We present a dynamic model in which firms accumulate wealth to avoid bankruptcy and to overcome financing constraints that affect their fixed operational costs and the costs of becoming an exporter. Financing constraints not only affect firms directly when they are binding, but also indirectly, through precautionary saving and the selection of firms via entry and exit of the domestic and export markets. We calibrate the model and test some of its predictions using a rich dataset of Italian manufacturing firms for the period 1995–2003. Financing constraints reduce the aggregate productivity gains induced by trade liberalization by 25 percent by distorting the incentives of the most productive firms to self-select into exporting.

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1. Introduction

Trade barriers and financing frictions have been highlighted as two important sources of misallocation. The trade literature on heterogeneous firms following Melitz (2003) shows how trade barriers reduce the market scope of the most productive firms, lowering foreign competition and allowing low productivity firms to operate. Similarly, financing frictions can also result in the misallocation of capital and talent by affecting the entry of firms and distorting their relative levels of investment (Holtz-Eakin et al., 1994; Buera, 2009; Buera et al., 2011). The interaction between these two sources of misallocation can also be important, as firms may not be able to take advantage of a reduction in trade barriers when they are financially constrained. However, the existing literature that explores the decision to export in the presence of financing constraints is quite limited and generally restricted to static models. This paper fills this gap by developing a dynamic industry model with heterogeneous firms and financing constraints, where we introduce firm dynamics and costly bankruptcy. In standard models based on Melitz (2003) export status is entirely determined by firm productivity; a lowering...
of trade barriers increases the number of exporting firms and displaces less productive firms.\(^2\) We show that, in the presence of financing frictions and bankruptcy costs, these positive reallocation effects are dampened.\(^3\) Financing frictions affect the export decision, entry and exit in the home market and the riskiness of operating firms. These factors determine a joint endogenous distribution of firms across productivity, volatility and financial wealth, which implies a reduction in the aggregate productivity gains of trade liberalization relative to a model without financing frictions.

The structure of the model follows Melitz (2003). Firms are heterogeneous in their productivity and subject to idiosyncratic shocks in a monopolistic competition setting. They are required to pay fixed per-period costs and one-off costs to access the domestic and export markets. We introduce two additional elements with respect to the standard Melitz (2003) framework. The first is that firms have limited access to external funding and must pay their fixed production costs in advance. When firms cannot pay these costs, they are forced to go bankrupt and are liquidated. Bankruptcy is inefficient; hence firms accumulate internal funds as a form of precautionary saving. Financing constraints directly affect the decision to become an exporter when firms lack sufficient internal funds to pay the costs required to begin exporting. More importantly, financing frictions also indirectly affect the export decision when firms delay exporting as a form of precautionary saving to prevent a costly bankruptcy. The second departure from the standard model is that we introduce additional heterogeneity in fixed costs and the volatility of firm profits. Financing constraints make the idiosyncratic component of profit volatility important when determining the firm’s export status. Bankruptcy and profit uncertainty distort the distribution of firms engaging in exporting, making the gains induced by trade liberalization smaller than in the benchmark model. More specifically, financing frictions and bankruptcy risk affect the distribution of firms in three different ways. First, becoming an exporter adds volatility to a firm’s profits and, initially, reduces the firm’s financial wealth; hence the risk of a costly bankruptcy increases. This risk is particularly important for very productive firms for which the difference between continuing operations and liquidation is largest. As a result, they delay the decision to begin exporting until they have accumulated a large cushion of precautionary wealth. Second, financing constraints also alter the firms’ decisions to begin producing in the home market, inducing a positive correlation between their idiosyncratic risk and productivity. This endogenous correlation amplifies the precautionary effect of financing constraints, as very productive firms are also, on average, riskier. Finally, the above mentioned effects and the endogenous bankruptcy of firms change the pool of competitors, inducing further misallocation as relatively inefficient firms with low profit volatility enter the market.

We quantify these effects by calibrating an industry that matches a sample of Italian manufacturing firms. This dataset covers the period from 1995 to 2003 and contains detailed information on balance sheet data, the firms’ trade policies, and their self-declared financing constraints. We consider a benchmark model with both liquidity constraints and costly bankruptcy. We also report the properties of two alternative models, one with liquidity constraints affecting export decisions but without costly bankruptcy, and one without financing frictions altogether. All three models are calibrated to target the empirical sample along several dimensions, such as the size distribution of firms, the cross sectional and time series volatility of profits, the employment turnover, and the percentage of exporting firms. Next, we show that the benchmark model matches some key features of the data, that are not targeted in the calibration, much better than the two alternative models; namely, the correlation between productivity and volatility, and the heterogeneity of exporting firms with respect to size and productivity. The comparison of the different models shows that the presence of costly bankruptcy, which induces precautionary saving and distorts both selection into entry and into export, plays an important role for these results. Finally, we conduct a counterfactual experiment where we compute the industry equilibrium without trade and we quantify the reallocation gains from trade in the three calibrated models. We show that, relative to the model without financing constraints, the presence of liquidity constraints without costly bankruptcy reduces the productivity gains from trade by only 9%. Conversely, the presence of both liquidity constraints and costly bankruptcy reduces the productivity gains by 25%.

The model is related to an extensive literature on firm-level export decisions started by Dixit (1989), Baldwin and Krugman (1989), and Roberts and Tybout (1997). This literature has expanded substantially since Melitz (2003) embedded its standard assumptions into a general equilibrium model. A sustained assumption of this family of models is that firms are heterogeneous in their productivity levels and decide to become exporters by paying a fixed initial cost. Based on this framework, Chaney (2005) and Manova (forthcoming) introduce exogenous financing constraints as an additional source of heterogeneity across firms. The risk of going bankrupt plays an important role in our model, and a related argument can be found in Garcia-Vega and Guariglia (2007), which modifies the Melitz (2003) model to introduce idiosyncratic volatility and financing constraints. More recently, models with financing frictions and exports have been studied by Mayneris (2010) and Berman and Hericourt (2010).\(^4\) More generally, many different frictions, not only financial ones may limit the amount

\(^2\) Using Colombian plant data, Eslava et al. (2013) show that when trade barriers are lowered, more productive plants are more likely to survive and increase their productivity and market shares.

\(^3\) Suwantaradon (2012) develops a model in which firms accumulate wealth to overcome financing constraints. However in her model, there is no bankruptcy and only binding financing constraints matter. Our paper is also related to the work of Hopenhayn (1992a, 1992b), which focuses on how endogenous entry and exit generate a distribution of firms that influences individual firm decisions.

\(^4\) A number of recent papers have empirically examined the impact of financing constraints on exports: Manova (2008, forthcoming), Muûls and Pisu (2009), Mayneris (2010), Berman and Hericourt (2010), Bellone et al. (2010) and Greenaway et al. (2007), among others. Minetti and Zhu (2011) use a subset of the database of Italian firms we employ in this paper to analyze the effect of financing frictions on the intensive and extensive export margins. In addition to employing a larger dataset, our empirical analysis differs from that of Minetti and Zhu (2011) in that we test the specific predictions of our dynamic model regarding the direct and indirect implications of financing frictions for the distribution of wealth and the productivity–export relationship.
of reallocation induced by trade liberalizations. Kambourov (2009) presents a model in which labor market frictions limit reallocation, and Armenter and Koren (2009) introduce generic latent frictions to match the observed size differentials between exporting and non-exporting firms. Finally, Caggese and Cuñat (2008) show how hiring and firing costs interact with financing constraints and distort the firms’ employment policies.

One important novel element of our paper is the introduction of firm dynamics. All of the above papers study industries in which firms are not allowed to retain earnings or change their capital structure in response to financing frictions. Conversely, our paper studies a fully dynamic industry in which the joint distribution of firms across productivity, volatility and wealth arises endogenously. We show that this endogenous distribution is very important to understanding both firm-level export decisions and their consequences for aggregate dynamics after a trade liberalization. Finally, the paper is also related to the investment literature, which has shown that firms’ investment decisions, and especially the timing of large fixed investments, are affected by the presence of borrowing constraints (see, for example, Whited, 2006), and the firm-dynamics literature, which has recently emphasized the contribution of inter-firm reallocation to industry productivity and growth (see for example Restuccia and Rogerson, 2008, and Hsieh and Klenow, 2009).

The remainder of the paper is organized as follows: Section 2 presents the setting of the theoretical model; Section 3 calibrates the full model and alternative models in which some of the restrictions on financing constraints and bankruptcy risk are relaxed, and it compares these models with the empirical data; Section 4 measures the impact of financial frictions on the reallocation gains from trade liberalizations; finally, Section 5 concludes.

2. The model

We follow Melitz (2003) and study an industry in which heterogeneous firms are allowed to produce at home and export to a foreign market. We consider a monopolistic competition model in which each firm in the industry produces a variety of a consumption good. There is a continuum of varieties \( w \in \Omega \). Consumers’ preferences over the varieties in the industry exhibit constant elasticity of substitution (C.E.S.) with elasticity \( \sigma > 1 \). Since there are no aggregate shocks, the C.E.S. price index \( P \) is constant in equilibrium and equal to:

\[
P = \left( \int_w p(w)^{1-\sigma} \right)^{\frac{1}{1-\sigma}}.
\]

And the associated quantity of the aggregated differentiated good \( Q \) is:

\[
Q = \left( \int_w q(w)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}},
\]

where \( p(w) \) and \( q(w) \) are the price and quantity consumed of the individual varieties \( w \), respectively. The overall demand for the differentiated good \( Q \) is generated by:

\[
Q = A p^{1-\eta},
\]

where \( A \) is an exogenous demand parameter and \( \eta < \sigma \) is the industry price elasticity of demand. From (2) and (3), the demand for an individual variety \( w \) is:

\[
q(w) = A \frac{p^{\sigma-\eta}}{p(w)^{\sigma}}.
\]

Each variety is produced by a firm using labor, and the wage rate is normalized to one. The productivity parameter \( \nu \) determines the marginal cost of production, which is equal to \( 1/\nu \). The profits from the domestic activity of a firm with productivity \( \nu \), and variety \( w \) are given by:

\[
\pi^D = p(w)q(w) - \frac{q(w)}{\nu} - F,
\]

\[
F = \mathcal{F} + \varepsilon^D,
\]

where \( \mathcal{F} \) are the overhead costs of production that have to be paid every period and \( \varepsilon^D \) is a profit shock. The shock \( \varepsilon^D \) introduces uncertainty in profits, and it plays an important role in the presence of financing frictions. Note that \( \varepsilon^D \) does not affect the firm’s decision on the optimal price \( p \) and quantity produced \( q \). The firm is risk-neutral and chooses \( p \) to maximize \( \pi^D \). The first-order condition yields the standard pricing function as in Dixit and Stiglitz (1977):

\[
p = \frac{\sigma}{\sigma-1} \frac{1}{\nu}.
\]

\(^5\) A multiplicative shock of the type \( \varepsilon^D p(w)q(w) \) would not change the qualitative results of the model, but it would have two main consequences. First, it would imply that the optimal quantity produced \( q(w) \) would be a function of the intensity of financing frictions, thus making the solution of the problem both more difficult to compute and less comparable to the Melitz (2003) benchmark model. Second, it would imply that expected profits are a function of the volatility of \( \varepsilon \).
Firms can export. To do so, they have to pay an initial one-off sunk cost $S^X$, a per-unit variable trade cost $\tau$, and a per-period overhead cost to export $F^X$ which is proportional to $F$. We define

$$F^X = F^X + \varepsilon^X,$$

where the profit shocks $\varepsilon^X$ is correlated to $\varepsilon^D$, because they are both linear functions of a common shock $\varepsilon$ that follows a two state symmetric persistent stochastic process in which $\varepsilon \in \{-\theta, \theta\}$, with the probability of remaining in the same state equal to $\rho_{\varepsilon}$ and the probability of changing state equal to $1 - \rho_{\varepsilon}$. The parameter $\theta$ is a positive constant. Firms are heterogeneous, and their individual permanent characteristics can be summarized by the variables included in $\mu = (v, F, \theta)$.

Assuming a symmetric two-country world, profits from domestic and from export activity can be easily shown to be as follows:

$$\pi^D(\varepsilon, \mu) = \frac{(\sigma - 1)^{\sigma - 1}}{\sigma} \kappa^\sigma v^{\sigma - 1} - F,$$

$$\pi^X(\varepsilon, \mu) = \frac{(\sigma - 1)^{\sigma - 1}}{\sigma} \kappa^\sigma \left( \frac{v}{\tau} \right)^{\sigma - 1} - F^X.$$

### 2.1. Wealth and financial frictions

In this subsection, we define the evolution of the firm’s financial wealth and its financing frictions. At the beginning of period $t$, the firm observes the realization of the shock $\varepsilon_t$, and generates profits. Its financial wealth $a_t$ is determined as follows:

$$a_t = R[a_{t-1} - S^X I^X_{t-1}(a_{t-1}, \varepsilon_{t-1}, \mu)] + \pi^D(\varepsilon_t, \mu) + X_t\pi^V(\varepsilon_t, \mu).$$

The decision variable $I^X_{t-1}(a_{t-1}, \varepsilon_{t-1}, \mu)$ takes value one if the firm decided to become an exporter in period $t - 1$ and zero otherwise. Beginning to export in period $t$ requires the firm to pay a fixed cost $S^X$ in $t - 1$. The variable $X_t$ is equal to one if the firm is an exporter in period $t$, and zero otherwise, and is determined as follows:

$$X_t = (1 - X_{t-1}) I^X_{t-1}(a_{t-1}, \varepsilon_{t-1}, \mu) + X_{t-1} C^X_{t-1}(a_{t-1}, \varepsilon_{t-1}, \mu),$$

where $C^X_{t-1}(a_{t-1}, \varepsilon_{t-1}, \mu)$ takes value one if a firm already exporting in period $t - 1$ decides to continue to export in period $t$, and zero otherwise. A firm that decides to stop exporting has to pay $S^X$ again if it decides to restart exporting in the future. Eq. (11) implies that the firm pays no dividends and all revenues are reinvested: profits are distributed only once the firm stops its activity and liquidates its assets. Since we assume that the firm is risk-neutral, and that it discounts future profits at the real interest rate $R$, the decision not to distribute dividends is optimal when the firm faces financing frictions. When the firm accumulates enough wealth to become financially unconstrained, it also becomes indifferent about distributing or retaining earnings. Therefore, this assumption does not limit the analysis in any important way.

After realizing earnings and determining $a_t$, with probability $\delta < 1$, the technology of the firm becomes useless. In this case, the firm ends its activities and distributes its assets $a_t$ as dividends. With probability $1 - \delta$, the firm continues to operate. At this point the firm decides whether to start exporting, if it currently only produces in the home market, or it decides whether it will continue to export, if it is already doing so.

Financing frictions are introduced by assuming that the firm cannot borrow. While it can pay wages and the profit shocks with the stream of revenues generated by its sales, it has to pay the fixed production costs in advance. Therefore $F$ in period $t + 1$, plus $F^X$ if the firm is an exporter, have to be paid with the firm’s internal funds $R(a_t - S^X I^D_{t})$ at the beginning of the period, before $t + 1$ revenues are generated. As it will be shown in the next section, low asset levels constrain the firm to operate only domestically, and very low levels force the firm to go bankrupt and exit permanently from production.

### 2.2. Firms’ decisions

We define $V^D(a_t, \varepsilon_t, \mu)$ as the value function of a firm that operates in the home market alone, and $V^X(a_t, \varepsilon_t, \mu)$ as the value function of a firm that operates both in the home and export markets. Both functions are evaluated at time $t$ after time $t$ profits are realized, and before choosing whether to export in period $t + 1$. The superscript $D$ indicates that the firm is currently only producing for the home market, and superscript $X$ indicates that the firm is currently an exporter. Given these definitions and the law of motion of assets (11), $I^X_t(a_t, \varepsilon_t, \mu)$ is equal to one (i.e., the firm begins exporting in period $t + 1$), when the following conditions are satisfied:

$$X_t = 0,$$

$$Et \left[ \pi^D_{t+1}(\varepsilon_{t+1}, \mu) + \pi^X_{t+1}(\varepsilon_{t+1}, \mu) \mid \varepsilon_t \right] > Et \left[ + V^D(R a_t - S^X I^D_{t}) + \pi^D_{t+1}(\varepsilon_{t+1}, \mu) | \varepsilon_t \right].$$

$$R(a_t - S^X) \geq F + F^X$$

$$Et \left[ \pi^D_{t+1}(\varepsilon_{t+1}, \mu) | \varepsilon_t \right] > Et \left[ + V^D(R a_t + \pi^D_{t+1}(\varepsilon_{t+1}, \mu), \varepsilon_{t+1}, \mu) | \varepsilon_t \right].$$
and is equal to zero otherwise. The left hand side of (14) is the expected value, conditional on $e_t$, of choosing to start exporting. It includes expected time $t + 1$ profits plus the expected value of the firm $V^X(a_{t+1}, \varepsilon_{t+1}, \mu)$, where $a_{t+1}$ is

It includes expected time $t + 1$ profits plus the expected value of the firm $V^X(a_{t+1}, \varepsilon_{t+1}, \mu)$, where $a_{t+1}$ is reduced by the export cost $S^X$ paid in period $t$. Condition (14) is satisfied if the left hand side is greater than the expected value of continuing only in the domestic market on the right hand side. This condition is written under the assumption that, once a firm decides in period $t$ to export in the next period, it cannot reverse this decision, and produce only domestically in period $t + 1$, after observing the realization of $\varepsilon_{t+1}$. Condition (15) requires that the firm has enough wealth to fund the fixed costs for both the home production and exports, as well as the sunk cost $S^X$. So it is feasible to start exporting.

Similarly, we also define the decision of continuing to export $C^X(a_t, \varepsilon_t, \mu)$, which is equal to 1 if the following conditions are satisfied:

$$X_t = 1,$$

$$E_t \left[ \pi^D_{t+1}(\varepsilon_{t+1}, \mu) + \pi^X_{t+1}(\varepsilon_{t+1}, \mu) + V^X(Ra_t) \bigg| e_t \right] > E_t \left[ \pi^D_{t+1}(\varepsilon_{t+1}, \mu) + V^D(Ra_t + \pi^D_{t+1}(\varepsilon_{t+1}, \mu), e_{t+1}, \mu) \bigg| e_t \right],$$

$$Ra_t \geq F + F^X,$$

and is zero otherwise. Given these conditions, we now determine the value functions $V^D(a_t, \varepsilon_t, \mu)$ and $V^X(a_t, \varepsilon_t, \mu)$ as the net present value of future expected dividends. It is important to note that they are derived under the assumption that firms always seek to continue their activities and only exit either because of default, or because their technology becomes useless. Voluntary liquidation would be optimal when the value of $V^D(a_t, \varepsilon_t, \mu)$ or $V^X(a_t, \varepsilon_t, \mu)$ is negative – for example, conditional on a negative realization of $\varepsilon$ when its persistence $\rho_\varepsilon$ is very high. However, this outcome never occurs under the calibrated parameters, therefore, to simplify the analysis and notation, we choose not to consider it in the derivation of the model. The value of a home-producing firm is:

$$V^D(a_t, \varepsilon_t, \mu) = 1(Ra_t \geq F) \frac{1}{R} \left[ \delta E_t \left[ R^D(a_t - S^X) + \pi^D(\varepsilon_{t+1}, \mu) + \pi^X(\varepsilon_{t+1}, \mu) \right] + (1 - \delta) E_t \left[ V^X(a_{t+1}, \varepsilon_{t+1}, \mu) \right] \right] + 1(Ra_t < F)a_t.$$

$1(Ra_t \geq F)$ is an indicator function that is equal to one if the argument is true and equal to zero otherwise. A value of zero implies that the firm cannot continue its activities in the next period because its internal funds are too low to finance the fixed costs of home production. In this case, the firm is forced to liquidate immediately and loses the net present value of its future profits. Symmetrically, the value of an exporting firm is:

$$V^X(a_t, \varepsilon_t, \mu) = 1(Ra_t \geq F) \frac{1}{R} \left[ \delta E_t \left[ Ra_t + \pi^D(\varepsilon_{t+1}, \mu) + \pi^X(\varepsilon_{t+1}, \mu) \right] + (1 - \delta) E_t \left[ V^X(a_{t+1}, \varepsilon_{t+1}, \mu) \right] \right] + 1(Ra_t < F)a_t.$$

The previous expressions clarify the effect of financing frictions. Low financial wealth violates conditions (15) and (18) and it constrains the firm to only operate in the domestic market. If wealth decreases further below the minimum level $F$, the firm is forced to exit and to distribute its remaining assets as a dividends. These assumptions generate realistic firm dynamics, and can be interpreted as a shortcut for more-realistic models of firm dynamics with financing frictions. Limitation of these assumptions is that they do not allow financing frictions to affect the firms’ investments in variable inputs. This limitation keeps the model considerably easier to solve and more comparable to the standard model of Melitz (2003).

### 2.3. Entry decision

There is free entry in every period. New entrants have to pay an initial cost $S^C$ to “establish the firm”. This can be interpreted as a research and development cost to determine the characteristics of the product the firm will produce and sell. After paying this cost, firm $i$ observes its type $\mu_i = [\nu_i, F_i, \theta]$. At this point, the firm decides whether to pay a one-off fixed cost $S^I$ to start the actual production process. Furthermore, we assume, for simplicity, that the firm always starts

---

6 In other words, if conditional on $s_{t+1}$ exporting becomes unprofitable, the firm will decide to stop exporting at the end of period $t + 1$, and it will produce only domestically in period $t + 2$.  
7 For instance, Clementi and Hopenhayn (2006) derive the optimal long-term bank-firm contract under asymmetric information. They show that under the optimal contract, firms are initially financially constrained. If successful, they gradually grow and become unconstrained. However, if unsuccessful they may go bankrupt even though their projects are profitable. Moreover, they also show that even with an i.i.d. shock to firm revenues, the model generates persistence and path dependence in firm dynamics. We will show in Section 3 that our model generates firm dynamics similar to those in Clementi and Hopenhayn (2006).
production in the home country only. The firm will decide to start production if the net present value of the business
\( \Psi_i(a_0, \mu_i) \) is profitable:

\[
E[V^D(a_0, \varepsilon_0, \mu_i)] - S^B_i \equiv \Psi_i(a_0, \mu_i) > 0,
\]

(21)

where \( a_0 \), the initial wealth of the firm, is a function of \( v_i \), and the operator \( E \) refers to the expectation concerning
the initial shock \( \varepsilon_0 \), which has equal probability to be either \( \theta \) or \( -\theta \). The free-entry condition requires that, ex-ante, the
expected value of paying \( S^C \) and learning \( \mu_i \) is zero:

\[
E[\Psi_i(a_0, \mu_i) | (\Psi_i(a_0, \mu_i) > 0)] - S^C = 0,
\]

(22)

where the operator \( E \) refers to the expectation concerning \( \mu_i \). In equilibrium these equations determine the number and
type of firms that enter the economy. This two-tiered structure allows us to study how expected financial constraints, which
depend on the expected volatility of profits, influence the firms’ decisions to pay \( S^B \) and begin production. In equilibrium,
it induces some positive correlation between firm-level productivity and risk. This correlation is not essential for the main
results of the model, but it provides an amplification mechanism, as more-productive firms are also those with higher
bankruptcy risk. Importantly, we verify that this correlation is present in our empirical dataset of Italian manufacturing
firms, and, consistent with the predictions of the model, is amplified by the presence of financing constraints.

2.4. Industry equilibrium

The steady state equilibrium is characterized by an aggregate price \( P \), an aggregate quantity \( Q \), and time invariant
distributions of operating firms and of new entrants over the values of \( v, F, a \) and \( \theta \) such that: Firm value is defined as the
net present value of dividends and is determined by Eqs. (19) and (20). Existing firms that only produce domestically decide
to start exporting according to Eqs. (14) and (15). Existing firms that are already exporting decide to continue exporting
according to Eqs. (17) and (18). New entrants satisfy conditions (21) and (22).

The mass and distribution of firms over \( v \) determines the mass and distribution of prices. Aggregating these prices into
the C.I.S. price index must yield the equilibrium aggregate price \( P \). The presence of the exogenous exit probability \( \delta \) ensures
that the distribution of wealth across firms is non-degenerate.

3. Calibration

The solution of the model is obtained using a numerical method (see Appendix A for details), and the time period is one
year. We calibrate the parameters of the model to closely match the firm dynamics in our sample of Italian manufacturing
firms.

3.1. Empirical dataset

The moments used to calibrate the model and the reminder of the empirical analysis come from the Mediocredito Centrale
surveys. The dataset is an unbalanced panel with annual balance-sheet data and profit and loss statements from 1995
to 2003, as well as qualitative information from three surveys conducted in 1997, 2000, and 2003. Each survey reports infor-
information about the activity of the firms in the three previous years and, in particular, it includes detailed information on
exports and financing constraints. The dataset is intended to be a comprehensive sample of firms of all sizes and sectors.
However, large firms are overrepresented and small firms are underrepresented relative to the Italian population of firms.
Therefore, when calculating all moments and regressions we weight each firm according to its size measured by number
of employees to ensure that the weighted sample is representative of the population. We construct population weights to
match the Italian National Statistical Institute (ISTAT) census database in terms of the distribution of employees per firm.
ISTAT contains the census of Italian firms. Because Mediocredito Centrale contains very few firms with fewer than ten em-
ployees, we censor both databases below ten employees. The calibration should, therefore, be interpreted as representative
of the population of Italian firms with more than ten employees. The median number of employees in the sample is 34
with an average of 111 employees. On average, firms sell goods worth 10.7 million euros per year and have a total average
book value of assets of 27 million euros. The fraction of firms exporting at least 5% of their output is 57 percent. Overall
the database contains 6776 firms and 33,399 firm–year observations. See Appendix B for details.

Both in the calibration and to test the predictions of the model, we use a proxy for self-declared financing constraints.
Firms report whether they had a loan application turned down recently, whether they desire more credit at the market
interest rate and whether they would be willing to pay a higher interest rate than the market rate to obtain credit. We
aggregate these three variables into a single variable \textit{constrained} that takes value one if the firm answers yes to any of these
three questions. According to this measure, 17 percent of the firms declare themselves to be financially constrained.8

---

8 Most of the variation of the financing constraints status is cross sectional. After averaging the constrained variable at a firm level, the standard deviation
across firms is 0.29. If, instead, we calculate the within firm standard deviation it has an average across firms of 0.07.
Fifth, we model \( \epsilon^D \) and \( \epsilon^X \) as proportional to \( \epsilon \) and to the firm’s fixed cost: \( \epsilon^D = F \epsilon \) and \( \epsilon^X = F^X \epsilon \). The parameter \( \theta \) that determines the volatility of \( \epsilon \) is drawn for new firms from a uniform distribution with support \([0, \theta]\).

Table 1 illustrates the choice of parameter values. All parameters are chosen simultaneously to match all empirical moments, even though each parameter is especially relevant for a subset of such moments. The parameters \( \nu \) and \( \lambda \) determine the probability distribution of the marginal productivity of new firms. Eqs. (4) and (7) imply a direct relationship between size and marginal productivity; therefore, these two coefficients are set to match two moments of the size distribution of

---

**Table 1**

Benchmark calibration.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Moment to match</th>
<th>Empirical moment</th>
<th>Simulated moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \lambda )</td>
<td>Aggregate sales</td>
<td>2510</td>
<td>10,700</td>
</tr>
<tr>
<td>( \delta )</td>
<td>Share of exiting firms</td>
<td>0.035</td>
<td>8.2%</td>
</tr>
<tr>
<td>( r )</td>
<td>Average real interest rate</td>
<td>0.02</td>
<td>2%</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Median ratio fixed costs/labor costs</td>
<td>0.05</td>
<td>0.29</td>
</tr>
<tr>
<td>( \theta )</td>
<td>Cross sectional volatility</td>
<td>0.075</td>
<td>0.060</td>
</tr>
<tr>
<td>( \sigma_i^2 )</td>
<td>Time series volatility of ( v_i )</td>
<td>0.441</td>
<td>0.034</td>
</tr>
<tr>
<td>( \rho )</td>
<td>Autocorrelation of ( \epsilon_i )</td>
<td>0.7</td>
<td>0.21</td>
</tr>
<tr>
<td>( \nu )</td>
<td>Mean size divided by median size</td>
<td>0.5</td>
<td>1.54</td>
</tr>
<tr>
<td>( \lambda )</td>
<td>Size of 75% percentile/median size</td>
<td>4</td>
<td>2.10</td>
</tr>
<tr>
<td>( \hat{S}_i )</td>
<td>Weighted average of R&amp;D/fixed costs</td>
<td>0.01</td>
<td>0.13</td>
</tr>
<tr>
<td>( s )</td>
<td>Median ( \epsilon_i ) of 50% smaller firms</td>
<td>0.885</td>
<td>0.023</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>Median ( \epsilon_i ) of 50% larger firms</td>
<td>2.0075</td>
<td>0.025</td>
</tr>
<tr>
<td>( \tau )</td>
<td>Ratio of exports to domestic sales</td>
<td>1.25</td>
<td>51%</td>
</tr>
<tr>
<td>( \mu )</td>
<td>Exit rate from exporting</td>
<td>0.52</td>
<td>4.9%</td>
</tr>
<tr>
<td>( S^X )</td>
<td>Fraction of exporters</td>
<td>0.02</td>
<td>57%</td>
</tr>
<tr>
<td>( a )</td>
<td>Fraction of financially constrained firms</td>
<td>1.3</td>
<td>17%</td>
</tr>
</tbody>
</table>

Other parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Moment to match</th>
<th>Empirical moment</th>
<th>Simulated moments</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \eta )</td>
<td>From Costantini and Melitz (2007)</td>
<td>1.5</td>
<td>From Costantini and Melitz (2007)</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>From Costantini and Melitz (2007)</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

---

\( a \) Exporters defined as exports \( > 5\% \) sales.

\( b \) Firms that declare financing problems.

\( c \) Firms with not enough financial wealth to start exporting \((w_i < S^X + F + F^X)\).

3.2. Benchmark calibration

Our strategy is to calibrate the model to ensure that it not only matches the size distribution of firms in the empirical sample but also the dynamics of firms’ financial assets conditional on the size distribution. We place a special emphasis on financial wealth because it determines how current and future expected financing constraints affect firms’ decisions. To fit the model with the empirical data, we make the following assumptions. First, we assume that the constant value of the productivity of a new firm \( v_i \), is equal to a common fixed component \( v \) plus a firm-specific component \( v_i^{\text{exp}} \) drawn from an exponential distribution with mean \( \frac{1}{\lambda} \) and variance \( \frac{1}{\lambda^2} \).

\[
v_i = v + v_i^{\text{exp}}.
\]

Second, we assume that the fixed component of the per period cost of a new firm \( \hat{F}_i \), is drawn from a uniform distribution with mean \( \hat{F}_i = F v_i^{\text{exp}} \) and support \([\hat{F}_i - \theta, \hat{F}_i + \theta]\). A value of \( \gamma \) equal to zero would make the parameters \( \nu \) and \( \lambda \) orthogonal. However, in this case, we would generate implausibly large profits of large firms relative to small firms. Therefore, in the calibration, a positive value of \( \gamma \) is necessary to match the profitability of firms conditional to size, and a positive value of \( \theta \) is necessary to match the cross-sectional distribution of profitability across firms.

Third, we assume that initial wealth \( a_{0i} \) and the sunk cost to begin production \( S^B_i \) are proportional to \( \hat{F}_i \):

\[
a_{0i} = \alpha_{0i} \hat{F}_i \quad \text{and} \quad S^B_i = \beta_{0i} \hat{F}_i.
\]

The assumptions regarding \( a_{0i} \) and \( S^B_i \) are necessary to generate realistic distributions of financing frictions and of volatility among both small and large firms.

Fourth, we assume that the per-period cost of exporting \( F^X_i \) is a constant fraction of \( \hat{F}_i \):

\[
F^X_i = \kappa \hat{F}_i.
\]

Fifth, we model \( \epsilon^D \) as proportional to \( \epsilon \) and to the firm’s fixed cost: \( \epsilon^D = F \epsilon \) and \( \epsilon^X = F^X \epsilon \). The parameter \( \theta \) that determines the volatility of \( \epsilon \) is drawn for new firms from a uniform distribution with support \([0, \theta]\).
firms: the mean size and the size of the 75th percentile, both relative to the median size. This feature is important because size is, empirically, one of the main determinants of the export decision. Conditional on $\gamma$ and $\lambda$, a second set of parameters determines the profitability of firms; the parameter $s$ matches the average profitability in the industry, and the parameter $y$ matches how profitability varies with size.

The parameters $\theta$, $\theta$ and $\rho$, determine the variability of profits both across firms and for each firm over time, as well as their persistence. As mentioned above, these parameters are of fundamental importance in the presence of financing frictions. The level and the volatility of profits shape the distribution of financial assets across firms, and determine current and future expected financing constraints and the probability of default.

The next set of parameters, $F$, $\delta$, $\mu$, $S^X$ and $\tau$, match entry and exit, in both the home and foreign markets, and average investment rates. The parameter $F$ matches the ratio of fixed overhead costs over labor cost. We follow Costantini and Melitz (2007) and proxy the fixed overhead costs using the aggregate wages of white collar workers. In our dataset we have the information about the number of white and blue collar workers, but only total wages. Manasse et al. (2004) study a sample of Italian manufacturing firms and report an average wage premium of 20% in 1997 for skilled vs. non-skilled workers. Given that we have the same disaggregation of worker types that Manasse et al. do, we can use this wage premium to calculate an estimate of the wage of white collar workers in our sample. The result is a median ratio of wages of white collar over total wages that ranges from 30% (data from the 1995 survey) to 31% (data from the 2003 survey).

The parameter $\delta$, the probability of exogenous exit, matches the share of employment held by entrants and exiting firms for Italian manufacturing firms. As we do not have entry and exit in our empirical sample, we obtain this information from Bartelsman et al. (2005), who analyze a similar sample of Italian manufacturing firms. The parameters $\mu$, $S^X$ and $\tau$ jointly match the fraction of exporting firms, the fraction of exports over total output, and the weighted average of firms that stop exporting.

Among the other parameters, the scale parameter $A$ matches total sales; the initial endowment parameter $a$ matches the fraction of financially-constrained firms; and the average real interest rate is set equal to two percent, which is consistent with the average short-term real interest rates in Italy in the sample period. Finally, the parameter $S^C$, the initial cost to observe the firm’s type $\mu_i$, is calibrated by assuming that it is a “research and development” (R&D) cost to determine the characteristics of the new product to be produced. We proxy this cost with the R&D information present in the surveys of our sample, where firms not only report how much they spent on R&D but also how much of this expenditures is directed to the development and introduction of new products. Therefore, we calibrate $S^C$ so that the weighted average of the ratio of R&D expenditure to introduce new products divided by fixed overhead costs. This empirical moment is likely to be a noisy proxy of the effective cost of R&D for a new firm, and we also provide some sensitivity analysis of the main results for values of $S^C$ equal to 50% and 200% of the calibrated value.

For the two remaining elasticity parameters we follow Costantini and Melitz (2007) and we set $\sigma$ equal to 4 and $\eta$ equal to 1.5.\footnote{The value of $\sigma$, the elasticity of substitution between varieties, is in line with Bernard et al. (2003), who calculate a value of 3.79 using plant level data. The difference between this value and the value of $\eta$, the industry price elasticity of demand, is consistent with Broda and Weinstein (2006), who estimate that the elasticity of substitution falls between 33% to 67% moving from the highest to the lowest level of disaggregation in industry data.}

Table 1 shows that the parameter values allow the model to match the chosen moments reasonably well. Importantly, the moments in the calibrated model are very similar to the empirical ones concerning the level, volatility and cross-sectional dispersion of profits, the size distribution, and the entry and exit dynamics. The only moment that proved difficult to match accurately is the fraction of firms ceasing to export. Our simulated industries generate too few firms ceasing to export, as is the case in several other papers in the literature.\footnote{Ruhl and Willis (2008) show that the fraction of firms exiting the export market and becoming domestic firms is very difficult to match in the standard models of heterogeneous firms. They propose a model in which export demand grows over time to attenuate this problem. An alternative approach, based on experimentation with exports can be found in Albornoz et al. (2012).}

3.3. Main channels and alternative calibrations

We use the calibrated model to analyze value functions, policy functions, and the distribution of firms. In the model, financial factors affect firms by limiting their ability to invest, and determining entry and the industry equilibrium. Because of these multiple interacting factors, it is useful to define four distinct effects:

(i) First, there is a direct effect. If the liquidity constraint (15) is not satisfied, a firm is unable to begin exporting because of insufficient internal funds.

(ii) Second, there is a “precautionary saving effect”. Even if a firm has enough internal funds to begin exporting, it may wait to do so for precautionary reasons, fearing future financing constraints and bankruptcy risk in the event of future negative profits shocks.

(iii) Third, there is a “bankruptcy selection effect”. Financing frictions generate a positive correlation between productivity and volatility among operating firms. After paying the initial R&D cost $S^C$, firms learn their productivity and risk levels. The higher the expected volatility of profits, the higher the probability of going bankrupt during the first years of operations and the higher the minimum level of productivity required by firms to decide to pay the setup cost $S^B$ and start production.
Alternative calibrations.

<table>
<thead>
<tr>
<th>Parameter calibration</th>
<th>Moment to match</th>
<th>Empirical moment</th>
<th>Simulated industry</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Financial frictions, costless default</strong></td>
<td><strong>Without financial frictions</strong></td>
<td><strong>Financial frictions, costless default</strong></td>
<td><strong>Without financial frictions</strong></td>
</tr>
<tr>
<td>$A$</td>
<td>2510</td>
<td>2505</td>
<td>Aggregate sales</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.03</td>
<td>0.041</td>
<td>Employment share of exiting firms</td>
</tr>
<tr>
<td>$\tau$</td>
<td>0.02</td>
<td>0.02</td>
<td>Average real interest rate</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>0.05</td>
<td>0.05</td>
<td>Median ratio fixed costs/labor costs</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.055</td>
<td>0.075</td>
<td>Cross sectional volatility (to match the volatility of operating firms); a smaller value of $\sigma$</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.381</td>
<td>0.331</td>
<td>Time series volatility (to match the volatility of operating firms); a smaller value of $\sigma$</td>
</tr>
<tr>
<td>$\rho_\mu$</td>
<td>0.7</td>
<td>0.7</td>
<td>Autocorrelation of $\rho_\mu$</td>
</tr>
<tr>
<td>$\nu$</td>
<td>0.5</td>
<td>0.5</td>
<td>Mean size divided by median size</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>4</td>
<td>3.5</td>
<td>Size of 75% percentile/median size</td>
</tr>
<tr>
<td>$\lambda_\epsilon$</td>
<td>0.01</td>
<td>0.01</td>
<td>Weighted average of R&amp;D/average cost</td>
</tr>
<tr>
<td>$s$</td>
<td>1.15</td>
<td>1.05</td>
<td>Median total variable costs 50% smaller firms</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>2.0075</td>
<td>2.0127</td>
<td>Median total variable costs 50% larger firms</td>
</tr>
<tr>
<td>$\tau$</td>
<td>1.25</td>
<td>1.25</td>
<td>Ratio of exports to domestic sales</td>
</tr>
<tr>
<td>$\lambda_\mu$</td>
<td>0.522</td>
<td>0.53</td>
<td>Exit rate from exporting</td>
</tr>
<tr>
<td>$\lambda_F$</td>
<td>0.02</td>
<td>0.02</td>
<td>Fraction of exporters</td>
</tr>
<tr>
<td>$a$</td>
<td>1.35</td>
<td>100</td>
<td>Fraction of financially constrained firms</td>
</tr>
</tbody>
</table>

* Exporters defined as exports > 5% sales.
* Firms that declare financing problems.
* Firms with not enough financial wealth to start exporting ($w_t < S^F + F + F^S$).

(iv) Fourth, there is a “competition” effect induced by the patterns of entry and endogenous exit via bankruptcy. Firms face some probability of inefficient default in the early stage of their operation. This discourages entry, reduces competition, and increases expected profits. In other words, the expected profits in the industry are upward-sloping. Lower profits expected at an early stage, because of the default risk, are compensated by reduced competition and higher profits for the firms that survive their early periods of activity and accumulate enough wealth to avoid future default.

We are able to partially disentangle these different effects by comparing the calibrated benchmark model, in which all four effects are active, with two alternative calibrations: first, an industry with “financial frictions and costless default”. In this industry, we assume that default is not costly, because defaulting firms can be sold at their continuation value (the net present value of profits), which is distributed as dividends. In this industry the direct effect (i) is active, while effects (ii), (iii) and (iv) are not, hence there is no role for precautionary saving and the selection of firms is not driven by bankruptcy risk.13 Second, an industry without financial frictions. In this case initial wealth $q_0$ is set sufficiently high to ensure that no firm is ever financially constrained and no firm ever goes bankrupt during its lifetime. In this calibration none of the four effects are present, although there are still firm selection effects that are not driven by the risk of bankruptcy. Financing imperfections are responsible for both default risk and for binding liquidity constraints, and therefore the comparison between the benchmark model and the model without financing frictions is the most appropriate to quantify their importance. However, in some of the figures and tables of the next sections, we will also focus on the intermediate model with financing frictions and costless default. This is useful to disentangle the direct effect (i) from the indirect effects (ii)–(iv).

The details for these two alternative calibrations are shown in Table 2. All parameters are re-calibrated, such that all of the empirical moments matched in the benchmark calibration are also matched in these alternative models. The main differences in the parameters of the industry without financing frictions, with respect to the benchmark calibration, are: a smaller value of $F$ (to match the volatility of operating firms); a larger value of $\theta$ (to match the size distribution of firms); a large value of $\mu$ (to match the frequency of exporting firms); and a larger value of $\sigma$ (to match entry and exit dynamics). In Section 4, we will calculate the changes in aggregate productivity for each of these simulated industries when trade is not allowed. Recalibrating the model in each case is, therefore, important to quantify the consequences of financing frictions and default risk on the reallocation gains from trade liberalization.

### 3.4. Value functions, export decision rules and the distribution of firms in the simulated industry

The value functions (net of financial assets $a$) and export status of two firms are shown in Fig. 1 as functions of firms’ assets $a$ over the average industry sales. One firm has low productivity and low profit volatility, while the other is a high productivity and high-risk firm.14 The low-risk low-productivity firm begins exporting as soon as the financing constraint (15) is satisfied. It also has a relatively flat value function, as the risk of going bankrupt is low for all values of $a$. Conversely,

---

13 This comparison model shares some features with Suwantaradon (2012).
14 In this example we measure productivity as the average cost of production. A similar example can be constructed by measuring it with marginal productivity. Note also that the non-smooth profile of the value function for the high risk firm is caused by the discrete nature of the profits shock $\varepsilon$. 

when assets are low, the high productivity high-volatility firm may go bankrupt, so it accumulates wealth above what is required by (15) before beginning to export as a form of precautionary saving. Bankruptcy risk is especially important for more productive firms because the wedge between their expected discounted profits and their liquidation value is higher. Moreover, selection makes more productive firms, on average, more volatile.

Fig. 2 shows the implied dynamics at the firm level. Fig. 2A illustrates the accumulation of financial assets over time for the benchmark industry with financial frictions and costly default. As active firms never pay dividends, wealth increases over time. More volatile firms accumulate wealth more rapidly because they are on average more productive due to the bankruptcy selection effect. Fig. 2B shows the probability of exporting for the same firms. Low-volatility firms begin exporting earlier even though they are less profitable and accumulate wealth slowly. This effect is caused by precautionary saving and the direct effect of the financing constraint (15). To isolate this direct effect, Fig. 2C compares the high-volatility firms in Fig. 2B with the high-volatility firms in the calibrated industry with financial frictions and costless default. The figure shows that most of the effect in Fig. 2B is due to precautionary saving as firms start to export much earlier when default is not costly.

Fig. 3 illustrates the distributional features of the simulated industries. Panel A shows the fraction of exporting firms for each marginal productivity in the benchmark industry and the industry without financing frictions. In both industries, more productive firms are more likely to start exporting. However financing frictions worsen the selection of firms into exporting. More productive firms export less and, simultaneously, some low-risk/low-productivity firms, which would not export in the absence of financing frictions, export in the industry with default risk due to lower competition from productive firms. Panel B shows the distribution of firms over volatility levels. For the financially-constrained industry, volatility reduces the incentives both to start activity in the home sector and to start exporting. Conversely, for the industry without financing constraints, both the fraction of active firms and the fraction of exporting firms increase in volatility. This is an “option value” effect: Because the profit shock is persistent, higher volatility also means higher expected profits from exports when the shock is positive.

3.5. Comparison between the model and the empirical dataset

In this section we evaluate the model using our empirical dataset, and we show that financing frictions and default risk play an important role in generating some key features of the data. In the first two columns of Table 3, we empirically verify the positive correlation between firm productivity and its volatility generated by the selection effect. The measure of firm productivity is the TFP variable, described in Appendix B. The measure of volatility is the firm-level standard deviation of this same measure. The volatility variable is then interacted with measures of the predicted intensity of financing constraints at the sector level. These are the dummy variables highconstrained, midconstrained, accounting for the top and middle third of sectors in terms of predicted financing constraints. (lowconstrained being the omitted group).

The results in columns 1 and 2 show that, indeed, productivity is correlated with its volatility and that this correlation is higher for those firms that are more constrained.
In column 3 of Table 3 we provide empirical evidence consistent with the prediction that financing frictions distort selection into export away from the most productive firms. More specifically, we test whether financing constraints lower the predictive power of firm productivity on exports. Confirming the predictions of the model, the coefficient of productivity on the export status of the firm becomes smaller as financing constraints become tighter, beginning with a positive coefficient and ending with an insignificant one for the most constrained firms (i.e. the composition of the coefficients of TFP and highconstrained $x$ TFP). Finally, column 4 confirms the prediction that volatility is more positively related to export for unconstrained than for constrained firms.

In the simulated data, we estimate productivity using average production costs. We regress the standard deviation of productivity calculated for each firm on the average productivity for the same firm and a constant. Because the time series is short in the empirical data, in the simulated data we only consider a time series of 10 years for each firm.
Table 3
Regression results.

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>1 sdTFP</th>
<th>2 sdTFP</th>
<th>3 Export</th>
<th>4 Export</th>
</tr>
</thead>
<tbody>
<tr>
<td>TFP</td>
<td>−0.004*</td>
<td>0.010***</td>
<td>0.060***</td>
<td>TFP</td>
</tr>
<tr>
<td></td>
<td>[0.002]</td>
<td>[0.002]</td>
<td>[0.010]</td>
<td>[0.007]</td>
</tr>
<tr>
<td>Midconstrained × TFP</td>
<td>0.019***</td>
<td>0.00049</td>
<td>0.018</td>
<td>sdTFP</td>
</tr>
<tr>
<td></td>
<td>[0.003]</td>
<td>[0.003]</td>
<td>[0.012]</td>
<td>[0.009]</td>
</tr>
<tr>
<td>Highconstrained × TFP</td>
<td>0.030***</td>
<td>0.013***</td>
<td>−0.059***</td>
<td>Midconstrained × sdTFP</td>
</tr>
<tr>
<td></td>
<td>[0.004]</td>
<td>[0.003]</td>
<td>[0.012]</td>
<td>[0.009]</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>Highconstrained × sdTFP</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>−0.741***</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>[0.091]</td>
</tr>
<tr>
<td>Controlsa</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Controls</td>
</tr>
<tr>
<td>Observations</td>
<td>33,375</td>
<td>33,375</td>
<td>29,237</td>
<td>29,236</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.005</td>
<td>0.448</td>
<td>0.161</td>
<td>0.094</td>
</tr>
</tbody>
</table>

TFP is total factor productivity, measured as the residual of a translog specification that uses capital and employment and is estimated at a 2 digit NACE sector level on 3 year windows. sdTFP corresponds to the standard deviation of the TFP measure calculated at a 2 digit sector–year level. Midconstrained is a dummy for the middle third quantile of the predicted financing constrained firms using the relationship lending instrumental variables. Constraints are aggregated at a sector level for columns 3 and 4. Similarly Highconstrained is a dummy variable for the top third quantile of predicted financing constraints. Controls in columns 2, 3 and 4 include the log of real total assets, age, age squared, year dummies, and regional dummies. Column 3 also includes 2 digit sector dummies. Standard errors in brackets. ***, **, * denote significance at a 1%, 5% and 10% level respectively.

Table 4 shows the extent to which the simulated industries are able to match some selected empirical moments that were not targeted in the calibration. The table shows, first, the exit hazard rate. The benchmark industry with financing frictions and default risk replicates well the hump shape in the hazard rate, with a peak probability of exit in year two of 9.3 percent versus 11 percent in the empirical data. The higher empirical value likely reflects reasons other than financing problems, such as learning about the firm’s own productivity, that lead firms to exit early. The industry with financing frictions and costless default also replicates the hump shape, with generally higher exit rates. Since default is not costly in this industry, more firms enter and more young firms default every period. Conversely in the industry without frictions, firms only exit with the exogenous probability $\delta$ and there is not a hump shaped age profile of exit. The lower section of Table 4 shows the relation of export with size, age and productivity. In the empirical sample, the intensity of exports does not change with age, while is instead very heterogeneous in the size and productivity dimensions. This feature cannot be replicated by the unconstrained simulated industry or the constrained industry with costless default, as exporting is.

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18 We take the information on the probability of exit of Italian manufacturing firms from Audretsch et al. (1999), who analyze a dataset similar to ours.

19 On the one hand bankruptcy is ruled out by construction in this industry, as firms can always pay their fixed costs. On the other hand, although voluntary exit is in theory possible for these firms, it never happens under the set of calibrated parameters.

20 The lack of positive relation between age and export intensity in the data is because the Mediocredito surveys primarily targeted firms that had been in operation for at least four years at the time of each survey, and therefore underestimated young firms. For example, Cirillo (2009) documents that in 1998, in the Italian manufacturing industry, 24 percent of firms were younger than six years old. In our sample, this percentage is only 9.5 percent.
Table 4
Model validation.

<table>
<thead>
<tr>
<th>Moments</th>
<th>Empirical data</th>
<th>Simulated data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Full sample</td>
<td>Industry with financing frictions and costly default</td>
</tr>
<tr>
<td>Hazard rate 1 year after starting</td>
<td>9.4%</td>
<td>3.7%</td>
</tr>
<tr>
<td>Hazard rate 2 years after starting</td>
<td>11.0%</td>
<td>9.3%</td>
</tr>
<tr>
<td>Hazard rate 3 years after starting</td>
<td>8.3%</td>
<td>6.2%</td>
</tr>
<tr>
<td>Hazard rate 4 years after starting</td>
<td>9.4%</td>
<td>5.0%</td>
</tr>
<tr>
<td>Hazard rate 5 years after starting</td>
<td>8.6%</td>
<td>4.5%</td>
</tr>
<tr>
<td>Hazard rate 6 years after starting</td>
<td>5.8%</td>
<td>4.2%</td>
</tr>
<tr>
<td>% of exporters, firms &lt; 33% size percentile</td>
<td>49%</td>
<td>8.8%</td>
</tr>
<tr>
<td>% of exporters, firms between 33% and 66% size</td>
<td>56%</td>
<td>74.9%</td>
</tr>
<tr>
<td>% of exporters, firms between 66% and 100% size</td>
<td>74%</td>
<td>83.1%</td>
</tr>
<tr>
<td>% of exporters, firms &lt; 33% percentiles of age</td>
<td>57%</td>
<td>25.2%</td>
</tr>
<tr>
<td>% of exporters, firms between 33% and 66% of age</td>
<td>57%</td>
<td>68.3%</td>
</tr>
<tr>
<td>% of exporters, firms between 66% and 100% of age</td>
<td>58%</td>
<td>76.9%</td>
</tr>
<tr>
<td>% of exporters, firms &lt; 33% productivity percentile</td>
<td>49%</td>
<td>25.1%</td>
</tr>
<tr>
<td>% of exporters, firms between 33% and 66% prod.</td>
<td>57%</td>
<td>70.2%</td>
</tr>
<tr>
<td>% of exporters, firms between 66% and 100% prod.</td>
<td>65%</td>
<td>77.4%</td>
</tr>
</tbody>
</table>

Table 5
Increase in productivity after a trade liberalization, in different industries.

<table>
<thead>
<tr>
<th></th>
<th>No financing frictions</th>
<th>Financing frictions and costless default</th>
<th>Financing frictions and costly default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Panel A: Reallocation gains from trade</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>% change in weighted average of average cost</td>
<td>3.38%</td>
<td>3.31%</td>
<td>2.55%</td>
</tr>
<tr>
<td>(% change relative to unconstrained industry)</td>
<td>(100%)</td>
<td>(91.2%)</td>
<td>(75.5%)</td>
</tr>
<tr>
<td>% change in weighted average of marginal cost</td>
<td>3.61%</td>
<td>3.33%</td>
<td>2.78%</td>
</tr>
<tr>
<td>(% change relative to unconstrained industry)</td>
<td>(100%)</td>
<td>(91.8%)</td>
<td>(77.0%)</td>
</tr>
<tr>
<td>Panel B: Selected statistics for the industries with no trade relative to the open industries</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average profits</td>
<td>117%</td>
<td>116%</td>
<td>114%</td>
</tr>
<tr>
<td>Number of active firms</td>
<td>146%</td>
<td>141%</td>
<td>140%</td>
</tr>
<tr>
<td>Std dev of profits</td>
<td>98.6%</td>
<td>98.4%</td>
<td>108%</td>
</tr>
<tr>
<td>Prob. of default for firms with age (\leq 2) years</td>
<td>n.a.</td>
<td>70.7%</td>
<td>94.5%</td>
</tr>
<tr>
<td>Prob. of default for firms with 2 years &lt; age (\leq 9) years</td>
<td>n.a.</td>
<td>51.9%</td>
<td>77.1%</td>
</tr>
<tr>
<td>Prob. of default for firms with age (\geq 10) years</td>
<td>n.a.</td>
<td>41.3%</td>
<td>60.6%</td>
</tr>
</tbody>
</table>

a Cross sectional average.

b Average of the standard deviation of calculated for each firm over a 10 periods time series.

centralized among the largest and most productive firms. Instead the heterogeneity of exporting firms is higher and much closer to the empirical data in the industry with financing frictions and costly default, especially regarding the relation between exports and productivity.

4. Financial frictions and the reallocation gains from trade

In the previous sections we illustrated the channels through which financial frictions affect firm dynamics in the model industry. In this section we quantify the aggregate implications of these distortions. We perform a counterfactual experiment where we increase marginal trade costs \(\tau\) and we completely shut down trade in the benchmark calibrated industry. We calculate the aggregate productivity in this closed industry and we compare it to the aggregate productivity in the same industry with the calibrated value of \(\tau\). The difference measures the productivity gains generated by the reallocation across firms when the industry moves from no trade to the observed levels of exports.

Then we perform the same counterfactual experiment with the same increase in \(\tau\) in the calibrated industry with “no financing frictions”. This experiment measures the difference in productivity that trade liberalization would generate if firms in the Italian industry were not subject to financing frictions. By comparing these two experiments we quantify how much the reallocation gains from trade are reduced because of the distortion in the selection into export caused by financing frictions. Finally, we apply the same increase in \(\tau\) to the calibrated industry with financing frictions and costless default. This additional experiment allows us to disentangle the contribution of the different financing frictions channels to reallocation.
Table 6
Increase in productivity after a trade liberalization, sensitivity analysis.

<table>
<thead>
<tr>
<th></th>
<th>$S_c = 0.005$</th>
<th></th>
<th>$S_c = 0.02$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No financing</td>
<td>Financing</td>
<td>No financing</td>
<td>Financing</td>
</tr>
<tr>
<td></td>
<td>frictions</td>
<td>frictions</td>
<td>frictions</td>
<td>frictions</td>
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<tr>
<td></td>
<td>and costly</td>
<td>and costly</td>
<td>and costly</td>
<td>and costly</td>
</tr>
<tr>
<td></td>
<td>default</td>
<td>default</td>
<td>default</td>
<td>default</td>
</tr>
<tr>
<td>Weighted average of</td>
<td></td>
<td>4.6%</td>
<td>5.1%</td>
<td></td>
</tr>
<tr>
<td>R&amp;D/fixed costs</td>
<td></td>
<td></td>
<td>21.1%</td>
<td>25.2%</td>
</tr>
<tr>
<td>% change in weighted</td>
<td></td>
<td>5.71%</td>
<td>4.42%</td>
<td></td>
</tr>
<tr>
<td>average of average</td>
<td></td>
<td>(100%)</td>
<td>(73.9%)</td>
<td></td>
</tr>
<tr>
<td>cost</td>
<td></td>
<td></td>
<td>2.97%</td>
<td>1.55%</td>
</tr>
<tr>
<td>% change in weighted</td>
<td></td>
<td>5.94%</td>
<td>4.46%</td>
<td></td>
</tr>
<tr>
<td>average of marginal</td>
<td></td>
<td>(100%)</td>
<td>(75.0%)</td>
<td></td>
</tr>
<tr>
<td>cost</td>
<td></td>
<td></td>
<td>2.85%</td>
<td>1.74%</td>
</tr>
<tr>
<td>(% change relative</td>
<td></td>
<td></td>
<td>(100%)</td>
<td>(58.2%)</td>
</tr>
<tr>
<td>to unconstrained</td>
<td></td>
<td></td>
<td>(100%)</td>
<td>(58.2%)</td>
</tr>
<tr>
<td>industry)</td>
<td></td>
<td></td>
<td>(100%)</td>
<td>(58.2%)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(100%)</td>
<td>(58.2%)</td>
</tr>
</tbody>
</table>

The results of these experiments are reported in Panel A of Table 5. The first column of Panel A reports the reallocation gains in the unconstrained industry. The percentage increase in average productivity measures the difference in productivity between the closed industry with high $\tau$ and the open calibrated industry. We report both a measure of reallocation based on marginal productivity, $\nu$, and on average productivity $q / \nu + F$, because the latter better represents firms efficiency, given that firms are heterogeneous with respect to $F$. The next two columns of Panel A report the result of a similar experiment for the two constrained industries. The reallocation gains in the benchmark industry with both, financing frictions and costly default are 75% of the gains in the unconstrained industry. This is consistent with the evidence shown in the previous sections, where financing frictions distort the selection into export of the most productive firms. It is important to note that the reduction in reallocation is smaller in the case of the industry with financing frictions and costless default. The precautionary saving effect, the bankruptcy selection effect and the competition effect, which indirectly affect export decisions in the benchmark industry, are not present in this industry, where only liquidity constraints affect export decisions. Consistent with the evidence shown in Fig. 2C, this confirms that the selection and precautionary behavior induced by the default risk are more important in distorting export decision than binding liquidity constraints.

Panel B of Table 5 shows some selected statistics for the simulated closed industries relative to the open ones. The relative number of active firms and average profitability are higher in the industry without financial frictions, meaning that in this industry trade liberalization reduces them more than in the other industries. This result is determined by the greater selection and reallocation happening in this industry once firms start to export. Panel B also shows that the riskiness of firms significantly decreases in the industry with costly default when it opens up to trade, while it slightly increases in the other two industries. Because export activity increases default risk, when default is costly trade liberalization induces the selection, both into entry and into export, of less risky firms. This selection effect also explains the last statistic shown in Panel B, the probability of exit in the constrained industries. This is on average lower in the closed than in the open industries, especially for the industry with costless default, because of the additional risk of export activity.

Table 6 reports the productivity gains for different value of $S_c$, the cost the firms pay to learn their own type $\mu$. We report results for values 50% and 200% of the calibrated value. Both cases confirm a significantly smaller productivity gain in the industry with financing frictions and costly default relative to the unconstrained industry.

Finally, Fig. 4 compares the reallocation gains from trade for the benchmark industry and the industry without financing frictions, for different values of $\tau$. In each line in the figure all parameters are constant except $\tau$. The lines start for a value of $\tau$ large enough so that there is no trade. then $\tau$ is progressively lowered to increase exports. Fig. 4 relates the reallocation gains from trade to the percentage of exporting firms. It shows that, as the industries become more open to trade, for a given percentage of exporting firms the selection into export is more efficient in the unconstrained industry, and gains from trade are larger than in the industry with financing frictions and costly default. The difference in reallocation between the
two industries peaks around a fraction of exporting firms equal to 50 percent, at which point reallocation gains are around 30 percent smaller in the constrained industry with respect to the unconstrained one.

5. Conclusions

We present a dynamic model in which firms accumulate wealth to avoid bankruptcy and to overcome financing constraints that affect their fixed operational costs and the one-off costs associated with becoming an exporter. Firms decide to begin exporting according to their accumulated wealth and their idiosyncratic demand shocks. Financially-constrained firms, which would become exporters in an unconstrained model, may not export because they cannot pay the fixed costs associated with exporting. Moreover, firms may postpone the decision to export, even when financing constraints are not currently binding, for precautionary reasons to avoid increasing their bankruptcy risk. This second effect is quantitatively more important in the calibrated model and it is especially strong for more productive firms, for which the wedge between continuing producing and liquidating is higher. The risk of bankruptcy also influences the entry decision. Firms with volatile profits enter only if they are also very productive. This generates a positive correlation between productivity and risk that amplifies the precautionary delay of exports. Finally, endogenous entry and bankruptcy induce an upward-sloping profile of the discounted value of future profits with age. This induces some firms with low risk and low productivity to begin production and exporting. These firms would not enter or export in an unconstrained economy, but the decrease in competition induced by financing constraints and bankruptcy induces them to do so.

The model shows that, in equilibrium, financing frictions reduce the aggregate productivity gains generated by trade liberalization by 25 percent. This occurs because the selection of firms into exporting is severely distorted by the presence of financing frictions. As a consequence, the predictive power of productivity in terms of determining exports gets reduced whenever a substantial number of firms face financial constraints.

Finally, two important policy implications can be drawn from this paper and put in perspective with the existing literature. First, if an aggregate financial shock hits, leading to reduced access to credit for all firms, this will affect the composition of exporting and non-exporting firms in the future. In particular, the positive effects of trade in terms of allocating production to the most productive set of firms will be attenuated whenever access to credit is scarce. We therefore elicit one important channel through which misallocation grows in financial crises, as documented in Oberfield (2013). Second, when a country opens up to trade, the effects will be most beneficial in those sectors with better access to financial markets. Those with poor access will not be able to take advantage of the opening to trade and may, in fact, face a higher risk of bankruptcy. Thus our results reinforce other contributions in the literature that show how different frictions can substantially reduce the gains from trade liberalizations. The nature of these frictions can be financial, as in our paper (see also Manova, forthcoming), but can also be related to labor market mobility (Kambourov, 2009; Cuñat and Melitz, 2012) or to poor contract enforcement (Nunn, 2007) among other frictions.

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Appendix A. Calibration

In order to obtain a numerical solution for the value functions \( V^P_t(q_t, \varepsilon_t, \mu) \) and \( V^X_t(q_t, \varepsilon_t, \mu) \), we consider values of \( q_t \) in the interval between 0 and \( \bar{q} \), where \( \bar{q} \) is a sufficiently high level of assets such that the firm is never financially constrained now or in the future. We then discretize this interval in a grid of 800 points. The shock \( \varepsilon_t \) is modeled as a two-state symmetric Markov process. The exponential distribution of \( \varepsilon^\text{exp}_t \) is discretized in a grid of 10 points; the uniform distribution of \( \theta \) in a grid of 12 points, and the uniform distribution of \( \mathcal{F}_t \) in a grid of 30 points. We conduct several experiments with these grid dimensions to be sure that the choice of the grid does not significantly influence the quantitative results of the simulations.

We first make an initial guess of the equilibrium aggregate price \( P \). Based on this guess, we calculate the optimal values of \( V^P_t(q_t, \varepsilon_t, \mu) \) and \( V^X_t(q_t, \varepsilon_t, \mu) \) using an iterative procedure. We then apply the zero profits condition (22) and we update the guess of \( P \) accordingly. We repeat this procedure until the solution converges to the equilibrium. Then we simulate an artificial industry in which every period, the total number of new entrants ensures that condition (1) is satisfied.
Appendix B. Data

For the calibration and empirical parts of the paper, we use data from the Mediocredito Centrale surveys that sample Italian manufacturing firms. It is an incomplete panel with information on: (i) annual balance-sheet data and profit and loss statements from 1995 to 2003; and (ii) qualitative information from three surveys conducted in 1997, 2000, and 2003; covering the firms’ activity over the three previous years and, in particular, detailed information on exports and financing constraints. The sample is selected balancing representativeness with continuity. Every three years two thirds of the sample is replaced. Relative to the population of Italian firms, small firms are underrepresented and large firms are overrepresented. Furthermore, there are very few firms of less than ten employees. We censor both databases at ten employees and for all our empirical analysis we use population weights obtained from ISTAT. All our results should be interpreted as representative of Italian firms with ten or more employees.21

This dataset is particularly well suited for our analysis. It contains firm-level detailed information about financing constraints, exports and standard accounting data. Once we restrict ourselves to the observations with valid information, it contains 6776 firms and 33,399 firm–year observations. The export variable is a dummy variable that measures whether the firm exports at least 5 percent of its production (export dummy).

To measure financing constraints, we use the firm’s responses to the following questions in the survey: (i) whether it had a loan application turned down recently, (ii) whether it desires more credit at the market interest rate, and (iii) whether it would be willing to pay an higher interest rate than the market rate in order to obtain credit. The variable constrained takes value 1 for a given period if a firm answers yes to any of the questions (i) to (iii) and takes value zero otherwise. According to this measure, 17 percent of the firms declare to be financially constrained. This is a much more reliable measure of financing constraints than measures based on balance-sheet information or financial outcomes. However a possible concern is that this variable could be correlated with productivity shocks that are likely determinants of trade outcomes. For this reason, we use an instrumental variables approach using as IVs variables that are unlikely to be correlated with productivity shocks. The instrumental variables, based on the relationship lending literature, are the share of loans of the main lending bank (percentage loans with main bank), the number of years that the firm has been operating with this bank (length of main bank relationship) and the square of this same variable (length of main bank relationship squared).22 We then construct sector-specific measures of the intensity of financing constraints. More specifically, we first predict at the firm level the probability of declaring financing problems using a set of instrumental variables. The prediction is then used to generate three dummy variables – highconstrained, midconstrained, and lowconstrained – that, depending on the specification, split firms or sectors in the empirical sample into roughly three equal parts according to the predicted financing constraints of the sector. In the regression analysis we introduce as controls the size of the firm measured as the log of its real total assets (log real total assets), the age of the firm in years (age (years)) and age squared (age squared) and the productivity level of the firm (TFP).

Productivity is measured as the average residual from a regression model in which total production is explained by a translog specification of a Cobb Douglas production model that includes fixed capital and total employment. The coefficients of the model are allowed to vary at a two digit-sector level and on three-year windows.23 In principle, a strict interpretation of our model would involve productivity being captured by a combination of sector and time dummies. However, the model is quite stylized and, in particular, it does not explicitly include capital use, although the firm-level variation of fixed costs could be interpreted as a reduced form of capital use. Having a simple functional form within a sector and three-year windows allows for some extra flexibility. The results are however robust to the use of a combination of fixed effects as a measure of productivity.

References


21 See Pfefferman (1993) for a good summary of this weighting procedure. The weights rebalance the importance of firms according to their number of employees. They range from 2.49 for firms with employees between ten and 20 to 0.17 for firms above 250 employees.
22 All three variables are statistically different from zero in the first-stage regression with T-statistics ranging from 4 to 9. The partial R² of the omitted variables is 1%. Details are available from the authors.
23 In particular productivity is measured by $e_t$ in the following model $log(y_{it}) = \sum_{j=1}^s \sum_{t=1}^T \alpha_j log(K_{it}) + \beta_j log(L_{it}) + \gamma_j log(K_{it}) + log(L_{it}) + \eta_t + \mu_t + e_{it}$ where $s$ refers to 2 digit sector levels, $j$ to 3 year intervals and $t$ to the years 1995 to 2003.


