Heterogeneity, Selection and Labor Market Disparities[*]

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Abstract

We propose a model in which differences in socioeconomic and labor market outcomes between ex-ante identical countries can be generated as multiple equilibria sustained by different beliefs on the value of effort for finding jobs. To do so, we study the incentive to improve ability in a model where heterogeneous firms and workers interact in a labor market characterized by matching frictions and costly screening. When effort in improving ability raises both the mean and the variance of the resulting ability distribution, a complementarity between workers' choices and firms' hiring strategies can give rise to multiple equilibria. In the high-effort equilibrium, heterogeneity in ability is larger and induces firms to screen more intensively workers, thereby confirming the belief that effort is important for finding good jobs. In the low-effort equilibrium, ability is less dispersed and firms screen less intensively, which confirms the belief that effort is not so important. The model has novel implications for wage inequality, the distribution of firm characteristics, productivity, sorting patterns between firms and workers, and unemployment rates that can help explain observed differences across countries.

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1 Introduction

Countries at similar stages of development differ markedly in a number of social and economic indicators. For instance, wage inequality, labor productivity, school attainment and employment rates are all higher in the United States than in Southern Europe. The population of active firms differs too, with a relatively larger number of small and less productive firms in the latter group of countries. While understanding these differences is important both from a positive and a normative standpoint, their origin remains largely an open question. One strand of literature attributes them to distortions, but typically does not explain how they arose in the first place.[1] Another strand of literature emphasizes the role of cultural values.[2] Yet, the mechanisms through which values and beliefs translate into different economic outcomes, even in places that started with similar conditions, are still poorly understood.

The objective of this paper is to show that significant differences in socioeconomic and labor market outcomes can emerge as alternative equilibria sustained by different, and yet rational, beliefs on the role played by ability and effort in determining individual economic success. We will argue that the mechanism we identify has implications for wage inequality, the distribution of firm characteristics, sorting patterns between firms and workers, and unemployment rates that can help to explain the cross-country variation observed in the data.

To this end, we study the incentives to invest in ability in a model where heterogeneous firms and workers interact in a labor market with matching frictions. Ability is unobservable, but firms can use a screening technology to select the best workers. As in Helpman, Itskhoki and Redding (2010), the combination of these features yields realistic distributions of firms and wages. We then allow workers to invest costly effort to improve their ability assuming that effort and exogenous talent are complementary. As a result, exerting effort increases average ability, but also its dispersion in the population. This is consistent with standard human capital accumulation functions (e.g., Heckman, Lochner and Taber, 1998) and with the observation that wage inequality is higher among skilled workers (Lemieux, 2006, Flinn and Mullins, 2014). More importantly, this assumption introduces a novel complementarity between firms' and workers' strategies. On the one hand, the returns to screening are higher when ability is more dispersed, i.e., when effort is high. On the other hand, investing effort pays out more when firms screen workers more intensively.

This complementarity can give rise to two equilibria. In the high-effort equilibrium, heterogeneity in ability is higher and this induces firms to be more selective when hiring

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[1]See for example Bartelsman, Haltiwanger and Scarpetta (2013), Bartelsman, Gautier and de Wind (2011), Restuccia and Rogerson (2008), and Wasmer (2006).

[2]See, for example, Guiso, Sapienza, and Zingales (2006), Tabellini (2010), Giavazzi, Schiantarelli and Serafinelli (2013), and Moriconi and Peri (2015).

workers. In turn, this makes ability, and hence effort, more important for finding good jobs, thereby confirming the initial belief. In the low-effort equilibrium, instead, since ability is less dispersed, firms screen less intensively and hence the probability of finding jobs depends more on luck rather than merit, which confirms the initial belief on the low value of effort. Relative to the alternative scenario, in the high-effort equilibrium ability is higher and more dispersed, firms are more productive, and a stronger sorting pattern between firms and workers generates more inequality among firms. Wage inequality is also typically higher. Our aim is to show that this mechanism can replicate several salient differences observed between countries such as the United States, Italy and Spain.[3]

First, regarding perceptions, the existing evidence suggests that Americans believe in individual merit, work ethic and competition more than Southern Europeans. For instance, according to the 1981-2000 World Values Survey, 26.4% of Americans strongly agree with the statement that "hard work brings success", against a share of 14.6% in Italy and 12.2% in Spain. Those who instead strongly believe that success "is a matter of luck and connections" represent 2.3%, 8.9% and 7.8% of respondents in the three countries, respectively. Similarly, 43.3% of Americans think that "hard work is an important quality that a child should learn", against 26.8% in Italy. More broadly, 29.6% of Americans strongly believe that "competition is good", as opposed to 19.2% of Italians and 15.6% of Spaniards.

Second, these beliefs come together with significant differences in investment in education. Available data on the quality and quantity of schooling indicate that Americans attach a higher value to education than people from Southern Europe. For instance, in 2010 the working-age population with tertiary schooling was 41% in the United States against 15% in Italy and 32% in Spain (OECD, 2013).[4] Investment in education, both private and total, is also higher in the United States. For instance, total expenditure on tertiary education as a percentage of GDP is 2.8%, 1% and 1.3% in the three countries respectively. Regarding outcomes, U.S. students outperform those from Italy and Spain in all major international comparisons, but also exhibit more dispersion in the results. For example, the standard deviation of IALS test scores is about 22% higher in the United Sates than in Italy (Cebreros, 2014).[5] Finally, the United States also score higher than Souther European countries in reported measures of discipline at school, which may be a proxy for effort (OECD, 2010a).

Third, the differential value attached to education and effort is also reflected in measures

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[3]We refer mostly to these three countries because they seem the most natural and economically important examples of the equilibria we have in mind. More generally, our model can be useful to understand differences between Anglo-Saxon and Southern European countries, but also between regions with segmented labor markets and different cultural values. Nordic countries, where governments play a particularly important role, are outside the scope of the paper.

[4]The same figures for the cohort aged 25-34 are 43%, 21% and 39%, respectively.

[5]Note that our model has clear predictions about the mean and variance of ability across equilibria, not necessarily across countries with possibly different fundamentals.

of wage inequality and other labor market outcomes. In particular, the college premium relative to the earnings of workers with secondary education is around 1.8 in the United States against 1.5 in Italy and 1.4 in Spain (OECD, 2013). Broader measures of wage inequality display similar patterns: as reported in Krueger et al. (2010), for instance, the total variance of the logarithm of U.S. wages is above 0.4 and around 0.2 in the other two countries. Unemployment is also lower in the United States, especially for skilled workers. For example, the unemployment rate of U.S. college graduate is about half of that of the total workforce, while in Italy it is about 70% of the mean (OECD 2013). As a result, the different composition of the workforce alone contributes to generate a significantly lower unemployment rate in the former country.

Fourth, there are also large cross-country differences in firm-level outcomes. Available data suggest U.S. firms to be on average bigger and more productive, and their size distribution to be more dispersed than their European counterparts. For example, the standard deviation of log sales among manufacturing firms is 0.66 in the United States, 0.53 in Italy and 0.49 in Spain.[6] Interestingly, there is also evidence that American markets are more selective: for example, the survival rate for new firms is about 10% lower in the United States than in Italy (Bartelsman, Haltiwanger and Scarpetta, 2009). Finally, regarding the covariance between size and productivity, Bartelsman, Haltiwanger and Scarpetta (2013) find that, within the typical U.S. manufacturing industry, labor productivity is almost 50% higher than it would be if employment was allocated randomly and that this measure of allocative efficiency is much lower on average in European countries.

Fifth, existing data suggest that American firms value selecting talent more. From their survey on managerial practices around the world, Bloom and Van Reenen (2010) build a synthetic measure of how strongly firms value selection, based on the answers to questions on the importance of attracting and keeping talented people to the company. In their sample of 17 countries, U.S. firms have the highest average score, while Italian firms have the lowest one.[7] Moreover, consistently with our hypothesis on differences in hiring strategies, only 13% of U.S. workers claim to have found their job through personal contacts against 25.5% in Italy and 45% in Spain (Pellizzari, 2010).

To our knowledge, our theory is the first to be able to match all these observations without referring to exogenous differences in preferences and/or institutions. Despite this being a remarkable result, it is important to stress that we do not believe the multiplicity of equilibria identified in this paper to be the only or even the most important source of these socioeconomic differences. Rather, our theory illustrates a simple and yet powerful mecha-

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[6] These figures have been computed for 2007. See Section 4 for more details.

[7] We refer to the measure Talent1 available in the 2006 dataset used in the paper. Spain is not in the sample.

nism through which large differences in economic outcomes can arise even when countries have access to the same technologies and share similar market and political institutions. The success at replicating some of the salient differences between the two sides of the Atlantic makes us more confident that the model is capturing real-world phenomena. In particular, we present numerical exercises suggesting that multiple equilibria can account for a significant part of the observed differences between Italy and the United States. Moreover, given that labor markets are often segmented regionally, we believe that our model can be useful for understanding disparities in firm- and labor-market outcomes between regions of the same country, such as the North and South of Italy, which share the same broad institutions and policies.[8]

Our paper is related to several lines of research. First, it contributes to a set of papers that study the role of social beliefs in explaining the main differences in economic performance and inequality observed between the United States, Europe and other countries. Several important contributions show how alternative sets of beliefs can sustain equilibria with high and low levels of inequality. In Benabou (2000), Alesina and Angeletos (2005), Hassler et al. (2005) and Benabou and Tirole (2006), this happens through the endogenous determination of the political support for redistributive policies; in Piketty (1998) through a status motive. In other papers multiple equilibria arise through endogenous preference formation (e.g., Francois and Zabojnik, 2005, Doepke and Zilibotti, 2014). Differently from these works, we focus on a complementarity between workers' effort decisions and the hiring strategies of heterogeneous firms. This approach seems well-suited for our aim of studying especially differences in the distribution of wages, workers and firms, which are believed to be of first-order importance.

The paper is also related to the large literature on the role of human capital, broadly defined, for economic development. Several contributions have shown how multiple equilibria and poverty traps can arise in the presence of increasing returns due to human capital externalities (e.g., Azariadis and Drazen, 1990), non-convexities coupled with credit frictions (e.g., Galor and Zeira, 1993), or a complementarity between talent and technological change (e.g., Hassler and Rodriguez Mora, 2000). Differently from these works, technological increasing returns to human capital or credit frictions are not needed in our approach to generate multiple equilibria. Moreover, none of the above mentioned papers examines the interaction between workers and firm heterogeneity.

Closer to our spirit, Acemoglu (1996) and Burdett and Smith (2002) show that human capital externalities may arise naturally when labor markets are characterized by search fric-

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[8]We do not push this interpretation further because it is more difficult to obtain the relevant data. Yet, a cursory look at existing evidence suggests regional disparities within Italy and Spain to be broadly consistent with the two equilibria described in this paper.

tions. Similarly to our model, agents choose human capital depending on their job prospects and firms choose jobs depending on the average human capital of the workforce.[9] Differently from our framework, however, these papers abstract from firm heterogeneity and selection through screening. The importance of the allocation of talent is stressed by many papers, including Acemoglu (1995), Hsieh et al. (2013) and Bonfiglioli and Gancia (2014).[10] None of them, however, studies its interplay with the hiring strategies of heterogeneous firms which is at the core of our theory.

Finally, the paper builds on the literature on wage inequality in models with imperfect labor markets and firm heterogeneity. Acemoglu (1997) shows how search frictions *à la* Mortensen and Pissarides (1994) can generate and shape wage inequality.[11] Helpman, Itskhoki and Redding (2008, 2010) combine search frictions, firm heterogeneity (as in Melitz, 2003) and worker heterogeneity to study wage dispersion, wage-size premia and unemployment in open and closed economy. Our model builds on these frameworks by adding an endogenous ability distribution and by exploring how the novel equilibrium multiplicity that arises can help explain some of the observed cross-county differences in the distribution of wages, firm characteristics and unemployment rates.

The rest of the paper is organized as follows. In Section 2 we lay down the model and derive the conditions for equilibrium multiplicity. In Section 3 we compare labor market outcomes, firms and welfare across equilibria. In Section 4 we explore the quantitative implications of the model by comparing numerical simulations to data for the United States and Italy. Section 5 concludes.

2   THE MODEL

We build a model where heterogeneous firms and workers meet in a labor market characterized by matching frictions along the lines of Helpman, Itskhoki and Redding (2010). For ease of comparison, we borrow their notation whenever possible. Firms are matched randomly with workers of unknown ability although they can use a screening technology to select them. The profitability of screening is proportional to the heterogeneity among workers. Moreover, ability is relatively more beneficial for more productive firms, which have an incentive to screen more intensively. Within this framework, we make the ability distribution endogenous by adding a stage in which workers can invest costly effort to improve their ability. We then show that when investing effort raises both the mean and the variance of the abil-

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[9]A similar source of equilibrium multiplicity is present in models of statistical discrimination. See, for example, Samuelson, Mailath and Shaked (2000). Fang and Moro (2010) provide a recent survey. Our application is however very different.

[10]See also, Bhattacharya, Guner and Ventura (2013) and Caselli and Gennaioli (2013).

[11]See also Decreuse and Zylberberg (2011) for a more recent example.

ity distribution, multiple equilibria may arise due to a complementarity between effort and screening decisions. We derive the implications of the model for the equilibrium distribution of firms, wages, sorting patterns, the unemployment rate and welfare. For simplicity, we focus on a static model which can however be interpreted as the long-run equilibrium of a dynamic model.

2.1 PREFERENCES AND TECHNOLOGY

The economy is populated by a unit measure of identical households of size $\bar{L}$ with quasi-linear preferences over the consumption of two homogenous goods, q and Q:[12]

$$U = q + \frac{Q^\zeta}{\zeta}, \quad \zeta \in (0,1).$$

The demand for Q, which we refer to as the advanced good, is

$$Q = P^{-\frac{1}{1-\zeta}},$$

where P is its price. The demand for q is residual. Hence, we call q the residual good and take it as the numeraire ($p = 1$). The indirect utility is

$$W = E + \frac{1-\zeta}{\zeta}Q^\zeta, \tag{1}$$

with E denoting expenditure. For simplicity, we normalize the size of the economy to one, $\bar{L} = 1$.

The residual good is produced by employing labor with a constant returns to scale technology and is sold in a perfectly competitive market.[13] The advanced good is also homogeneous, but it is produced by heterogeneous firms employing labor subject to decreasing returns to scale. Firms entering the market incur a sunk cost, $f_e > 0$, expressed in terms of the numeraire. Once the firm has paid the entry cost, it observes its productivity θ, which is drawn from a Pareto distribution with support on $[1, \infty)$, shape parameter $z > 1$, and c.d.f. $G(\theta) = 1 - (\theta)^{-z}$. After observing θ, the firm can decide whether to exit or to produce. Exit does not require any additional cost, while production entails a fixed cost of $f > 0$ units of the residual good. The mass of entering firms is endogenously determined by free entry.

Output of a firm with productivity θ, employing a measure h of workers with average

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[12] Households are infinitesimal but any idiosyncratic risk faced by individual workers is diversified at the household level. Alternatively, we could have assumed complete insurance markets.

[13] We assume that parameters are such to guarantee positive demand for the residual good in equilibrium.

ability $\bar{a}$ is given by:

$$y(\theta) = \theta h^{\gamma} \bar{a}, \quad \gamma \in (0, 1).$$

This technology has the following important features. First, $\gamma < 1$ implies that there are decreasing returns to hiring more workers as, for example, in Lucas' (1978) span of control model. Second, the productivity of a worker depends on the average ability of the entire team.[14] Third, there is a complementarity between firms' productivity and workers' ability. As we will see, these assumptions imply that firms face a trade-off between the quantity and quality of hired workers and that ability matters relatively more for more productive firms.[15]

We first characterize the production and hiring decisions for a given distribution of workers' ability. Ability is assumed to be independently distributed and drawn form a Pareto distribution with support on $[1, \infty)$, shape parameter $k > 1$ and c.d.f. $I(a) = 1 - (a)^{-k}$. For now, we take the parameter k of this distribution as given and assume it to be common knowledge. In Section 2.4, instead, we study how the distribution of ability depends on an endogenous binary choice of effort.

The labor market is characterized by search frictions. A firm has to pay bn units of the numeraire to be matched randomly with a measure n of workers.[16] We assume that ability is match specific and it is unknown both to the firm and to the worker. However, once the match is formed, the firm has access to a screening technology which allows it to identify workers with ability below a^* at the cost of $c(a^*)^{\delta}/\delta$ units of the numeraire, with $c > 0$ and $\delta > 1$. Given the distribution of ability, $I(a)$, a firm matched with n workers and screening at the cutoff a^* will hire a measure h of workers, where

$$h = n(1/a^*)^k, \tag{2}$$

with an average ability of $\bar{a} = a^* k/(k-1)$. With these results, the production function can be rewritten as a function of n and a^*:

$$y(\theta) = \frac{k}{k-1} \theta n^{\gamma} (a^*)^{1-\gamma k}.$$

Note that if $\gamma < 1/k$, output of a firm is increasing in the ability cutoff, a^*. When this condition is satisfied, there are sufficiently strong diminishing returns relative to the dispersion of ability that a firm can increase its output by not hiring the least productive workers.

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[14]See Bandiera, Barankay and Rasul (2010) for supportive empirical evidence.

[15]See Helpman, Itskhoki and Redding (2008) for possible microfoundations of this production function. Eeckhout and Kircher (2012) study the trade-off between quantity and quality of workers in a more general setting.

[16]For simplicity, we take the matching cost b as exogenous. Making b a function of labor market conditions, as in Helpman, Itskhoki and Redding (2010), does not affect the main results. We followed this alternative strategy in a previous working paper version, Bonfiglioli and Gancia (2014b).

When $\gamma > 1/k$, instead, no firm wants to screen because employing even the least productive worker raises the firm's output and revenue, while screening is costly. To rule out the less interesting case in which screening is never profitable, from now on we assume $\gamma < 1/k$.

Wages in the advanced sector are determined through strategic bargaining between the firm and workers. Since in the bargaining stage only average ability is known, the firm retains a fraction of revenues equal to the Shapley value, $1/(1+\gamma)$, and pays the rest to the workers.[17] Using $P = Q^{\zeta-1}$, we can express revenue as:

$$r\left(\theta\right) = Q^{\zeta-1}\frac{k\theta n^{\gamma}\left(a^{*}\right)^{1-\gamma k}}{k-1}. \tag{3}$$

2.2 THE FIRM'S PROBLEM

Firms choose how many workers to interview, n, and the cutoff ability for hiring workers, a^{*}, so as to maximize profit:

$$\pi\left(\theta\right) = \max_{n>0,a^{*}\geq 1}\left\{\frac{r\left(\theta\right)}{1+\gamma} - bn - c\frac{\left(a^{*}\right)^{\delta}}{\delta} - f\right\}, \tag{4}$$

where $r\left(\theta\right)$ is given by (3).

The first-order conditions for an interior solution are:

$$\frac{\gamma}{1+\gamma}r\left(\theta\right) = bn\left(\theta\right) \tag{5}$$

$$\frac{1-\gamma k}{1+\gamma}r\left(\theta\right) = ca^{*}\left(\theta\right)^{\delta}. \tag{6}$$

Combining the first-order conditions, it is immediate to show that firms that interview more workers (higher n) screen more intensively (higher a^{*}) and therefore hire workers with higher average ability. Moreover, if $\delta > k$, as we will assume, firms that screen to a higher ability cutoff also hire more workers.

Substituting equation (2) into (5), and using the result that the wage is a share $\gamma/(1+\gamma)$ of revenue per hired worker, we obtain:

$$w\left(\theta\right) \equiv \frac{\gamma}{1+\gamma}\frac{r\left(\theta\right)}{h\left(\theta\right)} = ba^{*}\left(\theta\right)^{k}. \tag{7}$$

Hence, the wage is equal to the replacement cost of a worker, which is proportional to the search cost b and increasing in the screening cutoff $a^{*}\left(\theta\right)$.[18] Recall that $a^{*}\left(\theta\right)$ increases in

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[17]See Helpman, Itskhoki and Redding (2008) for a formal derivation.

[18]Note that the expected wage conditional on being interviewed by a firm is constant: $w\left(\theta\right)h\left(\theta\right)/n\left(\theta\right) = b$.

$h(\theta)$ when $\delta > k$. Thus, under this assumption firms hiring more workers also pay higher wages, making the model consistent with the evidence of a positive size premium (see for example, Oi and Idson 1999 and Troske, 1999). To match this empirical regularity, from now on we restrict the analysis to the case $\delta > k$. Note that the model yields wage variation across firms, but the assumption of unobservable worker heterogeneity implies that wages are the same across all workers within a given firm. Still, average wages conditional on ability vary across workers, because high-ability workers are more likely to be hired by firms paying higher wages.

Profit can be rewritten as a fraction of revenue by replacing n and a^* from (5) and (6) into (4):

$$\pi(\theta) = \frac{\Gamma}{1+\gamma} r(\theta) - f \tag{8}$$

with

$$\Gamma \equiv 1 - \gamma - \frac{1-\gamma k}{\delta} > 0. \tag{9}$$

Revenue, in turn, is an increasing function of productivity. To see this, substitute (5) and (6) into (3):

$$r(\theta) = (1+\gamma)^{1-\frac{1}{\Gamma}} \left(\theta \frac{kQ^{\zeta-1}}{k-1} \frac{\gamma^\gamma}{b^\gamma} \right)^{\frac{1}{\Gamma}} \left(\frac{1-\gamma k}{c} \right)^{\frac{1-\gamma k}{\delta \Gamma}}. \tag{10}$$

Since revenue is continuously increasing in productivity and there is a fixed production cost, the least productive firms would make negative profits and hence exit the market. The cutoff productivity θ^* below which firms exit is defined by the condition:

$$\pi(\theta^*) = \frac{\Gamma}{1+\gamma} r(\theta^*) - f = 0. \tag{11}$$

In what follows, we assume that parameters are such that $a^*(\theta^*) > 1$.[19]. Then, the relative revenue of any two firms only depends on their relative productivity, $r(\theta)/r(\theta^*) = (\theta/\theta^*)^{1/\Gamma}$. This result, combined with (8) and (11), allows us to express profit of firms with productivity θ as

$$\pi(\theta) = f \left[\left(\frac{\theta}{\theta^*} \right)^{\frac{1}{\Gamma}} - 1 \right]. \tag{12}$$

We can obtain all firm-level equilibrium variables as a function of productivity relative

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This also implies that workers have no incentives to direct their search.

[19]The required parameter restriction, $(1-\gamma)c/(1-\gamma k) < f$, can be derived by imposing zero profit in (8), replacing the corresponding revenue into (6) and setting $a^* > 1$ in the right-hand side.

to the exit cutoff:[20]

$$r\left(\theta\right) = \frac{1+\gamma}{\Gamma} f\left(\frac{\theta}{\theta^*}\right)^{\frac{1}{\Gamma}} \tag{13}$$

$$n\left(\theta\right) = \frac{\gamma}{\Gamma}\frac{f}{b}\left(\frac{\theta}{\theta^*}\right)^{\frac{1}{\Gamma}} \tag{14}$$

$$a^*\left(\theta\right) = \left(\frac{1-\gamma k}{\Gamma}\frac{f}{c}\right)^{\frac{1}{\delta}}\left(\frac{\theta}{\theta^*}\right)^{\frac{1}{\delta\Gamma}} \tag{15}$$

$$h\left(\theta\right) = \frac{\gamma}{\Gamma}\frac{f}{b}\left(\frac{\theta}{\theta^*}\right)^{\frac{1}{\Gamma}} a^*\left(\theta\right)^{-k}. \tag{16}$$

Hence, more productive firms are larger in terms of revenues, interviewed and hired workers. They are also more selective (i.e., screen at a higher ability cutoff) and pay higher wages.

2.3 INDUSTRY EQUILIBRIUM

To determine the equilibrium of the advanced sector, we solve for the cutoff productivity, θ^*, and the overall consumption of advanced good, Q.[21]

First, we pin down the cutoff productivity, θ^*, from the free-entry condition. In particular, expected profits must be equal to the entry cost:

$$f_e = \int_{\theta^*}^{\infty} \pi\left(\theta\right)\mathrm{d}G\left(\theta\right) = f\int_{\theta^*}^{\infty}\left[\left(\frac{\theta}{\theta^*}\right)^{1/\Gamma} - 1\right]\mathrm{d}G\left(\theta\right), \tag{17}$$

where the second equality uses (12). After replacing $G\left(\theta\right) = 1-\theta^{-z}$ and $\mathrm{d}G\left(\theta\right) = \left(z\theta^{-z-1}\right)\mathrm{d}\theta$ into (17), we obtain the equilibrium value for θ^*:

$$\theta^* = \left[\left(\frac{1}{z\Gamma - 1}\right)\frac{f}{f_e}\right]^{1/z}. \tag{18}$$

We assume that parameters are such that the least productive firms exit, i.e. $\theta^* > 1$. This is equivalent to requiring f to be large enough relative to the entry cost f_e, and $z\Gamma$ to be higher than one.[22]

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[20] To derive the expression for profit, we replace (10) into (8) and use condition (11). As regards the other variables, we used the definition of profit to obtain $r\left(\theta\right)$, (5) and (6) to express $n\left(\theta\right)$ and $a^*\left(\theta\right)$ as functions of $r\left(\theta\right)$. Given $n\left(\theta\right)$ and $a^*\left(\theta\right)$, we obtain $h\left(\theta\right)$ from (2).

[21] In the Appendix we also derive the measure of entering firms.

[22] All the required parameter restrictions are stated explicitly in the Appendix.

Next, we obtain Q by substituting θ^* into (10) and (11):

$$Q = \left[\frac{1}{1+\gamma} \frac{k}{k-1} \left(\frac{\Gamma}{f} \right)^{\Gamma} \left(\frac{\gamma}{b} \right)^{\gamma} \left(\frac{1-\gamma k}{c} \right)^{\frac{1-\gamma k}{\delta}} \theta^* \right]^{1/(1-\zeta)}. \tag{19}$$

2.4   Effort, Ability and Multiple Equilibria

The equilibrium of the advanced sector was derived for given shape parameter of the ability distribution, k. In this section, we solve for the allocation of workers between the two sectors and endogenize the ability distribution. This will allow us to close the model.

Consider the problem of workers. Agents must first choose the sector of occupation. They can decide to enter the residual sector, where ability is irrelevant, in which case they have a payoff ω_q.[23] Alternatively, they can seek employment in the advanced sector, where ability matters. We assume that working in the advanced sector requires formal education (e.g., some college degree) which can be acquired at the cost of ε units of the residual good.[24] Hence, we denote with $L \leq 1$ the share of job seekers in the advanced sector and we identify them with (college) educated workers. We also refer to them as the "skilled workers".

Ability in the advanced sector depends on an exogenous component, $\tilde{a}$, but also on the effort put in acquiring education. The exogenous component $\tilde{a}$, which is firm-worker specific, is exponentially distributed, with $\Pr[\tilde{a} > x] = e^{-x}$. Effort is a binary choice, and we denote it with the indicator function $\mathbb{I}_{\eta} \in \{0,1\}$, taking value one if workers put high effort and zero otherwise. Putting high effort costs η units of the residual good and allows workers to improve their ability. More precisely, we assume that the ability of a worker is

$$\ln a = \tilde{a}/k,$$

where

$$k = \begin{cases} k_0 < 1/\gamma & \text{if } \mathbb{I}_{\eta} = 0 \\ k_1 < k_0 & \text{if } \mathbb{I}_{\eta} = 1. \end{cases}$$

These equations embed a common notion of complementarity between the exogenous component of ability, $\tilde{a}$, and effort: since $k_1 < k_0$, effort ($\mathbb{I}_{\eta} = 1$) raises ability more when $\tilde{a}$ is high.

We restrict attention to pure-strategy equilibria, in which all L workers, who are *ex-ante* identical, make the same effort decision.[25] Under these assumptions, the equilibrium effort

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[23]The payoff ω_q of seeking job in the residual sector will be discussed later.

[24]Since the model is static, the choice of education and of the sector of occupation are simultaneous. Yet, a two-period OLG model in which education is chosen before seeking employment would deliver similar results.

[25]For simplicity, we do not consider explicitly mixed-strategy equilibria in which only a fraction of workers chooses effort. This is without much loss of generality, since equilibria where only a fraction of workers put

choice (yet to be solved for) pins down the ability distribution of the population of workers, which is Pareto with shape parameter k, as can be seen from $\Pr\left[a > x\right] = \Pr\left[\tilde{a} > k \ln x\right] = x^{-k}$. Note that, intuitively, a lower k (high effort) increases the probability that ability be above any given level x. By shifting probability mass in the right tail of the distribution, it also increases the fraction of high-ability workers and inequality. Hence, effort raises both average ability:

$$\mathbb{E}\left[\ln a\right]\big|_{\mathbb{I}_\eta=1} = (k_1)^{-1} > (k_0)^{-1} = \mathbb{E}\left[\ln a\right]\big|_{\mathbb{I}_\eta=0},$$

and the variance, $\mathbb{V}$, of its realizations:

$$\mathbb{V}\left[\ln a\right]\big|_{\mathbb{I}_\eta=1} = (k_1)^{-2} > (k_0)^{-2} = \mathbb{V}\left[\ln a\right]\big|_{\mathbb{I}_\eta=0}.$$

Finally, we assume that the effort level exerted by a single worker is unobservable, although firms know the overall ability distribution.

We are now in the position to solve for the workers' choices. In equilibrium, workers must be indifferent between entering the residual sector or acquiring education and looking for a job in the advanced sector. This requires the probability of being interviewed, N/L, times the expected wage conditional on being interviewed, $w\left(\theta\right)h\left(\theta\right)/n\left(\theta\right) = b$, to be equal to the payoff ω_q in the residual sector plus the education cost ε and the effort cost η if $\mathbb{I}_\eta = 1$:

$$\omega_q + \varepsilon + \mathbb{I}_\eta\eta = \frac{N}{L}b.$$

Rearranging:
$$\frac{N}{L} = \frac{\omega_q + \varepsilon + \mathbb{I}_\eta\eta}{b}. \tag{20}$$

Equation (20) implies that workers reallocate between sectors so that the average interviews per job seeker, N/L, are higher when $\mathbb{I}_\eta = 1$. This is because workers have to be compensated for their effort by a higher interview probability. Put differently, a high cost of education discourages workers from entering the advanced sector.

Consider now the effort choice and define $\Delta_k \equiv k_0 - k_1 > 0$. As stated formally in the next proposition, under the conditions $k_1 < k_0 < 1/\gamma$ and

$$\frac{z\delta\Gamma_0 a^*\left(\theta_0^*, k_0\right)^{\Delta_k}}{z\delta\Gamma_0 - \Delta_k} < 1 + \frac{\eta}{\omega_q + \varepsilon} < \frac{1 + \Delta_k}{\delta z\Gamma_1}a^*\left(\theta_1^*, k_1\right)^{\Delta_k}, \tag{21}$$

two pure-strategy equilibria exist, sustained by different beliefs on the screening strategy of firms in the advanced sector.

Proposition 1 *Assume $k_1 < k_0 < 1/\gamma$ and that (21) is satisfied. Then there exist only two*

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effort are unstable.

pure-strategy equilibria sustained by different workers' beliefs on firms' screening decisions. In the high-effort equilibrium all job seekers in the advanced sectors exert effort ($\mathbb{I}_\eta = 1$) and firms with productivity θ screen workers at a cutoff ability $a^*(\theta, k_1)$, while in the low-effort equilibrium, investment in effort is zero ($\mathbb{I}_\eta = 0$) and firms screen workers at a lower cutoff for any productivity level, $a^*(\theta, k_0) < a^*(\theta, k_1)$.

To prove Proposition 1, consider first the behavior of firms. Firms know k and choose screening intensity according to (15). Hence, if workers put effort the distribution of ability will be more dispersed ($k_1 < k_0$) and this induces firms to screen at a higher cutoff, and vice versa if $\mathbb{I}_\eta = 0$. This is because the value of screening is proportional to the heterogeneity in ability. Next, we consider the choice of workers and we study under what conditions, given the optimal screening cutoff conditional on k, a deviation is not individually profitable.

In the low-effort equilibrium, a worker who deviates draws ability from a distribution that stochastically dominates that of other workers. As a result, effort yields a higher probability of being hired by any firm, conditional on being interviewed. Such a deviation is not profitable if the higher expected wage is not enough to compensate the cost of effort. Formally, if all workers choose $k = k_0$, firms screen at $a^*(\theta, k_0)$ and the expected wage is bN_0/L_0. The payoff of a deviating worker is instead $\mathbb{E}[w(k_1, a^*(k_0))] - \eta$, where $\mathbb{E}[w(k_1, a^*(k_0))]$ is the expected wage of a worker drawing ability from a distribution with $k = k_1$ when firms screen at the cutoff $a^*(\theta, k_0)$. The value of such a deviation depends on the screening policy of firms. To see this, use (7) to compute:

$$
\begin{aligned}
\mathbb{E}[w(k_1, a^*(k_0))] &= \frac{\int_{\theta_0^*}^\infty w(\theta, k_0) a^*(\theta, k_0)^{-k_1} \, \mathrm{d}G(\theta)}{1 - G(\theta_0^*)} \frac{N_0}{L_0} \\
&= \frac{z\delta\Gamma_0 a^*(\theta_0^*, k_0)^{\Delta_k}}{z\delta\Gamma_0 - \Delta_k} b \frac{N_0}{L_0},
\end{aligned}
$$

where subscript 0 (1) denotes variables in the equilibrium with $\mathbb{I}_\eta = 0$ ($\mathbb{I}_\eta = 1$). Note that $\mathbb{E}[w(k_1, a^*(k_0))] > bN_0/L_0$.[26] More importantly, the expected wage gain from putting effort is increasing in $a^*(\theta_0^*, k)$. Intuitively, higher ability is more valuable if firms screen more intensively, which in turn depends on the aggregate effort choice (see 15). Hence, by lowering the incentive to screen workers, the choice of low effort can be self-sustaining. In sum, deviating from the low-effort equilibrium is not profitable if:

$$
b\frac{N_0}{L_0} > \mathbb{E}[w(k_1, a^*(k_0))] - \eta. \tag{22}
$$

Consider next the high-effort equilibrium. If workers choose $k = k_1$ and firms screen at the

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[26] The assumptions $\delta > k_0$ and $z\Gamma_0 > 1$ guarantee that the denominator is always positive.

cutoff $a^*(\theta, k_1)$, the expected wage is bN_1/L_1. An individual worker who deviates $(k = k_0)$ saves the cost η but faces a lower probability to be hired and hence a lower expected wage. Such a deviation is not profitable if

$$b\frac{N_1}{L_1} - \eta > \mathbb{E}\left[w(k_0, a^*(k_1))\right],\tag{23}$$

where $\mathbb{E}\left[w(k_0, a^*(k_1))\right]$ is the expected wage of a worker choosing $k = k_0$ when firms screen at the cutoff $a^*(\theta, k_1)$. As before, we can compute:

$$\begin{aligned}
\mathbb{E}\left[w(k_0, a^*(k_1))\right] &= \frac{\int_{\theta_1^*}^{\infty} w(\theta, k_1) a^*(\theta, k_1)^{-k_0}\, \mathrm{d}G(\theta)}{1 - G(\theta_1^*)}\frac{N_1}{L_1}\\
&= \frac{z\delta\Gamma_1 a^*(\theta_1^*, k_1)^{-\Delta_k}}{\Delta_k + z\delta\Gamma_1}b\frac{N_1}{L_1}.
\end{aligned}$$

Note that $\mathbb{E}\left[w(k_0, a^*(k_1))\right] < bN_1/L_1$. Moreover, the punishment for low effort is proportional to the screening intensity, captured by $a^*(\theta_1^*, k_1)$. Hence, similarly to the previous case, by raising the incentive to screen workers, the choice of high effort can be self-sustaining.

Replacing $bN/L = \omega_q + \varepsilon + \mathbb{I}_\eta\eta$ and combining (22) and (23) yields condition (21). When this condition is satisfied, which requires the cost of effort to be neither too high nor too low, the two equilibria coexist. We now study the parameter space compatible with this multiplicity. Define $A_{\min}$ and $A_{\max}$ as the left-hand side and the right-hand side of condition (21), respectively. Multiplicity requires the interval $[A_{\min}, A_{\max}]$ to be non empty. This is so if and only if $A \equiv A_{\max}/A_{\min} \geq 1$, where:

$$A = \frac{1 + \Delta_k(\delta\Gamma_1 z)^{-1}}{\left[1 - \Delta_k(\delta\Gamma_0 z)^{-1}\right]^{-1}}\left[\frac{\Gamma_0(1 - \gamma k_1)}{\Gamma_1(1 - \gamma k_0)}\right]^{\Delta_k/\delta}.\tag{24}$$

To start with, note that $A = 1$ when $k_0 = k_1$. Of course, high effort is never an equilibrium if effort has no effect. Moreover, simple algebra shows that A increases with $k_0 - k_1$ and $A \to \infty$ when $k_0 \to 1/\gamma$. Equation (24) also shows that multiplicity is more likely (higher A) the smaller Γ_1 is compared to Γ_0. Since $d\Gamma/dk = \gamma/\delta$, this suggests that a high γ/δ expands the parameter space compatible with multiplicity. These results are intuitive, because γ/δ governs the complementarity between effort and screening intensity which is what sustains the two equilibria. If γ is high, there are weaker diminishing returns and this makes $a^*(\theta)$ lower but also more sensitive to k (see 6). On the other hand, if δ is high, the screening cost is very elastic to the ability cutoff, implying that firms' screening policies do not react much. Indeed, $A \to 1$ if $\delta \to \infty$. These results are illustrated in Figure 1, which shows how A vary with k_0 for different parameter values. The dashed line corresponds to higher δ and
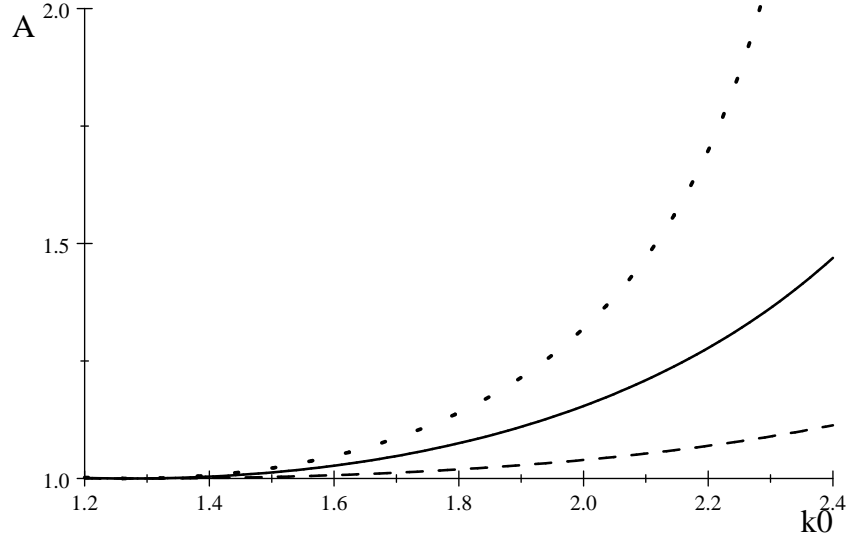
Figure 1: The Figure plots A as a function of k_0. The orizonthal axis starts at k_1. The solid line is the benchmark case. The dashed line corresponds to a higher value of δ. The dotted line corresponds to a higher value of γ.

the dotted line to higher γ compared to the benchmark case of the solid line.

2.5 Labor Markets and the Unemployment Rate

We can now compute the unemployment rate in the advanced sector, which is given by:

$$u = 1 - \frac{N}{L}\frac{H}{N},$$

where N/L is the number of interviews per job seeker and H/N is the fraction of interviewed workers that is actually hired. To find u, we first solve for H/N by integrating $h\left(\theta\right)$ and $n\left(\theta\right)$, from (16) and (14) respectively, across active firms:

$$\frac{H}{N} = \frac{\int_{\theta^*}^{\infty} h\left(\theta\right)\mathrm{d}G\left(\theta\right)}{\int_{\theta^*}^{\infty} n\left(\theta\right)\mathrm{d}G\left(\theta\right)} = \frac{a^*\left(\theta^*\right)^{-k}\left(\delta z\Gamma - \delta\right)}{\delta z\Gamma - \left(\delta - k\right)}. \tag{25}$$

Substituting this expression and N/L from (20) into u yields:

$$u = 1 - \frac{a^*\left(\theta^*\right)^{-k}\left(\delta z\Gamma - \delta\right)}{\delta z\Gamma - \left(\delta - k\right)} \cdot \frac{\omega_q + \varepsilon + \mathbb{I}_\eta \eta}{b}. \tag{26}$$

16

The unemployment rate is increasing in the search cost, b, and in the screening intensity chosen by firms.[27]

Finally, we consider the labor market of the residual sector. To make the model more realistic and obtain richer results through compositional effects, we allow for unemployment also in the residual sector. Following Helpman and Itskhoki (2009), we show in the Appendix that search frictions imply that job seekers in the residual sector face an unemployment rate u_q and a wage w_q, which depend on workers' bargaining power, the cost of posting vacancies and the matching function.[28] Hence, we can express the expected income of a worker seeking a job in the residual sector, ω_q, as the wage times the probability of being hired:

$$\omega_q = w_q \left(1 - u_q\right).$$

The aggregate unemployment rate is then:

$$\bar{u} = Lu + (1 - L) u_q. \tag{27}$$

where the share of job seekers in the advanced sector, L, is found imposing their *ex-ante* expected wage, $L\left(\omega_q + \varepsilon + \mathbb{I}_\eta \eta\right)$, to be equal to the wage bill:

$$L\left(\omega_q + \varepsilon + \mathbb{I}_\eta \eta\right) = \frac{\gamma}{1+\gamma} Q^\zeta. \tag{28}$$

In sum, given $\mathbb{I}_\eta \in \{0, 1\}$, the equilibrium values of θ^*, Q, L, u and $\bar{u}$ are given by (18), (19), (28), (26) and (27), respectively. Given θ^*, all firm-level outcomes are given by (7) and (13)-(16).

3   COMPARING EQUILIBRIA

In this section, we compare the predictions of the model for a number of variables of interest in the two equilibria. In what follows, we use subindexes 1 and 0 to denote the equilibrium with high and low effort, respectively, and we state explicitly the variables that are functions of k, when needed to avoid confusion. Since some comparisons are ambiguous, in the next Section we complement the analysis with numerical examples under plausible parametrizations.

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[27]We assume that b is sufficiently high to have positive unempolyment.

[28]These are all exogenous parameters. Hence, u_q and w_q do not depend on the equilibrium in the advanced sector.

We first compare the main outcomes within the advanced sector: firm-level productivity, revenue, wages, the size of the sector and the mass of educated workers.

3.1.1   Firm-Level Productivity

Productivity in the advanced sector is higher in the high-effort equilibrium for three reasons. First, active firms are more productive because the cutoff for exit is higher:

$$\theta^* (k) = [(z\Gamma (k) - 1) f_e/f]^{-1/z}$$

is decreasing in k since Γ is increasing in it. An increase in the dispersion of ability (lower k) benefits disproportionately more productive firms, which screen more intensively due to the complementarity between θ and a. This can be seen from (12), showing that $\pi(\theta)$ is a steeper function of productivity when k (and hence Γ) is lower. For a given exit cutoff, this increase in the profitability of more productive firms induces entry, thereby making it harder for the less productive firms to survive.

Second, higher effort has a direct positive effect on the average ability of workers seeking a job in the advanced sector, $\mathbb{E}[a] = k/(k-1)$.

Third, the average ability of hired workers, $\bar{a}$, is even higher since all firms screen at a higher cutoff. Analytically, the average ability of hired workers is

$$\bar{a} = a^* (\theta^* (k), k) \cdot \frac{k}{k-1} \cdot \frac{k + \delta (\Gamma (k) z - 1)}{k + \delta (\Gamma (k) z - 1) - 1},$$

where, from (15), $a^* (\theta^* (k), k) = [(1 - \gamma k)f/(\Gamma (k) c)]^{1/\delta}$. The first two factors (both decreasing in k) represent the average ability of employed workers if all firms had the same productivity $\theta^* (k)$, and the third term (also decreasing in k) accounts for the fact that more productive firms screen more intensively.

3.1.2   Average Revenue and Dispersion

Before comparing revenue across equilibria, we derive its distribution as a function of k. Given that revenue is a power function of productivity, which is Pareto distributed, it will also inherit the same type of distribution. In particular, using (13) and the properties of the

Pareto distribution we obtain:[29]

$$F_r\left(r,k\right) = 1 - \left(\frac{r^*\left(k\right)}{r}\right)^{\Gamma(k)z} \qquad \text{for } r \geq r^*\left(k\right) = \frac{1+\gamma}{\Gamma\left(k\right)}f, \qquad (29)$$

In the high-effort equilibrium firms are larger in terms of revenue. First, the revenue of the smallest surviving firm is higher, since

$$r\left(\theta^*\left(k\right),k\right) = \frac{1+\gamma}{\Gamma\left(k\right)}f$$

is decreasing in k. Moreover, as argued above, screening makes revenue a steeper function of productivity. Thus, average revenue is even higher:

$$\bar{r} = r\left(\theta^*\left(k\right),k\right)\frac{z\Gamma\left(k\right)}{z\Gamma\left(k\right)-1}.$$

Revenue is also more dispersed across firms when $\mathbb{I}_\eta = 1$. This can be shown computing the standard deviation of log-revenue, $SD\left[\ln r\right]$. From (29) and using the properties of the Pareto distribution, we obtain:

$$SD\left[\ln r\right] = 1/z\Gamma\left(k\right),$$

which is decreasing in k. Intuitively, more heterogeneity in worker ability amplifies the differences in output for a given productivity, thereby increasing the dispersion of revenue.

3.1.3 Wage Dispersion

In order to compare wage inequality across equilibria, we need to derive the equilibrium distribution of wages in the advanced sector. In the Appendix, we show that this is also Pareto, with c.d.f.

$$F_w\left(w,k\right) = 1 - \left(\frac{w^*\left(k\right)}{w}\right)^{1+\frac{\delta}{k}\left(\Gamma(k)z-1\right)} \qquad \text{for } w \geq w^*\left(k\right) = ba^*\left(\theta^*\left(k\right),k\right)^k. \qquad (30)$$

As in Helpman Itskhoki and Redding (2010), depending on parameter values, wages may be more or less dispersed in the high-effort equilibrium. This can be shown by computing the standard deviation of the log of wages within the advanced sector from (30) and using the

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[29]If θ follows a Pareto(θ^*, z), then $x \equiv \log\left(\theta/\theta^*\right)$ is distributed as an exponential with parameter z. Then, any power function of θ of the type $A\theta^B$, with A and B constant, is distributed as a Pareto$(A\left(\theta^*\right)^B, z/B)$, since $A\theta^B = A\left(\theta^*\right)^B e^{Bx}$ with $Bx \sim Exp(z/B)$, by the properties of the exponential distribution.

properties of the Pareto distribution:

$$SD\left[\ln w\right] = \frac{k}{k + \delta\left(\Gamma\left(k\right)z - 1\right)}.$$

Simple algebra shows that $SD\left[\ln w\right]$ is decreasing in k if and only if $\delta^{-1} + z^{-1} + \gamma > 1$. This ambiguity reflects the fact that the dispersion in ability affects the wage paid by more productive firms and fraction of workers hired by these firms in opposite ways.

3.1.4 Size of the Advanced Sector

Using (19), we derive the relative output of the advanced sector under the two equilibria:

$$\left(\frac{Q_1}{Q_0}\right)^{1-\zeta} = \frac{k_1\left(k_0 - 1\right)}{\left(k_1 - 1\right)k_0} \cdot \frac{a^*\left(\theta_1^*\right)^{1-\gamma k_1}}{a^*\left(\theta_0^*\right)^{1-\gamma k_0}} \cdot \frac{\theta_1^*}{\theta_0^*} \cdot \left(\frac{\Gamma_1}{\Gamma_0}\right)^{1-\gamma}.$$

It is easy to prove that Q is larger in the high-effort equilibrium because workers and firms are more productive. The product of the first three factors in the expression above, equal to the relative output per interviewed worker of the marginal firm, is greater than one because: (i) workers draw ability from a distribution with a higher mean, (ii) firms screen more intensively, and (iii) the marginal firm has higher productivity. The last term, accounting for the resources invested by firms in screening, is lower than one. However, as we formally prove in the Appendix, this cost is more than compensated by the benefit of screening, captured by the preceding factors. This is intuitive, since screening is chosen to maximize firms' profit.

We can also solve for the allocation of workers between the two sectors, which gives also the fraction of the population with high education. Equation (28) implies:

$$\frac{L_1}{L_0} = \frac{\omega_q + \varepsilon}{\omega_q + \varepsilon + \eta}\left(\frac{Q_1}{Q_1}\right)^{\zeta}.$$

On the one hand, the higher productivity in the advanced sector in the high-effort equilibrium $(Q_1/Q_0 > 1)$ tends to attract more workers to this sector. On the other hand, the cost of effort tends to discourage job seeking in the advanced sector. The former effect dominates if ζ is sufficiently high.

3.2 Aggregate Outcomes

3.2.1 Unemployment

We first compare the unemployment rate in the advanced sector, u, in the two equilibria by using equation (26), which is rewritten here for convenience:

$$u = 1 - a^* \left(\theta^* \left(k \right), k \right)^{-k} \frac{z\Gamma \left(k \right) - 1}{z\Gamma \left(k \right) - \left(1 - k/\delta \right)} \cdot \frac{\omega_q + \varepsilon + \mathbb{I}_\eta \eta}{b}.$$

Note that, in principle, u can be lower or higher in the high-effort equilibrium. This ambiguity stems from various mechanisms working in opposite directions. First, screening generates unemployment. Workers with ability below $a^* \left(\theta^* \left(k \right), k \right)$ are never hired. More dispersion in ability increases this screening cutoff, but it also raises the probability to draw ability above any given cutoff. The net effect is given by the factor $a^* \left(\theta^* \left(k \right), k \right)^{-k}$, which, as in Helpman Itskhoki and Redding (2008), is non monotonic in k. Workers with ability above $a^* \left(\theta^* \left(k \right), k \right)$ are hired with some probability, which depends on how fast screening increases with productivity. Since higher wages compensate unemployment risk, a higher dispersion in ability increases this component of the unemployment rate if an only if it also raises wage dispersion, i.e., when $\delta^{-1} + z^{-1} + \gamma > 1$. Finally, to compensate workers for the effort cost, η, the probability of getting an interview (N/L) needs to be higher when $\mathbb{I}_\eta = 1$, which tends to lower unemployment.

Next, we compare overall unemployment,

$$\bar{u} = L \left(k \right) u \left(k \right) + \left(1 - L \left(k \right) \right) u_q.$$

As shown above, both the unemployment rate and the mass of job-seekers in the advanced sector vary across equilibria. Assuming that the unemployment rate is higher among unskilled workers, as the data suggest, the reallocation of workers towards the advanced sector tends to lower overall unemployment.

3.2.2 Wage Inequality

We have already studied wage dispersion within the advanced sector. We now consider other measures of wage inequality which also account for variation across sectors: the skill premium and the overall Theil index of wages.

First, we define the skill premium as the average wage in the advanced sector, $\bar{w}$, relative to the wage in the residual sector, w_q. To obtain $\bar{w}$, note that the expected wage of a job seeker in the advanced sector, bN/L, is equal to the average wage multiplied by the hiring

probability, $1 - u$. Using (20), we can then express the skill premium as:

$$\frac{\bar{w}(k)}{w_q} = \frac{\omega_q + \varepsilon + \mathbb{I}_\eta \eta}{w_q (1 - u(k))}.$$

Note that the skill premium must compensate workers for the cost of entering the advanced sector and for any unemployment differential relative to the residual sector. Using (26) we can rewrite:

$$\frac{\bar{w}(k)}{w_q} = a^* (\theta^*(k), k)^k \cdot \frac{z\Gamma(k) - (1 - k/\delta)}{z\Gamma(k) - 1} \cdot \frac{b}{w_q}. \tag{31}$$

Comparing (31) to (26) shows that the skill premium behaves like the component of unemployment due to screening. Hence, if effort raises screening unemployment, it will also increase the skill premium. However, since the unemployment rate also depends on the ratio of job seekers per interviews, which falls with η, the high-effort equilibrium can potentially exhibit both a higher skill premium and a lower unemployment rate in the advanced sector.

We now consider a more comprehensive measure of inequality, the Theil index of wages. The advantage of this measure is that it can be decomposed into a within-sector (T_W) and a between-sector (T_B) component, $T = T_W + T_B$. As in Helpman, Itskhoki and Redding (2010), the within-sector component is:

$$T_W(k) = s(k)[\mu(k) - \ln(1 + \mu(k))],$$

where $\mu(k) \equiv \frac{k/\delta}{z\Gamma(k)-1}$ and $s(k)$ denotes the wage share of the advanced sector:

$$s(k) = \frac{(\omega_q + \varepsilon + \mathbb{I}_\eta \eta) L(k)}{\omega_q + (\varepsilon + \mathbb{I}_\eta \eta) L(k)}.$$

Note that the term in brackets (i.e., the Theil index of wages in the advanced sector) behaves like $SD(\ln w)$, while contribution from the residual sector is zero. The between-sector component is instead:

$$T_B(k) = s(k) \ln \bar{w}(k) + (1 - s(k)) \ln w_q - \ln[\omega_q + L(k)(\varepsilon + \mathbb{I}_\eta \eta)],$$

where $\omega_q + (\varepsilon + \mathbb{I}_\eta \eta) L(k)$ is the average wage in the economy. Overall inequality in the high-effort equilibrium can be higher if workers are reallocated to the advanced sector, where wages are higher and heterogeneous.

3.2.3 Welfare

Taking indirect utility as our measure of welfare, it is easy to show that it is an increasing function of output of the advanced sector, Q. To see this, note first that expenditure E is equal to the sum of wages minus investment in human capital:

$$E = (1 - L(k))\omega_q + (\omega_q + \varepsilon + \mathbb{I}_s\eta) L(k) - (\mathbb{I}_s\eta + \varepsilon) L(k) = \omega_q$$

This is intuitive, since the *ex-ante* average wage must be ω_q. Using (1), we obtain utility as:

$$W(k) = \omega_q + \frac{1-\zeta}{\zeta} Q(k)^\zeta. \tag{32}$$

Since $Q_1/Q_0 > 1$, welfare is necessarily higher in the high-effort equilibrium. This result stems from the fact that workers do not internalize the positive effect of their effort choice on average ability. Hence, the low-effort equilibrium is inefficient.

4 COMPARING LABOR MARKET OUTCOMES: NUMERICAL EXAMPLES

In this section we complement the qualitative comparison between equilibria presented in Section 3 with some numerical examples. The goal is twofold. First, given that the model predictions for some outcomes are potentially ambiguous, it is useful to explore them using plausible parameter values. Second, we would like to have a sense of how much of the observed cross-country differences in economic outcomes can be accounted for by our theory. Note that, since we have already discussed the effects of single parameters, here we are more interested in assessing the overall performance of the model under plausible configurations.

We proceed as follows. We identify Italy as a country in the low-effort equilibrium and the United States as representative of the other scenario. Hence, we calibrate the model with $\mathbb{I}_\eta = 0$ so as to match key observations in Italy. With this set of parameters, we change the value of k so as to match the ability distribution observed in the United States and compute the range of η compatible with both distributions being two equilibria of the same model. We then compute the predicted differences in the main variables of interest across the two equilibria and compare them with the data. For parameters that cannot be identified easily, we consider a range of possible values.

First, we match labor-market statistics. Since we normalized the total labor supply to one, we impose $L_0 = 0.15$, equal to the fraction of working-age population with tertiary schooling in Italy in 2010.[30] The total unemployment rate in Italy in 2006 was 6.8%, and 4.8% for workers with a college degree (7.2% for the rest). Hence, we choose b so as to have

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[30] The fixed cost f can be set to get $L = 0.15$.

$u_0 = 0.048$ in the advanced sector and impose $u_q = 0.072$ in the residual sector. We set labor productivity in the residual sector to yield $w_q = 1$. In this way, the skill premium is simply the average wage in the advanced sector, which is $\bar{w}_0 = (1 - u_q + \varepsilon)/(1 - u_0)$ in the equilibrium with low effort. Given u_0 and u_q, we set ε to have $\bar{w}_0 = 1.5$, corresponding to the college premium in Italy.

To calibrate the shape parameter of productivity, z, we match the firm size distribution in Italy. In particular, we compute the standard deviation of the log of value added across manufacturing establishments in Italy and, for comparison, the United States using data from the SDBS Structural Business Statistics (OECD, 2010b) and the U.S. Census. Since the data are aggregated into size categories, we follow Helpman, Melitz and Yeaple (2004) in assuming that all establishments falling within the same bin have the same value added as the group mean and using the number of firms in each size category as weights.[31] Doing so, we find that the standard deviation of log value added in 2007 is 0.53 for Italy and 0.66 for the United States. Since the standard deviation of the log of revenue in the advanced sector is $1/(z\Gamma)$, we impose $z = 1.88/\Gamma_0$.

In the absence of direct evidence on the cost of screening, we set the parameter c so that the marginal firm hires all the workers it interviews in the low-effort equilibrium (i.e., we impose $a^*(\theta_0^*) = 1$ in equation (15)), which seems a natural benchmark. We calibrate ζ from the elasticity of demand for good Q, which is equal to $1/(1 - \zeta)$. Estimates of the demand elasticity for skill-intensive goods are typically around 2 (e.g., Gancia, Mueller and Zilibotti, 2013). We therefore set $\zeta = 0.5$.

Next, we need to set the key parameters γ, δ, k_0 and k_1. As discussed extensively, some important results depend on the difference between Γ_0 and Γ_1, which is given by $(k_0 - k_1)\gamma/\delta$. Thus, for the model to produce any interesting effects, γ/δ and Δ_k must be sufficiently high. To calibrate the shape parameters of the Pareto distributions for ability, k_0 and k_1, we follow Cebreros (2014) who documents properties of the skill distribution across countries based on test scores from the IALS survey.[32] Using data on the minimum, the maximum and the standard deviation of test scores, we obtain $k_0 = 1.68$ for Italy and $k_1 = 1.27$ the United States.[33]

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[31] We use five size categories: 0-9, 10-19, 20-49, 50-250, over 250 employees. Helpman, Melitz and Yeaple (2004) show that this methodology to compute dispersion yields results that are highly correlated with direct measures based on the entire population (when available).

[32] The same data have been used by Bombardini, Gallipoli and Pupato (2012) to show that variation in skill dispersion across countries is a source of comparative advantage.

[33] Since test scores are bounded, we use the formula for a truncated Pareto distribution with shape parameter k and support $[a_{\min}, a_{\max}]$. We obtain k from:

$$SD(a) = \left[\frac{k}{k-2} \frac{1 - \tilde{a}^{k-2}}{1 - \tilde{a}^k} - \left(\frac{k}{k-2} \frac{1 - \tilde{a}^{k-1}}{1 - \tilde{a}^k} \right)^2 \right]^{1/2} a_{\min}$$

Having pinned down k_0 and $z\Gamma_0$, one possibility is to calibrate δ from the wage dispersion in the advanced sector, $SD\left[\ln w\right] = k_0/\left[k_0 + \delta\left(\Gamma_0 z - 1\right)\right]$. Note that, in the model, variation in wages across skilled workers does not depend on observable characteristics. Hence, we identify $SD\left[\ln w\right]$ with the standard deviation of residual wages, after controlling for observable characteristics, of workers with tertiary education. We compute this measure of residual wage dispersion using the data in Jappelli and Pistaferri (2010) for Italy in 2006, and find a value of 0.41, which implies $\delta = 2.7$. Since attributing the entire observed wage dispersion to the mechanism in the model may be excessive, we also experiment with higher values of δ. Finally, regarding γ, we also experiment with a range of values. The restriction $k_0 < 1/\gamma$, which is needed for screening to be profitable, requires $\gamma < 0.59$. Hence, we consider three possibilities, $\gamma \in \{0.3, 0.4, 0.5\}$.[34]

Finally, the cost of effort, η, is unobservable and cannot be inferred easily from the data. Recall from condition (21) that when η is too low, the low-effort equilibrium disappears and, conversely, the high-effort equilibrium vanishes when η is too high. Since we are interested in the behavior of the model within the range of multiple equilibria, we want η to satisfy condition (21). Moreover, a low η could imply a possibly implausibly low cost of improving ability. Then, to be conservative, we set η equal to its highest admissible value, so as to maximize the cost of switching to the U.S. equilibrium. Yet, to gauge the plausibility of equilibrium multiplicity, we report for all parameter configurations the range of η consistent with condition (21). Besides that, for most of the model's predictions, the exact value of η is not very important.[35]

A first set of results is reported in Table 1. The first two columns show the values of the main variables of interest computed from the data for Italy (column 1) and, for comparison, for the United States (column 2). Besides the variables already discussed, the first row reports the average firm revenue relative to Italy, computed from manufacturing data from the SDBS Structural Business Statistics (OECD, 2010b) and the U.S. Census, and the Theil index of the entire distribution of wages.[36] The remaining columns report the model's predictions under the benchmark case $\delta = 2.7$ and $\gamma = 0.3$. In column (3), the model is solved using the calibration for Italy, with no investment in effort and $k_0 = 1.68$.

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where $\tilde{a} = a_{\min}/a_{\max}$, and taking $SD(a)$, $a_{\min}$ and $a_{\max}$ from the data. From Cebreros (2014), for the United States, $SD(a) = 71.13$, $a_{\min} = 40.93$ and $a_{\max} = 437.9$; for Italy $SD(a) = 57.94$, $a_{\min} = 48.26$ and $a_{\max} = 395$.

[34]Note that the parameter γ captures the overall curvature of the revenue function. In more general models, it would depend on diminishing returns, managerial span of control and also product differentiation. Still, values of γ below 0.3 may appear extreme.

[35]The value of η only affects (marginally) the allocation of labor between the two sectors and the unemployment rate.

[36]The Theil index is computed on wages from Jappelli and Pistaferri (2010) for Italy and from the CPS for the United States.

	(1)	(2)	(3)	(4)
			$\gamma = 0.3$; $\delta = 2.7$	
	ITA	USA	$\mathbb{I}_\eta = 0$	$\mathbb{I}_\eta = 1$
Average Revenue, $\bar{r}$	1	1.94	1	1.23
Sales Dispersion, $SD\,[\ln r]$	0.53	0.66	0.53	0.58
Skilled Share, L	0.15	0.41	0.15	0.26
Skilled Unemployment (%), u	4.8	2.5	4.8	3.5
Total Unemployment (%), $\bar{u}$	6.8	4.6	6.8	6.2
Skill Premium, $\bar{w}$	1.5	1.8	1.5	1.69
Wage Dispersion, $SD\,[\ln w]$	0.41	0.53	0.41	0.40
Theil Index	0.16	0.19	0.05	0.09
$\eta_{\max}$	-	-	0.20	0.20
$\eta_{\min}$	-	-	0.12	0.12

Note: $\bar{r}$ is relative to ITA. Columns (1)-(2) report the data.

Table 1: Comparing Labor Market Outcomes

By construction, it matches exactly the values in column (1) except for the Theil index of overall wage inequality, which was not targeted. As expected, since the model abstracts from wage dispersion in the low-skill sector and given the size of this sector, the predicted overall inequality is lower than in the data. In column (4), we keep the same parameters but solve the model in the high-effort equilibrium, with $\mathbb{I}_\eta = 1$ and $k_0 = 1.27$.

Comparing column (4) to column (2) shows that equilibrium multiplicity has the potential to explain a significant fraction of the observed differences between Italy and the United States. Consider firm-level variables first. Average revenue per firm, $\bar{r}$, is 23% larger in the high-effort equilibrium. The model also predicts the standard deviation of log revenue ($SD\,[\ln r]$) to be almost 10% higher in the high-effort equilibrium. Compared to the US data, the model can account for about 25% of the difference in firm size and about 40% of the difference in dispersion. Moving to labor market outcomes, the higher productivity in the advanced sector in the equilibrium with high effort induces a substantial reallocation of workers towards that sector. As a result, the fraction of college-educated workers increases from 15% to 26%, accounting for more than 40% of the difference between Italy and the United States. In this calibration, the unemployment rate in the advanced sector is also significantly lower with high effort (3.5% instead of 4.8%) and the overall employment rate falls from 6.8% to 6.2%. Inequality is also higher. The skill premium rises from 1.5 to 1.69, accounting for more than 60% of the gap between the United States and Italy. Moreover, although wage dispersion in the advanced sector ($SD\,[\ln w]$) does not change much, the higher skill premium and the expansion of the high-inequality sector leads to a sizeable increase in the overall Theil index, from 0.05 to 0.09. Although the level of this index is too low, its absolute difference across equilibria is comparable to the variation observed in the

data. Finally, the last two rows show that the predictions in column 3 and 4 can correspond to multiple equilibria for a non-negligible range of the cost of effort, $\eta \in [\eta_{\min}, \eta_{\max}]$. The admissible values of η also look reasonable, in that they are comparable in magnitude to the increase in the skill premium across equilibria.

In Table 2 we do some robustness checks. First, we study what happens if we increase γ to 0.4. The results are reported in column (1), which shows the new predictions for the variables of interest in the high-effort equilibrium only (the low effort equilibrium is unchanged). With a higher γ, the complementarity between effort and screening becomes significantly stronger. As a result, all measures of inequality grow larger and $SD[\ln w]$ is now higher with $\mathbb{I}_\eta = 1$ than with $\mathbb{I}_\eta = 0$. The downside is that the firms become so selective in hiring workers that the unemployment rate becomes too high. To compensate this unemployment risk, the skill premium jumps to 1.96, well above the U.S. level. As we show next, however, these predictions become much less extreme when δ is also higher.

Recall that $\delta = 2.7$ was calibrated so as to match exactly (residual) wage dispersion in the advanced sector. Yet, in reality there might be several sources of wage heterogeneity and attributing all of it to the mechanism in the model may yield an excessive value of δ. Hence, in columns (2)-(4) we double the value of δ to 5.4. We then experiment with $\gamma = 0.3$ (as in Table 1), $\gamma = 0.4$ and $\gamma = 0.5$. For comparison, in the first two rows we report $SD[\ln w]$ and the overall Theil index in the equilibrium with low effort ($\mathbb{I}_\eta = 0$), which are now different. As expected, the model does not match wage dispersion in the advanced sector anymore, but it still accounts for a significant fraction of it (0.26 instead of 0.41). Given our calibration strategy, the model still matches the remaining variables for Italy.

A higher δ weakens the complementarity between effort and screening. As a result, with $\delta = 5.4$ effort has a slightly smaller effect on inequality. Also, as expected, the effect on inequality becomes stronger as we raise γ. For example, the skill premium increases modestly if $\gamma = 0.3$ (1.52), but it matches the U.S. level (1.8) if $\gamma = 0.5$. The Theil index is smaller, but it still increases by between 0.01 and 0.04 relative to the case with $\mathbb{I}_\eta = 0$. Reallocations of workers to the advanced sector are instead comparable to the previous cases. Differently from column (1), effort lowers the unemployment rate, except with the highest value of γ. Finally, in column (6) we keep $\gamma = 0.5$ and raise δ even further, to three times the baseline level, i.e., $\delta = 8.1$. While the effects of effort on inequality are somewhat muted and the unemployment rate rises slightly, the model can still reproduce significant differences in firm characteristics, in the skill premium and in the Theil index.

Before concluding, we mention briefly the model's implication for welfare. We know already that welfare is unambiguously higher in the high-effort equilibrium. In terms of magnitude, the simulations in this section suggest that the gain from switching to the "good" equilibrium can be substantial, varying between +40% and +50%. These numbers should

	(1)	(2)	(3)	(4)	(5)
	$\gamma = 0.4$	$\gamma = 0.3$	$\gamma = 0.4$	$\gamma = 0.5$	$\gamma = 0.5$
	$\delta = 2.7$	$\delta = 5.4$	$\delta = 5.4$	$\delta = 5.4$	$\delta = 8.1$
Wage Dispersion, $\mathbb{I}_\eta = 0$	-	0.26	0.26	0.26	0.19
Theil Index, $\mathbb{I}_\eta = 0$	-	0.02	0.02	0.02	0.02
Average Revenue $\bar{r}$	1.37	1.09	1.14	1.21	1.13
Sales Dispersion, $SD\,[\ln r]$	0.61	0.55	0.56	0.58	0.56
Skilled Share, L	0.26	0.28	0.27	0.27	0.28
Skilled Unemployment (%), u	13.8	2.0	4.3	12.1	8.2
Total Unemployment (%), $\bar{u}$	8.9	5.2	6.4	8.5	7.5
Skill Premium, $\bar{w}$	1.96	1.52	1.61	1.81	1.67
Wage Dispersion, $SD\,[\ln w]$	0.42	0.22	0.23	0.24	0.18
Theil Index	0.12	0.03	0.04	0.06	0.04
$\eta_{\max}$	0.26	0.09	0.11	0.17	0.11
$\eta_{\min}$	0.12	0.06	0.06	0.06	0.04

Note: $\bar{r}$ is relative to ITA; the first two rows refer to the low-effort equilibrium.

Table 2: Comparing Labor Market Outcomes, Robustness

however be interpreted with caution. In particular, the simplifying assumption of quasi-linear utility and risk neutrality may be restrictive for welfare comparison, especially since total inequality is significantly higher in the high-effort equilibrium.[37]

Overall, these numerical exercises yield results that are broadly consistent across parametrizations. Although stylized, the model can replicate reasonably well differences in wages, wage inequality and firm productivity. The simulations show that, despite the more selective hiring strategies of firms, effort can lower the unemployment rate. However, given that the model sometimes generates excessive unemployment rates in the high-effort equilibrium, we believe that adding differences in labor market rigidities would be important to better fit the data. The model can also replicate significant differences in the distribution of firms. These results, albeit suggestive, are remarkable considering that they are obtained without imposing any exogenous difference in rigidities, entry costs or any other structural parameter, which are certainly important in the real world.

Our simulations also suggest that differences in the ability distribution, especially its variance, can go a significant way towards explaining disparities in firms and labor market outcomes.[38] We think that this is an interesting and to our knowledge novel point. Yet, the contribution of the paper goes far beyond it. It shows that such differences in the

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[37]The welfare gain of switching to the high-effort equilibrium is also sensitive to the elasticity of substitution between the advanced and the residual sector. Yet, if $1/(1-\zeta) = 1.5$ ($\zeta = 1/3$), the gain is still substantial (around +30%).

[38]Recall however that the cost of effort is needed to match both a higher skill premium and a lower unemployment rate in the advanced sector in the high-effort equilibrium.

ability distribution can be endogenous and represent multiple equilibria under reasonable parameter values. The motivating evidence on social perceptions in Italy and the United States presented in the Introduction is also consistent with the beliefs needed to sustain the two equilibria.

5 Conclusions

We have proposed a model that explains disparities in several economic and labor market outcomes across similar countries based on multiple equilibria sustained by different beliefs on the value of effort and ability. In particular, when effort raises the dispersion of workers' ability and firms have access to a costly screening technology, two equilibria arise: in the "American" equilibrium, workers expect firms to screen more intensively and hence invest effort to improve their job prospects. This raises the dispersion in workers' ability and induces firms to be more selective when hiring. The opposite occurs in the "Southern European" equilibrium, where workers believe effort to be less important and do not invest in it, thereby inducing firms to screen less intensively. A numerical exercise shows that the two equilibria can generate variation in wage inequality, the distribution of firm characteristics, productivity and unemployment rates that can account for a significant fraction of the observed differences between the United States and Italy, even when they share the same structural parameters.

In addition to these positive results, the model yields useful policy insights. It suggests that governments could play an important role in trying to transition the economy towards the good equilibrium. For instance, to make the high-effort equilibrium possible, the technology for improving ability has to be sufficiently efficient. Making academic achievement, which are notoriously imperfect predictors of future wages, better signals of attributes that are valued in the labor market would also lower the screening costs. Moreover, measures could be taken to strengthen the social perception of effort and meritocracy in such a way to coordinate workers and firms on the desirable equilibrium.

Although we kept the model simple to obtain analytical results, the analysis could be extended in a number of interesting directions. As it is common in this class of models, we left the problem of equilibrium selection entirely outside the analysis. Allowing for learning dynamics (for instance, as in Blume, 2006) may help understand persistence and the role of policy in equilibrium selection.[39] Introducing dynamics and shocks in the model could yield novel implications for the cyclical properties of labor market outcomes in the different equilibria. Endogenizing policies may help explain why countries in the "American" equilibrium also tend to have more flexible labor and product market regulations.[40]

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[39]See Binder and François (2012) and Francois and Zabojnik (2005) for interesting examples of models studying the dynamic evolution of cultural values and institutions.

[40]Empirical work by Alesina et al. (2015) shows that labor market regulations have deep cultural roots.

Finally, while in this paper we endogenized workers' ability distribution, in Bonfiglioli, Crinò and Gancia (2016a, 2016b) we let the productivity distribution be chosen by firms, and show that trade and financial frictions can be an important determinant of the equilibrium degree of heterogeneity. In a similar vein, opening the economy in this model may provide interesting insights on the effects of trade between countries in different equilibria. For example, it may help explain why globalization seems to be associated to increased inequality more in the United States than in Southern European countries. We leave these questions to future research.

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6   Appendix

6.1   Parameter Restrictions

Parameters have to satisfy the following restrictions:

1. $\delta > k > 1$
2. $z\Gamma > 1$
3. $z\Gamma < 1 + f/f_e$
4. $\frac{f}{c} > \frac{1-\gamma}{1-\gamma k} - \frac{1}{\delta}$
5. $\frac{b}{\omega_q + \varepsilon + \mathbb{I}_\eta \eta} > \left(\frac{\Gamma}{1-\gamma k}\frac{c}{f}\right)^{k/\delta} \frac{(\delta z\Gamma - \delta)}{\delta z\Gamma - (\delta-k)}$
6. $0 < \zeta < 1$

The first two inequalities in restriction 1. guarantee that firms that sample more workers also hire more and pay higher wages, while the third one is needed for the mean of the ability distribution to be finite. Restriction 2. is needed for the mean of firm size distributions to be finite. The third restriction makes sure that there is firm selection ($\theta^* > 1$). Restriction 4. guarantees that all firms effectively screen ($a^*(\theta^*) > 1$). The fifth restriction makes sure that the unemployment rate in the advanced sector is positive.

6.2   Industry Equilibrium: Mass of Entrants

We derive the equilibrium relationship between consumption of the advanced good Q and the measure of entrants M by imposing market clearing, i.e. that expenditure in the advanced good be equal to total revenues of the sector, and substituting for $PQ = Q^\zeta$:

$$Q^\zeta = M \int_{\theta^*}^\infty r(\theta)\,\mathrm{d}G(\theta) = f\frac{1+\gamma}{\Gamma}M \int_{\theta^*}^\infty \left(\frac{\theta}{\theta^*}\right)^{1/\Gamma}\mathrm{d}G(\theta),$$

where we used $r(\theta) = f(\theta/\theta^*)^{1/\Gamma}(1+\gamma)/\Gamma$.[41]   Substituting $\mathrm{d}G(\theta)$ as above delivers the equilibrium mass of entrants,

$$M = \frac{1}{1+\gamma}\frac{Q^\zeta}{zf_e},$$

and hence the measure of surviving firms, $M[1 - G(\theta^*)] = M(1/\theta^*)^z$.

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[41]The market-clearing condition can be analogously expressed in terms of real quantities, i.e., $M\int_{\theta^*}^\infty y(\theta)\mathrm{d}G(\theta) = Q$, with $y(\theta)$ replaced from the expression for equlibrium revenue, $r(\theta) = Q^{-(1-\zeta)}y(\theta)$.

6.3 Labor Market in the Residual Sector

Following Helpman and Itskhoki (2009), we introduce search frictions through a Cobb-Douglas matching function that gives the mass of hired workers, N_q, as a function of vacancies posted by firms, V_q, and the measure of job seekers, $L_q = 1 - L$:

$$N_q = V_q^\chi L_q^{1-\chi},$$

where $\chi \in (0,1)$. This implies that the probability that a firm fills a vacancy is $N_q/V_q = (N_q/L_q)^{(\chi-1)/\chi}$. We assume that firms can freely enter by paying the cost of posting a vacancy, v, and they exit if not matched with any worker. A firm employing n_q workers produces and has revenue equal to n_q, which is split with the workers through Nash bargaining. Hence, the wage is equal to the bargaining power of workers $w_q = 1 - \Gamma_q$ and profit is $\Gamma_q n_q$. Free entry, driving expected profits to zero, requires $(N_q/L_q)^{(\chi-1)/\chi} \Gamma_q = v$. This pins down the tightness of the labor market:

$$\frac{N_q}{L_q} = \left(\frac{v}{\Gamma_q}\right)^{\frac{-\chi}{1-\chi}}.$$

Note that we need $v > \Gamma_q$ in order to have $N_q/L_q < 1$. The unemployment rate in the residual sector is just $u_q = 1 - N_q/L_q$.

6.4 Wage Distribution

To find the equilibrium distribution of wages, notice that a measure $h(\theta)$ of workers in each firm with productivity θ receive the same wage. Hence, the cumulated density of w is

$$F_w(w) = \frac{\int_{\theta^*}^{\theta_w(w)} h(\theta)\, dG(\theta)}{\int_{\theta^*}^{\infty} h(\theta)\, dG(\theta)} = 1 - \frac{\int_{\theta_w(w)}^{\infty} h(\theta)\, dG(\theta)}{\int_{\theta^*}^{\infty} h(\theta)\, dG(\theta)} \quad \text{for } w > w(\theta^*),$$

where $\theta_w(w)$ is the productivity of firms paying wages w, $\theta_w(w) = \theta^* [w/w(\theta^*)]^{\delta\Gamma/k}$ and $w(\theta^*) = ba^*(\theta^*)^k$. This expression can be simplified by replacing $h(\theta)$ from (16) and $dG(\theta) = (z\theta^{-z-1})d\theta$ to yield

$$F_w(w) = 1 - \left[\frac{w(\theta^*)}{w}\right]^{1+\frac{\delta}{k}(\Gamma z - 1)} \quad \text{for } w > ba^*(\theta^*)^k.$$

6.5 Comparing the Advanced Sector Size

The relative size of the advanced sector, $Q_1^{1-\varsigma}/Q_0^{1-\varsigma}$,

$$\frac{k_1}{k_1 - 1}\frac{k_0 - 1}{k_0}\left(\frac{\Gamma_1}{\Gamma_0}\right)^{1-\gamma}\frac{a^*\left(\theta_1^*\right)^{1-\gamma k_1}}{a^*\left(\theta_0^*\right)^{1-\gamma k_0}}\frac{\theta_1^*}{\theta_0^*},$$

is greater that one, since $a^*\left(\theta_1^*\right) > a^*\left(\theta_0^*\right)$ and $\theta_1^* > \theta_0^*$, as shown in section 3, and

$$\frac{k_1}{k_1 - 1}\frac{k_0 - 1}{k_0}\left(\frac{\Gamma_1}{\Gamma_0}\right)^{1-\gamma} > 1.$$

To prove the latter, we first notice that

$$\frac{k_1}{k_1 - 1}\frac{k_0 - 1}{k_0}\left(\frac{\Gamma_1}{\Gamma_0}\right)^{1-\gamma} > \frac{k_1}{k_1 - 1}\Gamma_1\left(\frac{k_0}{k_0 - 1}\Gamma_0\right)^{-1}$$

since $\gamma \in (0,1)$ and $\Gamma_1 < \Gamma_0$. Next, note that:

$$
\begin{aligned}
\frac{\partial}{\partial k}\left(\frac{k}{k-1}\Gamma\right) &= \frac{\gamma}{\delta}\frac{k}{k-1} - \frac{\Gamma}{(k-1)^2} < \frac{\gamma}{\delta}\frac{k}{k-1} - \frac{1}{(k-1)}\frac{1}{k} \\
&< \left(\frac{\gamma k - 1}{k}\right)\frac{1}{k-1} < 0,
\end{aligned}
$$

since $\delta > k_0$ and $k_0 < 1/\gamma$ imply that $\Gamma > (k-1)/k$. Hence,

$$\frac{k_1}{k_1 - 1}\frac{k_0 - 1}{k_0}\frac{\Gamma_1}{\Gamma_0} > 1.$$