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LOOSE COMMITMENT IN MEDIUM-SCALE MACROECONOMIC MODELS: THEORY AND APPLICATIONS

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This paper proposes a method and a toolkit for solving optimal policy with imperfect commitment. As opposed to the existing literature, our method can be employed in the medium- and large-scale models typically used in monetary policy. We apply our method to the Smets and Wouters model [*American Economic Review* 97(3), 586–606 (2007)], for which we show that imperfect commitment has relevant implications for interest rate setting, the sources of business cycle fluctuations, and welfare.

Keywords: Commitment, Discretion, Monetary Policy

1. INTRODUCTION

In the modern macroeconomic literature, economic outcomes result from the interactions between policy makers and rational firms and households. A common feature of these models is that economic decisions (e.g., consumption, hours worked, prices) depend on expectations about future policies (e.g., taxes, interest rates, tariffs). As shown by Kydland and Prescott (1977), optimal policy plans in this class of models are subject to time inconsistency.

The modern literature has taken different approaches to addressing this problem. One possibility is to assume that policy makers can fully commit—a single optimization is undertaken and the chosen policies are then implemented in all subsequent periods. This approach is known as full commitment or simply

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commitment. An alternative, often referred to as discretion or no commitment, assumes that policy makers cannot commit and that policy plans always need to be time-consistent. Although many types of time-consistent equilibria can be studied, one of the most common approaches is to solve for Markov-perfect equilibria, for which policy functions only depend on payoff relevant state variables.

Both the full commitment and discretion approaches are to some extent unrealistic. Commitment does not match the observation that governments and other institutions have defaulted on past promises. Discretion rules out the possibility that governments achieve the benefits of making and keeping a promise, despite the ex post incentive to renege. Roberds (1987) developed an approach—recently extended by Schaumburg and Tambalotti (2007) and Debortoli and Nunes (2010)—that escapes the "commitment vs. discretion" dichotomy. Policy makers are endowed with a commitment technology, but with some exogenous and common-knowledge probability they may succumb to the temptation to revise their plans. This approach has been labeled *quasi-commitment* or *loose commitment*.

Several questions can be addressed with the loose commitment approach. What are the gains of achieving more credibility? How does the possibility of future reoptimizations affect current outcomes and promises? What are the consequences of revising policy plans? How do occasional reoptimizations affect the shock propagation, volatilities, and cross correlations between relevant variables? To answer these questions and derive the associated positive and normative implications, one must depart from the frameworks of commitment and discretion and consider loose commitment instead.

Nevertheless, because of some technical difficulties, the loose commitment approach has so far been limited to relatively simple and stylized models. The goal of this paper is to overcome this limitation. We propose a simple and relatively general algorithm to solve for the optimal policy plan under loose commitment in medium- and large-scale models typically used for monetary policy analysis. We show how these types of problems reduce to solving systems of linear difference equations, and do not present any additional challenges with respect to the commitment or discretion cases.

Our framework allows us not only to address the questions posed in complex monetary policy models, but also to pose new questions and examine how additional economic features interact with imperfect commitment. For instance, central banks often and carefully devise communication strategies in which future actions may be revealed to the public. In one of our applications, we distinguish the shocks that require more commitment and may call for a more detailed planning and communication strategy.

Assuming plans' revisions to be stochastic events, rather than endogenous decisions, is clearly a simplification analogous in spirit to the Calvo pricing model. Although more complex credibility settings can easily be imagined (e.g., an endogenous timing of reoptimizations), such complexity may become prohibitive in medium- and large-scale models. In those type of models, the tractable though simplified approach employed here is particularly valuable.

This paper is related to the literature on optimal monetary policy in linear–quadratic frameworks. Solution algorithms for full commitment, together with a discussion about the computational aspects, have been developed by Currie and Levine (1993) and Söderlind (1999), among others. Methods of solving for (Markov-perfect) time-consistent equilibria are described in Backus and Driffill (1985), Söderlind (1999), and Dennis (2007). The main contribution of our paper is to extend these methodologies to address problems under loose commitment. To illustrate the benefits of our approach, the methodology is then applied to analyze the effects of commitment in the medium-scale model of Smets and Wouters (2007), which has arguably become one of the benchmark models in the dynamic stochastic general equilibrium literature.[1]

The paper continues as follows. In Section 2 we introduce the general formulation of the model. In Section 3 we study the optimal-policy problem and describe the solution algorithm. Section 4 discusses the role of commitment in the Smets and Wouters (2007) model, and Section 5 concludes. We provide as supplementary material a collection of codes and documentation that implement our algorithm in a variety of models.

2. GENERAL FORM OF THE MODELS

Consider a general linear model, whose structural equations can be cast in the form

$$A_{-1}y_{t-1} + A_0 y_t + A_1 E_t y_{t+1} + B v_t = 0, \quad \forall t, \tag{1}$$

where y_t indicates a vector of endogenous variables and v_t is a vector of serially uncorrelated exogenous disturbances with zero mean and $E v_t v_t' = \Sigma_v$. The vast majority of the models used for monetary policy analysis can be mapped into such a formulation.

The common approach in the monetary policy literature is to assume that central banks have a quadratic loss function,

$$\sum_{t=0}^{\infty} \beta^t y_t' W y_t. \tag{2}$$

In some cases, a purely quadratic objective function is consistent with a second-order approximation of a general time-separable utility function around an efficient steady state [see, e.g., Woodford (2003a)].[2] Moreover, quadratic loss functions have been shown to describe a central bank's behavior realistically, even if they do not necessarily reflect the preferences of the underlying society.[3] In fact, and following Rogoff (1985), appointing a central banker who is more averse toward inflation than the overall public may be desirable in the limited-commitment settings considered here.

Throughout the analysis, we therefore maintain the assumption that the central bank's loss function is purely quadratic and may or may not reflect social preferences. Besides obvious tractability considerations, this feature guarantees that our methodology is flexible and directly applicable to most of the models used for monetary policy analysis.[4]

3. OPTIMAL POLICY UNDER LOOSE COMMITMENT

In a loose commitment setting, it is assumed that policy makers have access to a commitment technology but may occasionally revise their plans. More formally, suppose that the occurrence of a reoptimization is driven by a two-state Markov stochastic process,

$$\eta_t = \begin{cases} 1 & \text{with probability } \gamma \\ 0 & \text{with probability } 1-\gamma. \end{cases} \tag{3}$$

At any given point in time, if $\eta_t = 1$, previous commitments are honored. This event occurs with probability $0 \le \gamma \le 1$. If instead $\eta_t = 0$, previous promises are reneged on and a new policy plan is formulated. This formulation nests both the full commitment and discretion approaches as limiting cases for which $\gamma = 1$ and $\gamma = 0$, respectively. More importantly, this formulation also spans the continuum between those two extremes.

Considering stochastic reoptimizations is a necessary simplification to address large-scale models. Such an assumption also seems justified if the timing of plan revisions can be uncorrelated with the state of the economy. One possible candidate for such events is a change in the dominating view within a central bank because of time-varying composition of its decision-making committee. Another candidate is outside pressures of varying intensity exerted by politicians and the financial industry.[5] Alternatively, our approach can be interpreted as the reduced form of a model in which commitment to a policy is sustained by the threat of punishment in case of reoptimization. If the punishment requires a priori coordination among private agents and in some random periods cannot be implemented, then such a model may bear similarities with our approach.[6] These are reasons for which our model can bear similarities to one in which the reoptimization decision is endogenous. Whether our approach is plausible from an empirical perspective would require an estimation exercise. In later sections we do contrast our model with the data.

Following Schaumburg and Tambalotti (2007) and Debortoli and Nunes (2010), the policy maker's problem can be written as

$$y_{-1}' P y_{-1} + d = \min_{\{y_t\}_{t=0}^{\infty}} E_{-1} \sum_{t=0}^{\infty} (\beta\gamma)^t \left[y_t' W y_t + \beta(1-\gamma)(y_t' P y_t + d) \right] \tag{4}$$

$$\text{s.t.} \quad A_{-1} y_{t-1} + A_0 y_t + \gamma A_1 E_t y_{t+1} + (1-\gamma) A_1 E_t y_{t+1}^r + B v_t = 0 \quad \forall t \ge 0.$$

The terms $y_{t-1}' P y_{t-1} + d$ summarize the value function at time t, when a reoptimization occurs ($\eta_t = 0$). Because the problem is linear–quadratic, the value function is quadratic and summarized by the state variables y_{t-1} and a term reflecting the stochastic nature of the problem. The matrix P and the scalar d have to be obtained in the solution procedure, as shown in later sections.[7] The Appendix discusses additional details related to the problem defined.

The objective function is given by an infinite sum discounted at the rate $\beta\gamma$, summarizing the history in which reoptimizations never occur. Each term in the summation is composed of two parts. The first part is the period loss function. The second part indicates the value the policy maker obtains if a reoptimization occurs in the next period.

The policy maker faces a sequence of constraints, for which in any period t expectations of future variables are an average between two terms. The first term (y_{t+1}), with weight γ, relates to the allocations prevailing when current plans are honored. The second term, y^r_{t+1}, with weight $(1-\gamma)$, refers to the choices made in period $t+1$ if a reoptimization occurs (i.e. if $\eta_{t+1}=0$). As in the Markov-perfect literature, we assume that expectations about choices following a reoptimization depend only on state variables:

$$E_t y^r_{t+1} = \tilde{H} y_t. \tag{5}$$

The policy maker cannot decide directly on the allocations implemented if a reoptimization occurs, and therefore the matrix $\tilde{H}$ is taken as given.

For any $\tilde{H}$, the policy maker's problem can be solved using recursive methods. We follow the approach of Kydland and Prescott (1980) and Marcet and Marimon (2009) and write the Lagrangian associated with the optimal policy problem

$$\begin{aligned}\mathcal{L} \equiv E_{-1} \sum_{t=0}^{\infty} (\beta\gamma)^t \Big\{ & y_t' \left[W + (1-\gamma)\beta P\right] y_t + \lambda_{t-1}' \beta^{-1} A_1 y_t \\ & + \lambda_t' \left[A_{-1} y_{t-1} + \left(A_0 + (1-\gamma) A_1 \tilde{H}\right) y_t + B v_t\right] \Big\}, \end{aligned} \tag{6}$$

$$\lambda_{-1} = 0,$$

$$\tilde{H}, y_{-1} \text{ given.}$$

This Lagrangian can be written recursively by expanding the state of the economy to include the Lagrange multiplier vector λ_{t-1}. The solution to the problem is then characterized by a time-invariant policy function

$$\begin{bmatrix} y_t \\ \lambda_t \end{bmatrix} = \begin{bmatrix} H_{yy} & H_{y\lambda} \\ H_{\lambda y} & H_{\lambda\lambda} \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \end{bmatrix} + \begin{bmatrix} G_y \\ G_\lambda \end{bmatrix} v_t, \tag{7}$$

where the matrices H and G depend on the unknown matrix $\tilde{H}$.

When a reoptimization occurs in a given period t, the vector λ_{t-1} must be reset to zero. This result, formally proved by Debortoli and Nunes (2010), has an intuitive interpretation. A reoptimization implies that all the past promises regarding current and future variables are no longer binding.

According to equation (7) and setting $\lambda_{t-1} = 0$, $y_t^r = H_{yy} y_{t-1} + G_y v_t$. Moving this equation forward one period and taking expectations, one obtains $E_t y_{t+1}^r = H_{yy} y_t$. For this expression to be consistent with equation (5), it must be that in a rational expectations equilibrium

$$H_{yy} = \tilde{H}. \tag{8}$$

Given our formulation, the optimal policy under loose commitment can be found as the solution of a fixed-point problem in the matrix H. In what follows, we propose an algorithm to solve for that fixed point.

3.1. Solution Algorithm

We start by writing the first-order conditions of the Lagrangian (6):

$$\frac{\partial \mathcal{L}}{\partial \lambda_t} = \left[A_0 + (1-\gamma) A_1 H_{yy}\right] y_t + \gamma A_1 E_t y_{t+1} + A_{-1} y_{t-1} + B v_t = 0, \tag{9}$$

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial y_t} &= 2W y_t + \beta (1-\gamma) A_{-1}' E_t \lambda_{t+1}^r + \left[A_0 + (1-\gamma) A_1 H_{yy}\right]' \lambda_t \\ &\quad + \mathcal{I}_\gamma \beta^{-1} A_1' \lambda_{t-1} + \beta \gamma A_{-1}' E_t \lambda_{t+1} = 0. \end{aligned} \tag{10}$$

The vector equation (9) corresponds to the structural equation (1), where we have used equations (5) and (8) to substitute for the term $E_t y_{t+1}^r$. As a result, the unknown matrix H_{yy} enters equation (9). That matrix also enters equation (10), reflecting that y_t can be used to affect the expectations of y_{t+1}^r. The term λ_{t+1}^r in equation (10) constitutes the derivative of the value function w.r.t. y_t. This derivative can be obtained using the envelope condition

$$\frac{\partial y_t' P y_t}{\partial y_t} = 2P y_t = A_{-1}' E_t \lambda_{t+1}^r. \tag{11}$$

Finally, the term $\mathcal{I}_\gamma$ in equation (10) is an indicator function,

$$\mathcal{I}_\gamma = \begin{cases} 0, & \text{if } \gamma = 0 \\ 1 & \text{otherwise,} \end{cases} \tag{12}$$

and is used for convenience so that equation (10) is also valid under discretion ($\gamma = 0$), where the term $\beta^{-1} A_1' \lambda_{t-1}$ would not appear.[8]

There are many methods for solving linear rational expectation systems such as (9)–(10), and standard routines are widely available [e.g., Klein (2000), Collard and Juillard (2001), Sims (2002)]. Our computational implementation is based on the method of undetermined coefficients.

For a given guess of the matrix H, the law of motion (7) can be used to compute the expectations terms

$$E_t y_{t+1} = H_{yy} y_t + H_{y\lambda} \lambda_t, \tag{13}$$

$$E_t \lambda_{t+1} = H_{\lambda y} y_t + H_{\lambda\lambda} \lambda_t, \tag{14}$$

$$E_t \lambda^r_{t+1} = H_{\lambda y} y_t, \tag{15}$$

where the last equation follows from resetting the Lagrange multiplier λ_t to zero because of the reoptimization at $t+1$. Substituting these formulas into (9) and (10), one obtains

$$\Gamma_0 \begin{bmatrix} y_t \\ \lambda_t \end{bmatrix} + \Gamma_1 \begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \end{bmatrix} + \Gamma_v v_t = 0, \tag{16}$$

with

$$\Gamma_0 \equiv \begin{bmatrix} A_0 + A_1 H_{yy} & \gamma A_1 H_{y\lambda} \\ 2W + \beta A'_{-1} H_{\lambda y} & A'_0 + (1-\gamma)\, H'_{yy} A'_1 + \beta\gamma A'_{-1} H_{\lambda\lambda} \end{bmatrix},$$

$$\Gamma_1 \equiv \begin{bmatrix} A_{-1} & 0 \\ 0 & \beta^{-1} \mathcal{I}_\gamma A'_1 \end{bmatrix}, \qquad \Gamma_v \equiv \begin{bmatrix} B \\ 0 \end{bmatrix}.$$

The resulting law of motion is

$$\begin{bmatrix} y_t \\ \lambda_t \end{bmatrix} = -\Gamma_0^{-1}\Gamma_1 \begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \end{bmatrix} - \Gamma_0^{-1}\Gamma_v v_t, \tag{17}$$

where we are assuming the matrix Γ_0 to be nonsingular.

The final step consists in verifying that this law of motion coincides with the initial guess, i.e., $H = -\Gamma_0^{-1}\Gamma_1$. If not, the guess-and-verify procedure is repeated until convergence. In summary, the algorithm proceeds as follows:

1. Using a guess H_{guess}, form Γ_0 and Γ_1.
2. Compute $H = -\Gamma_0^{-1}\Gamma_1$.
3. Check if $||H - H_{\text{guess}}|| < \xi$, where $||.||$ is a distance measure and $\xi > 0$. If the guess and the solution have converged, proceed to step 4. Otherwise, update the guess as $H_{\text{guess}} = H$ and repeat steps 1–3 until convergence.
4. Finally, form Γ_v and compute $G = -\Gamma_0^{-1}\Gamma_v$.

Clearly, there are many alternative algorithms to the one proposed. For example, for a given H, the system of equations (9)–(10) could be solved using a generalized Schur decomposition, as in Blanchard and Kahn (1980), or solving a quadratic matrix equation, as in Uhlig (1995). For this reason, the nonsingularity of the matrix Γ_0 is not essential. Also, the solution of the fixed-point problem on the matrix H could be performed using a Newton-type method. Nevertheless, the procedure described earlier proved to be computationally more efficient.

The main message of our analysis is that solving for an optimal policy problem under loose commitment only requires solving a fixed-point problem, which in a linear–quadratic framework is as simple as solving a system of linear equations. In addition, a loose commitment approach nests the full commitment and discretion cases.

There are other practical advantages. For instance, Blake and Kirsanova (2010) show that some linear–quadratic models may display multiple equilibria under discretion. Those models may thus also exhibit multiple equilibria for intermediate commitment settings, depending on the initial guess for H. The advantage of our loose commitment approach, as implemented in the companion toolkit, is that there is a natural initial guess: the full commitment solution, which is typically unique. The probability of commitment is then gradually reduced from full commitment to discretion, using as a guess the solution from the previous iteration.

In these iterations, the gradual reductions from $\gamma = 1$ to $\gamma = 0$ can be arbitrarily small, and this procedure can be viewed as a potential selection device among multiple discretionary equilibria.[9] Finally, even though multiple equilibria are a theoretical possibility, we found a unique solution in all the applications considered.[10]

3.2. Simulations and Impulse Responses

Once the matrices H and G have been obtained, it is straightforward to simulate the model for different realizations of the shocks and compute second moments and impulse response functions. For given initial conditions y_{-1}, λ_{-1}, and histories of the shocks $\{v_t, \eta_t\}_{t=0}^{T}$, the model simulation follows the formula

$$\begin{bmatrix} y_t \\ \lambda_t \end{bmatrix} = H \begin{bmatrix} y_{t-1} \\ \eta_t \lambda_{t-1} \end{bmatrix} + G v_t. \tag{18}$$

The peculiarity of the loose commitment setting is that a history of the shock driving the reoptimizations (η_t) should also be specified. Whenever $\eta_t = 0$, the Lagrange multiplier λ_{t-1} is reset to zero.

3.3. Welfare

For any initial condition $[y'_{t-1} \quad \lambda'_{t-1}]$ the welfare measure, unconditional on the first realization of v_0, is given by

$$\begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \end{bmatrix}' \widehat{P} \begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \end{bmatrix} + d. \tag{19}$$

The matrix $\widehat{P}$ can be obtained by taking the derivative of the recursive formulation of the Lagrangian (6), thus obtaining

$$\widehat{P} = \frac{1}{2} \begin{bmatrix} 0 & A'_{-1} \\ \beta^{-1} A_1 & 0 \end{bmatrix} H. \tag{20}$$

Notice that in the most pertinent case with initial conditions $\lambda_{t-1} = 0$ the only relevant term would be the upper left block of $\widehat{P}$, which equals $A'_{-1} H_{\lambda y}$.

The constant d is given by

$$d = \frac{1}{1-\beta} tr \left[\Sigma_v \left(G' \widetilde{V} G + G' \begin{bmatrix} 0 \\ B \end{bmatrix} \right) \right] \tag{21}$$

with

$$\widetilde{V} = \begin{bmatrix} W & 0 \\ A_0 + (1-\gamma) A_1 H_{yy} & 0 \end{bmatrix} + \beta (1-\gamma) \begin{bmatrix} A'_{-1} H_{\lambda y} & 0 \\ 0 & 0 \end{bmatrix} + \beta \gamma \widehat{P}.^{11} \tag{22}$$

Alternatively, one can compute welfare conditional on the first realization of the shock, which is defined as follows:

$$\begin{aligned} \begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \\ v_t \end{bmatrix}' \widetilde{P} \begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \\ v_t \end{bmatrix} + \widetilde{d} &= y_t' W y_t \\ &+ \beta \gamma E_t \left(\begin{bmatrix} y_t \\ \lambda_t \\ v_{t+1} \end{bmatrix}' \widetilde{P} \begin{bmatrix} y_t \\ \lambda_t \\ v_{t+1} \end{bmatrix} + \widetilde{d} \right) \\ &+ \beta (1-\gamma) E_t \left(\begin{bmatrix} y_t \\ 0 \\ v_{t+1} \end{bmatrix}' \widetilde{P} \begin{bmatrix} y_t \\ 0 \\ v_{t+1} \end{bmatrix} + \widetilde{d} \right). \end{aligned} \tag{23}$$

By the definition of conditional welfare, it must be that

$$E_t \left(\begin{bmatrix} y_t \\ \lambda_t \\ v_{t+1} \end{bmatrix}' \widetilde{P} \begin{bmatrix} y_t \\ \lambda_t \\ v_{t+1} \end{bmatrix} + \widetilde{d} \right) = \left(\begin{bmatrix} y_t \\ \lambda_t \end{bmatrix}' \widehat{P} \begin{bmatrix} y_t \\ \lambda_t \end{bmatrix} + d \right), \tag{24}$$

and equation (23) can be rewritten as

$$\begin{aligned} &\begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \\ v_t \end{bmatrix}' \widetilde{P} \begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \\ v_t \end{bmatrix} + \widetilde{d} \\ &= \left(H \begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \end{bmatrix} + G v_t \right)' \widetilde{V} \left(H \begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \end{bmatrix} + G v_t \right) \\ &+ \left(H \begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \end{bmatrix} + G v_t \right)' \left(\begin{bmatrix} 0 & \beta^{-1} A_1' \\ A_{-1} & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ \lambda_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ B \end{bmatrix} v_t \right) + \beta d. \end{aligned} \tag{25}$$

We can thus obtain the conditional welfare, for any given initial condition, by just evaluating the right-hand side of this last expression.

In these derivations we have computed welfare using the recursive formulation of the Lagrangian (6). As mentioned earlier, that formulation is equivalent to the original problem (4) only after the initial condition $\lambda_{-1} = 0$ is imposed. If one wants to evaluate the welfare according to the original formulation of equation (2), but for a different value of λ_{-1}, one needs to subtract $\lambda_{-1}\beta^{-1}A_1E_{-1}y_0$ and $\lambda_{-1}\beta^{-1}A_1y_0$ from equations (19) and (25), respectively.[12]

4. APPLICATION: A MEDIUM-SCALE CLOSED ECONOMY MODEL

In this section, we apply our methodology to the Smets and Wouters (2007) model. Needless to say, our purpose is neither to match business cycle properties nor to test the empirical plausibility of alternative commitment settings. We instead focus on examining the role of commitment in this benchmark medium-scale model.

The model includes nominal frictions in the form of sticky price and wage settings allowing backward inflation indexation. It also features real rigidities—habit formation in consumption, investment adjustment costs, variable capital utilization, and fixed costs in production. The dynamics are driven by six orthogonal shocks: total factor productivity, two shocks affecting the intertemporal margin (risk premium and investment-specific technology shocks), two shocks affecting the intratemporal margin (wage and price-markup shocks), and an exogenous government spending shock. The model equations are omitted here for brevity and all parameters are calibrated to the posterior mode as reported in Smets and Wouters (2007).

Unlike Smets and Wouters (2007), we do not consider a specific interest rate rule nor the associated monetary-policy shock. Instead, we assume that the central bank solves an optimal policy problem. By doing so, we exemplify how the degree of commitment and the reoptimization shocks affect the behavior of the central bank. We are not dismissing interest rate rules either from a normative or from a positive perspective. In fact, it is widely known that optimal policy plans can be implemented in a variety of ways, including targeting rules and instrument rules, of which interest rate rules are a subcase.

We explore the implications of two purely quadratic loss functions commonly used in the literature. The benchmark formulation is given by

$$U_t^b = w_\pi \pi_t^2 + w_y y_t^2 + w_i^b (i_t - i_{t-1})^2, \tag{26}$$

where π_t, y_t, and i_t denote respectively price inflation, output gap, and the nominal interest rate. The alternative specification takes the form

$$U_t^a = w_\pi \pi_t^2 + w_y y_t^2 + w_i^a i_t^2. \tag{27}$$

Following Woodford (2003b), we set the parameters $w_\pi = 1$, $w_y = 0.003$, $w_i^b = 0.0176$, and $w_i^a = 0.0048$. The plausibility of these formulations and of the corresponding calibration is discussed in the following sections, where we analyze the importance of commitment from different perspectives. As explained in Section 2, deriving and carrying out the analysis with a micro-founded utility function is an interesting approach but goes beyond the scope of this paper.[13] The

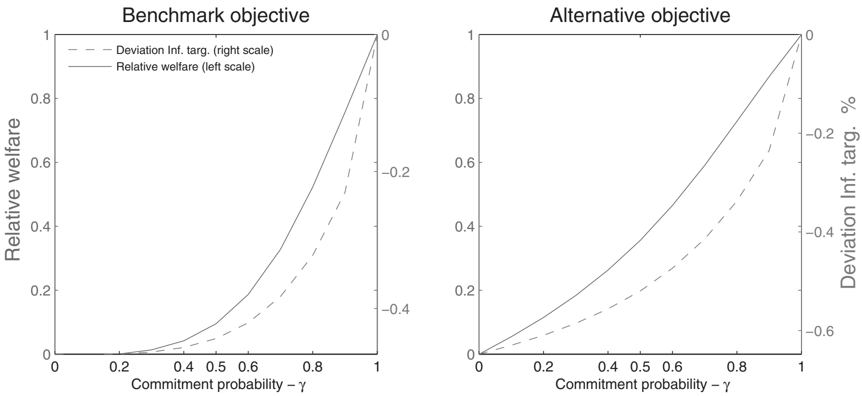

FIGURE 1. Welfare. The figure plots the welfare gains from commitment for the benchmark (left panel) and the alternative (right panel) objective function. The continuous line (left scale) indicates the relative gains from full discretion to a degree of commitment γ, i.e., $(V_\gamma - V_{\gamma=0})/(V_{\gamma=1} - V_{\gamma=0})$. This measure corresponds to conditional welfare and the results are robust to unconditioning on the shocks. The dashed line (right scale) indicates equivalent permanent deviation from the inflation target according to equation (28), i.e., $m^2 = (1-\beta)\,(V_{\gamma=1} - V_\gamma)/w_\pi$. We plot the negative of $-m$ for the convenience of plotting an increasing function in γ.

type of loss functions considered in this paper are used widely in central banks [e.g., Norges Bank (2011)] and in the literature describing or characterizing central bank behavior [see, e.g., Rogoff (1985), Svensson (1999), Dennis (2004), Ilbas (2012)].

4.1. What Are the Gains from Commitment?

In Figure 1, we plot the conditional welfare gains obtained for different levels of credibility. The continuous line (left axis) standardizes welfare by the total gains of changing credibility from discretion to full commitment. This standardization has the advantage that any affine transformation of the central bank's objective function would leave this welfare measure unchanged.

As expected, higher credibility leads to higher welfare.[14] More importantly, the figure suggests that if a central bank has low credibility to start with, a partial enhancement of its credibility will not deliver much of the welfare gains that credibility can potentially offer. On the other hand, a central bank with high credibility should be especially cautious. It will face severe welfare losses if its credibility is deemed to have been minimally affected. These results contrast with those obtained by Schaumburg and Tambalotti (2007) using a more stylized monetary policy model.

Figure 1 also considers another welfare measure that is useful in gauging losses for the objective functions employed by central banks and is described, for

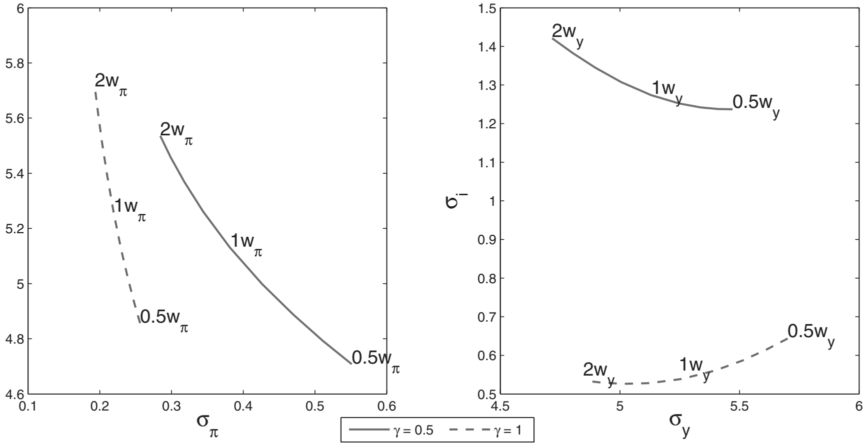

FIGURE 2. Credibility and volatility. The figure plots the volatilities of inflation, output gap, and interest rate for different credibility levels. The left and right panel change the weight on inflation and output gap, respectively. The two panels plot several weights from half to double the benchmark value. The solid and dashed lines assume the probability of commitment to be 0.5 and 1, respectively.

instance, in Jensen (2002). This measure (m) is the permanent deviation in the inflation target that would leave the central bank indifferent between full commitment and another credibility level γ,

$$E_{-1}\sum_{t=0}^{\infty}\beta^t\left[w_\pi\left(\pi_{t,\gamma=1}-m\right)^2+w_y\left(y_{t,\gamma=1}\right)^2+w_i^b(i_{t,\gamma=1}-i_{t-1,\gamma=1})^2\right]$$

$$= E_{-1}\sum_{t=0}^{\infty}\beta^t\left[w_\pi\pi_{t,\gamma}^2+w_y y_{t,\gamma}^2+w_i^b(i_{t,\gamma}-i_{t-1,\gamma})^2\right]. \qquad \textbf{(28)}$$

In other words, this measure plots the permanent increase or decrease in the inflation level relative to the target of zero that would leave the central bank indifferent between the two credibility cases. A complete loss of credibility would be equivalent to a permanent change in the inflation rate of around 0.47%.[15]

Credibility may also affect the relative contribution of inflation and output-gap volatilities to the overall welfare loss. A higher credibility level translates into better management of the policy trade-offs because forward guidance is more effective as a policy tool. Therefore one might conjecture that higher credibility would reduce the volatilities of all welfare-relevant variables. Figure 2 illustrates that such a conjecture does not always hold. The figure shows that for a given relative weight in the objective function, a loss in credibility leads to a rise in inflation volatility but a reduction in output-gap volatility. The reason is that stabilizing inflation is the most important welfare objective. A central bank with high credibility can achieve higher welfare by promising to stabilize inflation even if doing so implies more output-gap volatility.

Figure 2 also discriminates among the points on the policy frontiers associated with doubling or halving w_π or w_y relative to the baseline calibration. Even considering such extreme calibrations of the welfare function does not change the results qualitatively. The finding that a loss in credibility increases inflation volatility but reduces output-gap volatility holds for those extreme calibrations as well.

4.2. Loose Commitment and Simple Interest Rate Rules

The optimal policy under loose commitment can be implemented through targeting rules or through an appropriately defined interest rate rule.[16] In dynamic stochastic general equilibrium (DSGE) monetary policy models it is instead common to adopt simple reduced-form interest rate rules to describe the central bank's behavior. Clearly, such behavior is affected by the degree of commitment γ. An open question is how changes in γ are captured by the parameters of a simple rule. To address this question, we perform a Monte Carlo exercise, taking our model as the pseudo-true data generating process but estimating the interest rate rule

$$i_t = \phi_i i_{t-1} + \phi_\pi \pi_t + \phi_y y_t + \epsilon_t, \tag{29}$$

where ϵ_t is assumed to be i.i.d. and normally distributed.

As a clarification, this exercise does not aim at finding the coefficients ϕ_i, ϕ_π, ϕ_y that would maximize welfare, which implies commitment to a simple interest rate rule. That is also an interesting approach, followed for instance in Levine et al. (2008a). Here, we generate data from the model for several degrees of commitment and, as an econometrician would do, estimate the coefficients ϕ_i, ϕ_π, ϕ_y.

Table 1 presents the regression results. The coefficient estimates are similar to those found using actual data. In most cases, the coefficient on output gap is small (and in some cases not significant), the coefficient on inflation is plausible, and there is a considerable degree of interest rate smoothing.[17] Most of the motive for interest rate smoothing comes from commitment. Commitment implies that past policies matter for current allocations, thus introducing history dependence.[18] As a result, when commitment is high, the estimated values of ϕ_i are high even under the alternative loss function, for which per se there is no interest rate–smoothing motive. Overall, the coefficient ϕ_i is more plausible for relatively loose commitment settings rather than with full commitment.

Simple interest rate rules have been widely adopted to study central bank behavior across different periods of time. In that respect, our exercise shows that a change in the interest rate parameters $(\phi_i, \phi_\pi, \phi_y)$ should not necessarily be interpreted as a change in the central bank's preferences. Even if preferences remain unaltered, the reduced-form interest rate parameters may change because of a loss of credibility.

TABLE 1. Interest rate regressions

	Benchmark loss function				Alternative loss function				U.S. Data
	1	0.90	0.50	0	1	0.90	0.50	0	(1970–2008)
ϕ_π	0.241	0.207	1.204	1.914	0.175	0.057	0.725	2.334	0.128
	(0.047)	(0.103)	(0.141)	(0.048)	(0.043)	(0.138)	(0.312)	(0.072)	(0.039)
ϕ_y	0.002	−0.003	0.059	0.105	0.002	−0.010	−0.030	0.12	0.042
	(0.003)	(0.007)	(0.014)	(0.005)	(0.002)	(0.009)	(0.033)	(0.008)	(0.009)
ϕ_i	0.971	0.926	0.875	0.75	0.972	0.843	0.503	0.159	0.926
	(0.022)	(0.033)	(0.038)	(0.015)	(0.022)	(0.06)	(0.062)	(0.027)	(0.028)
R^2	0.923	0.865	0.843	0.977	0.921	0.759	0.416	0.930	0.947

Notes: The table displays the coefficients and standard deviations corresponding to estimating equation (29) in the original model. The Monte Carlo exercise is composed of 1,000 estimations of 200 periods each (roughly corresponding to the size of actual samples). The average standard deviations across simulations are reported in parentheses. The last row displays R^2. The panels on the left and in the center correspond to the benchmark and alternative welfare functions, respectively. The sample regarding the U.S. data goes from 1970:Q1 until 2008:Q3, where the latest data are determined by the beginning of the zero–lower bound period. The output-gap data correspond to the CBO measure.

The simple rule (29) captures fairly well the interest rate behavior, as signaled by the high value of R^2. This R^2 is plausible but lower at intermediate degrees of commitment. The reason is that reoptimizations imply a nonlinear change in the policy setting that the linear regression is not capturing well. The reoptimization uncertainty vanishes with full commitment or discretion, and therefore those two cases can be better described by a linear rule. Also, R^2 is lower for the alternative specification of the loss function. In that case, the absence of an interest rate–smoothing motive in the objective function causes the interest rate to change more abruptly when reoptimizations occur. This result suggests that our benchmark loss function is more consistent with available estimates of the central bank behavior.

4.3. Business Cycle Properties under Loose Commitment

We now analyze the effects of commitment on business cycle properties. To that end, the probability of commitment is set to $\gamma = .90$, implying that policy reoptimizations occur on the average every 10 quarters. That specific value is the one that minimizes the (weighted) difference between the (benchmark) model and the data, with respect to the Taylor-rule coefficients reported in Table 1, and the statistics summarized in Table 2.[19]

Impulse responses to different shocks are reported in Figures 3–5. The solid line considers the specific history where reoptimizations do not occur over the reported horizon ($\eta_t = 1, \forall t$). On impact, the sign of the responses does not change with the commitment assumption. However, for each of the shocks considered, after about six quarters the response of the nominal interest rate does not lie between full commitment (dashed line) and discretion (dash-dotted line). These differences arise because of the uncertainty about future reoptimizations, a feature unique to loose commitment settings.

TABLE 2. Effects of loose commitment on second moments

	Model			
	Full comm.	Loose comm. $\gamma = 0.9$	Discr.	U.S. Data (1970–2008)
		Standard deviation (w.r.t. output)		
Output gap	0.83	0.84	0.83	0.74
Price inflation	0.04	0.04	0.07	0.21
Wage inflation	0.08	0.08	0.09	0.26
Interest rate	0.09	0.15	0.18	0.29
		Cross correlations with output		
Output gap	0.87	0.88	0.86	0.90
Price inflation	0.05	−0.17	−0.70	−0.13
Wage inflation	0.21	0.13	−0.38	0.05
Interest rate	−0.34	−0.49	−0.56	−0.32

Notes: The table displays several statistics for the output gap, price inflation, wage inflation, and the interest rate. The model statistics are computed with 1,000 simulations of 200 periods each. The sample regarding the U.S. data goes from 1970:Q1 until 2008:Q3, where the latest data are determined by the beginning of the zero–lower bound period. The output-gap data correspond to the CBO measure.

For example, the interest rate response to a positive wage-markup shock, shown in Figure 3, peaks after about 10 quarters—as opposed to a negligible response at a similar horizon under both full commitment and discretion. In turn, the output-gap response is more prolonged, whereas both price and wage inflation are close to the values prevailing under commitment. Intuitively, the promise of a deeper and longer recession dampens inflation expectations and helps achieve higher welfare. When the central bank reoptimizes (line with crosses), it reneges upon past promises. It then reduces the interest rate, causing inflation to increase and the output gap to become closer to the target. The bottom right panel shows that the welfare gain of reoptimizing in a given quarter—a measure of the time inconsistency at each moment in time—is maximum after roughly nine quarters. The central bank is fulfilling the promise of a deep recession, which becomes especially costly at that time because inflation is already below target and the output gap is at its lowest level.

Similar reasoning also applies to productivity and government spending shocks.[20] In response to the latter shocks—as well as to other demand-type shocks—the output gap and the two measures of inflation are well stabilized. This occurs regardless of the degree of commitment, and as long as the central bank sets its policy optimally. This suggests that commitment would not be very important if these shocks were the main sources of business cycle fluctuations.[21] Also, the time-inconsistency problem, measured by the gains from reoptimizations (bottom right panel), is much smaller in response to technology and government spending shocks than in response to wage markup shocks.

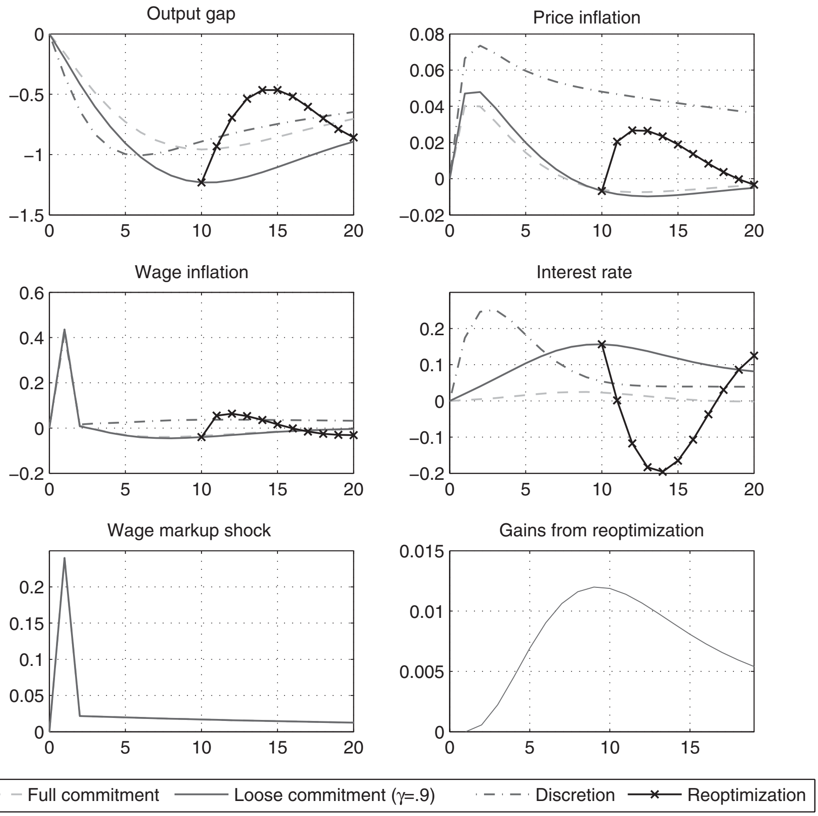

FIGURE 3. Impulse responses to a wage markup shock. The figure plots the impulse responses to a one–standard deviation shock under different commitment settings. The solid line refers to a particular history in which the probability of commitment $\gamma = .90$ and reoptimizations do not occur ($\eta_t = 1, \forall t$). The line with crosses refers to a particular history where the probability of commitment $\gamma = .90$ and a single reoptimization occurs after 10 quarters ($\eta_{10} = 0, \eta_t = 1, \forall t \neq 10$). For any quarter, the gains from reoptimization are computed as the welfare difference between keeping the announced plan and reoptimizing in that particular quarter.

Table 2 shows how commitment affects the second moments for some relevant variables. The correlation of output with the two measures of inflation is positive under full commitment and becomes negative at intermediate degrees of commitment. The reason is that under full commitment, output and inflation are positively correlated not only conditionally on demand shocks, but also conditionally on technology and markup shocks. In response to the latter shocks, output and inflation move in opposite directions on impact, but after about five quarters they comove. With loose commitment, especially if a reoptimization has occurred, inflation and output move in opposite directions for a longer horizon. As a result,

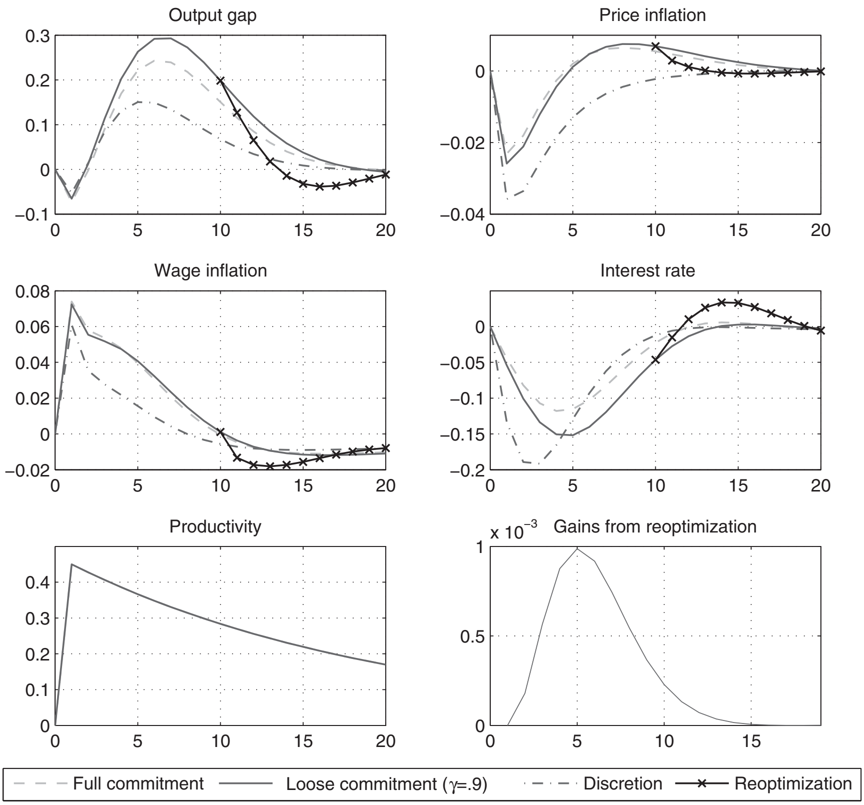

FIGURE 4. Impulse responses to a productivity shock. The figure plots the impulse responses to a one–standard deviation shock under different commitment settings. The solid line refers to a particular history where the probability of commitment $\gamma = .90$ and reoptimizations do not occur ($\eta_t = 1, \forall t$). The line with crosses refers to a particular history where the probability of commitment $\gamma = .90$ and a single reoptimization occurs after 10 quarters ($\eta_{10} = 0, \eta_t = 1, \forall t \neq 10$). For any quarter, the gains from reoptimization are computed as the welfare difference between keeping the announced plan and reoptimizing in that particular quarter.

the correlation between inflation and output conditional on nondemand shocks, as well as the unconditional counterpart, changes sign with even a small departure from the full commitment assumption.[22]

Table 2 also shows that in the data the correlation between output and price inflation is mildly negative, whereas the correlation between output and wage inflation is mildly positive—a feature that the loose-commitment model with $\gamma = .90$ matches quite well. In addition, the relative volatility of interest rates is also more plausible with limited-commitment settings.

Finally, loose commitment changes the relative contribution of alternative shocks to business cycle fluctuations, as summarized in Figure 6. This pattern

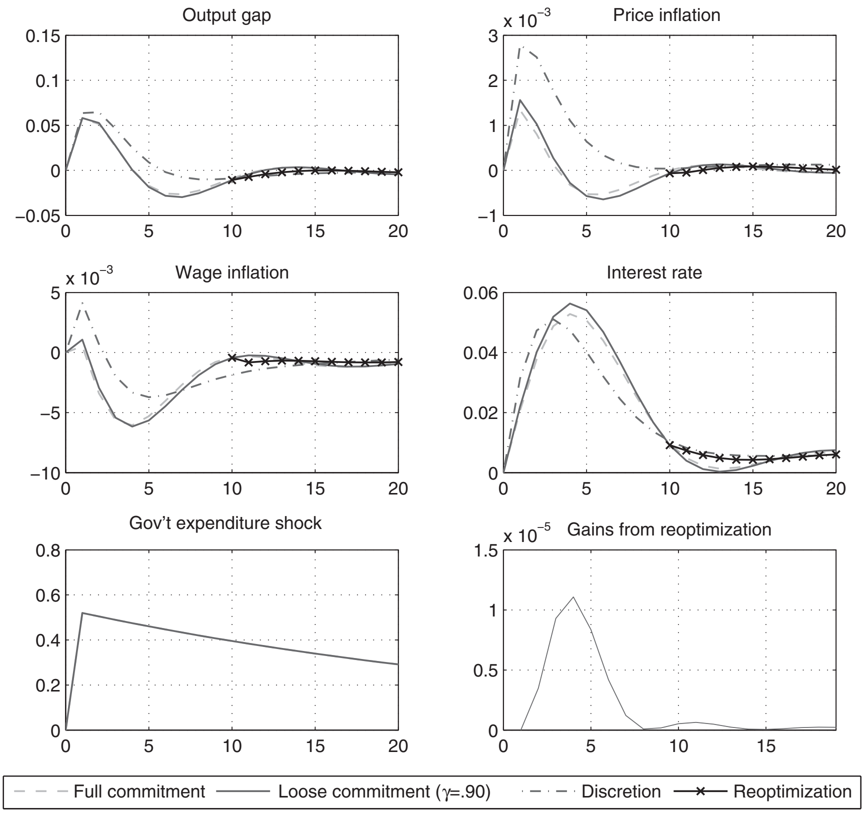

FIGURE 5. Impulse responses to a government spending shock. The figure plots the impulse responses to a one–standard deviation shock, under different commitment settings. The solid line refers to a particular history where the probability of commitment $\gamma = .90$ and reoptimizations do not occur ($\eta_t = 1, \forall t$). The line with crosses refers to a particular history where the probability of commitment $\gamma = .90$ and a single reoptimization occurs after 10 quarters ($\eta_{10} = 0, \eta_t = 1, \forall t \neq 10$). For any quarter, the gains from reoptimization are computed as the welfare difference between keeping the announced plan and reoptimizing in that particular quarter.

is mostly evident for interest rate fluctuations. Under full commitment about 55% of the fluctuations can be attributed to demand shocks. A small loss of credibility ($\gamma = .90$) is enough for this proportion to drop dramatically to about 17%. The contribution of wage and price markup shocks increases from 43% to 72%. The reason is that the interest rate response to a demand shock does not change much with the degree of commitment. Instead, in response to markup shocks, the interest rate barely responds under commitment, whereas it increases and remains high for a long period in limited commitment settings. For almost all the other variables, when commitment is lower, price markup shocks lose importance and

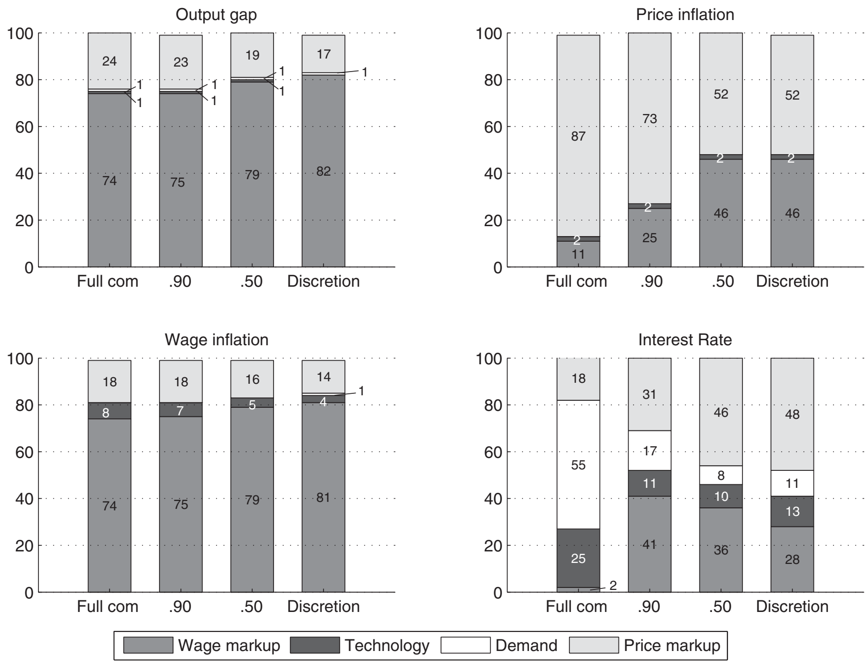

FIGURE 6. Variance decomposition. The figure displays the contributions of different shocks to the variance of our variables, under different commitment scenarios. For convenience, risk-premium, investment-specific, and government-spending shocks have been grouped as "demand" shocks. The model statistics are computed with 1,000 simulations of 200 periods each.

wage-markup shocks become more relevant. Hence, the variance decompositions and the earlier plots measuring time inconsistency suggest that commitment is particularly important to stabilize wage markup shocks.

In summary, loose commitment has large effects on price and wage inflation dynamics and nominal interest rates—the main variables for which the central bank is responsible. The impulse responses to different shocks, as well as the interest rate volatility, are not necessarily in between full commitment and discretion. Finally, small departures from full commitment change the sign of the correlation between output and inflation. In addition, the relative contribution of wage-markup shocks to business cycle fluctuations increases dramatically, especially for interest rates and inflation.

5. CONCLUSIONS

Imperfect commitment settings overcome the dichotomy between full commitment and discretion. In practice, policy makers have some degree of commitment that is not perfect—in some cases they keep a previously formulated policy plan, whereas in other cases they reformulate such plans. Recent proposals for imperfect commitment settings have been restricted to relatively simple and stylized models.

The contribution of this paper is to propose a method and a toolkit that extend the applicability of loose commitment to medium- and large-scale linear–quadratic models typically used in monetary policy. We exemplified the method in the Smets and Wouters (2007) model, where we posed a variety of questions that our method can address and that would otherwise remain unanswered.

Our easy-to-use toolkit permits several modeling extensions. For instance, it would be interesting to incorporate financial frictions, commodity price shocks, and unemployment dynamics and determine the importance of commitment in those cases. Because the optimal policy under loose commitment is not the average of the polar cases of full commitment and discretion, examining the policy responses to such shocks would be interesting per se and shed light on recent economic developments. Also, considering alternative intermediate-credibility settings is certainly desirable, but the technical and computational complexity of addressing the medium- and large-scale models considered here may become prohibitive. On a different note, our methodology could be exploited to analyze the plausibility of alternative commitment settings through an appropriate estimation exercise. We plan to pursue these projects in the near future.

NOTES

1. We have also tested our methodology with bigger models used for monetary policy analysis, such as the Norwegian Economy Model (NEMO) of the Norges Bank.

2. In the presence of steady state distortions, a purely quadratic objective can be obtained using a simple linear combination of the structural equations approximated to second order. However, as shown by Debortoli and Nunes (2006), this requires imposing the so-called "timeless perspective" assumption, which contrasts with the loose commitment settings considered in this paper. For an alternative approach, see Schmitt-Grohe and Uribe (2005).

3. See for example the empirical analysis of Dennis (2004) and Ilbas (2012).

4. In the companion code, models with more lags, leads, constants, and serially correlated shocks are automatically transformed to be consistent with the formulation in equations (1) and (2). Stochastic targets and preference shocks can also be incorporated by suitably expanding the vector y_t.

5. In the case of the United States, the reserve bank presidents serve one-year terms as voting members of the FOMC on a rotating basis, except for the president of the New York Fed. Furthermore, substantial turnover among the reserve bank presidents and the members of the Board of Governors arises because of retirement and outside options. With the (up to) seven members of the Board of Governors being nominated by the U.S. President and confirmed by the U.S. Senate, the composition of views in the FOMC may be affected by the views of the political party in power at the time of the appointment. Chappell et al. (1993) and Berger and Woitek (2005) find evidence of such effects in the United States and Germany, respectively.

6. Such a framework would build on the seminal contributions of Chari and Kehoe (1990), and Kehoe and Levine (1993). A related approach using a model of imperfect information is described in Sleet (2001). Most of these frameworks model the private sector as a representative household therefore avoiding the coordination problem.

7. The functional form of the value function is discussed, for instance, in Ljungqvist and Sargent (2004), (Ch. 5). In the initial period, the policy maker does not have to fulfil any previous promise, and this period is therefore equivalent to a reoptimization.

8. The indicator function is only needed because in deriving equation (10) we have divided all terms by $(\beta\gamma)^t$, which can be done only if $\gamma \neq 0$.

9. Dennis and Kirsanova (2010) propose alternative selection devices based on the concepts of robustness and learnability.

10. A recent work by Himmels and Kirsanova (2011) considers a model with multiple discretionary equilibria and shows that a minimal degree of commitment is enough to eliminate that multiplicity. The authors also propose a way to detect and compute multiple equilibria, which we view as a complement to our analysis.

11. The associated derivations, which follow the steps in Ljungqvist and Sargent (2004), (Ch. 5), are omitted for brevity and are available upon request.

12. Our sample codes incorporate these correction terms.

13. The reader is referred to Benigno and Woodford (2005), (2006), Levin et al. (2005), and Levine et al. (2008a, 2008b).

14. Debortoli and Nunes (2010) formally proved that welfare is increasing in the probability of commitment. Also, as discussed there, the shape of the relative welfare gains changes with the commitment metric. Here, we are considering and comparing results in the literature along the probability-of-commitment metric.

15. Computing the same measure relative to the output-gap target yields the value of 8.52%. This value is higher because the weight on output-gap stabilization is rather small. If, as stated in some central bank treaties such as the ECB, the only goal of the central bank is to stabilize inflation, then this value will be infinity. Also, note that this value is unrelated to consumption-equivalent gains computed with a micro-founded utility function.

16. Evans and Honkapohja (2003) discuss how interest rate rules can implement the optimal policy plan, whereas targeting rules are discussed by Giannoni and Woodford (2010) in a general framework and by Debortoli and Nunes (2011) in a loose commitment setting.

17. For comparability with some studies, the coefficients on inflation and output-gap should be adjusted as $\phi_\pi/(1-\phi_i)$ and $\phi_y/(1-\phi_i)$, respectively.

18. For example, an optimal policy plan under full commitment displays history dependence even when all the disturbances are i.i.d. and in the absence of natural state variables. See, e.g., Galí (2008, Ch. 5).

19. In particular, we chose the value of γ through the simulated method of moments, using the (inverse) of the estimated variance–covariance matrix of the statistics over the sample 1970:Q1–2008:Q3 as the weighting matrix. The resulting value of γ would be very similar if targeting only the Taylor-rule coefficients (.93), or with the alternative objective function (.94).

20. The responses to other shocks also present the same features and are omitted for brevity, but are available upon request.

21. However, this result is not obvious in the current model. The presence of both price and wage rigidities implies a trade-off between inflation and output stabilization, and thus a scope for commitment, even in response to demand and technology shocks.

22. The conditional cross correlations are omitted for brevity and are available upon request.

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APPENDIX

The problem of the central bank under full commitment is

$$V_0 = \min_{\{y_t\}_{t=0}^{\infty}} E_{-1} \sum_{t=0}^{\infty} \beta^t y_t' W y_t \tag{A.1}$$

$$\text{s.t.} \quad A_{-1} y_{t-1} + A_0 y_t + A_1 E_t y_{t+1} + B v_t = 0 \quad \forall t \geq 0,$$

where V_0 is the value function obtained at time 0. Treating the vector y_t as state variables and noting that the value function is quadratic, one obtains $V_0 = y_{-1}' P y_{-1} + d$, where the matrix P and the constant d need to be determined in the equilibrium solution.

The problem under limited commitment needs to be adapted because the central bank can only choose directly the allocations corresponding to histories where it retains commitment. If the commitment technology is broken in a certain time period, previous decisions are disregarded and policy is reoptimized—such an event is analogous to a new central bank or chairman taking over. Although the formulation (A.1) remains valid, before taking first-order conditions, it is helpful to write explicitly the allocations upon which the central bank appointed at $t = 0$ is deciding.

The treatment of the constraints is easier and needs to be adapted according to

$$A_{-1}y_{t-1} + A_0 y_t + A_1 \mathrm{Prob}(\eta_{t+1} = 1) E_t \left(y_{t+1} | \eta_{t+1} = 1\right) \\ + A_1 \mathrm{Prob}(\eta_{t+1} = 0) E_t \left(y^r_{t+1} | \eta_{t+1} = 0\right) + B v_t = 0, \tag{A.2}$$

where y^r_{t+1} refers to the allocations in case a reoptimization occurs. Given our assumptions on the distribution of the reoptimization shocks, this expression is simplified to

$$A_{-1}y_{t-1} + A_0 y_t + \gamma A_1 E_t y_{t+1} + (1-\gamma) A_1 E_t y^r_{t+1} + B v_t = 0, \tag{A.3}$$

where we simplify the notation on the expectations operator because we already distinguish y_{t+1} from y^r_{t+1}.

The objective function also needs to be adapted using similar steps. Whenever a reoptimization occurs, a new central bank takes over and the current central bank cannot decide on those allocations directly. However, the allocations decided by the new central bank still provide utility for the current central bank. Such lifetime utility in the case of reoptimization is conveniently summarized through a value function V^r. Writing a few terms of the central bank's objective function,

$$\begin{aligned} t &= 0 : y_0' W y_0, \\ t &= 1 : +\beta \left[\gamma y_1' W y_1 + (1-\gamma) V_1^r\right], \\ t &= 2 : +\beta^2 \left[\gamma^2 y_2' W y_2 + \gamma (1-\gamma) V_2^r\right], \\ t &> 2 : + \ldots. \end{aligned} \tag{A.4}$$

In period $t = 0$, the welfare terms are written explicitly because the current central bank decides directly on those. In period $t = 1$, discounted at rate β, the central bank chooses the allocations in case a reoptimization does not occur. This event has probability γ. With probability $(1-\gamma)$ a reoptimization occurs and the lifetime utility from that node onward is summarized by V_1^r. Period $t = 2$, as well as later periods, follows the same logic. Grouping all those terms together yields

$$\sum_{t=0}^{\infty} (\beta\gamma)^t \left[y_t' W y_t + \beta (1-\gamma) V^r_{t+1}\right]. \tag{A.5}$$

We solve for an equilibrium at which the problems of the current and future central banks coincide. In the initial period $t = 0$, the central bank does not have to fulfil any previous promises, and this period is therefore equivalent to a reoptimization. For these reasons, we obtain $V^r_{t+1} = y_t' P y_t + d$; the matrix P and scalar d are the same as before, but now one considers the state variables for the corresponding period. Making the relevant substitutions, the planner's problem is therefore given by (4).

Several details of this formulation are available in Debortoli and Nunes (2010). In that paper, we show in detail that all the nodes of the possible tree of events are covered. We also show that, given the value functions V^r_{t+1} and the policy functions when a reoptimization occurs ($E_t y^r_{t+1} = \tilde{H} y_t$), the problem is well posed and fits the framework of Marcet and Marimon (2009). As described in the main text, the solution procedure requires that V^r_{t+1} and $E_t y^r_{t+1}$ be consistent with the equilibrium (through the matrices P and $\tilde{H}$ and the scalar d).