independent of $L_{c,0}^T$. Taken together, (C.15) and (C.16) imply that in the short run debtor countries have a higher propensity to consume out of tradable income than creditors. Adding consumption demand from creditors and debtors gives the aggregate demand (AD) equation

$$C_{d,0}^T + C_{c,0}^T = (L_{d,0}^T)^{\alpha_T} + \frac{\bar{\kappa}}{R_0} - B_0 + \frac{C^T(\bar{\kappa})}{\beta R_0}, \quad (AD)$$

which implies a negative relationship between global demand for tradable consumption and the world interest rate. Intuitively, debtors' demand for consumption is decreasing in the interest rate because their borrowing, gross of future interest payments, is constrained by the limit $\bar{\kappa}$. Instead, a rise in the interest rate depresses creditors' demand for consumption because it increases their incentives to save. Figure 12 illustrates the downward-sloped relationship between R_0 and period 0 consumption implied by the AD curve.⁴⁴

The AD equation also shows that aggregate demand is increasing in the borrowing limit $\bar{\kappa}$. On the one hand, a drop in the borrowing limit forces debtors to devote a higher fraction of their income to debt repayment, driving resources away from consumption. On the other hand, since $C^{T'}(\bar{\kappa}) > 0$, a lower debt limit means that creditors' future assets and consumption will be lower, inducing creditor countries to reduce the present demand for consumption. Graphically, following a drop in the borrowing limit the AD curve shifts left to AD' . This generates a drop in the world interest rate R_0 that, at least partly, mitigates the negative impact of deleveraging on consumption demand. I will now show that the supply side of the economy plays a key role in determining the severity of the fall in R_0 in response to the deleveraging shock.

Let us start by considering a case in which supply does not respond to the deleveraging shock. In particular, let us assume that $L_{c,0}^T = L^T(B_0)$ and $L_{d,0}^T = L^T(-B_0)$, so that the period 0 production of tradables is the same as in the initial steady state. This case is depicted by the left panel of Figure 12, in which the short-run aggregate supply of tradables, the AS curve, is just a straight line corresponding to $Y_{c,0}^T + Y_{d,0}^T = (L^T(B_0))^{\alpha_T} + (L^T(-B_0))^{\alpha_T}$. As shown by the graph, the deleveraging shock, which moves the AD curve to AD' , produces a sharp fall in the interest rate, while leaving unchanged world production of tradable goods. Intuitively, the sharp fall in the

⁴⁴To construct the figure I have set $\beta = 0.9756$, $\omega = 0.2$, $\alpha_T = 0.65$, $\alpha_N = 0.65$, $B_0 = 0.9$ and $\bar{\kappa} = 0.675$.

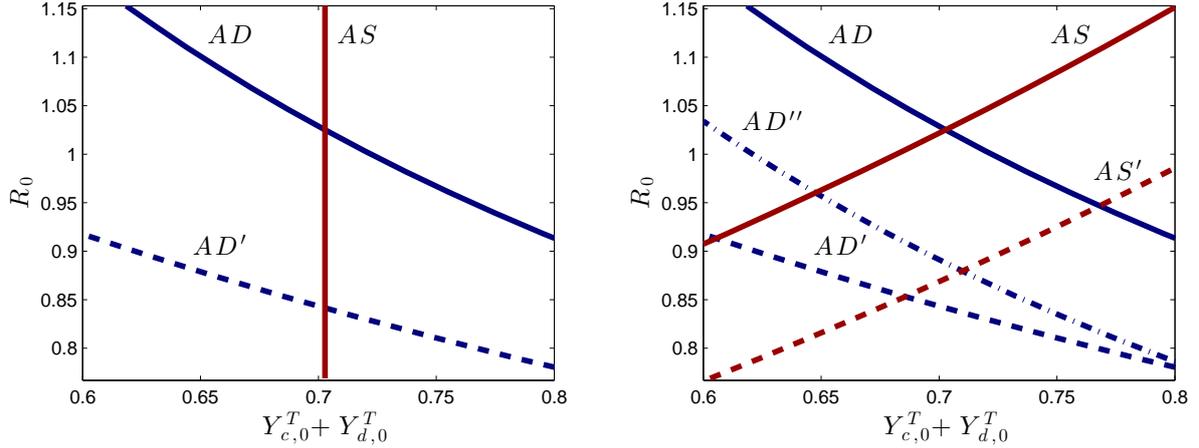


Figure 12: Short-run impact of deleveraging. Left panel: fix output. Right panel: baseline model with labor reallocation.

interest rate is needed to offset the impact of the deleveraging shock on demand, so that demand for consumption is strong enough to sustain the fixed supply of tradable goods.

In reality, aggregate supply does respond to the deleveraging shock. In fact, combining equation (C.13) with the consumption demands (C.15) and (C.16) gives

$$1 - L_{d,0}^T = \frac{\alpha_N}{\alpha_T} \frac{1 - \omega}{\omega} (L_{d,0}^T)^{1 - \alpha_T} \left((L_{d,0}^T)^{\alpha_T} + \frac{\bar{\kappa}}{R_0} - B_0 \right) \quad (\text{C.17})$$

$$1 - L_{c,0}^T = \frac{\alpha_N}{\alpha_T} \frac{1 - \omega}{\omega} \frac{C^T(\bar{\kappa})}{\beta R_0}. \quad (\text{C.18})$$

According to these equations, both $L_{d,0}^T$ and $L_{c,0}^T$ are increasing in R_0 and decreasing in $\bar{\kappa}$. This happens because of households' desire to consume a balanced basket of tradable and non-tradable goods. Hence, as consumption of tradables increases demand for non-tradables rises, leading to a reallocation of labor toward the non-tradable sector. Since, both for creditor and debtor countries, consumption of tradables is decreasing in R_0 and increasing in $\bar{\kappa}$, this effect explains why world production of tradables is increasing in R_0 and decreasing in $\bar{\kappa}$. Consequently, after a deleveraging shock, keeping R_0 constant, world production of tradables increases.

The case of endogenous production is illustrated by the right panel of Figure 12. The AS curve is upward sloped, capturing the positive relationship between R_0 and production of tradables. The fall in $\bar{\kappa}$ produces a shift right of the AS curve to AS' , capturing the expansion in production of tradables following the deleveraging shock. Moreover, the rise in tradable labor following the deleveraging shock also affects the AD curve. In fact, since debtor countries spend all their increase in tradable income in consumption, the rise in $L_{d,0}^T$ driven by the fall in $\bar{\kappa}$ sustains aggregate demand.⁴⁵ This effect is represented graphically by the shift of the AD curve from AD' to AD'' .

Compared to the fixed output case, the movement of the AS points toward a lower equilibrium

⁴⁵Instead, changes in $L_{c,0}^T$ do not have direct impact on creditors' demand, which is determined by their consumption in the final steady state, $C^T(\bar{\kappa})$, and by the interest rate R_0 .

interest rate, while the shift of the AD curve points toward a higher rate. However, it is possible to show that the demand effect dominates, so that the interest rate falls less in response to a deleveraging shock in the economy with endogenous production compared to the fixed output economy. To see this point, consider that using the AD equation we can express equilibrium on the market for tradables as

$$(L_{d,0}^T)^{\alpha_T} + \frac{\bar{\kappa}}{R_0} - B_0 + \frac{C^T(\bar{\kappa})}{\beta R_0} = (L_{d,0}^T)^{\alpha_T} + (L_{c,0}^T)^{\alpha_T}, \quad (\text{C.19})$$

which can be simplified and rearranged as

$$R_0 = \frac{\bar{\kappa} + C^T(\bar{\kappa})/\beta}{B_0 + (L_{c,0}^T)^{\alpha_T}}. \quad (\text{C.20})$$

This expression implies that in equilibrium the supply side of the economy affects R_0 only through $L_{c,0}^T$. Intuitively, since debtor countries have a propensity to consume out of income of 1, movements in $L_{d,0}^T$ have opposing effects on aggregate demand and supply that exactly cancel out in equilibrium. Instead, creditors' demand does not directly depend on $L_{c,0}^T$. Hence, when $L_{c,0}^T$ falls it contracts aggregate supply without any direct effect on demand, leading to a higher equilibrium rate. This implies that R_0 is higher in the economy with endogenous output compared to the economy with fixed output if $L_{c,0}^T < L^T(B_0)$, which turns out to be always the case.⁴⁶ Moreover, it is possible to show that debtor countries expand their production of tradables during deleveraging (i.e. $L_{d,0}^T > L(-B_0)$),⁴⁷ so that the response of total output is ambiguous, and depending on parameter values the deleveraging shock can produce a fall or rise in the world production of tradable goods.

Summing up, the deleveraging shock induces a reallocation of labor from the non-tradable to the tradable sector in debtor countries, while the opposite is true for creditors. Since debtor

⁴⁶ Suppose instead that $L_{c,0}^T > L^T(B_0)$. Then equation (C.13) implies that $C_{c,0}^T < C^T(B_0)$. Using the resource constraint this inequality can be written as

$$(L_{c,0}^T)^{\alpha_T} - \frac{\bar{\kappa}}{R_0} + B_0 < (L^T(B_0))^{\alpha_T} + B_0(1 - \beta),$$

which can be rearranged as

$$(L_{c,0}^T)^{\alpha_T} - (L^T(B_0))^{\alpha_T} < \frac{\bar{\kappa}}{R_0} - \beta B_0.$$

Notice that if $\beta B_0 > \bar{\kappa}/R_0$ this inequality holds only if $L_{c,0}^T < L^T(B_0)$, a contradiction. We are left to prove that $\beta B_0 > \bar{\kappa}/R_0$. Using (C.20) this condition can be written as

$$\frac{\bar{\kappa} + C^T(\bar{\kappa})/\beta}{B_0 + (L_{c,0}^T)^{\alpha_T}} > \frac{\bar{\kappa}}{\beta B_0}.$$

Combining this inequality with (C.14) evaluated at $B_i = \bar{\kappa}$ and simplifying gives

$$\frac{(L^T(\bar{\kappa}))^{\alpha_T}}{(L_{c,0}^T)^{\alpha_T}} > \frac{\bar{\kappa}}{B_0}.$$

The right-hand side of the inequality is smaller than one by assumption. Moreover, using the fact that $\beta R_0 < 1$ and creditors' Euler equation gives $C_{c,0}^T > C^T(\bar{\kappa})$. Since $L_{i,t}^T$ is strictly decreasing in $C_{i,t}^T$, this implies that $L^T(\bar{\kappa}) > L_{c,0}^T$, so that the inequality above always holds. This proves that $\beta B_0 > \bar{\kappa}/R_0$ and consequently that $L_{c,0}^T > L^T(B_0)$.

⁴⁷The proof follows the steps of footnote 46.

countries have a higher propensity to consume out of income than creditors, this reallocation of production unambiguously mitigates the fall in aggregate demand for tradable goods, and the drop in the world interest rate during deleveraging.

D Model with nominal rigidities

This section presents a detailed description of the model with nominal wage rigidities studied in sections 4 and 5. There are two key changes with respect to the baseline model of Section 2. First, there is monopolistic competition on the labor market. Second, there are frictions in the adjustment of nominal wages, which create a channel through which monetary policy can affect real variables.

As in the model of Section 2, the world is composed of a continuum of measure one of small open economies indexed by $i \in \{0, 1\}$. These economies are continuously hit by idiosyncratic productivity shocks. There is no uncertainty at the world level, and the only aggregate shock is a fully unexpected drop in the borrowing limit.

Households. The expected lifetime utility of the representative household in a generic country i is

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{C_{i,t}^{1-\gamma} - 1}{1-\gamma} - \frac{\int_0^1 L_{i,t}(j)^{1+\psi} dj}{1+\psi} \right) \right], \quad (\text{D.1})$$

with $\gamma \geq 1$ and $\psi \geq 0$. $E_t[\cdot]$ is the expectation operator conditional on information available at time t and $0 < \beta < 1$ is the subjective discount factor. Consumption $C_{i,t}$ is a Cobb-Douglas aggregate of a tradable good $C_{i,t}^T$ and a non-tradable good $C_{i,t}^N$:

$$C_{i,t} = (C_{i,t}^T)^\omega (C_{i,t}^N)^{1-\omega}, \quad (\text{D.2})$$

where $0 < \omega < 1$. Each household supplies a continuum of measure one of differentiated labor services indexed by $j \in \{0, 1\}$, so that $L_{i,t}(j)$ denotes labor effort of type j .

Each household can trade in one period, non-state contingent real and nominal bonds. Real bonds are denominated in units of the tradable consumption good and pay the gross interest rate R_t . The interest rate on real bonds is common across countries, and hence R_t can be interpreted as the *world interest rate*. Nominal bonds are denominated in units of the domestic currency and pay the gross nominal interest rate $R_{i,t}^n$. Notice that $R_{i,t}^n$ can differ across countries. To simplify the analysis, I assume that nominal bonds are traded only across households sharing the same domestic currency.

The household budget constraint in terms of the domestic currency is:

$$P_{i,t}^T C_{i,t}^T + P_{i,t}^N C_{i,t}^N + P_{i,t}^T \frac{B_{i,t+1}}{R_t} + \frac{B_{i,t+1}^n}{R_{i,t}^n} = \int_0^1 W_{i,t}(j) L_{i,t}(j) dj + P_{i,t}^T B_{i,t} + B_{i,t}^n + \Pi_{i,t}^n + T_{i,t}. \quad (\text{D.3})$$

The left-hand side of this expression represents the household's expenditure. $P_{i,t}^T$ and $P_{i,t}^N$ denote respectively the price of a unit of tradable and non-tradable good in terms of country i currency.

Hence, $P_{i,t}^T C_{i,t}^T + P_{i,t}^N C_{i,t}^N$ is the total nominal expenditure in consumption. $B_{i,t+1}$ and $B_{i,t+1}^n$ denote respectively the purchase of real and nominal bonds made by the household at time t , at prices $P_{i,t}^T/R_t$ and $1/R_{i,t}^n$. If $B_{i,t+1} < 0$ or $B_{i,t+1}^n < 0$ the household is holding a debt.

The right-hand side captures the household's income. $W_{i,t}(j)$ denotes the nominal wage paid to type j labor. Hence $\int_0^1 W_{i,t}(j)L_{i,t}(j)dj$ is the household's total labor income. Labor is immobile across countries and hence wages are country-specific. $P_{i,t}^T B_{i,t}$ and $B_{i,t}^n$ represent the gross returns on investment in bonds made at time $t-1$. $\Pi_{i,t}^n$ are the profits received from firms expressed in units of the domestic currency. All domestic firms are wholly owned by domestic households and equity holdings within these firms are evenly divided among them. Finally, $T_{i,t}$ is a lump-sum transfer, or tax if $T_{i,t} < 0$, that the household receives from the government.

The borrowing limit is such that expected debt repayment in terms of tradables cannot exceed the exogenous limit κ_t . Hence, the end-of-period bond position has to satisfy

$$B_{i,t+1} + E_t \left[\frac{B_{i,t+1}^n}{P_{i,t+1}^T} \right] \geq -\kappa_t. \quad (D.4)$$

Similar to the borrowing limit (3), this constraint captures a case in which the expected repayment that a household can credibly promise to lenders is equal to κ_t units of the tradable good.

In contrast with the baseline model of Section (2), here households do not choose directly how much labor to supply to the market. Instead, wages are negotiated by labor unions, to be described below, which act on behalf of the households. Once wages are set, households supply all the labor demanded by firms.

The household's optimization problem is to choose a sequence $\{C_{i,t}^T, C_{i,t}^N, B_{i,t+1}, B_{i,t+1}^n\}_{t \geq 0}$ to maximize the expected present discounted value of utility (D.1), subject to the consumption aggregator (D.2), the budget constraint (D.3) and the borrowing limit (D.4), taking the initial bond holdings $B_{i,0}$ and $B_{i,0}^n$, a sequence for income $\{\int_0^1 W_{i,t}(j)L_{i,t}(j)dj + \Pi_{i,t}^n\}_{t \geq 0}$, prices $\{R_t, R_{i,t}^n, P_{i,t}^T, P_{i,t}^N, W_{i,t}\}_{t \geq 0}$, and the path for the borrowing limit $\{\kappa_t\}_{t \geq 0}$ as given. The household's first-order conditions can be written as

$$P_{i,t}^N = P_{i,t}^T \frac{1 - \omega}{\omega} \frac{C_{i,t}^T}{C_{i,t}^N} \quad (D.5)$$

$$\frac{\lambda_{i,t}}{R_t} = \beta E_t [\lambda_{i,t+1}] + \mu_{i,t} \quad (D.6)$$

$$\frac{\lambda_{i,t}}{R_{i,t}^n} = \beta E_t \left[\lambda_{i,t+1} \frac{P_{i,t}^T}{P_{i,t+1}^T} \right] + \mu_{i,t} E_t \left[\frac{P_{i,t}^T}{P_{i,t+1}^T} \right] \quad (D.7)$$

$$B_{i,t+1} + E_t \left[\frac{B_{i,t+1}^n}{P_{i,t+1}^T} \right] \geq -\kappa_t, \quad \text{with equality if } \mu_{i,t} > 0, \quad (D.8)$$

where $\lambda_{i,t} \equiv \omega C_{i,t}^{1-\gamma} / C_{i,t}^T$ denotes the marginal utility from consumption of the tradable good, while $\mu_{i,t}$ is the non-negative Lagrange multiplier associated with the borrowing limit. As in the model

of Section 2, the optimality condition (D.5) equates the marginal rate of substitution of the two consumption goods, tradables and non-tradables, to their relative price, equation (D.6) is the Euler equation for real bonds, and equation (D.8) is the complementary slackness condition associated with the borrowing limit. In addition, equation (D.7) is the optimality condition for investment in nominal bonds.

Combining (D.6) and (D.7) gives a no arbitrage condition between real and nominal bonds

$$R_{i,t}^n = R_t \frac{\beta E_t [\lambda_{i,t+1}] + \mu_{i,t}}{\beta E_t \left[\lambda_{i,t+1} \frac{P_{i,t}^T}{P_{i,t+1}^T} \right] + \mu_{i,t} E_t \left[\frac{P_{i,t}^T}{P_{i,t+1}^T} \right]}.$$

Notice that in the absence of uncertainty this expression reduces to the familiar no arbitrage condition $R_{i,t}^n = R_t P_{i,t+1}^T / P_{i,t}^T$, equating the nominal interest rate to the real interest rate multiplied by expected inflation. Since real bonds are denominated in units of the tradable good, the relevant inflation rate is tradable price inflation.

Firms. In both sectors, identical firms rent labor from households and produce consumption goods under perfect competition. The two sectors are symmetric, so to streamline the exposition I will present the problem of a firm in a generic sector $x = \{T, N\}$. The production function is

$$Y_{i,t}^x = A_{i,t}^x (L_{i,t}^x)^{\alpha_x},$$

where $0 < \alpha_x < 1$ and A_i^x denote productivity in sector x . L_i^x is a CES aggregate of all the differentiated labor services supplied by households

$$L_{i,t}^x = \left(\int_0^1 (L_{i,t}^x(j))^{\frac{\epsilon-1}{\epsilon}} dj \right)^{\frac{\epsilon}{\epsilon-1}},$$

where $\epsilon > 1$, and $L_{i,t}^x(j)$ denotes the amount of labor service j purchased by firms in sector x .

Due to perfect labor mobility across sectors, every firm faces the same cost of labor. Hence, profits are equal to

$$\Pi_{i,t}^x = P_{i,t}^x Y_{i,t}^x - (1 - \vartheta) \int_0^1 W_{i,t}(j) L_{i,t}^x(j) dj,$$

where ϑ is an employment subsidy that every firm receives from the government. The minimum (pre-subsidy) cost of a unit of aggregate labor L_i^x is given by

$$W_{i,t} = \left(\int_0^1 W_{i,t}^{1-\epsilon}(j) dj \right)^{\frac{1}{1-\epsilon}},$$

which can be taken as the aggregate wage. Using this definition, profit maximization implies

$$\alpha_x P_{i,t}^x A_{i,t}^x (L_{i,t}^x)^{\alpha_x - 1} = (1 - \vartheta) W_{i,t},$$

while cost minimization implies the demand for type j labor

$$L_{i,t}^x(j) = \left(\frac{W_{i,t}}{W_{i,t}(j)} \right)^\epsilon L_{i,t}^x. \quad (\text{D.9})$$

The only purpose of the subsidy is to offset the impact of monopolistic distortions on the steady state of the economy. As it will become clear, this objective is reached if $\vartheta = 1/\epsilon$ and if the subsidy is financed with lump-sum taxes, so that $T_{i,t} = -\vartheta W_{i,t} (L_{i,t}^T + L_{i,t}^N)$. I will assume from now on that this is the case.

For future reference, notice that the total demand for labor of type j is equal to

$$L_{i,t}^T(j) + L_{i,t}^N(j) = \left(\frac{W_{i,t}}{W_{i,t}(j)} \right)^\epsilon (L_{i,t}^T + L_{i,t}^N). \quad (\text{D.10})$$

To complete the description of the labor market, it must be specified how wages are set. Before doing that, however, it is useful to state the market clearing conditions.

Market clearing. Goods market clearing implies that in every country i

$$C_{i,t}^N = Y_{i,t}^N \quad (\text{D.11})$$

$$C_{i,t}^T = Y_{i,t}^T + B_{i,t} - \frac{B_{i,t+1}}{R_t}$$

Moreover, since, given wages, households satisfy firms' labor demand, equilibrium on the labor market implies

$$L_{i,t}(j) = L_{i,t}^T(j) + L_{i,t}^N(j), \quad (\text{D.12})$$

for every labor type j .

Turning to the bonds market, I focus on equilibria in which nominal bonds are in zero net supply, so that $B_{i,t}^n = 0$ for all i and t .⁴⁸ Instead the market for real bonds clear at the world level, so that $\int_0^1 B_{i,t+1} di = 0$. By Walras' law, these two conditions imply that $\int_0^1 Y_{i,t}^T di = \int_0^1 C_{i,t}^T di$.

Labor unions and wage setting. For every labor type j there is a single labor union that sets the nominal wage $W_{i,t}(j)$. Every union acts on behalf of the households, and sets its nominal wage to maximize expected utility (D.1), subject to the budget constraint (D.3), and firms' demand for type j labor (D.10). Every period each union is free to reset its wage. Given this setting, the maximization problem of a generic union j can be written as

$$\max_{W_{i,t}(j)} E_t^j \left[\mathcal{L}_{i,t} W_{i,t}(j) L_{i,t}(j) - \frac{L_{i,t}(j)^{1+\psi}}{1+\psi} \right] \quad (\text{D.13})$$

⁴⁸This assumption is nearly without loss of generality. In fact, since trade in nominal bonds is allowed only across households sharing the same currency and since households are symmetric no trade in nominal bonds would occur in equilibrium under the flexible exchange rate regime. Trade in nominal bonds could occur inside a monetary union. However, since there is no uncertainty about the path of the price of the tradable good, it is easy to verify that in a monetary union real and nominal bonds are perfect substitutes. The only exception concerns the response of the economy to the unexpected shock to the borrowing limit in period 0. Since, this shock was previously unanticipated, it could engineer redistribution across countries and affect the equilibrium.

$$\text{s.t.} \quad L_{i,t}(j) = \left(\frac{W_{i,t}}{W_{i,t}(j)} \right)^\epsilon L_{i,t}, \quad (\text{D.14})$$

where $\mathcal{L}_{i,t} \equiv \omega C_{i,t}^{1-\gamma} / (P_{i,t}^T C_{i,t}^T)$ denotes the households' marginal utility from nominal wealth, and E_t^j denotes expectations with respect to the information set of labor union j , to be defined below. The solution is

$$W_{i,t}(j) = \frac{\epsilon}{\epsilon - 1} \frac{E_t^j [L_{i,t}(j)^\psi]}{E_t^j \left[\frac{\omega C_{i,t}^{1-\gamma}}{P_{i,t}^T C_{i,t}^T} \right]}. \quad (\text{D.15})$$

This expression implies that, under perfect information, every union would set a wage equal to

$$\frac{W_{i,t}(j)}{P_{i,t}^T} = \frac{\epsilon}{\epsilon - 1} \frac{L_{i,t}(j)^\psi}{\frac{\omega C_{i,t}^{1-\gamma}}{C_{i,t}^T}} = w_{i,t}, \quad (\text{D.16})$$

where the second equality is obtained from the fact that every union sets the same wage and that $w_{i,t} \equiv W_{i,t}/P_{i,t}^T$. This expression differs from the labor supply equation (2) only because of the presence of the wage markup $(\epsilon - 1)/\epsilon$. However, in equilibrium the markup is exactly offset by the employment subsidy. In fact, it is easy to show that under perfect information the real side of this model is exactly isomorphic to the model of Section 2. As a corollary, under perfect information monetary policy can only affect nominal variables, but not real ones.

I now introduce nominal wage rigidities by deviating from this perfect information benchmark. In particular, I allow for the possibility that unions might update their information set infrequently. As a result, unions may set current nominal wages based on outdated information, and so nominal wages might not respond immediately to unexpected shocks or to changes in monetary policy. This creates a channel through which monetary policy can influence the real economy.

To implement this idea, I adopt a variant of the Mankiw and Reis (2002) model of imperfect information. Start by noticing that in a generic period t , to operate under perfect information, it is sufficient for unions to observe the values of $B_{i,t}$, $A_{i,t}^T$, $A_{i,t}^N$, $\Psi_t(B, A^T, A^N)$, and the path of the borrowing limit $\{\kappa_t\}_{t \geq 0}$. Now let us split these variables in two groups. The first one is composed by the country-level, or idiosyncratic, variables $\xi_{i,t} = \{B_{i,t}, A_{i,t}^T, A_{i,t}^N\}$. The second group contains the world-level, or aggregate, variables $\Xi_t = \{\Psi_t(B, A^T, A^N), \{\kappa_t\}_{t \geq 0}\}$. I introduce a simple form of imperfect information by assuming that every period each union observes $\xi_{i,t}$ with probability one, while Ξ_t is observed with probability $\phi < 1$. In words, unions constantly update their information about idiosyncratic country-specific variables, while they update infrequently their information about the world-level variables. Hence, the information set of union j at time t can be written as $S_{i,t}^j \equiv (\xi_{i,t}, E_{\tau(j)}[\Xi_t])$, where $\tau(j) \leq t$ denotes the last period in which union j updated its information about the aggregate variables.

This setting captures an environment in which wage setters pay more attention to the idiosyncratic shocks that hit their country frequently, rather than to the rare shocks hitting the global economy. More broadly, this asymmetric information structure is meant to capture an environment in which there is enough wage flexibility to deal with normal business cycle fluctuations driven by

the productivity shocks. Instead, wages fail to adjust immediately to large and rare shocks, such as the one-time previously unexpected drop in the borrowing limit considered in our deleveraging experiment.⁴⁹

It turns out that, given that the only aggregate shock considered is a one-time fully unanticipated shock to the borrowing limit κ_t , the equilibrium behavior of wage setters takes a very simple form. In fact, both in the initial and final steady states, which are characterized by a constant κ_t , wage setters have perfect information about the state of the economy. Hence, in steady state the allocations correspond to the perfect-information benchmark discussed in Section 2. Instead, during the transition from the initial to the final steady state some wage setters are not immediately informed about the global deleveraging shock, and so they act on the basis of outdated information.

Specifically, during the transition in any period t wage setters can be divided in two groups. First, a fraction $1 - (1 - \phi)^t$ of wage setters have updated their information about the new path of the borrowing limit, and so they set their wage under full information. Instead, a fraction $(1 - \phi)^t$ of wage setters set their wages under an outdated information set, that is under the belief that the borrowing limit κ_t is still constant and equal to its value in the initial steady state. Effectively, these uninformed wage setters set their wage according to the pricing rule characterizing the initial steady state. Thus, the aggregate nominal wage evolves according to

$$W_{i,t} = \left((1 - (1 - \phi)^t) (W_{i,t}^{in})^{1-\epsilon} + (1 - \phi)^t (W_{i,t}^{un})^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}, \quad (\text{D.17})$$

where $W_{i,t}^{in}$ denotes the wage of informed unions, defined as

$$W_{i,t}^{in} = \frac{\epsilon}{\epsilon - 1} \frac{\left(L_{i,t}^{in} \right)^\psi}{\frac{\omega C_{i,t}^{1-\gamma}}{P_{i,t}^T C_{i,t}^T}}, \quad (\text{D.18})$$

where $L_{i,t}^{in}$ denotes labor effort from members of informed unions. Instead, $W_{i,t}^{un}$ denotes the wage of uninformed unions which set the wage according to

$$W_{i,t}^{un} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \left[\left(L_{i,t}^{un} \right)^\psi \mid S_t^{un} \right]}{E_t \left[\frac{\omega C_{i,t}^{1-\gamma}}{P_{i,t}^T C_{i,t}^T} \mid S_t^{un} \right]}, \quad (\text{D.19})$$

where $S_{i,t}^{un} \equiv (\xi_{i,t}, E_{-1}[\Xi_t])$ and $L_{i,t}^{un}$ denotes labor effort from members of uninformed unions.

Monetary policy. I consider two types of exchange rate regimes. First, I consider a world in which all the countries have their own currency and in which exchange rates are flexible. Second, I consider a world in which all the countries are part of a single monetary union. To make the

⁴⁹To be clear, this assumption is not made because wage rigidities are unimportant to explain normal business cycle fluctuations, but rather to isolate the interactions between wage rigidities and the transitional dynamics triggered by a global deleveraging shock.

two regimes comparable, I assume that in both cases the monetary authorities follow a policy of inflation targeting.⁵⁰

More formally, under flexible exchange rates each country i has its own currency, and consequently its own central bank. Under this regime, in every country i the central bank targets a value for consumer price inflation (CPI). The CPI, or price of the consumption basket, can be written as

$$P_{i,t} = \left(\frac{P_{i,t}^T}{\omega} \right)^\omega \left(\frac{P_{i,t}^N}{1-\omega} \right)^{1-\omega}. \quad (\text{D.20})$$

Defining CPI inflation as $\pi_{i,t} \equiv P_{i,t}/P_{i,t-1}$, the objective of the central bank is to set $\pi_{i,t} = \bar{\pi}$. This policy captures in a simple way the price stability objective typical of monetary authorities in advanced economies.

To see what this implies for the exchange rate, consider that by the law of one price the exchange rate between country i and country j can be written as

$$S_{i,j,t} = \frac{P_{j,t}^T}{P_{i,t}^T}. \quad (\text{D.21})$$

This expression implies that insofar as the price stability objective implies different tradable inflation rate across the two countries the exchange rate has to adjust.

The second regime considered is a monetary union. In this case all the countries share the same currency. In line with the inflation objective of the European Central Bank, I assume that the central bank of the currency union targets the average inflation across the member countries, that is

$$\pi_t = \int_0^1 \pi_{i,t} di. \quad (\text{D.22})$$

For comparability, I assume that the central bank of the union has the same inflation target as the central banks under flexible exchange rates $\pi_t = \bar{\pi}$. Notice that in a currency union, by the law of one price, all the countries must share the same price for the traded good.

Definition 2 *An equilibrium of the economy with nominal wage rigidities is a sequence of the world interest rate $\{R_t\}_{t \geq 0}$, a sequence of pricing functions $\{P_t^T(B, A^T, A^N), P_t^N(B, A^T, A^N)\}_{t \geq 0}$, a sequence of policy rules $\{C_t^T(B, A^T, A^N), C_t^N(B, A^T, A^N), W_t(B, A^T, A^N), L_t^T(B, A^T, A^N), L_t^N(B, A^T, A^N)\}_{t \geq 0}$, and a sequence of joint distributions for bond holdings and productivity $\{\Psi_t(B, A^T, A^N)\}_{t \geq 0}$, such that given the initial distribution $\Psi_0(B, A^T, A^N)$, a sequence of the borrowing limit $\{\kappa_t\}_{t \geq 0}$ and prices $P_{-1}^T(B, A^T, A^N)$ and $P_{-1}^N(B, A^T, A^N)$*

- $C_t^T(B, A^T, A^N), C_t^N(B, A^T, A^N), L_t^T(B, A^T, A^N), L_t^N(B, A^T, A^N)$ and $W_t(B, A^T, A^N)$ satisfy households', firms' and wage setters' optimality conditions.

⁵⁰One can think of monetary policy as being implemented through a policy rule in which the central bank sets $R_{i,t}^n$ as a function of the deviations of inflation from its target. To give the central bank control over $R_{i,t}^n$, it is common to assume that private agents have a demand for money which depends on the nominal interest rate. By changing the quantity of money in circulation the central bank can then set $R_{i,t}^n$. In the paper, I focus on the cashless limit of the economy, in which the quantity of money in circulation is approximately zero.

- *Markets for consumption clear in every country*

$$\frac{B_{t+1}(B, A^T, A^N)}{R_t} = A^T (L_t^T(B, A^T, A^N))^{\alpha_T} - C_t^T(B, A^T, A^N) + B$$

$$C_t^N(B, A^T, A^N) = A^N (L_t^N(B, A^T, A^N))^{\alpha_N}$$

- *Given the wage, households supply all the labor demanded by firms.*
- $\Psi_t(B, A^T, A^N)$ *is consistent with the decision rules.*
- *The market for bonds clears at the world level*

$$\int B d\Psi_t(B, A^T, A^N) = 0.$$

- $\{P_t^T(B, A^T, A^N), P_t^N(B, A^T, A^N)\}_{t \geq 0}$ *are such that $\pi_{i,t} = \bar{\pi}$ for every i and t if exchange rates are flexibles, or $\int_0^1 \pi_{i,t} di = \bar{\pi}$ in the case of the monetary union.*

D.1 The zero lower bound in a monetary union

With the zero lower bound the definition of the monetary union equilibrium must be modified as follows. Define \hat{R}_t^n as the gross interest rate on nominal bonds consistent with the central bank's inflation target. The presence of the zero lower bound implies $R_t^n \geq 1$. The central bank is assumed to follow the rule $R_t^n = \max(\hat{R}_t^n, 1)$. The definition of an equilibrium is then as in definition 2, with the exception that monetary policy is captured by the conditions

$$(R_t^n - 1) \left(\int_0^1 \pi_{i,t} di - \bar{\pi} \right) = 0, \quad R_t^n \geq 1, \quad \int_0^1 \pi_{i,t} di \leq \bar{\pi}.$$

E Data appendix

This appendix provides details on the construction of the series used in the calibration. The countries in the sample are Austria, Belgium, Finland, Germany, Greece, Ireland, Italy, Netherlands, Portugal and Spain. The euro area is defined as the aggregate of the sample countries.

1. **Share of tradable goods in consumption.** The consumption share of tradables is proxied by the share of tradable production in total value added. The tradable sector is defined as the aggregate of agriculture, mining and manufacturing. This procedure yields an average tradable share of 19 percent for the euro area during the period 1995-2013. The series used are yearly and come from Eurostat.
2. **Labor share in production.** The labor share is computed as workers' compensation as a fraction of total value added. To adjust for self-employed workers, following Gollin (2002), I use as a measure of workers' compensation the average employee compensation, obtained by dividing total employee compensation by the number of employees, multiplied by the total

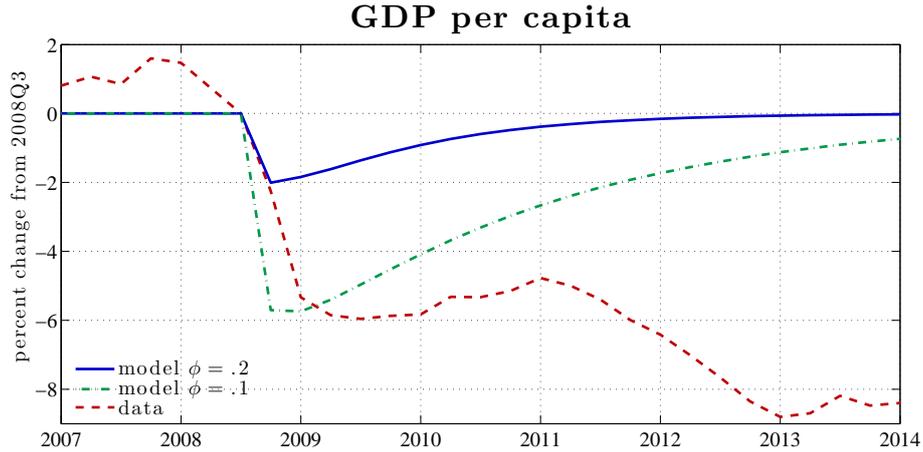


Figure 13: Comparison with euro area response to 2008 financial crisis. Notes: the dashed lines show detrended GDP per capita in the euro area in deviations from its value in 2008Q3.

number of employed workers. This adjustment is based on the assumption that on average self-employed workers earn the same return to labor as employees. Since this assumption is unlikely to hold for agriculture and mining, I proxy the tradable sector by manufacturing, while the rest of the economy less agriculture, mining and manufacturing captures the non-tradable sector. This procedure gives an average labor share for the euro area of 65 percent in both sectors for the period 1995-2013. The series used are yearly and come from Eurostat.

3. **Labor productivity.** I approximate (log) labor productivity as $\log(A_{i,t}) = \log(GDP_{i,t}) - 0.65 \log(L_{i,t})$, where $GDP_{i,t}$ denotes GDP at constant prices, while $L_{i,t}$ is total employment. For each country in the sample I obtained the cyclical component of labor productivity by subtracting a log-linear trend from the actual series. I then computed for each country the autocorrelation and standard deviations of de-trended labor productivity. The averages across the sample countries are 0.92 for the autocorrelation and 0.024 for the standard deviation. The series used are quarterly for the period 1999Q1-2014Q4, and come from the OECD.

F Comparison with euro area data

This appendix compares the behavior of the model with the euro area experience in the aftermath of the 2008 global financial crisis. I set the start of the crisis to 2008Q4. My objective is to estimate the cumulative output loss in the two years following the start of the crisis. To do so, I computed euro area GDP per capita using quarterly data from the OECD. I obtained the cyclical component by subtracting a log-linear trend calculated over the period 1999Q1-2015Q4. I then computed for every quarter between 2008Q4 and 2010Q3 the output loss as the log-deviation of detrended per capita GDP from its value in 2008Q3. Summing up gives a cumulative output loss equal to 41.7 percent of 2008Q3 GDP per capita.

Figure 13 provides a visual comparison between euro area detrended GDP per capita and the response of the model to the deleveraging shock. The figure plots both the response under the

benchmark parametrization ($\phi = 0.2$) and to an alternative parametrization with higher wage rigidities ($\phi = 0.1$). Qualitatively, the model captures fairly well the behavior of output during the first two years following the financial crisis. Quantitatively, the model under the benchmark parametrization underestimates the output drop. This can be explained with the fact that the euro area was hit by other shocks, such as the collapse in global demand due to the global financial crisis, over the same period. Another possibility is that the benchmark parametrization might underestimate the wage rigidities characterizing the euro area in the aftermath of the financial crisis. In fact, increasing the degree of wage rigidities to $\phi = 0.1$ brings the model significantly closer to the data. Moreover, the model predicts a faster recovery than in the data. Indeed, in the euro area the start of the financial crisis seems to coincide with a change in the trend growth rate. This change in trend growth can be attributed to the factors emphasized by the secular stagnation literature, such as a slowdown in population growth (Eggertsson and Mehrotra, 2014), or a fall in productivity growth (Benigno and Fornaro, 2015). Unsurprisingly, the model does not capture the double-dip recession that started in 2011Q1 after the arise of turmoil on the sovereign debt markets.

G Derivation of uncooperative constrained-efficient allocation

To prove that from the perspective of a single country the equilibrium of the baseline model attains the constrained efficient allocation I characterize the solution to the social planner problem for a single country. Importantly, the social planner in a single country takes the world interest rate as given, since a single country is too small to influence the world interest rate.

The social planner in a generic country i chooses a sequence $\{C_{i,t}^T, C_{i,t}^N, L_{i,t}, L_{i,t}^T, L_{i,t}^N, B_{i,t+1}\}_{t \geq 0}$ taking the path for the interest rate $\{R_t\}_{t=0}^\infty$ and the initial bond position $B_{i,0}$ as given, to maximize expected utility⁵¹

$$E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{\left((C_{i,t}^T)^\omega (C_{i,t}^N)^{1-\omega} \right)^{1-\gamma} - 1}{1-\gamma} - \frac{L_{i,t}^{1+\psi}}{1+\psi} \right) \right],$$

subject to the resource constraints

$$C_{i,t}^T = A_{i,t}^T (L_{i,t}^T)^{\alpha_T} + B_{i,t} - \frac{B_{i,t+1}}{R_t} \quad (\text{G.1})$$

$$C_{i,t}^N = A_{i,t}^N (L_{i,t}^N)^{\alpha_N} \quad (\text{G.2})$$

$$L_{i,t}^T + L_{i,t}^N = L_{i,t}, \quad (\text{G.3})$$

⁵¹For completeness, in the case of the model with differentiated labor services introduced in Section 4 the planner chooses for every $j \in \{0, 1\}$ the sequence $\{L_{i,t}(j), L_{i,t}^T(j), L_{i,t}^N(j)\}_{t \geq 0}$. However, due to the symmetric nature of the problem, it is optimal for the planner to set $L_{i,t}(j) = L_{i,t}$, $L_{i,t}^T(j) = L_{i,t}^T$, $L_{i,t}^N(j) = L_{i,t}^N$ for any $j \in \{0, 1\}$.

and the borrowing constraint

$$B_{i,t+1} \geq -\kappa_t. \quad (G.4)$$

The first order conditions are

$$\omega \frac{C_{i,t}^{1-\gamma}}{C_{i,t}^T} = \lambda_{i,t}^T$$

$$(1 - \omega) \frac{C_{i,t}^{1-\gamma}}{C_{i,t}^N} = \lambda_{i,t}^N$$

$$L_{i,t}^\psi = \lambda_{i,t}^L$$

$$\lambda_{i,t}^T \alpha_T A_{i,t}^T (L_{i,t}^T)^{\alpha_T - 1} = \lambda_{i,t}^L$$

$$\lambda_{i,t}^N \alpha_N A_{i,t}^N (L_{i,t}^N)^{\alpha_N - 1} = \lambda_{i,t}^L$$

$$\frac{\lambda_{i,t}^T}{R_t} = \beta E_t [\lambda_{i,t+1}^T] + \mu_{i,t}$$

$$B_{i,t+1} \geq -\kappa_t, \quad \text{with equality if } \mu_{i,t} > 0,$$

where $\lambda_{i,t}^T$, $\lambda_{i,t}^N$, $\lambda_{i,t}^L$ and $\mu_{i,t}$ are the Lagrange multipliers associated respectively with constraints (G.1), (G.2), (G.3) and (G.4).

Defining

$$w_{i,t} = \frac{\lambda_{i,t}^L}{\lambda_{i,t}^T}$$

$$p_{i,t} = \frac{\lambda_{i,t}^N}{\lambda_{i,t}^T},$$

it is easy to verify that the social planner allocation coincide with the equilibrium conditions of the baseline model presented in Section 2.