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Monetary Regime Switches and Central Bank Preferences

Monetary policy objectives and targets are not necessarily constant over time. The regime-switching literature has typically analyzed and interpreted changes in policymakers' behavior through simple interest rate rules. This paper analyzes policy regime switches by explicitly modeling policymakers' behavior and objectives. We show that changes in the parameters of simple rules do not necessarily correspond to changes in policymakers' preferences. In fact, capturing and interpreting regime changes in preferences through interest rate rules can lead to misleading results.

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THE ANALYSIS OF REGIME-SWITCHING policy has been central to several economic problems. The debate on the existence and sources of the Great Moderation is a clear example.¹ Regime switches have also been examined in the context of rational expectations determinacy (e.g., Davig and Leeper 2007, Farmer, Waggoner, and Zha 2009) and dynamic stochastic general equilibrium estimation (e.g., Owyang and Ramey 2004, Davig and Doh 2008, Bianchi 2013). Building

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1. See, for instance, Cogley and Sargent (2002), Stock and Watson (2003), Sims and Zha (2006), Canova and Gambetti (2009), and the literature reviewed therein.

on relatively standard New Keynesian models, this literature has typically modeled policymakers' behavior with time-varying or Markov-switching simple interest rate rules.

The extensive use of simple interest rate rules in both theoretical and empirical studies is justified by several reasons: their simplicity and potential for practical use, their fairly good performance when compared to the optimal policy, and their robustness across several model specifications.² However, simple rules are reduced-form representations of policymakers' behavior and cannot be used to identify the structural sources of behavioral changes. Reduced-form representations obscure the differences between factors the central bank can and cannot control. This limitation is relevant when distinguishing between monetary regimes and assessing the central bank's performance (e.g., "good policy" versus "good luck"). For these reasons, it is unclear if the effects of regime switches in central banks' objectives are similar to switches in simple interest rate rules.

A natural source of regime switches is a change in policymakers' preferences, in terms of the relative weight assigned to different objectives and the desired targets. Objectives and targets can change over time due to a variety of reasons. Policy objectives may change with appointments of governors and central bank staff, who may have different views from their predecessors. Also, even among academic economists, there is scope for different opinions and evolving theories on the benefits of output versus inflation stabilization, and what is, in practice, the exact output level that should be targeted. The inflation target itself is also subject to debate, as discussed, for instance, in Blanchard, Dell'Ariccia, and Mauro (2010).

We first consider two regimes differing in the relative weight assigned to inflation stabilization, and where regime switches are governed by an exogenous process. The economy is, in fact, affected by alternative regimes through an expectation effect. The possibility of a future dovish regime increases inflation expectations and induces the hawkish regime to increase the inflation response to markup shocks through an accommodation effect. To control inflation, the hawkish regime also induces an output contraction, which is the opposite of what the dovish regime is aiming for.

Interestingly, in our baseline case, the optimal policy is invariant to whether the central bank actively internalizes future regime switches or not. In that sense, our results are robust to considering an anticipated utility framework (as in, e.g., Kreps 1998) rather than expected utility.

We also examine alternative frameworks. In particular, we consider cases where the output-gap target or the inflation target are subject to regime switches. We obtain similar results to the baseline case but the results are more pronounced since the regime switches affect directly a permanent target rather than a response to a temporary shock. Finally, we examine the case in which regime switches cannot occur in every period, and therefore the current regime has time to "prepare for the handover."

2. Dennis (2004, 2006) are two exceptions considering that monetary policy is set optimally in the estimation exercise. For a recent paper on the importance of expectations and optimal allocations, see Carboni and Ellison (2009).

Characterizing policies through an optimal decision process enables us to examine whether the switches in simple interest rate rules identified in the data are likely to stem from changes in policymaker's preferences. Imposing a structure on the policymakers' decision process amounts to imposing restrictions on the possible switches in the resulting policies.

We find that changes in simple rules parameters cannot be interpreted solely as changes in policymakers' preferences. Intuitively, changes in policymaker's preferences imply a movement along the policy frontier, where reducing the volatility of one variable implies increasing the volatility of another variable. Instead, switches in simple interest rate rules often imply that the volatilities of inflation and output move in the same direction. In addition, using simple rules to capture changes in policymakers' objectives can lead to misleading results: the presence of regime switches can be wrongly rejected and indeterminacy can be wrongly detected. Altogether, our findings restrict the possible interpretations of what are the deep sources of the existing estimates of regime switches in policymakers' behavior and the associated normative implications.

Our paper is partially related to the literature on political economy and monetary policy (see, e.g., Alesina, Roubini, and Cohen 1997), but our goal is not to provide a partisan analysis of monetary policy. In fact, we shed light on the difficulty of the partisan empirical literature to match election dates and political parties with effective changes in monetary policy—in our model, a future dovish regime implies an increase in inflation even if the current regime remains hawkish.³

The paper is organized as follows. Section 1 introduces the model. Section 2 presents the model of regime switches in central bank's objectives. Section 3 analyzes the relationship with simple rules, and Section 4 concludes. The appendices contains additional derivations.

1. THE MODEL

We base our analysis on a simple monetary model. Inflation dynamics are described by a New Keynesian Phillips curve (NKPC). As is well known, the NKPC is a linear approximation of the relationship between inflation and output in an economy with monopolistic competition and staggered price setting

$$\pi_t = \kappa y_t + \beta E_t \pi_{t+1} + u_t, \quad (1)$$

3. Theoretical models in this literature did not contemplate this possibility. On the empirical side, Alesina, Roubini, and Cohen (1997) point out several empirical successes of political cycle models, whereas Faust and Irons (1999) conclude that partisan effects in U.S. macroeconomic data are fragile, and that there is little evidence that the partisan effects on the economy operate through changes in monetary policy.

where π_t denotes price inflation and y_t measures the output gap, that is, the difference between actual output and the output level that would prevail under flexible prices.⁴ The term u_t constitutes an exogenous markup shock, introducing a trade-off between inflation and output stabilization and following the process $u_t = \rho_u u_{t-1} + e_t^u$, with $e_t^u \sim N(0, \sigma_{e^u})$ being an i.i.d. disturbance.

As is standard in the optimal monetary policy literature, we abstract from the choice of a specific policy instrument. In addition, we assume that the central bank minimizes a weighted average of deviations of inflation and output gap from their respective targets

$$U_t^i = \frac{1}{2} \left[\pi_t^2 + w^i (y_t - \tilde{y}^i)^2 \right]. \quad (2)$$

The parameter w^i measures the relative importance of output stabilization and the superscript i denotes the specific regime. The inflation target is normalized to zero, while $\tilde{y}^i \geq 0$ represents the (exogenously given) output gap target. The target \tilde{y}^i can be interpreted as the difference between the efficient level of output and the output that would prevail under flexible prices.

We model changes in the objective function in a straightforward way that allows for analytical solutions. In any period, current objectives can persist or change with probability q and $1 - q$, respectively. We consider the objectives of the central bank to be either *dovish* (d) or *hawkish* (h), that is, $i = \{d, h\}$. The term *dovish regime* refers to a case where the output gap target or the relative weight to output stabilization are higher than in a hawkish regime, that is, $\tilde{y}^d > \tilde{y}^h$ or $w^d > w^h$. In this model, expectations of future policy matter for current outcomes. Therefore, whether the central bank can or cannot commit to future policies affects the outcomes in the economy. The literature assumes that the central bank operates either under discretion or under commitment. In the former case, the central bank is assumed to never make credible commitments; in the latter case, the central bank can make credible commitments regarding all future periods and states of nature. In this paper, we assume an intermediate setting of imperfect commitment.

We assume that the central bank can only make credible commitments about future policies as long as objectives remain unchanged. For instance, a hawkish central bank starting in period 0 can commit to policies in future periods t , but such commitments are only implemented if objectives have not switched to dovish. When objectives change, previous commitments are disregarded and a new policy plan is set that maximizes the new objectives. In other words, every time that objectives change the central bank can disregard previous policy plans and can just freely reoptimize. This assumption can be justified on the grounds that if objectives change, the central bank

4. The theoretical framework underlying such relationship is described in Yun (1996), Woodford (2003), and Galí (2008). This specification of the NKPC holds in a neighborhood of a zero inflation steady state. Throughout our analysis, we abstract from the changes that may derive from having a different steady-state level of inflation.

will adopt the best possible policy to fulfill its new objectives, and thus will disregard the plans made when priorities were different.

Details on this type of policy formulation are available in Roberds (1987), Schaumburg and Tambalotti (2007), and Debortoli and Nunes (2010a). In this context, it can be shown that under regime i , policy plans solve the following problem:

$$V^{ii}(u_0) = \max_{\{\pi_t, y_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} (\beta q)^t \left\{ -\frac{1}{2} \left[\pi_t^2 + w^i (y_t - \tilde{y}^i)^2 \right] \right. \quad (3)$$

$$\left. + \beta(1 - q)V^{ij}(u_t) \right\},$$

s.t. $\pi_t = \kappa y_t + \beta q E_t \pi_{t+1} + \beta(1 - q)E_t \pi_{t+1}^j + u_t \quad \forall t,$

$$u_t = \rho_u u_{t-1} + \epsilon_t^u \quad \forall t,$$

where to economize on notation, we have suppressed the superscript i whenever possible.

The objective function is given by an infinite sum discounted at the rate βq , summarizing the events in which objectives remain unchanged. Each term in the summation is composed of two parts. The first part, in square brackets, is the period loss function. The second part is the value function V^{ij} , summarizing the utility the central bank obtains if next period objectives change. Since when objectives change, the central bank loses its commitment, such terms are summarized by a value function that depends on the policies of the alternative regime.

The central bank faces a sequence of constraints represented by the NKPC, where in any period t , inflation expectations are an average between two terms. The first term, with weight q , is the inflation that would prevail under the current regime (π_{t+1}) and upon which there is commitment. The second term, with weight $(1 - q)$, is the inflation that would be implemented under the alternative regime j (π_{t+1}^j), which is taken as given by the current central bank. As stated in the equilibrium definition, such a level of inflation is determined by solving a symmetric problem to the one described above.

DEFINITION 1. *A Markov perfect equilibrium with regime switches in objectives must satisfy the following condition. For any i and $j \neq i$, given the sequence $\{\pi_t^j, y_t^j\}_{t=0}^{\infty}$*

- (1) *The sequence $\{\pi_t^i, y_t^i\}_{t=0}^{\infty}$ is optimal.*
- (2) *The value function V^{ij} satisfies equation*

$$V^{ij}(u_0) \equiv E_0 \sum_{t=0}^{\infty} (\beta q)^t \left[-\frac{1}{2} \left((\pi_t^j)^2 + w^i (y_t^j)^2 \right) + \beta(1 - q)V^{ii}(u_t) \right].$$

- (3) *The sequence $\{\pi_t^j, y_t^j\}_{t=0}^{\infty}$ is optimal, solving the symmetric problem of regime j .*

The first requirement imposes the optimality of the policy functions given $\{\pi_t^j, y_t^j\}_{t=0}^\infty$ and V^{ij} . The second part defines the value function V^{ij} as the continuation value in case the regime changes. The value function V^{ij} takes into account that regime i may become relevant again in the future. The first two conditions in the definition leave the sequence $\{\pi_t^j, y_t^j\}_{t=0}^\infty$ and the institutional setting of regime j unspecified. The third part of the definition states that regime j solves a symmetric problem. We refer to Markov perfect equilibrium because in a reoptimization period—the initial period in which the regime changes—policy depends only on natural state variables. The setup discussed above is convenient to understand the problem and obtain analytical solutions. Importantly, one must solve the model using standard solution techniques from the regime-switching literature as discussed, for instance, in Costa, Fragoso, and Marques (2004) and Farmer, Waggoner, and Zha (2009). Those methods require solving the two regimes simultaneously rather than independently. Using fixed-regime methods to solve a Markov switching model can produce incorrect results. As discussed by Costa, Fragoso, and Marques (2004) and Farmer, Waggoner, and Zha (2009), using mean square stability as a stability concept, stability of each regime does not imply stability of the entire economy, and instability of one regime does not imply instability overall.

Throughout our analysis, we compare the regime-switching model with two benchmark scenarios. The first one is the standard full-commitment case, where there are no regime switches (i.e., $\tilde{y}^i = \tilde{y}^j$, $w^i = w^j$, and $q = 1$). The second scenario is limited commitment as in Roberds (1987), Schaumburg and Tambalotti (2007), and Debortoli and Nunes (2010a). With limited commitment, objectives do not change, but previous promises are disregarded with probability $1 - q$ (i.e., $\tilde{y}^i = \tilde{y}^j$, $w^i = w^j$, but $0 < q < 1$).

2. THE EFFECTS OF REGIME-SWITCHING OBJECTIVES

2.1 Regime Switches in the Relative Weight of Output

The baseline case for our analysis is one where regimes only differ in the relative weight assigned to output stabilization, and the output-gap target is set to $\tilde{y}^i = \tilde{y}^j = 0$. Appendix D shows that arranging the first-order conditions of problem (4) yields

$$\pi_t = -\frac{w^i}{\kappa} y_t + \frac{w^i}{\kappa} y_{t-1}, \quad (4)$$

where y_{-1} is set to zero. Equation (4) can be interpreted as a targeting rule, as, for example, in Giannoni and Woodford (2010). This rule holds conditionally on being in a certain regime i . When objectives switch to type j , a new plan is made and the term on lagged output is discarded. Since the targeting rule does not depend on the parameters of alternative regimes, it is robust to the presence of regime-switching objectives.

Moreover, rule (4) implements the optimal policy regardless of whether the central bank does or does not internalize future objectives changes. Therefore, for our purposes, it would be irrelevant if the central bank minimizes the expected or the anticipated (see Kreps 1998) loss function. We should clarify that this only happens because in problem (4), there are no endogenous state variables. The lack of endogenous state variables implies that policy in the current period is not affected by a strategic incentive to affect future policymakers. Since bygones are bygones in this purely forward-looking model, the actions of the current regime do not enter the optimization of future regimes. For these reasons, within this simple setting, it is only necessary that private agents internalize the possibility of regime switches, which follows directly from the assumption that expectations are formed rationally.⁵

In order to find an analytical solution to the model, we focus on the stable solution under each regime. Using the standard techniques of Costa, Fragoso, and Marques (2004) and Farmer, Waggoner, and Zha (2009), we have then checked numerically that such a solution is indeed the unique stable minimal state variable solution.⁶ Combining (1) and (4), the evolution of inflation and output is determined by

$$\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = \begin{bmatrix} \frac{w^i}{\kappa} (1 - \psi_y^i) & \psi_u^i \\ \psi_y^i & -\frac{\kappa}{w^i} \psi_u^i \end{bmatrix} \begin{bmatrix} y_{t-1} \\ u_t \end{bmatrix}, \tag{5}$$

with $\psi_y^i \equiv 1/\gamma^i$, $\psi_u^i \equiv (1 + \beta(1 - q)\psi_u^j \rho_u)/(\gamma^i - \beta q \rho_u)$, and the term

$$\gamma^i \equiv \frac{1}{2} \left[1 + \frac{\kappa^2}{w^i} + \beta q + \sqrt{\left(1 + \frac{\kappa^2}{w^i} + \beta q\right)^2 - 4\beta q} \right] > 1 \tag{6}$$

is increasing in q and decreasing in w^i . The persistence of output ψ_y^i is not affected by the parameters of the alternative regime. In contrast, the inflation response to markup shocks ψ_u^i depends on ψ_u^j . The more likely is the regime switch (the lower is q) and the more persistent are the markup shocks, the stronger are the spillovers between alternative regimes.⁷ The difference in the initial response then propagates over time through the state variable y_{t-1} . The Appendix shows that in a nonstochastic steady state where a given regime i continues indefinitely, inflation converges to zero. This

5. In Debortoli and Nunes (2010b) we checked that the presence of endogenous state variables does not affect the main point of our paper—regime switches in objectives are not equivalent to regime switches in simple interest rate rules. For an analysis of regime-switching models with learning, see Nunes (2009) and Branch, Davig, and McGough (2013).

6. To do so note that there are four regimes in this economy: (1) new hawkish regime where y_{t-1} is irrelevant, (2) continued hawkish regime, (3) new dovish regime where y_{t-1} is irrelevant, and (4) continued dovish regime. The transition probabilities with four regimes are still just dependent upon q , but with the restriction that there is zero probability of moving from regimes 3 and 4 to regime 2, and zero probability of moving from 1 and 2 to regime 4.

7. Zampolli (2006) analyzes exchange rate regime switches and optimal policy in a model with backward looking expectations where these types of interactions are not present. In our forward-looking model, such spillovers would be absent only in the very particular case of i.i.d. markup shocks and no endogenous state variables.

result shows that, even with regime switches, long-run price stability continues to be a main feature of this model.

As a benchmark for our analysis, we first analyze the standard case with full-commitment and constant objectives (of type i) as discussed, for instance, in Woodford (2003, ch. 4) and Galí (2008, ch. 5). In that case, and denoting the corresponding variables with an upper bar, the dynamics are described by equation (5), where the relevant parameters are given by $\bar{\psi}_u^i \equiv 1/(\bar{\gamma}^i - \beta\rho_u)$, $\bar{\psi}_y^i \equiv 1/(\bar{\gamma}^i)$, and $\bar{\gamma}^i$ is the value taken by equation (6) when $q = 1$.

With regime-switching objectives ($0 \leq q < 1$ and $w^i \neq w^j$), and assuming that regime j solves a symmetric problem, it can be shown that ψ_u^i is given by

$$\psi_u^i = \Gamma^i \left(\frac{\bar{\gamma}^i - \beta\rho_u}{\gamma^i - \beta\rho_u} \right) \bar{\psi}_u^i, \quad (7)$$

where

$$\Gamma^i \equiv \frac{(\gamma^i - \beta q\rho_u)(\gamma^j - \beta q\rho_u) - \beta\rho_u(1-q)(\beta\rho_u(1-q) + \gamma^j - \gamma^i)}{(\gamma^i - \beta q\rho_u)(\gamma^j - \beta q\rho_u) - [\beta\rho_u(1-q)]^2} > 0. \quad (8)$$

In equation (7), the term in parentheses is always bigger than one, since $\bar{\gamma}^i > \gamma^i$.⁸ Instead, it holds that $\Gamma^i > 1$ if and only if $\gamma^j > \gamma^i$ (or equivalently $w^i < w^j$).

For a hawkish regime with $w^h < w^d$, being $\Gamma^h > 1$, equation (7) univocally implies that $\psi_u^h > \bar{\psi}_u^h$. This means that regime-switching objectives force the hawkish regime into a stronger inflation hike and a sharper output contraction in response to a positive markup shock, relative to the constant objectives counterpart. In other words, the hawkish regime faces a worse contemporaneous trade-off caused by the possibility of a future change to a dovish regime.

The results for a dovish regime are less clear-cut, since in that case, $\Gamma^d < 1$. The relation between ψ_u^d and $\bar{\psi}_u^d$ depends on the exact parameterization. If the hawkish regime assigns a sufficiently low weight to output (i.e., w^h is close to 0), regime-switching objectives may improve the trade-off faced by the dovish regime. This feature differentiates the effects of regime-switching objectives from those of limited commitment and constant objectives, as in Schaumburg and Tambalotti (2007). Indeed, limited commitment *per se* always worsens the trade-off in the markup shock response.

Dynamic response. After the initial inflation surge in response to a positive markup shock, inflation is reduced in subsequent periods. The possibility of regime switches impacts expectations and consequently the optimal speed at which inflation is reduced. As shown in Appendix A.1 the (absolute) inflation change, in comparison to

8. This follows from the definition of $\bar{\gamma}^i$ together with the fact that γ^i is increasing in q .

TABLE 1
CALIBRATION

Parameter	Value	Description
Structural		
β	0.99	Discount factor
κ	0.1275	Slope of Phillip's curve
σ	1.0	Real rate elasticity of output
Policymakers		
w^h	0.0213	Weight on output-gap hawkish regime
w^d	0.5	Weight on output-gap dovish regime
q	0.9	Probability of objectives remaining constant
Shocks		
ρ_u	0.2	Autocorrelation of markup shocks
ρ_r	0.9	Autocorrelation of real rate shocks
$\sigma_{e^u}(\%)$	0.2	Std. dev. markup innovation
$\sigma_{e^r}(\%)$	0.2	Std. dev. real rate innovation

the constant objective case, is given by

$$|E_0\pi_1 - \pi_0| - |E_0\bar{\pi}_1 - \bar{\pi}_0| = \left[\left(2 - \frac{1}{\gamma^i} - \rho_u \right) \Gamma^i \frac{\bar{\gamma}^i - \beta\rho_u}{\gamma^i - \beta\rho_u} - \left(2 - \frac{1}{\bar{\gamma}^i} - \rho_u \right) \right] \bar{\psi}_u^i u_0. \tag{9}$$

For a hawkish regime, because $\Gamma^h > 1$, a sufficient condition for (9) to be positive is

$$\rho_u < \frac{2\bar{\gamma}^h \gamma^h - \bar{\gamma}^h - \gamma^h}{\bar{\gamma}^h \gamma^h - \beta}. \tag{10}$$

Under standard calibrations of the relative weight of output stabilization (w), the latter condition is satisfied even in the limiting case with $\rho_u = 1$.⁹ This means that the hawkish regime is led to reduce inflation more rapidly under regime-switching objectives.¹⁰ The intuition for this result is the following. The possibility that the dovish regime takes place next period increases inflation expectations. Besides the optimal impact response described in equation (7), if the hawkish regime is in place, then next-period inflation is reduced faster. This anchoring effect on inflation expectations dampens the negative impact on current variables.

To provide a quantitative illustration of our results, we adopt a quarterly calibration that is summarized in Table 1. The structural parameters β , κ , and σ follow the calibration of Galí (2008).

9. Section A.1 in Appendix A explores more extreme calibrations for other parameters.

10. Using a similar argument as above, the sign of (9) depends on the parameterization and cannot be univocally determined for the dovish regime.

TABLE 2
SECOND MOMENTS AND WELFARE

	Full commitment	Limited commitment	Regime switches
Hawkish			
Output	3.395	3.540	3.579
Inflation	0.519	0.535	0.540
Av. period loss	0.536	0.575	0.588
Dovish			
Output	0.558	0.656	0.649
Inflation	1.028	1.072	1.061
Av. period loss	1.265	1.424	1.393

NOTES: The table reports the standard deviations of the variables (in relative terms of the standard deviation of the markup shock), as well as the implied welfare loss, conditional on being under the hawkish regime (upper panel) or the dovish regime (lower panel).

We set w^h according to the utility-based welfare criterion implied by those parameters. The value assigned to $w^d = 0.5$ implies that the dovish regime assigns to output stabilization half of the weight assigned to price stabilization. The persistence of policy objectives is measured by the parameter $q = 0.9$, which is in the range of recent estimates of Markov-switching New Keynesian models.¹¹ The autocorrelation of the markup shock is set to $\rho_u = 0.2$. There is no widespread consensus on the value of ρ_u . Values found in the literature range from the i.i.d. case considered in Rabanal and Rubio-Ramirez (2005) to 0.96 found in Ireland (2004). As discussed earlier, the choice of a low degree of persistence reduces the spillovers between policymakers. Finally, the standard deviation of the markup shock is set to 0.2% in line with Adam and Billi (2006), Davig and Doh (2008), and Bianchi (2013) in similar small-scale New Keynesian models.

Second moments and welfare are reported in Table 2. The upper and lower panels are conditional on the hawkish and dovish regime, respectively. Moving from full-commitment to regime-switching objectives leads to an increase in the volatility of output and inflation both for the hawkish and the dovish regime.¹² However, with regime-switching objectives, a switch from a dovish to a hawkish regime implies lower volatility of inflation but higher volatility of output.

2.2 Regime Switches in Output-Gap Targets

In this section, we consider two regimes that have different output-gap targets. This difference in targets can obviously be the consequence of disparate views on what kind of distortions and events should be accounted for by the central bank, as has been

11. See, for example, Davig and Doh (2008) and Bianchi (2013). Results are amplified when considering more frequent switches, or a central bank with a dual mandate ($w^d = 1$).

12. This result is due in part to a loss in credibility, as can be seen in Table 2 by comparing the volatilities with full and limited commitment. However, regime-switching objectives in comparison to limited commitment increase the volatilities for the hawk but reduce them for the dove. Accordingly, the hawk induces a positive welfare externality, whereas the dove introduces a negative externality with respect to the limited-commitment case.

quite evident in the recent crisis. In addition, another likely source of disagreement on the output-gap target lies in the measurement of the output level prevailing if prices would be flexible. Even if a consensus would exist that the output-gap target should be zero, as long as the flexible-price output level is not perfectly observed and is subject to mismeasurement, substantial disagreement on the operative output gap target can emerge. This issue is not a mere theoretical curiosity and is actually quite likely to occur in practice. Orphanides (2001, 2002) and several related papers show that structural breaks in productivity can be hard to detect and do lead to dramatically different concepts regarding the output-gap target and the conduct of monetary policy. Chari, Kehoe, and McGrattan (2009) also discuss that the interpretation of shocks is not always straightforward, which can lead to different views on the flexible-price output level.¹³

We consider the dovish and hawkish regimes to be characterized by $\tilde{y}^d > \tilde{y}^h \geq 0$ and $w^d = w^h$. The case with regime switches in inflation targets delivers similar results, and is considered in Appendix A.1. Also, for the brevity of exposition, we will now abstract from markup shocks, as such analysis was already carried out above. The dynamics are characterized by the system

$$\begin{bmatrix} \pi_t \\ y_t - \tilde{y}^i \end{bmatrix} = \begin{bmatrix} \frac{w}{\kappa} (1 - \psi_y) & \frac{1}{\gamma - \beta} \\ \psi_y & -\frac{\kappa}{w} \frac{1}{\gamma - \beta} \end{bmatrix} \begin{bmatrix} y_{t-1} - \tilde{y}^i \\ \left((1 - \Phi)\kappa\tilde{y}^i + \Phi\kappa \frac{\tilde{y}^j + \Phi\tilde{y}^i}{(1 + \Phi)} \right) \end{bmatrix}, \quad (11)$$

where $\Phi \equiv (\beta - \beta q)/(\gamma - \beta q) < 1$, $(\partial\Phi)/(\partial q) < 0$, and the system is initialized at $y_{-1} = \tilde{y}^i$ as described in the Appendix. The dynamics coincide with those described in Section 2.1, for the particular case of markup shocks with a unitary root ($\rho_u = 1$). The coefficient on the lagged output gap to target difference ($y_{t-1} - \tilde{y}^i$) remains unchanged. In addition, the coefficient $1/(\gamma - \beta)$ is equivalent to ψ_u^i , as given by equation (7), after imposing $\gamma^i = \gamma^j$ holding in the present case, and setting $\rho_u = 1$. That coefficient now multiplies a constant term that is increasing on the output-gap targets of both regimes. Under the plausible assumption of positive output-gap targets, that term is always positive.

In the Appendix, we show that even in this case, long-run inflation converges to zero under both regimes. This result confirms the robustness of the price-stability principle, and indicates that there is no long-run exploitation of the Phillips curve (even though $\beta < 1$).

There is one difference between regime switches in relative weights and in output-gap targets. When regimes only differ in the relative weight of output, the nonstochastic steady state of the economy is identical across regimes (i.e., $\pi_t = 0$ and $y_t = 0$).

13. Policymakers themselves seem to be aware of such issues; answering a question on “the so-called ‘natural rate’ of unemployment” Alan Greenspan on June 22, 1994 said “[w]hile the idea of a national ‘threshold’ at which short-term inflation rises or falls is statistically appealing, it is very difficult, in practice, to arrive at useful estimates that would identify such a natural rate.” (see Greenspan 1994). For a discussion, see Ball and Mankiw (2002) and for recent attempts to estimate the efficient level of output see, for example, Gali, Gertler, and Lopez-Salido (2007) and Justiniano, Primiceri, and Tambalotti (2013).

Starting from that steady state, fluctuations are entirely driven by the markup shocks, while switches to a different regime *per se* would produce no effect. When, instead, regimes have different output gap targets, the steady state under the two regimes is different (with $\pi^d = \pi^h = 0$ and $y^d > y^h > 0$). At that steady state, switching to a distinct regime gives rise to transition dynamics, even in the absence of shocks.

Despite the different source of the dynamics, regime switches in relative weights and targets display some similarities as illustrated in Figure 1. The first two rows in Panel (a) refer to regime switches in relative weights. Those graphs plot the impulse responses to a markup shock for the values of $\rho_u = 0.2$ and $\rho_u = 0.99$. One can read in the left and right columns the evolution of the economy if the hawkish or dovish regime remain in place, respectively.

The last row in Panel (a) plots a slightly different exercise related to regime switches in output-gap targets. The last row displays the transition dynamics after a regime switch has occurred. For instance, if the hawkish regime is in place, one can read the evolution of the economy in the left columns. Should there be a change to the dovish regime, one can read the evolution of the economy in the right columns, starting from period 0 onward.

In all graphs in Panel (a), the regime-switching scenario is compared with two benchmark cases: full commitment (in which case there are no regime switches) and limited commitment (in which case there are reoptimizations but no regime switches). The lines in each panel are directly comparable and one can examine how different assumptions affect the evolution of the economy. For ease of inspection, Panel (b) plots the difference between limited commitment and full commitment, and between regime switches and limited commitment.

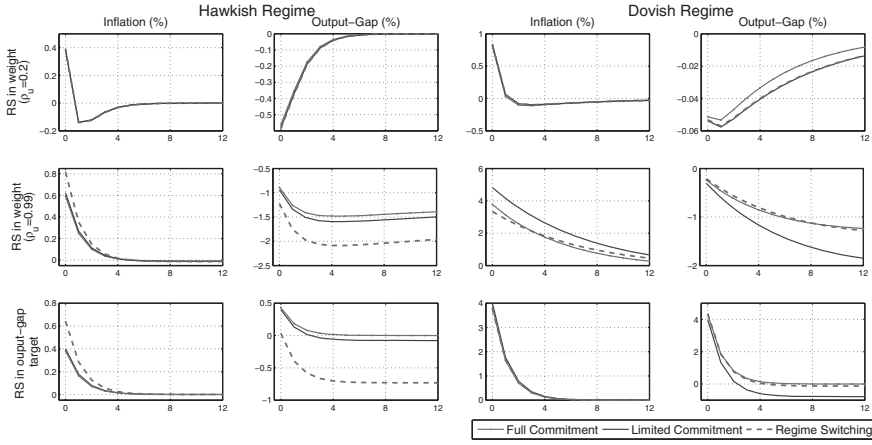
Figure 1 shows that regime switches in relative weights and in targets produce similar deviations from the full-commitment benchmark. For instance, the presence of a dovish regime induces the hawk to increase inflation and reduce output in the first period, and to reduce inflation faster in the following periods.

For a dovish regime, the effects are less pronounced and the overall effects are ambiguous in sign. This ambiguity can be better understood by decomposing two effects, as shown in Panel (b). With regime switches, there is both an effect due to limited commitment and an effect due to the presence of a hawkish regime. Limited commitment *per se* worsens the inflation–output trade-off and leads to an increase in inflation and a reduction in output (line with circles). But the possibility of switching to a hawkish regime (line with crosses) has the opposite effect, reducing inflation and increasing output. The net effect thus depends on the relative magnitude of the two opposite forces.¹⁴

A comparison between the first two rows of Panel (a) also shows that the effects of regime switches are more pronounced when markup shocks are more persistent. When persistence is low ($\rho_u = 0.2$), the dynamics under regime switches

14. Also, notice that if one only measures the effects of regime switches relative to limited commitment, then there is only one force at play. As expected, the effects of the hawk and the dove are mostly symmetric. This effect can be seen by comparing the lines with crosses for the hawk and the dove in Panel (b).

(a) Dynamics of Inflation and Output-Gap in Levels



(b) Decomposition of the Effects Limited Commitment vs Different Regimes

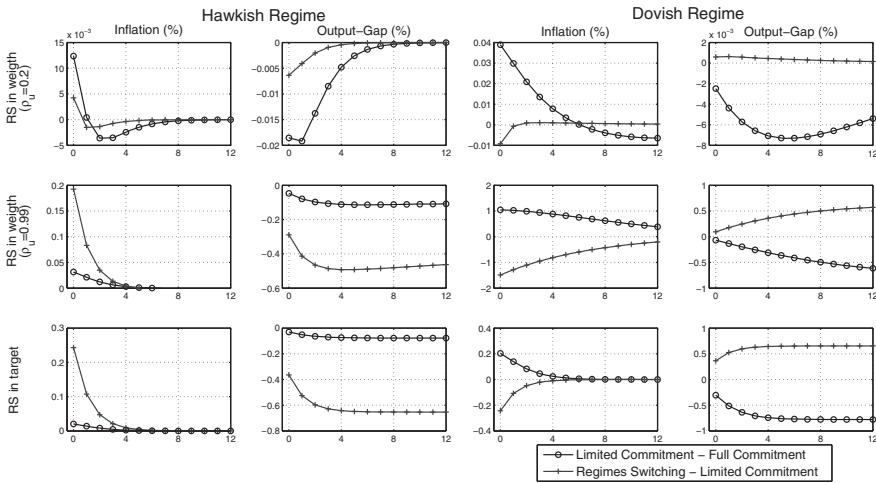


FIG. 1. Regime Switches in Weights versus Targets.

NOTES: The figure compares the dynamics of inflation and output gap under the hawkish (first two columns) and dovish (last two columns) regimes. Panel (a) displays the dynamics for three scenarios: full commitment, limited commitment, and regime switches. Panel (b) decomposes the effects of limited commitment (i.e., $x^{\text{Limited-commitment}} - x^{\text{Full-commitment}}$) from those of possible switches to an alternative regime ($x^{\text{Regime-switches}} - x^{\text{Limited-commitment}}$). In each panel, the first two rows refer to the model with switches in the output gap weights, and plot the effects on the impulse response function to markup shocks. In each panel, the last row refers to the model with switches in the output gap target, and plots the effects on the transition dynamics after a regime switch. All the values are in percentages and inflation is annualized. In all the exercises, the initial condition is set to $\lambda_{-1} = 0$ (or $y_{-1} = \bar{y}$), as previous commitments are not binding.

resemble those under full commitment because the spillovers across regimes are relatively small. That parameterization is then used in the Monte Carlo experiments of the next section, as it should facilitate the identification of the differences between hawkish and dovish regimes.¹⁵ When markup shocks are very persistent ($\rho_u = 0.99$), the economy behaves similarly to the case with positive output gap targets.

These considerations show that our results of Section 2.1 do not necessarily depend on the particular class of regime switches considered. Similarly, to markup shocks, regime switches in output gap targets create a wedge between inflation and the output gap.¹⁶

2.3 Regime Switches Every T Periods

It may not be entirely plausible to assume that regimes can switch in every period. For instance, central banks' presidents and committees have tenures that last for several years. We investigate whether incorporating this feature introduces additional interactions among policy regimes. We assume that objectives remain unchanged with certainty for T periods. Every T periods, current objectives can eventually change ($t = T, 2T, 3T, \dots$). In this setup, the current regime has a few periods to "prepare for the handover" and the problem of the central bank can be written as

$$V^i = \max_{\{\pi_t, y_t\}_{t=0}^{\infty}} E_0 \sum_{m=0}^{\infty} (\beta^T q)^m \left[-\frac{1}{2} \sum_{t=0}^{T-1} \beta^t [\pi_{m+t}^2 + w^i (y_{m+t} - \tilde{y}^i)^2] + \beta^T (1-q) V^{ij} \right], \quad (12)$$

$$s.t. \quad \pi_{mT+t} = \kappa y_{mT+t} + \beta E_{mT+t}(\pi_{mT+t+1}) \quad t = 0, 1, \dots, T-2, \quad (13)$$

$$\pi_{mT+t} = \kappa y_{mT+t} + (1-q)\beta E_{mT+t}(\pi_{mT+t+1}^j) + q\beta E_{mT+t}(\pi_{mT+t+1}^i) \\ t = T-1, \forall m = 0, \dots, \infty \quad (14)$$

15. We confirm this conjecture in the next section. The hawkish and dovish regimes behave differently. If there are no interactions between the two regimes (ρ_u is low), it is easier to distinguish them.

16. Since the microfoundations of cost-push/markup shocks are not well understood, we regard this result as a contribution in itself. However, there is one important difference between the effects we identify and the traditional cost-push/markup shocks. The downward pressure in output and upward pressure in inflation occurs in response to the anticipation of future dovish objectives, and not in response to a current change in objectives. When the output gap target switches and becomes dovish, both inflation and output expand. Therefore, it is the anticipation and not the realization of the regime switch that resembles the traditional (positive) cost-push/markup shock.

where m indexes the sequence of regimes each lasting for T periods.¹⁷ In order to solve problem (12), we first write its recursive formulation. To do so, we apply the technique of Marcet and Marimon (1998) and write the problem as a saddle point functional equation that generalizes the usual Bellman equation. The proof of that result requires considering each tenure as one fictional big period, and then applying the results of Debortoli and Nunes (2010a) to address the probabilistic switch at the end of each tenure. Proposition 1 in Appendix D proves this result in detail. As stated in Proposition 2 in Appendix D, the solution can be characterized as tenure-invariant functions of the Lagrange multipliers associated with constraints (13) and (14).

Figure 2 plots the optimal policy functions with regime-switching objectives (continuous line). The upper and lower panels correspond to the hawkish and dovish regime with output gap targets of 0.01 and 0.1, respectively.¹⁸ Each regime implements the policy functions shown in each period until the regime is changed. We calibrate the model such that regimes can only change with probability $q = 0.5$ every $T = 4$ periods (signaled with continuous vertical lines). For comparison, Figure 2 also plots the policy functions that occur in a limited commitment setting without regime changes (dashed line).

The hawkish regime implements a low inflation level immediately after knowing that the dovish regime has dissipated and objectives will not change in the following four periods (periods 5, 9, and 13 in the graph). This pattern produces an anchoring effect on inflation expectations. Different from the model where regime changes can occur in every period, the strengths of the accommodation and anchoring effects are not constant over time. These two effects explain why the hawk starts with low inflation and then increases it.

This model puts in evidence the interactions between the two regimes and the potential difficulties in identifying them through simple processes—it is difficult to distinguish whether the accommodation effect is making a hawk increase inflation, or if, in fact, the regime already became dovish. Our findings are qualitatively robust to two alternative specifications. First, following Galí and Gertler (1999), we have also examined the results with a hybrid NKPC ($\pi_t = \alpha\beta E_t\pi_{t+1} + (1 - \alpha)\beta\pi_{t-1} + \kappa y_t + u_t$), in which case the value function derivative enters the first-order conditions. Second, we assumed that objectives may change but the central bank never reoptimizes, and therefore makes state contingent promises regarding the objectives.¹⁹

17. This model is different from considering that there is a news shock that hits every period t announcing that in period $t + T$, objectives may change. That setup is similar to the one being considered here in the sense that the current regime has a few periods to “prepare for the handover.”

18. Schaumburg and Tambalotti (2007) also consider an output gap target of 0.1.

19. In this case, the term with lagged output gap or lagged Lagrange multiplier is not reset to zero. The corresponding derivations are omitted for brevity and available in the working paper version (Debortoli and Nunes 2010b).

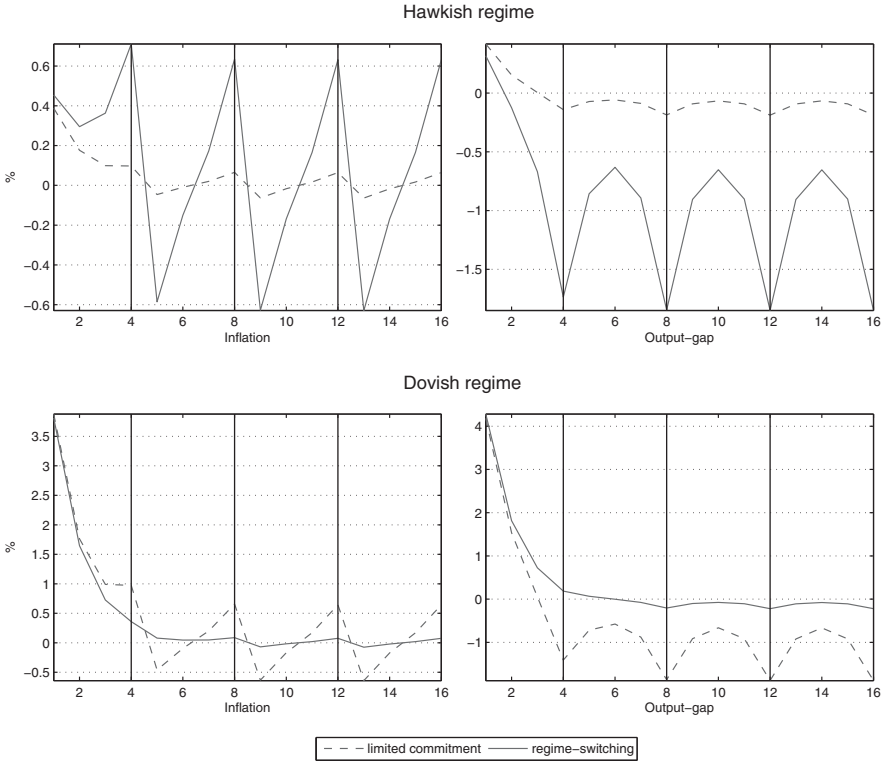


FIG. 2. Regime Switches in T Periods.

NOTES: This figure refers to the model where objectives can only change every T period. The upper panel plots the evolution of the economy (inflation and output gap) for a hawkish regime, and the lower panel refers to a dovish regime. Changes in objectives can only occur every four periods—periods marked with continuous vertical lines. The case with regime switches is plotted with a continuous line. The case with limited commitment and without regime switches is plotted with a dashed line. In all panels, the horizontal axis refers to the number of periods elapsed after the last regime change or reoptimization. Values are in percentages and inflation is annualized. The graphs are truncated at period 16 in the figure for convenience.

3. REGIME SWITCHES IN OBJECTIVES AND IN INTEREST RATE RULES

The most common approach followed in the monetary policy literature is to model regime switches as exogenous changes in the parameters of an interest rate rule. Some empirical studies support the view that interest rate rule parameters have changed over time.²⁰ Such changes are typically interpreted as a signal of switches between “good” and “bad” policies.

20. For instance, Clarida, Galí, and Gertler (2000) and Lubik and Schorfheide (2004) find evidence of monetary regime switches, whereas Sims and Zha (2006) do not after controlling for stochastic volatility.

The goal of this section is to understand if there is a relationship between changes in the parameters of an interest rate rule (the common approach in the literature) and the changes in policy preferences modeled in the previous sections. We proceed in two different directions. First, we compare some qualitative implications of the two approaches. Second, we perform a Monte Carlo experiment, where we check if changes in policy objectives would be detected by estimating an interest rate rule with Markov-switching coefficients.

3.1 Qualitative Effects on Volatilities

Consider the interest rate rule

$$ir_t = \rho + \phi_\pi \pi_t + \phi_y y_t, \tag{15}$$

where the parameters ϕ_π and ϕ_y measure the response of the interest rate to changes in inflation and output gap, respectively.²¹ In order to show how this policy rule affects the behavior of inflation and output, we need to supplement the NKPC (1) with an IS equation describing the demand side of the economy

$$y_t = E_t y_{t+1} - \frac{1}{\sigma} (ir_t - E_t \pi_{t+1} - r_t^n), \tag{16}$$

where the term r_t^n represents a real interest rate shock, which may result either from demand or supply shocks, and is assumed to follow an AR(1) process $r_t^n = \rho_r r_{t-1}^n + e_t^r$. The calibration of the real rate shock follows standard values described in Table 1. For simplicity and without loss of generality, we assume the innovation to the real rate shock, e_t^r , and the innovation to the markup shock, e_t^u , to be uncorrelated.²²

The solution of the three-equation system (1), (15), and (16) is given by

$$\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = \begin{bmatrix} (\sigma(1 - \rho_u) + \phi_y)\Lambda_u & \kappa \Lambda_r \\ -(\phi_\pi - \rho_u)\Lambda_u & (1 - \beta\rho_r)\Lambda_r \end{bmatrix} \begin{bmatrix} u_t \\ r_t^n \end{bmatrix} \equiv H \begin{bmatrix} u_t \\ r_t^n \end{bmatrix}, \tag{17}$$

where $\Lambda_u \equiv 1/((1 - \beta\rho_u)[\sigma(1 - \rho_u) + \phi_y] + \kappa(\phi_\pi - \rho_u))$ and $\Lambda_r \equiv 1/((1 - \beta\rho_r)[\sigma(1 - \rho_r) + \phi_y] + \kappa(\phi_\pi - \rho_r))$.

As opposed to optimal policy, here, inflation and output also respond to the real rate shock r_t^n . The intensity of the response to r_t^n depends on the magnitude of ϕ_π and ϕ_y through the parameter Λ_r . In particular, since $(\partial \Lambda_r)/(\partial \phi_\pi) < 0$ and $(\partial \Lambda_r)/(\partial \phi_y) < 0$, an increase in the policy parameters ϕ_π or ϕ_y leads to a simultaneous reduction in the volatility of both inflation and output, conditional on the shock r_t^n . Unconditional

21. We have explored numerically that the results obtained are robust to many alternative interest rate rules displaying forward- and backward-looking terms in inflation, output gap, and interest rate.

22. This assumption greatly simplifies our algebra without affecting qualitatively the results. A nonzero correlation between r_t^n and u_t arises when the flexible price equilibrium is not attainable. For example, in a model with sticky prices and sticky wages, the term u_t would be an endogenous variable, which is a linear function of the shock r_t^n .

volatilities may follow a similar pattern, as long as the volatility of the real rate shocks is sufficiently higher than the volatility of markup shocks, as seems more plausible from an empirical viewpoint (see Rabanal and Rubio-Ramirez 2005, Adam and Billi 2006). The Appendix shows that a change in the policy parameters ϕ_π and ϕ_y drive the volatilities of inflation and output gap in the same direction, as long as

$$\frac{\sigma_r^2}{\sigma_u^2} > \max \left(\frac{\Lambda_u}{\Lambda_r} \frac{H_{11}}{H_{12}} \frac{1 - (1 - \beta\rho_u)H_{11}}{(1 - \beta\rho_r)H_{12}}, \frac{\Lambda_u}{\Lambda_r} \frac{H_{21}}{H_{22}} \frac{1 + \kappa H_{21}}{\kappa H_{22}} \right). \quad (18)$$

This inequality is easily satisfied. According to our calibration, for the above condition to be violated, the volatility of the innovations to markup shocks should be more than 10 times than that of real shocks.

We can then conclude that changes in simple rule parameters cannot always be interpreted as changes in policy objectives. Changes in simple rules may and usually drive the volatilities of output gap and inflation in the same direction.²³ On the contrary, we have shown that a change in the central banks' preferences should produce an increase in the volatility of one variable, but a decrease in the volatility of the other.

Empirical studies have typically found that both the volatility of inflation and output were reduced after the Volcker disinflation period. This finding cannot be solely associated with a reduction in the relative weight assigned to output stabilization. Instead, as shown in Table 2, a simultaneous decrease in the volatilities of inflation and output gap can be associated with a change in the probability of regime switches (e.g., moving from the third to the first column), or a change in the perception of the alternative regime objectives. Both the explanations are not related to the preferences of the incumbent regime or to factors fully under its control. It is then unclear to what extent changes in simple rule parameters can be interpreted as "good" or "bad" policies for which the current central bank is responsible.

3.2 Are Regime-Switching Objectives Detected by Markov-Switching Simple Rules?

In this section, we examine whether changes in policy objectives could be identified in the data as changes in interest rate rule coefficients. We try to address this issue through a Monte Carlo exercise. We simulate our baseline model of Section 2.1 for different realizations of the markup shock, the real rate shock, and the regime-switching shock. We then use the resulting series to estimate the following (standard) Taylor-type interest rate rule

$$ir_t = \alpha + \phi_{ir} ir_{t-1} + \phi_\pi \pi_t + \phi_y y_t + \epsilon_t^{ir}, \quad (19)$$

23. Our analysis focuses mainly on simple interest rate rules because these are usually employed in empirical studies. Giannoni and Woodford (2010) discuss targeting rules implementing the optimal policy with similar form to simple interest rate rules. In some cases, those rules can be invariant to regime-switching objectives. However, the targeting rule parameters depend on the preferences. If the implied cross restrictions on the parameters are ignored, similar issues arise.

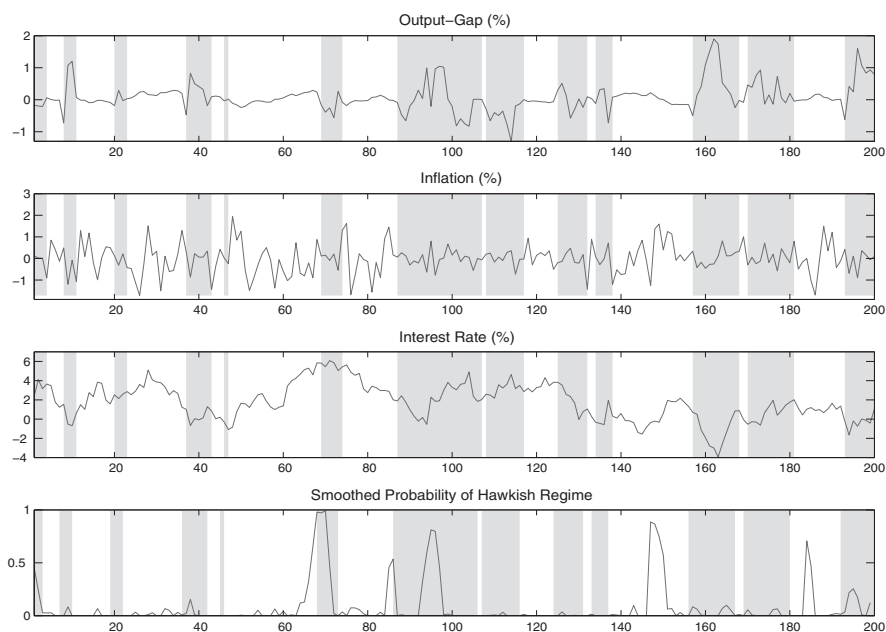


FIG. 3. Sample Simulation.

NOTES: The upper three panels show the evolution of the economy in a sample simulation. Shaded areas indicate when the hawkish regime is in place. The last panel shows the probabilities of the hawkish regime identified by the estimation.

where ϵ_t^{ir} is an unobservable residual, assumed to be uncorrelated with the regressors.²⁴ Equation (19) includes a lagged interest rate term, not only because of its empirical plausibility, but also because in our model, all the variables display an endogenous persistence component due to the presence of past commitments (see, e.g., Woodford 2003). Following Hamilton (1989) and Kim and Nelson (1999), we estimate equation (19) by maximum likelihood allowing for ϕ_{ir} , ϕ_π , ϕ_y , and ϵ_t^{ir} to follow a two-state Markov-switching process.²⁵

Figure 3 shows a sample simulation of the model. Shaded areas indicate when the hawkish regime is in place. The time series for all variables seem plausible. The last panel shows the probabilities of the hawkish regime as identified by the estimation. As seen in the figure, the regime-switching estimation is not very successful in capturing regime switches in objectives through regime switches in simple

24. The Monte Carlo exercise uses 1,000 histories of 200 periods each, which is comparable to the number of quarters available using actual data. The exercise also assumes that the econometrician knows all the parameters except those of the simple rule. Removing that assumption gives less information to the econometrician and may imply that the misspecification problem biases the estimation of the other structural parameters. Examining such bias is interesting but goes beyond the scope of this paper.

25. We chose maximum likelihood estimation because we need to estimate the model many times, and adopting Bayesian methods would increase the computation time dramatically.

TABLE 3
 MARKOV-SWITCHING MAXIMUM LIKELIHOOD ESTIMATION OF A SIMPLE INTEREST RATE RULE

	Baseline	$\sigma_u = \sigma_r$	$\rho_u = 0$			
% of MSC(2) < MSC(1)	0.13	0.36	0.43			
% of Right Regime	0.60	0.72	0.77			
<i>Means of parameters estimates</i>						
	Hawk	Dove	Hawk	Dove	Hawk	Dove
ϕ_{ir}	0.8577 (0.0185)	0.8452 (0.0319)	0.7526 (0.0238)		0.8417 (0.0299)	
ϕ_π	0.2270 (0.0635)	0.1469 (0.0792)	0.7200 (0.1066)	-0.1124 (0.0547)	2.3285 (0.2294)	-0.2578 (0.1232)
ϕ_y	-0.1954 (0.0154)	-0.1780 (0.0288)	-0.1525 (0.0193)	-0.1703 (0.0426)	-0.0775 (0.0428)	-0.3482 (0.1159)
σ_e^{ir} (%)	0.1396 (0.040)	0.1470 (0.049)	0.2007 (0.1030)	0.1980 (0.0843)	0.1664 (0.0686)	0.1841 (0.0762)
q	0.6191 (0.0692)	0.6525 (0.0711)	0.7685 (0.1097)	0.7287 (0.0881)	0.8413 (0.0913)	0.8400 (0.0795)
Determinacy	Yes	No	Yes	No	Yes	No

NOTES: The Markov-switching criterion was computed for the two-state model (MSC(2)) and the alternative specification with constant parameter across regimes (MSC(1)). The second row displays the average fraction of periods the estimation correctly identifies the regime in power. For each parameter, the table presents the mean estimate over 1,000 simulations, each of 200 observations. Standard errors are reported in parentheses. The convergence to global maxima was checked using alternative initial conditions.

interest rate rules. Given the theoretical results derived earlier, this result is largely expected.

Table 3 shows that according to the Markov-switching criterion developed by Smith, Naik, and Tsai (2006), the two-regime model is preferred to a single-regime specification only in 13% of the cases. In addition, the algorithm identifies correctly the regime in place in a certain period only in 60% of the cases, a relatively small improvement over the 50% probability of being right without any information.

The mean estimates seem plausible. The coefficient on the lagged interest rate is in accordance with empirical studies. The coefficients ϕ_π and ϕ_y are also plausible for the hawk.²⁶ The two regimes differ in an important dimension. While the policy rule followed by the hawkish regime implies a determinate equilibrium (when combined with the other equations of the model), the dovish regime implies an indeterminate one. This result is consistent with many empirical studies arguing that monetary policy became more hawkish leading to equilibrium determinacy. But our results are due to a misinterpretation of the source of regime switches rather than the determinacy characteristics. Indeterminacy is not a feature of the data-generating

26. The estimates in some studies should be changed to $\phi_\pi/(1 - \phi_{ir})$ for direct comparability.

process—both regimes follow an optimal policy delivering a determinate solution.²⁷

The results in Table 3 show the risks associated with estimating simple policy rules to draw conclusions about the underlying objectives of the central bank. The presence of regime switches may be wrongly rejected, the specific regime in power may be hard to identify, and the determinacy properties that each regime would imply may be erroneous.

We performed many alternative exercises to check the robustness of our results, as reported in the last two columns of Table 3. In these exercises, we constrained ϕ_{ir} to be constant across regimes, which simplifies the algorithm task to identify switches in the interest rate response to inflation and the output gap. Also, we increased the markup volatility to the level of the real rate volatility, which we see as an upper bound. As discussed in Section 3.1, such calibration allows the simple interest rate rule to better capture the optimal policy data-generating process. The performance of the estimation algorithm improves, but the main conclusions reached with the baseline calibration are still valid.

Finally, we checked the dependence of the results on the presence of a strategic interaction between regimes. To do so, we simulated the model imposing $\rho_u = 0$, thus eliminating the spillovers between the regimes. The performance of the estimation algorithm improves significantly, since it detects the presence of regime switches in 43% of the cases, and the correct regime is identified in 77% of the periods. This result, however, does not undermine the main conclusions obtained above, but rather highlights why explicitly modeling the strategic interactions is important. Strategic interactions between different policy regimes would be present in any economy with endogenous state variables, like private capital or public debt. In those cases, estimating simple policy rules that ignore such interactions may lead to erroneous conclusions about the underlying monetary policy decision process.

4. CONCLUSIONS

Regime shifts in macroeconomic relationships in general and central bank behavior in particular have been identified in the data. We study the effects of regime switching in objectives as a potential source of regime changes, and characterize policy choices and allocations in a variety of models and specifications.

The paper illustrates some perils of using switches in simple interest rate rules for positive and normative analysis, and identifies the conditions under which such analyses are less prone to error. We show that changes in simple rules cannot be

27. In the work of Davig and Leeper (2007), considering the dove in isolation would deliver an indeterminate solution, but the presence of the hawk renders the solution determinate. Here, the problem is of a different nature. Indeterminacy is detected even though the hawk and the dove always deliver a determinate equilibrium both in isolation and jointly. The calculation of determinacy conditions with regime switches and especially when the dynamic system has lagged variables is not clear and is subject to debate (see Davig and Leeper 2007, Farmer, Waggoner, and Zha 2009, among others). Due to this reason, we have calculated the determinacy conditions ignoring the possibility of regime changes.

interpreted solely as changes in policy objectives, but are potentially related to factors not under the central bank's control. Similarly, it may be difficult to detect the presence of regime switches, the regime in place, and the determinacy conditions through changes in simple rules.

It can be argued that central banks do not behave optimally, and that changes in simple rules reflect the central banks' ability to approach optimal policy. Our intuitions do not require a strictly optimal behavior, only that central banks recognize a trade-off between inflation and output stabilization. While switches in preferences should lead to movements in the volatilities of two variables in opposite directions, usually, switches in simple interest rate rules lead to movements in volatilities in the same direction.

Finally, it is not our claim that regime-dependent objectives are the main source of regime switches. Other factors not considered in the paper may also play a role, like structural factors, or the information available to central banks. Combining those features with regime-switching objectives would constitute a crucial step toward the identification and interpretation of the sources of monetary regime switches.

APPENDIX A: REGIME SWITCHES IN THE RELATIVE WEIGHT OF OUTPUT-GAP STABILIZATION

Here, we consider a particular case of problem (4), where $\tilde{y}^i = \tilde{y}^j = 0$, $w^i \neq w^j$, and $0 < q \leq 1$. For notational convenience, we suppress the superscript i , and indicate with the superscript j the variables related to the alternative regime. The first-order conditions of the problem are given by

$$\pi_t = -\lambda_t + \lambda_{t-1}, \quad (\text{A1})$$

$$y_t = \frac{\kappa}{w} \lambda_t, \quad (\text{A2})$$

$$\pi_t = \kappa y_t + \beta q E_t \pi_{t+1} + \beta(1 - q) E_t \pi_{t+1}^j + u_t, \quad (\text{A3})$$

where λ_t is the Lagrange multiplier associated with the NKPC, and $\lambda_{-1} = 0$ following the framework of Marcet and Marimon (1998). Combining equations (A1) and (A2) yields equation (4); the condition $\lambda_{-1} = 0$ is matched with $y_{-1} = 0$.

Since the model is linear-quadratic, we guess that expected inflation prevailing when objectives change is given by a linear rule $E_t \pi_{t+1}^j = \psi_u^j \rho_u u_t$, where ψ_u is a coefficient to be determined. Rearranging equations (A1)–(A3), the following second-order difference equation is obtained:

$$\left[\beta q L^{-2} - \left(1 + \beta q + \frac{\kappa^2}{w} \right) L^{-1} + 1 \right] \lambda_{t-1} = [1 + \beta(1 - q) \psi_u^j \rho_u] u_t, \quad (\text{A4})$$

whose solution is given by

$$(1 - \gamma_2 L^{-1})(1 - \gamma L^{-1})\lambda_{t-1} = [1 + \beta(1 - q)\psi_u^j \rho_u] u_t, \tag{A5}$$

where

$$\gamma = \frac{(1 + \beta q + \frac{\kappa^2}{w}) + \sqrt{(1 + \beta q + \frac{\kappa^2}{w})^2 - 4\beta q}}{2}, \tag{A6}$$

$$\gamma_2 = \frac{(1 + \beta q + \frac{\kappa^2}{w}) - \sqrt{(1 + \beta q + \frac{\kappa^2}{w})^2 - 4\beta q}}{2}. \tag{A7}$$

Note that $\gamma\gamma_2 = \beta q$ and $\gamma + \gamma_2 = (1 + \beta q + \kappa^2/w)$ and $0 < \gamma_2 < 1 < \gamma$.²⁸ Moreover,

$$\frac{\partial \gamma}{\partial q} = \frac{\beta}{2} \left(1 + \frac{(\gamma + \gamma_2) - 2}{\gamma - \gamma_2} \right) = \beta \left(\frac{\gamma - 1}{\gamma - \gamma_2} \right) > 0, \tag{A8}$$

and $(\partial \gamma)/(\partial w) < 0$. The unique stable solution to (A5) is given by the expression

$$\lambda_t = \psi_y \lambda_{t-1} - \psi_u u_t, \tag{A9}$$

where $\psi_y \equiv 1/\gamma$ and $\psi_u \equiv (1 + \beta(1 - q)\psi_u^j \rho_u)/(\gamma - \beta q \rho_u)$. Combining (A9) with (A1), using (A2) to eliminate the Lagrange multiplier, and imposing the initial condition $\lambda_{-1} = y_{-1} = 0$, equation (5) in the main text is obtained.

In the limiting case where $q = 0$, equations (A2) and (A3) remain unaltered, while (A1) takes the form $\pi_t = -\lambda_t$. Solving for the equilibrium, it holds

$$\lambda_t = -\frac{1 + \beta\psi_u^j \rho_u}{1 + \frac{\kappa^2}{w}} u_t. \tag{A10}$$

The resulting law of motion is similar to (A9). Indeed, the coefficient multiplying u_t is the same as ψ_u^i in (A9) after imposing the condition $q = 0$. However, none of the variables depend on λ_{t-1} (or equivalently on y_{t-1}).

Now we show that in a nonstochastic steady state where a given regime i continues, inflation converges to zero ($\pi_t \rightarrow 0$).

PROOF. Since $\gamma^i > 1$ and $\psi_y^i \equiv 1/(\gamma^i)$, it follows that $\psi_y^i < 1$. To determine the nonstochastic steady state where a given regime i continues forever, one needs to

28. The solution is always a real number since $(1 + \beta q + \kappa^2/w)^2 - 4\beta q > (1 + \beta q)^2 - 4\beta q = (1 - \beta q)^2 > 0$.

iterate forward on equation (5). Since we are interested in a nonstochastic steady state, we set all future shocks to zero ($\{u_t\}_0^\infty = 0$). Because $\psi_y^i < 1$, it is easy to see in the law of motion of y_t (second row of equation (5)) that $y_{ss} = 0$. Then, it also follows from the first row of equation (5) that $\pi_{ss} = 0$. \square

A.1 Slope of the Impulse Response Function

Given the above law of motion, it follows that in response to a markup shock

$$\begin{aligned}\pi_0 &= \psi_u u_0, \\ E_0 \pi_1 &= -(1 - \psi_y - \rho_u) \psi_u u_0.\end{aligned}\tag{A11}$$

Following a positive markup shock, the (absolute) initial change in inflation is therefore given by

$$E_0 |\pi_1 - \pi_0| = (2 - \psi_y - \rho_u) \psi_u u_0.\tag{A12}$$

An identical expression holds for the (standard) full-commitment and constant objectives case, whose corresponding variables are indicated with an upper bar. In comparison to that case, the slope of the impulse response function is then given by

$$\begin{aligned}|E_0 \pi_1 - \pi_0| - |E_0 \bar{\pi}_1 - \bar{\pi}_0| &= [(2 - \psi_y - \rho_u) \psi_u - (2 - \bar{\psi}_y - \rho_u) \bar{\psi}_u] u_0 \\ &= \left[\left(2 - \frac{1}{\gamma} - \rho_u\right) \Gamma \frac{\bar{\gamma} - \beta \rho_u}{\gamma - \beta \rho_u} - \left(2 - \frac{1}{\bar{\gamma}} - \rho_u\right) \right] \bar{\psi}_u^i u_0,\end{aligned}\tag{A13}$$

which coincides with (9), and where the second equality is obtained using equation (7) and the definition of ψ_y and $\bar{\psi}_y$. For that expression to be positive, and setting $\Gamma = 1$ —the lowest possible value taken by that parameter for a hawkish regime—it must be that

$$\left(2 - \frac{1}{\gamma} - \rho_u\right) (\bar{\gamma} - \beta \rho_u) > \left(2 - \frac{1}{\bar{\gamma}} - \rho_u\right) (\gamma - \beta \rho_u),\tag{A14}$$

or equivalently,

$$\begin{aligned}(\bar{\gamma} - \gamma) [2\bar{\gamma}\gamma - \bar{\gamma} - \gamma - \rho_u(\bar{\gamma}\gamma - \beta)] &> 0 \\ \Rightarrow \rho_u &< \frac{2\bar{\gamma}\gamma - \bar{\gamma} - \gamma}{\bar{\gamma}\gamma - \beta}.\end{aligned}\tag{A15}$$

The right-hand side of (A15) only depends on the parameters β and q and on the ratio κ^2/w . Fixing κ and β to the values described in Table 1, Figure 4 shows that condition

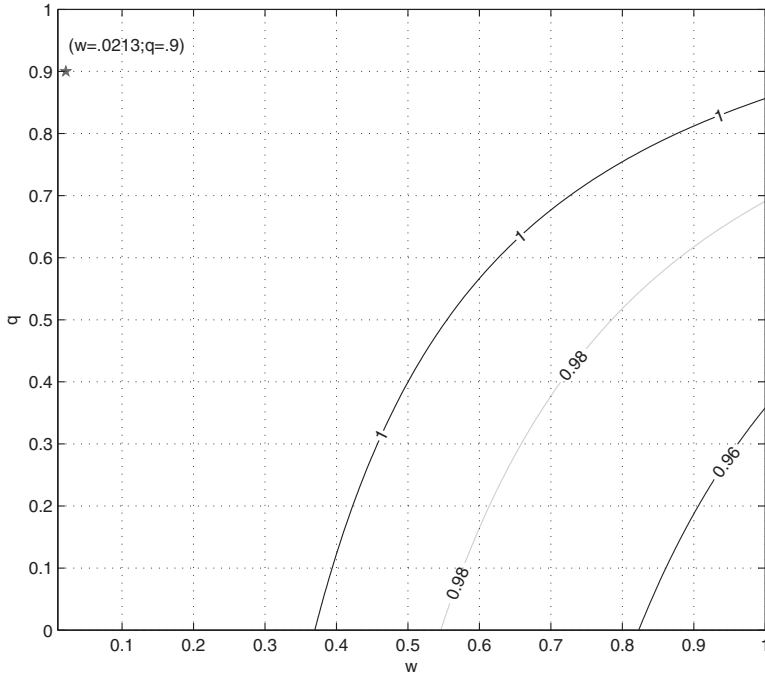


FIG. 4. Parameters Satisfying Condition (A15).

NOTES: For any value of ρ_u , the regions above the corresponding contour line indicate the values of w and q satisfying condition (A15). The dot on the top-left corner indicates our baseline calibration for those parameters. The parameters κ and β are set to the values described in Table 1.

(A15) is easily satisfied for standard calibrations. The condition is not satisfied only if one considers an extreme calibration in several parameters simultaneously—very low credibility coupled with a very high persistence of markup shocks and a weight ω well above microfounded values.

APPENDIX B: REGIME SWITCHES IN OUTPUT-GAP TARGETS

Consider that $\tilde{y}^j > \tilde{y}^i$, $w^j = w^i$, and for simplicity that the markup shock is not present, then the first-order conditions of regime i are

$$\pi_t = -\lambda_t + \lambda_{t-1}, \tag{B1}$$

$$(y_t - \tilde{y}) = \frac{\kappa}{w} \lambda_t, \tag{B2}$$

$$\pi_t = \kappa y_t + \beta q \pi_{t+1} + \beta (1 - q) \pi_0^j. \tag{B3}$$

Substituting (B1) and (B2) into (B3), and solving the resulting second-order difference equation, the following expression is obtained:

$$\lambda_t = \frac{1}{\gamma} \lambda_{t-1} - \frac{1}{\gamma - \beta q} \left(\kappa \tilde{y} + \beta (1 - q) \pi_0^j \right), \tag{B4}$$

where γ is defined as in equation (A6). For convenience, define $\Phi \equiv \beta (1 - q) / (\gamma - \beta q) < 1$, and notice that $(\partial \Phi) / (\partial q) < 0$. Assuming regime j is solving a symmetric problem, equations (B4) and (B1), together with the initial condition $\lambda_{-1} = 0$, imply

$$\pi_0^j = \frac{\Phi}{(1 - \Phi)} \frac{\kappa}{\beta (1 - q)} \frac{\tilde{y}^j + \Phi \tilde{y}}{(1 + \Phi)}. \tag{B5}$$

Substituting the last expression into (B4), and using the fact that $(1 - \Phi)(\gamma - \beta q) = \gamma - \beta$, the law of motion of the Lagrange multiplier is given by

$$\lambda_t = \frac{1}{\gamma} \lambda_{t-1} - \frac{1}{\gamma - \beta} \left((1 - \Phi) \kappa \tilde{y} + \Phi \kappa \frac{\tilde{y}^j + \Phi \tilde{y}}{(1 + \Phi)} \right). \tag{B6}$$

Substituting this expression into (B1) and (B2), and imposing the initial condition $\lambda_{-1} = 0$ (or equivalently $y_{-1} = \tilde{y}$), the law of motion in equation (11) in the text is obtained. Notice that in the law of motion of the endogenous variables π_t and $(y_t - \tilde{y})$, the coefficients coincide with the case of the previous section in the limiting case where $\rho_u = 1$, but are responding to different variables. Indeed, the previous term u_t is now replaced by a regime weighted output gap measure.

We now show that in a nonstochastic steady state where a given regime i continues, inflation converges to zero $\pi_{ss}^i = 0$.

PROOF. Using the second row of equation (11) to solve for the steady-state equilibrium $y_{ss}^i - \tilde{y}^i$, one obtains:

$$y_{ss}^i - \tilde{y}^i = -\frac{1}{(1 - \psi_y)} \frac{\kappa}{w} \frac{1}{\gamma - \beta} \left((1 - \Phi) \kappa \tilde{y}^i + \Phi \kappa \frac{\tilde{y}^j + \Phi \tilde{y}^i}{(1 + \Phi)} \right). \tag{B7}$$

Substituting $y_{ss}^i - \tilde{y}^i$ in the law of motion for π_t^i yields:

$$\begin{aligned} \pi_{ss}^i = & -\frac{w}{\kappa} (1 - \psi_y) \frac{1}{(1 - \psi_y)} \frac{\kappa}{w} \frac{1}{\gamma - \beta} \left((1 - \Phi) \kappa \tilde{y}^i + \Phi \kappa \frac{\tilde{y}^j + \Phi \tilde{y}^i}{(1 + \Phi)} \right) \\ & + \frac{1}{\gamma - \beta} \left((1 - \Phi) \kappa \tilde{y}^i + \Phi \kappa \frac{\tilde{y}^j + \Phi \tilde{y}^i}{(1 + \Phi)} \right) = 0. \end{aligned} \tag{B8}$$

□

APPENDIX C: REGIME SWITCHES IN INFLATION TARGETS

In a problem where regimes differ only in their inflation target, that is, $\tilde{\pi}^j > \tilde{\pi}^i$, and abstracting from the presence of markup shocks, the first-order conditions of regime i are:

$$\pi_t - \tilde{\pi} = -\lambda_t + \lambda_{t-1}, \tag{C1}$$

$$y_t = \frac{\kappa}{w} \lambda_t, \tag{C2}$$

$$\pi_t = \kappa y_t + \beta q \pi_{t+1} + \beta (1 - q) \pi_0^j. \tag{C3}$$

Substituting (C1) and (C2) into (C3), and solving the resulting second-order difference equation, the following expression is obtained:

$$\left[\beta q L^{-2} - \left(1 + \beta q + \frac{\kappa^2}{w} \right) L^{-1} + 1 \right] \lambda_{t-1} = \beta (1 - q) \pi_0^j - (1 - \beta q) \tilde{\pi}, \tag{C4}$$

or equivalently

$$(1 - \gamma_2 L^{-1}) (1 - \gamma L^{-1}) \lambda_{t-1} = \beta (1 - q) \pi_0^j - (1 - \beta q) \tilde{\pi}. \tag{C5}$$

The unique stable solution to the latter expression is given by

$$\lambda_t = \frac{1}{\gamma} \lambda_{t-1} - \frac{1}{\gamma - \beta q} \left[\beta (1 - q) \pi_0^j - (1 - \beta q) \tilde{\pi} \right], \tag{C6}$$

where γ is defined as in equation (A6). For convenience, define $\Phi \equiv \beta (1 - q)$ ($\gamma - \beta q < 1$), and notice that $(\partial \Phi) / (\partial q) < 0$. Assuming that regime j is solving a symmetric problem, equations (C6) and (C1), together with the initial condition $\lambda_{-1} = 0$, imply

$$\pi_0^j = \tilde{\pi}^j + \Phi \pi_0^i - \frac{1 - \beta q}{\gamma - \beta q} \tilde{\pi}^j, \tag{C7}$$

$$\pi_0^i = \tilde{\pi}^i + \Phi \pi_0^j - \frac{1 - \beta q}{\gamma - \beta q} \tilde{\pi}^i, \tag{C8}$$

and combining the two equations

$$\pi_0^j = \frac{\Phi}{1 - \Phi} \frac{\gamma - 1}{\beta (1 - q)} \frac{\tilde{\pi}^j + \Phi \tilde{\pi}^i}{(1 + \Phi)}. \tag{C9}$$

Substituting the last expression into (C6) and using the fact that $(1 - \Phi)(\gamma - \beta q) = \gamma - \beta$, the law of motion of the Lagrange multiplier is given by

$$\lambda_t = \frac{1}{\gamma} \lambda_{t-1} + \frac{1}{\gamma - \beta} \left[\Phi (1 - \gamma) \frac{\tilde{\pi}^j + \Phi \tilde{\pi}^i}{(1 + \Phi)} + (1 - \Phi)(1 - \beta q) \tilde{\pi} \right], \quad (\text{C10})$$

which is equivalent to (A9) in the limiting case of a fully-persistent markup shock (i.e., when $\rho_u = 1$). Notice that in the present case, the equilibrium variables respond to a constant term, which is increasing on the inflation target of both regimes. Under the plausible assumption of positive inflation targets, that term is always positive.

APPENDIX D: REGIME SWITCHES EVERY T PERIOD

For notational convenience only, we consider a purely forward-looking Phillips curve and we abstract from the presence of uncertainty other than the one regarding the policy objective changes. Results in the presence of a hybrid Phillips curve are available in the working paper version (Debortoli and Nunes 2010b). The problem is

$$V^i = \max_{\{\pi_t, y_t\}_{t=0}^{\infty}} E_0 \sum_{m=0}^{\infty} (\beta^T q)^m \left[-\frac{1}{2} \sum_{t=0}^{T-1} \beta^t [\pi_{m+t}^2 + w^i (y_{m+t} - \tilde{y}^i)^2] + \beta^T (1 - q) V^{ij} \right], \quad (\text{D1})$$

$$s.t. \quad \pi_{mT+t} = \kappa y_{mT+t} + \beta E_{mT+t}(\pi_{mT+t+1}) \quad t = 0, 1, \dots, T - 2, \quad (\text{D2})$$

$$\pi_{mT+t} = \kappa y_{mT+t} + (1 - q)\beta E_{mT+t}(\pi_{mT+t+1}^j) + q\beta E_{mT+t}(\pi_{mT+t+1}^i) \quad (\text{D3}) \\ t = T - 1, \forall m = 0, \dots, \infty$$

PROPOSITION 1. *Letting λ be the vector of lagrange multipliers associated with the constraints (D2) and (D3), problem (D1) can be written as a saddle point functional equation (SPFE) as follows:*

$$W(\gamma) = \min_{\lambda \geq 0} \max_{\{\pi_t, y_t\}_{t=0}^{T-1}} \{h^m(\{\pi_t, y_t\}_{t=0}^{T-1}, \lambda, \gamma)\} + \beta(1 - q)V^{ij} + \beta q W(\gamma') \quad (\text{D4})$$

$$s.t. \quad \gamma' = \lambda, \gamma_0 = 0,$$

where

$$h^m(\{\pi_t, y_t\}_{t=0}^{T-1}, \lambda, \gamma) \equiv \ell(\{\pi_t, y_t\}_{t=0}^{T-1}) + \lambda g_1(\{\pi_t, y_t\}_{t=0}^{T-1}) + \gamma g_2(\{\pi_t, y_t\}_{t=0}^{T-1}),$$

$$\ell(\{\pi_t, y_t\}_{t=0}^{T-1}) \equiv \sum_{t=0}^{T-1} \beta^t [\pi_t^2 + w^i(y_t - \tilde{y})^2],$$

$$g_1(\{\pi_t, y_t\}_{t=0}^{T-1}) \equiv \left[(\pi_0 - \kappa y_0 - \beta \pi_1), \dots, (\pi_{T-1} - \kappa y_{T-1} - \beta(1-q)\pi_T^j) \right]', \quad \text{and}$$

$$g_2(\{\pi_t, y_t\}_{t=0}^{T-1}) \equiv [0, \dots, 0, \pi_0]'$$

PROOF. Define the real valued function $r(\cdot)$ as follows:

$$r(\{\pi_t, y_t\}_{t=0}^{T-1}) \equiv -\frac{1}{2} \sum_{t=0}^{T-1} \beta^t [\pi_t^2 + w^i(y_t - \tilde{y})^2] + \beta^T (1-q)V^{\tilde{y}}. \quad (\text{D5})$$

Moreover, $g_1(\cdot)$ and $g_2(\cdot)$ are defined as in the second part of the proposition. Problem (D1) is therefore equivalent to:

$$V^i = \max_{\{\pi_t, y_t\}_{t=0}^{\infty}} E_0 \sum_{m=0}^{\infty} (\beta^T q)^m r(\{\pi_{mT+t}, y_{mT+t}\}_{t=0}^{T-1}). \quad (\text{D6})$$

$$\text{s.t. } g_1(\{\pi_{mT+t}, y_{mT+t}\}_{t=0}^{T-1}) + g_2(\{\pi_{(m+1)T+t}, y_{(m+1)T+t}\}_{t=0}^{T-1}) \geq 0$$

$$\forall m = 0, 1, \dots, \infty.$$

This formulation fits the definition of Program 1 in Marcet and Marimon (1998). We can therefore write the problem as a saddle point functional equation in the sense that there exists a unique function satisfying:

$$W(\gamma) = \min_{\lambda \geq 0} \max_{\{\pi_t, y_t\}_{t=0}^{T-1}} h(\{\pi_t, y_t\}_{t=0}^{T-1}, \lambda, \gamma) + \beta q W(\gamma'), \quad (\text{D7})$$

$$\text{s.t. } \gamma' = \lambda, \gamma_0 = 0,$$

where

$$h(\{\pi_t, y_t\}_{t=0}^{T-1}, \lambda, \gamma) = r(\{\pi_t, y_t\}_{t=0}^{T-1}) + \lambda g_1(\{\pi_t, y_t\}_{t=0}^{T-1}) + \gamma g_2(\{\pi_t, y_t\}_{t=0}^{T-1}), \quad (\text{D8})$$

or in a more intuitive formulation, define

$$h^m(\{\pi_t, y_t\}_{t=0}^{T-1}, \lambda, \gamma) \equiv \ell(\{\pi_t, y_t\}_{t=0}^{T-1}) + \lambda g_1(\{\pi_t, y_t\}_{t=0}^{T-1}) + \gamma g_2(\{\pi_t, y_t\}_{t=0}^{T-1}), \quad (\text{D9})$$

$$\ell(\{\pi_t, y_t\}_{t=0}^{T-1}) \equiv \sum_{t=0}^{T-1} \beta^t [\pi_t^2 + w^i(y_t - \tilde{y})^2], \quad (\text{D10})$$

and the saddle point functional equation is

$$\begin{aligned}
 W(\gamma) &= \min_{\lambda \geq 0} \max_{\{\pi_t, y_t\}_{t=0}^{T-1}} \{h^m(\{\pi_t, y_t\}_{t=0}^{T-1}, \lambda, \gamma)\} \\
 &\quad + \beta(1 - q)V^{ij} + \beta q W(\gamma'), \\
 \text{s.t. } \gamma' &= \lambda, \gamma_0 = 0
 \end{aligned} \tag{D11}$$

□

PROPOSITION 2. *For any type of policy objectives $i = \ell, c$ the solution of problem (D1) is a tenure invariant function $\psi(\gamma)$ such that*

$$\begin{aligned}
 \psi(\gamma) &= \arg \min_{\lambda \geq 0} \max_{\{\pi_t, y_t\}_{t=0}^{T-1}} \{h^m(\{\pi_t, y_t\}_{t=0}^{T-1}, \lambda, \gamma)\} \\
 &\quad + \beta(1 - q)V^{ij} + \beta q W(\gamma'), \\
 \gamma' &= \lambda, \gamma_0 = 0
 \end{aligned} \tag{D12}$$

PROOF. Using Proposition 1, this proof follows directly from the results of Marcat and Marimon (1998). □

APPENDIX E: SIMPLE INTEREST RATE RULES AND VOLATILITY

Consider a simple New Keynesian economy characterized by an dynamic IS equation (16), an NKPC (1), and where monetary policy is conducted according to the simple interest rate rule (15). The solution of this model is given by

$$\begin{bmatrix} \pi_t \\ y_t \end{bmatrix} = \begin{bmatrix} (\sigma(1 - \rho_u) + \phi_y)\Lambda_u & \kappa\Lambda_r \\ -(\phi_\pi - \rho_u)\Lambda_u & (1 - \beta\rho_r)\Lambda_r \end{bmatrix} \begin{bmatrix} u_t \\ r_t^n \end{bmatrix} \equiv H \begin{bmatrix} u_t \\ r_t^n \end{bmatrix}, \tag{E1}$$

where $\Lambda_u \equiv 1/((1 - \beta\rho_u)[\sigma(1 - \rho_u) + \phi_y] + \kappa(\phi_\pi - \rho_u))$ and $\Lambda_r \equiv 1/((1 - \beta\rho_r)[\sigma(1 - \rho_r) + \phi_y] + \kappa(\phi_\pi - \rho_r))$, corresponding to equation (17) in the main text.²⁹ We are assuming a standard calibration with stationary shocks ($0 < \rho_u, \rho_r < 1$), positive interest rate rule coefficients ($\phi_\pi > 0, \phi_y > 0$), and a unique rational expectations stationary equilibrium $\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0$.

29. The associated derivations are standard and available upon request.

It is now possible to analyze how the responses of our variables to the different shocks, and the implied conditional volatilities, are affected by changes in policy parameters. It can be noticed that

$$\frac{\partial H}{\partial \phi_\pi} = \begin{bmatrix} -\kappa \Lambda_u H_{11} & -\kappa \Lambda_r H_{12} \\ -\Lambda_u(1 + \kappa H_{21}) & -\kappa \Lambda_r H_{22} \end{bmatrix}, \tag{E2}$$

$$\frac{\partial H}{\partial \phi_y} = \begin{bmatrix} \Lambda_u(1 - (1 - \beta\rho_u)H_{11}) - (1 - \beta\rho_r)\Lambda_r H_{12} \\ -(1 - \beta\rho_u)\Lambda_u H_{21} & -(1 - \beta\rho_r)\Lambda_r H_{22} \end{bmatrix}. \tag{E3}$$

The following properties are then satisfied:

- (1) In response to a markup shock (u_t):
 - an increase in ϕ_π dampens the response of inflation and magnifies the response of output,
 - an increase in ϕ_y magnifies the response of inflation and dampens the response of output.
- (2) In response to a real interest rate shock (r_t^n), the response of both inflation and output is dampened by increasing ϕ_π and ϕ_y .

The composite effects on the unconditional volatilities of our variables thus depend on the volatilities of the shocks σ_u , σ_r as well as on their correlation $\sigma_{u,r}$.

The (unconditional) volatility of inflation and output is given by

$$\text{var}(\pi_t) = (H_{11})^2 \sigma_u^2 + (H_{12})^2 \sigma_r^2 + 2H_{11}H_{12}\sigma_{ur}, \tag{E4}$$

$$\text{var}(y_t) = (H_{21})^2 \sigma_u^2 + (H_{22})^2 \sigma_r^2 + 2H_{21}H_{22}\sigma_{ur}, \tag{E5}$$

where σ_u^2 , σ_r^2 , and σ_{ur} are, respectively, the variances of the shocks u_t , r_t , and their contemporaneous correlation.

Taking the derivatives of (E4) and (E5), and assuming that the two shocks are uncorrelated ($\sigma_{ur} = 0$), the following hold:

$$\frac{\partial \text{var}(\pi_t)}{\partial \phi_\pi} = -2\kappa [\Lambda_u(H_{11})^2 \sigma_u^2 + \Lambda_r(H_{12})^2 \sigma_r^2] < 0, \tag{E6}$$

$$\frac{\partial \text{var}(y_t)}{\partial \phi_y} = -2 [\Lambda_u(1 - \beta\rho_u)(H_{21})^2 \sigma_u^2 + (1 - \beta\rho_r)(H_{22})^2 \sigma_r^2] < 0. \tag{E7}$$

In other words, regardless of the relative volatility of the underlying shocks, the unconditional volatility of inflation is decreasing in ϕ_π and the unconditional volatility of output is decreasing in ϕ_y .³⁰ Moreover,

$$\frac{\partial \text{var}(y_t)}{\partial \phi_\pi} = -2\kappa [\Lambda_u(H_21)^2\sigma_u^2 + \Lambda_r(H_{22})^2\sigma_r^2] - 2\Lambda_u H_{21}\sigma_u^2 < 0, \quad (\text{E8})$$

$$\iff \frac{\sigma_r^2}{\sigma_u^2} > -\frac{\Lambda_u H_{21}}{\Lambda_r H_{22}} \frac{1 + \kappa H_{21}}{\kappa H_{22}}, \quad (\text{E9})$$

and

$$\begin{aligned} \frac{\partial \text{var}(\pi_t)}{\partial \phi_y} &= -2[\Lambda_u(1 - \beta\rho_u)(H_{11})^2\sigma_u^2 + \Lambda_r(1 - \beta\rho_r)(H_{12})^2\sigma_r^2] \\ &\quad + 2\Lambda_u H_{11}\sigma_u^2 < 0, \end{aligned} \quad (\text{E10})$$

$$\iff \frac{\sigma_r^2}{\sigma_u^2} > \frac{\Lambda_u H_{11}}{\Lambda_r H_{12}} \frac{1 - (1 - \beta\rho_u)H_{11}}{(1 - \beta\rho_r)H_{12}}. \quad (\text{E11})$$

We can then conclude that as long as condition (18) holds, a change in policy parameters leads to the volatility of both inflation and output to move in the same direction.

APPENDIX F: OPTIMAL POLICY AND VOLATILITY

If objectives are constant over time and the central bank is behaving optimally, the implied paths of inflation and output are described by equations (5) and (6) for the case where $q = 1$. The (unconditional) variances of inflation and output are given by

$$\text{var}(\pi_t) = 2(1 - \rho_u)\Sigma\sigma_u^2, \quad (\text{F1})$$

$$\text{var}(y_t) = \left(\frac{\kappa}{w}\right)^2 \frac{\gamma + \rho_u}{\gamma - 1} \Sigma\sigma_u^2, \quad (\text{F2})$$

with

$$\Sigma \equiv \frac{\gamma^2}{(\gamma + 1)(\gamma - \rho_u)(\gamma - \beta\rho_u)^2}, \quad (\text{F3})$$

30. This holds also in the more general cases with a positive correlation between u_t and r_t^n ($\sigma_{ur} > 0$).

$$\frac{\partial \Sigma}{\partial \gamma} = -\frac{\Sigma}{\gamma} \left(\frac{\gamma}{(\gamma + 1)} + \frac{\gamma}{(\gamma - \rho_u)} + \frac{2\beta\rho_u}{(\gamma - \beta\rho_u)} \right) < 0. \quad (\text{F4})$$

In this case, a change in policy parameters is given by a change in the relative weight of output stabilization (w). Noticing that $(\partial\gamma)/(\partial w) < 0$, it follows that

$$\frac{\partial \text{var}(\pi_t)}{\partial w} = 2(1 - \rho_u)\sigma_u^2 \frac{\partial \Sigma}{\partial \gamma} \frac{\partial \gamma}{\partial w} > 0, \quad (\text{F5})$$

$$\begin{aligned} \frac{\partial \text{var}(y_t)}{\partial w} &= \sigma_u^2 \left[2 \left(\frac{\kappa}{w} \right)^2 \left(-\frac{1}{w} \right) \frac{\gamma + \rho_u}{\gamma - 1} \Sigma + \left(\frac{\kappa}{w} \right)^2 \frac{\partial \gamma}{\partial w} \left(\frac{\partial \Sigma}{\partial \gamma} \frac{\gamma + \rho_u}{\gamma - 1} - \Sigma \frac{1 + \rho_u}{(\gamma - 1)^2} \right) \right] \\ &= -\frac{\text{var}(y_t)}{w} \left[2 - \frac{(\gamma - \beta)(\gamma - 1)}{\gamma^2 - \beta} \left(\frac{\gamma}{\gamma + 1} + \frac{\gamma}{\gamma - \rho_u} + \frac{2\beta\rho_u}{\gamma - \beta\rho_u} + \frac{\gamma(1 + \rho_u)}{(\gamma - 1)(\gamma + \rho_u)} \right) \right] \\ &< -\frac{\text{var}(y_t)}{w} \left[2 - \frac{(\gamma - \beta)(\gamma - 1)}{\gamma^2 - \beta} \left(2 - \frac{\gamma^2 - \beta}{(\gamma - \beta)(\gamma - 1)} \right) \right] = 0, \quad (\text{F6}) \end{aligned}$$

where the last inequality follows from noticing that the term in the round brackets is increasing in ρ_u and taking its limit as $\rho_u \rightarrow 1$. This clarifies that a change in w , as opposed to changes in simple rule parameters, always drives the volatility of inflation and output in opposite directions.

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