ONLINE APPENDIX FOR "INTERNATIONAL DEBT DELEVERAGING"

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Appendix A: A model with interest rate spreads

In the model presented in the main text all the agents, and countries, are subject to the same interest rate. However, in many cases tight credit conditions manifest themselves with high interest rates. In fact, often countries whose access to the international credit markets is restricted are charged a spread over the interest rate paid by unconstrained countries. This appendix shows how it is possible to reconcile this fact with the model without changing any of the results. In particular, in this appendix I present a model in which the borrowing limit is enforced through interest rate spreads and show that this model is isomorphic to the framework studied in the main text. The discussion draws on Uribe (2006).

For simplicity I focus on the economy without nominal rigidities described in Section 2, but the results can be extended to the case of a monetary union with nominal wage rigidities. Suppose that the representative household in country i is charged the country-specific interest rate $R_{i,t}$, potentially different from the world interest rate R_t. Suppose also that there is no limit to how much the household can borrow at the interest rate $R_{i,t}$. The Euler equation then writes

$$\lambda_{i,t} = R_{i,t} \beta E_t \left[\lambda_{i,t+1} \right],$$

where $\lambda_{i,t}$ is the Lagrange multiplier on the budget constraint.

In the model in the main text, in which the household is constrained by the borrowing limit (3) and it is charged the world interest rate R_t, the Euler equation can instead be written as

$$\lambda_{i,t} = \frac{R_t}{1 - \frac{\mu_{i,t} R_t}{\lambda_{i,t}}} \beta E_t \left[\lambda_{i,t+1} \right],$$

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where $\mu_{i,t}$ is the Lagrange multiplier on the borrowing constraint.

Notice that if the household is charged the interest rate

$$R_{i,t} = \frac{R_t}{1 - \frac{\mu_{i,t} R_t}{\lambda_{i,t}}},$$

the two Euler equations coincide. Moreover, we have that $R_{i,t} = R_t$ when $\mu_{i,t} = 0$, while $R_{i,t} > R_t$ when $\mu_{i,t} > 0$.[1] Intuitively, investors can make sure that an household respects the borrowing limit by charging a positive spread over the world interest rate anytime the household would violate the constraint if charged the world interest rate. In equilibrium, we would thus observe that high-debt constrained countries are charged a positive spread over the world interest rate.

To obtain a version of the model with interest rate spreads isomorphic to the model in the main text, we must make sure that the resource constraint of the household is not affected by the interest rate spreads. Indeed, whenever the constraint is binding there is a financial rent given by the difference between the cost of funds for the investor and the interest rate that the borrower would like to pay. In the model in the main text this rent accrues to the borrower, since constrained borrowers are charged the world interest rate, that is the cost of funds for investors. We must then ensure that financial rents go fully to the borrower also in the version of the model with interest rate spreads. Following Uribe (2006), this can be done by assuming the existence of domestic financial intermediaries that borrow at the world interest rate R_t and lend to households at the interest rate $R_{i,t}$. Assuming that the profits of the domestic financial intermediaries are fully rebated to households in a lump sum fashion, we obtain that the economy with the borrowing constraint described in the main text is isomorphic to the economy with spreads described in this appendix.

Appendix B: Numerical solution method

To solve the model numerically I employ the method proposed by Guerrieri and Lorenzoni (2017).

Computing the steady state of the model involves finding the interest rate that clears the bond market at the world level. The first step consists in deriving the optimal policy functions $C^T(B,A)$, $C^N(B,A)$ and $L(B,A)$ for a given interest rate R. To compute the optimal policy functions I discretize the endogenous state variable B using a grid with 600 points, and then iterate on the Euler equation and on the intratemporal optimality conditions using the endogenous gridpoints method of Carroll (2006). Using the optimal policies, it is possible to derive the inverse of the bond accumulation policy $g(B,A)$. This is used to update the conditional bond distribution $\Psi(B,A)$ according to the formula $\Psi_\tau(B,A) =$

1. Using the fact that $\mu_{i,t} \geq 0$ and rearranging the Euler equation in the main text, it is easy to check that $\mu_{i,t} R_t / \lambda_{i,t} < 1$.

$\sum_A \Psi_{\tau-1}\left(g(B,\tilde{A}),\tilde{A}\right) P(A|\tilde{A})$ for all $B \leq -\kappa$, where τ is the τ-th iteration and $P(A|\tilde{A})$ is the probability that $A_{t+1} = A$ if $A_t = \tilde{A}$. The bond accumulation function is not invertible at $B = -\kappa$, but the formula above holds if $g(-\kappa,A)$ is defined as the largest B such that $B' = -\kappa$ is optimal. Once the bond distribution has converged to the stationary distribution, I check whether the market for bonds clears. If not, I update the guess for the interest rate.

To compute the transitional dynamics, I first derive the initial and final steady states. I then choose a T large enough so that the economy has approximately converged to the final steady state at $t = T$ (I use $T = 150$, increasing T does not affect the results reported). The next step consists in guessing a path for the interest rate. I then set the policy functions for consumption in period T equal to the ones in the final steady state and iterate backward on the Euler equation and on the intratemporal optimality conditions to find the sequence of optimal policies $\{C_t^T(B,A), C_t^N(B,A), L_t(B,A)\}$. Next, I use the optimal policies to compute the sequence of bond distributions $\Psi_t(B,A)$ going forward from $t = 0$ to $t = T$, starting with the distribution in the initial steady state. Finally, I compute the world demand for bonds in every period and update the path for the interest rate until the market clears in every period.

To compute the transitional dynamics with wage rigidities I follow a similar method. The only difference is that in every period $t \geq 0$ uninformed wage setters behave according to their policy functions in the initial steady state. However, since in period $T = 150$ the measure of uninformed wage setters is essentially 0, we can still set the policy functions for consumption in period T equal to the ones in the final steady state.

Appendix C: Analytic example

This appendix presents a simplified version of the baseline model, useful to sharpen intuition about the adjustment triggered by an episode of international deleveraging. To enhance the tractability of the model, here I study a perfect foresight economy with a stylized form of initial wealth heterogeneity, in which deleveraging takes place in a single period.

The equilibrium conditions are the same as in Section 2. I restate them here for convenience:

$$\left(C_{i,t}^T\right)^{\omega(1-\gamma)-1}\left(C_{i,t}^N\right)^{(1-\omega)(1-\gamma)} = R_t\left(\beta E_t\left[\left(C_{i,t+1}^T\right)^{\omega(1-\gamma)-1}\left(C_{i,t+1}^N\right)^{(1-\omega)(1-\gamma)}\right] + \frac{\mu_{i,t}}{\omega}\right) \tag{C.1}$$

$$B_{i,t+1} \geq -\kappa_t, \quad \text{with equality if } \mu_{i,t} > 0, \tag{C.2}$$

$$p_{i,t}^N = \frac{1-\omega}{\omega}\frac{C_{i,t}^T}{C_{i,t}^N} \tag{C.3}$$

$$L_{i,t}^{\psi} = w_{i,t}\left(C_{i,t}^T\right)^{\omega(1-\gamma)-1}\left(C_{i,t}^N\right)^{(1-\omega)(1-\gamma)} \tag{C.4}$$

$$\alpha_T A^T_{i,t} \left(L^T_{i,t}\right)^{\alpha_T - 1} = w_{i,t}. \tag{C.5}$$

$$p^N_{i,t} \alpha_N A^N_{i,t} \left(L^N_{i,t}\right)^{\alpha_N - 1} = w_{i,t}. \tag{C.6}$$

$$C^N_{i,t} = A^N_{i,t} \left(L^N_{i,t}\right)^{\alpha_N} \tag{C.7}$$

$$C^T_{i,t} = A^T_{i,t} \left(L^T_{i,t}\right)^{\alpha_N} + B_{i,t} - \frac{B_{i,t+1}}{R_t} \tag{C.8}$$

$$L_{i,t} = L^T_{i,t} + L^N_{i,t} \tag{C.9}$$

$$\int_0^1 B_{i,t+1} \, \mathrm{d}i = 0. \tag{C.10}$$

I make some parametric assumptions to simplify the derivations. I thus assume that $\gamma = 1$, so that utility from consumption is logarithmic, and focus on the limit $\psi \to +\infty$, so that labor supply is infinitely inelastic. This last assumption and equation $(C.4)$ imply that labor supply is constant and equal to 1. Moreover, to abstract from uncertainty and precautionary savings, I assume that in every country productivity in both sectors is constant and normalized to 1, so that $A^T_{i,t} = A^N_{i,t} = 1$ for all t.

For future reference, notice that under these assumptions the Euler equation $(C.1)$ becomes

$$\frac{1}{C^T_{i,t}} = \frac{\beta R_t}{C^T_{i,t+1}} + R_t \mu_{i,t}. \tag{C.11}$$

In addition, using $(C.3)$, $(C.5)$ and $(C.6)$ gives

$$L^N_{i,t} = \frac{\alpha_N}{\alpha_T} \frac{1-\omega}{\omega} \left(L^T_{i,t}\right)^{1-\alpha_T} C^T_{i,t}. \tag{C.12}$$

Finally, the expression above can be combined with $(C.9)$ to obtain

$$1 - L^T_{i,t} = \frac{\alpha_N}{\alpha_T} \frac{1-\omega}{\omega} \left(L^T_{i,t}\right)^{1-\alpha_T} C^T_{i,t}. \tag{C.13}$$

The economy starts from a steady state in which half of the countries are creditors and half are debtors. In what follows I denote creditor and debtor countries respectively with subscripts c and d. In the initial steady state each creditor (debtor) country holds assets $B_0 > 0$ $(-B_0 < 0)$. In period 0 there is a permanent drop in the borrowing limit that forces debtor countries to reduce their debt. In particular, starting from period 0 onward the borrowing limit is equal to $\bar{\kappa}$, with $\bar{\kappa} < B_0$. This implies that deleveraging takes place in a single period after which the economy reaches its final steady state. Hence, we can divide the analysis into a short run (period 0) and a long run (periods $t \geq 1$).

Initial and final steady states. In steady state consumption of both creditor and debtor countries is constant, and the borrowing constraint is not binding for creditors. Hence, by creditors' Euler equation $(C.1)$, the steady state interest rate is $R = 1/\beta$. Moreover, in steady state each country rolls over its stock of foreign assets, so that by

$(C.8)$ steady state tradable consumption in a generic country i is

$$C_i^T = \left(L_i^T\right)^{\alpha_T} + B_i\left(1-\beta\right), \tag{C.14}$$

where the absence of a time subscript denotes the value of a variable in steady state. In steady state the labor market clears in every country. Hence, combining $(C.14)$ with $(C.12)$ and $(C.13)$ gives the steady state values of C_i^T, $L_{i,}^T$ and L_i^N as a function of B_i. For future reference, notice that $(C.14)$ and $(C.13)$ imply that $C_i^T = C^T(B_i)$ with $C^{T\prime}(\cdot) > 0$, and $L_i^T = L^T(B_i)$ with $L^{T\prime}(\cdot) < 0$. In words, in steady state tradable consumption (labor) is strictly increasing (decreasing) in foreign assets.

The economy starts from a steady state in which $B_c = B_0$ and $B_d = -B_0$. In period 1 the economy converges to the final steady state in which $B_c = \bar{\kappa}$ and $B_d = -\bar{\kappa}$. Recalling that $\bar{\kappa} < B_0$, the analysis above implies that comparing the initial steady state to the final one debtor countries increase their consumption of tradables and decrease the fraction of labor allocated to the tradable sector. This happens because in the post-deleveraging final steady state debtor countries have higher financial wealth compared to the initial one. The opposite is true for creditor countries. I now turn to the short-run adjustment triggered by a deleveraging shock.

Short-run response to deleveraging shock. I now derive the short run adjustment to the deleveraging shock. To gain intuition, it is useful to recast the equilibrium in terms of an aggregate demand/aggregate supply diagram. To derive an aggregate demand equation for tradable goods, start by noticing that in period 0 debtor countries are borrowing constrained, so their demand for tradables is

$$C_{d,0}^T = \left(L_{d,0}^T\right)^{\alpha_T} + \frac{\bar{\kappa}}{R_0} - B_0. \tag{C.15}$$

Effectively, in period 0 debtor countries act as hand-to-mouth consumers, and consume all their production of tradables net of the amount needed to reduce external debt, so as to satisfy the new borrowing limit. Instead, creditors are on their Euler equation, and so their demand for consumption is

$$C_{c,0}^T = \frac{C^T\left(\bar{\kappa}\right)}{\beta R_0}. \tag{C.16}$$

Importantly, in the short run creditors' demand for consumption does not depend directly on their production of tradable goods, since $C^T(\bar{\kappa})$ is independent of $L_{c,0}^T$. Taken together, $(C.15)$ and $(C.16)$ imply that in the short run debtor countries have a higher propensity to consume out of tradable income than creditors. Adding consumption demand from creditors and debtors gives the aggregate demand (AD) equation

$$C_{d,0}^T + C_{c,0}^T = \left(L_{d,0}^T\right)^{\alpha_T} + \frac{\bar{\kappa}}{R_0} - B_0 + \frac{C^T\left(\bar{\kappa}\right)}{\beta R_0}, \tag{AD}$$

which implies a negative relationship between global demand for tradable consumption and the world interest rate. Intuitively, debtors' demand for consumption

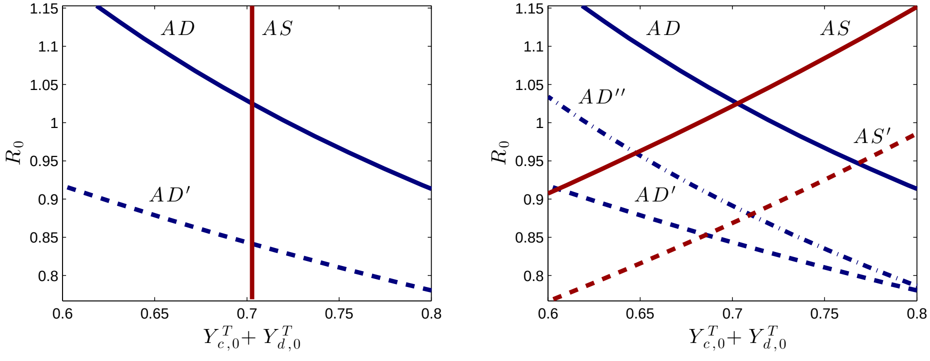

FIGURE C.1. Short-run impact of deleveraging. Left panel: fix output. Right panel: baseline model with labor reallocation.

is decreasing in the interest rate because their borrowing, gross of future interest payments, is constrained by the limit $\bar{\kappa}$. Instead, a rise in the interest rate depresses creditors' demand for consumption because it increases their incentives to save. Figure $C.1$ illustrates the downward-sloped relationship between R_0 and period 0 consumption implied by the AD curve.[2]

The AD equation also shows that aggregate demand is increasing in the borrowing limit $\bar{\kappa}$. On the one hand, a drop in the borrowing limit forces debtors to devote a higher fraction of their income to debt repayment, driving resources away from consumption. On the other hand, since $C^{T\prime}(\bar{\kappa}) > 0$, a lower debt limit means that creditors' future assets and consumption will be lower, inducing creditor countries to reduce the present demand for consumption. Graphically, following a drop in the borrowing limit the AD curve shifts left to AD′. This generates a drop in the world interest rate R_0 that, at least partly, mitigates the negative impact of deleveraging on consumption demand. I will now show that the supply side of the economy plays a key role in determining the severity of the fall in R_0 in response to the deleveraging shock.

Let us start by considering a case in which supply does not respond to the deleveraging shock. In particular, let us assume that $L^T_{c,0} = L^T(B_0)$ and $L^T_{d,0} = L^T(-B_0)$, so that the period 0 production of tradables is the same as in the initial steady state. This case is depicted by the left panel of Figure $C.1$, in which the short-run aggregate supply of tradables, the AS curve, is just a straight line corresponding to $Y^T_{c,0} + Y^T_{d,0} = \left(L^T(B_0)\right)^{\alpha_T} + \left(L^T(-B_0)\right)^{\alpha_T}$. As shown by the graph, the deleveraging shock, which moves the AD curve to AD′, produces a sharp fall in the interest rate, while leaving unchanged world production of tradable goods. Intuitively, the sharp fall in the interest rate is needed to offset the impact of the deleveraging shock on

2. To construct the figure I have set $\beta = 0.9756$, $\omega = 0.2$, $\alpha_T = 0.65$, $\alpha_N = 0.65$, $B_0 = 0.9$ and $\bar{\kappa} = 0.675$.

demand, so that demand for consumption is strong enough to sustain the fixed supply of tradable goods.

In reality, aggregate supply does respond to the deleveraging shock. In fact, combining equation $(C.13)$ with the consumption demands $(C.15)$ and $(C.16)$ gives

$$1-L_{d,0}^{T}=\frac{\alpha_N}{\alpha_T}\frac{1-\omega}{\omega}\left(L_{d,0}^{T}\right)^{1-\alpha_T}\left(\left(L_{d,0}^{T}\right)^{\alpha_T}+\frac{\bar{\kappa}}{R_0}-B_0\right) \tag{C.17}$$

$$1-L_{c,0}^{T}=\frac{\alpha_N}{\alpha_T}\frac{1-\omega}{\omega}\frac{C^T\left(\bar{\kappa}\right)}{\beta R_0}. \tag{C.18}$$

According to these equations, both $L_{d,0}^{T}$ and $L_{c,0}^{T}$ are increasing in R_0 and decreasing in $\bar{\kappa}$. This happens because of households' desire to consume a balanced basket of tradable and non-tradable goods. Hence, as consumption of tradables increases demand for non-tradables rises, leading to a reallocation of labor toward the non-tradable sector. Since, both for creditor and debtor countries, consumption of tradables is decreasing in R_0 and increasing in $\bar{\kappa}$, this effect explains why world production of tradables is increasing in R_0 and decreasing in $\bar{\kappa}$. Consequently, after a deleveraging shock, keeping R_0 constant, world production of tradables increases.

The case of endogenous production is illustrated by the right panel of Figure $C.1$. The AS curve is upward sloped, capturing the positive relationship between R_0 and production of tradables. The fall in $\bar{\kappa}$ produces a shift right of the AS curve to AS$'$, capturing the expansion in production of tradables following the deleveraging shock. Moreover, the rise in tradable labor following the deleveraging shock also affects the AD curve. In fact, since debtor countries spend all their increase in tradable income in consumption, the rise in $L_{d,0}^{T}$ driven by the fall in $\bar{\kappa}$ sustains aggregate demand.[3] This effect is represented graphically by the shift of the AD curve from AD$'$ to AD$''$.

Compared to the fixed output case, the movement of the AS points toward a lower equilibrium interest rate, while the shift of the AD curve points toward a higher rate. However, it is possible to show that the demand effect dominates, so that the interest rate falls less in response to a deleveraging shock in the economy with endogenous production compared to the fixed output economy. To see this point, consider that using the AD equation we can express equilibrium on the market for tradables as

$$\left(L_{d,0}^{T}\right)^{\alpha_T}+\frac{\bar{\kappa}}{R_0}-B_0+\frac{C^T\left(\bar{\kappa}\right)}{\beta R_0}=\left(L_{d,0}^{T}\right)^{\alpha_T}+\left(L_{c,0}^{T}\right)^{\alpha_T}, \tag{C.19}$$

which can be simplified and rearranged as

$$R_0=\frac{\bar{\kappa}+C^T\left(\bar{\kappa}\right)/\beta}{B_0+\left(L_{c,0}^{T}\right)^{\alpha_T}}. \tag{C.20}$$

3. Instead, changes in $L_{c,0}^{T}$ do not have direct impact on creditors' demand, which is determined by their consumption in the final steady state, $C^T\left(\bar{\kappa}\right)$, and by the interest rate R_0.

This expression implies that in equilibrium the supply side of the economy affects R_0 only through $L^T_{c,0}$. Intuitively, since debtor countries have a propensity to consume out of income of 1, movements in $L^T_{d,0}$ have opposing effects on aggregate demand and supply that exactly cancel out in equilibrium. Instead, creditors' demand does not directly depend on $L^T_{c,0}$. Hence, when $L^T_{c,0}$ falls it contracts aggregate supply without any direct effect on demand, leading to a higher equilibrium rate. This implies that R_0 is higher in the economy with endogenous output compared to the economy with fixed output if $L^T_{c,0} < L^T(B_0)$, which turns out to be always the case.[4] Moreover, it is possible to show that debtor countries expand their production of tradables during deleveraging (i.e. $L^T_{d,0} > L(-B_0)$),[5] so that the response of total output is ambiguous, and depending on parameter values the deleveraging shock can produce a fall or rise in the world production of tradable goods.

Summing up, the deleveraging shock induces a reallocation of labor from the non-tradable to the tradable sector in debtor countries, while the opposite is true for creditors. Since debtor countries have a higher propensity to consume out of income than creditors, this reallocation of production unambiguously mitigates the fall in aggregate demand for tradable goods, and the drop in the world interest rate during deleveraging.

4. Suppose instead that $L^T_{c,0} > L^T(B_0)$. Then equation $(C.13)$ implies that $C^T_{c,0} < C^T(B_0)$. Using the resource constraint this inequality can be written as

$$\left(L^T_{c,0}\right)^{\alpha_T} - \frac{\bar{\kappa}}{R_0} + B_0 < \left(L^T(B_0)\right)^{\alpha_T} + B_0(1-\beta),$$

which can be rearranged as

$$\left(L^T_{c,0}\right)^{\alpha_T} - \left(L^T(B_0)\right)^{\alpha_T} < \frac{\bar{\kappa}}{R_0} - \beta B_0.$$

Notice that if $\beta B_0 > \bar{\kappa}/R_0$ this inequality holds only if $L^T_{c,0} < L^T(B_0)$, a contradiction. We are left to prove that $\beta B_0 > \bar{\kappa}/R_0$. Using $(C.20)$ this condition can be written as

$$\frac{\bar{\kappa} + C^T(\bar{\kappa})/\beta}{B_0 + \left(L^T_{c,0}\right)^{\alpha_T}} > \frac{\bar{\kappa}}{\beta B_0}.$$

Combining this inequality with$(C.14)$ evaluated at $B_i = \bar{\kappa}$ and simplifying gives

$$\frac{\left(L^T(\bar{\kappa})\right)^{\alpha_T}}{\left(L^T_{c,0}\right)^{\alpha_T}} > \frac{\bar{\kappa}}{B_0}.$$

The right-hand side of the inequality is smaller than one by assumption. Moreover, using the fact that $\beta R_0 < 1$ and creditors' Euler equation gives $C^T_{c,0} > C^T(\bar{\kappa})$. Since $L^T_{i,t}$ is strictly decreasing in $C^T_{i,t}$, this implies that $L^T(\bar{\kappa}) > L^T_{c,0}$, so that the inequality above always holds. This proves that $\beta B_0 > \bar{\kappa}/R_0$ and consequently that $L^T_{c,0} > L^T(B_0)$.

5. The proof follows the steps of footnote 4.

Appendix D: Model with nominal rigidities

This section presents a detailed description of the model with nominal wage rigidities studied in Sections 4 and 5. There are two key changes with respect to the baseline model of Section 2. First, there is monopolistic competition on the labor market. Second, there are frictions in the adjustment of nominal wages, which create a channel through which monetary policy can affect real variables.

As in the model of Section 2, the world is composed of a continuum of measure one of small open economies indexed by $i \in [0,1]$. These economies are continuously hit by idiosyncratic productivity shocks. There is no uncertainty at the world level, and the only aggregate shock is a fully unexpected drop in the borrowing limit.

Households. The expected lifetime utility of the representative household in a generic country i is

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_{i,t}^{1-\gamma}-1}{1-\gamma}-\frac{\int_0^1 L_{i,t}(j)^{1+\psi}\,dj}{1+\psi}\right)\right], \tag{D.1}$$

with $\gamma \geq 1$ and $\psi \geq 0$. $E_t[\cdot]$ is the expectation operator conditional on information available at time t and $0<\beta<1$ is the subjective discount factor. Consumption $C_{i,t}$ is a Cobb-Douglas aggregate of a tradable good $C_{i,t}^T$ and a non-tradable good $C_{i,t}^N$:

$$C_{i,t} = \left(C_{i,t}^T\right)^{\omega}\left(C_{i,t}^N\right)^{1-\omega}, \tag{D.2}$$

where $0<\omega<1$. Each household supplies a continuum of measure one of differentiated labor services indexed by $j \in [0,1]$, so that $L_{i,t}(j)$ denotes labor effort of type j.

Each household can trade in one period, non-state contingent real and nominal bonds. Real bonds are denominated in units of the tradable consumption good and pay the gross interest rate R_t. The interest rate on real bonds is common across countries, and hence R_t can be interpreted as the *world interest rate*. Nominal bonds are denominate in units of the domestic currency and pay the gross nominal interest rate $R_{i,t}^n$. Notice that $R_{i,t}^n$ can differ across countries. To simplify the analysis, I assume that nominal bonds are traded only across households sharing the same domestic currency.

The household budget constraint in terms of the domestic currency is:

$$P_{i,t}^T C_{i,t}^T + P_{i,t}^N C_{i,t}^N + P_{i,t}^T\frac{B_{i,t+1}}{R_t}+\frac{B_{i,t+1}^n}{R_{i,t}^n} = \int_0^1 W_{i,t}(j)L_{i,t}(j)dj + P_{i,t}^T B_{i,t} + B_{i,t}^n + \Pi_{i,t}^n + T_{i,t}. \tag{D.3}$$

The left-hand side of this expression represents the household's expenditure. $P_{i,t}^T$ and $P_{i,t}^N$ denote respectively the price of a unit of tradable and non-tradable good in terms of country i currency. Hence, $P_{i,t}^T C_{i,t}^T + P_{i,t}^N C_{i,t}^N$ is the total nominal expenditure in consumption. $B_{i,t+1}$ and $B_{i,t+1}^n$ denote respectively the purchase of real and nominal bonds made by the household at time t, at prices $P^T_{i,t}/R_t$ and $1/R^n_{i,t}$. If $B_{i,t+1} < 0$ or $B^n_{i,t+1} < 0$ the household is holding a debt.

The right-hand side captures the household's income. $W_{i,t}(j)$ denotes the nominal wage paid to type j labor. Hence $\int_0^1 W_{i,t}(j)L_{i,t}(j)dj$ is the household's total labor income. Labor is immobile across countries and hence wages are country-specific. $P^T_{i,t}B_{i,t}$ and $B^n_{i,t}$ represent the gross returns on investment in bonds made at time $t-1$. $\Pi^n_{i,t}$ are the profits received from firms expressed in units of the domestic currency. All domestic firms are wholly owned by domestic households and equity holdings within these firms are evenly divided among them. Finally, $T_{i,t}$ is a lump-sum transfer, or tax if $T_{i,t} < 0$, that the household receives from the government.

The borrowing limit is such that expected debt repayment in terms of tradables cannot exceed the exogenous limit κ_t. Hence, the end-of-period bond position has to satisfy

$$B_{i,t+1} + E_t\left[\frac{B^n_{i,t+1}}{P^T_{i,t+1}}\right] \geq -\kappa_t. \tag{D.4}$$

Similar to the borrowing limit (3), this constraint captures a case in which the expected repayment that a household can credibly promise to lenders is equal to κ_t units of the tradable good.

In contrast with the baseline model of Section 2, here households do not choose directly how much labor to supply to the market. Instead, wages are negotiated by labor unions, to be described below, which act on behalf of the households. Once wages are set, households supply all the labor demanded by firms.

The household's optimization problem is to choose a sequence $\{C^T_{i,t}, C^N_{i,t}, B_{i,t+1}, B^n_{i,t+1}\}_{t\geq 0}$ to maximize the expected present discounted value of utility $(D.1)$, subject to the consumption aggregator $(D.2)$, the budget constraint $(D.3)$ and the borrowing limit $(D.4)$, taking the initial bond holdings $B_{i,0}$ and $B^n_{i,0}$, a sequence for income $\{\int_0^1 W_{i,t}(j)L_{i,t}(j)dj + \Pi^n_{i,t}\}_{t\geq 0}$, prices $\{R_t, R^n_{i,t}, P^T_{i,t}, P^N_{i,t}, W_{i,t}\}_{t\geq 0}$, and the path for the borrowing limit $\{\kappa_t\}_{t\geq 0}$ as given. The household's first-order conditions can be written as

$$P^N_{i,t} = P^T_{i,t}\frac{1-\omega}{\omega}\frac{C^T_{i,t}}{C^N_{i,t}} \tag{D.5}$$

$$\frac{\lambda_{i,t}}{R_t} = \beta E_t\left[\lambda_{i,t+1}\right] + \mu_{i,t} \tag{D.6}$$

$$\frac{\lambda_{i,t}}{R^n_{i,t}} = \beta E_t\left[\lambda_{i,t+1}\frac{P^T_{i,t}}{P^T_{i,t+1}}\right] + \mu_{i,t}E_t\left[\frac{P^T_{i,t}}{P^T_{i,t+1}}\right] \tag{D.7}$$

$$B_{i,t+1} + E_t\left[\frac{B^n_{i,t+1}}{P^T_{i,t+1}}\right] \geq -\kappa_t, \quad \text{with equality if } \mu_{i,t} > 0, \tag{D.8}$$

where $\lambda_{i,t} \equiv \omega C^{1-\gamma}_{i,t}/C^T_{i,t}$ denotes the marginal utility from consumption of the tradable good, while $\mu_{i,t}$ is the non-negative Lagrange multiplier associated with the borrowing limit. As in the model of Section 2, the optimality condition $(D.5)$ equates the marginal rate of substitution of the two consumption goods, tradables and non-tradables, to their relative price, equation $(D.6)$ is the Euler equation for real bonds, and equation $(D.8)$ is the complementary slackness condition associated with the borrowing limit. In addition, equation $(D.7)$ is the optimality condition for investment in nominal bonds.

Combining $(D.6)$ and $(D.7)$ gives a no arbitrage condition between real and nominal bonds

$$R_{i,t}^{n} = R_t \frac{\beta E_t\left[\lambda_{i,t+1}\right] + \mu_{i,t}}{\beta E_t\left[\lambda_{i,t+1}\frac{P_{i,t}^{T}}{P_{i,t+1}^{T}}\right] + \mu_{i,t} E_t\left[\frac{P_{i,t}^{T}}{P_{i,t+1}^{T}}\right]}.$$

Notice that in the absence of uncertainty this expression reduces to the familiar no arbitrage condition $R_{i,t}^{n} = R_t P_{i,t+1}^{T}/P_{i,t}^{T}$, equating the nominal interest rate to the real interest rate multiplied by expected inflation. Since real bonds are denominated in units of the tradable good, the relevant inflation rate is tradable price inflation.

Firms. In both sectors, identical firms rent labor from households and produce consumption goods under perfect competition. Each sector is populated by a continuum of measure one of identical firms. The two sectors are symmetric, so to streamline the exposition I will present the problem of a firm in a generic sector $x = \{T, N\}$. The production function is

$$Y_{i,t}^{x} = A_{i,t}^{x}\left(L_{i,t}^{x}\right)^{\alpha_x},$$

where $0 < \alpha_x < 1$ and A_i^x denote productivity in sector x. L_i^x is a CES aggregate of all the differentiated labor services supplied by households

$$L_{i,t}^{x} = \left(\int_0^1 \left(L_{i,t}^{x}(j)\right)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}},$$

where and $\varepsilon > 1$, and $L_{i,t}^{x}(j)$ denotes the amount of labor service j purchased by firms in sector x.

Due to perfect labor mobility across sectors, every firm faces the same cost of labor. Hence, profits are equal to

$$\Pi_{i,t}^{x} = P_{i,t}^{x} Y_{i,t}^{x} - (1-\vartheta)\int_0^1 W_{i,t}(j) L_{i,t}^{x}(j) dj,$$

where ϑ is an employment subsidy that every firm receives from the government. The minimum (pre-subsidy) cost of a unit of aggregate labor L_i^x is given by

$$W_{i,t} = \left(\int_0^1 W_{i,t}^{1-\varepsilon}(j) dj\right)^{\frac{1}{1-\varepsilon}},$$

which can be taken as the aggregate wage. Using this definition, profit maximization implies

$$\alpha_x P_{i,t}^{x} A_{i,t}^{x}\left(L_{i,t}^{x}\right)^{\alpha_x - 1} = (1-\vartheta) W_{i,t},$$

while cost minimization implies the demand for type j labor

$$L_{i,t}^{x}(j)=\left(\frac{W_{i,t}}{W_{i,t}(j)}\right)^{\varepsilon}L_{i,t}^{x}. \tag{D.9}$$

The only purpose of the subsidy is to offset the impact of monopolistic distortions on the steady state of the economy. As it will become clear, this objective is reached if $\vartheta=1/\varepsilon$ and if the subsidy is financed with lump-sum taxes, so that $T_{i,t}=-\vartheta W_{i,t}\left(L_{i,t}^{T}+L_{i,t}^{N}\right)$. I will assume from now on that this is the case.

For future reference, notice that the total demand for labor of type j is equal to

$$L_{i,t}^{T}(j)+L_{i,t}^{N}(j)=\left(\frac{W_{i,t}}{W_{i,t}(j)}\right)^{\varepsilon}\left(L_{i,t}^{T}+L_{i,t}^{N}\right). \tag{D.10}$$

To complete the description of the labor market, it must be specified how wages are set. Before doing that, however, it is useful to state the market clearing conditions.

Market clearing. Goods market clearing implies that in every country i

$$C_{i,t}^{N}=Y_{i,t}^{N} \tag{D.11}$$

$$C_{i,t}^{T}=Y_{i,t}^{T}+B_{i,t}-\frac{B_{i,t+1}}{R_{t}}$$

Moreover, since, given wages, households satisfy firms' labor demand, equilibrium on the labor market implies

$$L_{i,t}(j)=L_{i,t}^{T}(j)+L_{i,t}^{N}(j), \tag{D.12}$$

for every labor type j.

Turning to the bonds market, I focus on equilibria in which nominal bonds are in zero net supply, so that $B_{i,t}^{n}=0$ for all i and t.[6] Instead the market for real bonds clear at the world level, so that $\int_0^1 B_{i,t+1}\,\mathrm{d}i=0$. By Walras' law, these two conditions imply that $\int_0^1 Y_{i,t}^{T}di=\int_0^1 C_{i,t}^{T}di$.

Labor unions and wage setting. For every labor type j there is a single labor union that sets the nominal wage $W_{i,t}(j)$. Every union acts on behalf of the households, and sets its nominal wage to maximize expected utility $(D.1)$, subject to the budget constraint $(D.3)$, and firms' demand for type j labor $(D.10)$. Every period each union is free to reset its wage. Given this setting, the maximization problem of a generic

6. This assumption is nearly without loss of generality. In fact, since trade in nominal bonds is allowed only across households sharing the same currency and since households are symmetric no trade in nominal bonds would occur in equilibrium under the flexible exchange rate regime. Trade in nominal bonds could occur inside a monetary union. However, since there is no uncertainty about the path of the price of the tradable good, it is easy to verify that in a monetary union real and nominal bonds are perfect substitutes. The only exception concerns the response of the economy to the unexpected shock to the borrowing limit in period 0. Since, this shock was previously unanticipated, it could engineer redistribution across countries and affect the equilibrium.

union j can be written as

$$\max_{W_{i,t}(j)} E_t^j \left[\mathscr{L}_{i,t} W_{i,t}(j) L_{i,t}(j) - \frac{L_{i,t}(j)^{1+\psi}}{1+\psi} \right] \quad \text{(D.13)}$$

$$\text{s.t.} \quad L_{i,t}(j) = \left(\frac{W_{i,t}}{W_{i,t}(j)} \right)^{\varepsilon} L_{i,t}, \quad \text{(D.14)}$$

where $\mathscr{L}_{i,t} \equiv \omega C_{i,t}^{1-\gamma} / (P_{i,t}^T C_{i,t}^T)$ denotes the households' marginal utility from nominal wealth, and E_t^j denotes expectations with respect to the information set of labor union j, to be defined below. The solution is

$$W_{i,t}(j) = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t^j \left[L_{i,t}(j)^{\psi} \right]}{E_t^j \left[\frac{\omega C_{i,t}^{1-\gamma}}{P_{i,t}^T C_{i,t}^T} \right]}. \quad \text{(D.15)}$$

This expression implies that, under perfect information, every union would set a wage equal to

$$\frac{W_{i,t}(j)}{P_{i,t}^T} = \frac{\varepsilon}{\varepsilon - 1} \frac{L_{i,t}(j)^{\psi}}{\frac{\omega C_{i,t}^{1-\gamma}}{C_{i,t}^T}} = w_{i,t}, \quad \text{(D.16)}$$

where the second equality is obtained from the fact that every union sets the same wage and that $w_{i,t} \equiv W_{i,t}/P_{i,t}^T$. This expression differs from the labor supply equation (**??**) only because of the presence of the wage markup $(\varepsilon - 1)/\varepsilon$. However, in equilibrium the markup is exactly offset by the employment subsidy. In fact, it is easy to show that under perfect information the real side of this model is exactly isomorphic to the model of Section 2. As a corollary, under perfect information monetary policy can only affect nominal variables, but not real ones.

I now introduce nominal wage rigidities by deviating from this perfect information benchmark. In particular, I allow for the possibility that unions might update their information set infrequently. As a result, unions may set current nominal wages based on outdated information, and so nominal wages might not respond immediately to unexpected shocks or to changes in monetary policy. This creates a channel through which monetary policy can influence the real economy.

To implement this idea, I adopt a variant of the Mankiw and Reis (2002) model of imperfect information. Start by noticing that in a generic period t, to operate under perfect information, it is sufficient for unions to observe the values of $B_{i,t}$, $A_{i,t}^T$, $A_{i,t}^N$, $\Psi_t \left(B, A^T, A^N\right)$, and the path of the borrowing limit $\{\kappa_t\}_{t \geq 0}$. Now let us split these variables in two groups. The first one is composed by the country-level, or idiosyncratic, variables $\xi_{i,t} = \{B_{i,t}, A_{i,t}^T, A_{i,t}^N\}$. The second group contains the world-level, or aggregate, variables $\Xi_t = \{\Psi_t \left(B, A^T, A^N\right), \{\kappa_t\}_{t \geq 0}\}$. I introduce a simple form of imperfect information by assuming that every period each union observes $\xi_{i,t}$ with probability one, while Ξ_t is observed with probability $\varphi < 1$. In words, unions constantly update their information about idiosyncratic country-specific variables, while they update infrequently their information about the world-level variables.

Hence, the information set of union j at time t can be written as $S^j_{i,t} \equiv \left(\xi_{i,t}, E_{\tau(j)}\left[\Xi_t\right]\right)$, where $\tau(j) \leq t$ denotes the last period in which union j updated its information about the aggregate variables.

This setting captures an environment in which wage setters pay more attention to the idiosyncratic shocks that hit their country frequently, rather than to the rare shocks hitting the global economy. More broadly, this asymmetric information structure is meant to capture an environment in which there is enough wage flexibility to deal with normal business cycle fluctuations driven by the productivity shocks. Instead, wages fail to adjust immediately to large and rare shocks, such as the one-time previously unexpected drop in the borrowing limit considered in our deleveraging experiment.[7]

It turns out that, given that the only aggregate shock considered is a one-time fully unanticipated shock to the borrowing limit κ_t, the equilibrium behavior of wage setters takes a very simple form. In fact, both in the initial and final steady states, which are characterized by a constant κ_t, wage setters have perfect information about the state of the economy. Hence, in steady state the allocations correspond to the perfect-information benchmark discussed in Section 2. Instead, during the transition from the initial to the final steady state some wage setters are not immediately informed about the global deleveraging shock, and so they act on the basis of outdated information.

Specifically, during the transition in any period t wage setters can be divided in two groups. First, a fraction $1-(1-\varphi)^t$ of wage setters have updated their information about the new path of the borrowing limit, and so they set their wage under full information. Instead, a fraction $(1-\varphi)^t$ of wage setters set their wages under an outdated information set, that is under the belief that the borrowing limit κ_t is still constant and equal to its value in the initial steady state. Effectively, these uninformed wage setters set their wage according to the pricing rule characterizing the initial steady state. Thus, the aggregate nominal wage evolves according to

$$W_{i,t} = \left(\left(1-(1-\varphi)^t\right)\left(W^{in}_{i,t}\right)^{1-\varepsilon} + (1-\varphi)^t\left(W^{un}_{i,t}\right)^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}}, \tag{D.17}$$

where $W^{in}_{i,t}$ denotes the wage of informed unions, defined as

$$W^{in}_{i,t} = \frac{\varepsilon}{\varepsilon-1}\frac{\left(L^{in}_{i,t}\right)^{\psi}}{\frac{\omega C^{1-\gamma}_{i,t}}{P^T_{i,t}C^T_{i,t}}}, \tag{D.18}$$

7. To be clear, this assumption is not made because wage rigidities are unimportant to explain normal business cycle fluctuations, but rather to isolate the interactions between wage rigidities and the transitional dynamics triggered by a global deleveraging shock.

where $L^{in}_{i,t}$ denotes labor effort from members of informed unions. Instead, $W^{un}_{i,t}$ denotes the wage of uninformed unions which set the wage according to

$$W^{un}_{i,t} = \frac{\varepsilon}{\varepsilon - 1} \frac{E_t\left[\left(L^{un}_{i,t}\right)^{\psi} | S^{un}_t\right]}{E_t\left[\frac{\omega C^{1-\gamma}_{i,t}}{P^T_{i,t} C^T_{i,t}} | S^{un}_t\right]}, \tag{D.19}$$

where $S^{un}_{i,t} \equiv (\xi_{i,t}, E_{-1}[\Xi_t])$ and $L^{un}_{i,t}$ denotes labor effort from members of uninformed unions.

Monetary policy. I consider two types of exchange rate regimes. First, I consider a world in which all the countries have their own currency and in which exchange rates are flexible. Second, I consider a world in which all the countries are part of a single monetary union. To make the two regimes comparable, I assume that in both cases the monetary authorities follow a policy of inflation targeting.[8]

More formally, under flexible exchange rates each country i has its own currency, and consequently its own central bank. Under this regime, in every country i the central bank targets a value for consumer price inflation (CPI). The CPI, or price of the consumption basket, can be written as

$$P_{i,t} = \left(\frac{P^T_{i,t}}{\omega}\right)^{\omega} \left(\frac{P^N_{i,t}}{1-\omega}\right)^{1-\omega}. \tag{D.20}$$

Defining CPI inflation as $\pi_{i,t} \equiv P_{i,t}/P_{i,t-1}$, the objective of the central bank is to set $\pi_{i,t} = \bar{\pi}$. This policy captures in a simple way the price stability objective typical of monetary authorities in advanced economies.

To see what this implies for the exchange rate, consider that by the law of one price the exchange rate between country i and country j can be written as

$$S_{ij,t} = \frac{P^T_{j,t}}{P^T_{i,t}}. \tag{D.21}$$

This expression implies that insofar as the price stability objective implies different tradable inflation rate across the two countries the exchange rate has to adjust.

The second regime considered is a monetary union. In this case all the countries share the same currency. In line with the inflation objective of the European Central Bank, I assume that the central bank of the currency union targets the average inflation

8. One can think of monetary policy as being implemented through a policy rule in which the central bank sets $R^n_{i,t}$ as a function of the deviations of inflation from its target. To give the central bank control over $R^n_{i,t}$, it is common to assume that private agents have a demand for money which depends on the nominal interest rate. By changing the quantity of money in circulation the central bank can then set $R^n_{i,t}$. In the paper, I focus on the cashless limit of the economy, in which the quantity of money in circulation is approximately zero.

across the member countries, that is

$$\pi_t = \int_0^1 \pi_{i,t} di. \tag{D.22}$$

For comparability, I assume that the central bank of the union has the same inflation target as the central banks under flexible exchange rates $\pi_t = \bar{\pi}$. Notice that in a currency union, by the law of one price, all the countries must share the same price for the traded good.

DEFINITION D.D.1. *An equilibrium of the economy with nominal wage rigidities is a sequence of the world interest rate $\{R_t\}_{t\geq 0}$, a sequence of pricing functions $\{P_t^T(B,A^T,A^N), P_t^N(B,A^T,A^N)\}_{t\geq 0}$, a sequence of policy rules $\{C_t^T(B,A^T,A^N)$, $C_t^N(B,A^T,A^N)$, $W_t(B,A^T,A^N)$, $L_t^T(B,A^T,A^N)$, $L_t^N(B,A^T,A^N)\}_{t\geq 0}$, and a sequence of joint distributions for bond holdings and productivity $\{\Psi_t(B,A^T,A^N)\}_{t\geq 0}$, such that given the initial distribution $\Psi_0(B,A^T,A^N)$, a sequence of the borrowing limit $\{\kappa_t\}_{t\geq 0}$ and prices $P_{-1}^T(B,A^T,A^N)$ and $P_{-1}^N(B,A^T,A^N)$*

- *$C_t^T(B,A^T,A^N), C_t^N(B,A^T,A^N), L_t^T(B,A^T,A^N), L_t^N(B,A^T,A^N)$ and $W_t(B,A^T,A^N)$ satisfy households', firms' and wage setters' optimality conditions.*
- *Markets for consumption clear in every country*

$$\frac{B_{t+1}(B,A^T,A^N)}{R_t} = A^T\left(L_t^T(B,A^T,A^N)\right)^{\alpha_T} - C_t^T(B,A^T,A^N) + B$$

$$C_t^N(B,A^T,A^N) = A^N\left(L_t^N(B,A^T,A^N)\right)^{\alpha_N}$$

- *Given the wage, households supply all the labor demanded by firms.*
- *$\Psi_t(B,A^T,A^N)$ is consistent with the decision rules.*
- *The market for bonds clears at the world level*

$$\int B\, \mathrm{d}\Psi_t(B,A^T,A^N) = 0.$$

- *$\{P_t^T(B,A^T,A^N), P_t^N(B,A^T,A^N)\}_{t\geq 0}$ are such that $\pi_{i,t} = \bar{\pi}$ for every i and t if exchange rates are flexibles, or $\int_0^1 \pi_{i,t} di = \bar{\pi}$ in the case of the monetary union.*

D.1. The zero lower bound in a monetary union

With the zero lower bound the definition of the monetary union equilibrium must be modified as follows. Define $\hat{R}_t^n$ as the gross interest rate on nominal bonds consistent with the central bank's inflation target. The presence of the zero lower bound implies $R_t^n \geq 1$. The central bank is assumed to follow the rule $R_t^n = \max\left(\hat{R}_t^n, 1\right)$. The definition of an equilibrium is then as in definition $D.D.1$, with the exception that monetary policy is captured by the conditions

$$\left(R_t^n - 1\right)\left(\int_0^1 \pi_{i,t} di - \bar{\pi}\right) = 0, \quad R_t^n \geq 1, \quad \int_0^1 \pi_{i,t} di \leq \bar{\pi}.$$

Appendix E: Data appendix

This appendix provides details on the construction of the series used in the calibration and to construct Figure 10.

E.1. Data used in the calibration

The countries in the sample are Austria, Belgium, Finland, Germany, Greece, Ireland, Italy, Netherlands, Portugal and Spain. The euro area is defined as the aggregate of the sample countries.

1. *Share of tradable goods in consumption.* The consumption share of tradables is proxied by the share of tradable production in total value added. The tradable sector is defined as the aggregate of agriculture, mining and manufacturing. This procedure yields an average tradable share of 19% for the euro area during the period 1995-2013. The series used are yearly and come from Eurostat.
2. *Labor share in production.* The labor share is computed as workers' compensation as a fraction of total value added. To adjust for self-employed workers, following Gollin (2002), I use as a measure of workers' compensation the average employee compensation, obtained by dividing total employee compensation by the number of employees, multiplied by the total number of employed workers. This adjustment is based on the assumption that on average self-employed workers earn the same return to labor as employees. Since this assumption is unlikely to hold for agriculture and mining, I proxy the tradable sector by manufacturing, while the rest of the economy less agriculture, mining and manufacturing captures the non-tradable sector. This procedure gives an average labor share for the euro area of 65% in both sectors for the period 1995-2013. The series used are yearly and come from Eurostat.
3. *Labor productivity.* I approximate (log) labor productivity as $\log(A_{i,t}) = \log(GDP_{i,t}) - 0.65\log(L_{i,t})$, where $GDP_{i,t}$ denotes GDP at constant prices, while $L_{i,t}$ is total employment. For each country in the sample I obtained the cyclical component of labor productivity by subtracting a log-linear trend from the actual series. I then computed for each country the autocorrelation and standard deviations of de-trended labor productivity. The averages across the sample countries are 0.92 for the autocorrelation and 0.024 for the standard deviation. The series used are quarterly for the period 1999Q1-2014Q4, and come from the OECD.

E.2. Data used to construct Figure 10

1. *GDP per capita.* Euro area real GDP divided by total population. Both series are quarterly and come from the OECD. Real GDP is seasonally adjusted. The figure shows the cyclical component, obtained by subtracting a log-linear trend calculated over the period 1999Q1-2015Q4.

2. *Nominal rate.* European Central Bank discount rate. Monthly series from the IMF International Financial Statistics database.
3. *CPI inflation.* Growth rate of Harmonized Index of Consumer Prices: All Items for Euro area (19 countries). Quarterly series obtained from FRED.
4. *Real wages.* Nominal labor cost index divided by Harmonized Index of Consumer Prices. The nominal labor cost index is provided by the OECD and is defined as the hourly compensation of employees plus taxes minus subsidies. The index is quarterly, and is computed for the Euro area (12 countries) aggregate.

Appendix F: Derivation of noncooperative constrained-efficient allocation

To prove that from the perspective of a single country the equilibrium of the baseline model attains the constrained efficient allocation I characterize the solution to the social planner problem for a single country. Importantly, the social planner in a single country takes the world interest rate as given, since a single country is too small to influence the world interest rate.

The social planner in a generic country i chooses a sequence $\{C_{i,t}^T, C_{i,t}^N, L_{i,t}, L_{i,t}^T, L_{i,t}^N, B_{i,t+1}\}_{t\geq 0}$ taking the path for the interest rate $\{R_t\}_{t=0}^{\infty}$ and the initial bond position $B_{i,0}$ as given, to maximize expected utility[9]

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{\left(\left(C_{i,t}^T\right)^{\omega}\left(C_{i,t}^N\right)^{1-\omega}\right)^{1-\gamma}-1}{1-\gamma}-\frac{L_{i,t}^{1+\psi}}{1+\psi}\right)\right],$$

subject to the resource constraints

$$C_{i,t}^T = A_{i,t}^T\left(L_{i,t}^T\right)^{\alpha_T} + B_{i,t} - \frac{B_{i,t+1}}{R_t} \tag{F.1}$$

$$C_{i,t}^N = A_{i,t}^N\left(L_{i,t}^N\right)^{\alpha_N} \tag{F.2}$$

$$L_{i,t}^T + L_{i,t}^N = L_{i,t}, \tag{F.3}$$

and the borrowing constraint

$$B_{i,t+1} \geq -\kappa_t. \tag{F.4}$$

The first order conditions are

$$\omega\frac{C_{i,t}^{1-\gamma}}{C_{i,t}^T} = \lambda_{i,t}^T$$

9. For completeness, in the case of the model with differentiated labor services introduced in Section 4 the planner chooses for every $j \in [0,1]$ the sequence $\{L_{i,t}(j), L_{i,t}^T(j), L_{i,t}^N(j),\}_{t\geq 0}$. However, due to the symmetric nature of the problem, it is optimal for the planner to set $L_{i,t}(j) = L_{i,t}$, $L_{i,t}^T(j) = L_{i,t}^T$, $L_{i,t}^N(j) = L_{i,t}^N$ for any $j \in [0,1]$.

$$(1-\omega)\frac{C_{i,t}^{1-\gamma}}{C_{i,t}^{N}} = \lambda_{i,t}^{N}$$

$$L_{i,t}^{\psi} = \lambda_{i,t}^{L}$$

$$\lambda_{i,t}^{T}\alpha_T A_{i,t}^{T}\left(L_{i,t}^{T}\right)^{\alpha_T-1} = \lambda_{i,t}^{L}$$

$$\lambda_{i,t}^{N}\alpha_N A^{N}\left(L_{i,t}^{N}\right)^{\alpha_N-1} = \lambda_{i,t}^{L}$$

$$\frac{\lambda_{i,t}^{T}}{R_t} = \beta E_t\left[\lambda_{i,t+1}^{T}\right] + \mu_{i,t}$$

$$B_{i,t+1} \geq -\kappa_t, \quad \text{with equality if } \mu_{i,t} > 0,$$

where $\lambda_{i,t}^{T}$, $\lambda_{i,t}^{N}$, $\lambda_{i,t}^{L}$ and $\mu_{i,t}$ are the Lagrange multipliers associated respectively with constraints $(F.1)$, $(F.2)$, $(F.3)$ and $(F.4)$.

Defining

$$w_{i,t} = \frac{\lambda_{i,t}^{L}}{\lambda_{i,t}^{T}}$$

$$p_{i,t}^{N} = \frac{\lambda_{i,t}^{N}}{\lambda_{i,t}^{T}},$$

it is easy to verify that the social planner allocation coincide with the equilibrium conditions of the baseline model presented in Section 2.

Appendix G: Additional tables and figures

TABLE G.1. Welfare losses (% permanent consumption equivalent).

	Mean	5th perc.	10th perc.	25th perc.	50th perc.	75th perc.
Benchmark	0.06	0.32	0.08	0.00	0.00	0.00
$\bar{\pi} = 1.04^{1/4}$	0.05	0.27	0.05	0.00	0.01	0.01
Transfer	0.04	0.18	0.02	-0.01	0.01	0.02

Notes: The welfare losses are computed as the proportional increase in consumption for all possible future histories that agents living in a monetary union must receive, in order to be indifferent between remaining in the monetary union and switching to the baseline frictionless economy.

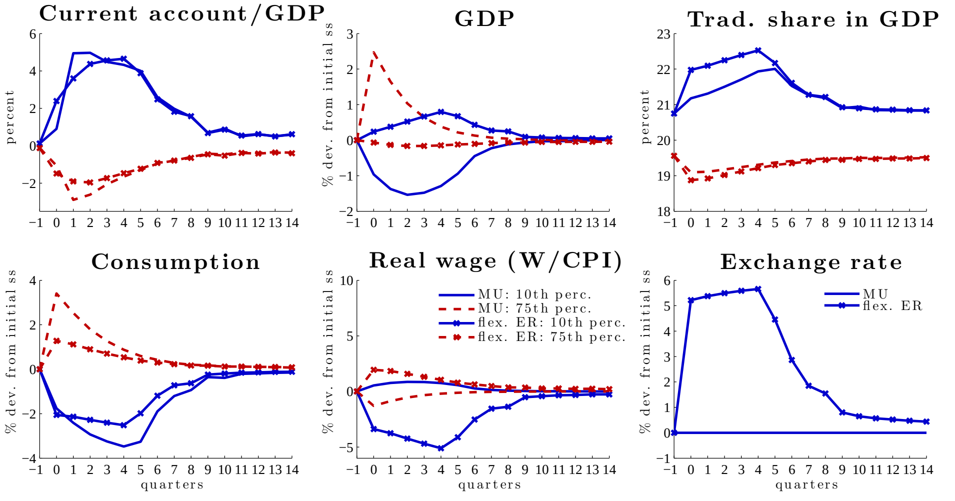

FIGURE G.1. Response to deleveraging shock across the NFA distribution with nominal rigidities. Notes: GDP and consumption are the value of production and consumption at constant prices. Non-tradable goods are weighted using the unconditional mean of p^N in the initial steady state. The real wage is the wage in units of the consumption basket. The exchange rate is defined as the units of the 10th percentile country's currency needed to buy one unit of the currency of the 75th percentile country.

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