Information Aggregation in Dynamic Markets with Adverse Selection

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Abstract

How effectively does a decentralized marketplace aggregate information that is dispersed throughout the economy? We study this question in a dynamic setting, in which sellers have private information that is correlated with an unobservable aggregate state. We first characterize equilibria with an arbitrary finite number of informed traders. A common feature is that each seller's trading behavior provides an informative and conditionally independent signal about the aggregate state. We then ask whether the state is revealed as the number of informed traders goes to infinity. Perhaps surprisingly, the answer is no. We provide generic conditions under which the amount of information revealed is necessarily bounded and does not reveal the aggregate state. When these conditions are violated, there may be coexistence of equilibria that lead to aggregation with those that do not. We discuss the implications for policies meant to enhance information dissemination in markets. Reporting lags combined with segmented trading platforms can be an effective way to ensure information aggregation without sacrificing welfare. In general, a partially revealing information policy can increase trading surplus.

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1 Introduction

Since the seminal work of Hayek (1945), the question of whether markets effectively aggregate dispersed information has been a central one in economics. Formal investigations of this question are typically conducted in a setting with a single (perhaps divisible) asset about which traders have dispersed information. Whether information is aggregated then usually boils down to whether the equilibrium price reveals the value of the asset conditional on the union of traders' information.[1] This broad class of models is quite natural for many applications and trading environments from static common-value auctions to dynamic trading of securities in financial markets. For other applications (e.g., real estate, OTC markets), dispersed information arises due to dispersion in ownership and one is interested in the extent to which aggregate trading behavior across many different assets reveals information about the underlying state of the economy. In this paper, we explore such a setting.

More specifically, we investigate the question of information aggregation in a dynamic setting with many assets, whose values are independently and identically drawn from a distribution that depends on an underlying aggregate state. The value of each asset is privately observed by its seller, who receives offers each period from competitive buyers. We ask whether the history of all transactions reveals the aggregate state as the number of informed sellers in the economy (denoted by N) grows large.

To answer this question, we begin by characterizing the set of equilibria for arbitrary but finitely many N. Due to a complementarity between the amount of information collectively revealed by others and the optimal strategy of an individual seller, multiple equilibria can exist. A feature common to all equilibria is that each individual seller's trading behavior provides an informative and conditionally independent signal about the aggregate state. Intuitively, one might then expect that, by the law of large numbers, the state would then be revealed as the number of sellers tends to infinity.

Our first main result (Theorem 1) shows that this intuition is incorrect. We provide necessary and sufficient conditions under which there does not exist any sequence of equilibria that reveal the state as $N \to \infty$. The reason why aggregation fails is that the information content of each individual seller's behavior tends to zero at a rate of $1/N$, just fast enough to offset the additional number of observations. As a result, some information is revealed by the limiting trading behavior, but not enough to precisely determine the underlying state. Roughly speaking, the conditions for non-aggregation require that the correlation of asset values is sufficiently high and that agents are sufficiently patient. Intuitively, these conditions

[1] Seminal works on this topic include Grossman (1976), Wilson (1977) and Milgrom (1979). More recent progress on this question has been made by Pesendorfer and Swinkels (1997), Kremer (2002), Albagli et al. (2012), Lauermann and Wolinsky (2013), Siga (2013), Bodoh-Creed (2013), and Axelson and Makarov (2014).

guarantee that if the aggregate state were to be revealed with certainty tomorrow, then the option value of delaying trade is large enough that a seller will prefer not to trade today.

When these conditions are not satisfied, there exists a sequence of equilibria such that information about the state is aggregated as $N \to \infty$. However, even when the conditions are not satisfied, information aggregation is not guaranteed. Our second main result shows that there exists a region of the parameter space in which there is coexistence of equilibria that reveal the state with equilibria that do not. The key difference across the two types of equilibria is the rate at which trade declines as the number of informed sellers grows. In the non-aggregating equilibria, trade declines at rate $1/N$ whereas in aggregating equilibria, the rate of trade declines slower than $1/N$. We are not aware of analogous coexistence results in the literature.

These two results give rise to natural questions about market regulation and design. For instance, how should a regulator disclose trading activity to market participants in order to ensure that aggregation obtains? Under what circumstances is concealing information desirable? Is there a trade-off between maximizing trading surplus and information revelation? We argue that introducing reporting "lags" is a simple mechanism that can be used to ensure that information is aggregated. Creating a delay between when a trade happens and when it is publicly disclosed to market participants prevents the rate of trade from converging to zero, which ensures that the information content of each individual trade remains non-trivial. Clearly then as $N \to \infty$, information is aggregated.

While this simple instrument ensures information is (eventually) aggregated, it uniformly delays revelation and thereby limits the scope for mitigating the adverse selection problem. A social planner can ensure aggregation while increasing trading surplus by revealing some information without delay and some information with delay. One way to accomplish such a revelation policy is to arrange market participants on segmented trading platforms, each with a finite number of traders. Within a platform, traders can observe all trading activity in real time, but across platforms trades are disclosed only with a lag. This structure balances a trade-off between providing the market with information to overcome adverse selection while not providing too much information that trading activity becomes sufficiently uninformative that aggregation fails.

Reporting lags and segmented trading platforms are simple and seemingly empirically relevant.[2] Yet, one could consider a richer class of information revelation policies and characterize the policy that reveals as much information as early as possible while simultaneously maxi-

[2]For instance, TRACE delays disseminating information about transactions for certain types of securities. See Hendershott and Madhavan (2015) for a discussion of the various venues on which trading financial securities are traded.

mizing gains from trade. In solving this problem, one must account for a novel feedback effect. Namely, that the revelation policy influences how market participants behave and therefore the information content of their trading behavior and therefore the information content of whatever is in fact disclosed. Though a complete analysis of such a problem is left for future work, we are able to provide sufficient conditions under which the Pareto optimal mechanism involves concealing some information.

Recently, there has been a strong regulatory push towards making financial markets more transparent (i.e., disclosing more information about trading activity to market participants). For example, one of the stated goals of the Dodd-Frank Act of 2010 is to increase transparency and information dissemination in the financial system. The European Commission is considering revisions to the Markets in Financial Instruments Directive (MiFID), in part to improve the transparency of European financial markets. Our results highlight a potential trade-off for such policies and provide a justification for limiting the amount of information available to market participants (or at least, delaying its disclosure).

The introduction of benchmarks, which reveal some aggregate trading information has also received recent attention by policy makers and academics. Duffie et al. (2014) analyze the role of benchmarks (e.g., LIBOR) in revealing information about fundamentals and suggest that the introduction of benchmarks is welfare enhancing. Our analysis highlights an important consideration absent in their setting. Namely, that the informational content of the benchmark may change once it is published due to endogenous responses by market participants.

1.1 Related Literature

Our paper contributes to two strands of literature. The first is the literature on information aggregation cited earlier. Our model differs from this literature along several important dimensions. First, we consider a setting with many indivisible assets with heterogeneous values rather than a single asset. Second, our model is dynamic and each seller has an incentive to delay the sale of their asset to signal quality.

Kyle (1985) studies a dynamic insider trading model and shows that the insider fully reveals his information as time approaches the end of the trading interval. Foster and Viswanathan (1996) and Back et al. (2000) extend this finding to a model with multiple strategic insiders with different information. Ostrovsky (2012) further generalized these findings to a broader class of securities and information structures. He considers a dynamic trading model with finitely many partially informed traders and provides necessary and sufficient conditions on security payoffs for information aggregation to obtain. Our paper differs from these works in that we study a setting with heterogeneous but correlated assets owned by privately informed sellers. We ask whether information aggregates as the number of sellers becomes arbitrarily large. Despite the fact that we look at the limit as $N \to \infty$, the strategic considerations do not vanish in our model due to the fact that there is an idiosyncratic component to the value of each asset.

Golosov et al. (2014) consider an environment in which a fraction of agents has private information about an asset while the other fraction are uninformed. Agents trade in a decentralized anonymous market through bilateral matches, i.e., signaling with trading histories is not possible. They find that information aggregation obtains in the long run. In contrast, in our setting observing trading histories plays a crucial role: signaling through delay diminishes the amount of trade thus reducing the information content of the market, and potentially allowing for the possibility that information aggregation fails.

Lauermann and Wolinsky (2016) study information aggregation in a search market, in which an informed buyer sequentially solicits offers from sellers who have noisy information about the buyer's value. They provide conditions under which information aggregation fails, and they trace this failure to a strong form of winner's curse that arises in a search environment. Although our setups are substantially different, we share the common feature that the fear of adverse selection hinders trade and thus reduces information generation in markets.

Babus and Kondor (2016) explore how the network structure affects information diffusion in a static OTC model with a single divisible asset. They show that strategic considerations do not influence the degree of information diffusion. However, the network structure combined with a private value component leads to an informational externality that constrains the informativeness of prices and hence the informational efficiency of the economy.

Finally, our paper is related to a growing literature that studies dynamic markets with adverse selection (e.g., Janssen and Roy (2002), Hörner and Vieille (2009), Fuchs and Skrzypacz (2012), Fuchs et al. (2015), Daley and Green (2012, 2016)). Our innovation is the introduction of asset correlation into an otherwise standard framework, which allows us to study the information aggregation properties of these markets. This paper builds upon our previous work, Asriyan et al. (2017). Indeed, we share a similar setup, but that model features two informed agents, whereas in this paper we focus on the information aggregation properties of markets in a setting with many informed agents and study policies regarding information design.

2 The Model

There are $N + 1$ sellers indexed by $i \in \{1, ..., N + 1\}$, with $N \geq 1$. Each seller is endowed with an indivisible asset and is privately informed of her asset's type, denoted by $\theta_i \in \{L, H\}$.

Seller i has a value c_{θ_i} for her asset, where $c_L < c_H$. The value of a type-θ asset to a buyer is v_θ and there is common knowledge of gains from trade, $v_\theta > c_\theta$. One can interpret c_θ and v_θ as the present value of the flow payoffs from owning the asset to the seller and the buyer respectively.

There are two trading periods: $t \in \{1, 2\}$.[3] In each period, multiple competing buyers make offers to each seller. A buyer whose offer is rejected gets a payoff of zero and exits the game.[4] The payoff to a buyer who purchases an asset of type θ at price p is $v_\theta - p$. Sellers discount future payoffs by a factor $\delta \in (0, 1)$. The payoff to a seller with an asset of type θ, who agrees to trade at a price p in period t is

$$\left(1 - \delta^{t-1}\right) c_\theta + \delta^{t-1} p.$$

If the seller does not trade at either date, his payoff is c_θ. All players are risk neutral.

The key feature of the model is that asset values are correlated with an unobservable underlying state, S, that takes values in $\{l, h\}$. The unconditional distribution of θ_i is $\mathbb{P}(\theta_i = H) = \pi \in (0, 1)$. Assets are mutually independent conditional on the state, but their conditional distributions are given by $\mathbb{P}(\theta_i = L | S = l) = \lambda \in (1 - \pi, 1)$. To allow for arbitrarily high level of correlation, we set $\mathbb{P}(S = h) = \pi$.

Importantly, our correlation structure introduces the possibility that a trade of one asset contains relevant information about the aggregate state and therefore the value of other assets. To capture this possibility, we assume that all transactions are observable. Therefore, prior making offers in the second period, buyers observe the set of assets that traded in the first period. For convenience, we assume that offers are made privately (i.e., the level of rejected offers is not observed by other buyers).

Notice that by virtue of knowing her asset quality, each seller has a private and conditionally independent signal about the aggregate state of nature. Thus, if each seller were to report her information truthfully to a central planner, then the planner would learn the aggregate state with probability one as $N \to \infty$. Our interest is to explore under what conditions the same information can be gleaned from the transaction data of a decentralized market. Though we do not model them explicitly, one can imagine a variety of reasons why information aggregation is a desirable feature (e.g., better allocation of capital). To ensure that strategic interactions remain relevant, we focus on primitives which satisfy the following assumptions.

Assumption 1. $\pi v_H + (1 - \pi) v_L < c_H$.

[3]We extend the analysis to an arbitrary number of trading periods in Section 4.

[4]That buyers are "short-lived" (i.e., make offers in only one period) is a fairly standard assumption in this literature (e.g., Swinkels, 1999; Kremer and Skrzypacz, 2007; Hörner and Vieille, 2009). Our results remain similar even with long-lived buyers if offers are publicly observable,.

Assumption 2. $v_L < (1-\delta)c_L + \delta v_H$.

The first assumption, which we refer to as the "lemons" condition, asserts that the adverse selection problem is severe enough to rule out the efficient equilibrium in which all sellers trade immediately. In this equilibrium, trade is uninformative about the underlying state (regardless of N). The second assumption implies a lower bound on the discount factor and ensures that dynamic considerations remain relevant. Our main results do not rely on this assumption but it simplifies exposition and rules out "fully separating" equilibrium (also independent of N).

2.1 Strategies and Equilibrium Concept

A strategy of a buyer is a mapping from his information set to a probability distribution over offers. In the first period (i.e., at $t = 1$), a buyer's information set is empty. In the second period, buyers know whether each asset traded in the first period. If asset i trades in the first period, then it is efficiently allocated and it is without loss to assume that buyers do not make offers for it in the second period (Milgrom and Stokey, 1982). The strategy of each seller is a mapping from her information set to a probability of acceptance. Seller i's information includes her type, the set of previous and current offers as well as the information set of buyers.

We use Perfect Bayesian Equilibria (PBE) as our solution concept. This has three implications. First, each seller's acceptance rule must maximize her expected payoff at every information set taking buyers' strategies and the other sellers' acceptance rules as given (*Seller Optimality*). Second, any offer in the support of the buyer's strategy must maximize his expected payoff given his beliefs, other buyers' strategy and the sellers' strategy (*Buyer Optimality*). Third, given their information set, buyers' beliefs are updated according to Bayes' rule whenever possible (*Belief Consistency*).

2.2 Updating

Let σ_i^θ denote the probability that seller i trades in the first period if her asset is type θ. There are two ways in which the prior about seller i is updated between the first and second periods. First, conditional on rejecting the offer in the first period, buyers' interim belief is given by

$$\pi_{\sigma_i} \equiv \mathbb{P}(\theta_i = H | \text{reject at } t = 1) = \frac{\pi(1-\sigma_i^H)}{\pi(1-\sigma_i^H) + (1-\pi)(1-\sigma_i^L)} \tag{1}$$

Second, before making offers in the second period, buyers learn about any other trades that took place in the first period. How this information is incorporated into the posterior depends on the trading strategy of the other sellers (i.e., σ_j^θ, $j \neq i$). Let $z^j \in \{0, 1\}$ denote the indicator for whether seller j trades in the first period, and let $\mathbf{z} = (z^j)_{j=1}^{N+1}$ and $\mathbf{z}_{-i} = (z^j)_{j \neq i}$. Denote the probability of $\mathbf{z}_{-i}$ conditional on seller i being of type θ by $\rho^i_\theta(\mathbf{z}_{-i})$, which can be written as

$$\rho^i_\theta(\mathbf{z}_{-i}) \equiv \sum_{s \in \{l,h\}} \mathbb{P}\left(S = s | \theta_i = \theta\right) \cdot \prod_{j \neq i} \mathbb{P}\left(z^j | S = s\right), \tag{2}$$

where $\mathbb{P}\left(z^j = 1 | S = s\right) = \sum_{\theta \in \{L,H\}} \sigma^\theta_j \cdot \mathbb{P}\left(\theta_j = \theta | S = s\right)$ is the probability that seller j traded in state s. Provided there is positive probability that i rejects the bid at $t = 1$ and $\mathbf{z}_{-i}$ is realized, we can use equations (1) and (2) to express the posterior probability of seller i being high type conditional on these two events:

$$\pi_i(\mathbf{z}_{-i}; \sigma_i, \sigma_{-i}) \equiv \mathbb{P}(\theta_i = H | z^i = 0, \mathbf{z}_{-i}) = \frac{\pi_{\sigma_i} \cdot \rho^i_H(\mathbf{z}_{-i})}{\pi_{\sigma_i} \cdot \rho^i_H(\mathbf{z}_{-i}) + (1 - \pi_{\sigma_i}) \cdot \rho^i_L(\mathbf{z}_{-i})}. \tag{3}$$

To conserve on notation, we often suppress arguments of π_i.

2.3 Equilibrium Properties

Asriyan et al. (2017) analyze the model with two sellers (i.e., $N = 1$), and they establish several properties that must hold in any equilibrium. These properties extend to the model studied here with an arbitrary number of sellers.

In order to introduce them, we will use the following definitions and notation. We refer to the *bid* for asset i at time t as the maximal offer made across all buyers for asset i at time t. Let $V(\tilde{\pi}) \equiv \tilde{\pi} v_H + (1 - \tilde{\pi}) v_L$ denote buyer's expected value for an asset given an arbitrary belief $\tilde{\pi}$. Let $\bar{\pi} \in (\pi, 1)$ be such that $V(\bar{\pi}) = c_H$, and recall that π_i denotes the probability that buyers assign to $\theta_i = H$ prior to making offers in the second period.

Property 1 (Second period) *If seller i does not trade in the first period, then in the second period:*

(i) If $\pi_i > \bar{\pi}$ then the bid is $V(\pi_i)$, which the seller accepts w.p.1.

(ii) If $\pi_i < \bar{\pi}$ then the bid is v_L, which the high type rejects and the low type accepts w.p.1.

(iii) If $\pi_i = \bar{\pi}$, then the bid is $c_H = V(\pi_i)$ with some probability $\phi_i \in [0, 1]$ and v_L otherwise.

Note that a high type will only accept a bid higher than c_H. When the expected value of the asset is below c_H (as in (i)), buyers cannot attract both types without making a loss. Thus, only the low type will trade and competition pushes the bid to v_L. When the expected value of the asset is above c_H (as in (ii)), competition forces the equilibrium offer to be the expected value. Finally, when the expected value of the asset is exactly c_H (as in (iii)), buyers are indifferent between offering c_H and trading with both types or offering v_L and only trading with the low type.

Property 2 (First period) *In the first period, the bid for each asset is v_L. The high-type rejects the first period bid with probability 1. The low-type seller accepts with probability $\sigma_i \in [0, 1)$.*

Any offer that is acceptable to a high type in the first period is accepted by the low type w.p.1. Assumption 1 implies that any such offer yields negative profits for the buyers. Hence, in equilibrium only low types trade in the first period and competition pushes the bid to v_L. Finally, if $\sigma_i = 1$, then the bid in the second period must be v_H (Property 1). But then the low-type seller i would strictly prefer to delay trade to the second period (Assumption 2), a contradiction.

Notice that Property 1 implies a second period payoff to a type-θ seller i as a function of (π_i, ϕ_i), which we denote by $F_\theta(\pi_i, \phi_i)$, where

$$F_H(\pi_i, \phi_i) \equiv \max\{c_H, V(\pi_i)\}, \tag{4}$$

and

$$F_L(\pi_i, \phi_i) \equiv \begin{cases} v_L & \text{if } \pi_i < \bar{\pi} \\ \phi_i c_H + (1 - \phi_i) v_L & \text{if } \pi_i = \bar{\pi} \\ V(\pi_i) & \text{if } \pi_i > \bar{\pi}. \end{cases} \tag{5}$$

Properties 1 and 2 also imply that an equilibrium can be characterized by $\{\sigma_i, \phi_i\}_{i=1}^{N+1}$.

From seller i's perspective, the strategy of seller $j \neq i$ in the first period is relevant because it influences the distribution of news $\mathbf{z}_{-i}$ and therefore the distribution of π_i. In particular, the (expected) continuation value of a seller from rejecting an offer in the first period can be written as

$$Q_\theta^i(\sigma_i, \sigma_{-i}, \phi_i) \equiv (1 - \delta)c_\theta + \delta \sum_{\mathbf{z}_{-i}} \rho_\theta^i(\mathbf{z}_{-i}) F_\theta(\pi_i(\mathbf{z}_{-i}), \phi_i). \tag{6}$$

where σ_{-i} denotes the vector of $\{\sigma_j\}_{j \neq i}$. It is worthwhile to note that because the posterior conditional on rejection is increasing in σ_i, so does Q_L^i. These observations naturally lead to a third useful property that equilibria must satisfy.

Property 3 (Symmetry) *In any equilibrium, $\sigma_i = \sigma > 0$ for all i. If buyer mixing is part of the equilibrium then $\phi_i = \phi$ for all i.*

The key step to prove symmetry is to show that if $\sigma_i > \sigma_j \geq 0$, then $Q_L^i > Q_L^j$. This follows from the fact that, due to imperfect correlation, π_i (and therefore Q_L^i) is more sensitive to i's own trading probability than it is to that of the other players. Note that if $Q_L^i > Q_L^j$, then the low-type seller i strictly prefers to wait, which contradicts $\sigma_i > 0$ being consistent with an equilibrium. That there must be strictly positive probability of trade then follows: if $\sigma_i = 0$ for all i, then no news arrives and buyers' in the second period would have the same beliefs as buyer's in the first period. This would imply that the second period bid is v_L but in that case the low-type sellers would be strictly better off by accepting v_L in the first period, which contradicts $\sigma_i = 0$.

2.4 Equilibria

Given Properties 1–3, we can drop the subscripts and denote a candidate equilibrium by the pair (σ, ϕ). Because all equilibria are symmetric, any information about seller i that is contained in news $\mathbf{z}_{-i}$ does not depend on the identity of those who sold but only on the number (or fraction) of other sellers that traded. For example, suppose that $\mathbf{z}_{-i} = \mathbf{z}(K)$ where $\mathbf{z}(K)$ is such that $\sum_{j \neq i} z^j = K \leq N$. Then

$$\rho_\theta^i(\mathbf{z}(K)) = \sum_{s \in \{l,h\}} p_s^K \cdot (1 - p_s)^{N-K} \cdot \mathbb{P}(S = s | \theta_i = \theta),$$

where $p_s \equiv \sigma \cdot \mathbb{P}(\theta_i = L | S = s)$ is the probability that any given seller trades in state s. Naturally, the probability of observing K trades among sellers $j \neq i$ is $\binom{N}{K} \cdot \rho_\theta^i(\mathbf{z}(K))$.

Furthermore, since any equilibrium involves $\sigma \in (0, 1)$, a low-type seller must be indifferent between accepting v_L in the first period and waiting until the second period. The set of equilibria can thus be characterized by the solutions to

$$Q_L(\sigma, \sigma, \phi) = v_L. \tag{7}$$

As we show in the next proposition, there can be multiple solutions to (7) and hence multiple equilibria.

Proposition 1 (Existence and Multiplicity) *Fix N. An equilibrium always exists. If λ and δ are sufficiently large, there exist multiple equilibria.*

Intuitively, a higher σ has two opposing effects on the seller's continuation value. On the one hand, the posterior beliefs and thus prices in the second period are increasing in σ, which increases the expected continuation value Q_L. On the other hand, as other low types trade more aggressively, the distribution over buyers' posterior shifts towards lower posteriors, thus decreasing Q_L. The latter force generates complementarities in sellers' trading strategies, which results in multiple equilibria when the correlation between assets is high and traders care sufficiently about the future.

We now turn to our main question, specifically, whether information about the underlying state is aggregated as the number of informed participants grows large. To understand the essence of this question, first notice that the trading behavior of each seller provides an informative signal about the aggregate state. If the seller trades in the first period, than she reveals her asset's type is L, which is more likely when the aggregate state is l than when it is h. Conversely, if the seller does not trade in the first period, then buyers update their beliefs about the asset toward H and their belief about the aggregate state toward h. Moreover, the amount of information revealed in the first period is increasing in the low-type's trading probability, which we now denote by σ_N (in order to explicitly indicate its dependence on the number of other informed participants).

If the information content of each individual trade were to converge to some positive level (i.e., $\lim_{N\to\infty} \sigma_N = \bar{\sigma} > 0$), then information about the state would aggregate. The reason is that by the law of large numbers the fraction of assets traded would concentrate around its population mean $\bar{\sigma} \cdot \mathbb{P}(\theta_i = L|S = s)$, which is strictly greater when the aggregate state is l than when it is h. If, on the other hand, σ_N decreases to zero at a rate weakly faster than $1/N$ (i.e., $\lim_{N\to\infty} N \cdot \sigma_N < \infty$), then information would not aggregate. In this case, despite having arbitrarily many signals about the state, the informativeness of each signal goes to zero fast enough that the overall amount of information does not reveal the true state.

Of course, the equilibrium trading behavior of each individual seller is determined endogenously. Therefore, in order to establish information aggregation properties of equilibria, we need to understand how the set of equilibrium values of σ_N changes with N. Moreover, since different equilibria have different σ_N, the limiting information aggregation properties could be different for different sequences of equilibria. As we will see in the next section, neither of the two cases mentioned above is pathological.

3 Information Aggregation

We begin by studying the information aggregation properties of equilibria in the first period. Consider a sequence of economies indexed by N (standing for $N+1$ assets), and let σ_N denote an equilibrium trading probability in the first period and π_N^{State} be the buyers' posterior belief that the aggregate state is h, conditional on having observed the outcome of trade in the first period. That is, given a first period trading history $\mathbf{z} = (z^j)_{j=1}^{N+1}$, $\pi_N^{State}(\mathbf{z}) \equiv \mathbb{P}(S = h|\mathbf{z})$. We say that:

Definition 1 *There is information aggregation along a given sequence of equilibria if along this sequence* $\pi_N^{State} \to^p 1_{\{S=h\}}$ *as* $N \to \infty$.

Our notion of information aggregation requires that, upon observing the trading history, buyers (or the econometrician, who observes only whether and when an asset trades) learn all the information available in the market that is relevant to infer the aggregate state. Asymptotically, this is equivalent to asking whether agents' beliefs about the aggregate state become degenerate at the truth.[5]

3.1 A 'Fictitious' Economy

Before presenting our main results, it will be useful to consider a 'fictitious' economy in which buyers observe the true state S via an exogenous signal before making second period offers. This benchmark economy is useful because it approximates the information revealed in the true economy if there is information aggregation. We proceed by deriving a necessary and sufficient condition under which the fictitious economy supports an equilibrium with trade in the first period (Lemma 1). We then show that the same condition is necessary, though not sufficient, for information aggregation (Theorem 1). Intuitively, information aggregation requires trade. But if the fictitious economy does not support an equilibrium with trade, then (by continuity) there cannot exist a sequence of equilibria along which information aggregates.

First, note that Properties 1 and 2 trivially extend to the fictitious economy. Second, observe that conditional on knowing the true state, the information revealed by other sellers is irrelevant for buyers when forming beliefs about seller i. That is, buyer's posterior belief about seller i following a rejection in the first period and observing the true state is s is given by

$$\pi_i^{fict}(s) = \frac{\pi_{\sigma_i} \cdot \mathbb{P}(\theta_i = H|S = s)}{\pi_{\sigma_i} \cdot \mathbb{P}(\theta_i = H|S = s) + (1 - \pi_{\sigma_i}) \cdot \mathbb{P}(\theta_i = L|S = s)}.$$

This implies that seller $i's$ continuation value in the fictitious economy, which we denote by $Q_L^{i,fict}(\sigma_i, \phi_i)$ is independent of the trading strategies of the other sellers. Analogous to (6), the continuation value is given by

$$Q_L^{i,fict}(\sigma_i, \phi_i) = (1-\delta)c_L + \delta\left(\lambda F_L(\pi_i^{fict}(l), \phi_i) + (1-\lambda)F_L(\pi_i^{fict}(h), \phi_i)\right)$$

Since there are no complementarities, the fictitious economy has a unique equilibrium, which must be symmetric. As in Daley and Green (2012), due to the exogenous arrival of

[5] That our definition involves convergence in probability is standard in the literature (see e.g., Kremer (2002)).

information, it is possible that the equilibrium of the fictitious economy will involve zero probability of trade in the first period.

Lemma 1 *The unique equilibrium of the fictitious economy involves zero probability of trade in the first period (i.e., $\sigma^{fict} = 0$) if and only if*

$$Q_L^{i,fict}(0,0) \geq v_L. \tag{C1}$$

Furthermore, (C1) *holds if and only if λ and δ satisfy the following:*

$$\lambda \geq \bar{\lambda} \equiv 1 - \frac{\pi(1-\bar{\pi})}{1-\pi} \tag{8}$$

and

$$\delta \geq \bar{\delta}_\lambda \equiv \frac{v_L - c_L}{\lambda v_L + (1-\lambda)V\left(1 - \frac{(1-\lambda)(1-\pi)}{\pi}\right) - c_L}. \tag{9}$$

This result is intuitive. The equilibrium of the fictitious economy features no trade whenever the low type's option value from delaying trade to the second period is high. This occurs when both the information revealed in the second period is sufficiently informative about the seller's type (i.e., $\lambda \geq \bar{\lambda}$) and for a given correlation the future is sufficiently important (i.e., $\delta \geq \bar{\delta}_\lambda$).

3.2 Main Results

We now establish our first main result, which shows that (C1) is also the crucial determinate of the information aggregation properties of equilibria.

Theorem 1 (Aggregation Failure) *If* (C1) *holds with strict inequality, then information aggregation fails along any sequence of equilibria. If* (C1) *does not hold, then there exists a sequence of equilibria along which information aggregates.*

The proof of the first statement uses the observation that if information were to aggregate, then for N large enough the continuation payoffs of the sellers to the continuation payoffs in the fictitious economy. Thus, when (C1) holds strictly, delay is also uniquely optimal when there are a large but finite number of assets. But this contradicts Property 3, which states that $\sigma_N \in \{0, 1\}$ cannot be part of an equilibrium for any finite N. In fact, when (C1) holds strictly, the trading probability σ_N is positive but must go to zero at a rate proportional to $1/N$, which is fast enough to prevent information from aggregating. The rate is also slow enough to ensure that the market does not become completely uninformative in the limit, in which case the bid for any asset in the second period would be v_L with probability arbitrarily close to one, and the low types would strictly prefer to trade in the first period (implying $\sigma_N = 1$), which would contradict Property 3.[6]

On the other hand, when the fictitious economy has an equilibrium with positive trade, we can explicitly construct a sequence of equilibria in which the trading probability σ_N is bounded away from zero. Clearly, information is aggregated along such a sequence. Nevertheless, even when information aggregating equilibria exist, as our next result shows there is no guarantee that information will aggregate along *every* sequence of equilibria.

Theorem 2 (Aggregation and Coexistence) *If either $\lambda < \bar{\lambda}$ or δ is sufficiently small, then information aggregates along any sequence of equilibria. There exists a $\widehat{\delta} < 1$ such that whenever $\delta \in (\widehat{\delta}, \bar{\delta}_\lambda)$ and λ is sufficiently large, there is coexistence of sequences of equilibria along which information aggregates with sequences of equilibria along which aggregation fails.*

For the first part, consider the buyers' posterior belief in the second period about a given seller i. When $\lambda < \bar{\lambda}$, the maximal information available from other trades is insufficient by itself to push the buyers' posterior about seller i above $\bar{\pi}$, which would imply an offer of v_L for sure in the second period, inconsistent with equilibrium. Thus, in the first period, seller i must trade with probability σ_N bounded away from zero. But since this is true of all sellers, information must aggregate. The intuition for δ is simpler. Clearly, if Assumption 2 were violated, then low types would trade in the first period w.p.1, implying aggregation. Note, however, that Assumption 2 assumes that the low type seller gets a payoff of v_H upon waiting. But in fact, in any non-aggregating equilibrium, the low type's payoff in the second period will be strictly below $V(\pi) < v_H$. Thus, even if Assumption 2 is satisfied but δ is sufficiently small, in any non-aggregating equilibrium, a low type seller would want to deviate and trade in the first period w.p.1, contradicting Property 3.

To prove the coexistence result, we first note that for a given $\delta < 1$, if λ is sufficiently large, then we must have $\delta < \bar{\delta}_\lambda$ and thus by Theorem 1 aggregating equilibria must exist. We then show that if we fix δ above a certain threshold, then for a sufficiently large λ, also non-aggregating equilibria must exist. In particular, we explicitly construct a sequence of equilibria in which the second period bid is v_L for all histories except the one in which no seller has traded in the first period. In these equilibria, the probability of the event that *no seller has traded in the first period* remains bounded away from zero, in *both* states of nature. Thus, even as $N \to \infty$, the uncertainty about the state of nature does not vanish.

Figure 1 summarizes our main results by illustrating the regions of the parameter space for which aggregation holds and fails as well as the region of coexistence.

[6] Indeed, we can show that as $N \to \infty$, the distribution of trades in state s becomes Poisson with mean $\kappa \cdot \mathbb{P}(\theta_i = L | S = s)$, where $\kappa \cdot N^{-1}$ is the asymptotic rate at which σ_N goes to zero.

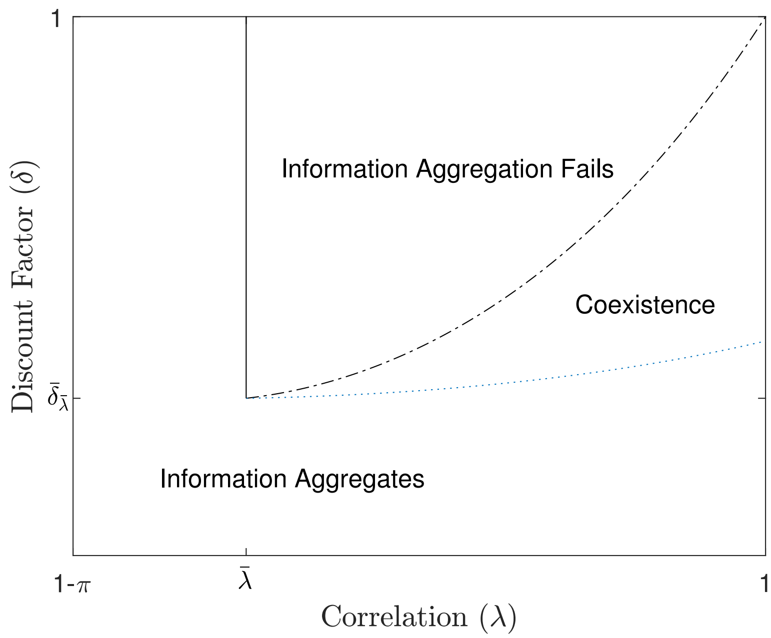

Figure 1: **When does Information Aggregate?** Illustrates the regions of the parameter space over which information aggregation obtains or fails. In the bottom-left region, all sequences of equilibria aggregate information as $N \to \infty$. In the top-right region, there do not exist sequences of equilibria that aggregate information. In the middle-right region, sequences in which information aggregates coexist with sequences in which information aggregation fails.

Thus far, we considered the information aggregation properties of equilibria conditional on the trading history in the first period. To consider aggregation in the second period, we simply extend Definition 1 by requiring that the convergence of buyers' beliefs be conditional on the history of trade over two periods (rather than the first period). Clearly, if information were already aggregated by the first period, it would also be aggregated in the second period. But what if information does not aggregate in the first period? Will trading behavior in the second period provide the additional information necessary to identify the true state?

The answer is that such an outcome is indeed possible; there can exist sequences of equilibria in which information is not aggregated based only on first period trading behavior, but is successfully aggregated from both first and second period trading behavior.[7] This can happen because, there being no further opportunities to trade after the second period, a low-type

[7]To illustrate this possibility in more detail, suppose that there exist a sequence of equilibria that achieves the lower bound (i.e., $\pi_i(\mathbf{z}(0)) = \bar{\pi}$) for all N. Clearly, $\pi_i(z(k)) < \bar{\pi}$ for all $k > 0$. Hence, if at least one seller trades in the first period, then the second-period bid will be v_L for all sellers and all the remaining low-type sellers will reveal their type by accepting, which aggregates information (since all and only low types trade over the two periods). Now suppose that no sellers trade in the first period (i.e., $\mathbf{z}_{-i} = z(0)$). Then, in the second period, buyers will mix between a pooling bid with probability ϕ_N and a separating one with probability $(1 - \phi_N)$. If the buyer mixing is independent across sellers the fraction of sellers who trade in the second period will converge to $\lim_{N\to\infty} \phi_N + (1 - \phi_N)\mathbb{P}(\theta = L|S)$, which also reveals the state provided $\lim_{N\to\infty} \phi_N < 1$.

seller would accept an offer of v_L w.p.1 at that date, thus revealing her type and information about the aggregate state. Instead, if there were additional trading opportunities, then such behavior could not be part of an equilibrium. As we show in Theorem 3, with a longer trading horizon, our main results regarding the information aggregation properties of equilibria hold for all periods save for the last.

4 Longer Trading Horizon

In this section, we extend our main results to a setting with an arbitrary number $T > 1$ of trading periods. Intuitively, one might expect that with more trading periods there are more opportunities to learn from trading behavior and hence more information will be revealed. However, there is a countervailing force; there are more opportunities for (strategic) sellers to signal through delay. It turns out that two factors essentially cancel each other out and we obtain (non-)aggregation and coexistence under remarkably similar conditions.

Characterizing the set of all possible equilibria in such a model is more difficult because the set of relevant histories is more complex.[8] Hence, we will focus on information aggregation properties of symmetric equilibria.[9] Within this class, we show that our main results can be generalized.

First, we must generalize our notion of information aggregation. Let $\pi_{t,N}^{State}$ denote the buyers' posterior belief that the state is high, conditional on having observed the trading history in periods 1 through t in an economy with $N + 1$ sellers.

Definition 2 *There is information aggregation along a given sequence of equilibria in period t if along this sequence $\pi_{t,N}^{State} \to^p 1_{\{S=h\}}$ as $N \to \infty$.*

In the previous section we argued that, because the second period was also the last trading opportunity, information in that period could aggregate independently of whether condition (C1) holds. The following theorem shows that, given an arbitrary trading horizon $T > 1$, the condition (C1) is indeed essential to understand the information aggregation properties of equilibria in all periods but the last.

Theorem 3 *If (C1) holds with strict inequality, then for any $t < T$ information aggregation fails along any sequence of symmetric equilibria. If (C1) does not hold, then there exists a*

[8] In principle, the path of play can depend on sellers' beliefs about the quality of other sellers' assets, the distribution of buyers' beliefs about the quality of each seller's asset, the buyers' and the sellers' beliefs about the aggregate state, the number of assets remaining on the market as well as the number of trading periods remaining.

[9] It is straightforward to show that such equilibria exist.

sequence of equilibria along which information aggregates. Furthermore, there exists a $\widehat{\delta} < 1$ *such that whenever* $\delta \in (\widehat{\delta}, \bar{\delta}_\lambda)$ *and* λ *is sufficiently large, there is coexistence of sequences of equilibria along which information aggregates with sequences of equilibria along which aggregation fails for any* $t < T$.

The proof hinges on arguments similar to those for the two-period economy. For the non-existence result, we show that the first (earliest) period in which information about the state is supposed to aggregate is essentially like the first period in a two-period economy. Thus, given that (C1) holds with strict inequality, the option value of waiting for the state to be revealed is sufficiently high to make it profitable for the seller to prefer not to trade at all. Furthermore, the belief about the remaining seller types is also evolving with time. Thus, Assumption 1 may fail to hold in some non-terminal histories on the equilibrium path of play, which can give rise to other types of equilibria (absent in the two-period economy). Nevertheless, our proof shows that even if these other equilibria arise, they fail to aggregate information as long as (C1) holds with strict inequality.

In order to establish the existence and co-existence results, we construct a class of equilibria that essentially share the information aggregation properties of the two-period economy. A feature of this class is that once the belief about the seller weakly exceeds $\bar{\pi}$, all future bids are pooling. When either $\lambda < \bar{\lambda}$ or $\delta < \bar{\delta}_\lambda$ is sufficiently small, we show that such equilibria exist and that there is at least one equilibrium sequence within this class along which information aggregates. Then, following arguments similar to those for the proof of Theorem 2, we show that if also δ is sufficiently high (but still below $\bar{\delta}_\lambda$), there also exists another sequence of equilibria in which aggregation fails.

5 Policy Implications and Market Design

In this section we explore the implications of the model for market design. To do so, we first consider a social planner who wishes to ensure that information aggregates and can decide what information about trading activity should be revealed to whom and when.[10] We then ask whether there is a trade-off between achieving information aggregation and maximizing trading surplus.

[10]We leave the motivation for the social planner's objective unspecified though there are a numerous reasons that have been proposed for why information aggregation is a desirable feature of an economy. See Bond et al. (2012) for a summary of both theoretical and empirical work on this topic.

5.1 Reporting Lags

One simple way that a planner can ensure aggregation is to introduce a reporting lag. That is, reveal all information about trading activity to all participants in the economy, but only *after* trading has taken place in the second period.[11] By doing so, a low-type seller will trade *as if* there are no other seller's in the market (i.e., as if $N = 0$) and therefore with a strictly positive probability in the first period (by Property 2) that is independent of N. Clearly then, as $N \to \infty$ information is aggregated at the end of the second period when participants observe the fraction of seller's that traded in the first period.[12]

Proposition 2 *If the planner introduces a reporting lag for all trades then there exists a unique equilibrium for any N. Moreover as $N \to \infty$, information is aggregated.*

One potential downside to this approach is that in the $N = 0$ equilibrium, there are no other sources of information to mitigate the adverse selection problem. This leads to a high probability of costly delays and market failures, which reduces the overall trading surplus realized relative to an economy with $N > 0$ (see the left-panel of Figures 2 and 3). Thus, there is a potential trade-off between information aggregation and trading surplus.[13]

Another potential downside of this approach is that it uniformly delays the revelation of information to all market participants. To the extent that the timing of information revelation and aggregation is important (e.g., in order to make unmodeled investment decisions), a uniform reporting lag for all trading activity is likely too blunt of an instrument.

5.2 Segmented Trading Platforms

A less drastic alternative to a uniform reporting lag for all trades is to reveal some information without a lag and some information with a lag. One way to accomplish this information revelation policy is to arrange traders on different *platforms*. Buyers and sellers can observe trading behavior of others on the same platform in real time (i.e., immediately after it occurs and before the second trading period), but across platforms trading behavior is only revealed with a lag. Suppose that there are a total of $M + 1$ sellers assigned to each platform. Then, within the platform, each seller will behave *as if* $N = M$. Again by Property 2, with only a finite number of other traders, each low-type seller i will trade with strictly positive probability

[11] Therefore, a buyer interested in purchasing seller i's asset will not observe trading activity of other sellers $j \neq i$ until after the second period.

[12] A similar policy works with a longer trading horizon though it may require a reporting lag of more than one period.

[13] In any equilibrium, the expected payoff to a low-type seller is v_L and buyers make zero profit. Therefore, total welfare can be measured by a high-type seller's equilibrium payoff. Moreover, any increase in total welfare corresponds to a Pareto improvement.

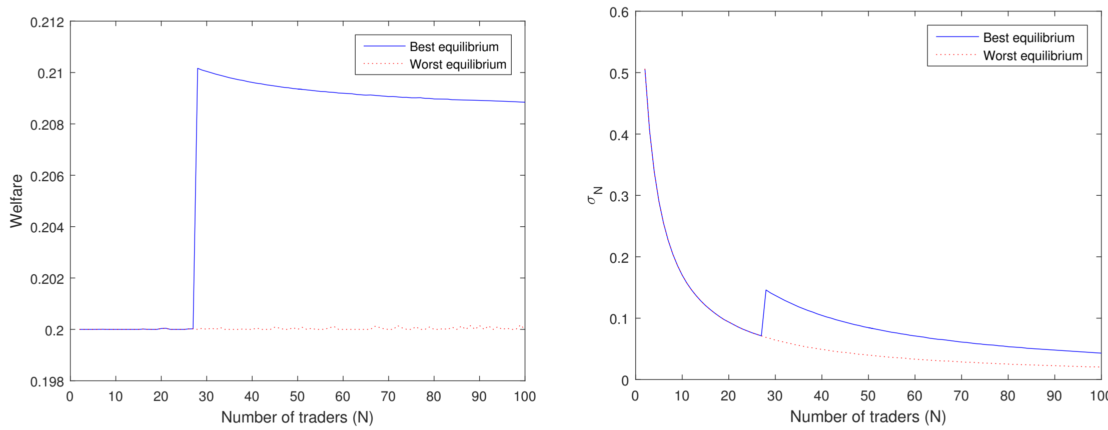

Figure 2: **Welfare and Trade when Information does not Aggregate**. The left panel illustrates how the welfare per trader in the best equilibrium of the economy depends on the number of traders. The parameters are such that (C1) holds and hence aggregating equilibria do not exist. The right panel shows the corresponding strategy of the seller in the first period.

in the first-period and therefore information will aggregate when the trading activity across all platforms is revealed after the second period.

The advantage of this approach is twofold. First, traders will have access to at least some information prior to the second period. And perhaps more importantly, the planner can increase total trading surplus by appropriately choosing the number of traders on each platform. Figure 2 illustrates an example in which the optimal number of traders on each platform is finite. In this example, it is also the case that information aggregation fails absent some form of intervention (i.e., C1 holds). Therefore, by arranging market participants on segmented trading platforms and introducing a reporting lag across platforms, the social planner can *both* improve welfare and achieve information aggregation (albeit with delay).

That there is an interior optimal number of traders per platform for maximizing welfare is driven by the fact that more information revelation leads to an endogenous response by sellers. Namely, for large enough N the seller trades less aggressively (see the right panel of Figure 2), which has a negative effect on welfare.

On the other hand, Figure 3 shows that the effect is not always negative. More specifically, it illustrates that, for a different set of parameters, increasing the number of traders still leads to a response by sellers, but the effect is positive: as N increases, sellers trade more aggressively, which leads to higher welfare. In this case, the parameters are such that information aggregates along all sequences of equilibria even absent intervention and therefore it is suboptimal to use any form of reporting lags.

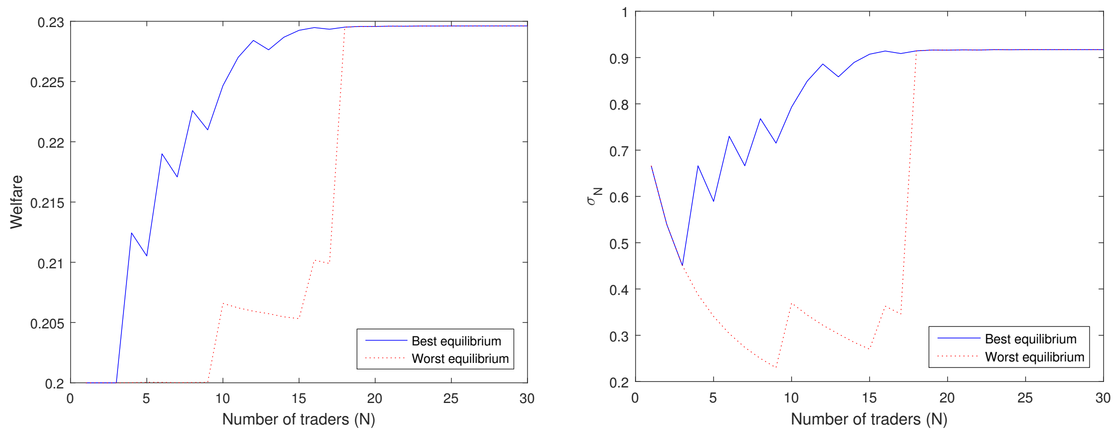

Figure 3: **Welfare and Trade when Information Aggregates**. The left panel illustrates how the welfare per trader in the best equilibrium of the economy depends on the number of traders. The right panel shows the corresponding strategy of a low-type seller in the first period. The parameters are such that only aggregating equilibria exist.

5.3 Information Design

In the previous two subsections, we considered several specific types of information structures. More generally, one might be interested in considering a broader class of information revelation policies and find the policy that maximizes welfare subject to revealing as much information as early as possible. While a complete analysis of such a problem is beyond the scope of this paper, we are able to provide sufficient conditions under which maximizing welfare necessitates concealing some information.

Before doing so, it is useful to compare this problem to the literature on Bayesian persuasion (Kamenica and Gentzkow, 2011; Rayo and Segal, 2010) and "information design" problems more generally (Bergemann and Morris, 2013, 2016).[14] On one hand, the problems are quite similar. Both involve designing an information revelation policy to induce other players to take certain desired actions. On the other hand, the planner's problem in our setting must take into account a novel feedback effect. Namely, that the policy influences how market participants behave and therefore the information content of their trading behavior and therefore the information content of whatever is in fact disclosed. In short, the statistical properties of the information the planner can reveal, which is typically exogenous in a Bayesian persuasion setting, depends on the rule itself.

[14] Bergemann and Morris (2017) provide a more general treatment of information design problems drawing a distinction between whether the designer has an informational advantage (as in Bayesian persuasion) or not (as in communication games). In our model, the planner has no informational advantage ex-ante but has a technology for acquiring one in the interim. Another important distinction of our setting is that the planner has only limited means by which she can elicit information.

Theorem 4 *Suppose that (C1) is violated (so that aggregating equilibria exist) and δ is sufficiently large. Then, when N is large enough, the social planner can (generically) increase the trading surplus with a partially revealing information policy.*

We establish the result in two steps. First, we show that if the social planner knew the state, then it would be sub-optimal for her to fully reveal it, i.e., she would prefer to reveal a noisy signal of the true state. A simple policy that the planner can use to increase surplus is as follows. When the true state is low, the planner makes a report of 0. When the true state is high, the planner makes a report of 1 with probability $1 - \gamma$ and a report of 0 with probability γ. In the proof of Proposition 4, we show that there exists a $\gamma > 0$ such that this policy leads to strictly higher trading surplus. Denote this policy by Γ. Intuitively, the reason why suppressing information can increase surplus is similar to the intuition for the example in Figure 2. By doing so, the planner reduces the low-type's payoff from delaying trade, which thereby induces the seller to trade with a higher probability in the first period and reduces costly delays.

Second, we show that there exists a disclosure policy such that information aggregates along all sequences of equilibria and the posterior beliefs of traders converge to the ones under Γ. This disclosure policy involves a threshold such that when the total fraction of sellers who trade in the first period is above the threshold, the planner makes a report of 0 and when the total fraction of sellers who trade is below the threshold, the planner makes a report of 1 with probability $1 - \gamma$ and makes a report of 0 with probability γ.[15] An interesting feature of this revelation policy is that information is in fact aggregated given the planners information even though it does not aggregate based on what is publicly revealed.

6 Concluding Remarks

We study the information aggregation properties of decentralized dynamic markets in which traders have private information about the value of their asset, which is correlated with some underlying 'aggregate' state of nature. We provide necessary and sufficient conditions under which information aggregation necessarily fails. Further, we show that when these conditions are violated, there can be a coexistence of non-trivial equilibria in which information about the state aggregates with equilibria in which aggregation fails. Our findings suggest there are important differences in the aggregation properties of multi-asset decentralized markets (as studied here) and single-asset centralized markets as typically explored in the literature.

[15] Notice that this disclosure rule ensures that non-aggregating equilibria cannot persist, since in that case the fraction of trades converges to zero with probability one and hence no information would be revealed in the limit.

We argue that our theory has implications for policies meant to enhance information dissemination in asset markets. In particular, a social planner seeking information aggregation (when it otherwise would fail) can do so by introducing a reporting lag. Reporting lags come at the cost of reducing trading activity and therefore trading surplus. A preferable alternative is to reveal some information in real time and some information with a lag, which can be accomplished by organizing traders on segmented trading platforms. More generally, we show that there is indeed a trade-off between ensuring information aggregation and maximizing total welfare: a planner can increase total surplus through an information design policy that partially obscures the true aggregate state.

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Appendix A - Proofs for Section 3

Proof of Property 1. See Lemma 1 in Asriyan et al. (2017). ■

Skimming Property. Since $c_H > c_L$ and $F_H \geq F_L$, the continuation value of the low type seller from rejecting the bid v_L in the first period satisfies:

$$\begin{aligned} Q_L^i &= (1-\delta) \cdot c_L + \delta \cdot \mathbb{E}_L\{F_L(\pi_i, \phi_i)\} \\ &< (1-\delta) \cdot c_H + \delta \cdot \mathbb{E}_L\{F_L(\pi_i, \phi_i)\} \\ &\leq (1-\delta) \cdot c_H + \delta \cdot \mathbb{E}_L\{F_H(\pi_i, \phi_i)\}. \end{aligned}$$

Therefore, in order to prove that $Q_H^i > Q_L^i$, it is sufficient to show that $\mathbb{E}_H\{F_H(\pi_i, \phi_i)\} \geq \mathbb{E}_L\{F_H(\pi_i, \phi_i)\}$. Recall that F_H is increasing in π_i and independent of ϕ_i. Hence, the desired inequality is implied by proving that conditional on $\theta_i = H$, the random variable π_i (weakly) first-order stochastically dominates π_i conditional on $\theta_i = L$.

Note that the distribution of π_i in the second period is a function of the trading probabilities of the seller i and of the realization of news from sellers $j \neq i$, $z_i^j \in \{0,1\}$. Fix the interim belief π_{σ_i}, and consider news $\mathbf{z}'_{-i}$ and $\mathbf{z}''_{-i}$ (which occur with positive probability) such that the posterior π_i satisfies $\pi_i(\mathbf{z}'_{-i}) \geq \pi_i(\mathbf{z}''_{-i})$, i.e., $\mathbf{z}'_{-i}$ is "better news" for seller i than $\mathbf{z}''_{-i}$. But note that:

$$\frac{\pi_{\sigma_i} \cdot \rho_H^i(\mathbf{z}'_{-i})}{\pi_{\sigma_i} \cdot \rho_H^i(\mathbf{z}'_{-i}) + (1-\pi_{\sigma_i}) \cdot \rho_L^i(\mathbf{z}'_{-i})} = \pi_i(\mathbf{z}'_{-i}) \geq \pi_i(\mathbf{z}''_{-i}) = \frac{\pi_{\sigma_i} \cdot \rho_H^i(\mathbf{z}''_{-i})}{\pi_{\sigma_i} \cdot \rho_H^i(\mathbf{z}''_{-i}) + (1-\pi_{\sigma_i}) \cdot \rho_L^i(\mathbf{z}''_{-i})},$$

which implies that $\frac{\rho_H(\mathbf{z}'_{-i})}{\rho_L(\mathbf{z}'_{-i})} \geq \frac{\rho_H(\mathbf{z}''_{-i})}{\rho_L(\mathbf{z}''_{-i})}$, i.e. the ratio of distributions $\frac{\rho_H(\cdot)}{\rho_L(\cdot)}$ satisfies the monotone likelyhood ratio property. This in turn implies that $\rho_H(\cdot)$ first-order stochastically dominates $\rho_H(\cdot)$, which establishes the result. ■

Proof of Property 2. See Lemma 3 in Asriyan et al. (2017). ■

Proof of Property 3. The proof that all equilibria involve strictly positive probability of trade in the first period is in the text. We show here that all equilibria must be symmetric. In search of a contradiction, assume there exists an equilibrium in which $\sigma_A > \sigma_B \geq 0$ for some $A, B \in \{1, ..., N\}$. We establish the result by first showing that the beliefs for seller A are more favorable than for seller B, following all news realizations; then we show that good news about seller A are more likely to arrive than good news about seller B.

Consider the posterior belief about seller $i \in \{A, B\}$ following some news $\mathbf{z}_{-i} = \left(z_i^j\right)_{j \neq i}$:

$$\pi_i\left(\mathbf{z}_{-i}\right) = \frac{\pi_{\sigma_i} \cdot \rho_H^i\left(\mathbf{z}_{-i}\right)}{\pi_{\sigma_i} \cdot \rho_H^i\left(\mathbf{z}_{-i}\right) + (1-\pi_{\sigma_i}) \cdot \rho_L^i\left(\mathbf{z}_{-i}\right)}$$

where we can express $\rho^i_\theta(\mathbf{z}_{-i})$ as:

$$\rho^i_\theta(\mathbf{z}_{-i}) = \sum_{s\in\{l,h\}} \mathbb{P}(S=s|\theta_i=\theta)\cdot\mathbb{P}\left((z^j_i)_{j\neq i,i'}|S=s\right)\cdot\mathbb{P}(z^{i'}_i|S=s)$$

for $i,i'\in\{A,B\}$ and $i'\neq i$. Note that $\rho^i_\theta(\mathbf{z}_{-i})$ depends on $\sigma_{i'}$ only through the term $\mathbb{P}\left(z^{i'}_i|S\right)$. We now show that $\sigma_A>\sigma_B$ implies that:

$$\frac{1-\pi_{\sigma_A}}{\pi_{\sigma_A}}\cdot\frac{\rho^A_L(\mathbf{z}_{-i})}{\rho^A_H(\mathbf{z}_{-i})} < \frac{1-\pi_{\sigma_B}}{\pi_{\sigma_B}}\cdot\frac{\rho^B_L(\mathbf{z}_{-i})}{\rho^B_H(\mathbf{z}_{-i})}, \tag{10}$$

which will establish that $\pi_A(\mathbf{z}_{-i})>\pi_B(\mathbf{z}_{-i})$ for all news $\mathbf{z}_{-i}$. There are two cases to consider, depending on whether $z^{i'}_i=0$ or $z^{i'}_i=1$.

If $z^{i'}_i=1$, then $\mathbb{P}\left(z^{i'}_i=1|S=s\right)=\sigma_{i'}\cdot\mathbb{P}(\theta_{i'}=L|S=s)$ and the likelihood ratio $\frac{1-\pi_{\sigma_i}}{\pi_{\sigma_i}}\cdot\frac{\rho^i_L(\mathbf{z}_{-i})}{\rho^i_H(\mathbf{z}_{-i})}$ decreases in σ_i but is independent of $\sigma_{i'}$. Intuitively, if seller i' traded, her type is revealed to be low, and the intensity with which she trades is irrelevant for updating. But then inequality (10) follows because π_{σ_i} is increasing in σ_i.

If $z^{i'}_i=0$, then $\mathbb{P}\left(z^{i'}_i=0|S=s\right)=1-\sigma_{i'}\cdot\mathbb{P}(\theta_{i'}=L|S=s)$, and now the likelihood ratio $\frac{1-\pi_{\sigma_i}}{\pi_{\sigma_i}}\cdot\frac{\rho^i_L(\mathbf{z}_{-i})}{\rho^i_H(\mathbf{z}_{-i})}$ decreases in both σ_i and $\sigma_{i'}$. However, given that both i and i' did not trade (both are good news for i), inequality (10) follows because the assets i and i' are imperfectly correlated and $\frac{1-\pi_{\sigma_i}}{\pi_{\sigma_i}}\cdot\frac{\rho^i_L(\mathbf{z}_{-i})}{\rho^i_H(\mathbf{z}_{-i})}$ is more sensitive to trading probability σ_i than to $\sigma_{i'}$.

Finally, note that $\sigma_A>\sigma_B$ also implies that the probability that seller B trades and releases bad news about seller A is lower than the probability that seller A trades and releases bad news about seller B. Since the posteriors following good news are higher than following bad news, this establishes the result. ■

Proof of Proposition 1. To prove existence of an equilibrium, it suffices to show there exists a $(\sigma,\phi)\in[0,1]^2$ such that equation (7) holds, i.e., $Q_L(\sigma,\sigma,\phi)=v_L$ where the second argument states that all other sellers also trade with intensity σ. Note that by varying σ from 0 to 1, Q_L ranges from $[(1-\delta)c_L+\delta v_L,(1-\delta)c_L+\delta v_H]$. By continuity of Q_L and Assumption 2, the intermediate value theorem gives the result.

Consider the following two candidate equilibria. We will refer to an equilibrium in which the posterior belief about the seller satisfies $\pi_i(\mathbf{z}(0))=\bar{\pi}$ as a *low trade* equilibrium, and when the posterior belief about the seller satisfies $\pi_i(\mathbf{z}(N))=\bar{\pi}$ as a *high trade* equilibrium. Although there can be other equilibria as well, we do not focus on them. We will now show that the *high trade* and the *low trade* equilibria coexist when λ and δ are large enough.

1. Low trade. Note that there is at most one low trade equilibrium since the trading intensity σ in this category is fully pinned down by the requirement that $\pi_i(\mathbf{z}(0))=\bar{\pi}$. Let x be

the value of σ such that $\pi_i(\mathbf{z}(0); x, x) = \bar{\pi}$ (x denotes the trading probability of all $N+1$ sellers). As ϕ varies from 0 and 1, $Q_L(x, x, \phi)$ varies continuously from $(1-\delta)c_L + \delta v_L$ to $(1-\delta)c_L + \delta(\rho_L^i(\mathbf{z}(0))v_L + (1-\rho_L^i(\mathbf{z}(0)))c_H)$ where $\rho_L^i(\mathbf{z}(0)) > 0$. Hence, there exists a $\bar{\delta}_\lambda < 1$, such that $Q_L(x, x, 1) = v_L$. Clearly, a low trade equilibrium exists if $\delta > \bar{\delta}_\lambda$. Moreover, it is straightforward to show that $\sup_\lambda \rho_L^i(\mathbf{z}(0)) < 1$. Hence, this equilibrium exists if δ is larger than $\bar{\delta} \equiv \sup_{\lambda \in (1-\pi, 1)} \widehat{\delta}_\lambda < 1$.

2. High trade. Note that there is at most one high trade equilibrium since the trading intensity σ is fully pinned down by the requirement that $\pi_i(\mathbf{z}(N)) = \bar{\pi}$. Let y be the value of σ such that $\pi_i(\mathbf{z}(N); y, y) = \bar{\pi}$. As ϕ varies from 0 to 1, Q_L varies continuously from $(1-\delta)c_L + \delta\left(\rho_L^i(\mathbf{z}(N))v_L + \sum_{\mathbf{z}_{-i} \neq \mathbf{z}(N)} \rho_L^i(\mathbf{z}_{-i}) V(\pi_i(\mathbf{z}_{-i}; y, y))\right)$ to $(1-\delta)c_L + \delta\left(\rho_L^i(\mathbf{z}(N))c_H + \sum_{\mathbf{z}_{-i} \neq \mathbf{z}(N)} \rho_L^i(\mathbf{z}_{-i}) V(\pi_i(\mathbf{z}_{-i}; y, y))\right)$. Hence, we have $\lim_{\lambda \to 1} \rho_L^i(\mathbf{z}(N)) = 1$, and it follows that the range of Q_L converges to the interval $((1-\delta)c_L + \delta v_L, (1-\delta)c_L + \delta c_H]$ as λ goes to 1. If $v_L < (1-\delta)c_L + \delta c_H$, which is the case when $\delta > \bar{\delta}$, then v_L is in this interval. This establishes the existence of the threshold $\bar{\lambda}_\delta$ such that the high trade equilibrium exists whenever $\delta > \bar{\delta}$ and $\lambda > \bar{\lambda}_\delta$.

Thus, we conclude that multiple equilibria exist when $\delta > \bar{\delta}$ and $\lambda > \bar{\lambda}_\delta$. ■

Appendix B - Proofs for Section 3

Proof of Lemma 1. Uniqueness of equilibrium follows from the fact that $Q_L^{i,fict} = (1-\delta)c_L + \delta v_H > v_L$ when $\sigma_i = 1$, and because $Q_L^{i,fict}$ is monotonically increasing in σ_i, and in ϕ_i when buyer mixing is part of an equilibrium. Hence, the unique equilibrium must feature no trade if $Q_L^{i,fict}(0,0) \geq v_L$. Finally, it is straightforward to check that $Q_L^{i,fict}(0,0) \geq v_L$ holds if and only if $\lambda \geq \bar{\lambda}$ and $\delta \geq \bar{\delta}_\lambda$, as given by (8) and (9). ■

For the proof of Theorem 1, it will be useful to reference the following lemma, which is straightforward to verify so the proof is omitted. Let $\pi_i(s; \sigma)$ denote the buyers' posterior posterior about seller i following a rejection, conditional on observing that the state is s. Then, for $s \in \{l, h\}$, we have:

$$\pi_i(s; \sigma) = \frac{\pi_\sigma \cdot \mathbb{P}(S = s | \theta_i = H)}{\pi_\sigma \cdot \mathbb{P}(S = s | \theta_i = H) + (1 - \pi_\sigma) \cdot \mathbb{P}(S = s | \theta_i = L)},$$

where as before π_σ is the interim belief.

Lemma 2 *Given a sequence $\{\sigma_N\}_{N=1}^\infty$ of trading probabilities along which information aggregates, we also have convergence of posteriors: $\pi_i(\mathbf{z}_{-i}; \sigma_N) \to^p \pi_i(S; \sigma_N)$ as $N \to \infty$.*

Proof of Theorem 1. *Non-aggregation result.* Suppose to the contrary that (C1) holds with strict inequality, but that information aggregation obtains. Recall that in equilibrium, for any N, we must have:

$$v_L = Q_L^i(\sigma_N, \phi_i) = (1-\delta)\, c_L + \delta \sum_{\mathbf{z}_{-i}} \rho_L^i(\mathbf{z}_{-i}) \cdot F_L(\pi_i(\mathbf{z}_{-i}; \sigma_N), \phi_i),$$

where

$$\begin{aligned}\sum_{\mathbf{z}_{-i}} \rho_L^i(\mathbf{z}_{-i}) \cdot F_L(\pi_i(\mathbf{z}_{-i}; \sigma_N), \phi_i) &= \sum_{s=l,h} \mathbb{P}(S = s|\theta_i = L) \sum_{\mathbf{z}_{-i}} \mathbb{P}(\mathbf{z}_{-i}|S = s) \cdot F_L(\pi_i(\mathbf{z}_{-i}; \sigma_N), \phi_i) \\ &> \lambda \cdot v_L + (1-\lambda) \cdot \sum_{\mathbf{z}_{-i}} \mathbb{P}(\mathbf{z}_{-i}|S = h) \cdot F_L(\pi_i(\mathbf{z}_{-i}; \sigma_N), \phi_i).\end{aligned}$$

Since by Lemma 2, $\pi_i(\mathbf{z}_{-i}; \sigma_N) \to^p \pi_i(h; \sigma_N)$ when the state is h, and because (C1) holding strictly implies that $\pi_i(h; \sigma_N) > \pi_i(h; 0) > \bar{\pi}$, we have that for a given $\epsilon > 0$, if N is large enough, then:

$$\sum_{\mathbf{z}_{-i}} \mathbb{P}(\mathbf{z}_{-i}|S = h) \cdot F_L(\pi_i(\mathbf{z}_{-i}; \sigma_N), \phi_i) > V(\pi_i(h; \sigma_N)) - \epsilon.$$

Therefore, we conclude that for sufficiently large N:

$$\begin{aligned}v_L = Q_L^i(\sigma_N, \phi_i) &> (1-\delta)\, c_L + \delta \cdot (\lambda \cdot v_L + (1-\lambda) \cdot V(\pi_i(h; \sigma_N))) - \delta \cdot (1-\lambda) \cdot \epsilon \\ &> (1-\delta)\, c_L + \delta \cdot (\lambda \cdot v_L + (1-\lambda) \cdot V(\pi_i(h; 0))) - \delta \cdot (1-\lambda) \cdot \epsilon.\end{aligned}$$

Since ϵ was arbitrary, it must be that:

$$v_L \geq (1-\delta)\, c_L + \delta \cdot (\lambda \cdot v_L + (1-\lambda) \cdot V(\pi_i(h; 0))),$$

which violates (C1) holding with strict inequality, a contradiction.

Aggregation result. If (C1) does not hold, then in the fictitious economy, the unique equilibrium trading probability in the first period must satisfy $\sigma^* > 0$. We next construct an equilibrium sequence $\{\sigma_N\}$ of the actual model such that the sequence is uniformly bounded away from zero, which clearly implies that information aggregates. First, consider a sequence $\{\widehat{\sigma}_N\}$, not necessarily an equilibrium one, such that $\widehat{\sigma}_N = \widehat{\sigma} \in (0, \sigma^*)$, i.e., this is a sequence of constant trading probabilities that are positive but are strictly below σ^*. Along such a sequence, clearly information aggregation holds and, by Lemma 2, $\pi_i(\mathbf{z}_{-i}, \widehat{\sigma}_N) \to^p \pi_i(S, \widehat{\sigma}_N)$. Therefore, combined with the fact that $\pi_i(\mathbf{z}_{-i}, \widehat{\sigma}_N) = \pi_i(\mathbf{z}_{-i}, \widehat{\sigma}) < \pi_i(\mathbf{z}_{-i}, \sigma^*)$, there exists an N^* such that for $N > N^*$, we have:

$$\mathbb{E}_L\left\{F_L\left(\pi_i\left(\mathbf{z}_{-i}, \widehat{\sigma}_N\right), \phi_i\right)\right\} < \mathbb{E}_L^{fict}\left\{F_L\left(\pi_i\left(S, \sigma^*\right)\right)\right\} = \frac{v_L - (1-\delta)\cdot c_L}{\delta},$$

where the last equalty holds since $\sigma^* > 0$ implies that, in the fictitious economy, the low type must be indifferent to trading at $t = 1$ and delaying trade to $t = 2$. The correspondence $\mathbb{E}_L\left\{F_L\left(\pi_i\left(\mathbf{z}_{-i}, \sigma\right), \cdot\right)\right\}$ is upper hemicontinuous in σ for each N, and has a maximal value of v_H that is strictly greater than $\mathbb{E}_L^{fict}\left\{F_L\left(\pi_i\left(S, \sigma^*\right)\right)\right\}$. Hence, for each $N > N^*$, we can find a σ_N such that $\sigma_N \geq \widehat{\sigma}_N > 0$ and $\mathbb{E}_L\left\{F_L\left(\pi_i\left(\mathbf{z}_{-i}, \sigma_N\right), \phi_i\right)\right\} = \frac{v_L-(1-\delta)\cdot c_L}{\delta}$. This delivers the desired equilibrium sequence $\{\sigma_N\}$ along which information aggregates. ■

Proof of Theorem 2. We first establish the conditions under which only aggregating equilibria exist. Suppose that $\lambda < \bar{\lambda}$ and assume to the contrary that information aggregation fails along some sequence of equilibria, and pick a subsequence of equilibria with $\sigma_N \to 0$ as N goes to ∞ (See Lemma 4 for the existence of such a subsequence). But note that for each N, we have $\pi_i(\mathbf{z}_{-i}, \sigma_N) \leq \pi_i(h, \sigma_N)$, i.e., the posterior beliefs must be weakly lower than if the state were revealed to be high. Since $\pi_i(h, \sigma_N)$ is continuous in σ_N, and since $\lambda < \bar{\lambda}$ implies that $\pi_i(h, 0) < \bar{\pi}$, it follows that for N large enough all posterior beliefs are strictly below $\bar{\pi}$. But then for N large, $Q_L^i < v_L$ and therefore $\sigma_N = 1$, contradicting Property 2.

Next, consider $\hat{\delta}$ defined by $v_L = (1-\hat{\delta})c_L + \hat{\delta}V(\pi)$, and assume that $\delta < \hat{\delta}$ (Note that Assumption 2 can still be satisfied). Suppose to the contrary that information aggregation fails along a sequence of equilibria, and again pick a subsequence of equilibria with $\sigma_N \to 0$ as N goes to ∞. By continuity, we must also have that $\pi_{\sigma_N} \to \pi$ along this subsequence. But, note that for each N along this subsequence, it must be that:

$$\begin{aligned} v_L = Q_L^i(\sigma_N, \phi_i) &= (1-\delta)c_L + \delta\mathbb{E}_L\{F_L(\pi_i(\mathbf{z}_{-i}, \sigma_N), \phi_i)\} \\ &\leq (1-\delta)c_L + \delta\mathbb{E}_L\{V(\pi_i(\mathbf{z}_{-i}, \sigma_N))\} \\ &\leq (1-\delta)c_L + \delta V(\pi_{\sigma_N}). \end{aligned}$$

where the first inequality follows immediately from (5) and the second from the fact that V is linear function and $\pi_i(\mathbf{z}_{-i}, \sigma_N)$ is a supermartingale conditional on $\theta_i = L$. Because $\delta < \hat{\delta}$ and $V(\pi_{\sigma_N}) \to V(\pi)$, the last expression is strictly lower than v_L for N large enough, a contradiction.

Next, we establish the conditions for the coexistence of aggregating and non-aggregating equilibria. To do so, we first show that if $\lambda > \bar{\lambda}$, there exists a $\delta_2(\lambda) < 1$ such that non-aggregating equilibria exist if $\delta > \delta_2(\lambda)$. Second, we show that for λ large enough $\delta_2(\lambda) < \bar{\delta}_\lambda$.

Therefore both non-aggregating and aggregating equilibria exist if $\delta \in (\delta_2(\lambda), \bar{\delta}_\lambda)$, since (C1) is violated (see Theorem 1).

Consider a candidate sequence of equilibria with trading probabilities $\{\sigma_N\}$, such that $\sigma_N = \kappa_N \cdot N^{-1}$ and:

$$\pi_i \left(\mathbf{z}(0); \kappa_N \cdot N^{-1}\right) = \bar{\pi} \tag{11}$$

Solving (11) for κ_N and taking the limit as $N \to \infty$ gives $\kappa_N \to \kappa$ where

$$\kappa \equiv \frac{1}{\lambda - \frac{(1-\lambda)(1-\pi)}{\pi}} \cdot \log \left(\frac{\lambda - \left(\frac{1-\bar{\pi}}{\bar{\pi}} \cdot \frac{\pi}{1-\pi}\right) \cdot \frac{(1-\lambda)(1-\pi)}{\pi}}{\left(\frac{1-\bar{\pi}}{\bar{\pi}} \cdot \frac{\pi}{1-\pi}\right) \cdot \left(1 - \frac{(1-\lambda)(1-\pi)}{\pi}\right) - (1-\lambda)} \right) \in (0, \infty). \tag{12}$$

Seller i expects to receive an offer of v_L in all events other than $\mathbf{z}(0)$ and an expected offer $\phi_i c_H + (1-\phi_i) v_L$ for some $\phi_i \in [0,1]$ in the event $\mathbf{z}(0)$. Therefore, the sequence of trading probabilities defined above constitutes an equilibrium if δ is sufficiently high and the probability of the event $\mathbf{z}(0)$ conditional on the seller's type being low is bounded away from zero. To establish the latter, note that:

$$\begin{aligned}
\mathbb{P}\left(\mathbf{z}(0) | \theta_i = L\right) &= \sum_{s=l,h} \mathbb{P}\left(S = s | \theta_i = L\right) \cdot \left(1 - \sigma_N \cdot \mathbb{P}\left(\theta_i = L | S = s\right)\right)^N \\
&= \sum_{s=l,h} \mathbb{P}\left(S = s | \theta_i = L\right) \cdot \left(1 - \kappa_N \cdot N^{-1} \cdot \mathbb{P}\left(\theta_i = L | S = s\right)\right)^N \\
&\to \sum_{s=l,h} \mathbb{P}\left(S = s | \theta_i = L\right) \cdot e^{-\kappa \cdot \mathbb{P}(\theta_i = L | S = s)} > 0,
\end{aligned}$$

where the last limit follows from Lemma 3. In these equilibria, information fails to aggregate because as a result $\mathbb{P}\left(\mathbf{z}(0) | S = s\right)$ is bounded away from zero in both states of nature (see Lemma 5). Thus, for each $\lambda > \bar{\lambda}$, we have established the existence of a $\delta_2(\lambda) < 1$ such that non-aggregating equilibria exist whenever $\delta > \delta_2(\lambda)$.

Finally, from (12) we have that:

$$\lim_{\lambda \to 1} \sum_{s=l,h} \mathbb{P}\left(S = s | \theta_i = L\right) \cdot e^{-\kappa \cdot \mathbb{P}(\theta_i = L | S = s)} = \frac{1-\bar{\pi}}{\bar{\pi}} \cdot \frac{\pi}{1-\pi} \in (0,1),$$

and hence $\lim_{\lambda \to 1} \delta_2(\lambda) < 1$. Letting $\widehat{\delta} = \lim_{\lambda \to 1} \delta_2(\lambda)$ and noting that $\lim_{\lambda \to 1} \bar{\delta}_\lambda = 1$ implies the result. ■

Lemma 3 *Let $\{\alpha_x\}$ be any non-negative sequence of real numbers such that $\alpha_x \to \alpha$ as $x \to \infty$ where $\alpha \in (0,1)$. Then $\left(\frac{x - \alpha_x}{x}\right)^x \to e^{-\alpha}$ as $x \to \infty$.*

Proof. Assume that for any $\gamma \in (0,1)$, $\left(\frac{x-\gamma}{x}\right)^x \to e^{-\gamma}$ as $x \to \infty$. Then, given $\epsilon > 0$ so that $\epsilon < \alpha < 1-\epsilon$, if x is large enough then $|\alpha_x - \alpha| < \epsilon$, $\left(\frac{x-\alpha-\epsilon}{x}\right)^x \geq e^{-\alpha-\epsilon} - \epsilon$, and $\left(\frac{x-\alpha+\epsilon}{x}\right)^x \leq e^{-\alpha+\epsilon} + \epsilon$. This in turn implies that:

$$e^{-\alpha-\epsilon} - \epsilon \leq \left(\frac{x-\alpha-\epsilon}{x}\right)^x \leq \left(\frac{x-\alpha_x}{x}\right)^x \leq \left(\frac{x-\alpha+\epsilon}{x}\right)^x \leq e^{-\alpha+\epsilon} + \epsilon.$$

Since ϵ is arbitrary, we conclude that $\left(\frac{x-\alpha_x}{x}\right)^x \to e^{-\alpha}$ as $x \to \infty$. Next, we prove the supposition that for any $\gamma \in (0,1)$, $\left(\frac{x-\gamma}{x}\right)^x \to e^{-\gamma}$ as $x \to \infty$. Note that $\left(\frac{x-\gamma}{x}\right)^x = e^{x \cdot \log\left(\frac{x-\gamma}{x}\right)}$ and by L'Hospital's rule:

$$\lim_{x\to\infty} x \cdot \log\left(\frac{x-\gamma}{x}\right) = \lim_{x\to\infty} \frac{\log\left(\frac{x-\gamma}{x}\right)}{x^{-1}} = -\lim_{x\to\infty} \frac{\gamma \cdot x}{x-\gamma} = -\gamma.$$

By continuity, $\lim_{x\to\infty} e^{x\cdot\log\left(\frac{x-\gamma}{x}\right)} = e^{-\gamma}$. ■

Lemma 4 *Suppose that there is a sequence of equilibria $\{\sigma_N\}$ along which information aggregation fails. Then there exist real numbers $0 < \underline{\kappa} < \bar{\kappa} < \infty$ and a subsequence $\{\sigma_{N_m}\}$ such that $\underline{\kappa} < \sigma_{N_m} N_m < \bar{\kappa}$ for all m.*

Proof. Suppose for contradiction that $\lim_{N\to\infty} \sigma_N N = +\infty$. Let X_i denote the indicator that takes value of 1 if seller i has traded in the first period. Define $Y_N = N^{-1} \cdot \sum_{i=1}^N X_i$ be the fraction of sellers who have traded in the first period, and note that conditional on the state being s, Y_N has a mean $p_{s,N}$ and variance $N^{-1} \cdot p_{s,N} \cdot (1-p_{s,N})$, where recall that $p_{s,N} = \sigma_N \cdot \mathbb{P}(\theta_i = L|S=s)$. Since $p_{l,N} > p_{h,N}$,

$$\mathbb{P}\left(Y_N \geq \frac{p_{h,N}+p_{l,N}}{2} | S=h\right) = \mathbb{P}\left(Y_N - p_{h,N} \geq \frac{p_{l,N}-p_{h,N}}{2} | S=h\right)$$
$$\leq \mathbb{P}\left((Y_N - p_{h,N})^2 \geq \left(\frac{p_{l,N}-p_{h,N}}{2}\right)^2 | S=h\right)$$

And by Chebyshev's inequality:

$$\mathbb{P}\left((Y_N - p_{h,N})^2 \geq \left(\frac{p_{l,N}-p_{h,N}}{2}\right)^2 | S=h\right) \leq \frac{\mathbb{E}\left\{(Y_N - p_{h,N})^2 | S=h\right\}}{\left(\frac{p_{l,N}-p_{h,N}}{2}\right)^2}$$
$$= \frac{N^{-1} \cdot p_{h,N} \cdot (1-p_{h,N})}{\left(\frac{p_{l,N}-p_{h,N}}{2}\right)^2}$$
$$= 4 \cdot \frac{\sigma_N \cdot \mathbb{P}(\theta_i = L|S=h) - \sigma_N^2 \cdot \mathbb{P}(\theta_i = L|S=h)^2}{N \cdot \sigma_N^2 \cdot (\mathbb{P}(\theta_i = L|S=l) - \mathbb{P}(\theta_i = L|S=h))^2}$$

which by our assumption tends to 0 as $N \to \infty$. By a similar reasoning, we have that:

$$\begin{aligned}
\mathbb{P}\left(Y_N < \frac{p_{h,N} + p_{l,N}}{2} | S = l\right) &= \mathbb{P}\left(p_{l,N} - Y_N > \frac{p_{l,N} - p_{h,N}}{2} | S = l\right) \\
&\leq \mathbb{P}\left((p_{l,N} - Y_N)^2 > \left(\frac{p_{l,N} - p_{h,N}}{2}\right)^2 | S = l\right) \\
&\leq \frac{\mathbb{E}\left\{(Y_N - p_{l,N})^2 | S = l\right\}}{\left(\frac{p_{l,N} - p_{h,N}}{2}\right)^2} \\
&= \frac{N^{-1} \cdot p_{l,N} \cdot (1 - p_{l,N})}{\left(\frac{p_{l,N} - p_{h,N}}{2}\right)^2} \\
&= 4 \cdot \frac{\sigma_N \cdot \mathbb{P}(\theta_i = L|S = l) - \sigma_N^2 \cdot \mathbb{P}(\theta_i = L|S = l)^2}{N \cdot \sigma_N^2 \cdot (\mathbb{P}(\theta_i = L|S = l) - \mathbb{P}(\theta_i = L|S = h))^2}
\end{aligned}$$

which again tends to 0 as $N \to \infty$. Combining these two observations, we conclude that information about the state must aggregate along the sequence $\{\sigma_N\}$, a contradiction.

Next, suppose for contradiction that $\lim_{N \to \infty} \sigma_N N = 0$. Then, given any finite $\epsilon > 0$ and N large enough, we have:

$$(1 - \sigma_N \cdot \mathbb{P}(\theta_i = L|S = s))^N = \left(\frac{N - \sigma_N \cdot N \cdot \mathbb{P}(\theta_i = L|S = s)}{N}\right)^N \geq \left(\frac{N - \epsilon}{N}\right)^N$$

for $s \in \{l, h\}$, where the last expression converges to $e^{-\epsilon}$ by Lemma 3. Since ϵ is arbitrary, $(1 - \sigma_N \cdot \mathbb{P}(\theta_i = L|S = s))^N$ goes to 1 as $N \to \infty$. Hence, we have that for $\theta \in \{L, H\}$:

$$\mathbb{P}(Y_N = 0|\theta_i = \theta) = \sum_{s=l,h} \mathbb{P}(S = s|\theta_i = \theta) \cdot (1 - \sigma_N \cdot \mathbb{P}(\theta_i = L|S = s))^N \to 1.$$

Now, consider the posterior belief about the seller conditional on event that no seller has traded. For any N, since the low type must expect offers above v_L with positive probability and since $\mathbf{z}(0)$ is the best possible news, it must be that:

$$\begin{aligned}
&\pi_i(\mathbf{z}(0), \sigma_N) \geq \bar{\pi} \\
&\qquad \iff \frac{\pi_{\sigma_N} \cdot \mathbb{P}(Y_N = 0|\theta_i = H)}{\pi_{\sigma_N} \cdot \mathbb{P}(Y_N = 0|\theta_i = H) + (1 - \pi_{\sigma_N}) \cdot \mathbb{P}(Y_N = 0|\theta_i = L)} \geq \bar{\pi}.
\end{aligned}$$

But note that, since $\sigma_N \to 0$ and π_{σ_N} is continuous, the left-hand side converges to $\pi < \bar{\pi}$, a contradiction. ■

Lemma 5 *Consider a sequence of equilibria with trading probabilities* $\{\sigma_N\}$ *such that* $\sigma_N N < \bar{\kappa}$ *for some* $\bar{\kappa} < \infty$*. Then* $\mathbb{P}(Y_N = 0|S = s)$ *is bounded away from zero, uniformly over* N*, for* $s \in \{l, h\}$*.*

Proof. We have that $\mathbb{P}(Y_N = 0|S = s) = (1 - p_{s,N})^N$ for $s \in \{l, h\}$. By assumption, $p_{s,N} \leq N^{-1} \cdot \bar{\kappa} \cdot \mathbb{P}(\theta_i = L|S = s)$. Therefore,

$$\mathbb{P}(Y_N = 0|S = s) \geq \left(1 - N^{-1} \cdot \bar{\kappa} \cdot \mathbb{P}(\theta_i = L|S = s)\right)^N$$

and by Lemma 3,

$$\lim_{N\to\infty} \left(1 - N^{-1} \cdot \bar{\kappa} \cdot \mathbb{P}(\theta_i = L|S = s)\right)^N = e^{-\bar{\kappa}\cdot\mathbb{P}(\theta_i = L|S=s)} > 0.$$

■

Appendix C - Proofs for Section 4

Proof of Theorem 3. We prove the result in two steps. First, we prove that if (C1) does not hold, there exist (sequences of) equilibria that aggregate information and, if δ is large, these equilibria coexist with others that do not aggregate information. We do this by considering a class of equilibria that we label class-P. The key property of equilibria in class-P is that once the posterior π_t^i about the seller is weakly above $\bar{\pi}$, the market stops revealing any information until the last period. Let $\pi_{i,t}$ denote the buyers' belief about the seller at time t. Then,

Definition 3 *An equilibrium is said to belong to class-P if it has the following features:*

1. *Equilibrium play at* $t < T$ *is given by:*
   - *(i) If* $\pi_{i,t} < \bar{\pi}$*, then the bid is* v_L*, the low type accepts w.p.* $\sigma_t \in (0, 1)$ *whereas the high type rejects it.*
   - *(ii) If* $\pi_{i,t} > \bar{\pi}$*, then the bid is* $V(\pi_{i,t})$ *and both types accept it.*
   - *(iii) If* $\pi_{i,t} = \bar{\pi}$*, then the bid is* $V(\pi_{i,t})$ *w.p.* ϕ_t *(and both types accept it) and is* v_L *w.p.* $1 - \phi_t$ *(and both types reject it).*
2. *Equilibrium play at* $t = T$ *is given by:*
   - *(i) If* $\pi_{i,T} < \bar{\pi}$*, then the bid is* v_L*, the low type accepts w.p.1 whereas the high type rejects it.*
   - *(ii) If* $\pi_{i,T} > \bar{\pi}$*, then the bid is* $V(\pi_{i,t})$ *and both types accept it.*

(iii) If $\pi_{i,T} = \bar{\pi}$, then the bid is $V(\pi_{i,T})$ w.p. ϕ_T (and both types accept it) and is v_L w.p. $1 - \phi_T$ (and only low type accepts).

Second, we establish the non-existence of (sequences of) aggregating equilibria when (C1) holds with strict inequality. We use techniques that are similar to those in the proof of Theorem 1, with the exception that, in the longer horizon economy, in order to study the aggregation properties at any given date, we need to keep track of the dynamic incentives to trade since beliefs about the state evolve over time.

When C1 does not hold

The following lemma states that the class-P of equilibria is non-empty and that it contains equilibria in which information about the state aggregates.

Lemma 6 *The class-P is non-empty. If either condition (1) or (2) is reversed, then there exists a sequence of equilibria in class-P along which information about the state aggregates in the first period.*

Proof. Let $\mathbf{z}^t$ denote the state variable that summarizes the history of trade up to period t, which we will define recursively as $\mathbf{z}^{t+1} = (\mathbf{z}_{t+1}, \mathbf{z}^t)$ where $\mathbf{z}_t$ summarizes the trading behavior in period $t-1$, with $\mathbf{z}_1 \equiv \emptyset$ signifying that at $t = 1$ the information set is given by prior beliefs. The beliefs about the seller and about the state following history $\mathbf{z}^t$ are denoted by $\pi_{i,t}(\mathbf{z}^t) = \mathbb{P}(\theta_i = H|\mathbf{z}^t)$ and $\lambda_{\theta,t}(\mathbf{z}^t) = \mathbb{P}(S = l|\mathbf{z}^t, \theta_i = \theta)$ respectively. Note that these beliefs are computed using Bayes' rule and the equilibrium trading strategies of the low and the high type from periods 1 through t (i.e., $\{\sigma_s^\theta\}_{s=1,\theta=L,H}^{t-1}$); for brevity we omit the dependence on the latter when it is clear. Let $\mathbf{z}_{-i}^t$ denote the history of trade of sellers other than i up to period t and, analogously to equation (2), let $\rho_{\theta,t}\left(\mathbf{z}_{-i,t+1}|\mathbf{z}_{-i}^t\right) = \mathbb{P}\left(\mathbf{z}_{-i,t+1}|\theta_i = \theta, \mathbf{z}_{-i}^t\right)$ be the conditional distribution over other sellers' trading behavior (or news) in $t+1$, given that the seller i is of type θ and given that the history of other sellers' trades up to date t is $\mathbf{z}_{-i}^t$ (as well as that seller i has not traded until period t).

Consider the candidate equilibria in Definition 3. We begin with period $t = T$. That the proposed play at $t = T$ is part of an equilibrium follows from the same reasoning as that for Property 1. From (2), the period-T payoffs to the low and the high type are (just as in equations (4) and (5) of Section 2) given by:

$$F_{H,T}(\pi_{i,T}, \phi_{i,T}) = \max\{c_H, V(\pi_{i,T})\} \tag{13}$$

and

$$F_{L,T}\left(\pi_{i,T},\phi_{i,T}\right)=\begin{cases}V\left(\pi_{i,T}\right) & \text{if}\pi_{i,T}>\bar{\pi}\\ \phi_{i,T}c_H+\left(1-\phi_{i,T}\right)v_L & \text{if}\pi_{i,T}=\bar{\pi}\\ v_L & \text{if}\pi_{i,T}<\bar{\pi}\end{cases} \tag{14}$$

We can use the payoffs in (13) and (14) to construct the expected continuation value $Q^i_{\theta,T-1}$ of the θ type seller from rejecting a bid v_L in $t=T-1$. This continuation value depends on the posterior $\pi_{i,T-1}$. In particular, if $\pi_{i,T-1}>\bar{\pi}$, from (1) the bid is $V\left(\pi_{i,T-1}\right)$ and both sellers' accept. In this case, we need to specify the off-equilibrium belief in case a seller deviates and rejects this bid. Setting $\pi_{i,T}$ so that $v_L=Q^i_{L,T-1}=(1-\delta)\,c_L+\delta\left(\pi_{i,T}v_H+\left(1-\pi_{i,T}\right)v_L\right)$ suffices to deter such deviations. If $\pi_{i,T-1}=\bar{\pi}$, the bid is $c_H=V\left(\bar{\pi}\right)$ w.p. ϕ_{T-1} (and both types accept) and the bid is v_L w.p. $1-\phi_{T-1}$ and both types reject. Setting ϕ_T to satisfy $v_L=Q^i_{L,T-1}=(1-\delta)\,c_L+\delta\left(\phi_Tc_H+\left(1-\phi_T\right)v_L\right)$ suffices to ensure that the low type does not want to accept bid v_L at $T-1$; clearly, the high type rejects v_L. Finally, if $\pi_{i,T-1}<\bar{\pi}$, the bid is v_L and only the low type accepts w.p. σ_{T-1}. In this case, the low type's expected continuation value upon rejection is:

$$Q^i_{L,T-1}=(1-\delta)\,c_L+\delta\sum_{\mathbf{z}_{-i,T}|\mathbf{z}^{T-1}_{-i}}\rho_{L,T-1}\left(\mathbf{z}_{-i,T}|\mathbf{z}^{T-1}_{-i}\right)\cdot F_{L,T}\left(\pi_{i,t}\left(\mathbf{z}^{T}_{-i}\right),\phi_T\right), \tag{15}$$

which is the period-$T-1$ analogue of equation (7). For the low type to accept bid v_L with interior probability, we must have $Q^i_{L,T-1}=v_L$. By arguments similar to those in the two period economy, we can show that there exists a pair $\left(\sigma_{T-1},\phi_T\right)$ that solves equation (15): (a) $Q^i_{L,T-1}$ is upper hemicontinuous in σ_{T-1}, (b) $Q^i_{L,T-1}$ is lower than v_L when evaluated at $\sigma_{T-1}=0$, and (c) $Q^i_{L,T-1}$ is greater than v_L when evaluated at $\sigma_{T-1}=1$. We have thus verified that the proposed play at $T-1$ is part of an equilibrium.

Using similar reasoning and working back, we can show that for any $t\in\{2,T-1\}$: (a) $F_{\theta,t}$ is given by equations (13) and (14) but where we replace $\left(\pi_{i,T},\phi_T\right)$ with $\left(\pi_{i,t},\phi_t\right)$, and (b) the low type's expected continuation value from rejecting the bid at $t-1$ satisfies $Q^i_{L,t-1}=v_L$: (i) if $\pi_{i,t-1}>\bar{\pi}$, the off-equilibrium belief is as before $\pi_{i,t}=\pi_{i,T}$, (ii) if $\pi_{i,t-1}=\bar{\pi}$, then ϕ_t satisfies $v_L=Q^i_{L,t-1}=(1-\delta)\,c_L+\delta\left(\phi_tc_H+\left(1-\phi_t\right)v_L\right)$ and, finally, (iii) if $\pi_{i,t-1}<\bar{\pi}$, the pair $\left(\sigma_{t-1},\phi_t\right)$ satisfies:

$$v_L=Q^i_{L,t-1}=(1-\delta)\,c_L+\delta\sum_{\mathbf{z}_{-i,t}|\mathbf{z}^{t-1}_{-i}}\rho_{L,t-1}\left(\mathbf{z}_{-i,t}|\mathbf{z}^{t-1}_{-i}\right)\cdot F_{L,t}\left(\pi_{i,t}\left(\mathbf{z}^{t-1}_{-i}\right),\phi_t\right). \tag{16}$$

At $t=1$, the initial history is $\mathbf{z}^1_{-i}=\emptyset$ and thus the belief about the seller is the prior $\pi_{i,1}=\pi<\bar{\pi}$, where the latter inequality holds by Assumption (1). This establishes the fact that class-P is non-empty. We next show that there is a sequence of equilibria within this class that aggregates information.

Since the beliefs $\pi_{i,2}\left(\mathbf{z}_{-i}^{2}\right)$ and $\rho_{\theta,1}\left(\mathbf{z}_{-i,2}|\mathbf{z}_{-i}^{1}\right)$ are as in equations (2) and (3) in Section 2, the expected continuation value of the low type seller from rejecting a bid v_L in the first period is exactly as in equation (6). Therefore, the trading behavior and the information aggregation properties of these equilibria in the first period are as in the first period of the two period economy, i.e., they are independent of trading horizon T. Thus, since when either condition (1) or (2) is reversed, in the two period economy, there exists a sequence of equilibria which aggregates information in the first period, this must also be true of the T period economy. ■

The following lemma states that if in addition δ is sufficiently large, then class-P also contains equilibria in which aggregation fails.

Lemma 7 *If either condition (1) or (2) is reversed, then there exists a sequence of equilibria in class-P along which information aggregation fails for any $t < T$.*

Proof. In Proposition 2, we showed that in the two period economy, when either condition is reversed and δ is sufficiently large, there exists a sequence of equilibria in which aggregation fails. From the class-P equilibria in Definition 3, we can conclude that if information does not aggregate in the first period of the two period economy, then it does not aggregate in the first period of the T period economy. In the T horizon economy, however, we must also show that information also does not aggregate in $t > 1$. But this is straightforward, since the equilibrium condition $v_L = Q_{L,t-1}^{i}$ implies that there must be a positive probability that the posterior in the second period is above $\bar{\pi}$: otherwise, from (1), the bid would be v_L in the second period implying that $Q_{L,t-1}^{i} < v_L$, a contradiction. But then, in equilibria in class-P, once the posterior is weakly above $\bar{\pi}$, the market stops revealing any additional information for any $t < T$. This establishes the result. ■

When C1 holds with strict inequality

When the buyers' belief about the seller $\pi_{i,t}$ is below $\bar{\pi}$, just as in class-P equilibria, the only possible equilibrium play in period t must involve a bid of v_L, the low type accepting it with some probability and the high type rejecting it.

On the other hand, when the belief is $\pi_{i,t} \geq \bar{\pi}$, the class-P equilibria featured a pooling bid that was accepted by both types, thus precluding any further information revelation. There is, however, another possibility. There can potentially exist equilibria in which (despite the fact that $\pi_{i,t} \geq \bar{\pi}$) the bid is still v_L and only the low type accepts it with some probability. For these type of equilibria to exist, it must be that the high type expects sufficiently good information to arrive from other sellers in the next period (i.e., there must be further trading opportunities and information arrival from other sellers); otherwise, there would a profitable deviation for buyers, who could offer slightly less than the pooling bid in the current period, attract both seller types and make positive profits. Together with the equilibria in class-P, these exhaust the set of all possible symmetric equilibria. Rather than prove the conditions under which these type of equilibria can arise, we show that even if such equilibrium play were possible, conditions (1) and (2) still guarantee information non-aggregation for any $t < T$.

Our strategy for the proof is the following. As in the proof of Theorem 1, we study the low type's incentive to delay trade from period t to period $t+1$, when she anticipates information about the aggregate state to be (essentially) revealed in period $t+1$. As in Section 3, we compare the trading incentives in our economy with those of the 'fictitious' two period economy in which the state is revealed in the second period w.p.1. There are, however, two differences. First, since period $t+1$ may not be the terminal date, the continuation value of the low type seller at t will depend on more than one period ahead payoffs. Second, conditions (1) and (2) use the prior beliefs about the seller and the state, but in the longer horizon economy these beliefs evolve as a function of past trading behavior. Nevertheless, we show that conditions (1) and (2) are sufficient to understand the information aggregation properties of equilibria in any period but the last. We prove this result by contradiction.

Consider any period $t < T$ and suppose that information has not aggregated until that period, but that it aggregates at t. This must be true if information is to aggregate in any period t. Clearly, there must still be sellers present at t to trade and reveal information; in fact, the number of sellers at this date must grow with N in order for aggregation to be possible. Furthermore, if the bid were pooling at this date and both seller types were to accept it with probability bounded away from zero as N grows large, then there would also be a positive probability (bounded away from zero) that no additional information gets revealed, which would contradict aggregation at t. Thus, it must be the case that with probability going to 1 as N grows large, the equilibrium play at t has the following feature: the bid is v_L, the low type accepts the bid w.p. $\sigma_t \in (0,1)$ (we can again rule out $\sigma_t \in \{0,1\}$), and the high type rejects. In these equilibria, it must be the case that v_L is equal to the expectation continuation value $Q^i_{L,t}$ of the low type seller upon rejecting the bid v_L at t. In what follows, our strategy is to show that information aggregation at t implies that the probability that the continuation value $Q^i_{L,t}$ strictly exceeds v_L remains bounded away from zero. But, this would lead to a contradiction, since in such histories for large but finite N, the sellers would strictly prefer not to trade at t, contradicting equilibrium requirement that σ_t be positive for all N.

The next lemma provides a lower bound on the low type's continuation value as a function of low type's beliefs about the state of nature.

Lemma 8 *Suppose that information has not aggregated before period $t < T$, but that it aggregates in period t. Then the probability that the low type's expected continuation value $Q^i_{L,t}$ satisfies*

$$Q^i_{L,t} \geq \bar{Q}(\lambda_{L,t}) \equiv (1-\delta)\, c_L + \delta \left(\lambda_{L,t} \cdot v_L + (1-\lambda_{L,t}) \cdot V\left(1 - \frac{(1-\lambda)\,(1-\pi)}{\pi}\right)\right) \tag{17}$$

goes to 1 as N goes to ∞, where $\lambda_{L,t}$ is the low type's period t belief that the state is low.

Proof. We show in Lemma 10 below that, when the state is high, information aggregation at t implies that if the seller were to reject bid v_L at t, the probability that the bid in $t+1$ is pooling goes to 1 as N becomes large. The reason is that, conditional on the state being high, the probability that the seller is a high type drifts upwards over time. Because by condition (1) this probability exceeds $\bar{\pi}$ in the first period, it must exceed it for all t. In addition, since information aggregation at t implies that there are (essentially) no additional news arriving from other sellers in periods after t, equilibria with delay are no longer possible at $t+1$. But, if the equilibrium were pooling in the high state, then by the convergence of posteriors the bid would converge to $V(\pi_{i,t+1}(h))$, which is greater than $V\left(1 - \frac{(1-\lambda)(1-\pi)}{\pi}\right)$, where $\pi_{i,t+1}(h)$ is the posterior about the seller conditional on the state being revealed to be high in $t+1$. Proceeding along the same steps as in the proof of Theorem 1 and replacing the bids in the low state by the lowest possible bid v_L, we have the desired inequality. ■

Since $\lambda_{L,1} = \lambda$, condition (2) states that $\bar{Q}(\lambda_{L,1}) > v_L$. In the proof of Theorem 1, we showed that this implies that if information were to aggregate in the first period, then trade in the first period must have collapsed to zero for some large but finite N, which led to a contradiction. We use a similar argument in the longer horizon economy. While we cannot guarantee that $\bar{Q}(\lambda_{L,t}) > v_L$ for all $t < T$, we do not need to: we only need to find histories in which this inequality continues to hold. The following lemma shows that the probability that the lower bound $\bar{Q}(\lambda_{L,t})$ in (17) exceeds v_L is bounded away from zero.

Lemma 9 *Suppose that information has not aggregated before period $t < T$, but that it aggregates in period t. Then the probability of the event that $v_L < \bar{Q}(\lambda_{L,t})$ is bounded away from zero, uniformly over N.*

Proof. By condition (2), we have $v_L < \bar{Q}(\lambda_{L,1})$. Thus, information cannot aggregate in the first period. Lemma 5 shows that non-aggregation implies that the probability of the event that no seller trades before $t = 2$ remains bounded away from zero, uniformly over N in both states of nature. Since this event is good news about the state, we have $\lambda_{L,2} < \lambda_{L,1}$ and thus $v_L < \bar{Q}(\lambda_{L,2})$. But then information cannot aggregate in the second period either, and again by an argument similar to that in Lemma 5 the probability of the event that no seller trades before $t = 3$ must remain bounded away from zero, uniformly over N in both states of nature. Again, this event is good news about the state and we have $\lambda_{L,3} < \lambda_{L,2}$. Using this argument repeatedly, we can construct histories that occur with probability bounded away from zero, in which uncertainty about the aggregate state does not vanish for any $t < T$. ■

Thus, combining Lemmas 8 and 9, we can use arguments similar to those in the proof of Theorem 1 to show that if conditions (1) and (2) hold, then non-aggregation until period t but aggregation at t implies that trade at t must collapse to zero for some large but finite N, contradicting equilibrium requirement that trade be positive for any N. Hence, information aggregation must fail for any $t < T$, establishing the part of Theorem 3 that assumes conditions (1) and (2) hold. ■

The following lemma was used in the proof of Lemma 8.

Lemma 10 *Suppose that the state is h. Suppose that information has not aggregated before period $t < T$, but that it aggregates at t. Then the probability that after period t the bid is pooling and both types accept it goes to 1 as N goes to ∞.*

Proof. We prove this by backwards induction. Assume that the state is high. Let us start in the final period and look at histories in which at least one seller arrives to T. If these histories had zero probability, it would imply that pooling occured w.p.1 before T and we can simply consider the last period such that at least one seller is still present with positive probability. By reasoning analogous to that for Property 2, there is a unique equilibrium at this date, and it is given by (2) as in Definition 3. Next, let us go to period $t = T - 1$, and assume that information has aggregated before this period. We argue next that, if it were the case that the belief about the seller were $\pi_{i,T-1} > \bar{\pi}$, then the bid at $T-1$ must (essentially) be $V(\pi_{i,T-1})$. To this end, consider histories z^{T-1} in which we have $\pi_{i,T-1}\left(\mathrm{z}^{T-1}\right) > \bar{\pi}$, the bid is v_L and only the low type accepts with some probability. The low type's continuation value conditional on rejecting this bid is given by equation (15). We can rule out $\sigma_{T-1} = 1$ as before. We can rule out $\sigma_{T-1} = 0$, as then the belief would remain unchanged from $T-1$ to T and there would be a profitable deviation for buyers at $T-1$: a buyer could offer a bid slightly smaller than pooling, attract both types and make positive profits. Hence, we must have that $Q^i_{L,T-1} = v_L$ and $\sigma_{T-1} \in (0,1)$. Next, note that the fact that information has aggregated before $T-1$ implies that for large N, the posterior about the seller must be close (in probability) to the posterior about the seller conditional on the state being high, $\pi_{i,T}(h)$. But the latter posterior only drifts upwards over time as the low types trade with higher probability that the high type at any t. Since by condition (1) $\pi_{i,1}(h) > \bar{\pi}$, we must have that $V(\pi_{i,T}(h)) > c_H$ and therefore by the convergence of posteriors:

$$Q^i_{L,T-1} = (1-\delta)\, c_L + \delta \cdot V\left(\pi_{i,T}\left(h\right)\right) + \epsilon \tag{18}$$

where ϵ can be made small by making N large. Since $V\left(\pi_{i,T}\left(h\right)\right) > c_H$, by Assumption 2, the low type would reject the bid v_L at $T-1$ when N is sufficiently large but finite, a contradiction to the fact that for any N, any equilibrium with delay must feature $\sigma_{T-1} \in (0,1)$. But also note that because the posteriors conditional on the high state drift upwards, $\pi_{i,T-1}\left(h\right) \geq \pi_{i,1}\left(h\right) > \bar{\pi}$, information aggregation before $T-1$ also implies that, the probability of histories in which $\pi_{i,T-1} > \bar{\pi}$ goes to one as N becomes large. Thus, we conclude that if information has aggregated before $T-1$ and if the state is high, then the probability that the bid is pooling at $T-1$ goes to 1 with N. Using the above argument repeatedly, we can conclude that if the state were high and information were to aggregate before t, then the probability that the bid after t would be pooling would go to 1 as N gets large. ■

Appendix D - Proofs for Section 5

Proof of Proposition 2. If trades are only reported at the end of the second period, then the offers made at the beginning of the second period cannot be conditioned on the trading activity of any other seller. Hence, the equilibrium trading strategies (σ, ϕ) are the same as in a model with $N = 1$, and given by the solution to:

$$v_L = Q^i_L(\sigma_i, \phi_i) = (1-\delta)c_L + \delta F_L(\pi_{\sigma_i}, \phi_i), \tag{19}$$

where we omit the argument σ_i from the continuation value since Q^i_L does not depent on it. Notice, because there is no information received before offers are made in the second period, the posterior belief in the second period is simply given by the interim belief π_{σ_i} that is given in equation (1). Since Q^i_L is monotonic in σ_i, and in ϕ_i when buyer mixing is part of equilibrium, the equilibrium must be unique, with $\sigma_i = \sigma > 0$ by Property 3. Because each low-type seller trades with strictly positive probability independent of N, information is (trivially) aggregated when trades are reported at the end of the second period. ■

Proof of Proposition 4. From Theorem 1, we know that if the fictitious economomy features a strictly positive probability of trade in the first period, then in the true economy, sequences of equilibria along which information aggregates exist. Furthermore, it is straightforward to show that all aggregating sequences of equilibria (if there are multiple) yield the same asymptotic welfare, and that the welfare in such equilibria is higher than welfare in any non-aggregating equilibria (if the latter also exist). Thus, without loss of generality, we focus improving welfare along sequences of equilibria which aggregate information.

There are two cases to consider, depending on whether λ is greater or smaller than $\bar{\lambda}$. In both cases, we will choose δ large enough but still ensure that (C1) is violated.

Case (1). Suppose that in the unique equilibrium of the fictitious economy there is full pooling in the second period when $s = h$. Then

$$v_L = Q_L^{i,fict} = (1-\delta)\, c_L + \delta \cdot \left(\mathbb{P}(S=l|\theta_i = L) \cdot v_L + (1 - \mathbb{P}(S=l|\theta_i=L)) \cdot V\left(\pi_i\left(h;\sigma_i\right)\right)\right), \tag{20}$$

with $\sigma_i > 0$. For this to be the equilibrium, it must be that $\lambda > \bar{\lambda}$ and $\delta \in (\hat{\delta}, \bar{\delta}_\lambda)$ for some $\hat{\delta} < \bar{\delta}_\lambda$ where $\bar{\delta}_\lambda$ is given in (9). Whereas the low type's welfare is v_L, the high type's welfare is given by:

$$Q_H^{i,fict} = (1-\delta)\, c_H + \delta \cdot \left(\mathbb{P}(S=l|\theta_i = H) \cdot c_H + (1 - \mathbb{P}(S=l|\theta_i=H)) \cdot V\left(\pi_i\left(h;\sigma_i\right)\right)\right). \tag{21}$$

Step 1. Suppose that the social planner knows the state and consider the following revelation policy at the beginning of the second period, before offers are made. The planner's report is given by a random variable ω^{fict} that takes values in $\{0,1\}$, and satisfies the following property. When the state is low, then $\omega^{fict} = 0$. When the state is high, then $\omega^{fict} = 1$ w.p. $1-\gamma$ and $\omega^{fict} = 0$ w.p. γ. When $\gamma = 0$, the planner fully reveals the state, whereas when $\gamma = 1$ she fully conceals it.

As in Section 3.1, we can define a fictitious economy in which agents who trade for asset i do not observe whether other assets traded, but they observe the realization of the random variable ω after first period trade but before making offers in the second period. From type θ seller's perspective, the probability that $\omega = 0$ is:

$$\mathbb{P}(\omega^{fict} = 0|\theta_i = \theta) = \mathbb{P}\left(S = l|\theta_i = \theta\right) + \gamma \cdot \left(1 - \mathbb{P}\left(S = l|\theta_i = \theta\right)\right). \tag{22}$$

If γ is small, it is straightforward to show that the unique equilibrium of this fictitious economy must feature $v_L = Q_L^{i,fict}(\gamma)$, where:

$$Q_L^{i,fict}(\gamma) = (1-\delta)\, c_L + \delta \cdot \big(\mathbb{P}(\omega^{fict} = 0|\theta_i = L) \cdot v_L + \big(1 - \mathbb{P}(\omega^{fict} = 0|\theta_i = L)\big) \cdot V\left(\pi_i\left(1;\hat{\sigma}_i\right)\right)\big), \tag{23}$$

for some $\hat{\sigma}_i > 0$, where note that posterior belief is conditioned on the event that $\omega^{fict} = 1$. We make the dependence of the continuation value on γ explicit, as it will be useful in what follows. Again, the low type's welfare is v_L, whereas the high type's welfare is:

$$Q_H^{i,fict}(\gamma) = (1-\delta)\, c_H + \delta \cdot \left(\mathbb{P}(\omega^{fict} = 0 | \theta_i = H) \cdot c_H + \left(1 - \mathbb{P}(\omega^{fict} = 0 | \theta_i = H)\right) \cdot V\left(\pi_i\left(1; \hat{\sigma}_i\right)\right)\right). \tag{24}$$

Next, we show that in the fictitious economy, it can be Pareto improving to conceal some information; namely, that there exists a $\gamma > 0$ such that $Q_H^{i,fict}(\gamma) > Q_H^{i,fict}(0)$.

Since in the equilibrium of the fictitious economy, the low type's value is v_L for any γ, it must be that in equilibrium $\left(1 - \mathbb{P}(\omega^{fict} = 0 | \theta_i = L)\right) \cdot \pi_i\left(1; \hat{\sigma}_i\right)$ is equal to some $x \in (0,1)$ that is independent of γ, when γ is sufficiently small. We can express the low type's indifference condition in equation (23) as:

$$\left(1 - \left(\lambda + \gamma \cdot (1-\lambda)\right)\right) \cdot \pi_i\left(1; \hat{\sigma}_i\right) = \frac{(1-\delta) \cdot (v_L - c_L)}{\delta \cdot (v_H - v_L)}, \tag{25}$$

which uniquely pins down $\hat{\sigma}_i$ and the belief $\pi_i\left(1; \hat{\sigma}_i\right)$, as a function of γ. Furthermore,

$$\frac{d\pi_i\left(1, \hat{\sigma}_i\right)}{d\gamma}|_{\gamma=0} = \pi_i(1, \hat{\sigma}_i) > 0. \tag{26}$$

Thus, we have that:

$$\delta^{-1}\left(Q_H^{i,fict}(\gamma) - (1-\delta)\, c_H\right) = \left(\frac{(1-\lambda)(1-\pi)}{\pi} + \gamma \cdot \left(1 - \frac{(1-\lambda)(1-\pi)}{\pi}\right)\right) \cdot c_H$$
$$+ (1-\gamma) \cdot \left(1 - \frac{(1-\lambda)(1-\pi)}{\pi}\right) \cdot V\left(\pi_i\left(1; \hat{\sigma}_i\right)\right),$$

which is differentiable in γ and, using (26), we have:

$$\frac{dQ_H^{i,fict}(\gamma)}{d\gamma}|_{\gamma=0} = \delta \cdot \left(1 - \frac{(1-\lambda)(1-\pi)}{\pi}\right) \cdot (c_H - v_L) > 0. \tag{27}$$

This establishes that $Q_H^{i,fict}(\gamma) > Q_H^{i,fict}(0)$ for $\gamma \in (0, \bar{\gamma})$ for some small $\bar{\gamma} > 0$. Therefore, in the fictitious economy, concealing some information is strictly Pareto improving.

Step 2. In the actual economy, with arbitrarily many but a finite number of assets, the planner does not observe the state S. Instead, she observes the outcome of trade in the first period, i.e., how many sellers have traded. Along sequences of information aggregating equilibria, however, observation of trades is asymptotically equivalent to endowing the planner with the observation of the state.

To see this, consider the following policy. Fix $\gamma \in (0, \bar{\gamma})$. Fix N and suppose that the equilibrium trading probability in the first period is $\tilde{\sigma}_N \in (0,1)$. Suppose that the planner reports to the traders a binary signal $\omega_N \in \{0,1\}$ with the following properties. If planner observes more than $\tau_N \equiv \tilde{\sigma}_N \cdot \frac{\mathbb{P}(\theta_i=L|S=h)+\mathbb{P}(\theta_i=L|S=l)}{2} \cdot N$ trades, then $\omega_N = 0$. If she observes weakly less than τ_N trades, then $\omega_N = 1$ w.p. $1-\gamma$ and $\omega_N = 0$ w.p. γ. In any equilibrium with finitely many assets, it must be the case that $v_L = Q^i_{L,\omega_N}(\gamma)$, where the continuation value $Q^i_{\theta,\omega_N}(\gamma)$ of type θ is computed using the posterior beliefs induced by the probability distribution over ω_N:

$$Q^i_{\theta,\omega_N}(\gamma) = (1-\delta)c_\theta + \delta \sum_{\omega_N \in \{0,1\}} \mathbb{P}(\omega_N|\theta_i=\theta) \cdot F_\theta(\pi_i(\omega_N,\tilde{\sigma}_N),\phi_i). \tag{28}$$

Along sequences of equilibria in which information aggregates, $\mathbb{P}(\omega_N = 0|\theta_i = \theta) \to \mathbb{P}(\omega^{fict} = 0|\theta_i = \theta)$ and $\pi_i(\omega_N, \tilde{\sigma}_N) \to^p \pi_i(\omega^{fict}, \tilde{\sigma}_N)$, where recall that the random variable ω^{fict} is the planner's report in the fictitious economy. Hence, given $\epsilon > 0$, for N large enough, we have:

$$\left|(1-\delta)c_L + \delta\left(\mathbb{P}(\omega^{fict}=0|\theta_i=L)v_L + (1-\mathbb{P}(\omega^{fict}=0|\theta_i=L))V(\pi_i(\omega^{fict},\tilde{\sigma}_N)\right) - v_L\right| < \epsilon, \tag{29}$$

Since ϵ is arbitrary and posteriors are continuous in $\tilde{\sigma}_i$. this implies that $\tilde{\sigma}_N$ converges to $\hat{\sigma}_i$ and $Q^i_{H,\omega_N}(\gamma) \to Q^{i,fict}_H(\gamma)$.

Case (2). Consider next the case in which in the unique equilibrium of the fictitious economy, we have $\pi_i(h;\sigma_i) = \bar{\pi}$ and $v_L = Q^{i,fict}_L$, where

$$Q^{i,fict}_L = (1-\delta)\,c_L + \delta \cdot (\mathbb{P}(S=l|\theta_i=L) \cdot v_L + (1-\mathbb{P}(S=l|\theta_i=L)) \cdot (\phi_i c_H + (1-\phi_i)v_L))\,, \tag{30}$$

for some $\phi_i < 1$. For this to be the equilibrium, it must be that $\lambda < \bar{\lambda}$ and $\delta > \frac{v_L - c_L}{\lambda v_L + (1-\lambda)c_H - c_L}$. Thus, in this equilibrium, the low type's welfare is v_L, whereas the high type's welfare is $Q^{i,fict}_H = c_H$.

Step 1. Consider the same revelation policy as in Case (1). If γ is large enough, i.e., above some threshold $\hat{\gamma}$, it is straightforward to show that the unique equilibrium of this fictitious economy must feature $v_L = Q^{i,fict}_L(\gamma)$, where:

$$\begin{aligned} Q^{i,fict}_L(\gamma) &= (1-\delta)\,c_L + \\ &+ \delta \cdot \left(\mathbb{P}(\omega^{fict}=0|\theta_i=L) \cdot (\phi_i c_H + (1-\phi_i)v_L) + \left(1-\mathbb{P}(\omega^{fict}=0|\theta_i=L)\right) \cdot V(\pi_i(1,\hat{\sigma}_i))\right) \end{aligned}$$

for some $\hat{\sigma}_i > 0$ such that $\pi_i(0,\hat{\sigma}_i) = \bar{\pi}$ and $\phi_i \in (0,1)$. But then, since the offer following $\omega^{fict} = 1$ is strictly above c_H, the high type's welfare must satisfy $Q^{i,fict}_H(\gamma) > c_H = Q^{i,fict}_H(0)$.

Step 2. Next, consider the actual economy with arbitrary many but a finite number of assets. Fix γ above $\hat{\gamma}$. Fix N and suppose that the equilibrium trading probability in the first period is $\tilde{\sigma}_N \in (0,1)$. Consider the same policy as in Case (1). Along sequences of equilibria in which information aggregates, we again have that $\mathbb{P}(\omega_N = 0|\theta_i = \theta) \to \mathbb{P}(\omega^{fict} = 0|\theta_i = \theta)$ and $\pi_i(\omega_N, \tilde{\sigma}_N) \to^p \pi_i(\omega^{fict}, \tilde{\sigma}_N)$. If $\gamma = 0$, then it must be that for some large but finite N, $\pi_i(1, \tilde{\sigma}_N) = \bar{\pi}$; otherwise, for large but finite N, $Q^i_{L,\omega_N}(0)$ would be either strictly greater or strictly smaller than v_L, which cannot occur in equilibrium. Hence, $Q^i_{H,\omega_N}(0) = c_H$ for N large enough. On the other hand, if $\gamma > \hat{\gamma}$, then for large but finite N, the posterior $\pi_i(1, \tilde{\sigma}_N)$ must be strictly larger than $\bar{\pi}$, implying that $Q^i_{H,\omega_N}(\gamma)$ is strictly greater than c_H if N is large enough. ■