

OPTIMAL TIME-CONSISTENT GOVERNMENT DEBT MATURITY*

DAVIDE DEBORTOLI
RICARDO NUNES
PIERRE YARED

This article develops a model of optimal government debt maturity in which the government cannot issue state-contingent bonds and cannot commit to fiscal policy. If the government can perfectly commit, it fully insulates the economy against government spending shocks by purchasing short-term assets and issuing long-term debt. These positions are quantitatively very large relative to GDP and do not need to be actively managed by the government. Our main result is that these conclusions are not robust to the introduction of lack of commitment. Under lack of commitment, large and tilted debt positions are very expensive to finance ex ante since they exacerbate the problem of lack of commitment ex post. In contrast, a flat maturity structure minimizes the cost of lack of commitment, though it also limits insurance and increases the volatility of fiscal policy distortions. We show that the optimal time-consistent maturity structure is nearly flat because reducing average borrowing costs is quantitatively more important for welfare than reducing fiscal policy volatility. Thus, under lack of commitment, the government actively manages its debt positions and can approximate optimal policy by confining its debt instruments to consols. *JEL Codes:* E62, H21, H63.

*For helpful comments we are grateful to the editor and three anonymous referees; to Marina Azzimonti, Marco Bassetto, Luigi Bocola, Fernando Broner, Francisco Buera, V.V. Chari, Isabel Correia, Jordi Galí, Mike Golosov, Patrick Kehoe, Alessandro Lizzeri, Guido Lorenzoni, Albert Marcet, Jean-Baptiste Michau, Juan Pablo Nicolini, Facundo Piguillem, Ricardo Reis, Andrew Scott, Pedro Teles, Joaquim Voth, Iván Werning, and seminar participants at Bank of Portugal, Boston College, Cambridge, Católica Lisbon School of Business and Economics, Columbia, CREI, EIEF, EUI, Fed Board of Governors, HKUST, London Business School, NBER, Nova School of Business and Economics, Northwestern, NYU, Paris School of Economics, Princeton, and Stonybrook. Sunny Liu provided excellent research assistance. The views expressed in this article are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Federal Reserve Bank of Boston, the Board of Governors of the Federal Reserve System, or of any other person associated with the Federal Reserve System. Debortoli acknowledges financial support from the Marie Curie FP7-PEOPLE-2013-IIF grant 628855 - MONFISCPOL and the Spanish Ministry of Economy and Competitiveness grant ECO2014-55555-P.

© The Author(s) 2016. Published by Oxford University Press, on behalf of President and Fellows of Harvard College. All rights reserved. For Permissions, please email: journals.permissions@oup.com

The Quarterly Journal of Economics (2017), 55–102. doi:10.1093/qje/qjw038.
Advance Access publication on October 13, 2016.

I. INTRODUCTION

How should government debt maturity be structured? Two seminal papers by [Angeletos \(2002\)](#) and [Buera and Nicolini \(2004\)](#) argue that the maturity of government debt can be optimally structured so as to completely hedge the economy against fiscal shocks. This research concludes that optimal debt maturity is tilted long, with the government purchasing short-term assets and selling long-term debt. These debt positions allow the market value of outstanding government liabilities to decline when spending needs and short-term interest rates increase. Moreover, quantitative exercises imply that optimal government debt positions, both short and long, are large (in absolute value) relative to GDP. Finally, these positions are constant and do not need to be actively managed since the combination of constant positions and fluctuating bond prices delivers full insurance.

In this article, we show that these conclusions are sensitive to the assumption that the government can fully commit to fiscal policy. In practice, a government chooses taxes, spending, and debt sequentially, taking into account its outstanding debt portfolio, as well as the behavior of future governments. Thus, a government can always pursue a fiscal policy which reduces (increases) the market value of its outstanding (newly issued) liabilities *ex post*, even though it would not have preferred such a policy *ex ante*. Moreover, the government's future behavior is anticipated by households lending to the government, which affects its *ex ante* borrowing costs. We show that once the lack of commitment by the government is taken into account, it becomes costly for the government to use the maturity structure of debt to completely hedge the economy against shocks; there is a trade-off between the cost of funding and the benefit of hedging.¹ Our main result is that, under lack of commitment, the optimal maturity structure of government debt is quantitatively nearly flat, so that the government owes the same amount to households at all future dates. Moreover, debt is actively managed by the government.

We present these findings in the dynamic fiscal policy model of [Lucas and Stokey \(1983\)](#). This is an economy with public spending shocks and no capital in which the government

1. Our framework is consistent with an environment in which the legislature sequentially chooses a primary deficit and the debt management office sequentially minimizes the cost of financing subject to future risks, which is what is done in practice (see the [IMF report \[2001\]](#)).

chooses linear taxes on labor and issues public debt to finance government spending. Our model features two important frictions. First, as in [Angeletos \(2002\)](#) and [Buera and Nicolini \(2004\)](#), we assume that state-contingent bonds are unavailable, and that the government can only issue real noncontingent bonds of all maturities. Second, and in contrast to [Angeletos \(2002\)](#) and [Buera and Nicolini \(2004\)](#), we assume that the government lacks commitment to policy.

The combination of these two frictions leads to an inefficiency. The work of [Angeletos \(2002\)](#) and [Buera and Nicolini \(2004\)](#) shows that, even in the absence of contingent bonds, an optimally structured portfolio of noncontingent bonds can perfectly insulate the government from all shocks to the economy. Moreover, the work of [Lucas and Stokey \(1983\)](#) shows that, even if the government cannot commit to a path of fiscal policy, an optimally structured portfolio of contingent bonds can perfectly induce a government without commitment to pursue the ex ante optimally chosen policy ex post.² Even though each friction by itself does not lead to an inefficiency, the combination of the two frictions leads to a non-trivial trade-off between market completeness and commitment in the government's choice of maturity.

To get an intuition for this trade-off, consider the optimal policy under commitment. This policy uses debt to smooth fiscal policy distortions in the presence of shocks. If fully contingent claims were available, there would be many maturity structures that would support the optimal policy. However, if the government only has access to noncontingent claims, then there is a unique maturity structure which replicates full insurance. As has been shown in [Angeletos \(2002\)](#) and [Buera and Nicolini \(2004\)](#), such a maturity structure is tilted in a manner that guarantees that the market value of outstanding government liabilities declines when the net present value of future government spending rises. If this occurs when short-term interest rates rise—as is the case in quantitative examples with Markovian fiscal shocks—then the optimal maturity structure requires that the government purchases short-term assets and sells long-term debt. Because interest rate movements are quantitatively small, the tilted debt positions required for hedging are large.

2. This result requires the government to lack commitment to taxes or to spending but not to both. See [Rogers \(1989\)](#) for more discussion.

Under lack of commitment, such large and tilted positions are very costly to finance *ex ante* if the government cannot commit to policy *ex post*. The larger and more tilted the debt position, the greater a future government's benefit from pursuing policies *ex post* which changes bond prices to relax the government's budget constraint. To relax its budget constraint, the government can either reduce the market value of its outstanding long-term liabilities by choosing policies that increase short-term interest rates, or it can increase the market value of its newly issued short-term liabilities by choosing policies that reduce short-term interest rates. If the government's debt liabilities are mostly long term, then the government will follow the former strategy *ex post*. If its liabilities are mostly short term, then the government will pursue the latter strategy *ex post*. Households purchasing government bonds *ex ante* internalize the fact that the government will pursue such policies *ex post*, and they therefore require higher interest rates to lend to the government the more tilted is the government's debt maturity.

For this reason, the flatter the debt maturity—meaning the smaller the difference between short-term and long-term debt—the lower the cost of funding for the government. Such a flat maturity maximizes the government's commitment to future fiscal policies by minimizing the benefit of any future deviations. However, a flatter debt maturity comes at the cost of lower insurance for the government; the flatter the debt maturity, the smaller the fluctuation in the market value of outstanding government liabilities, and the more exposed is the government to fiscal shocks.

To assess optimal policy in light of this trade-off, we analyze the Markov perfect competitive equilibrium of our model in which the government dynamically chooses its policies at every date as a function of payoff relevant variables: the fiscal shock and its outstanding debt position at various maturities. Because a complete analysis of such an equilibrium in an infinite horizon economy with an infinite choice of debt maturities is infeasible, we present our main result in three exercises.

In our first exercise, we show that optimal debt maturity is exactly flat in a three-period example as the volatility of future shocks goes to zero or as the persistence of future shocks goes to 1. In both of these cases, a government under commitment financing a deficit in the initial date chooses a negative short-term debt position and a positive long-term debt position which are large in magnitude. However, a government under lack of commitment

chooses an exactly flat debt maturity with a positive short-term and long-term debt position which equal each other.

In our second exercise, we show that the insights of the three-period example hold approximately in a quantitative finite horizon economy under fiscal shocks with empirically plausible volatility and persistence. We consider a finite horizon economy since this allows the government's debt maturity choices to also be finite. We find that, despite having the ability to choose from a flexible set of debt maturity structures, the optimal debt maturity is nearly flat, and the main component of the government's debt can be represented by a consol with a fixed nondecaying payment at all future dates.

In our final exercise, we consider an infinite horizon economy, and we show that optimal policy under lack of commitment can be quantitatively approximated with active consol management, so that the optimal debt maturity is again nearly flat. An infinite horizon analysis allows us to more suitably capture quantitative features of optimal policy and to characterize policy dynamics, but it also comes at a cost of not being able to consider the entire range of feasible debt maturity policies by the government. We consider a setting in which the government has access to two debt instruments: a nodecaying consol and a decaying perpetuity. Under full commitment, the government holds a highly tilted debt maturity, where each position is large in absolute value and constant. In contrast, under lack of commitment, the government holds a negligible and approximately constant position in the decaying perpetuity, and it holds a positive position in the consol which it actively manages in response to fiscal shocks. We additionally show that our conclusion that optimal debt maturity is approximately flat is robust to the choice of volatility and persistence of fiscal shocks, to the choice of household preferences, and to the introduction of productivity and discount factor shocks.

Our results show that structuring government debt maturity to resolve the problem of lack of commitment is more important than structuring it to resolve the problem of lack of insurance. It is clear that a flat debt maturity comes at a cost of less hedging. However, substantial hedging requires massive and tilted debt positions. When the government lacks commitment, financing these large positions can be very expensive in terms of average fiscal policy distortions. Moreover, under empirically plausible levels of volatility of public spending, the cost of lack of insurance under a flat maturity structure is small. Therefore, the optimal policy

pushes in the direction of reducing average fiscal policy distortions versus reducing the volatility of distortions, and the result is a nearly flat maturity structure.³

Our analysis implies that government debt management in practice is much closer to the theoretically optimal policy under lack of commitment versus that under full commitment. In the United States, for example, government bond payments across the maturity spectrum are all positive, small relative to GDP, actively managed, and with significant comovement across maturities. All these features are consistent with optimal policy under lack of commitment. Nevertheless, while the optimal policy under lack of commitment prescribes the issuance of consols, the highest bond maturity for the U.S. government is 30 years. Determining whether a maturity extension would move the U.S. government closer to an optimal policy is a complicated question. The answer depends in part on how to measure the maturity structure of the government's overall liabilities, which can additionally include partial commitments to future transfers such as Social Security and Medicare. Such an analysis goes beyond the scope of this paper and is an interesting avenue for future research.

I.A. Related Literature

This paper is connected to several literatures. As discussed, we build on the work of [Angeletos \(2002\)](#) and [Buera and Nicolini \(2004\)](#) by introducing lack of commitment.⁴ Our model is most applicable to economies in which the risks of default and surprise in inflation are not salient, but the government is still not committed to a path of deficits and debt maturity issuance. [Arellano et al. \(2013\)](#) study a similar setting to ours but with nominal frictions and lack of commitment to monetary policy.⁵ In contrast to [Aguiar](#)

3. The conclusion that the welfare benefit of smoothing economic shocks is small relative to that of improving economic levels is more generally tied to the insight in [Lucas \(1987\)](#).

4. Additional work explores government debt maturity maintaining the assumption of full commitment, in environments with less debt instruments than states ([Shin 2007](#)), in models with habits, productivity shocks and capital ([Faraglia, Marcet, and Scott 2010](#)), in the presence of nominal rigidities ([Lustig, Sleet, and Yeltekin 2008](#)), or in a preferred habitat model ([Guibaud, Nosbusch, and Vayanos 2013](#)).

5. In addition, [Alvarez, Kehoe, and Neumeyer \(2004\)](#) and [Persson, Persson, and Svensson \(2006\)](#) consider problems of lack of commitment in an environment with real and nominal bonds of varying maturity where the possibility of surprise inflation arises. [Alvarez, Kehoe, and Neumeyer \(2004\)](#) find that to minimize

and Amador (2014), Arellano and Ramanarayanan (2012), and Fernandez and Martin (2015)—who consider small open economy models with the possibility of default—we focus on lack of commitment to taxation and debt issuance, which affects the path of risk-free interest rates. This difference implies that, in contrast to their work, short-term debt does not dominate long-term debt in minimizing the government’s lack of commitment problem. In our setting, even if the government were to only issue short-term debt, the government *ex post* would deviate from the *ex ante* optimal policy by pursuing policies which reduce short-term interest rates below the *ex ante* optimal level.⁶

More broadly, our article is also tied to the literature on optimal fiscal policy which explores the role of incomplete markets. A number of papers have studied optimal policy under full commitment when the government issues one-period noncontingent bonds, such as Barro (1979) and Aiyagari *et al.* (2002).⁷ Bhandari *et al.* (2015) generalize the results of this work by characterizing optimal fiscal policy under commitment whenever the government has access to any limited set of debt securities. As in this work, we find that optimal taxes respond persistently to economic shocks, though in contrast to this work, this persistence is due to the lack of commitment by the government as opposed to the incompleteness of financial markets due to limited debt instruments.

Other work has studied optimal policy in settings with lack of commitment, but with full insurance (e.g., Krusell, Martin, and Ríos-Rull 2006; Debortoli and Nunes 2013). We depart from this work by introducing long-term debt, which in a setting with full insurance can imply that the lack of commitment friction no longer introduces any inefficiencies.

Our article proceeds as follows. In Section II, we describe the model and define the equilibrium. In Section III, we show that the

incentives for surprise inflation, the government should only issue real bonds. Barro (2003) comes to a similar conclusion.

6. In a small open economy with default, the risk-free rate is exogenous and the government’s *ex post* incentives are always to issue more debt, increasing short-term interest rates (which include the default premium) above the *ex ante* optimal level. For this reason, short-term debt issuance *ex ante* can align the incentives of the government *ex ante* with those of the government *ex post*. Niepelt (2014), Chari and Kehoe (1993a, 1993b), and Sleet and Yeltekin (2006) also consider the lack of commitment under full insurance, though they focus on settings that allow for default.

7. See also Farhi (2010).

optimal debt maturity is exactly flat in a three-period example. In [Section IV](#), we show that the optimal debt maturity is nearly flat in a finite horizon economy with unlimited debt instruments and in an infinite horizon economy with limited debt instruments. [Section V](#) concludes. The Appendix and the [Online Appendix](#) provide all of the proofs and additional results not included in the text.

II. MODEL

II.A. Environment

We consider an economy identical to that of [Lucas and Stokey \(1983\)](#) with two modifications. First, we rule out state-contingent bonds. Second, we assume that the government cannot commit to fiscal policy. There are discrete time periods $t = \{1, \dots, \infty\}$ and a stochastic state $s_t \in S$ which follows a first-order Markov process. s_0 is given. Let $s^t = \{s_0, \dots, s_t\} \in S^t$ represent a history, and let $\pi(s^{t+k}|s^t)$ represent the probability of s^{t+k} conditional on s^t for $t+k \geq t$.

The resource constraint of the economy is

$$(1) \quad c_t + g_t = n_t,$$

where c_t is consumption, n_t is labor, and g_t is government spending.

There is a continuum of mass 1 of identical households that derive the following utility:

$$(2) \quad \mathbb{E} \sum_{t=0}^{\infty} \beta^t [u(c_t, n_t) + \theta_t(s_t) v(g_t)], \beta \in (0, 1).$$

$u(\cdot)$ is strictly increasing in consumption and strictly decreasing in labor, globally concave, and continuously differentiable. $v(\cdot)$ is strictly increasing, concave, and continuously differentiable. Under this representation, $\theta_t(s_t)$ is high (low) when public spending is more (less) valuable. In contrast to the model of [Lucas and Stokey \(1983\)](#), we have allowed g_t in this framework to be chosen by the government, as opposed to being exogenously determined. We allow for this possibility to also consider that the government may not be able to commit to the ex ante optimal level of public spending. In our analysis, we also consider the [Lucas and Stokey \(1983\)](#) environment in which there is no discretion over government spending, and we show that all of our results hold.

Household wages equal the marginal product of labor (which is 1 unit of consumption) and are taxed at a linear tax rate τ_t . $b_t^{t+k} \geq 0$ represents government debt purchased by a representative household at t , which is a promise to repay 1 unit of consumption at $t+k > t$, and q_t^{t+k} is its price at t . At every t , the household's allocation $\{c_t, n_t, \{b_t^{t+k}\}_{k=1}^\infty\}$ must satisfy the household's dynamic budget constraint

$$(3) \quad c_t + \sum_{k=1}^{\infty} q_t^{t+k} (b_t^{t+k} - b_{t-1}^{t+k}) = (1 - \tau_t) n_t + b_{t-1}^t.$$

$B_t^{t+k} \geq 0$ represents debt issued by the government at t with a promise to repay 1 unit of consumption at $t+k > t$. At every t , government policies $\{\tau_t, g_t, \{B_t^{t+k}\}_{k=1}^\infty\}$ must satisfy the government's dynamic budget constraint

$$(4) \quad g_t + B_{t-1}^t = \tau_t n_t + \sum_{k=1}^{\infty} q_t^{t+k} (B_t^{t+k} - B_{t-1}^{t+k}).^8$$

The economy is closed, which means that the bonds issued by the government equal the bonds purchased by households:

$$(5) \quad b_t^{t+k} = B_t^{t+k} \quad \forall t, k.$$

Initial debt $\{B_{-1}^{k-1}\}_{k=1}^\infty$ is exogenous.⁹ We assume that there exist debt limits to prevent Ponzi schemes:

$$(6) \quad B_t^{t+k} \in [\underline{B}, \overline{B}].$$

8. We follow the same exposition as in [Angeletos \(2002\)](#) in which the government restructures its debt in every period by buying back all outstanding debt and then issuing fresh debt at all maturities. This is without loss of generality. For example, if the government at $t-k$ issues debt due at date t of size B_{t-k}^t which it then holds to maturity, then all future governments at date $t-k+l$ for $l=1, \dots, k-1$ will choose $B_{t-k+l}^t = B_{t-k}^t$, implying that $B_{t-k}^t = B_{t-1}^t$.

9. Our model implicitly allows the government to buy back the long-term bonds from the private sector. While ruling out bond buybacks is interesting, 85% of countries conduct some form of bond buyback and 32% of countries conduct them on a regular basis (see the OECD report by [Blommestein, Elmadag, and Ejsing 2012](#)). Note furthermore, that even if bond buyback is not allowed in our environment, a government can replicate the buyback of a long-term bond by purchasing an asset with a payout on the same date (see [Angeletos 2002](#)). See [Faraglia et al. \(2014\)](#) for a discussion of optimal policy under commitment in the absence of buybacks.

We let \underline{B} be sufficiently low and \overline{B} be sufficiently high so that [equation \(6\)](#) does not bind in our theoretical and quantitative exercises.

A key friction in this environment is the absence of state-contingent debt, since the value of outstanding debt B_t^{t+k} is independent of the realization of the state s_{t+k} . If state-contingent bonds were available, then at any date t , the government would own a portfolio of bonds $\{\{B_{t-1}^{t+k}|s^{t+k}\}_{s^{t+k} \in S^{t+k}}\}_{k=0}^{\infty}$, where the value of each bond payout at date $t+k$ would depend on the realization of a history of shocks $s^{t+k} \in S^{t+k}$. In our discussion, we will refer back to this complete market case.

The government is benevolent and shares the same preferences as the households in [equation \(2\)](#). We assume that the government cannot commit to policy and therefore chooses taxes, spending, and debt sequentially.

II.B. Definition of Equilibrium

We consider a Markov perfect competitive equilibrium (MPCE) in which the government must optimally choose its preferred policy—which consists of taxes, spending, and debt—at every date as a function of current payoff-relevant variables: the current shock and current debt outstanding. The government takes into account that its choice affects future debt and thus affects the policies of future governments. Households rationally anticipate these future policies, and their expectations are in turn reflected in current bond prices. Thus, in choosing policy today, a government anticipates that it may affect current bond prices by impacting expectations about future policy. We provide a formal definition of the equilibrium in the Appendix.

While we assume for generality that the government can freely choose taxes, spending, and debt in every period, we also consider cases throughout the draft in which the government does not have discretion in either setting spending or in setting taxes. These special cases highlight how the right choice of government debt maturity can induce future governments to choose the commitment policy.

II.C. Primal Approach

Any MPCE must be a competitive equilibrium. We follow [Lucas and Stokey \(1983\)](#) by taking the primal approach to the characterization of competitive equilibria since this allows us to abstract

away from bond prices and taxes. Let

$$(7) \quad \{c_t(s^t), n_t(s^t), g_t(s^t)\}_{s^t \in S^t} \Big|_{t=0}^{\infty}$$

represent a stochastic sequence, where the resource constraint [equation \(1\)](#) implies

$$(8) \quad c_t(s^t) + g_t(s^t) = n_t(s^t).$$

We can establish necessary and sufficient conditions for [equation \(7\)](#) to constitute a competitive equilibrium. The household's optimization problem implies the following intratemporal and intertemporal conditions, respectively:

$$(9) \quad \begin{aligned} 1 - \tau_t(s^t) &= -\frac{u_{n,t}(s^t)}{u_{c,t}(s^t)} \text{ and} \\ q_t^{t+k}(s^t) &= \frac{\sum_{s^{t+k} \in S^{t+k}} \beta^k \pi(s^{t+k}|s^t) u_{c,t+k}(s^{t+k})}{u_{c,t}(s^t)}. \end{aligned}$$

Substitution of these conditions into the household's dynamic budget constraint implies the following condition:

$$(10) \quad \begin{aligned} &u_{c,t}(s^t) c_t(s^t) + u_{n,t}(s^t) n_t(s^t) \\ &+ \sum_{k=1}^{\infty} \sum_{s^{t+k} \in S^{t+k}} \beta^k \pi(s^{t+k}|s^t) u_{c,t+k}(s^{t+k}) B_t^{t+k}(s^t) \\ &= \sum_{k=0}^{\infty} \sum_{s^{t+k} \in S^{t+k}} \beta^k \pi(s^{t+k}|s^t) u_{c,t+k}(s^{t+k}) B_{t-1}^{t+k}(s^{t-1}). \end{aligned}$$

Forward substitution into the above equation and taking into account the absence of Ponzi schemes implies the following implementability condition:

$$(11) \quad \begin{aligned} &\sum_{k=0}^{\infty} \sum_{s^{t+k} \in S^{t+k}} \beta^k \pi(s^{t+k}|s^t) [u_{c,t+k}(s^{t+k}) c_{t+k}(s^{t+k}) \\ &+ u_{n,t+k}(s^{t+k}) n_{t+k}(s^{t+k})] \\ &= \sum_{k=0}^{\infty} \sum_{s^{t+k} \in S^{t+k}} \beta^k \pi(s^{t+k}|s^t) u_{c,t+k}(s^{t+k}) B_{t-1}^{t+k}(s^{t-1}). \end{aligned}$$

By this reasoning, if a stochastic sequence in [equation \(7\)](#) is generated by a competitive equilibrium, then it necessarily satisfies [equations \(8\)](#) and [\(11\)](#). We prove in [Online Appendix A](#) that the converse is also true, which leads to the following proposition that is useful for the rest of our analysis.

PROPOSITION 1 (Competitive Equilibrium). A stochastic sequence [equation \(7\)](#) is a competitive equilibrium if and only if it satisfies [equation \(8\)](#) $\forall s^t$ and $\exists \{\{B_{t-1}^{t+k}(s^{t-1})\}_{k=0}^{\infty}\}_{s^{t-1} \in S^{t-1}}\}_{t=0}^{\infty}$ which satisfy [equation \(11\)](#) $\forall s^t$.

A useful corollary to this proposition concerns the relevant implementability condition in the presence of state-contingent bonds, $B_t^{t+k}|s^{t+k}$, which provide payment at $t+k$ conditional on the realization of a history s^{t+k} .

COROLLARY 1. In the presence of state-contingent debt, a stochastic sequence [equation \(7\)](#) is a competitive equilibrium if and only if it satisfies [equations \(8\)](#) $\forall s^t$ and [\(11\)](#) for $s^t = s^0$ given initial liabilities.

If state-contingent debt is available, then the satisfaction of [equation \(11\)](#) at s^0 guarantees the satisfaction of [equation \(11\)](#) for all other histories s^t , since state-contingent payments can be freely chosen so as to satisfy [equation \(11\)](#) at all future histories s^t .

In the Appendix, we show how the primal approach can be used to represent the MPCE recursively.

III. THREE-PERIOD EXAMPLE

We turn to a simple three-period example to provide intuition for our quantitative results. This example allows us to explicitly characterize government policy both with and without commitment, making it possible to highlight how dramatically different optimal debt maturity is under the two scenarios.

Let $t = 0, 1, 2$ and define θ^L and θ^H with $\theta^H = 1 + \delta$ and $\theta^L = 1 - \delta$ for $\delta \in [0, 1)$. Suppose that $\theta_0 > \theta^H$, $\theta_1 = \theta^H$ with probability $\frac{1}{2}$ and $\theta_1 = \theta^L$ with probability $\frac{1}{2}$. In addition, let $\theta_2 = \alpha\theta^H + (1 - \alpha)\theta^L$ if $\theta_1 = \theta^H$ and $\theta_2 = \alpha\theta^L + (1 - \alpha)\theta^H$ if $\theta_1 = \theta^L$ for $\alpha \in [0.5, 1)$. Therefore, all of uncertainty is realized at date 1, with δ capturing the volatility of the shock and α capturing the persistence of the shock between dates 1 and 2.

Suppose that taxes and labor are exogenously fixed to some τ and n , respectively, so that the government collects a constant

revenue in all dates. Assume that the government's welfare can be represented by

$$(12) \quad \mathbb{E} \sum_{t=0,1,2} \beta^t [(1 - \psi) \log c_t + \psi \theta_t g_t]$$

for $\psi \in [0, 1]$. We consider the limiting case in which $\psi \rightarrow 1$, and we let $\beta = 1$ for simplicity. There is zero initial debt and all debt is repaid in the final period. Thus, the implementability conditions at date 0 and date 1 are given, respectively, by

$$(13) \quad \frac{c_0 - n(1 - \tau)}{c_0} + \mathbb{E} \left(\frac{c_1 - n(1 - \tau)}{c_1} + \frac{c_2 - n(1 - \tau)}{c_2} \right) \geq 0,$$

$$(14) \quad \frac{c_1 - n(1 - \tau)}{c_1} + \frac{c_2 - n(1 - \tau)}{c_2} \geq \frac{B_0^1}{c_1} + \frac{B_0^2}{c_2}.$$

In this environment, the government does not have any discretion over tax policy, and any ex post deviation by the government is driven by a desire to increase spending since the marginal benefit of additional spending always exceeds the marginal benefit of consumption.

III.A. Full Commitment

This section shows analytically that a government with commitment chooses highly tilted and large debt positions to fully insulate the economy from shocks. [Angeletos \(2002\)](#) proves that any allocation under state-contingent debt can be approximately implemented with noncontingent debt. This implies that there is no inefficiency stemming from the absence of contingent debt. Our example explicitly characterizes these allocations to provide a theoretical comparison with those under lack of commitment.

Let us consider an economy under complete markets. From [Corollary 1](#), the only relevant constraints on the planner are the resource constraints and the date 0 implementability constraint (13), which holds with equality. The maximization of social welfare under these constraints leads to the following optimality condition

$$(15) \quad c_t = \frac{1}{\theta_t^{\frac{1}{2}}} \frac{n(1 - \tau)}{3} \mathbb{E} \left(\sum_{k=0,1,2} \theta_k^{\frac{1}{2}} \right) \forall t.$$

Equation (15) implies that in the presence of full insurance, spending is independent of history and depends only on the state θ_t , which takes on two possible realizations at $t = 1, 2$.

This allocation can be sustained even if state-contingent bonds are not available. From Proposition 1, it suffices to show that the additional constraint (14) is also satisfied. This is possible by choosing appropriate values of B_0^1 and B_0^2 which simultaneously satisfy equation (14) (which holds with equality) and (15). It can also be shown that

$$B_0^1 < 0 \quad \text{and} \quad B_0^2 > 0.$$

Intuitively, the net present value of the government's primary surpluses at $t = 1$ is lower if the high shock is realized under the solution in equation (15). To achieve this full insurance solution with noncontingent debt, the government must choose the maturity structure so that the market value of the government's outstanding bond portfolio at $t = 1$ is lower if the high shock is realized. This market value at $t = 1$ is given by

$$(16) \quad B_0^1 + \frac{c_1}{c_2} B_0^2.$$

Since the shock is mean-reverting, it follows from equation (9) that the one-period bond price at $t = 1$, $\frac{c_1}{c_2}$ is lower if the shock is high. As such, choosing $B_0^1 < 0 < B_0^2$ provides insurance to the government. How large are the debt positions required to achieve full insurance? The below proposition shows that the magnitude of these positions can be very high.

PROPOSITION 2 (Full Commitment). The following characterizes the unique solution under full commitment:

- i. (deterministic limit) As $\delta \rightarrow 0$,

$$(17) \quad B_0^1 = -n(1 - \tau) \frac{\theta_0^{\frac{1}{2}} + 2}{3} \frac{(2\alpha - 1) + (1 - \alpha)}{1 - \alpha} < 0 \text{ and}$$

$$(18) \quad B_0^2 = n(1 - \tau) \frac{\theta_0^{\frac{1}{2}} + 2}{3} \frac{-(1 - \alpha)}{1 - \alpha} > 0.$$

ii. (full persistence limit) As $\alpha \rightarrow 1$,

$$B_0^1 \rightarrow -\infty \quad \text{and} \quad B_0^2 \rightarrow \infty.$$

The first part of Proposition 2 characterizes the optimal value of the short-term debt B_0^1 and the long-term debt B_0^2 as the variance of the shock δ goes to 0. There are a few points to note regarding this result. First, it should be highlighted that this is a limiting result. At $\delta = 0$, the optimal values of B_0^1 and B_0^2 are indeterminate. This is because there is no hedging motive, and any combination of B_0^1 and B_0^2 which satisfies

$$(19) \quad B_0^1 + B_0^2 = 2n(1 - \tau) \frac{\theta_0^{\frac{1}{2}} - 1}{3}$$

is optimal, since the market value of total debt—which is what matters in a deterministic economy—is constant across these combinations. Therefore, the first part of the proposition characterizes the solution for δ arbitrarily small, in which case the hedging motive still exists, leading to a unique maturity structure. Second, in the limit, the debt positions do not go to 0, and the government maintains a positive short-term asset position and a negative long-term debt position. This happens since, even though the need for hedging goes to 0 as volatility goes to 0, the volatility in short-term interest rates goes to 0 as well. The size of a hedging position depends in part on the variation in the short-term interest rate at date 1 captured by the variation in $\frac{c_1}{c_2}$ in the complete market equilibrium. The smaller this variation, the larger is the required position to generate a given variation in the market value of debt to generate insurance. This fact implies that the positions required for hedging do not need to go to 0 as volatility goes to 0. As a final point, note that the debt positions can be large in absolute value. For example, since $\theta_0 > 1$ and $\alpha \geq 0.5$, $B_0^1 < -n(1 - \tau)$ and $B_0^2 > n(1 - \tau)$, so that the absolute value of each debt position strictly exceeds the disposable income of households.

The second part of the proposition states that as the persistence of the shock between dates 1 and 2 goes to 1, the magnitude of the debt positions chosen by the government explodes to infinity, so that the government holds an infinite short-term asset position and an infinite long-term debt position. As we

discussed, the size of a hedging position depends in part on the variation in the short-term interest rate at date 1 captured by the variation in $\frac{c_1}{c_2}$ in the complete market equilibrium. As the persistence of the shock goes to 1, the variation in the short-term interest rate at date 1 goes to 0, and since the need for hedging does not go to 0, this leads to the optimality of infinite debt positions. Under these debt positions, the government can fully insulate the economy from shocks since [equation \(15\)](#) continues to hold.

The two parts of Proposition 2 are fairly general and do not depend on the details of our particular example. These results are a consequence of the fact that fluctuations in short-term interest rates should go to 0 as the volatility of shocks goes to 0 or the persistence of shocks goes to 1. To the extent that completing the market using maturities is possible, the reduced volatility in short-term interest rates is a force which increases the magnitude of optimal debt positions required for hedging. In addition, note that our theoretical result is consistent with the quantitative results of [Angeletos \(2002\)](#) and [Buera and Nicolini \(2004\)](#). These authors present a number of examples in which volatility is not equal to 0 and persistence is not equal to 1, yet the variation in short-term interest rates is very small, and optimal debt positions are very large in magnitude relative to GDP.

III.B. Lack of Commitment

We now show that optimal policy changes dramatically once we introduce lack of commitment. We solve for the equilibrium under lack of commitment by using backward induction. At date 2, the government has no discretion in its choice of fiscal policy, and it chooses $c_2 = n(1 - \tau) + B_1^2$.

Now consider government policy at date 1. The government maximizes its continuation welfare given B_0^1 and B_0^2 , the resource constraint, and the implementability condition [\(14\)](#). Note that if $n(1 - \tau) + B_0^t \leq 0$ for $t = 1, 2$, then no allocation can satisfy [equation \(14\)](#) with equality. Therefore, such a policy is infeasible at date 0 and is never chosen. The lemma below characterizes government policy for all other values of $\{B_0^1, B_0^2\}$.

LEMMA 1. If $n(1 - \tau) + B_0^t > 0$ for $t = 1, 2$, the date 1 government under lack of commitment chooses:

$$(20) \quad c_t = \frac{1}{2} \left(\frac{n(1 - \tau) + B_0^t}{\theta_t} \right)^{\frac{1}{2}} \times \left[\sum_{k=1,2} \theta_k^{\frac{1}{2}} (n(1 - \tau) + B_0^k)^{\frac{1}{2}} \right] \text{ for } t = 1, 2.$$

If $n(1 - \tau) + B_0^t \leq 0$ for either $t = 1$ or $t = 2$, the date 1 government can maximize welfare by choosing c_t arbitrarily close to 0 for $t = 1, 2$.

Given this policy function at dates 1 and 2, the government at date 0 chooses a value of c_0 and $\{B_0^1, B_0^2\}$ given the resource constraint and given [equation \(13\)](#) so as to maximize social welfare.¹⁰ We proceed by deriving the analog of Proposition 2 but removing the commitment assumption. We conclude by discussing optimal debt maturity away from those limiting cases.

1. Deterministic Limit. If we substitute [equation \(20\)](#) into the social welfare function [\(12\)](#) and date 0 implementability condition [\(13\)](#), we can write the government's problem at date 0 as:

$$(21) \quad \max_{c_0, B_0^1, B_0^2} -\theta_0 c_0 - \frac{1}{2} \mathbb{E} \left[\sum_{t=1,2} \theta_t^{\frac{1}{2}} (n(1 - \tau) + B_0^t)^{\frac{1}{2}} \right]^2$$

$$(22) \quad \text{s.t. } c_0 = \frac{n(1 - \tau)}{3 - 2n(1 - \tau) \mathbb{E} \left[\frac{\sum_{t=1,2} \theta_t^{\frac{1}{2}} (n(1 - \tau) + B_0^t)^{-\frac{1}{2}}}{\sum_{t=1,2} \theta_t^{\frac{1}{2}} (n(1 - \tau) + B_0^t)^{\frac{1}{2}}} \right]}.$$

i. Optimality of a Flat Maturity Structure. Proposition 3 states that as the volatility of the shock δ goes to 0, the unique

10. It is straightforward to see that the government never chooses $n(1 - \tau) + B_0^t \leq 0$ for either $t = 1$ or $t = 2$. In that case, c_t is arbitrarily close to 0 for $t = 1, 2$, which implies that [equation \(13\)](#) is violated since a positive value of c_0 cannot satisfy that equation. Therefore, date 0 policy always satisfies $n(1 - \tau) + B_0^t > 0$ for $t = 1, 2$ and [equation \(20\)](#) applies.

optimal solution under lack of commitment admits a flat maturity structure with $B_0^1 = B_0^2$. It implies that for arbitrarily low levels of volatility, the government will choose a nearly flat maturity structure, which is in stark contrast to the case of full commitment described in Proposition 2. In that case, debt positions take on opposing signs and are bounded away from 0 for arbitrarily low values of volatility.

PROPOSITION 3 (Lack of Commitment, Deterministic Limit). The unique solution under lack of commitment as $\delta \rightarrow 0$ satisfies

$$(23) \quad B_0^1 = B_0^2 = n(1 - \tau) \frac{\theta_0^{\frac{1}{2}} - 1}{3} = \frac{1}{2} \bar{B} > 0.$$

When δ goes to 0, the cost of lack of commitment also goes to 0. The reason is that, as in [Lucas and Stokey \(1983\)](#), the government utilizes the maturity structure of debt in order to achieve the same allocation as under full commitment characterized in [equation \(15\)](#). More specifically, while the program under commitment admits a unique solution for $\delta > 0$, when $\delta = 0$, any combination of B_0^1 and B_0^2 satisfying

$$B_0^1 + B_0^2 = \bar{B}$$

is optimal. Whereas the government with commitment can choose any such maturity, the government under lack of commitment must by necessity choose a flat maturity in order to achieve the same welfare.

Why is a flat maturity structure optimal as volatility goes to 0? To see this, let $\delta = 0$, and consider the incentives of the date 1 government. This government—which cares only about raising spending—would like to reduce the market value of what it owes to the private sector which from the intertemporal condition can be represented by

$$(24) \quad B_0^1 + \frac{c_1}{c_2} B_0^2.$$

Moreover, the government would also like to increase the market value of newly issued debt, which can be represented by

$$(25) \quad \frac{c_1}{c_2} B_1^2.$$

If debt maturity were tilted toward the long end, then the date 1 government would deviate from a smooth policy so as to reduce the value of what it owes. For example, suppose that $B_0^1 = 0$ and $B_0^2 = \bar{B}$. Under commitment, it would be possible to achieve the optimum under this debt arrangement. However, under lack of commitment, [equation \(20\)](#) implies that the government deviates from the smooth ex ante optimal policy by choosing $c_1 < c_2$. This deviation, which is achieved by issuing higher levels of debt B_1^2 relative to commitment, serves to reduce the value of what the government owes in [equation \(24\)](#), therefore freeing up resources to be utilized for additional spending at date 1.

Analogously, if debt maturity were tilted toward the short end, then the government would deviate from a smooth policy so as to increase the value of what it issues. For example, suppose that $B_0^1 = \bar{B}$ and $B_0^2 = 0$. As in the previous case, this debt arrangement would implement the optimum under commitment. However, rather than choosing the ex ante optimal smooth policy, the date 1 government lacking commitment chooses policy according to [equation \(20\)](#) with $c_1 > c_2$. This deviation, which is achieved by issuing lower levels of debt B_1^2 relative to commitment, serves to increase the value of what the government issues in [equation \(25\)](#), therefore freeing up resources to be utilized for additional spending at $t = 2$.

It is only when $B_0^1 = B_0^2 = \frac{\bar{B}}{2}$ that there are no gains from deviation. In this case, it follows from [equation \(20\)](#) that $B_1^2 = B_0^2$, and therefore any deviation's marginal effect on the market value of outstanding debt is perfectly outweighed by its effect on the market value of newly issued debt. For this reason, a flat debt maturity structure induces commitment.

ii. Trade-off between Commitment and Insurance. What this example illustrates is that, whatever the value of δ , the government always faces a trade-off between using the maturity structure to fix its problem of lack of commitment and using the maturity structure to insulate the economy from shocks. Under lack of commitment, the date 1 short-term interest rate captured by $\frac{c_2}{c_1}$ is rising in B_0^2 and declining in B_0^1 and this follows from [equation \(20\)](#). The intuition for this observation is related to our discussion in the previous section.¹¹

11. One natural implication of this observation is that the slope of the yield curve at date 0 is increasing in the maturity of debt issued at date 0. Formally,

A flat maturity structure minimizes the cost of lack of commitment. Equation (15) implies that the solution under full commitment requires $\frac{c_1}{c_2} = (\frac{\theta_2}{\theta_1})^{\frac{1}{2}}$. From equation (20), this can only be true under lack of commitment if $B_0^1 = B_0^2$ since in that case,

$$(26) \quad \frac{c_1}{c_2} = \left(\frac{\theta_2}{\theta_1}\right)^{\frac{1}{2}} \left[\frac{(n(1-\tau) + B_0^1)}{(n(1-\tau) + B_0^2)} \right]^{\frac{1}{2}}.$$

Therefore, the short-term interest rate at date 1 under lack of commitment can only coincide with that under full commitment if the chosen debt maturity is flat under lack of commitment.¹²

In contrast, a tilted maturity structure minimizes the cost of incomplete markets. To see this, let c_t^H and c_t^L correspond to the values of c at date t conditional on $\theta_1 = \theta^H$ and $\theta_1 = \theta^L$, respectively, under full commitment. From equation (15), under full commitment it is the case that $\frac{c_1^H}{c_1^L} = (\frac{\theta^L}{\theta^H})^{\frac{1}{2}}$ and $\frac{c_2^H}{c_2^L} = [(\frac{\alpha\theta^L + (1-\alpha)\theta^H}{\alpha\theta^H + (1-\alpha)\theta^L})]^{\frac{1}{2}}$. From equation (20), this cannot be true under lack of commitment if $B_0^1 = B_0^2$. The variance in consumption at date 1 under lack of commitment could only coincide with that under full commitment if the chosen debt maturity under lack of commitment is tilted.

Thus, the government at date 0 faces a trade-off. On the one hand, it can choose a flat maturity structure to match the short-term interest rate between dates 1 and 2 which it would prefer ex ante under full commitment. On the other hand, it can choose a tilted maturity structure to try to mimic the variance in consumption at dates 1 and 2 which it would prefer ex ante under full commitment. This is the key trade-off between insurance and commitment that the government considers at date 0. We formally analyzed this trade-off through a second-order approximation to welfare in a neighborhood of the deterministic case ($\delta = 0$). We found that, up to this approximation, for any value of the variance $\delta > 0$ the cost of lack of commitment is of higher order importance than the cost of lack of insurance. Thus, the debt maturity should

starting from a given policy, if we perturb B_0^1 and B_0^2 so as to keep the primary deficit fixed at date 0, one can show that $\frac{\theta_0^1}{\theta_0^2}$ is strictly increasing in B_0^2 . This result is in line with the empirical results of Guibaud, Nosbusch, and Vayanos (2013) and Greenwood and Vayanos (2014).

12. This observation more generally reflects the fact that, conditional on $B_0^1 = B_0^2$, the government under full commitment and the government under lack of commitment always choose the same policy at date 1.

be structured to fix the problem of lack of commitment, and should therefore be flat.¹³

2. *Full Persistence Limit.* In the previous section, we considered an economy in which the volatility of the shock is arbitrarily low, and we showed that optimal policy is a flat debt maturity which minimizes the cost of lack of commitment. In this section, we allow the volatility of the shock to take on any value, and we consider optimal policy as the persistence of the shock α goes to 1.

PROPOSITION 4 (Lack of Commitment, Full Persistence Limit). The unique solution under lack of commitment as $\alpha \rightarrow 1$ satisfies

$$(27) \quad B_0^1 = B_0^2 = n(1 - \tau) \frac{\theta_0^{\frac{1}{2}} - 1}{3} = \frac{1}{2} \bar{B} > 0.$$

This proposition states that as the persistence of the shock α goes to 1, the unique optimal solution under lack of commitment admits a flat maturity structure with $B_0^1 = B_0^2$. This means that for arbitrarily high values of persistence, the government will choose a nearly flat maturity structure, which is in stark contrast to the case of full commitment described in Proposition 2. In that case, debt positions are tilted and arbitrarily large in magnitude since B_0^1 diverges to minus infinity and B_0^2 diverges to plus infinity as α approaches 1. Given [equation \(15\)](#) which holds under full commitment and [equation \(20\)](#) which holds under lack of commitment, this proposition implies that under lack of commitment, the government no longer insulates the economy from shocks, since the level of public spending at dates 1 and 2 is no longer responsive to the realization of uncertainty at date 1. Therefore, as α goes to 1, the cost of lack of commitment remains positive.

The reasoning behind this proposition is as follows. As persistence in the shock between dates 1 and 2 goes to 1, the government at date 0 would prefer to smooth consumption as much as possible between dates 1 and 2. From [equation \(20\)](#), the only way to do this given the incentives of the government at date 1 is to choose a flat debt maturity with $B_0^1 = B_0^2$. Clearly, choosing $B_0^1 = B_0^2$ reduces hedging, since from [equation \(20\)](#) it implies that consumption, and therefore public spending, is unresponsive to the shock. If the

13. Details regarding this exercise are in [Online Appendix B](#).

government were to attempt some hedging as under commitment with $B_0^1 < 0$ and $B_0^2 > 0$, it would need to choose debt positions of arbitrarily large magnitude, since the variation in the short-term interest rate at date 1 across states diminishes as persistence goes to 1. From [Lemma 1](#), if $B_0^1 \leq -n(1 - \tau)$, this leads the date 1 government to choose c_1 and c_2 arbitrarily close to 0, but this is infeasible from the perspective of period 0 since there does not exist a level of c_0 high enough to satisfy [equation \(13\)](#) in that case.

Since any hedging has an infinite cost in the limit, the date 0 government chooses to forgo hedging altogether and instead chooses a flat debt maturity which induces the date 1 government to implement a smooth consumption path. While under commitment such a smooth consumption path could be implemented with a number of maturity structures, under lack of commitment it can only be implemented with a flat debt position. In doing so, the government minimizes the welfare cost due to lack of commitment.¹⁴

3. *Discussion.* The two limiting cases described provide examples in which the optimal debt maturity under lack of commitment is flat. In the case where the volatility of the shock goes to 0, the benefit of hedging goes to 0, and for this reason, the government chooses a flat maturity structure to minimize the cost of lack of commitment. A similar reasoning applies in the case where the persistence of the shock goes to 1, since the cost of any hedging becomes arbitrarily large. The optimal maturity under lack of commitment is thus in stark contrast to the case of full commitment. In that case, the government continues to hedge in the limit by choosing large and tilted debt positions.

Our examples more broadly show that any attempt to hedge by the government will be costly in terms of commitment. A tilted maturity creates a greater scope for deviation ex post, and this is costly from an ex ante perspective. Formally, a tilted maturity induces the date 1 government to deviate to a policy that reduces the right-hand side of [equation \(14\)](#); doing so causes the left-hand side of [equation \(13\)](#) to

14. One can easily show using numerical methods that the results in Propositions 3 and 4 do not depend on the particular preference structure. In general, in a three-period economy with exogenous tax rates or exogenous spending, a smooth policy between dates 1 and 2 can only be guaranteed with a flat maturity structure. Moreover, as persistence goes to 1, any hedging has an infinite cost in the limit. Our example allows us to show the optimality of a flat maturity theoretically since we are able to solve for the date 1 policy in closed form using [Lemma 1](#).

also become lower. Therefore, by relaxing the implementability condition at date 1, the date 1 government is tightening the implementability condition at date 0, which can directly reduce the ex ante welfare at date 0. In the following section we explore the quantitative implications of this insight once we move away from the limiting cases in our three-period example.

IV. QUANTITATIVE EXERCISE

We first consider a finite horizon economy. The advantage of a finite horizon over an infinite horizon is that it is computationally feasible to allow the government to choose any arbitrary debt maturity structure. We then move to consider an infinite horizon economy with limited debt instruments which allows us to more suitably capture the quantitative features of optimal policy and to characterize policy dynamics. We show in these exercises that the optimal debt maturity is nearly flat. We conclude by discussing the policy implications of our analysis.

We use the same parameterization as in [Chari, Christiano, and Kehoe \(1994\)](#). More specifically, we set the per period payoff of households to

$$(28) \quad \frac{c_t^{1-\sigma_c} - 1}{1 - \sigma_c} + \eta \frac{(1 - n_t)^{1-\sigma_l} - 1}{1 - \sigma_l} + \theta_t \frac{g_t^{1-\sigma_g} - 1}{1 - \sigma_g},$$

with $\sigma_c = \sigma_l = \sigma_g = 1$. $\eta = 3.33$ since this value implies that hours worked $n = 0.23$ under full commitment. Each period is a year, and hence $\beta = 0.9644$ such that the riskless rate is 4%. We consider an economy with two shocks θ^L and θ^H following a symmetric first-order Markov process. The levels and persistence of the shocks imply that, under full commitment, the average spending to output ratio is 0.18, the standard deviation of spending equals 7% of average spending, and the autocorrelation of spending is 0.89. All these values match the statistics and steady state values in [Chari, Christiano, and Kehoe \(1994\)](#). We set $\theta_0 = \theta^H$.

IV.A. Finite Horizon Analysis

We begin our quantitative analysis in a finite horizon economy with $t = 0, \dots, T$, where the set of available maturities is unrestricted. To compare our results with those of the three-period example of [Section III](#) in which a flat debt maturity (i.e., $B_0^1 = B_0^2$) is optimal, we allow the government at every date t to issue a

consol $B_t^L \geq 0$ which represents a promise by the government to pay a constant amount B_t^L at each date $t + k$ for $k = \{1, \dots, T - t\}$. In addition, the government can issue a set of zero-coupon bonds $\{B_t^{t+k}\}_{k=1}^{T-t-1}$. We can exclude T -period zero-coupon bonds B_t^T because these securities are redundant given the presence of the consol B_t^L .

It follows that the dynamic budget constraint of the government [equation \(4\)](#) for $t < T - 1$ can be rewritten as:

$$(29) \quad g_t + B_{t-1}^t + B_{t-1}^L = \tau_t n_t + \sum_{k=1}^{T-t-1} q_t^{t+k} (B_t^{t+k} - B_{t-1}^{t+k}) + q_t^L (B_t^L - B_{t-1}^L),$$

where q_t^L corresponds to the price of the consol. This budget constraint takes into account that, at date t , the government: (i) makes a flow payoff to households equal to $B_{t-1}^t + B_{t-1}^L$ according to their holdings of one-period bonds and consols, (ii) exchanges old zero-coupon bonds B_{t-1}^{t+k} for new zero-coupon bonds B_t^{t+k} at price q_t^{t+k} , and (iii) exchanges old consols B_{t-1}^L for new consols B_t^L at price q_t^L . In this environment, a flat debt maturity—which we found to be optimal in the theoretical example of [Section III](#)—corresponds to one in which $B_t^{t+k} = 0 \forall k$.¹⁵

We choose initial conditions such that, under full commitment, the value of debt equals 2.1% of the net present value of output, out of which 28% has a maturity of less than one year, and the rest is equally distributed across the remaining maturities.¹⁶ Our main results are unaffected by our choice of initial conditions, as we show below. All debt must be repaid in the terminal date. As in the theoretical example of [Section III](#), we let θ_T be deterministic from the point of view of the government at $T - 1$, and equal to its expected value conditional on the realization of θ_{T-1} . This modification implies that full hedging is possible under full commitment, so that any inefficiencies in our setting arise purely

15. At $t = T - 1$, the dynamic budget constraint is $g_t + B_{t-1}^t + B_{t-1}^L = \tau_t n_t + q_t^L (B_t^L - B_{t-1}^L)$, since there are no zero-coupon bonds that can be issued.

16. These values are consistent with our parameterization of the infinite-horizon economy which matches the U.S. data from 1988 to 2007 described in the next section. Given a discount factor $\beta = 0.9644$, a debt equal to 2.1% of the net present value of output corresponds to a debt to GDP ratio of roughly 60% in an infinite horizon economy.

from the lack of commitment. All of our results continue to hold if θ_T is instead stochastically determined.

Table I summarizes the main results. Panel A describes our results in a three-period economy, and Panel B describes our results in a four-period economy. In all cases, we display bond positions as a fraction of GDP, and with some abuse of notation in the text the bond positions B represent B normalized by GDP. Panel A describes the benchmark simulation under full commitment. In this case, $B_0^1 = -10,057$ and $B_0^L = 5,120$ (percent of GDP). These large magnitudes are consistent with the analysis of [Angeletos \(2002\)](#) and [Buera and Nicolini \(2004\)](#). In the case of lack of commitment, $B_0^1 = 0.07$ and $B_0^L = 2.32$, so that optimal debt maturity is nearly flat. This characterization is consistent with that of our theoretical three-period model in [Section III](#) in which the optimal debt maturity is exactly flat.

In Panel B, we find similar results if the horizon is extended to a four-period economy. In this circumstance, the optimal maturity structure at date 0 under commitment is indeterminate since there are more maturities than shocks. If confined to a one-period bond and a consol, the government chooses a one-period bond equal to $-7,317\%$ of GDP and a consol equal to $2,529\%$ of GDP. In contrast, under lack of commitment, $B_0^1 = -0.04$, $B_0^2 = 0.00$, and $B_0^L = 2.41$, so that the optimal maturity structure is nearly flat. Moreover, the optimal government debt maturity is even more flat at date 1, since $B_1^2 = 0.00$ and $B_1^L = 2.44$.

In the second, third, and fourth columns of **Table I**, we consider the robustness of our results as we increase the volatility and decrease the persistence of shocks, since this moves us further away from the limiting cases considered in [Section III](#). We find that the optimal debt maturity under lack of commitment remains nearly flat if the standard deviation of shocks is 2 and 4 times larger than in the benchmark simulation. We find the same result if shocks have 0 persistence and are i.i.d.

In the last two columns of **Table I**, we explore whether our results depend on the initial tilt of the maturity structure. We consider an extreme case where the majority of the debt consists of one-period bonds, so that these constitute 72% instead of 28% of liabilities, and the total amount of debt is unchanged. We find that under lack of commitment, the optimal debt maturity at date 0 remains nearly flat both in the three-period and four-period models, though it is less flat than in the benchmark case since

TABLE I
DEBT POSITIONS IN FINITE-HORIZON ECONOMIES

	Benchmark	Std. dev. ($\times 2$)	Std. dev. ($\times 4$)	i.i.d.	Initial debt tilted short	Initial debt flat
Panel A: Three-period model						
Commitment						
One-year bond	-10,057.38	-9,879.06	-9,500.32	-597.68	-9,941.93	-10,039.59
Consol	5,120.42	5,030.48	4,839.23	304.32	5,063.44	5,111.64
Lack of commitment						
One-year bond	0.07	0.06	0.02	-0.10	-0.41	-0.02
Consol	2.32	2.37	2.47	2.68	2.78	2.40
Panel B: Four-period model						
Commitment						
One-year bond	-7,317.73	-7,189.12	-6,914.89	-447.65	-7,230.17	-7,320.67
Consol	2,529.06	2,485.42	2,392.17	154.97	2,500.39	2,530.03
Lack of commitment (all maturities)						
Date 0 policies						
One-year bond	-0.04	-0.08	-0.10	-0.16	-0.45	-0.02
Two-year bond	0.00	0.00	0.00	0.00	-0.01	0.00
Consol	2.41	2.47	2.55	2.65	2.73	2.40
Date 1 policies						
One-year bond	0.00	-0.03	-0.10	-0.06	0.05	0.00
Consol	2.44	2.54	2.73	2.63	2.59	2.43
Lack of commitment (one-year and consol)						
Date 0 policies						
One-year bond	-0.04	-0.08	-0.09	-0.16	-0.45	-0.02
Consol	2.41	2.47	2.55	2.64	2.72	2.40
Date 1 policies						
One-year bond	-0.02	-0.04	-0.10	-0.07	0.06	-0.02
Consol	2.45	2.55	2.73	2.63	2.59	2.45

Notes. The table reports the debt positions (% of GDP) in three-period (Panel A) and four-period (Panel B) economies with and without commitment.

the one-period bond B_0^1 is larger in absolute value. This is in part because the initial debt position is itself highly tilted and there is a large flattening out which occurs during the initial period. In the four-period model, the optimal debt maturity becomes even more flat with time (date 1 policies involve a nearly flat maturity with $B_1^2 = 0.05$ and $B_1^L = 2.59$). In the last column, we consider the consequences of having initial debt be exactly flat, and we find that the optimal maturity structure under lack of commitment is nearly flat in all cases.

In the bottom of Panel B, we consider the consequences of restricting the set of maturities to a one-year bond and a consol. We find that our main results continue to hold in this case and that the optimal debt maturity is nearly flat even under these restricted set of debt instruments.

Our quantitative result from the finite horizon environment are in line with our theoretical results. The optimality of a flat debt maturity emerges because of the combination of two forces. First, substantial hedging requires massive and tilted debt positions, as has been shown in [Angeletos \(2002\)](#) and [Buera and Nicolini \(2004\)](#). Due to their size, financing these positions can be very expensive in terms of average tax distortions because of the lack of commitment by the government. Second, under empirically plausible levels of volatility of public spending, the cost of lack of insurance under a flat maturity structure is small. Therefore, the optimal policy pushes in the direction of reducing average tax and spending distortions versus reducing the volatility of these distortions, and the result is a nearly flat maturity structure.

IV.B. Infinite Horizon Analysis

The previous section suggested that quantitatively, a government lacking commitment should principally issue consols in a finite horizon economy. We now consider the robustness of this result in an infinite horizon. In an infinite horizon economy, the set of tradeable bonds is infinite, and to facilitate computation, we reduce the set of tradeable bonds in a manner analogous to the work of [Woodford \(2001\)](#) and [Arellano et al. \(2013\)](#). Namely, we consider an economy with two types of bonds: a decaying perpetuity and a nondecaying consol. We allow for a nondecaying consol since our analysis of the previous sections suggests that the optimal debt maturity is nearly flat. We then consider whether or not

the government makes use of the nondecaying perpetuity in its financing strategy.

Let $B_{t-1}^S \geq 0$ denote the value of the coupon associated with the decaying perpetuity issued by the government at $t - 1$. Moreover, let $B_{t-1}^L \geq 0$ denote the value of the coupon associated with the nondecaying consol issued by the government at $t - 1$. It follows that the dynamic budget constraint of the government becomes:

$$(30) \quad g_t + B_{t-1}^S + B_{t-1}^L = \tau_t n_t + q_t^S (B_t^S - \gamma B_{t-1}^S) + q_t^L (B_t^L - B_{t-1}^L).$$

The only difference relative to [equation \(29\)](#) relates to the decaying perpetuity. Besides the consol, the government exchanges nondecayed perpetuities γB_{t-1}^S for new perpetuities B_t^S at price q_t^S , where $\gamma \in [0, 1)$.

We focus on an MPCE in which the value and policy functions are differentiable. We cannot prove that this MPCE is unique, but we have verified that our computational algorithm converges to the same policy when starting from a large grid of many different initial guesses.¹⁷ In our benchmark simulation we let $\gamma = 0$, so that B^S represents a one-year bond. We choose initial debt positions to match the U.S. statistics for the period 1988–2007, with an average market value of total debt of 60% of GDP, out of which 28% has maturity of less than one year.¹⁸

1. Benchmark Simulation. [Figure I](#) displays the path of the one-year bond and the consol relative to GDP. The left panel shows the path of these quantities under full commitment. From $t \geq 1$ onward, the value of B^S is $-2,789\%$ of GDP and the value of B^L is 102% of GDP. The price of the consol is significantly higher than that of the one-year bond, which explains why the position is significantly lower; in fact, the market value of the consol is $2,858\%$ of GDP. These large and highly tilted quantities are consistent with previous results under commitment. These debt positions are not actively managed and are constant over time.

17. Further details regarding our computational method are available in the [Online Appendix](#).

18. This calculation ignores off-balance sheet liabilities, such as unfunded mandatory spending obligations which are significantly more long term. Taking this additional debt into account and changing initial conditions would not change our main conclusion that the optimal debt maturity under lack of commitment is nearly flat.

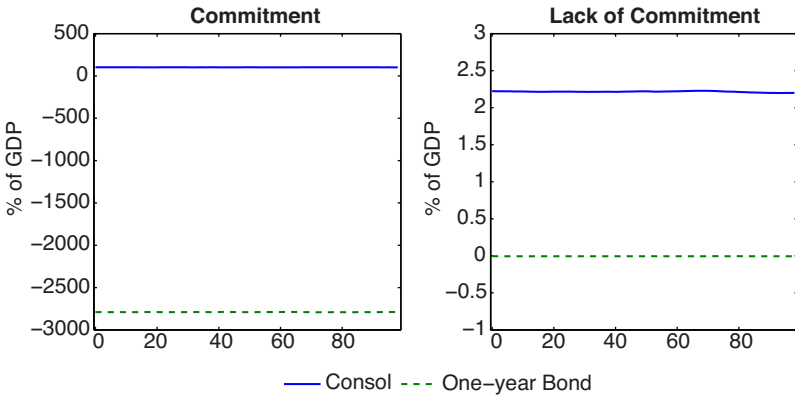


FIGURE I

Debt Positions with and without Commitment

The figure shows the optimal debt positions over time with commitment (left panel) and without commitment (right panel). For the case with lack of commitment we report averages across 1,000 simulations.

The right panel considers the economy under lack of commitment, and in this scenario debt is actively managed from $t \geq 1$ onward. Since it is actively managed, we plot the average value of debt for each time period taken from 1,000 simulations. Between $t = 1$ and $t = 100$, the average value of B^S is -0.01% of GDP and the average value of B^L is 2.22% of GDP.¹⁹ Therefore, the maturity structure of debt is approximately flat. Also, the total amount of debt maturing in one period (i.e., the value one-period bond plus the coupon payment of the consol) is positive and equals 2.21% of GDP. At the same horizon, a government with commitment would instead hold assets with a value of about 26 times the GDP.

Figure II considers an equilibrium sequence of shocks and shows that B^S is approximately 0 and constant in response to shocks, whereas B^L is actively managed. More specifically, the level of the consol rises (declines) during high (low) spending shocks. This pattern occurs because the government runs larger deficits (surpluses) when spending is high (low). Therefore, in contrast to the case of full commitment, the government actively manages its debt which primarily consists of consols.

19. We calculate the average starting from $t = 1$ rather than $t = 0$ since the simulation suggests that debt quickly jumps towards its long-run average between $t = 0$ and $t = 1$.

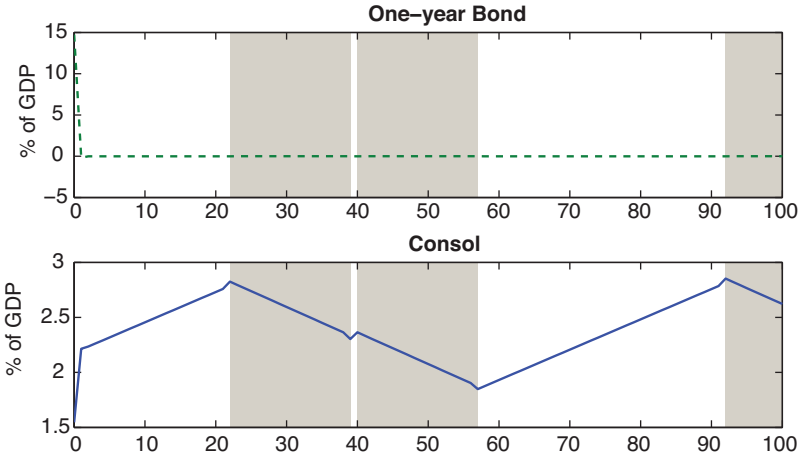


FIGURE II

Active Debt Management

The figure shows the evolution of debt positions for a particular sequence of shocks. The shaded areas indicate periods in which the fiscal shock is low.

Figure III presents the path of policy under this sequence of shocks. Whereas taxes are nearly constant under full commitment—which is consistent with the complete market results of Chari, Christiano, and Kehoe (1994)—they are volatile and respond persistently to shocks under lack of commitment. More specifically, during periods of high (low) expenditure, taxes jump up (down) and continue to increase (decrease) the longer the fiscal shock persists. Periods of high (low) expenditure are periods with lower (higher) primary surpluses in the case of full commitment and lack of commitment, but in contrast to the case of full commitment, under lack of commitment the surplus responds persistently to shocks. This persistence is reflected in the total market value of debt, which contrasts with the transitory response of the market value of debt in the case of full commitment.²⁰

20. Shin (2007) considers a model under full commitment and shows that if there are N possible states of the shock but at any moment only $N_1 < N$ can be reached, then N_1 bonds of different maturities can provide full insurance. Such a model would require active management of debt positions. Our model under lack of commitment also captures the active management of debt. This result, however, is not achieved by limiting the maturities available; instead it follows from the trade-off between hedging and the cost of borrowing.

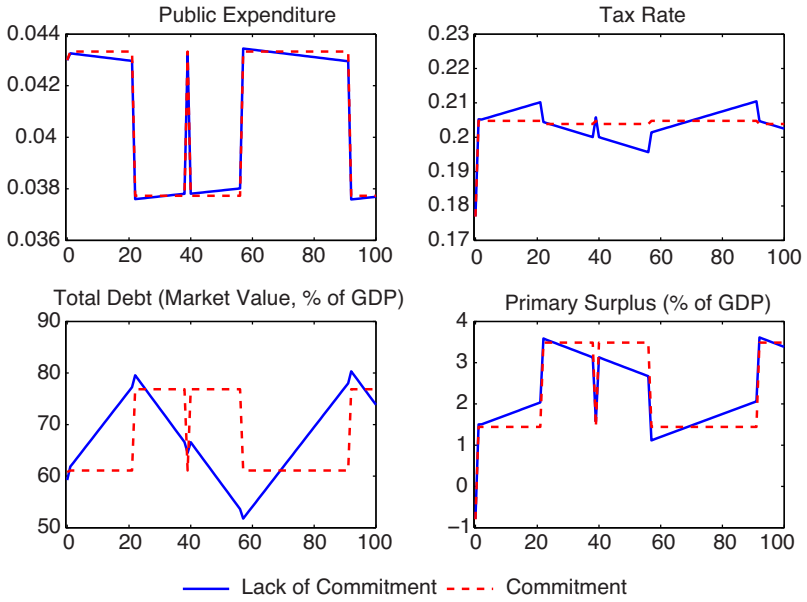


FIGURE III
Fiscal Policy without Commitment

The figure shows the evolution of public expenditure, tax rates, total debt, and primary surpluses for a particular sequence of shocks.

We can calculate the welfare cost of lack of commitment in this setting. In particular, we compare welfare under full commitment to that under lack of commitment and report the welfare difference in consumption equivalent terms. We find that this welfare cost is 0.0038%. As a comparison, the welfare cost of imposing a balanced budget on a government with full commitment is 0.04%, more than 10 times larger.²¹ These numbers mean that the welfare cost of lack of commitment is very low—as long as the maturity is chosen optimally which implies a nearly flat maturity.

In addition, we can compute the welfare cost of imposing a completely flat maturity. To do this, we compare welfare under lack of commitment when the government can freely choose B^S to that when B^S is constrained to 0 in all periods (so that debt issuance is exactly flat). We find that the difference in welfare

21. This corresponds to the cost of forcing a government to set $B_t^S = B_{-1}^S$ and $B_t^L = B_{-1}^L \forall t$.

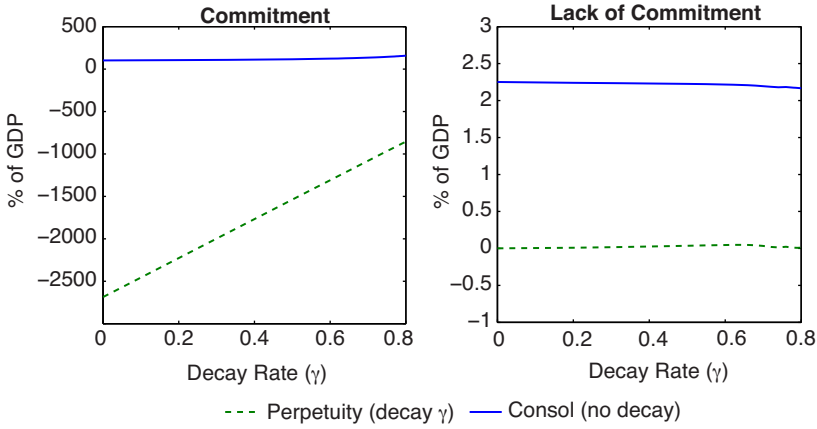


FIGURE IV

Debt Positions with Different Debt Maturities

The figure shows the optimal debt positions with commitment (left column) and without commitment (right column) under alternative values for the decay rate of the perpetuity. For the case with lack of commitment we report averages across 1,000 simulations of 200 periods.

is less than 0.00001%. This negligible welfare cost implies that optimal policy under lack of commitment can be approximated by constraining debt issuance to consols.

2. *Robustness: Alternative Debt Maturities.* One limitation of our infinite horizon analysis is that we have restricted the horizon of the short-term debt instrument. We now show that the optimal maturity structure is flat even if alternative horizons are considered. Figure IV displays the average values of B^S and B^L under commitment and under lack of commitment for different values of γ (the decay rate of the perpetuity B^S).²² Under full commitment, the optimal value of B^L is positive and nearly unchanged by different values of γ , whereas the optimal value of B^S is negative, large, and decreasing in magnitude as γ rises. The reason is that the higher is γ , the lower is the decay rate of B^S and the higher its price, implying that a smaller position is required for hedging. In contrast, under lack of commitment, the average value of the perpetuity B^S is 0 regardless of the value of γ , and

22. For this exercise, the initial conditions are calculated for each γ so as to keep fixed the market value of initial debt.

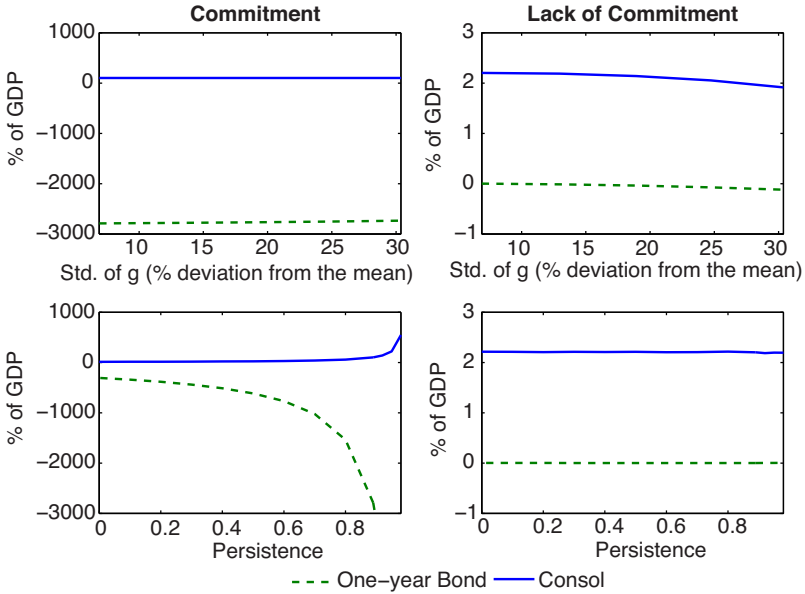


FIGURE V

Debt Positions under Alternative Variances and Persistences of Fiscal Shocks

The figure shows the optimal debt positions with commitment (left) and without commitment (right) under alternative values for the standard deviation (top row), and persistence (bottom row) of public expenditure. For the case with lack of commitment we report averages across 1,000 simulations of 200 periods.

the value of the consol B^L is large and unaffected by γ . As such, the optimal debt maturity remains flat, even when considering alternative debt maturities.

3. Robustness: Variance and Persistence of Fiscal Shocks.

The quantitative results are consistent with the theoretical results from the three-period model which considered the limiting cases as volatility declined to 0 and persistence increased to 1. A natural question concerns the degree to which our results depend on the parameterization of public spending shocks. To explore this question, we return to the benchmark environment with $\gamma = 0$ and choose different values of volatility and persistence for the public spending shock.

Figure V displays the average values of B^S and B^L under different assumptions for the shocks' process. In the case of full

TABLE II
DEBT POSITIONS WITH ALTERNATE SHOCKS

	Commitment	Lack of commitment			
	Benchmark	Benchmark	20 shocks	w/Fiscal shock	w/Prod. shock
Panel A: Fiscal shocks					
One-year bond	-2,789.46	-0.005	-0.005	—	—
Consol	101.76	2.22	2.23	—	—
Panel B: Productivity shocks					
One-year bond	-13.49	-0.007	-0.007	-0.028	—
Consol	2.71	2.21	2.24	2.15	—
Panel C: Discount factor shocks					
One-year bond	0.00	0.000	0.000	0.062	-0.014
Consol	2.26	2.26	2.36	2.24	2.15

Notes. The table reports the average debt position (% of GDP) over 1,000 simulations of 200 periods. The shock processes follow discrete Markov-chains with 2 states (first and second columns), 20 states (third column), and 4 states (last two columns).

commitment, debt positions are large and tilted independently of the volatility and persistence of the shocks. Moreover, consistent with our three-period example, debt positions become arbitrarily large as the autocorrelation of the shock goes to 1. In contrast, the optimal maturity structure under lack of commitment is nearly flat for all volatilities and persistence levels of public spending. We additionally find that the debt positions decrease in size as volatility increases, and this occurs because the volatility of the marginal utility of consumption increases, which facilitates hedging through the consol with a smaller position. As such, our result is robust to changes in the stochastic characteristics of fiscal shocks.

4. Robustness: Additional Shocks. We have thus far considered an economy in which the shocks to the economy are fiscal. In [Table II](#), we show that our main result—that the optimal debt maturity is flat—is robust to the introduction of productivity and discount factor shocks. We consider each shock in isolation in the first two columns. We then increase the number of realization of shocks in the third column (so that the number of shock realizations exceeds the number of debt instruments), and in the last two columns we consider combinations of different shocks.

Panel A reports the debt position in our benchmark model with fiscal shocks θ_t and replicates our results described in the previous sections. Panel B introduces a productivity shock in an environment in which θ_t is constant and equal to its average

value. More specifically, we replace n_t in the resource constraint equation (1) with $A_t n_t$, where A_t captures the productivity of labor and therefore equals the wage. Let $A_t = \{A^L, A^H\}$ follow a symmetric first-order Markov process with unconditional mean equal to 1. We choose A^L, A^H , and the persistence of the process so that, as in Chari, Christiano, and Kehoe (1994), the standard deviation of A_t equals 0.04 and the autocorrelation equals 0.81. The first column of Panel B shows that, consistently with the results in Buera and Nicolini (2004), under commitment the average debt positions are tilted, though the magnitudes of debt are smaller than those under fiscal shocks. In the second column, it is clear that optimal debt positions under lack of commitment are nearly flat.

Panel C of Table II introduces a discount factor shock in an economy in which θ_t and A_t are constant and equal to the average value. We replace the utility function in equation (28) with

$$\zeta_t \left[\frac{c_t^{1-\sigma_c} - 1}{1 - \sigma_c} + \eta \frac{(1 - n_t)^{1-\sigma_l} - 1}{1 - \sigma_l} + \theta_t \frac{g_t^{1-\sigma_g} - 1}{1 - \sigma_g} \right]$$

for some $\zeta_t = \{\zeta^L, \zeta^H\}$ which follows a first-order Markov process. ζ_t represents a discount factor shock, which can impact the variance of short-term interest rates without affecting the time series properties of other variables in the model. As discussed in Angeletos (2002) and Buera and Nicolini (2004), the large size of the debt positions required for hedging under commitment is driven in part by the fact that fluctuations in short-term interest rates are small in the benchmark economic environment. Introducing the discount factor shock allows us to increase the volatility of interest rates and determine whether the optimality of a flat debt maturity in our setting depends on the presence of low interest rate volatility.

To that end, we choose the stochastic properties of ζ_t so that under commitment, the mean of the one-year interest rate is 4%, its standard deviation is 0.73% of the mean, and its persistence is 0.78, which matches the properties of the real one-year interest rate in the United States from 1988 to 2007.²³ The first column of Panel C shows that in this situation, the maturity structure is exactly flat under commitment, and this is because optimal

23. The real interest rate is calculated as the difference between the nominal one-year rate and realized inflation (GDP deflator).

policy is smooth from date 1 onward. As such, a flat debt maturity allows the market value of the consol to fluctuate one-to-one with the present value of future surpluses. Analogous logic implies that optimal debt maturity is flat under lack of commitment, where a flat maturity also mitigates the commitment problem.

In the third column of Table II, we increase the number of shocks so that these exceed the number of debt instruments. In each panel, we extend the environment by allowing the shocks to take on 20 realizations that approximate a Gaussian AR(1) process. This exercise is performed while preserving the mean, standard deviation, and persistence of the shocks. We find that our results are unchanged and that the optimal debt maturity remains flat under lack of commitment.

The last two columns of Table II consider our results in environments with two types of shocks, where we take all combinations of the shocks previously analyzed. This allows us to analyze situations where the government may have greater incentives for hedging, even under lack of commitment. For instance, the combination of discount factor shocks with either fiscal or productivity shocks means that the fluctuations in the government's financing needs come hand in hand with larger fluctuations in short-term interest rates. These larger interest rate fluctuations imply that hedging does not require very large debt positions and is therefore less expensive. In fact, in all the situations considered, we find that the maturity is slightly more tilted, but it remains nearly flat.

In Panel B, we consider an environment with fiscal and productivity shocks, where we set $Corr(\theta_t, A_t) = -0.33$ so that our simulation matches the correlation between total factor productivity (TFP) and primary deficits in the United States from 1988 to 2007.²⁴ We find that the optimal debt maturity continues to be approximately flat, though it is a little more tilted in comparison to the case in which the impact of each shock is assessed separately.

In Panel C, we consider an environment with fiscal shocks and discount factor shocks. We set $Corr(\theta_t, \zeta_t) = -0.51$ so that our simulation matches the correlation between real interest rates and primary deficits in the United States from 1988 to 2007. In this case, the maturity structure is slightly more tilted than in the case which excludes the discount factor shock (the one-year bond is 0.068% of GDP), but the optimal debt maturity remains

24. The series of the TFP shock and the primary deficit are taken from the World Penn Table and the U.S. Office of Management and Budget, respectively.

TABLE III
DEBT POSITIONS WITH COMMITMENT TO SPENDING (LUCAS AND STOKEY 1983 MODEL)

	Commitment		Lack of commitment	
	Benchmark		Benchmark	20 Shocks
Panel A: Fiscal shocks				
One-year bond	-2,789.32		-0.006	-0.008
Consol	101.76		2.22	2.21
Panel B: Productivity shocks				
One-year bond	-90.06		-0.062	-0.060
Consol	5.54		2.24	2.31
Panel C: Discount factor shocks				
One-year bond	0		0	0
Consol	2.27		2.27	2.37

Notes. The table reports the average debt position (% of GDP) over 1,000 simulations of 200 periods, for a model with exogenous public expenditure. The shock processes follow discrete Markov-chain with 2 states (first and second columns) or 20 states (third column). In the model with fiscal shocks (Panel A), public expenditure takes the same values as in the model with endogenous spending of Table II under commitment. With different shocks (Panels B and C), public expenditure is fixed at the average of the values taken in the corresponding endogenous spending models under commitment.

essentially flat.²⁵ In the final column of Panel C, we consider an environment with productivity and discount factor shocks, and we set $\text{Corr}(A_t, \zeta_t) = -0.43$, so that our simulation matches the correlation between total factor productivity and real interest rates. We find that the maturity is slightly more tilted than in the case which excludes the discount factor shock, but it remains approximately flat.

5. Robustness: Commitment to Spending. We have so far considered an economy in which the government lacks commitment to taxes, spending, and debt issuance. Instead, in the economy of Lucas and Stokey (1983), public spending is exogenous and can therefore not be chosen by the government. Table III shows that our results hold, even if the government is able to commit to the level of spending, as is the case in their model. Under commitment the optimal maturity structure is tilted, and the optimal tilt

25. The optimal debt maturity is tilted to the short end in this case since there is a negative correlation between interest rates and the government's financing needs. We have also explored the extent to which one can put an upper bound on the degree of tilt in the government's debt maturity. For example, in the case when interest rates and fiscal shocks are perfectly positively correlated, the value of the one-year bond is -0.23 and the consol is 2.14% of GDP, so that even in this extreme case, the bulk of public debt is in the consol.

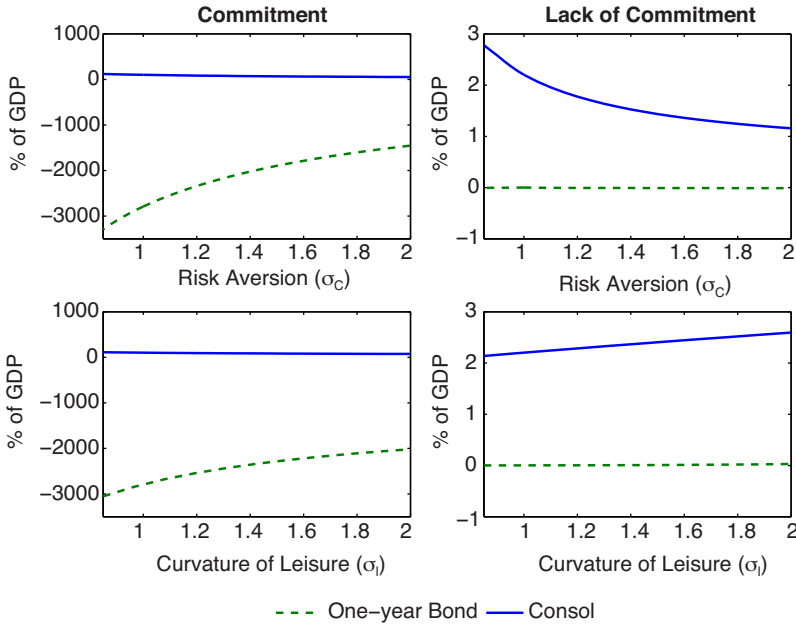


FIGURE VI

Debt Positions under Alternative Preferences

The figure shows the optimal debt positions with commitment (left) and without commitment (right) under alternative values for the risk aversion (top row) and curvature of leisure (bottom row). For the case with lack of commitment we report averages across 1,000 simulations of 200 periods.

is extremely sensitive to the particular type of shocks affecting the economy. Instead, with lack of commitment the maturity remains nearly flat under all types of shocks considered.

6. *Robustness: Alternate Preferences.* We now consider the robustness of our results to other preference specifications. The top panel of Figure VI considers the consequences of altering the coefficient of relative risk aversion σ_c . In the case of full commitment, lower values of σ_c generate larger and more tilted debt positions. A lower value of σ_c reduces the volatility in the marginal utility of consumption and therefore makes it more difficult to achieve significant hedging with smaller positions. In the case of lack of commitment, a similar force emerges since both the tilt and size of debt positions rise. Note however that, quantitatively, the

maturity structure remains nearly flat as σ_c declines. The reason is that even though more tilted positions are useful for hedging, more tilted positions also exacerbate the problem of lack of commitment, so that the best way to deal with this problem is to still choose a nearly flat maturity structure.

The exercise in the bottom panel considers the equilibrium under different values of σ_l , which relates to the curvature of the utility function with respect to leisure. We find that for all values of σ_l below 2, the optimal debt maturity under lack of commitment is essentially flat. The effect of higher value of σ_l is dual. On one hand, higher values of σ_l imply that it is socially costly to have volatility in labor supply, and consequently, oscillations in consumption play a greater role in absorbing public spending shocks. This force increases the volatility in the marginal utility of consumption and implies that smaller debt positions are required to generate hedging. On the other hand, higher values of σ_l also imply that it is more beneficial to engage in hedging as a way of smoothing out labor market distortions. This force implies larger debt positions since the value of hedging increases. In the case of full commitment, we find that, quantitatively, the first force dominates since debt positions become less tilted as σ_l increases. In the case of lack of commitment, we find that the second force dominates since the consol position become larger as σ_l increases, which facilitates hedging. It continues to be the case throughout, however, that the debt maturity is nearly flat under lack of commitment.

IV.C. Implications for Fiscal Policy and Debt Management

As in the work of [Barro \(1979\)](#), [Aiyagari et al. \(2002\)](#), and [Bhandari et al. \(2015\)](#), our analysis finds that optimal taxes are volatile and respond persistently to economic shocks. In contrast to this related work, this feature of optimal policy in our model is due to the lack of commitment by the government as opposed to the incompleteness of financial markets resulting from limited debt instruments. Moreover, this feature of optimal fiscal policy—which does not hold under commitment and sufficiently rich bond instruments as in [Angeletos \(2002\)](#) and [Buera and Nicolini \(2004\)](#)—is consistent with the dynamics of U.S. tax rates, as discussed in [Barro \(1979\)](#).

While the purpose of our analysis is normative, a natural question concerns the degree to which government debt maturity

in practice is consistent with the optimal government debt maturity in theory. In this regard, we can also show that government debt maturity in practice is much closer to the optimal government debt maturity under lack of commitment versus under full commitment.

To make this comparison, we can extend our framework as well as that of [Angeletos \(2002\)](#) and [Buera and Nicolini \(2004\)](#) to allow for a constant growth rate in labor productivity, a constant inflation rate, and nominal—as opposed to real—government bonds. Such an extension incorporates important features of the U.S. economy and it implies that nominal GDP grows at a constant long-run rate. The extension does not change the substance of our results or those of [Angeletos \(2002\)](#) and [Buera and Nicolini \(2004\)](#), and it facilitates a comparison to U.S. government debt maturity.

Under this extension, the government under full commitment holds a negative short-term nominal debt position and a positive long-term nominal debt position. Both positions are large relative to GDP, and both positions grow deterministically (in opposite directions) at the long-run rate of nominal GDP, without responding to shocks. In contrast, the government under lack of commitment actively manages a positive nominal consol position in response to shocks, and future consol payments are structured to grow at the long-run rate of nominal GDP. This characterization is the analog of a flat debt position in our theoretical framework once long-run nominal GDP growth is taken into account.²⁶

[Figure VII](#) displays the maturity structure of marketable U.S. federal nominal treasury bonds in 2007.²⁷ We display the sum of all nominal payments—coupons and principal—due at various horizons from the perspective of 2007 (i.e., “1” represents payments due in 2008, “2” represents payments due in 2009, etc.).²⁸ With some abuse of notation, we can refer to the sum of all of these nominal payments at horizon k as B_t^{t+k} . In a given year t in which we observe the government’s bond portfolio, we can construct a measure of the growth of these payments by calculating

26. For this extension, we let preferences satisfy [equation \(28\)](#), we set $\sigma_c = \sigma_l = \sigma_g = 1$, which is consistent with a balanced growth path.

27. A similar pattern emerges in more recent years. We chose 2007 for our display since it predates the maturity management performed by the Federal Reserve during periods of quantitative easing.

28. We exclude TIPS since we focus on nominal payments. We obtain similar patterns if we include TIPS and adjust for expected inflation.

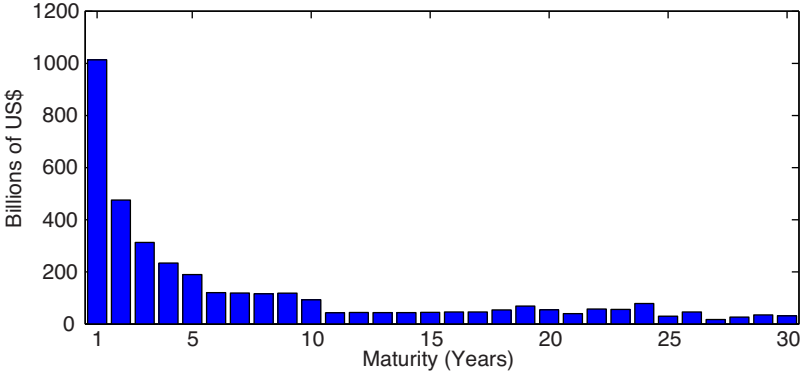


FIGURE VII

Maturity Structure of U.S. Government Liabilities in 2007

The figure displays the maturity structure of U.S. federal marketable debt (principal and coupons payments), excluding TIPS, calculated using CRSP data.

the average difference between $\log B_t^{t+k+1}$ and $\log B_t^{t+k}$ across all $k > 1$. This statistic relates to the decay rate in our analysis of the perpetuity in our simulations. In 2007, this average difference implies that payments decline at an average rate of 11% a year.

Clearly, the U.S. debt maturity is very different from the optimal maturity structure under full commitment, since in the data all debt positions are positive, and they are all quantitatively small relative to GDP.

Figure VIII displays the difference between $\log B_t^{t+k}$ and $\log B_{t-1}^{t+k-1}$ across different horizons k for t spanning 1985 to 2013. This statistic measures the change in the k -maturity bond issuance over time. The figure shows significant comovement across the maturity spectrum as overall debt rises and falls. This pattern is in contrast to optimal policy under commitment in which debt positions grow at a constant rate, with government assets and government debt becoming larger and offsetting each other.

In sum, Figures VII and VIII show that debt payments are positive across the maturity spectrum, payments are small relative to GDP, and importantly, payments change almost proportionately across the maturity spectrum in response to shocks. These features are all in line with the characterization of optimal debt management under lack of commitment.

Nonetheless, there are important differences. In particular, while the theoretically optimal maturity structure under lack of commitment involves the issuance of perpetuities, the maximum

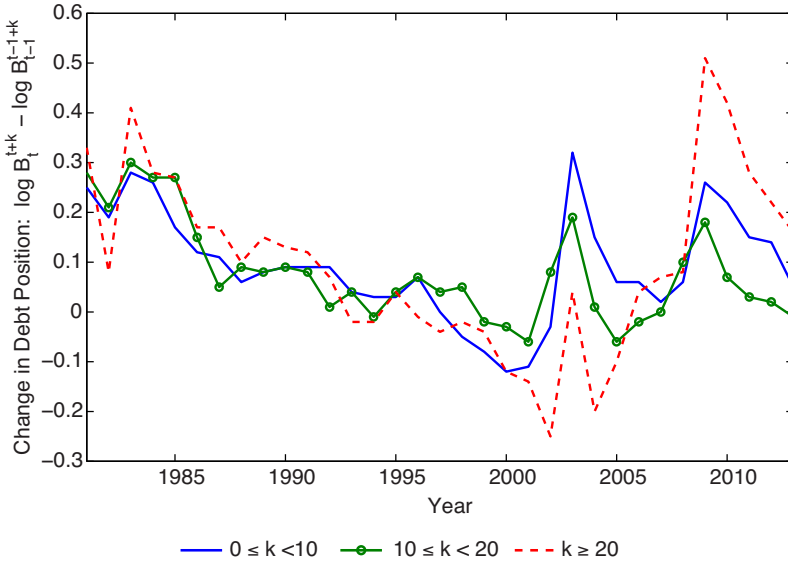


FIGURE VIII

Comovements across Maturities (1980–2013)

The figure displays the (log) change in the sum of the liabilities with maturity between 0 and 10 years (solid line), between 10 and 20 years (line with circles), and above 20 years (dashed line).

horizon of the U.S. government's official marketable liabilities is 30 years. Moreover, whereas optimal policy under lack of commitment requires future bond payments to grow at the rate of nominal GDP—which has averaged around 5% since 1985—debt payments in practice decline at a rate of 11%. Moreover, this pattern is general: across all years between 1985 and 2013, debt payments decline in the horizon, and the rate of decline is relatively stable, a pattern consistent with the comovement across maturities displayed in [Figure VIII](#).²⁹

Do these observations imply that U.S. government policy could be improved by increasing the maturity of U.S government debt? Based on our model—which excludes government transfers and in which the government cannot commit to taxes and

29. Payments continue to decline, but the pace of decline is reduced if we exclude bonds due in one-year—which often serve a liquidity purpose which is unmodeled in our setting—and if we also exclude bonds held by the Federal Reserve. Details available on request.

spending but only to repaying public debt—the answer to this question is yes.

However, in practice, the U.S. government does partially commit to mandatory government transfer programs such as Social Security and Medicare.³⁰ While our model is not equipped to address the issue of partial commitment, full commitment to such transfers in our model can be introduced in the form of an exogenous, nontradeable, and potentially stochastic debt portfolio at date 0 representing this stream of future mandatory obligations.

An implication of such an extension is that if these mandatory obligations grow faster than nominal GDP—which has been the case historically—then the government should choose the optimal maturity structure of marketable debt to offset this growth. Such an offsetting, which frontloads marketable debt payments, ensures that the path of payments from the government to the private sector—both marketable debt payments and mandatory old-age payments—grow at the same rate as nominal GDP. Taken from this light, optimal marketable debt payments from the government should decline in the horizon, and the answer as to whether lengthening U.S. government debt maturity would be an improvement is ambiguous. In sum, given the complexity in modeling the issue of partial commitment and in modeling the time path of expected mandatory spending obligations, we leave a full analysis of this question to future research.³¹

V. CONCLUSION

The current literature on optimal government debt maturity concludes that the government should fully insulate itself from economic shocks. This full insulation is accomplished by choosing a maturity heavily tilted toward the long end, with a constant short-term asset position and long-term debt position, both positions extremely large relative to GDP. In this article, we show

30. In principle, one can also consider other mandatory transfers from the U.S. government, such as unemployment compensation and child tax credits.

31. In a preliminary analysis of this question, we modeled mandatory old-age payments as deterministic and analyzed historical Social Security and Medicare payments as well as projections from the U.S. government. We found that future nominal marketable debt payments plus mandatory old-age payments from the perspective of a given year grow at a rate of 3% to 4% with the horizon, which is not too far from average nominal GDP growth of 5%. These findings suggest that U.S. government debt maturity is close to optimal. Details available on request.

that these conclusions strongly rely on the assumption of full commitment by the government. Once lack of commitment is taken into account, then full insulation from economic shocks becomes impossible; the government faces a trade-off between the benefit of hedging and the cost of funding. We show through a series of exercises that the optimal debt maturity structure under lack of commitment is nearly flat, with the government actively managing its debt in response to economic shocks. Thus, optimal policy can be approximately achieved by confining government debt instruments to consols.

Our analysis thus provides an argument for the use of consols in debt management based on the limited commitment of the government to the future path of fiscal policy. The use of consols has been pursued historically, most notably by the British government in the Industrial Revolution, when consols were the largest component of the British government's debt (see [Mokyr 2011](#)). Moreover, the reintroduction of consols has received some support in the press and in policymaking circles (e.g., [Leitner and Shapiro 2013](#); [Yglesias 2013](#); [Cochrane 2015](#)).

Our analysis leaves several interesting avenues for future research. First, our framework follows [Angeletos \(2002\)](#) and [Buera and Nicolini \(2004\)](#) and therefore ignores nominal bonds and the risk of surprise inflation. Taking this issue into account is important since it incorporates a monetary authority's ability to change the value of outstanding debt in response to shocks, and it also brings forward the issues of dual commitment to monetary and fiscal policy. We believe that our work is a first step in studying this more complicated problem. Second, our framework does not incorporate investment and financing frictions, which can be affected by the supply of public debt. It has been suggested that short-term government debt is useful in alleviating financial frictions (see, e.g., [Greenwood, Hanson, and Stein 2015](#)), and an open question regards how important this friction is quantitatively relative to the lack of commitment. Finally, our analysis ignores heterogeneity and the redistributive motive for fiscal policy (see, e.g., [Werning 2007](#); [Bhandari et al. 2013](#)). An interesting question for future research involves how incentives for redistribution can affect the maturity structure of public debt.

APPENDIX: EQUILIBRIUM DEFINITION AND RECURSIVE
REPRESENTATION

Definition of MPCE. Let $\mathbf{B}_t \equiv \{B_t^{t+k}\}_{k=1}^\infty$ and $\mathbf{q}_t \equiv \{q_t^{t+k}\}_{k=1}^\infty$. In every period t , the government enters the period and chooses a policy $\{\tau_t, g_t, \mathbf{B}_t\}$ given $\{s_t, \mathbf{B}_{t-1}\}$. Households then choose an allocation $\{c_t, n_t, \{b_t^{t+k}\}_{k=1}^\infty\}$. An MPCE consists of a government strategy $\rho(s_t, \mathbf{B}_{t-1})$ which is a function of (s_t, \mathbf{B}_{t-1}) ; a household allocation strategy $\omega((s_t, \mathbf{B}_{t-1}), \rho_t, \mathbf{q}_t)$ which is a function of (s_t, \mathbf{B}_{t-1}) , the government policy $\rho_t = \rho(s_t, \mathbf{B}_{t-1})$, and bond prices \mathbf{q}_t ; and a set of bond pricing functions $\{\varphi^k(s_t, \mathbf{B}_{t-1}, \rho_t)\}_{k=1}^\infty$ with $q_t^{t+k} = \varphi^k(s_t, \mathbf{B}_{t-1}, \rho_t) \forall k \geq 1$ which depend on (s_t, \mathbf{B}_{t-1}) and the government policy $\rho_t = \rho(s_t, \mathbf{B}_{t-1})$. In an MPCE, these objects must satisfy the following conditions $\forall t$:

- i. The government strategy $\rho(\cdot)$ maximizes [equation \(2\)](#) given $\omega(\cdot)$, $\varphi^k(\cdot) \forall k \geq 1$, and the government budget constraint [\(4\)](#),
- ii. The household allocation strategy $\omega(\cdot)$ maximizes [equation \(2\)](#) given $\rho(\cdot)$, $\varphi^k(\cdot) \forall k \geq 1$, and the household budget constraint [\(3\)](#), and
- iii. The set of bond pricing functions $\varphi^k(\cdot) \forall k \geq 1$ satisfy [equation \(5\)](#) given $\rho(\cdot)$ and $\omega(\cdot)$.

Recursive Representation of MPCE. We can use the primal approach to represent an MPCE recursively. Recall that $\rho(s_t, \mathbf{B}_{t-1})$ is a policy that depends on (s_t, \mathbf{B}_{t-1}) , and that $\omega((s_t, \mathbf{B}_{t-1}), \rho_t, \mathbf{q}_t)$ is a household allocation strategy which depends on (s_t, \mathbf{B}_{t-1}) , government policy $\rho_t = \rho(s_t, \mathbf{B}_{t-1})$, and bond prices \mathbf{q}_t , where these bond prices depend on (s_t, \mathbf{B}_{t-1}) and government policy. As such, an MPCE in equilibrium is characterized by a stochastic sequence in [equation \(7\)](#) and a debt sequence $\{\{B_t^{t+k}(s^t)\}_{k=1}^\infty\}_{s^t \in S^t}\}_{t=0}^\infty$, where each element depends only on s^t through (s_t, \mathbf{B}_{t-1}) , the payoff-relevant variables. Given this observation, in an MPCE, one can define a function $h^k(\cdot)$

$$(31) \quad h^k(s_t, \mathbf{B}_t) = \beta^k \mathbb{E} [u_{c,t+k} | s_t, \mathbf{B}_t]$$

for $k \geq 1$, which equals the discounted expected marginal utility of consumption at $t+k$ given (s_t, \mathbf{B}_t) at t . This function is useful since, in choosing \mathbf{B}_t at date t , the government must take into account how it affects future expectations of policy which in turn affect

current bond prices through expected future marginal utility of consumption.

Note furthermore that choosing $\{\tau_t, g_t, \mathbf{B}_t\}$ at date t is equivalent to choosing $\{c_t, n_t, g_t, \mathbf{B}_t\}$ from the perspective of the government, and this follows from the primal approach delineated in [Section II.C](#). Thus, we can write the government's problem recursively as

$$(32) \quad V(s_t, \mathbf{B}_{t-1}) = \max_{c_t, n_t, g_t, \mathbf{B}_t} u(c_t, n_t) + \theta_t(s_t) v(g_t) + \beta \sum_{s_{t+1} \in S} \pi(s_{t+1} | s_t) V(s_{t+1}, \mathbf{B}_t)$$

$$(33) \quad \text{s.t.} \quad c_t + g_t = n_t,$$

$$(34) \quad u_{c,t}(c_t - B_{t-1}^t) + u_{n,t} n_t + \sum_{k=1}^{\infty} h^k(s_t, \mathbf{B}_t) (B_t^{t+k} - B_{t-1}^{t+k}) = 0,$$

where [equation \(34\)](#) is a recursive representation of [equation \(10\)](#). Let $f(s_t, \mathbf{B}_{t-1})$ correspond to the solution to [equations \(32\)–\(34\)](#) given $V(\cdot)$ and $h^k(\cdot)$. It therefore follows that the function $f(\cdot)$ necessarily implies a function $h^k(\cdot)$ which satisfies [equation \(31\)](#). An MPCE is therefore composed of functions $V(\cdot)$, $f(\cdot)$, and $h^k(\cdot)$ which are consistent with one another and satisfy [equations \(31\)–\(34\)](#).

UNIVERSITAT POMPEU FABRA, CREI, AND BARCELONA GSE
 FEDERAL RESERVE BANK OF BOSTON
 COLUMBIA UNIVERSITY AND NBER

SUPPLEMENTARY MATERIAL

An Online Appendix for this article can be found at [The Quarterly Journal of Economics](#) online.

REFERENCES

Aguar, Mark, and Manuel Amador, "Take the Short Route: How to Repay and Restructure Sovereign Debt with Multiple Maturities," Working Paper, Princeton University, 2014.

- Aiyagari, Rao, Albert Marcet, Thomas Sargent, and Juha Seppala, "Optimal Taxation without State-Contingent Debt," *Journal of Political Economy*, 110 (2002), 1220–1254.
- Alvarez, Fernando, Patrick J. Kehoe, and Pablo Andrés Neumeyer, "The Time Consistency of Optimal Monetary and Fiscal Policies," *Econometrica*, 72 (2004), 541–567.
- Angeletos, George-Marios, "Fiscal Policy with Noncontingent Debt and the Optimal Maturity Structure," *Quarterly Journal of Economics*, 117 (2002), 1105–1131.
- Arellano, Cristina, Yan Bai, Patrick J. Kehoe, and Ananth Ramanarayanan, "Credibility and the Maturity of Government Debt," Manuscript, Federal Reserve Bank of Minneapolis, 2013.
- Arellano, Cristina, and Ananth Ramanarayanan, "Default and the Maturity Structure in Sovereign Bonds," *Journal of Political Economy*, 120 (2012), 187–232.
- Barro, Robert J., "On the Determination of the Public Debt," *Journal of Political Economy*, 87 (1979), 940–971.
- , "Optimal Management of Indexed and Nominal Debt," *Annals of Economics and Finance*, 4 (2003), 1–15.
- Bhandari, Anmol, David Evans, Mikhail Golosov, and Thomas J. Sargent, "Taxes, Debts, and Redistributions with Aggregate Shocks," *Quarterly Journal of Economics*, forthcoming.
- , "Fiscal Policy and Debt Management with Incomplete Markets," Working Paper, University of Minnesota, 2015.
- Blommestein, Hans J., Mehmet Emre Elmadağ, and Jacob Wellendorph Ejsing, "Buyback and Exchange Operations Policies, Procedures and Practices among OECD Public Debt Managers," OECD Working Papers on Sovereign Borrowing and Public Debt Management No. 5, 2012.
- Buera, Francisco, and Juan Pablo Nicolini, "Optimal Maturity of Government Debt without State Contingent Bonds," *Journal of Monetary Economics*, 51 (2004), 531–554.
- Chari, Varadarajan V., Lawrence J. Christiano, and Patrick J. Kehoe, "Optimal Fiscal Policy in a Business Cycle Model," *Journal of Political Economy*, 102 (1994), 617–52.
- Chari, Varadarajan V., and Patrick J. Kehoe, "Sustainable Plans and Debt," *Journal of Economic Theory*, 61 (1993), 230–261.
- , "Sustainable Plans and Mutual Default," *Review of Economic Studies*, 60 (1993), 175–95.
- Cochrane, John H., "A New Structure for US Federal Debt," in *The 13 Trillion Question: Managing the U.S. Government's Debt*, Wessel David ed. (Washington, DC: Brookings Institution Press, 2015).
- Debortoli, Davide, and Ricardo Nunes, "Lack of Commitment and the Level of Debt," *Journal of the European Economic Association*, 11 (2013), 1053–1078.
- Faraglia, Elisa, Albert Marcet, Rigas Oikonomou, and Andrew Scott, "Government Debt Management: The Long and the Short of It," CEPR Discussion Paper No. 10281, 2014.
- Faraglia, Elisa, Albert Marcet, and Andrew Scott, "In Search of a Theory of Debt Management," *Journal of Monetary Economics*, 57 (2010), 821–836.
- Farhi, Emmanuel, "Capital Taxation and Ownership When Markets Are Incomplete," *Journal of Political Economy*, 118 (2010), 908–948.
- Fernandez, Raquel, and Alberto Martin, "The Long and the Short of It: Sovereign Debt Crises and Debt Maturity," Barcelona GSE Working Paper, No. 818, 2015.
- Greenwood, Robin, Samuel G. Hanson, and Jeremy C. Stein, "A Comparative-Advantage Approach to Government Debt Maturity," *Journal of Finance*, 70 (2015), 1683–1722.
- Greenwood, Robin, and Dimitri Vayanos, "Bond Supply and Excess Bond Returns," *Review of Financial Studies*, 27 (2014), 663–713.

- Guibaud, Stéphane, Yves Nosbusch, and Dimitri Vayanos, "Bond Market Clienteles, the Yield Curve, and the Optimal Maturity Structure of Government Debt," *Review of Financial Studies*, 26 (2013), 1914–1961.
- IMF, "Guidelines for Public Debt Management," Report by the Staffs of the International Monetary Fund and the World Bank, 2001.
- Krusell, Per, Fernando Martin, and José-Victor Ríos-Rull, "Time-Consistent Debt," Society for Economic Dynamics 2006 Meeting Papers No. 210, 2006.
- Leitner, James, and Ian Shapiro, "Consols Could Avert Another Debt-Ceiling," *Washington Post*, November 14, 2013.
- Lucas, Robert E., *Models of Business Cycles* (New York: Basil Blackwell, 1987).
- Lucas, Robert E., and Nancy L. Stokey, "Optimal Fiscal and Monetary Policy in an Economy without Capital," *Journal of Monetary Economics*, 12 (1983), 55–93.
- Lustig, Hanno, Christopher Sleet, and Sevin Yeltekin, "Fiscal Hedging with Nominal Assets," *Journal of Monetary Economics*, 55 (2008), 710–727.
- Mokyr, Joel, *The Enlightened Economy: Britain and the Industrial Revolution, 1700–1850* (London: Penguin Books, 2011).
- Niepelt, Dirk, "Debt Maturity without Commitment," *Journal of Monetary Economics*, 68 (2014), S37–S54.
- Persson, Mats, Torsten Persson, and Lars E.O. Svensson, "Time Consistency of Fiscal and Monetary Policy: A Solution," *Econometrica*, 74 (2006), 193–212.
- Rogers, Carol, "Debt Restructuring with a Public Good," *Scandinavian Journal of Economics*, 91 (1989), 117–130.
- Shin, Yongseok, "Managing the Maturity Structure of Government Debt," *Journal of Monetary Economics*, 54 (2007), 1565–1571.
- Sleet, Christopher, and Sevin Yeltekin, "Optimal Taxation with Endogenously Incomplete Debt Markets," *Journal of Economic Theory*, 127 (2006), 36–73.
- Werning, Iván, "Optimal Fiscal Policy with Redistribution," *Quarterly Journal of Economics*, 122 (2007), 925–967.
- Woodford, Michael, "Fiscal Requirements for Price Stability," *Journal of Money Credit and Banking*, 33 (2001), 669–728.
- Yglesias, Matthew, "Don't Repay the National Debt," *Slate*, January 29, 2013.