Monetary Policy with Heterogeneous Agents: Insights from TANK models *

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Abstract

Heterogeneous agents New Keynesian (HANK) models are shown to differ from their representative agent (RANK) counterparts along two dimensions: differences in average consumption at any point in time *between* constrained and unconstrained households, and consumption heterogeneity *within* the subset of unconstrained households. These two factors are captured in a simple way by two "wedges" that appear in an aggregate Euler equation, and whose behavior can be traced in response to any aggregate shock, allowing us to assess their quantitative significance. A simple two-agent New Keynesian (TANK) model abstracts completely from heterogeneity within unconstrained agents, but is shown to capture reasonably well the implications of a baseline HANK model regarding the effects of aggregate shocks on aggregate variables. We discuss the implications of our findings for the design of optimal monetary policy.

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1 Introduction

A growing literature has emerged in recent years that aims at re-examining some important macro questions through the lens of monetary models with heterogenous agents. Models in this literature commonly assume the presence of idiosyncratic shocks to individuals' income, together with the existence of incomplete markets and borrowing constraints. Those features are combined with the kind of nominal rigidities and monetary non-neutralities that are the hallmark of New Keynesian models. Following Kaplan et al. (2016), we refer to those models as HANK models (for "Heterogenous Agent New Keynesian"models).

Two key lessons can be drawn from this literature. Firstly, taking into account agents' heterogeneity is important in order to understand the transmission of monetary policy, including the relative contribution of direct and indirect effects (Kaplan et al. (2016)) or its redistributive effects across income groups (Auclert (2016)). Secondly, the transmission of monetary policy and its aggregate effects may vary significantly depending on the prevailing fiscal policy, as the latter determines how the implementation of monetary policy affects the distribution of individual income and wealth among agents with different marginal propensities to consume.

As is well known, solving for the equilibria of HANK models requires the use of nontrivial computational techniques, given the need to keep track of the distribution of wealth, and the hurdles arising from the presence of occasionally binding borrowing constraints. The reliance on numerical techniques for the analysis of those models often presents a challenge when it comes to understanding the mechanisms underlying some of the findings, and may thus limit their use in the classroom or as an input in policy institutions.

The purpose of the present paper is to assess the merits of HANK models for our understanding of an economy's aggregate behavior, relative to a simpler alternative that assumes the existence of two types of consumers –"Ricardian" and "Keynesian"– with constant shares in the population, while allowing only for aggregate shocks (i.e. disregarding idiosyncratic shocks). Ricardian consumers are assumed to have full access to financial markets (including markets for stocks and bonds), while Keynesian consumers are assumed to behave in a "hand-to-mouth"fashion, consuming their current labor income at all times. This will be the case if they do not have access to financial markets, find themselves continuously against a binding borrowing borrowing constraint, or have a pure myopic behavior. Following Kaplan et al. (2016), we refer to those models as TANK models (short for "Two Agent New Keynesian").

HANK and TANK models share a key feature that is missing in representative agent models, namely, the fact that at any point in time a fraction of agents face a binding borrowing constraint (or behave as if they did), and thus do not adjust their consumption in response to changes in interest rates or variables other than current income. This fact generally implies that the economy's response to shocks and the possible role of monetary policy in shaping that response will differ from the standard NK model with a representative agent (henceforth, RANK, for short).

On the other hand, TANK models differ from HANK models in three important ways. First, the fraction of agents who are subject (or act as if subject) to a binding borrowing constraint does not change over time in response to shocks, whereas in HANK models that fraction is endogenous and may vary over time, as a result of the interaction of aggregate shocks and the distribution and composition of wealth at any point in time. Secondly, TANK models assume away the impact on agents' current decisions of the likelihood of being financially constrained in the future, whereas the latter possibility-–typically associated to the presence of idiosyncratic shocks—is a source of (time-varying) precautionary savings in HANK models. Finally, the analysis of TANK models is highly simplified relative to their HANK counterparts for there is no need to keep track of the wealth distribution and its changes over time. In fact, as we show below, the implied equilibrium conditions of a baseline TANK model can be reduced to a system of difference equations isomorphic to of the standard NK model.

A first goal of this paper is to assess the extent to which TANK models can be viewed as a tractable framework that captures well the main predictions of HANK models regarding the economy's response to aggregate shocks, monetary as well as non-monetary. In case of a favorable answer, TANK models would be an appealing option as a laboratory for the study of the effects of aggregate shocks, and the role of monetary and fiscal policy in shaping those effects, in the presence of heterogenous agents.

With that goal in mind, we propose a simple framework, related to Werning (2015), that allows us to identify two dimensions of heterogeneity that explain the differential behavior of a HANK economy relative to its RANK counterpart. The first dimension of heterogeneity is given by the difference in average consumption *between* constrained and unconstrained households at any point in time. The dispersion in consumption *within* the subset of unconstrained households at any point in time constitute the second relevant source of heterogeneity. HANK models generally account for both dimensions of heterogeneity, whereas TANK models only capture the first dimension. Furthermore, and as we show below, these two dimensions of heterogeneity are captured in a simple way by two "wedges" that appear in an Euler equation for aggregate consumption, and whose behavior can be traced in response to any aggregate shock, allowing us to assess their quantitative significance.

Using that framework as a reference, we seek to understand the differences and similarities in the predictions of HANK, TANK and RANK models with regard to the aggregate effects of aggregate shocks.

A key finding of our analysis is that a simple TANK model approximates well, both from a qualitative and a quantiative viewpoint, the aggregate dynamics of a canonical HANK model in response to aggregate shocks, monetary and non-monetary. This is because of two reasons. On the one hand, for standard calibrations of a HANK model, consumption heterogeneity *between* constrained and unconstrained households fluctuates significantly in response to aggregate shocks, and its fluctuations are well captured by a TANK model. On the other hand, consumption heterogeneity *within* the subset of unconstrained households remains roughly constant, since those agents are able to limit consumption fluctuations by borrowing and saving. Thus, ignoring this second form of heterogeneity –as in a TANK model– is largely inconsequential for determening the behaviour of aggregate variables. This result is robust to alternative calibrations of the borrowing limits, and specifications of the fiscal transfers, which in turn are among the main drivers of the differences between HANK and RANK models.

The previous finding suggests that a TANK model may be used to obtain analytical results that provide useful insights on the role of heterogeneity in more general HANK models. A first insight in that regard pertains to the role of heterogeneity for the transmission of monetary policy shocks. In particular, we show analytically that households' heterogeneity may amplify or dampen the effects of aggregate shocks depending on the size of constrained agents and the cyclicality of fiscal transfers, among other factors, but independently of the magnitude of nominal rigidities, or the way monetary policy is conducted.

A second insight pertains to the implications of heterogeneity for the design of monetary policy. We use a TANK model to show that central banks face a non-trivial policy trade-off between stabilizing inflation and a measure of consumption heterogeneity. That trade-off is, of couse, assumed away in standard RANK models. In our TANK model, the objective function of a benevolent central bank can be approximated with a simple (quadratic) loss function that penalizes fluctuations in consumption heterogeneity, in addition to fluctuations in inflation and output gap. A policy trade-off emerges whenever fluctuations in (the natural level of) output are not proportionally distributed across households. In that case, it is not possible to simultaneously stabilize inflation, the output-gap and the heterogeneity index —i.e. the "divine coincidence" does not hold. The central bank may then tolerate some deviation from inflation and output from their respective targets in order to avoid too large fluctuations in consumption dispersion. However, we find that for standard calibrations of the TANK model, the optimal policy still implies minimimal fluctuations in inflation and the output-gap, and is thus nearly identical to the one that would prevail in a RANK model.

The paper is related to two main strands of the literature. On the one hand, the emerging literature introducing New-Keynesian features into heterogeneous agent models with idiosyncratic risk and incomplete markets. Some examples are the works of McKay et al. (2016), Gornemann et al. (2016), Kaplan et al. (2016), McKay and Reis (2016), Werning (2015), Auclert (2017), and Ravn and Sterk (2014), among many others. The main difference with respect to that literature is the development of a simple TANK model with no idiosyncratic shocks, which admits an analytical solution, and the emphasis on the distinction between constrained and unconstrained households. On the other hand, the paper builds on the earlier literature on two-agent models, such as Campbell and Mankiw (1989), Galí et al. (2007), Bilbiie (2008), Bilbiie and Straub (2013) and Broer et. al. (2016). The main difference with respect to that literature is the comparison, both from a theoretical and a quantitative viewpoint, with more general heterogenous agent models.[1] Our analysis of optimal monetary policy in a TANK model is related to the previous works of Cúrdia and Woodford (2010) and Nisticó (2016), who neverthless emphasize the importance of financial frictions, rather than consumption heterogeneity as we do here. Finally, Bilbiie and Ragot (2017) study optimal monetary policy in an TANK model where money is used to provide liquidty to financially constrained agents. Differently from that work, we study the monetary policy trade-offs in a cashless economy.

The paper is organized as follows. In section 1 we introduce our proposed framework and discuss its potential usefulness. In section 2 we lay out a baseline TANK model and derive the corresponding equilibrium conditions. In section 3 we derive the main predictions of the model regarding the effects of three aggregate shocks –monetary policy, demand and technology– under

[1] In indedendent work, Bilbiie (2017) uses a TANK model to illustrate the "direct" and "indirect" effects of monetary policy shocks emphasized by Kaplan et al. (2016) in a more general HANK model. Also, Ravn and Sterk (2017) build a tractable heterogeneous agent model with nominal rigidities and labor market frictions, giving rise to endogenous unemployment risk.

alternative assumptions regarding the transfer policies in place, and discuss the main differences with respect to the predictions of a baseline RANK model. In section 4 we carry out a quantitative comparison of the predictions the baseline TANK model to those of a HANK model. Section 5 analyzes the optimal monetary policy in a TANK model, while section 6 summarizes our main findings and concludes.

2 Heterogeneity and Aggregate Consumption: An Organizing Framework

We start out by describing an equilibrium condition for aggregate consumption that we use as a simple organizing device to think about the implications of heterogeneity for aggregate demand. Our approach is related to Werning (2015), but differs from the latter in the emphasis we attach to the distinction between constrained and unconstrained households in our interpretation of the deviations from the standard Euler equation of a representative household model.

Consider an economy with a continuum of heterogeneous households, indexed by $s \in [0, 1]$. Each household seeks to maximize utility $\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t^s, N_t^s; Z_t)$, where Z_t is a preference shifter following a stationary process. We specialize the utility function to be of the form $U(C, N; Z) \equiv \left(\frac{C^{1-\sigma}-1}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}\right) Z$. Let $\Theta_t \subset [0, 1]$ denote the set of households that in period t have *unconstrained* access to a market for one-period bonds yielding a (gross) riskless real return R_t.[2]

Next we derive a generalized Euler equation for aggregate consumption for economies with heterogeneous households. We proceed in two steps. First we derive an Euler equation in terms of aggregate consumption for unconstrained households. Then we rewrite that Euler equation in terms of aggregate consumption.

[2]In the present section, we assume a riskless real bond for notational convenience. The analysis carries over to the case of a nominally riskless bond, as assumed below.

2.1 A Generalized Euler Equation for Unconstrained Households' Consumption

For any household $s \in \Theta_t$ the following Euler equation is satisfied:

$$Z_t (C_t^s)^{-\sigma} = \beta R_t \mathbb{E}_t \left\{ Z_{t+1} (C_{t+1}^s)^{-\sigma} \right\}. \tag{1}$$

Averaging over all $s \in \Theta_t$,

$$\frac{Z_t}{1-\lambda_t} \int_{s \in \Theta_t} (C_t^s)^{-\sigma} ds = \frac{\beta R_t}{1-\lambda_t} \mathbb{E}_t \left\{ \int_{s \in \Theta_t} Z_{t+1} (C_{t+1}^s)^{-\sigma} ds \right\} \tag{2}$$

where λ_t is the measure of *constrained* households in period t, i.e. households for which (1) is not satisfied.

Let $C_t^R \equiv \frac{1}{1-\lambda_t} \int_{s \in \Theta_t} C_t^s ds$ denote the average consumption of unconstrained households in period t. Taking a second order Taylor expansion of the left hand side of (2) around C_t^R we obtain (see Appendix for details):

$$\frac{Z_t}{1-\lambda_t} \int_{s \in \Theta_t} (C_t^s)^{-\sigma} ds \simeq Z_t (C_t^R)^{-\sigma} \left[1 + \frac{\sigma(1+\sigma)}{2} var_{s|t}\{c_t^s\} \right]$$

where $var_{s|t}\{c_t^s\} \equiv \frac{1}{1-\lambda_t} \int_{s \in \Theta_t} (c_t^s - c_t^R)^2 ds$, with $c_t^s \equiv \log C_t^s$ and $c_t^R \equiv \log C_t^R$.[3] Similarly,

$$\frac{Z_{t+1}}{1-\lambda_t} \int_{s \in \Theta_t} (C_{t+1}^s)^{-\sigma} ds \simeq Z_{t+1} \left(C_{t+1}^R \right)^{-\sigma} \Phi_t^{mean} \left[1 + \frac{\sigma(1+\sigma)}{2} var_{s|t}\{c_{t+1}^s\} \right]$$

where $var_{s|t}\{c_{t+1}^s\} \equiv \frac{1}{1-\lambda_t} \int_{s \in \Theta_t} (c_{t+1}^s - c_{t+1}^R)^2 ds$ and $\Phi_t^{mean} \equiv \left(\frac{C_{t+1|t}^R}{C_{t+1}^R} \right)^{-\sigma}$, with $C_{t+1|t}^R \equiv \frac{1}{1-\lambda_t} \int_{s \in \Theta_t} C_{t+1}^s ds$.

Combining the above results, we can write an approximate Euler equation in terms of average consumption for unconstrained households as:

$$Z_t (C_t^R)^{-\sigma} = \beta R_t \mathbb{E}_t \left\{ Z_{t+1} (C_{t+1}^R)^{-\sigma} V_{t+1} \right\} \tag{3}$$

[3]Note that, strictly speaking $\frac{1}{1-\lambda_t} \int_{s \in \Theta_t} (c_t^s - c_t^R)^2 ds$ is the cross-sectional variance of (log) consumption only approximately, since c_t^R is the cross-sectional mean of c_t^s only up to a second order approximation.

where

$$V_{t+1} \equiv \Phi_{t+1}^{mean} \Phi_{t+1}^{var}$$

with $\Phi_{t+1}^{var} \equiv \frac{2+\sigma(1+\sigma) var_{s|t}\{c_{t+1}^s\}}{2+\sigma(1+\sigma) var_{s|t}\{c_t^s\}}$.

Notice that Φ_t^{mean} and Φ_t^{var} and, as a result, the wedge V_t, capture different aspects of the dynamics of heterogeneity *within* unconstrained households. If the set of households that are unconstrained is time invariant *and* consumption is equalized among those households, then $\Phi_t^{mean} = \Phi_t^{var} = 1$ and, hence, $V_t = 1$ for all t. The previous condition is trivially satisfied in a TANK model like the one analysed below.

More generally, in economies with more complex heterogeneity dynamics, one would expect V_t to differ from unity and to vary over time in response to all kinds of shocks. To the extent that consumption of unconstrained households is not lower than that of constrained ones, then $C_{t+1|t}^R \leq C_{t+1}^R$ and $\Phi_{t+1}^{mean} \geq 1$. Similarly, one would expect the dispersion of consumption across households that are unconstrained in period t, given by $var_{s|t}\{c_t^s\}$, to be no larger than the corresponding dispersion in $t+1$ for the same subset of households, given by $var_{s|t}\{c_{t+1}^s\}$, since in period $t+1$ a fraction of those households may have become constrained. In that case $\Phi_{t+1}^{var} \geq 1$ will obtain. If one of the previous conditions is satisfied, as is likely, with strict inequality, then $V_{t+1} > 1$, which captures a precautionary savings motive among unconstrained households, i.e. it leads to lower consumption today, for any given interest rate and expected consumption next period.

Yet, it is an open question, both empirical and theoretical, whether deviations of the sequence $\{V_t\}$ from unity are *quantitatively* significant or not and, more precisely, whether they imply deviations between $\mathbb{E}_t\left\{Z_{t+1}(C_{t+1}^R)^{-\sigma}V_{t+1}\right\}$ and $\mathbb{E}_t\left\{Z_{t+1}(C_{t+1}^R)^{-\sigma}\right\}$ in response to shocks that are of the same order of magnitude as C_{t+1}^R/C_t^R.

2.2 A Generalized Euler Equation for Aggregate Consumption

Next we proceed to rewrite (3) in terms of aggregate consumption $C_t \equiv \int_0^1 C_t^s ds$ as follows:

$$Z_t H_t^{-\sigma}(C_t)^{-\sigma} = \beta R_t \mathbb{E}_t \left\{ Z_{t+1} H_{t+1}^{-\sigma}(C_{t+1})^{-\sigma} V_{t+1} \right\} \tag{4}$$

where H_t is an index of heterogeneity *between* constrained and unconstrained households defined as:

$$H_t \equiv \frac{C_t^R}{C_t}.$$

Consider a steady state with no aggregate shocks, $Z = 1$, constant Y, H and V and, hence, $\beta RV = 1$. Log-linearization of (4) around that steady state combined with the goods market clearing condition $Y_t = C_t$ yields:

$$y_t = \mathbb{E}_t\{y_{t+1}\} - \frac{1}{\sigma}\widehat{r}_t - \frac{1}{\sigma}\mathbb{E}_t\{\Delta z_{t+1}\} + \mathbb{E}_t\{\Delta h_{t+1}\} - \frac{1}{\sigma}\mathbb{E}_t\{\widehat{v}_{t+1}\},$$

where $y_t \equiv \log Y_t$, $h_t \equiv \log H_t$, $z_t \equiv \log Z_t$, $\widehat{r}_t \equiv \log(\beta V R_t)$, and $\widehat{v}_t \equiv \log(V_t/V)$.

Assuming $\lim_{T\to\infty} \mathbb{E}_t\{c_{t+T}\} = c$, $\lim_{T\to\infty} \mathbb{E}_t\{h_{t+T}\} = h$, and $\lim_{T\to\infty} \mathbb{E}_t\{z_{t+T}\} = 0$, we can derive the aggregate demand equation:

$$\widehat{y}_t = -\frac{1}{\sigma}\left(\widehat{r}_t^L - z_t\right) - \widehat{h}_t - \widehat{f}_t, \tag{5}$$

where $\widehat{r}_t^L$ and f_t are defined recursively as

$$\widehat{r}_t^L = \mathbb{E}_t\{\widehat{r}_{t+1}^L\} + \widehat{r}_t$$

$$\widehat{f}_t = \mathbb{E}_t\{\widehat{f}_{t+1}\} + \frac{1}{\sigma}\mathbb{E}_t\{\widehat{v}_{t+1}\}.$$

Thus, output is seen to depend on four factors: (i) the exogenous demand shifter z_t (ii) current and expected real interest rates, as summarized by $\widehat{r}_t^L$, and whose responses to shocks of all kinds will be determined by the monetary policy rule in place, (iii) the h-index of heterogeneity *between* constrained and unconstrained households, and (iv) variable $\widehat{f}_t$ which is a function of expected future distributions of consumption *within* unconstrained households.

Note that in the representative household model, $\widehat{h}_t = \widehat{v}_t = 0$ for all t. Thus, the extent to which aggregate demand and output in a HANK model behave differently from their RANK counterparts, conditional on the monetary policy stance (as summarized by $\widehat{r}_t^L$) will depend on the endogenous response of $\widehat{h}_t$ and $\widehat{v}_t$ to different aggregate shocks. In particular, an increase (decrease) in the h-index in response to a shock that raises output will dampen (amplify) that response relative to a RANK economy, ceteris paribus (i.e. conditional on a given real rate response). In other words, the aggregate effects of a given shock is seen to depend on the extent to which it leads towards a redistribution towards/from those unconstrained households. The extent of that redistribution may in turn be a consequence of (i) differential characteristics of unconstrained households (e.g. their participation in labor markets and/or ownership of firms) and/or (ii) the nature of the transfer/tax regime in place. An analogous argument can be made about variable f_t, though in the latter case only the extent of redistribution within the subset of unconstrained households at any point in time is relevant.

Given the above considerations, the analysis of the determinants of the dynamics of h_t and v_t and their response to aggregate shocks should contribute to our understanding of the implication of heterogeneity. Next we put that idea into practice when analyzing the properties of a particular instance of a HANK model, which we refer to as the baseline TANK model, and for which all deviations from its RANK counterpart are the result of variations in h_t, since the assumption of a representative unconstrained household implies that $v_t = 0$ for all t.

3 A Baseline TANK Model

We consider an economy with two types of households, a continuum of firms, a monetary authority and a fiscal authority.[4] This simple model is used as a laboratory to study the implications of

[4]Campbell and Mankiw (1989) introduced the two-agent framework to account for empirical deviations from the permanent-income hypothesis. Gal that framework into a New Keynesian model in order to re-examine the conditions for equiligrium uniqueness, and to account for the effects of government purchases on consumption. See

agents' heterogeneity for the behavior of aggregate variables in response to aggregate shocks. As in Werning (2015), our focus is on understanding the determinants of aggregate consumption. The supply side of the model is kept relatively simple, and such that it can be directly embedded in the richer HANK models considered below.

3.1 Households

All households are assumed to have identical preferences, given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t^s, N_t^s; Z_t)$$

where $C_t^s \equiv \left(\int_0^1 C_t^s(i)^{1-\frac{1}{\epsilon_p}} di\right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$ is a consumption index, with $C_t^s(i)$ denoting the quantities consumed of good $i \in [0,1]$. N_t^s denotes hours of work. The superindex $s \in \{R, K\}$ specifies the household type ("Ricardian" or "Keynesian"), as discussed below. $\beta \in (0,1)$ is the discount factor. Z_t is an exogenous preference shock, whose evolution is described by the log-normal process $z_t = \rho_z z_{t-1} + \varepsilon_t^z$, where $z_t \equiv \log Z_t$ and $\varepsilon_t^z \sim \mathcal{N}(0, \sigma_z^2)$. Period utility is specialized to be of the form:

$$U(C, N; Z) \equiv \left(\frac{C^{1-\sigma} - 1}{1-\sigma} - \frac{N^{1+\varphi}}{1+\varphi}\right) Z \tag{6}$$

A constant measure $1-\lambda$ of households, which we refer to as " Ricardian," have unconstrained access to financial markets. In particular, they can trade two types of assets: one-period nominally riskless bonds and shares in a fund that owns all firms. Their period budget constraint is given by

$$\frac{1}{P_t}\int_0^1 P_t(i) C_t^R(i) di + \frac{B_t^R}{P_t} + Q_t S_t^R = \frac{B_{t-1}^R(1+i_{t-1})}{P_t} + W_t N_t^R + (D_t + Q_t) S_{t-1}^R + T_t^R \tag{7}$$

where B_t^R and S_t^R denote, respectively, holdings of bonds and shares in a stock market fund. T_t^R denotes lump-sum tranfers (or taxes, if negative). Q_t is the price of a share in the stock fund,

also Bilb of a version of TANK closer to the one considered here (i.e. without physical capital).

while D_t denoting the corresponding dividends (all of them in terms of the consumption bundle). i_t the nominal yield on bonds. W_t is the (real) wage. $P_t \equiv \left(\int_0^1 P_t(i)^{1-\epsilon_p} di\right)^{\frac{1}{1-\epsilon_p}}$ is a price index.

A fraction λ of households, henceforth referred to as "Keynesian" just consume their labor income net of taxes (or transfers) each period, possibly (but not necessarily) because they do not have access to financial markets. Formally

$$\frac{1}{P_t} \int_0^1 P_t(i) C_t^K(i) di = W_t N_t^K + T_t^K \tag{8}$$

where C_t^K, N_t^K and T_t^K denote, respectively, consumption, hours worked and transfers for Keynesian households.

Importantly, and in contrast with many of the HANK models in the literature, the two types of agents do not face any form of idiosyncratic uncertainty. Furthermore, we assume that they take the wage as given and are happy to supply as much labor as demanded by firms (i.e. they also take N_t^R and N_t^K as given, more on this below).

Optimal allocation of expenditures across goods implies $C_t^s(i) = (P_t(i)/P_t)^{-\epsilon_p} C_t^s$ and $\int_0^1 P_t(i) C_t^s(i) di = P_t C_t^s$ for $s \in \{R, K\}$. In addition, the intertemporal optimality conditions for Ricardian households take the form

$$\frac{1}{1+i_t} = \mathbb{E}_t \left\{ \Lambda_{t,t+1} \frac{P_t}{P_{t+1}} \right\} \tag{9}$$

$$Q_t = \mathbb{E}_t \left\{ \Lambda_{t,t+1} (Q_{t+1} + D_{t+1}) \right\} \tag{10}$$

where $\Lambda_{t,t+1} \equiv \beta (Z_{t+1}/Z_t) \left(C_{t+1}^R / C_t^R\right)^{-\sigma}$ is the stochastic discount factor for one-period ahead (real) payoffs.

As emphasized above, our focus is on aggregate demand dynamics, so we keep the supply side of the model as simple as possible. In particular we assume a wage schedule

$$W_t = \mathcal{M}^w C_t^\sigma N_t^\varphi \tag{11}$$

for all $j \in [0,1]$, where $\mathcal{M}^w > 1$ can be interpreted as the gross average wage markup. Such a wage schedule would arise in a version of the RANK model with a continuum of labor types, with wages set in a decentralized way by each type of labor (or the union representing it) in a way consistent with household utility maximization and given an isoelastic demand for labor. Throughout we assume $W_t \geq (C_t^R)^\sigma N_t^\varphi \geq (C_t^K)^\sigma N_t^\varphi$ for all t, so that all households will be willing to supply the work hours demanded by firms.[5]

3.2 Firms

The supply side is standard, and consists of a continuum of identical monopolistically competitive firms $i \in [0,1]$, each producing a differentiated product, using a technology

$$Y_t(i) = A_t N_t(i)^{1-\alpha}$$

where $Y_t(i)$ denotes output, $N_t(i)$ is labor input, and A_t is a technology parameter. The latter evolves according to the process $a_t = \rho_a \log a_{t-1} + \varepsilon_t^a$, where $a_t \equiv \log A_t$ and $\varepsilon_t^a \sim N(0, \sigma_a^2)$. The demand for each good variety is isoelastic, and given by the function $Y_t(i) = (P_t(i)/P_t)^{-\epsilon} Y_t$, where Y_t denotes aggregate output.

As in Rotemberg (1983) we assume that each firm faces a quadratic price adjustment cost given by $\mathcal{C}(\cdot) \equiv \frac{\xi}{2} Y_t \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2$ where $\xi > 0$, and in terms of a CES goods bundle with elasticity of substitution ϵ_p. Since all firms are identical, profit maximization implies that all firms choose the same price, i.e. $P_t(i) = P_t$ for all $i \in [0,1]$, and hence produce the same output Y_t with the same labor input N_t. As a result, the (gross) price inflation rate $\Pi_t \equiv \frac{P_t}{P_{t-1}}$ evolves according to

$$\Pi_t(\Pi_t - 1) = \mathbb{E}_t \left\{ \Lambda_{t,t+1} \left(\frac{Y_{t+1}}{Y_t} \right) \Pi_{t+1} (\Pi_{t+1} - 1) \right\} + \frac{\epsilon}{\xi} \left(\frac{1}{\mathcal{M}_t^p} - \frac{1}{\mathcal{M}^p} \right) \tag{12}$$

[5] We have analyzed a version of such a model for a TANK economy, where each union maximizes a weighted average of the utilities of its Ricardian and Keynesian members, and derived a wage schedule slightly different from (11), where the consumption of the two types enters separately, complicating the algebra without yielding any significant additional insights on the issues of interest.

where $\mathcal{M}_t^p \equiv (1-\alpha)A_t N_t^{-\alpha}/W_t$ is the average gross markup, and with the latter's optimal value in the absence of price adjustment costs given by $\mathcal{M}^p \equiv \frac{\epsilon_p}{\epsilon_p - 1}$.

3.3 Fiscal and Monetary Policy

The cyclical behavior of taxes and transfers plays a crucial role in the determination of the effects of aggregate shocks. To illustrate that role we consider an environment where transfers to Keynesian households are given by

$$T_t^K = \tau_0 D + \tau_d (D_t - D) \tag{13}$$

where $0 \leq \tau_0 \leq 1$, $0 \leq \tau_d \leq 1$, and where a balanced budget condition holds

$$\lambda T_t^K + (1-\lambda) T_t^R = 0. \tag{14}$$

The previous specification nests as special cases *laissez-faire* ($\tau_0 = \tau_d = 0$) and full steady state redistribution ($\tau_0 = 1$ and $\tau_d = 0$). In addition to the previous two cases, below we also consider a " realistic" case with *partial* redistribution, both in steady state and period by period ($0 < \tau_0 < 1$ and $0 < \tau_d < 1$). The case of full redistribution period by period ($\tau_0 = 1$ and $\tau_d = 1$) can be checked to be equivalent to the RANK model.

Regarding the conduct of monetary policy, we consider alternative scenarios. We start by assuming that the central bank follows a simple interest rate rule, potentially including terms related to income heterogeneity. Then, in Section 5, we derive the optimal monetary policy.

3.4 Equilibrium

Goods market clearing requires

$$Y_t(i) = C_t(i) + X_t(i)$$

for all $i \in [0,1]$, where $X_t(i) = (P_t(i)/P_t)^{-\epsilon_p}(\xi/2)Y_t (\Pi_t - 1)^2$ captures the demand for good i to meet price adjustment costs. Noting that in equilibrium all firms set the same prices, thus implying

$Y_t(i) = Y_t$ and $C_t(i) = C_t$ for all $i \in [0,1]$, we can write

$$C_t = Y_t \Delta^p(\Pi_t) \tag{15}$$

where $\Delta_t^p(\Pi_t) \equiv 1 - (\xi/2)\left(\Pi_t - 1\right)^2$ and $C_t \equiv (1-\lambda)C_t^R + \lambda C_t^K$ denotes aggregate consumption. Furthermore, since all Ricardian households are identical, market clearing in the bonds and stock markets implies that $B_t^R = 0$ and $S_t^R = 1/(1-\lambda)$.

Aggregate employment is given by

$$N_t = (Y_t/A_t)^{\frac{1}{1-\alpha}} \tag{16}$$

and is assumed to be distributed uniformly among household types, so that $N_t^R = N_t^K = N_t$ for all t.

The Euler equation for Ricardian households can be rewritten in terms of aggregate consumption as:

$$1 = \beta(1+i_t)\mathbb{E}_t\left\{\left(C_{t+1}/C_t\right)^{-\sigma}\left(H_{t+1}/H_t\right)^{-\sigma}(Z_{t+1}/Z_t)\Pi_{t+1}^{-1}\right\} \tag{17}$$

where, as above, $H_t \equiv C_t^R/C_t$.

In equilibrium, and given the goods market clearing condition (15), we can write:

$$\begin{aligned}
H_t &= \frac{1}{\Delta^p(\Pi_t)}\left(\frac{W_tN_t}{Y_t} + \frac{1}{1-\lambda}\frac{D_t}{Y_t} + \frac{T_t^R}{Y_t}\right) \\
&= \frac{1}{\Delta^p(\Pi_t)}\left[\frac{1-\alpha}{\mathcal{M}_t^p} + \left(\frac{1-\lambda\tau_d}{1-\lambda}\right)\frac{D_t}{Y_t} - \frac{\lambda(\tau_0-\tau_d)}{1-\lambda}\frac{D}{Y_t}\right] \\
&= 1 + \frac{1}{\Delta^p(\Pi_t)}\left[\frac{\lambda(1-\tau_d)}{1-\lambda}\left(\Delta^p(\Pi_t) - \frac{1-\alpha}{\mathcal{M}_t^p}\right) - \frac{\lambda(\tau_0-\tau_d)}{1-\lambda}\left(1 - \frac{1-\alpha}{\mathcal{M}^p}\right)\frac{Y}{Y_t}\right]
\end{aligned} \tag{18}$$

where the derivation of the second and third equalities makes use of the transfer and balanced budget rules (13) and (14), the accounting identity $Y_t\Delta^p(\Pi_t) = W_tN_t + D_t$, and the definition of the average price parkup introduced earlier.

Note that as long as $\tau_d < 1$ (i.e. less than full redistribution) the heterogeneity index H_t is increasing in the markup, given output, for a rise in the latter implies a redistribution of income towards the unconstrained. The same is true for a rise in output, given the markup, as long as $\tau_0 > \tau_d$, since in that case a rise in output also implies a redistribution towards the unconstrained.

Equations (15) through (18), combined with the inflation equation (12) and a monetary policy rule determining the interest rate i_t describe the equilibrium conditions of the model. In the next sections we characterize the perfect foresight steady state with zero inflation and the log-linearized equilibrium dynamics around that steady state.

3.4.1 Steady State

In a perfect foresight, zero inflation steady state for the above economy we have $A = Z = 1$, $\Pi = 1$, $\Delta^p(1) = 1$ (implying $C = Y$), and $\mathcal{M}_t^p = \mathcal{M}^p$. Hence, evaluating (11) at that steady state and combining it with the aggregate production function (16) yields the following expression for steady state output:

$$Y = \left(\frac{1-\alpha}{\mathcal{M}}\right)^{\frac{1-\alpha}{\sigma(1-\alpha)+\varphi+\alpha}}$$

where $\mathcal{M} \equiv \mathcal{M}^w \mathcal{M}^p$ denotes the steady state "composite" markup. On the other hand, evaluating (18) at the zero inflation steady state we obtain a closed-form expression for H:

$$H = 1 + \frac{\lambda(1-\tau_0)}{1-\lambda}\left(1 - \frac{1-\alpha}{\mathcal{M}^p}\right) \geq 1.$$

3.4.2 Equilibrium Dynamics around a Steady State

Note that in a neighborhood of the zero inflation steady state, and up to a first order approximation, $\Delta_{p,t} \simeq 1$ for all t. Accordingly, we henceforth approximate the goods market equilibrium condition as

$$\widehat{y}_t = \widehat{c}_t \tag{19}$$

for all t, where $\widehat{y}_t \equiv \log(Y_t/Y)$ and $\widehat{c}_t \equiv \log(C_t/C)$. Log-linearization of (12) around the zero inflation steady state yields the inflation equation

$$\pi_t = \beta \mathbb{E}_t\{\pi_{t+1}\} - \delta \widehat{\mu}_t^p \tag{20}$$

where $\widehat{\mu}_t^p \equiv \log(\mathcal{M}_t^p/\mathcal{M}^p)$ and $\delta \equiv \epsilon_p/(\xi \mathcal{M}^p)$. The deviations of the (log) price markup from steady state can be written as:

$$\widehat{\mu}_t^p = \left(\frac{1+\varphi}{1-\alpha}\right) a_t - \left(\sigma + \frac{\alpha+\varphi}{1-\alpha}\right) \widehat{y}_t. \tag{21}$$

Note that by setting $\widehat{\mu}_t^p = 0$ all t we can solve for the natural (i.e. flexible price) level of output, $\widehat{y}_t^n$ (expressed in log deviations from steady state). In the illustrative model analyzed here, the latter variable is as a function of technology only, given by

$$\begin{aligned} \widehat{y}_t^n &= \frac{1+\varphi}{\sigma(1-\alpha)+\alpha+\varphi} a_t \\ &\equiv \psi_a a_t. \end{aligned}$$

More generally, however, $\widehat{y}_t^n$ may also depend on other exogenous shocks that may shift the markup-output schedule (21), including labor supply shocks and shocks to the desired markup, among others. Independently of the number and nature of the shocks affecting $\widehat{y}_t^n$, the following relation will generally hold:

$$\widehat{\mu}_t^p = -\left(\sigma + \frac{\alpha+\varphi}{1-\alpha}\right) \widetilde{y}_t \tag{22}$$

where $\widetilde{y}_t \equiv y_t - y_t^n$ is the output gap, i.e. the log deviation of output from its flexible price counterpart. Substituting (22) into (20) we obtain a version of the New Keynesian Phillips curve (NKPC, henceforth):

$$\pi_t = \beta \mathbb{E}_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t \tag{23}$$

where $\kappa \equiv \delta\left(\sigma + \frac{\alpha+\varphi}{1-\alpha}\right)$. Thus, under the assumptions of our baseline TANK model, the supply side is not affected by the presence of heterogeneity. This allows us to focus on its impact on aggregate demand (which coincides with aggregate consumption in our simple model) in the spirit of Werning (2015).

Note that log-linearization of (18) yields, after letting $\widehat{h}_t \equiv \log(H_t/H)$:

$$\begin{aligned}\widehat{h}_t &= \frac{\lambda}{(1-\lambda)H}\left[\frac{(1-\alpha)(1-\tau_d)}{\mathcal{M}^p}\widehat{\mu}_t^p + \left(1-\frac{1-\alpha}{\mathcal{M}^p}\right)(\tau_0-\tau_d)\widehat{y}_t\right] \\ &= \chi_y \widetilde{y}_t + \chi_n \widehat{y}_t^n \qquad (24)\end{aligned}$$

where

$$\chi_y \equiv \frac{\lambda}{(1-\lambda)H}\left[(\tau_0-\tau_d)\left(1-\frac{1-\alpha}{\mathcal{M}^p}\right) - \frac{(1-\tau_d)(1-\alpha)}{\mathcal{M}^p}\left(\sigma + \frac{\alpha+\varphi}{1-\alpha}\right)\right]$$

$$\chi_n \equiv \frac{\lambda(\tau_0-\tau_d)}{(1-\lambda)H}\left(1-\frac{1-\alpha}{\mathcal{M}^p}\right).$$

Log-linearizing the Euler equation for aggregate consumption (17), and imposing goods market clearing we obtain:

$$\widetilde{y}_t = \mathbb{E}_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma}(\widehat{i}_t - E_t\{\pi_{t+1}\}) + \mathbb{E}_t\{\Delta\widehat{h}_{t+1}\} + \frac{1}{\sigma}(1-\rho_z)z_t + \mathbb{E}_t\{\Delta\widehat{y}_{t+1}^n\}$$

which can be combined with (24) to yield the Dynamic IS equation (DIS):

$$\widetilde{y}_t = \mathbb{E}_t\{\widetilde{y}_{t+1}\} - \frac{1}{\sigma(1+\chi_y)}(\widehat{i}_t - E_t\{\pi_{t+1}\} - \widehat{r}_t^n) \qquad (25)$$

where $\widehat{r}_t^n \equiv (1-\rho_z)z_t + \sigma\,(1+\chi_n)\,\mathbb{E}_t\{\Delta\widehat{y}_{t+1}^n\}$.

Thus we see that the equilibrium conditions describing the non-policy block of the baseline TANK model can be reduced to two equations –the NKPC and the DIS– in terms of the output gap and inflation, that are isomorphic to those describing the equilibrium of the standard RANK model. Under our assumptions the NKPC is identical to that in the RANK model. The DIS, on the other hand, is affected by heterogeneity in two ways. First, the sensitivity of the output gap to interest rate changes is now given by $1/[\sigma(1+\chi_y)]$, which is smaller or larger than its counterpart in the RANK model ($1/\sigma$) depending on the sign of χ_y, the elasticity of h with respect to the output gap. Secondly, the impact of expected natural output growth on the natural rate of interest is potentially amplified by the presence of the term χ_n, the elasticity of h with respect to natural output.

Note that coefficients χ_y and χ_n are functions of the three "structural" parameters determining heterogeneity in the model, namely, λ, τ_0, and τ_d. Trivially, $\chi_y = \chi_n = 0$ if $\lambda = 0$. Conditional on $\lambda > 0$, two extreme cases are worth mentioning. Trivially, in the case of full redistribution ($\tau_0 = \tau_d = 1$) we also have $\chi_y = \chi_n = 0$, and, hence, equivalence to the RANK model, independently of λ. At the other extreme, in the "laissez-faire" case ($\tau_0 = \tau_d = 0$), we have $\chi_n = 0$ and $\chi_y < 0$, i.e. the natural rate is not affected by the presence of heterogeneity, but output is more sensitive to changes in interest rates, with that sensitivity increasing in the share of Keynesian households. More generally, in an intermediate (and more realistic) case of less than full redistribution ($0 < \tau_d \leq \tau_0 < 1$), the natural rate is more sentitive to changes in natural output growth ($\chi_n > 0$), while the impact of heterogeneity on the interest rate sensitivity of output is ambiguous ($\chi_y \gtrless 0$), with $\chi_y > 0$ if and only if

$$\frac{\tau_0 - \tau_d}{1 - \tau_d} > \frac{\sigma(1-\alpha) + \varphi + \alpha}{\mathcal{M}^p - 1 + \alpha}.$$

Intuitively, when Keynesian agents are high enough constant transfers (i.e. τ_0 is high), their income is in not much affect by changes in interest rates. As a result, aggregate output would be less sensitive to interest rates than in a representative agent economy (i.e. where all agents are Ricardian). Viceversa, when constant transfers are low, income of Keynesian agents fluctuates more, and thus the response of aggregate output is amplified. Thus, fiscal policy plays an important role in determining the effects of interest rate changes. On the other hand, note that χ_y and χ_n do not depend on the presence of nominal rigidities, or on the conduct of monetary policy.

The parameters χ_y and χ_n could alternatively be interpreted as reduced-form coefficients describing the cyclical behavior of the h-index, as implied by eq. (24). Under such interpretation, the two parameters could be calibrated to match the observed cyclicality of heterogeneity between constrained and unconstrained households, without the need of explicitly modeling the actual sources of consumption heterogeneity —e.g. labor income, employment status, wealth distribution, portofolio choices, etc. The resulting values could then be used to assess the effects of heterogeneity on the sensitivity of the output gap to interest rate changes.

In addition to the two equations describing the non-policy block of the TANK economy, (23) and (25), an additional equation describing how monetary policy is conducted is needed in order to close the model. Following convention, in the analysis below we assume a Taylor-type rule of the form:

$$\widehat{i}_t = \phi_\pi \pi_t + \phi_y \widehat{y}_t + \nu_t$$

where $\phi_\pi \geq 0$, $\phi_y \geq 0$, and where ν_t is an exogenous monetary policy shock, following an $AR(1)$ process with autoregressive coefficient $\rho_\nu \in [0, 1]$.

3.4.3 Equilibrium Dynamics: A Quantitative Illustration

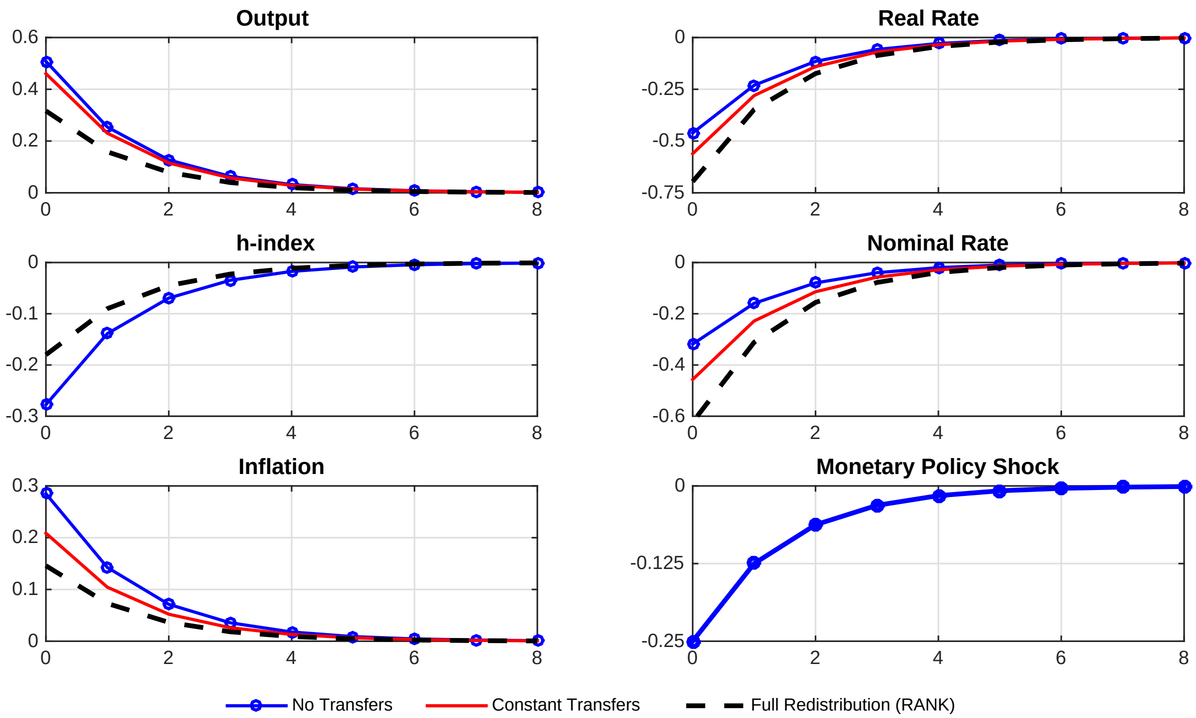
To provide a quantitative illustration of the role of heterogeneity for the effects of monetary policy shocks, we consider a calibration of the model where the share of Keynesian agents is set to 0.21, in the middle of range of values typically used in the HANK literature.[6] The remaining parameters are set to standard values.[7]

Figure 1 shows that in a TANK model with no transfers (solid line with "o" markers) the response of output to a monetary policy shock is amplified with respect to a RANK model, i.e. a

[6]For instance, the fraction of constrained agents equals 13% in McKay et. al. (2016), while Kaplan et. al. (2014) report that about one third of the U.S. households are hand-to-mouth, of which about one third is poor and two thirds are wealthy.

[7]Furthermore, we set the discount factor $\beta = 0.9925$, the coefficient of risk aversion $\sigma = 1$ (log-utility), the (inverse) Frisch elasticity of substitution $\phi = 1$, the labor income share $1 - \alpha = 3/4$, the elasticity of substitution among good varieties $\epsilon = 9$ and the adjstument cost parameter $\xi = 372.8$ (implying the same slope of the Phillips curve as a model with sticky prices a la Calvo with an average price duration of 1-year).

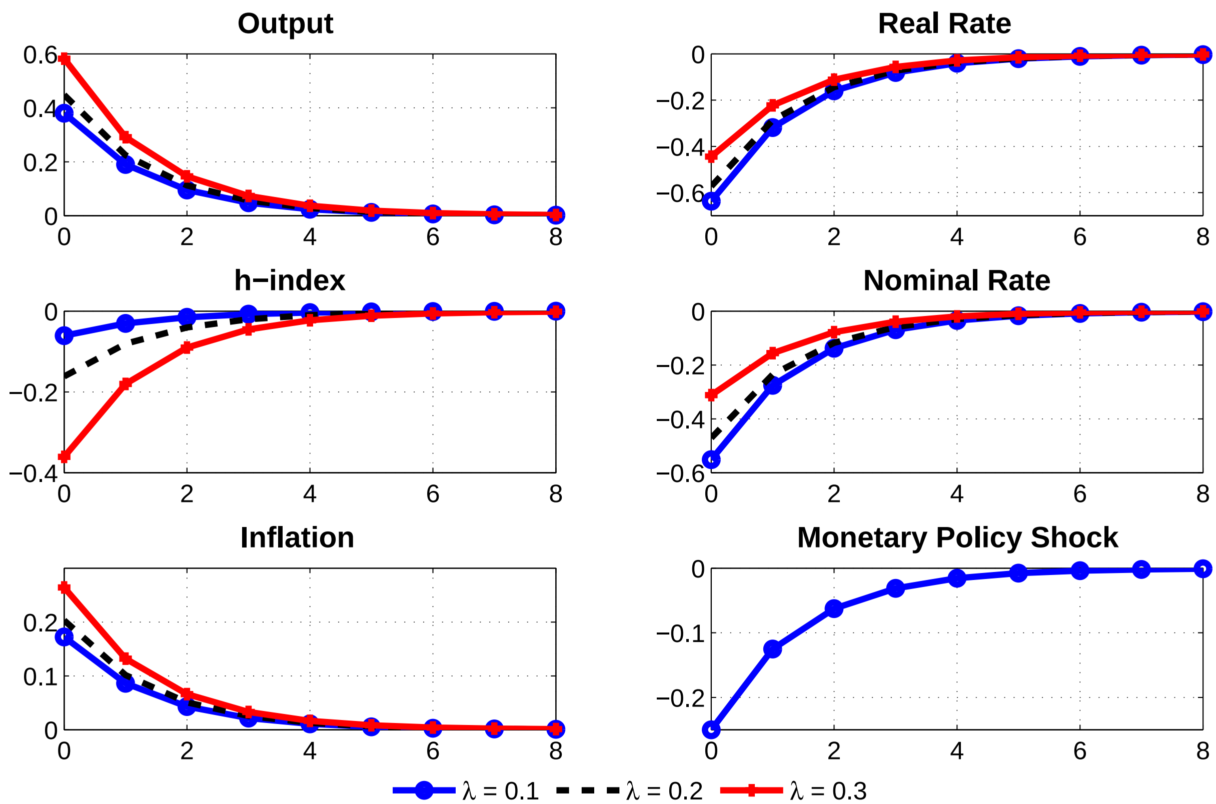
Figure 1: The Effects of Monetary Policy Shock in the TANK model under Alternative Transfer Policies



case where there is full redistribution (dashed line). In the absence of transfers, firms' profits constitute the only source of earnings heterogeneity. Since profits move countercyclically in response to demand shocks (a standard feature of New-Keynesian models), the income share of Ricardian agents –the firms' owners– is countercyclical, and thus the $h-$index is also countercyclical (i.e. $\chi_y < 0$). As a result, consumption of Keynesian agents responds more than consumption of Ricardian agents, and the aggregate effects are thus amplified with respect to an economy where all agents are Ricardian (as in RANK). A similar amplification effect also arises under in a case with constant transfers set to eliminate steady state inequality (solid lines).

Figure 2 analyzes the response to a monetary policy shocks when varying the fraction Keynesian agents λ from 0.1 to 0.3, while keeping the remaining parameters unchanged, and assuming a fiscal

Figure 2: The Effects of a Monetary Policy Shock in the TANK model under Alternative Size of Keynesian Agents



policy with constant transfers. As expected, the figure shows that the larger is the fraction of Keynesian agents, the larger is the response of output to a monetary policy shock.

4 HANK vs TANK: A Quantitative Comparison

The simple TANK model introduced in the previous section can at most account for the heterogeneity between constrained and unconstrained agents, but it ignores some key features of more realistic HANK models, such as the presence of idiosyncratic risk and the associated dynamics of the wealth distribution. Clearly, those features could potentially alter the aggregate implications of monetary policy shocks, or of aggregate shocks in general. In this section, we thus compare the predictions of our simple TANK model with those of a canonical HANK model, restricting our attention to the responses of aggregate variables to aggregate shocks.

The HANK model considered is a standard Bewley-Aiyagari-Huggett economy, populated by a continuum of ex-ante identical agents $s \in [0,1]$ with preferences given by eq. (6), and receiving in each period a labor income $\exp(e_t^s)W_tN_t$, where e_t^s denotes an idiosyncratic shock. Following McKay et. al. (2016) and Auclert (2016), it is assumed that the idiosyncratic income shocks follows an exogenous AR(1) process, with the persistence parameter $\rho_\epsilon = 0.966$ and standard deviation $\sigma_\epsilon = 0.017$. Agents cannot perfectly insure against idiosyncratic risk, since the only asset available to the agents is a riskless one-period nominal bond which is zero net supply, and each agent faces a (exogenous) borrowing limit $\underline{a} = \psi\bar{Y}$. In our baseline calibration we set $\psi = 100\%$, which implies that 21% of the agents are borrowing constrained in steady state. We also consider alternative calibrations with tighter ($\psi = 50\%$) and looser ($\psi = 200\%$) borrowing limits, implying a fraction of constrained agents of 36% and 11%, respectively.

The supply side is identical to the one described for the TANK model in Section 3. We instead consider several alternative transfer policies. In our baseline calibration, as for the TANK model, it is assumed that transfers to constrained agents are given by eq. (13).[8] As an alternative, we consider a more general transfer policy to take into account alternative degree of progressivity or cyclicality of transfers. In particular, and following Auclert (2016), we consider a formulation for transfers given by

$$\frac{T_t^s}{Y_t} = \frac{D}{Y} + \left(\frac{D_t}{Y_t} - \frac{D}{Y}\right)\left[\gamma + (1-\gamma)\, exp(e^s)\right]$$

and where the parameter γ measures the degree of progressivity of fiscal policy —i.e. a positive

[8]The implicit assumption is that the government takes all profits and redistribute them across households according to the specified transfer rule. Since the distribution of firms' profits is fully determined by the transfer policies, one could abstract from the households' portfolio choice between debt and equity. An alternative (and equally tractable) formulation is to assume that agents invest in a mutual funds choosing the asset allocation between households' credit and firms' equity –see e.g. Gornemann et. al. (2016).

Table 1: The Real Interest Rate Elasticity of Output (Baseline Transfer Policy)

	Transfer Regime			Borrowing Limit	
τ_0, τ_d, ψ	(1,0,1)	(1,0.5,1)	(1,1,1)	(1,0,0.5)	(1,0,2)
RANK	1.00	1.00	1.00	1.00	1.00
TANK	1.64	1.24	1.00	5.74	1.23
HANK	1.70	1.23	0.97	5.30	1.46
Implied λ	0.21	0.21	0.21	0.36	0.11

(negative) γ implies that the fiscal system is progressive (regressive). For instance, when $\gamma = 1$ all agents receive the same transfers in every period, regardless of the their labor income. Instead, when $\gamma = 0$ the transfers are proportional to the idiosyncratic shock. Finally, when γ is negative agents with high labor income receive a disproportionately larger share of profits. Clearly, this type of fiscal policy cannot be mapped directly into a TANK model with no idiosyncratic shocks. For this reason, and in order to compare the predictions with a TANK model, we first solve and simulate the HANK model, and then use the resulting series to estimate the parameters τ_0 and τ_d in (13) for the TANK model.

As in previous studies in the HANK literature, we first calculate the real interest rate elasticity of output $\partial y/\partial r$ on impact in response to a monetary policy shock.

Table I reports the results under the baseline transfer policy, under alternative values for the parameters describing fiscal transfers (columns 1-3) and the borrowing limit (columns 4 and 5), and where the response in the RANK model is normalized to one. For instance, in an economy with constant transfers ($\tau_0 = 1$ and $\tau_d = 0$, column 1) the response of output to a change in the real interest rate in HANK is about 70% larger than in a RANK economy. In other words, in this economy heteroegeneity has important implications for the dynamics of aggregate variables. Notably, the differences between HANK and RANK are well captured by the simple TANK model, where the output response is 64% larger than in RANK.[9] Similarly, the TANK model provides a

[9]This suggests that the large "indirect" (income) effects of monetary policy found by Kaplan et. al. (2016) in

Table 2: The Real Interest Rate Elasticity of Output (Alternative Fiscal Policy)

	Degree of Progressivity		
	$\gamma = -1$	$\gamma = 0$	$\gamma = 1$
RANK	1.00	1.00	1.00
TANK	1.44	1.17	1.00
HANK	1.50	1.18	0.97
τ_0	0.43	0.72	1
τ_d	0.35	0.67	1
R^2	0.75	0.98	1

good approximation to the richer HANK model also under alternative parametrization for fiscal transfers (columns 2 and 3), or under alternative values for the borrowing limit parameters (last two columns). Finally, as shown in Table 2, the same results also hold under an alternative transfer policy, for different value of tax progressivity (and cyclicality of income risk) as implied by the parameter γ in the HANK model, and approximated by the resulting estimates of τ_0 and τ_d in the TANK model.

A more complete description of the model dynamics is provided in Figures 3-5 containing, respectively, the impulse responses to monetary policy, preference and technology shocks, under our baseline calibration. As it can be seen in Figure 3, the response of output to monetary policy shocks is remarkably similar in TANK and HANK models, and different from the RANK model. The main reason for the similarity between TANK and HANK can be understood looking at the evolution of the h-index and v-index (second row). The TANK model approximates well the heterogeneity *between* constrained and unconstrained agents (h-index). Instead, the heterogeneity *within* unconstrained agents (v-index), which is constant by construction in a TANK model, remains nearly constant in the HANK model. Similar considerations also holds in response to preference shocks (Figure 4) and technology shocks (Figure 5), even though in the latter case the

an HANK model, are also captured by our simple TANK economy. Indeed the differences between TANK and RANK are entirely due to the presence of hand-to-mouth agents, who are not affected by the "direct" intertemporal substitution effect of monetary policy. See Bilbiie (2017) for a more thorough discussion on this issue.

differences between HANK and TANK are slightly larger.

In summary, our quantitative exercise shows that a simple TANK model may provide a good approximation of a prototypical HANK model, even when the latter generates predictions that are sizeably different from its RANK counterpart. Needless to say, TANK models might not constitute a good approximation for richer HANK models, or more importantly the actual data. This is likely to be the case for economies where aggregate shocks have large effects on consumption heterogeneity, and in particular on the heterogeneity *within* unconstrained households. That could be the case in economies with endogenous unemployment risk (e.g. Ravn and Sterk (2017)), or where financial market participants have heterogeneous portfolios of assets (as in Kaplan et. al. (2016), Bayer et. al. (2015) and Lütticke (2017)). Yet, assessing the quantitative significance of fluctuations in consumption heterogeneity, both between and within different groups of agents, remains a largely open question both from a theoretical and an empirical viewpoint.

5 Optimal Monetary Policy in a TANK model

Most of the existing literature on HANK models has focused on studying the aggregate effects of monetary policy in the presence of households' heterogeneity. However, and partly because of the computational difficulties associated with HANK models, an open question is whether the presence of households heterogeneity changes the prescriptions for the design of monetary policy resulting from standard RANK models. For instance, it is subject to debate whether central banks should be concerned or not with measures of heterogeneity. In what follows we analyze the implications heterogeneity for the design of optimal monetary policy within a TANK model, which given the

Figure 3: The Effects of a Monetary Policy Shock: HANK vs TANK

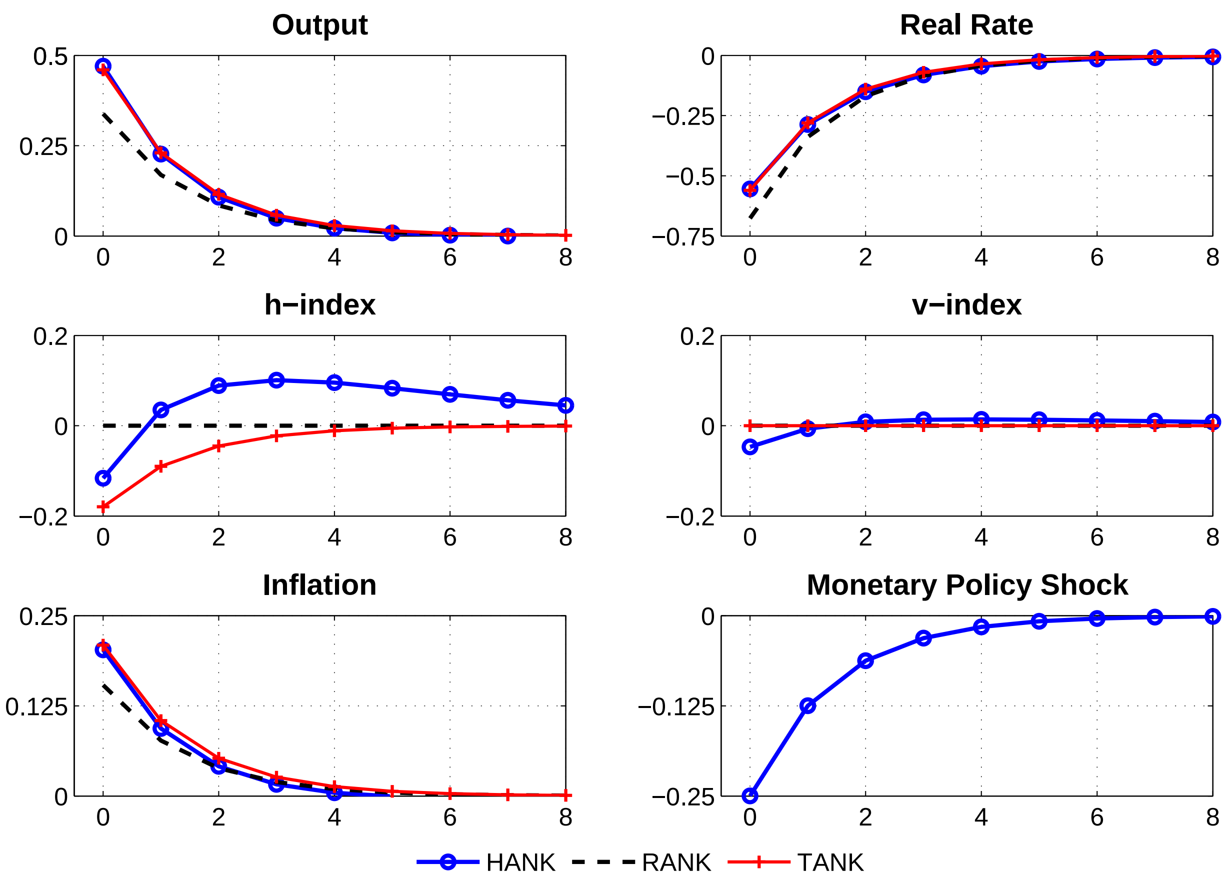


Figure 4: The Effects of a Preference Shock: HANK vs TANK

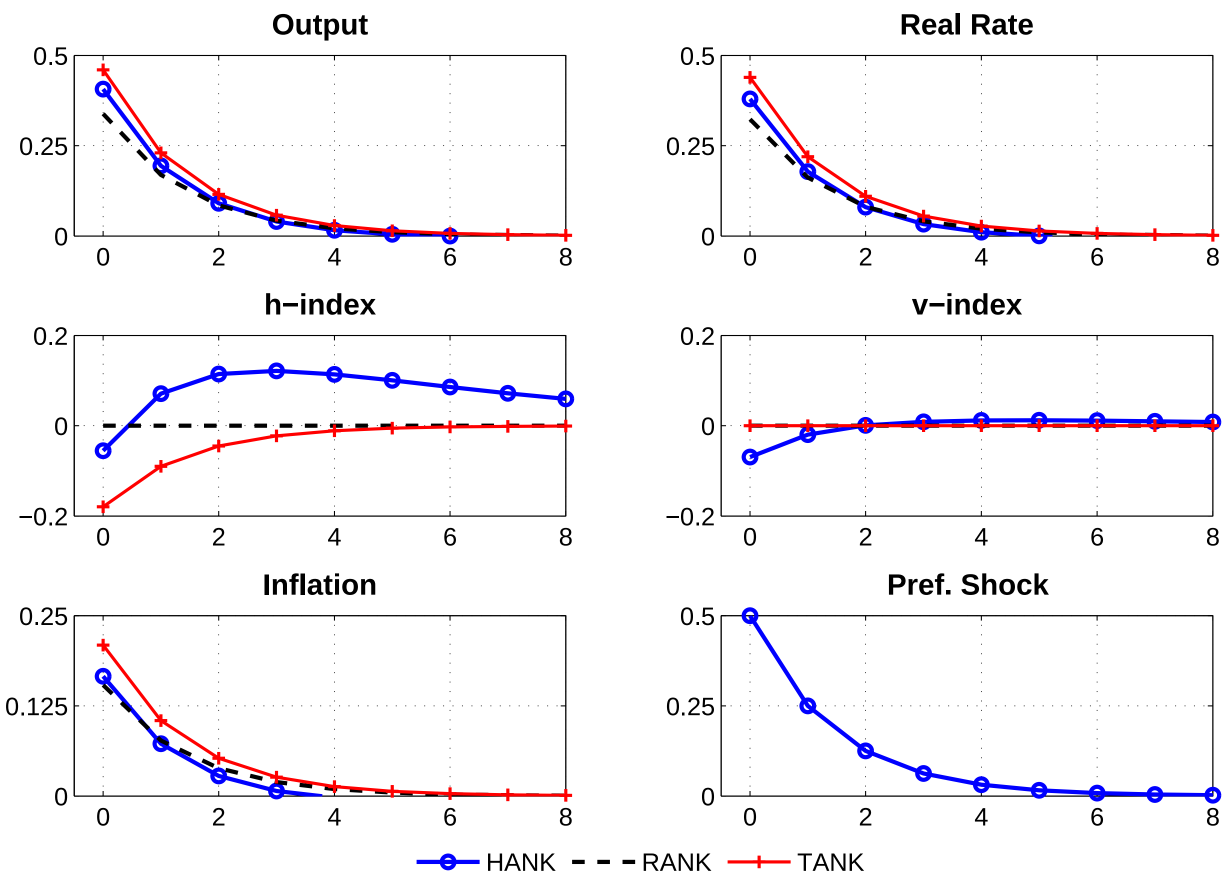
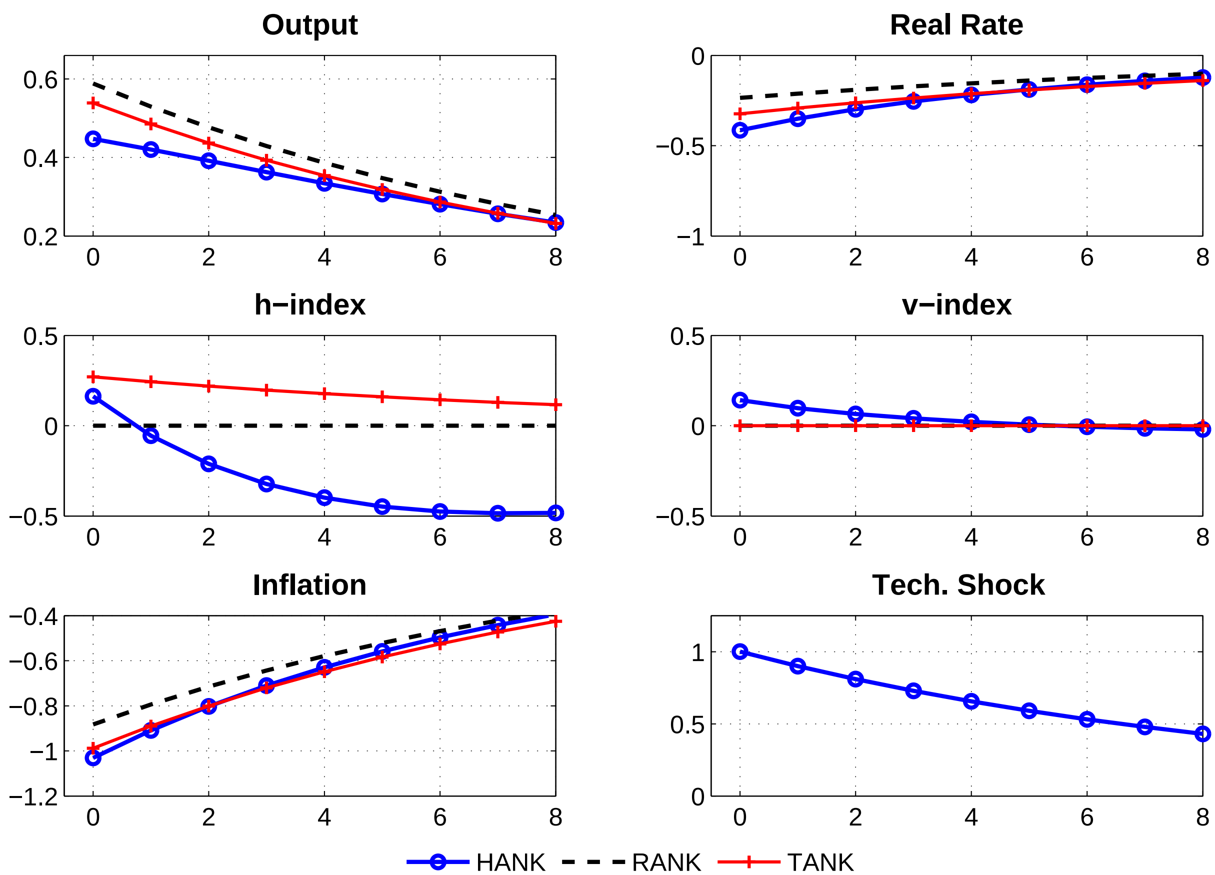


Figure 5: The Effects of a Technology Shock: HANK vs TANK



results of the previous section, could be viewed as a parsimonious representation of a richer HANK model.

In particular, we consider a utilitarian central bank, namely one assigning equal weight to the utility of all the agents. It can be shown that a second-order approximation to the central bank's welfare criterion around an efficient steady state (with no inequality) gives the central bank's loss function

$$\mathcal{L}_t \simeq \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \pi_t^2 + \alpha_y \tilde{y}_t^2 + \alpha_h \hat{h}_t^2 \right\} + t.i.p. \tag{26}$$

where $\alpha_y = \left(\sigma + \frac{\phi+\alpha}{1-\alpha}\right)\frac{1}{\xi}$ and $\alpha_h \equiv \frac{\sigma}{\xi}\left(\frac{1-\lambda}{\lambda}\right)$ and $t.i.p.$ indicates all the terms independent of policy. The only difference with respect to the loss function of a standard RANK model is the presence of the term $\alpha_h \hat{h}_t^2$. That term indicates that fluctuations in consumption heterogeneity generate welfare losses for the central bank. Intuitively, a benevolent central bank would like to spread to costs of fluctuations equally across all the agents, which implies no fluctuations in consumption heterogeneity.

The optimal monetary policy is then the solution to the following problem

$$\min \frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \pi_t^2 + \alpha_y \tilde{y}_t^2 + \alpha_h \hat{h}_t^2 \right\} \tag{27}$$

$$\text{s.t.} \quad \pi_t = \beta \mathbb{E}_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t \tag{28}$$

$$\hat{h}_t = \chi_y \widetilde{y}_t + \chi_n \widehat{y}_t^n. \tag{29}$$

Notably, eq. (29) signals the presence of a policy trade-off related to heterogeneity. In the absence of heterogeneity, the optimal policy with a representative agent would be to stabilize both inflation and the output-gap (i.e. $\pi_t = \tilde{y}_t = 0$). In our TANK model, the central bank would also like to stabilize heterogeneity. However, this might not be possible. This is because as long as $\chi_n \neq 0$, fluctuations in the natural level of output leads to fluctuations in the heterogeneity index $\hat{h}_t$. As a result, a central bank that wishes to stabilize output at its natural level would have to tolerate some fluctuations in heterogeneity. Similarly, a central bank that wishes to stabilize heterogeneity would have to tolerate fluctuations in inflation and the output-gap.

Using the optimality conditions to the above problem, the optimal monetary policy can be charactized by the following price level targeting rule

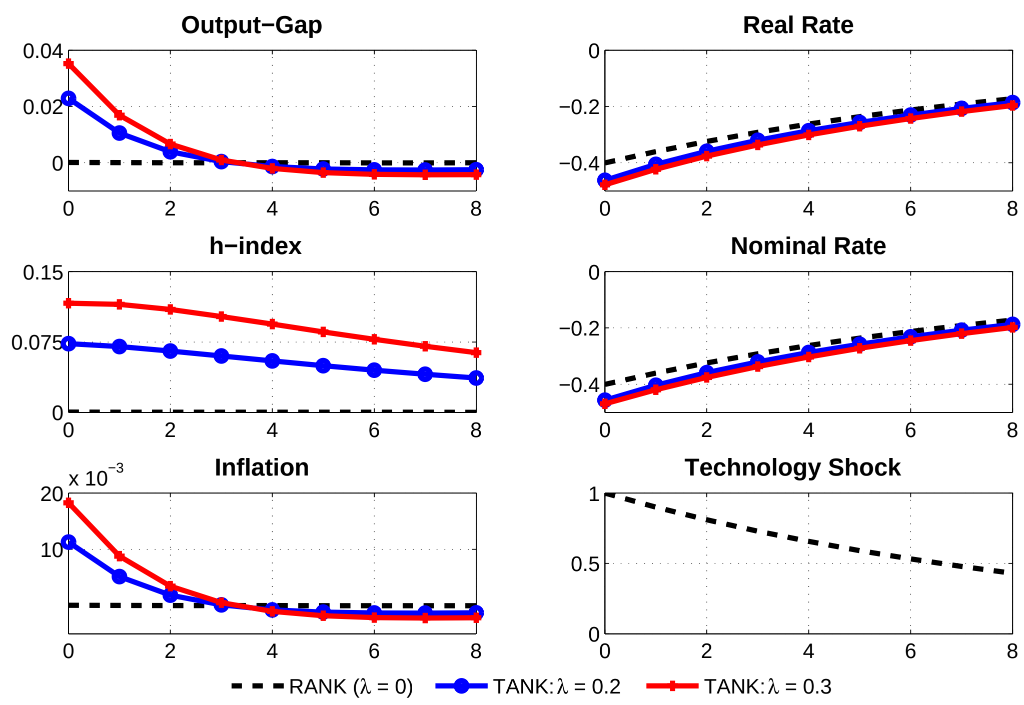
$$\hat{p}_t \equiv p_t - p_{-1} = -\frac{1}{\kappa}\left(\alpha_y \tilde{y}_t + \chi_y \alpha_h \hat{h}_t\right).$$

The latter equation, differently from a RANK model, indicates that the price level should respond to changes in heterogeneity. For example, in a situation where $\chi_y < 0$ (as under the fiscal policies considered above), the price level should increase (decrease) whenever there is an increase (decrease) in the h-index. In other words, the central bank should tolerate some inflation whenever consumption of Ricardian agents increases more than average consumption.

We finally examine the magnitude of the effects of hetereogeneity on the optimal policy. To that end, Figure 6 illustrates the impulse response to a 1% technology shock under the optimal monetary policy in a TANK model (solid lines) in comparison to a RANK model (dashed line), under the baseline calibration of Section 3. Clearly, in a RANK model inflation and the output gap remain constant in every period. Instead TANK model, we do observe an increase in both inflation and output gap, but of a neglible magnitude (less than 0.05%), as well as an increase in the h-index. Also, the response of the nominal rate is almost identical in a RANK and in a TANK model. Overall, these results suggest that from a quantitative viewpoint, heterogeneity seems to alter only minimally the monetary policy prescriptions obtained in a standard RANK model.

The main reason for that result is that, under our baseline calibration, the optimal weight on heterogeneity a_h in eq. (26) is very small and is less than 1% of the weight that should be assigned to inflation stabilization. Clearly, if central banks attached a higher weight to measures of heterogeneity, heterogeneity would have a larger quantitative effects on interest rates and inflation, and other variables.

Figure 6: Optimal Monetary Policy in TANK - The Effects of Technology Shocks



6 Conclusions

We identify two key dimensions of heterogeneity that explain the differential behavior of a Heterogeneous agents New-Keynesian economy (HANK) relative to its Representative Agent (RANK) counterpart: (i) the differences in average consumption *between* constrained and unconstrained households and (ii) consumption heterogeneity *within* the subset of unconstrained households. A tractable two-agent model (TANK), which only captures the first dimension, approximates reasonably well the implications of a baseline HANK model regarding the effects of aggregate shocks on aggregate variables. We then use the TANK model as a laboratory to study the design of optimal monetary policy, and show that heterogeneity introduces a non-trivial policy trade-off, but that under standard calibrations price stability is nearly optimal, as in standard RANK models.

Our results should not be interpreted as implying that existing HANK models are not useful, nor that households' heterogeneity is not important for the conduct of monetetary policy. On the contrary, HANK models seem essential to describe economies with large fluctuations in consumption heterogeneity *within* financial market participants. Also, our analysis highlights some aspects of heterogeneity that are relevant for monetary policy, but that are not captured by simple models, nor are well understood from an empirical viewpoint. For this reason, we hope that our results will constitute a useful input for future studies on monetary policy with heterogeneous agents.

The current TANK model can be extended along several dimensions. First, to introduce some form of idiosyncratic risk between Ricardian and Keynesian agents. One possibility is to consider that in each period, a fraction of Ricardian agent might become Keynesian, and viceversa, as e.g. in Nisticó (2016) and Bilbiie (2017). That extension of the model could be useful to address the so-called "forward-guidance" puzzle inherent in representative agent models, and that are also present in the current version of our TANK model. Second, it sthe comparison between TANK and HANK models, could be extended to alternative frameworks –e.g. with capital, government debt and other assets (liquid and illiquid)– to understand to what extent TANK models might be able to capture some of the defining features of the richer HANK models (e.g. Kaplan et. al. (2016)).

APPENDIX

A1. Some Approximations

Using the definition $C_t^R \equiv \frac{1}{1-\lambda_t}\int_{s\in\Theta_t} C_t^s ds$, a second-order approximation to the left hand side of (2) is given by

$$\begin{aligned}
\frac{Z_t}{1-\lambda_t}\int_{s\in\Theta_t}(C_t^s)^{-\sigma}ds &= \frac{Z_t}{1-\lambda_t}\left(C_t^R\right)^{-\sigma}\int_{s\in\Theta_t}\left(\frac{C_t^s}{C_t^R}\right)^{-\sigma}ds \\
&\simeq \frac{Z_t}{1-\lambda_t}\left(C_t^R\right)^{-\sigma}\left\{\int_{s\in\Theta_t}\left[1-\sigma\left(\frac{C_t^s}{C_t^R}-1\right)+\frac{\sigma\left(1+\sigma\right)}{2}\left(\frac{C_t^s}{C_t^R}-1\right)^2\right]ds\right\} \\
&= Z_t\left(C_t^R\right)^{-\sigma}\left[1+\frac{\sigma\left(1+\sigma\right)}{2}\frac{1}{1-\lambda_t}\int_{s\in\Theta_t}\left(\frac{C_t^s}{C_t^R}-1\right)^2 ds\right] \\
&= Z_t\left(C_t^R\right)^{-\sigma}\left[1+\frac{\sigma\left(1+\sigma\right)}{2}var_{s|t}\{c_t^s\}\right].
\end{aligned}$$

where $var_{s|t}\{c_t^s\} \equiv \frac{1}{1-\lambda_t}\int_{s\in\Theta_t}\left(c_t^s - c_t^R\right)^2 ds$ and where in the last step we have use the fact that up to a second order approximation $\left(\frac{C_t^s}{C_t^R}-1\right)^2 \simeq \left(c_t^s - c_t^R\right)^2 \equiv \left(\log\frac{C_t^s}{C_t^R}\right)^2$.

Similarly, using the definition $C_{t+1|t}^R \equiv \frac{1}{1-\lambda_t}\int_{s\in\Theta_t} C_{t+1}^s ds$ we can write the term inside the expectations on the right hand size of (2) as

$$\begin{aligned}
\frac{Z_{t+1}}{1-\lambda_t}\int_{s\in\Theta_t}(C_{t+1}^s)^{-\sigma}ds &\simeq \frac{Z_{t+1}}{1-\lambda_t}\left(C_{t+1|t}^R\right)^{-\sigma}\int_{s\in\Theta_t}\left(\frac{C_{t+1}^s}{C_{t+1|t}^R}\right)^{-\sigma}ds \\
&= \frac{Z_{t+1}}{1-\lambda_t}\left(C_{t+1}^R\right)^{-\sigma}\left(\frac{C_{t+1|t}^R}{C_{t+1}^R}\right)^{-\sigma}\left\{\int_{s\in\Theta_t}\left[1-\sigma\left(\frac{C_{t+1}^s}{C_{t+1|t}^R}-1\right)+\frac{\sigma\left(1+\sigma\right)}{2}\left(\frac{C_{t+1}^s}{C_{t+1|t}^R}-1\right)^2\right]ds\right\} \\
&= Z_{t+1}\left(C_{t+1}^R\right)^{-\sigma}\Phi_t^{mean}\left[1+\frac{\sigma(1+\sigma)}{2}var_{s|t}\{c_{t+1}^s\}\right]
\end{aligned}$$

where $var_{s|t}\{c_{t+1}^s\} \equiv \frac{1}{1-\lambda_t}\int_{s\in\Theta_t}\left(c_{t+1}^s - c_{t+1|t}^R\right)^2 ds$ and $\Phi_{t+1}^{mean} \equiv \left(\frac{C_{t+1|t}^R}{C_{t+1}^R}\right)^{-\sigma}$.

Combining these results, we get the Euler equation

$$Z_t(C_t^R)^{-\sigma} = \beta R_t \mathbb{E}_t\left\{Z_{t+1}(C_{t+1}^R)^{-\sigma}V_{t+1}\right\}$$

where

$$V_{t+1} \equiv \Phi_{t+1}^{mean} \Phi_{t+1}^{var}$$

where $\Phi_{t+1}^{var} \equiv \frac{2+\sigma(1+\sigma)var_{s|t}\{c_{t+1}^{s}\}}{2+\sigma(1+\sigma)var_{s|t}\{c_{t}^{s}\}}$, which corresponds to eq. (3) in the main text.

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