Balance Sheet Recessions
with Informational and Trading Frictions

Vladimir Asriyan
CREi, UPF, and Barcelona GSE*

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Abstract

Balance sheet recessions result from concentration of macroeconomic risks on the balance sheets of leveraged agents. In this paper, I argue that information dispersion about the future states of the economy combined with trading frictions in financial markets can explain why such concentration of risk may be privately but not socially optimal. I show that borrowers face a tradeoff between the insurance benefits of financing with macro contingent contracts and the illiquidity premia they need to pay creditors for holding such contracts. In aggregate, as borrowers sacrifice contingency in order to provide liquidity, the severity of macroeconomic fluctuations becomes endogenously linked to the magnitudes of information dispersion and trading frictions. In this setting, I study the policy implications of the theory and I find that imposing (or subsidizing) contingencies in private contracts is welfare improving.

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*Address: Ramon Trias Fargas, 25-27, Mercè Rodoreda Bldg., 08005 Barcelona, Spain; Email: vasriyan@crei.cat; Website: https://sites.google.com/site/vasriyan. I acknowledge financial support from the Spanish Ministry of Economy and Competitiveness Grant (ECO2014-54430-P) and the Barcelona GSE Seed Grant. I am highly indebted to William Fuchs, Yuriy Gorodnichenko, Pierre-Olivier Gourinchas, Demian Pouzo, and Jaume Ventura. I also thank Fernando Broner, Sebastian Di Tella, Brett Green, Christian Hellwig, Amir Kermani, Takeshi Murooka, Maury Obstfeld, Aniko Öery, Victoria Vanasco,, and seminar participants at Bocconi University, Barcelona GSE, CREi-Universitat Pompeu Fabra, Paris School of Economics, the Federal Reserve Board, the World Bank, EIEF - Rome, University of California at Berkeley, Universitat de Barcelona, the Society for Economic Dynamics, the 7th Joint French Macro Workshop, the fourth annual UTDT Economics Conference and the European Winter Meeting of the Econometric Society for helpful suggestions and comments. I thank Yimei Zou for excellent research assistance.

1 Introduction

The Great Recession has once again underscored the important role that financial frictions play in the amplification and propagation of macroeconomic shocks. We were reminded that the concentration of macroeconomic risks on the balance sheets of leveraged agents can produce powerful feedback effects and turn shocks of small magnitude into full-blown *balance sheet recessions*.[1] As an example, the disproportionate exposure of leveraged households to real estate risks is thought to have been responsible for the large and prolonged drop in consumption and employment in the recent recession, and similar arguments have been made of the contribution of the financial sector to the recession.[2] Although the literature has long recognized that balance sheet recessions can result from the combination of leverage and concentrated risks, we do not have a good understanding of why borrowers choose to retain so much risk on their balance sheets to begin with. Answering this question is particularly important for policy makers to design an effective framework for financial regulation.

Much of the existing literature on balance sheet recessions assumes that borrowers and creditors cannot write contracts contingent on the aggregate states of the economy. However, the typical agency-based explanations for why these contingencies may be limited do not apply to aggregate states: atomistic agents cannot influence aggregate outcomes and in many cases there are readily available indicators that can be used in contracting (e.g. GDP, real estate indices, inflation).[3] This is particularly important since our standard models predict that agents' ability to contract on aggregate states can generate sufficient risk-sharing so as to eliminate the balance sheet amplification mechanism altogether.[4] Consistent with this, there have been recent suggestions for policy makers to impose macro contingencies in households' and firms' liabilities.[5] Arguably, to even consider whether such interventions are desirable, we need to have a theory of why contracts with limited macro contingencies may arise endogenously. The goal of this paper is precisely to propose such a theory.

The main contribution of this paper is to show that information dispersion about the future states of the economy combined with imperfect competition in financial markets can explain why borrowers choose to retain

[1] Balance sheet recessions refer to downturns driven by feedback effects between borrowers' balance sheets and general economic activity (e.g. asset prices). The seminal papers are Bernanke and Gertler (1989), and Kiyotaki and Moore (1997). Some recent contributions are He and Krishnamurthy (2011) and Brunnermeier and Sannikov (2014).

[2] See Mian and Sufi (2010) and Adrian and Boyarchenko (2012) for the importance of household and financial firm balance sheets respectively in the Great Recession. Mishkin (1978), Bernanke (1983), and Olney (1997) provide support for the role of balance sheets in the Great Depression.

[3] See the seminal papers by Townsend (1979), Holmstrom (1979), and Myers and Majluf (1984).

[4] See the recent papers by Krishnamurthy (2003) and Di Tella (2014).

[5] Mian (2012) states that contingencies on real-estate indices in households' liabilities could have considerably ameliorated the negative repercussions of the deleveraging-aggregage demand cycle of the Great Recession. Similarly, Calomiris and Herring (2012) argue that contingent convertible bonds would have prevented the financial meltdown of 2008.

aggregate risks on their balance sheets rather than share them with creditors through contingent contracting. The key idea is that the interaction between informational and trading frictions generates illiquidity costs to contingent contracting: creditors will forgo informational rents if they have to sell contingent contracts before their maturity. Creditors will therefore pass these illiquidity costs onto borrowers who will then face a tradeoff between the insurance benefits of financing with macro contingent contracts and the illiquidity premia that they need to pay creditors for holding such contracts. In aggregate, as borrowers sacrifice contingency in order to provide liquidity, the severity of macroeconomic fluctuations becomes endogenously linked to the magnitudes of informational and trading frictions. In this setting, I study the policy implications of the theory and I find that imposing (or subsidizing) contingencies in private contracts is welfare improving.

I adopt a standard model of the balance sheet channel *à la* Kiyotaki and Moore (1997) (henceforth, KM), where entrepreneurs issue financial contracts to investors to finance long-term projects whose cash-flows are exposed to aggregate risk. When there is limited contingency in entrepreneurs' contracts, this framework features the possibility of balance sheet amplification: if in the bad state entrepreneurs' cash-flows fall short of their debt repayments, they will need to liquidate projects prematurely. In aggregate, such liquidations lead to 'fire-sales' and become endogenously amplified through the interaction between entrepreneurial balance sheets and asset prices. However, as noted by Krishnamurthy (2003), in such an environment the presence of contingent contracts allows entrepreneurs to write down some of their liabilities in the bad state, thus reducing premature liquidations and endogenously muting the impact of macroeconomic shocks.

To understand why such contingencies in contracts may be limited, I extend the baseline KM model by allowing investors to retrade contracts before maturity in financial markets. I introduce *trading frictions* by assuming that a seller of a contract can only trade with finitely many counterparties. This assumption is meant to capture a variety of frictions that may prevent markets from being perfectly competitive.[6] I introduce *informational frictions* by supposing that agents observe noisy private signals about the future states of the economy. This assumption is supported by a growing literature in macroeconomics that documents substantial disagreements among economic agents about a variety of macroeconomic variables; this literature further shows that an important reason for disagreements is the disparity of information among economic agents.[7] These ingredients

[6]The number of counterparties that a seller trades with can be thought of as a measure of market depth or development. Such trading frictions are typically larger in over-the-counter type markets or centralized markets that are relatively nascent, and smaller in well-organized deep/competitive exchanges. See Duffie (2010) for a discussion of the relevance of trading frictions in financial markets.

[7]Mankiw et al. (2004) find substantial heterogeneity in inflation forecasts among professional forecasters, economists, and consumers, and Dovern et al. (2009) show similar findings for GDP and prices in a cross-country study of surveys of professional forecasters. Coibion and Gorodnichenko (2012) provide extensive evidence that informational disagreements are pervasive across a variety of

deliver the main results of the paper.

More concretely, the model has three periods and two sets of agents, entrepreneurs (borrowers) and investors (creditors). In the first period, entrepreneurs issue financial contracts to investors to finance long-term projects. In the final period, these projects deliver cash-flows that are exposed to aggregate risk. There is a an aggregate state of nature that can be good or bad and entrepreneurial cash-flows are low in the latter. To motivate trade in financial markets, I suppose that in the intermediate period investors may experience idiosyncratic liquidity needs that they can fulfill by selling their contracts. Before trade, however, each investor receives a private signal about the future state of the economy – information dispersion. Finally, I model markets for contracts as follows: an investor who decides to sell his contract solicits price offers from a finite number of other investors who are willing to buy – trading friction. Modeling markets as auction-theoretic trading mechanisms allows me to deal with the well-known complications that arise in Walrasian markets with dispersed information. Furthermore, as argued by Milgrom (1981), even perfectly competitive markets can be thought of as limits of auction-like trading mechanisms. As a result, the second contribution of the paper is to provide a framework for general equilibrium asset pricing in the presence of dispersed information.

I show that contracts contingent on the state of the economy require an illiquidity premium because they are expected to trade at a discount in financial markets. As investors rationally anticipate future liquidity needs, they pass these illiquidity costs of contractual contingencies onto entrepreneurs, who thus face a tradeoff between insurance against aggregate fluctuations and the scale of their operations. This gives rise to a 'pecking order' for liability design. There is a threshold for shock severity such that for shocks below this threshold entrepreneurs prefer to borrow with non-contingent contracts,[8] and for more severe shocks introducing some contingency in contracts becomes desirable. In aggregate, the costs of contractual contingencies limit the extent to which macroeconomic fluctuations are endogenously stabilized through risk-sharing.

The interaction between information dispersion and imperfect competition in financial markets is key in generating the illiquidity cost to contingent contracting. The essential idea is that disagreements introduce a 'local' monopoly power that buyers are able to exploit but only to the extent that competition among them is imperfect. This insight is borrowed from the large literature on common value auctions which shows that in a variety of trading mechanisms (first-price, second-price, uniform-price, ascending bid auctions, etc.), sellers

population groups and macroeconomic variables.

[8]Entrepreneurs may have to issue a contingent contract (e.g., defaultable debt) if their collateral is insufficient to back a safe claim.

forgo informational rents when faced with buyers who disagree about the value of the asset posted for sale.[9] It is well-known that in the presence of information dispersion the notion of Walrasian markets has problems in reconciling agents' optimization with the process of price formation (e.g., information aggregation/acquisition paradoxes; Grossman and Stiglitz (1981)).[10] Modeling markets as a collection of auctions allows me to address these issues, and I further show how to adapt these trading mechanisms to general equilibrium analysis. Here, it is important to highlight the difference with the partial equilibrium models of noisy rational expectations, which introduce limits to arbitrage and noise traders to study asset pricing with heterogeneous information (e.g., Albagli, Hellwig, and Tsyvinsky (2011)). In general equilibrium, one needs an interpretation for noise traders, since these agents introduce additional resources to the economy by systematically losing to informed traders; moreover, the particular interpretation one has in mind can matter for asset pricing.[11] In a nutshell, auction-theoretic trading mechanisms allow me to do away with noise traders: all agents in my model are rational optimizers.

To study the policy implications of the theory, I consider the problem of a *constrained* social planner who can coordinate entrepreneurs' contractual choices, but who cannot violate the financial constraints faced by economic agents or interfere with the functioning of financial markets. The key finding is that the planner optimally chooses to have more contingency in financial contracts than would have been chosen by entrepreneurs. This finding is consistent with Shiller (1998) and Mian (2012) who argue that promoting macro-contingent contracts is essential for macroeconomic stability. As a result, policies that solely focus on borrowers' leverage, as it is typically suggested in the literature, are sub-optimal.[12] The novel feature is that my model generates a form of *contractual* externality that is different from the pecuniary *fire-sale* externality typically emphasized in the literature. The idea is that while each investor incurs loses when selling contingent contracts to satisfy his liquidity needs, these loses are rents earned by other investors without liquidity needs; in aggregate, these are just transfers across investors that do not result in a social loss. From a practical standpoint, I show that the planner's allocation can be implemented by an ex-ante subsidy to contingent contracts and an ex-ante transfer

[9] See, for example, the seminal works of Wilson (1977), and Milgrom and Weber (1982). To be precise, Cremer and MacLean (1985) show existence of mechanisms which can extract buyers' entire surplus. Such mechanisms are, however, rather complex and unrealistic for financial markets; for example, full surplus extraction breaks down if buyers have limited liability or if buyers who do not receive the asset cannot be forced to pay.

[10] Milgrom (1981) was the first to note that the problems of Walrasian markets stem from the ill-defined process of price formation; he further shows how one can use auction theory to microfound the notion of competitive markets.

[11] One way to introduce noise in the clearing process in my model is through correlated liquidity shocks to investors. However, this is tantamount to introducing one unhedgeable aggregate risk in order to explain why another aggregate risk may remain unhedged. This is conceptually unattractive for the purposes of this paper.

[12] See, for example, the papers by Lorenzoni (2008), Stein (2010), and Jeanne and Korinek (2011).

which ensures that the intervention is Pareto improving; the optimal subsidy is such that it precisely corrects the distortion due to the mis-pricing of contracts. These findings suggests that understanding the underlying reason why risk-sharing among borrowers and creditors may be limited is also crucial for the design of effective financial regulation.

Related Literature. Some recent papers have argued that risk-sharing between borrowers and creditors may be limited if financial constraints are sufficiently tight. For example, Krishnamurthy (2003) shows that a problem of commitment on the side of the *lenders* may prevent them from providing sufficient insurance against macroeconomic shocks; this allows for the balance sheet channel to kick in even when contingent contracting is feasible.[13] However, this argument does not explain why debt write-downs should be limited; lender commitment is inessential for borrowers to be able to repay less in bad states of the world.

Rampini and Viswanathan (2009) argue that risk-sharing may be limited due to a problem of commitment on the side of the *borrowers*, i.e., when borrowers' funding needs are very high ex-ante relative to the value of their collateral. They argue that their theory can potentially explain why poor households and small firms engage insufficiently in hedging macroeconomic risks. In contrast, the theory proposed in this paper applies also for borrowers whose funding needs do not override their needs for insurance. Furthermore, as noted by Krishnamurthy (2003), even if collateral constraints bind for borrowers, they may still allow for enough risk-sharing so as to eliminate amplification effects altogether.

There are also papers which show that the technological/preference features of the economy can be important in determining the extent of macroeconomic risk-sharing. For example, in a continuous time model of the balance sheet channel, Di Tella (2013) shows that the type of shock that hits the economy matters; in particular, he shows that shocks to idiosyncratic volatility can interact with wealth effects to create a 'hedging motive' that induces borrowers to become overly exposed to aggregate risk. Freeman and Tabellini (1998), on the other hand, provide conditions on the type of aggregate shocks and preferences for agents to refrain from hedging aggregate shocks; in their case those are relative and aggregate price level shocks. Finally, Korinek (2009) limits risk-sharing by assuming that lenders are systemically more risk-averse than borrowers. In contrast to these approaches, the underlying mechanism proposed in this paper does not depend on agents' preferences, and it depends on the type of shock that hits the economy only through the features of markets where contracts contingent on these shocks can be traded.

[13] The idea that a problem of commitment can limit risk-sharing is also present in the literature on asset pricing with limited commitment (e.g. Alvarez and Jermann (2000), Chien and Lustig (2010)).

The idea that asymmetric information can lead to mispricing of contingent contracts is also present in the literature on security design.[14] For example, a recent paper by Dang et al. (2010) argues that informational asymmetries can influence market liquidity of financial contracts and thus affect contract design. Beside my focus on general equilibrium and balance sheet recessions, a key difference with this literature is that I study contingencies on aggregate and not idiosyncratic states. The typical assumption in this literature is that one of the parties to trade has exclusive access to private information about the states underlying the contract. However, such an assumption is unrealistic for aggregate states; it is much more plausible that information is dispersed among economic agents. But then, as emphasized in this paper, market structure (e.g., extent of competition) matters for pricing contracts as well.

Finally, my paper contributes to the literature that studies the macroeconomic implications of heterogeneous information. Lucas (1972) is the seminal paper in this literature, and Angeletos and La'O (2009, 2011) and Lorenzoni (2009) are some of the recent contributions. My work adds to this literature by showing that, when interacted with trading frictions in financial markets, information dispersion can depress investment and amplify macroeconomic fluctuations by limiting the extent to which agents can share macroeconomic risks.

Organization. The paper is organized as follows. In Section 2, I describe the economic environment. In Section 3, I set up the economic agents' problems and I define the equilibrium of the economy. In Section 4, I characterize the equilibrium of the economy and I analyze the policy implications of the theory in Section 5. In Section 6, I conclude. All proofs are in the Appendix.

2 The Model

The model has three periods, $t = 0, 1, 2$. There are two sets of agents, entrepreneurs and investors, each of unit mass. The initial date is the financing stage of projects. The intermediate date is when investors retrade contracts in financial markets. The final date is when shocks to entrepreneurial projects arrive and entrepreneurs must decide whether to liquidate them.

Preferences and Endowments. Entrepreneurs have no endowments, are risk-neutral, and consume only in $t = 2$. An entrepreneur's preferences are given by $\mathbb{E}\{c_2^E\}$, where c_2^E is her consumption in $t = 2$. Investors receive

[14] See, for example, the papers by Myers and Majluf (1983), Nachman and Noe (1994), and DeMarzo and Duffie (1999).

endowment e in all periods, are also risk-neutral, but they may be subject to idiosyncratic liquidity needs in $t = 1$. An investor's preferences are given by $\mathbb{E}\{c_0^I + c_1^I + \beta c_2^I\}$, where c_t^I is his consumption in period t and where $\beta \in \{0, 1\}$ denotes his liquidity need. The preference parameter β is realized in $t = 1$ and is distributed iid across investors with $\mathbb{P}(\beta = 0) = \lambda \in (0, 1)$. Thus, λ also denotes the fraction of investors who experience liquidity needs in period 1.[15]

Entrepreneurial Technology. An entrepreneur can install k units of capital in $t = 0$ at a convex cost $\chi(k)$, and each unit of capital delivers a per unit cash-flow a at the *beginning* of $t = 2$ and a cash-flow A at the *end* of $t = 2$. The arrival of cash-flows in two stages is a simple way to introduce a tradeoff between liquidations of projects and their continuation.[16] Thus, the cash-flow A should be interpreted as the continuation payoff of the project. Entrepreneurial cash-flows are exposed to aggregate risk. The aggregate state of the economy is realized in $t = 2$ and is denoted by $s \in \{l, h\}$ with $\Pr(s = l) = \pi(l) \in (0, 1)$. This is the only source of aggregate uncertainty. The cash-flow a varies with the state and takes values in $\{a(l), a(h)\}$ with $a(l) < a(h)$, while the cash-flow A is for simplicity assumed to be deterministic.[17]

Liquidation and Fire-Sales. In $t = 2$, when in need of funds, an entrepreneur has the option to liquidate her capital in a competitive capital goods market. If an entrepreneur has begun her project at scale k and the price of capital is q, she can liquidate a fraction $z \in [0, 1]$ of her capital and receive qzk units of the consumption good. The remaining $(1 - z)k$ units of capital deliver $A(1 - z)k$ units of the consumption good at the end of the period. Thus, entrepreneurs may face a trade-off: they can liquidate capital prematurely to raise funds in the capital goods market or they can keep capital intact to receive a per unit return A in the future.

The units of capital that entrepreneurs liquidate are absorbed by a "traditional" sector, which is composed of a mass of competitive firms. Each of these firms can convert capital goods to consumption goods according to an increasing and concave production technology $g(\cdot)$ that satisfies $g'(0) = A$. Thus, the productivity of firms in this sector decreases below that of entrepreneurs as the units of capital employed by these firms increases.[18]

[15]These idiosyncratic preference shocks are a simple modeling device to introduce gains from trade in financial contracts at $t = 1$. For example, an investor may have new investment/consumption opportunity, or an institution may experience a 'run' and need to sell assets to pay its depositors. The qualitative results of the paper do not depend on this particular modeling approach.

[16]The results would not change if instead there were period $t = 3$ and A were the cash-flow in that period.

[17]The fluctuations in the intermediate cash-flow intend to capture transitory shocks to the entrepeneurs' ability to meet their liabilities. For example, firms may experience demand shocks, or financial institutions may experience losses on existing positions (e.g. mortgages).

[18]This technological assumption is a standard way to introduce amplification effects in the form of 'fire-sales'. See, for example, Kiyotaki and Moore (1997) for a similar modeling approach. The idea of 'fire-sales' in response to common industry shocks goes back to Shleifer and Vishny (1992).

Figure 1: **Timeline**

The profits (if any) from these firms' operations are rebated lump sum to investors.

Financial Contracts. The aggregate state of the economy is assumed to be verifiable at $t = 2$. On the other hand, investors' idiosyncratic preference shocks are non-contractable either because they are observable but non-verifiable or because they are unobservable; I will consider both cases. In period 0, to finance projects, entrepreneurs raise funds from investors by issuing financial contracts to them. An entrepreneur approaches an investor and makes him a take-it-or-leave-it contractual offer that specifies a desired loan amount L and state-contingent repayments $b(s)$ and $B(s)$ made at the beginning and at the end of period 2, respectively. The investor can accept or reject the offer and, if he rejects, the entrepreneur does not invest.

If entrepreneurs could fully pledge their project returns to investors, liquidations would never occur. To allow for the possibility of liquidations, I introduce a problem of limited commitment as in Lorenzoni (2008). An entrepreneur's promises of repayment must be backed by her capital plus a fraction $\theta < 1$ of the project's cash-flows. In particular, if the scale and the fraction of the project liquidated are k and $z(s)$ respectively, the financial contracts must satisfy the following commitment constraints for $s \in \{l, h\}$:

$$0 \leq b(s) + B(s) \leq (\theta a(s) + q(s))k$$

$$0 \leq B(s) \leq \theta A(1 - z(s))k$$

This friction can be microfounded by assuming that the entrepreneur can always renege on the contract and walk away with a fraction $1 - \theta$ of her cash-flows; this threat then puts a limit on the entrepreneur's borrowing capacity. These financial constraints state that in equilibrium agents do not want to renege on their contractual promises. The non-negativity constraints presume that investors lack collateral to back up their own promises.

Information Dispersion and Trading Frictions. The novel feature of my paper is to introduce information dispersion and trading frictions into secondary markets where investors trade financial contracts. The interaction of these two frictions will be key to introducing costs to contractual contingency and, therefore, to deriving the main results of the paper.

In period 1, investors receive signals about the future state of the economy. Investor $i \in [0, 1]$ observes a *private* signal $x_i \in [\underline{x}, \overline{x}]$; the signals $\{x_i\}$ are distributed independently across investors conditional on the state, with a continuously differentiable cdf denoted by $F_s(\cdot)$ for $s \in \{l, h\}$.[19] I assume that the conditional distributions are related by the monotone likelihood ratio property; that is, $\frac{f_h(x)}{f_l(x)}$ is increasing in x on $[\underline{x}, \overline{x}]$, where $f_s(\cdot)$ denotes the pdf of signal x_i conditional on state $s \in \{l, h\}$.[20] Thus, investors who receive higher signals are more optimistic about the future state of the economy.

After liquidity needs and signals are realized, investors participate in secondary financial markets where they can sell and/or buy financial contracts. If an investor chooses to sell his contract, he solicits price offers from an exogenous number $n \geq 2$ of buyers, and he commits to sell his contract to the buyer with the highest offer.[21] If an investor becomes a buyer, then he submits a price offer to a contacting seller, and he receives the contract if his offer is the highest. Trading frictions will be said to be small (large) when n is large (small). All trades are assumed to be executed simultaneously, which ensures that there are no informational spillovers across markets.[22] A more detailed description of investors' strategies and of the matching process is deferred to Section 3.2. Once all trades are executed, secondary markets close, and holders of financial contracts wait to period 2 to receive their repayments.

Timeline. The heuristic timeline of the economy is illustrated in Figure 1. To summarize, in period 0, entrepreneurs issue contracts in order to raise funds and invest. In period 1, after liquidity needs and signals are realized, investors are matched to trade in secondary markets. In period 2, the aggregate state is realized, entrepreneurs receive cash-flows, liquidate capital if needed, and meet their contractual promises to investors.

3 Agents' Problems and Equilibrium Definition

In this section, I setup the problems solved by entrepreneurs and investors, and then I define the equilibrium of the economy.

[19]The assumption that there are no public signals is only made for simplicity. Introducing public signals would not alter the qualitative results of the paper as long as these signals are imperfectly informative.

[20]In addition, I suppose that signals are boundedly informative: there exist positive constants $\underline{\phi}, \overline{\phi}$ such that $\underline{\phi} < \frac{f_h(\underline{x})}{f_l(\underline{x})} < \frac{f_h(\overline{x})}{f_l(\overline{x})} \leq \overline{\phi}$.

[21]The assumption of commitment to sell is made for simplicity. When investor liquidity needs are observable, this assumption is non-binding because only liquidity hit investors will sell in equilibrium and these agents' outside option is 0. On the other hand, when liquidity needs are unobservable, this assumption simplifies the analysis by allowing me to ignore the situation where investors ask for offers, learn others' information, and do not trade.

[22]Allowing for staggered trade and learning would considerably complicate the analysis but not change the qualitative results of the paper. See, for example, Duffie and Manso (2007) for a model of information percolation in markets with correlated assets.

3.1 Entrepreneurs' Problem

In period 0, an entrepreneur chooses how much to invest and raises funds by proposing contractual terms $\{L, b(s), B(s)\}$ to an investor. An investor accepts the contract if L, the loan amount, does not exceed the expected present value of the repayments to the investor, which in this section I take as given and denote by $\mathcal{L}(\{b(s), B(s)\})$, i.e., this is investor's participation constraint.

Entrepreneur takes the prices $\{q(s)\}$ of capital goods and the schedule $\mathcal{L}(\cdot)$ as given and chooses her investment and the contractual terms in order to maximize her expected period 2 consumption. Entrepreneur's problem is given by

$$\max_{\{k,L,b(s),B(s),z(s),c_2^E(s)\}} \mathbb{E}\{c_2^E(s)\}$$

subject to

$$c_2^E(s) = a(s)k + q(s)z(s)k - b(s) + A(1 - z(s))k - B(s) \tag{1}$$

$$\chi(k) = L \leq \mathcal{L}(\{b(s), B(s)\}) \tag{2}$$

$$b(s) \leq a(s)k + q(s)z(s)k \tag{3}$$

$$0 \leq b(s) + B(s) \leq (\theta a(s) + q(s))k \tag{4}$$

$$0 \leq B(s) \leq \theta A(1 - z(s))k \tag{5}$$

$$0 \leq z(s) \leq 1 \tag{6}$$

for $s = l, h$.

Constraint (1) is the entrepreneurial budget constraint in period 2, where entrepreneurs consume the returns of the project net of repayments. Constraint (2) is the period 0 budget constraint combined with the investor participation constraint. Because there are no informational asymmetries in period 0, it is without loss to assume that entrepreneurs issue contracts that investors accept. Constraint (3) states that the entrepreneur must be able to cover her repayment $b(s)$ from the intermediate cash-flow $a(s)k$ and capital goods liquidations $q(s)z(s)k$. Because state-contingent contracts are allowed, assuming that the entrepreneur does not raise new funds at the beginning of period 2 is without loss of generality. Constraints (4) and (5) are the limited commitment constraints and, finally, constraint (6) states that the fraction $z(s)$ of capital goods liquidated must lie between zero and one.

The prices of capital goods in period 2 are determined by the traditional firms' optimal demand for capital and the clearing condition in the capital goods market. The profits (if any) from these firms' operations are rebated lump sum to late investors. Before proceeding further, I make the following parametric assumptions that simplify the subsequent analysis. Let $k^{fb} \equiv \chi'^{-1}(\mathbb{E}\{a(s) + A\})$ be the scale of investment that would be optimal in a frictionless economy, then I assume that

Assumption 1 *(1) $(g'(x) - \theta A)x$ is increasing in x, (2) $\chi'(x) - \frac{\chi(x)}{x}$ is increasing in x, (3) $a(l) < A \leq a(h)$, and (4) $\underline{k} < k^{fb}$ where $\underline{k}$ is implicitly defined by $\chi(\underline{k}) = (a(l) + \theta A)\underline{k}$.*

Assumption 1.1 ensures equilibrium uniqueness in the capital goods market, Assumption 1.2 ensures that entrepreneurs face decreasing returns to scale, Assumption 1.3 implies that entrepreneurs will not need to liquidate capital in state h, and Assumption 1.4 will ensure that in equilibrium entrepreneurs face a meaningful tradeoff between the scale of their investment and premature liquidations of their projects.[23] An immediate result that follows from Assumption 1.1 is that, in equilibrium, the prices of capital goods satisfy

$$q(s) = g'(z(s)k) \in (\theta A, A] \tag{7}$$

for $s = l, h$.[24] Entrepreneurs thus face a downward sloping inverse demand schedule for capital that is bounded above by the return to capital at its efficient use, A, and below by the portion of this return that can be credibly promised to investors, θA.

3.2 Investors' Problem

In period 0, an investor's problem is to accept or reject the entrepreneur's contractual offer. If accepted, the contract will either be held to maturity to $t = 2$, or be sold in secondary markets at $t = 1$. To price financial contracts at $t = 0$, I solve the investors' problem by backwards induction.

[23] The first two assumptions are standard. See, for example, Lorenzoni (2008). The third assumption is only made for simplicity; the analysis is qualitatively the same if it is relaxed. The final assumption is made to eliminate the uninteresting case where entrepreneurs achieve first-best scale by borrowing with non-contingent contracts and never needing to liquidate.

[24] See Lemma 0 in Appendix.

3.2.1 Investors' Problem at t=1: Trading Contracts in Secondary Markets

Let $\mathcal{C} = \{b(s), B(s)\}$ denote the financial contract held by an investor in $t = 1$, and let $(x, \beta) \in [\underline{x}, \overline{x}] \times \{0, 1\}$ denote an investor's type, given by his signal x and his liquidity need β. An investor must decide (i) whether to post his contract for sale, and (ii) whether to buy contracts from other investors. Importantly, I allow each investor *both* to sell his contract and be a buyer of contracts from other investors.

I make the following indifference-breaking assumptions. First, an investor becomes a buyer only if he is willing to pay a positive price for some contract. An immediate result is that an investor becomes a buyer if and only if he has not experienced a liquidity need. Second, an investor posts his contract for sale only if he strictly prefers to do so.[25] I will denote an investor's posting strategy by γ, with $\gamma = 1$ if and only if the investor posts the contract for sale. I will focus on symmetric posting strategies in which an investor's posting decision only depends on his type and the contract he holds: $(x, \beta, \mathcal{C}) \mapsto \gamma(x, \beta, \mathcal{C})$.

Matching. After each investor makes his decisions whether to post his contract for sale and whether to buy other contracts, each posted contract is matched with n randomly selected buyers from the set of available buyers (investors with $\beta = 1$). For notational simplicity, I suppose that each buyer can participate in trade for at most one contract; I show in Appendix C that relaxing this assumption does not alter the results.[26]

Buyer's Payoff. When a buyer is matched to trade for a contract, he submits a price offer p for that contract. In the main analysis, I assume that buyers observe the liquidity need of the seller who has posted the contract. This assumption simplifies the illustration of the key mechanisms by eliminating adverse selection on the side of the seller (see Section 4.1.1 for the case with unobservable liquidity needs). I will focus on symmetric offer strategies, in which conditional on contract $\mathcal{C}$, the price offer that a buyer submits is only a function of his own signal x and the liquidity need β^S of the seller: $(x, \beta^S, \mathcal{C}) \mapsto p(x, \beta^S, \mathcal{C})$. In what follows, I omit the superscript S on the seller's liquidity need whenever this dependence is clear. If other buyers follow offer strategy $p(\cdot, \beta^S, \mathcal{C})$, the payoff to buyer j who has received signal $x_j^B = x$ and submits offer $\widehat{p}$ is given by

$$U^B(\widehat{p}|x, \beta^S, \mathcal{C}, p) = \mathbb{E}\{(b(s) + B(s) - \widehat{p}) \cdot \phi(\widehat{p}, \max_{-j}\{p(x_i, \beta^S, \mathcal{C})\})|x, \beta^S, \mathcal{C}\}$$

[25]The first assumption eliminates trivial offers from equilibrium, while the second assumption minimizes the ratio of sellers-to-buyers in the market; it can be rationalized by assuming that there is a small cost to posting contracts.

[26]In the benchmark model, a sufficient condition for such matching to be consistent with the ratio of buyers to sellers is that $n\lambda \leq 1 - \lambda$. In order to do comparative statics on n independently of the value of λ, I need to relax this assumption in Appendix C.

where $\phi(\cdot,\cdot)$ is the allocation rule that maps the offer of buyer j and the maximal offer of his opponents to a probability that buyer j receives the contract.[27] By symmetry, the offer strategy $p(\cdot, \beta^S, \mathcal{C})$ is optimal for buyers if $p(x, \beta^S, \mathcal{C}) \in \underset{\widehat{p}}{\text{argmax}}\, U^B(\widehat{p}|x, \beta^S, \mathcal{C}, p)$ for all $x \in [\underline{x}, \overline{x}]$.[28]

Seller's Payoff. A seller commits to sell his contract to the buyer with the highest offer. Let $p^{\max}(\beta, \mathcal{C}) \equiv \max_{j=1,\ldots,n}\{p(x_j, \beta, \mathcal{C})\}$ be the maximal offer that the seller with liquidity need β receives conditional on buyers having received signals $\{x_j\}_{j=1}^n$. The expected payoff to a seller who has received signal x and has a liquidity need β is thus given by

$$U^S(x, \beta, \mathcal{C}) = \mathbb{E}\{p^{\max}(\beta, \mathcal{C})|x\} - \beta\mathbb{E}\{b(s) + B(s)|x\}$$

Investor of type (x, β) will therefore post his contract for sale if only if he is strictly better off selling it than keeping it, i.e. $\gamma(x, \beta, \mathcal{C}) = 1$ if and only if $U^S(x, \beta, \mathcal{C}) > 0$.

Beliefs. Upon being matched with a seller, a buyer makes an inference about the seller's signal. The reason why a buyer cares about the seller's signal is that it helps him make an inference about the state of the economy and thus about the payoffs of the entrepreneurial contracts. Let $\widehat{F}(\cdot|x, \beta, \mathcal{C}) : [\underline{x}, \overline{x}] \to [0, 1]$ denote the belief (cdf) that a buyer with signal x holds over the seller's signal conditional on seller having a liquidity need β and holding contract $\mathcal{C}$. We are now ready to define an equilibrium in secondary financial markets:

Definition 1 *Given financial contract $\mathcal{C}$, an equilibrium in secondary markets is given by a posting strategy γ, price offer strategy p, and belief function $\widehat{F}$, such that*

1. *Seller Optimality: $\gamma(x, \beta, \mathcal{C})$ is optimal for an investor of type (x, β), given that buyers follow strategy p,*
2. *Buyer Optimality: $p(x, \beta, \mathcal{C})$ is optimal for a buyer with signal x who is matched with a seller with a liquidity need β, given that other buyers follow strategy p and given belief $\widehat{F}$, and*
3. *Belief Consistency: $\widehat{F}$ is derived from strategy γ using Bayes' rule where possible.*

[27] Buyer j gets the contract if his offer is highest and the contract is allocated randomly among highest bidders in case of a tie.
[28] Investor's endowment e is assumed to be large enough for his budget constraint not to bind when submitting offers.

3.2.2 Investors' Problem at t=0: Pricing Financial Contracts

In period 0, an investor decides whether to accept the contract $\mathcal{C}$ offered by the entrepreneur. If the investor accepts the contract, he can keep it to maturity and consume its cash-flows $\{b(s), B(s)\}$ in period 2, or he can sell the contract in secondary markets and consume the proceeds from this sale in period 1. Given the equilibrium strategies γ and p, the expected present value of the contract to the investor is given by

$$\mathcal{L}(\mathcal{C}) = \mathbb{E}\{\gamma(x, \beta, \mathcal{C})\mathbb{E}\{p^{max}(\beta, \mathcal{C})|x\} + (1 - \gamma(x, \beta, \mathcal{C}))\beta\mathbb{E}\{b(s) + B(s)|x\} \tag{8}$$

Equation (8) fully defines the investor participation constraint $\mathcal{L}(\cdot)$ as it holds for an arbitrary contract issued by the entrepreneurs.

3.3 Equilibrium Notion

In the previous sections, I described the entrepreneurial problem for given investor participation constraint $\mathcal{L}(\cdot)$ and prices of capital $\{q(s)\}$. I have also described how the equilibrium prices of capital and the investor participation constraint are determined. An equilibrium of this economy is then defined as follows,

Definition 2 *An equilibrium consists of an allocation $\{k, L, b(s), B(s), z(s), c_2(s)\}$, capital goods prices $\{q(s)\}$, an investor participation constraint $\mathcal{L}(\cdot)$, and a triple $\{\gamma, p, \widehat{F}\}$, such that*

1. *Entrepreneurs' Optimality: the allocation $\{k, L, b(s), B(s), z(s), c_2(s)\}$ is optimal for entrepreneurs, given the capital goods prices $\{q(s)\}$ and the contract price schedule $\mathcal{L}(\cdot)$,*
2. *Investors' Optimality: the triple $\{\gamma, p, \widehat{F}\}$ is an equilibrium in secondary markets, given financial contract $\mathcal{C} = \{b(s), B(s)\}$,*
3. *Market Clearing: the prices $\{q(s)\}$ and the schedule $\mathcal{L}(\cdot)$ are given by (7) and (8) respectively.*

In the definition above, I omitted the allocations of the investors. I have done this only for brevity, as these allocations affect the equilibrium variables of interest only through the prices of capital goods and of financial contracts. These prices in turn are fully summarized by the investor participation constraint and the traditional sector's optimal demand for capital. I now turn to the characterization of the equilibrium behavior of this economy.

4 Equilibrium Characterization

In this section, I characterize the equilibrium of the economy and derive the main results of the paper. In Section 4.1, I show how information dispersion and trading frictions imply discounting of contingent contracts in secondary financial markets. In Section 4.2, I use this result to derive a pecking order for entrepreneurial liability design, and I show how macroeconomic fluctuations become endogenously linked to the informational and trading frictions introduced in the paper. In Section 4.3, I provide comparative statics results and discuss the key implications of the model.

4.1 Equilibrium in Secondary Markets

The first immediate result of this section is that non-contingent contracts are always traded at their 'fair' value,

Lemma 1 (Non-Contingent Contracts) *If financial contract $\mathcal{C} = \{b(s), B(s)\}$ satisfies $b(l) + B(l) = b(h) + B(h) \equiv \overline{V}$, then the offer strategy*

$$p(x, \beta, \mathcal{C}) = \overline{V} \quad \text{for} \quad x \in [\underline{x}, \overline{x}] \quad \text{and} \quad \beta \in \{0, 1\}$$

is optimal for buyers for any belief $\widehat{F}$.

Intuitively, when the payoff of the underlying contract does not depend on the state of the economy, informational asymmetries are irrelevant for pricing. Competition among $n \geq 2$ buyers then forces them to bid the prices of such contracts up to their common expected value. As will be seen below, this logic does not extend to contingent contracts. In this case, information dispersion causes buyers to disagree about the expected payoffs and, thus, gives them an ability to exploit these disagreements to earn informational rents.

Consider now contracts that are contingent on the state of the economy. Let $\{x_1^B, ..., x_n^B\}$ denote the signals received by the n buyers who have been matched to trade for contract $\mathcal{C}$, and consider the problem of buyer 1. If the contract is positively correlated with the state, then buyer 1 receives the contract if his signal is higher than the signals of his opponents.[29] If we let y_1^+ denote the maximal signal among the signals $\{x_2^B, ..., x_n^B\}$,

[29]As is standard in the literature on common value auctions, I restrict my attention to offer strategies p that are monotonic and differentiable in buyers' signals (see Milgrom and Weber 1982).

then buyer 1's valuation of the contract conditional on his signal being $x_1^B = x$ and conditional on his signal being highest is

$$V^+(x) = \mathbb{E}\{b(s) + B(s)|x_1^B = x, y_1^+ < x\}$$

For a contract that is negatively correlated with the state, buyer 1's conditional valuation is

$$V^-(x) = \mathbb{E}\{b(s) + B(s)|x_1^B = x, y_1^- > x\}$$

where now y_1^- denotes the minimal signal among the signals $\{x_2^B, ..., x_n^B\}$ received by buyer 1's opponents. By symmetry, these are also the conditional valuations for any buyer with signal x.

In principle, when making offers, buyers should also take into account the seller's incentive for selling. As I show in Appendix A, however, investors will post their contracts for sale if and only if they have liquidity needs; buyers thus will not need to update their beliefs upon being matched. The reason for this is that buyers can condition their offers on the liquidity need of the seller and because there are no gains from trade between investors without liquidity needs. This reasoning is akin to the no-trade theorem of Milgrom and Stokey (1982). Hence, on equilibrium path, given contract $\mathcal{C}$, the only offer strategy that is relevant for pricing is $p(\cdot, \beta^S, \mathcal{C})$ with $\beta^S = 0$.[30]

The following lemma states that when the posted contract is contingent on the state of the economy, then it is optimal for buyers to shed their offers below their conditional valuations in order to earn informational rents:

Lemma 2 (Informational Rents) *Suppose that financial contract $\mathcal{C} = \{b(s), B(s)\}$ satisfies $b(l) + B(l) \neq b(h) + B(h)$, then in equilibrium the optimal offer strategy of the buyers satisfies*

$$p(x, 0, \mathcal{C}) \leq \begin{cases} V^+(x) & \text{if} \quad b(l) + B(l) < b(h) + B(h) \\ V^-(x) & \text{if} \quad b(l) + B(l) > b(h) + B(h) \end{cases}$$

for all $x \in [\underline{x}, \overline{x}]$, with strict inequalities when $x > \underline{x}$ and $x < \overline{x}$ respectively.

[30] We still need to specify buyers' beliefs off the equilibrium path when they are contacted by a seller without a liquidity need who was never supposed to sell his contract. To this end, I consider the simplest off-equilibrium beliefs: buyers believe that the seller is a pessimist, i.e., he has signal $\underline{x}$ if the contract is positively correlated with the state, and he is an optimist otherwise, i.e, he has signal $\overline{x}$. See Section 4.1.1 for the case where buyers cannot observe the liquidity need of the seller and where, thus, this no-trade result no longer holds.

As the expressions for the optimal offer strategies are rather cumbersome, they are relegated to the Appendix. The intuition for the above result is, however, the following. When submitting their offers, buyers trade off the probability of receiving the contract with the size of the expected rent earned conditional on having the highest offer. If a buyer submits an offer that is equal to his conditional valuation, he expects to earn zero profits; on the other hand, if a buyer sheds his offer below, though the probability of receiving the contract is reduced, he now makes a positive 'informational' rent. The reason why the probability of receiving the contract does not drop to zero when a buyer bids below his valuation is precisely due to information dispersion and finiteness of buyers per seller.

The key takeaway from this result is that because buyers shed their offers in order to earn informational rents, the expected resale price of a contingent contract to an investor is strictly below its expected value:

$$\mathbb{E}\{p^{max}(0, \mathcal{C})\} = \mathbb{E}\{max_{j=1}^{n}\{p(x_j^B, 0, \mathcal{C})\}\} < \mathbb{E}\{b(s) + B(s)\}$$

The last inequality is derived by integrating the offers over all possible signal realizations of the buyers. That contingent contracts are systematically discounted in secondary markets is a key driver of the results of the paper. As will be seen next, this is the precise reason why it is costly for entrepreneurs to introduce contingencies into financial contracts at the funding date. The following proposition yields the main result of this section which states that an investor's willingness to pay for a generic financial contract is given by the contract's expected value net of a discount that is proportional to the degree of contingency of that contract on the state of the economy:

Proposition 1 (Costs of Contingency) *The expected present value of a financial contract* $\mathcal{C} = \{b(s), B(s)\}$ *to an investor at* $t = 0$ *satisfies*

$$\mathcal{L}(\mathcal{C}) = \mathbb{E}\{b(s) + B(s)\} - \zeta_{\mathcal{C}} \cdot |b(h) + B(h) - b(l) - B(l)| \tag{9}$$

for some $\zeta_{\mathcal{C}} = \begin{cases} \zeta^{+} & \text{if } b(h) + B(h) \geq b(l) + B(l) \\ \zeta^{-} & \text{if } b(h) + B(h) < b(l) + B(l) \end{cases}$ *where* ζ^{+}, ζ^{-} *are positive constants.*

To understand why the participation constraint takes the form above, recall the definition of the schedule

$\mathcal{L}(\cdot)$ in equation (8) and note that given contract $\mathcal{C} = \{b(s), B(s)\}$ issued at the initial date, we have

$$\begin{aligned}\mathcal{L}(\mathcal{C}) &= \mathbb{E}\{\gamma(x,\beta,\mathcal{C})\mathbb{E}\{p^{\max}(\beta,\mathcal{C})|x\} + (1-\gamma(x,\beta,\mathcal{C}))\beta\mathbb{E}\{b(s)+B(s)|x\}\} \\ &= \mathbb{E}\{b(s)+B(s)\} - \lambda\left(\mathbb{E}\{b(s)+B(s)\} - \mathbb{E}\{p^{\max}(0,\mathcal{C})\}\right)\end{aligned}$$

The above expression follows from the fact that an investor sells his contract only when he has $\beta = 0$ (a probability λ event), and the fact that investor liquidity needs are idiosyncratic and therefore independent of the state of the economy. The precise form of the schedule $\mathcal{L}(\cdot)$ in Proposition 1 follows from the linearity of offer strategy p contractual payoffs $\{b(s), B(s)\}$ and because buyers earn rents only on contracts that are contingent.

In what follows, I analyze the implication of the above result for entrepreneurs' contractual choices ex-ante and for equilibrium fluctuations. Before proceeding further, however, I consider the case where investor liquidity needs are unobservable to show that the results also extend to that, arguably more plausible, economic environment.

4.1.1 Extending to Unobservable Liquidity Needs

Suppose that the contract $\mathcal{C}$ issued at $t = 0$ is positively correlated with the state; the case of negatively correlated contracts is similar and non-contingent contracts are priced as before. Now, pessimistic investors who have not experienced liquidity needs may also decide to sell their contracts. I will consider threshold posting strategies where investor of type $\beta = 1$ posts his contract for sale if and only if he has received signal below $\widehat{x} \in [\underline{x}, \overline{x}]$, where $\widehat{x} = \underline{x}$ if and only if these investors never sell. An equilibrium in financial markets is defined as before, except that now buyers cannot condition their offers on the liquidity need of the seller: a buyer with signal x submits an offer $p\left(x, \mathcal{C}, \widehat{x}\right)$, where $\widehat{x}$ appears in the argument because buyers will know the equilibrium threshold strategy.

Suppose as before that buyers of contract $\mathcal{C}$ have signals $x_1^B, ..., x_n^B$, and let y_1^+ denote the maximal signal among the signals $\{x_2^B, ..., x_n^B\}$ received by the opponents of buyer 1. Then, buyer 1's valuation of the contract conditional on his signal being $x_B^1 = x$ and conditional on receiving the contract is given by

$$V^+(x, \widehat{x}) = \mathbb{E}\{b(s) + B(s)|x_1^B = x, y_1^+ < x, x^S < \widehat{x}\}$$

In contrast to the case of observable liquidity needs, this valuation is adjusted to reflect the possible adverse selection on the side of the seller. A result similar to that of Lemma 2 yields that buyers will shed their offers strictly below their conditional valuations in order to earn informational rents: $p(x, \mathcal{C}, \widehat{x}) < V^{+}(x, \widehat{x})$ for $x > \underline{x}$ and a given $\widehat{x} \in [\underline{x}, \overline{x}]$.

Let $p^{max}(\mathcal{C}, x, \widehat{x})$ denote the maximal offer that an investor with signal x expects to receive for contract $\mathcal{C}$ when the equilibrium threshold strategy is $\widehat{x}$. Let x^* denote the signal of the marginal investor who is indifferent between selling the contract and keeping it, then we must have that

$$\mathbb{E}\{b(s) + B(s)|x^*\} \geq \mathbb{E}\{p^{max}(\mathcal{C}, x^*, \widehat{x})|x^*\}$$

with strict equality if $x^* > \underline{x}$. This indifference condition maps each possible threshold $\widehat{x}$ to a signal x^*, and the fixed point of this mapping defines the equilibrium threshold $x^*(\widehat{x}) = \widehat{x}$. Lemma 4 in Appendix C shows that such a fixed point exists, and Proposition 5 in Appendix C shows that the present value of financial contract $\mathcal{C}$ to an investor satisfies:

$$\mathcal{L}(\mathcal{C}) = \mathbb{E}\{b(s) + B(s)\} - \widehat{\zeta}_{\mathcal{C}} \cdot |b(h) + B(h) - b(l) - B(l)|$$

for some $\widehat{\zeta}_{\mathcal{C}} > 0$, where the magnitude of $\widehat{\zeta}_{\mathcal{C}}$ as before depends on the correlation of the contract with the state. The reasoning for why contingent contracts are discounted is the same as before: buyers adjust for adverse selection on the side of the seller and then shed their offers below their conditional valuations. Thus, the main insights of the paper also generalize to environments where investors liquidity needs are private information.

4.2 Equilibrium Contracts and Fluctuations

In this section, I solve the entrepreneurs' problem given the pricing of financial contracts in Proposition 1, and then I characterize the equilibrium fluctuations in the economy and how they are related to the informational and trading frictions. I solve entrepreneurs' problem by backwards induction.

In period 2, after cash-flows are realized, entrepreneurs decide how much capital to liquidate in order to meet their liabilities. Recall that $\{b(s), B(s)\}$ denote the contractual repayments that entrepreneurs have promised to make to investors in period 2, and let $d(s) \equiv \frac{b(s)+B(s)}{k}$ denote the 'per unit' equivalent of the total period

2 repayments. Since liquidations are costly, it is without loss to assume that entrepreneurs would have chosen to repay as much as possible at the end of period 2, i.e. $B(s) = \theta A(1 - z(s))k$ for $s \in \{l, h\}$. We can thus express entrepreneurial consumption in period 2 as

$$c_2^E(s) = (a(s) + q(s)z(s) + (1 - z(s))A - d(s))\, k$$

Entrepreneurs choose $z(s) \in [0, 1]$ to maximize $c_2^E(s)$ subject to the beginning of period resource constraint given in equation (3), which can be re-written as

$$0 \leq (a(s) + q(s)z(s) - d(s) + \theta A(1 - z(s)))\, k$$

Since entrepreneurs will only choose to liquidate capital if the resource constraint is violated at $z(s) = 0$, we can immediately make the following conclusion: in equilibrium, entrepreneurs liquidate capital in state s if and only if the cash-flows at hand plus the pledgeable portion of future cash-flows are insufficient to cover debt obligations, i.e. if $d(s) > a(s) + \theta A$. Given this, the following lemma characterizes how entrepreneurs' contractual choices made in period 0 translate into liquidations and equilibrium fluctuations in the prices of capital in period 2:

Lemma 3 *In equilibrium, entrepreneurs do not liquidate capital in state h, and they liquidate capital in state l if only if the financial contract features $d(l) > a(l) + \theta A$. The prices of capital goods satisfy*

$$\theta A < q(l) \leq q(h) = A$$

where $q(l) = g'(z(l)k)$ and $z(l) = \max\{0, \frac{d(l)-a(l)-\theta A}{q(l)-\theta A}\} \in [0, 1)$.

The above result illustrates how feedback effects between asset prices and entrepreneurial balance sheets can arise when contracts are insufficiently contingent. In particular, if entrepreneurs make 'excessive' repayments in the low state, $d(l) > a(l) + \theta A$, they need to liquidate in that state, $z(l) = \frac{d(l)-a(l)-\theta A}{q(l)-\theta A} > 0$. Liquidations by the entrepreneurial sector as a whole tend to depress asset prices, $q(l) = g'(z(l)k) < A$, and such a decline in asset prices means that entrepreneurs need to liquidate even more capital to meet their liabilities, and so on. It is through such feedback effects can rationalize how shocks of relatively modest magnitude can get amplified into larger scale recessions. However, it is important to note that whether such feedback effects arise depends critically on whether entrepreneurs find it optimal to expose themselves to costly liquidations when contracting ex-ante. I turn to this question next.

In period 0, entrepreneurs choose what financial contract to issue, taking as given the schedule $\mathcal{L}(\cdot)$ and the prices $\{q(s)\}$ of capital goods. These contractual choices then fully characterize the solution to the entrepreneurial problem as well as the equilibrium of the economy. In the following proposition, I show that the tradeoff between the costs of contingency, as described in Proposition 1, and the costs of liquidations, as described in Lemma 3, yields a pecking order for entrepreneurial liability design. Let $\underline{q}$ be a threshold price for capital goods defined by

$$\underline{q} = \left(\theta + (1-\theta)\frac{\pi(l)}{1-\pi(l)}\frac{1-\pi(l)-\zeta^{+}}{\pi(l)+\zeta^{+}}\right)\cdot A \in (\theta A, A)$$

then we have the following result:

Proposition 2 (Pecking Order) *Given the participation constraint $\mathcal{L}(\cdot)$ as in Proposition 1 and prices $\{q(s)\}$ of capital goods that satisfy $q(l) \leq q(h) = A$, the optimal financial contract falls into one of the following categories:*

- **Type I.** *If $q(l) > \underline{q}$, entrepreneurs prefer* **non-contingent** *contracts: $a(l) + \theta A < d(l) = d(h)$, and $d(l) < d(h)$ only if the financing constraint in the low state is exhausted, $d(l) = \theta a(l) + q(l)$.*
- **Type II.** *If $q(l) < \underline{q}$, entrepreneurs prefer* **contingent** *contracts: $a(l) + \theta A = d(l) < d(h)$, and $a(l) + \theta A < d(l)$ only if the financing constraint in the high state is exhausted, $d(h) = \theta a(h) + A$.*
- **Type III.** *If $q(l) = \underline{q}$, entrepreneurs are* **indifferent** *to the degree of contingency of their contracts.*

Thus, if the price of capital $q(l)$ in low state is above the threshold $\underline{q}$, then entrepreneurs prefer to borrow with non-contingent debt without introducing explicit contingencies in their contracts: they set their repayments to $d(l) = d(h)$ unless the borrowing capacity in state l is exhausted, i.e. if $d(l) = \theta a(l) + q(l)$. On the other hand, if the price $q(l)$ is below the threshold $\underline{q}$, then introducing contingency becomes desirable. In fact, entrepreneurs fully insure themselves against fluctuations as long as their borrowing capacity in the high state is not exhausted, i.e. if $d(h) < \theta a(h) + A$. Finally, when the price $q(l)$ is equal to the threshold $\underline{q}$, entrepreneurs are indifferent between the two types of contracts.

Notice the importance of informational and trading frictions for this result. If secondary markets were frictionless, $\zeta^+ = 0$, then entrepreneurs would begin insuring even the slightest fluctuations in the prices of capital in order to avoid liquidations ($\underline{q} < A \iff \zeta^+ > 0$). Thus, in such an economy, we would never be able to observe 'fire-sales', $q(l) < A$, and non-contingent contracts, $d(l) = d(h)$, at the same time. The result in Proposition 2 thus provides a potential rationale for why contingencies in financial contracts may be limited or missing, despite the fact that they cause borrowers to engage in costly liquidations of productive assets.

The above analysis was conducted in partial equilibrium, where the price of capital was set exogenously. This is a useful exercise when thinking of contract design from an atomistic entrepreneur's perspective who takes the prices of capital as given. In general equilibrium, however, there is an interaction between contractual design and asset prices that puts additional restrictions on what can happen. The following proposition combines the results of Proposition 2 with market clearing for capital goods to characterize the equilibrium behavior of the economy.

Proposition 3 (Equilibrium Fluctuations) *In equilibrium, entrepreneurs invest at a scale below first best, they liquidate a positive fraction of their capital in the low state, and the price of capital goods is depressed in the low state. Depending on parameters, the equilibrium contract may or may not be contingent and, if the financing constraint in the high state does not bind, then the prices of capital goods satisfy*

$$0 \leq \frac{q(h) - q(l)}{q(h)} \leq (1 - \theta) \left(1 - \frac{\pi(l)}{1 - \pi(l)} \frac{1 - \pi(l) - \zeta^+}{\pi(l) + \zeta^+}\right)$$

This result states that because contingencies are costly to introduce, in equilibrium entrepreneurs will choose to make 'excessive' repayments in the low state, $d(l) > a(l) + \theta A$, and they will thus liquidate capital in that state, $z(l) = \frac{d(l) - a(l) - \theta A}{q(l) - \theta A} > 0$. As a result, the equilibrium prices of capital will fluctuate, $q(l) < q(h) = A$, and the feedback effects between asset prices and balance sheets will indeed arise when the cash-flow shock materializes. The bounds on asset price fluctuations in essence provide a necessary condition for the balance sheet channel to kick in. In particular, the proposition states that if the financing constraints give entrepreneurs enough room for insurance, then the fluctuations in the prices of capital will be bounded by the cost of contractual contingency derived in the previous section: $\frac{q(h) - q(l)}{q(h)} > 0 \iff \zeta^+ > 0$.

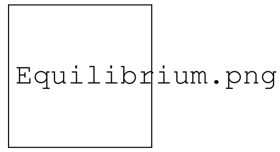

Figure 2: **Equilibrium Fluctuations (a)**

Calibration: $\chi(k) = \frac{\chi}{\alpha} k^{\alpha}$ with $(\chi, \alpha) = (1, 2)$; $g(k) = Ak - \frac{g}{\beta} k^{\beta}$ with $(g, \beta) = (0.1, 1.7)$; $\theta = 0.1$ and $(a(h), A) = (0.5, 0.5)$; $x_i \sim N(\mu(s), \sigma^2)$ on $[-0.5, 0.5]$ for $s = l, h$, with $\mu(l) = -0.2$ and $\mu(h) = 0.2$; $\pi(l) = 0.05$, $\lambda = 0.3$, $\sigma = 0.14$, $n = 10$.

4.3 Comparative Statics

In this section, I provide comparative statics results that relate the informational and trading frictions to the magnitudes of macroeconomic fluctuations. I focus on the case where in equilibrium the financial constraint is non-binding in the high state.[31] This allows me to illustrate my results most starkly because full-insurance will be obtained in the absence of the two frictions.

The first question that I ask is the following. Given the magnitude of information dispersion and trading frictions, what can we say about the behavior of the economy in response to shocks of different magnitudes? Figure 2 illustrates the answer to that question. Here, I plot the optimal contractual repayments (top diagram), the equilibrium prices of capital goods (middle diagram), and equilibrium liquidations (bottom diagram) as a function of cash-flows $a(l)$ in the low state. The parameterization of the remainder of the model is described in the figure.

As we can see, entrepreneurs borrow with non-contingent claims when the shocks to cash-flows are not too severe (Type I contract). In this region, entrepreneurial liquidations become larger and asset prices become more depressed as the cash-flow shock becomes more severe. However, when cash-flows in the low state become sufficiently low ($a(l) \approx 0.365$), introducing contingencies and limiting liquidations becomes optimal (Type III contract).[32] Note that, in equilibrium, this optimal response of entrepreneurs puts an endogenous lower bound ($\underline{q} \approx 0.469$) on the price of capital in the low state.

The second question that I ask is the following. Given the magnitude of economic shocks, what can we say about their macroeconomic impact for different levels of informational and trading frictions? Figure 3

[31] See Appendix for conditions on the primitives that ensure non-binding constraint in state h.

[32] A cash-flow of 0.365 should not be interpreted as a fall of 0.135 in cash-flows from 0.5; the reason is that fire-sales are triggered at a cash-flow of about 0.4; in particular, the results would be identical if $a(h) \approx 0.4$ and not 0.5.

Figure 3: **Equilibrium Fluctuations (b)**

Calibration: Same as in Figure 2(a), except that $a(l) = 0.3$, and that σ and n vary.

illustrates the answer to that question. On the horizontal axes, I plot the standard deviation σ of signals and on the vertical axes I plot $V(q) = \frac{q(h)-q(l)}{q(h)}$ as a measure of asset price fluctuations in this economy. The relationship is plotted for three different values of the trading friction n.[33] I calibrate the size of the shock so that the maximal implied asset price volatility is 12 percent. The parameterization of the remainder of the model is described in the figure.

As we can see, both disagreements and trading frictions are crucial for the model to generate significant fluctuations in asset prices. When disagreements are small (σ is small or large) or when trading frictions are small (n is large), then the volatility bounds of Proposition 3 impose endogenous limits to asset price fluctuations. On the other hand, when these frictions are large, then these bounds on volatility can be non-binding as entrepreneurs find contingencies in contracts extremely costly; here, economic shocks have their largest impact.

4.4 Discussion

The mechanism that generates costs to contractual contingency relies crucially on the interaction between information dispersion and imperfect competition in financial markets. Although the trading environment assumed in this paper is rather stylized, the insight that disagreements and limited competition allow buyers to earn rents is more general. In particular, the literature on common value auctions has shown that in a variety of trading mechanisms, sellers forgo informational rents when faced with buyers who disagree about the value of the object being offered for sale. While in general the magnitude of rents may depend on the particular trading environment, the logic that drives costly contractual contingencies extends well beyond the basic economic environment presented in this paper (see Milgrom and Weber (1982)). The results also do not rely on the absence of public signals, as long as these signals are imperfectly informative. In fact, a result known in

[33] I have assumed that buyers are matched with at most one seller; here, I need to relax this assumption to be able to vary n while keeping λ fixed. I show in Appendix C that the results are unchanged when I allow buyers to match with more than one seller.

the literature as the 'linkage principle' can be used to show that informational rents, and thus discounting of contingent contracts, will be smaller when there is an informative public signal. Since the qualitative results would be unchanged, the abstraction away from public signals is only made for simplicity.

The idea that information dispersion allows heterogeneously informed buyers to earn informational rents is not unique to auction-theoretic trading mechanisms; for example, this is also the prediction of the popular noisy rational expectation (noisy REE) models of financial markets (e.g., Grossman and Stiglitz (1980), Albagli et al. (2011)). While in auctions it is the finiteness of buyers that generates informational rents, in this literature that role is played by bounds on the risk capacity of traders combined with noise trader risk. However, it is not straight-forward to adapt noisy REE models to general equilibrium analysis: one needs an interpretation for the noise in the price formation and, depending on the application, this can matter for results. The advantage of using auction-theoretic trading mechanisms is that I do not need to introduce any noise in the clearing process in order to have a well-defined trading environment. Furthermore, as argued by Milgrom (1981), auctions give a microfoundation to the notion of competitive markets, especially in environments with dispersed information where this notion is ill-defined.[34]

The theory proposed here highlights the importance of understanding why macro-contingencies in contracts may be limited endogenously. It suggests that we need to be careful when using models that exogenously force contracts to be incomplete, since whether economic agents incorporate contingencies in their contracts will tend to depend on the magnitude of frictions in financial markets relative to the benefits of risk-sharing. In particular, the model predicts a relationship between magnitudes of economic fluctuations and financial market development. Thus, we should expect the impact of economic shocks to be large in environments where markets for contingent contracts are underdeveloped (e.g. contracts contingent on real estate indices); on the other hand, risk-sharing should mute the impact of economic shocks when agents can insure themselves with contracts that are traded in deep/competitive exchanges (e.g. contracts contingent on energy prices). This is consistent with the observation that real estate related shocks are associated with larger economic downturns and that economies with less developed financial markets tend to have more volatile business cycles. Furthermore, as I show in the next section, the theory yields normative predictions that would be difficult to obtain with models that force contracts to be exogenously incomplete.

[34]A perfectly competitive market can be thought of as the limit of auction-theoretic markets as the number n of traders goes to ∞.

5 Policy Implications

In this section, I consider the implications of the theory for financial regulation. In particular, I ask whether the contractual choices that entrepreneurs and investors make are socially optimal. To study this question, I consider the problem of a *constrained* social planner who can coordinate agents' contractual choices and whose objective is to maximize entrepreneurial welfare subject to leaving investors as well off as in the decentralized equilibrium; however, the planner must still respect the financial constraints faced by economic agents and she must still allow economic agents to freely trade in capital goods markets and in secondary markets for financial contracts.[35]

When intervening in agents' contractual choices at $t = 0$, the planner affects the prices of capital goods and the degree of contingency of all contracts issued in the economy, both of which affect investors' welfare. Thus, to ensure that the intervention is Pareto improving, the planner will also need to make a transfer T at $t = 0$ from the entrepreneurs to investors in order to guarantee that the latter are as well off at the planner's allocation as at the decentralized equilibrium. Let superscript SP denote the allocations chosen by the planner and superscript DE denote the allocations in the decentralized equilibrium. First, note that if the planner chooses to reduce liquidations in the low state, this will result in a loss of profits to the traditional sector firms. Because late investors are the ultimate owners of these firms, the planner will need to transfer the difference in expected profits from the entrepreneurs to investors: $\pi(l)\left(\Pi^{DE}(l) - \Pi^{SP}(l)\right)$ where $\Pi^{*}(l)$ denotes the profits of these firms in low state and in allocation $* \in \{DE, SP\}$. Second, at allocation $* \in \{DE, SP\}$, investors earn a rent from entrepreneurs equivalent to the total cost of contingency in that allocation: $R^{*} \equiv \zeta^{+} \cdot (d^{*}(h) - d^{*}(l))k^{*}$; the reason is that while it is privately costly for an investor to hold a contingent contract, in aggregate these are just transfers among investors. To leave investors as well off as in the decentralized allocation, the lump transfer T from the entrepreneurs to investors must therefore satisfy

$$T \geq \pi(l)\left(\Pi^{DE}(l) - \Pi^{SP}(l)\right) + R^{DE} - R^{SP}$$

i.e., it must compensate investors both for the change in profits of the 'traditional' sector firms and for the change in investors' rents associated with the change in the design of financial contracts.

The presence of the rent R^{*} turns out to be crucial for the form that the optimal intervention takes. In essence,

[35] It is straight-forward to show that a social planner could achieve first-best if she were unconstrained to eliminate the trading friction (take n to ∞) and circumvent the collateral constraints (take θ to 1).

the planner internalizes that more contingency in contracts results in larger aggregate rents for investors, rather than costs as each investor privately perceives. Hence, because the planner perceives a lower cost to contingent contracting than private agents, she chooses to have more contingency in liabilities and as a result less volatility in asset prices and output:

Proposition 4 (Constrained Optimal Policy) *The planner borrows with contingent contracts and she liquidates capital in the low state only if she exhausts the borrowing capacity in the high state: $z^{SP}(l) > 0$ only if $d^{SP}(h) = \theta a(h) + A$. If the financing constraint in the low state does not bind at DE, then (even if the planner liquidates) her liquidations are lower and capital goods prices are less volatile:*

$$q^{DE}(l) < q^{SP}(l) \leq q^{SP}(h) = q^{DE}(h) = A$$

The planner's allocation can be decentralized by imposing at $t = 0$: (i) a marginal subsidy to debt that pays only in the high state, and (ii) lump sum taxes/transfers to ensure that the subsidy is financed and the intervention is Pareto improving.

Let us understand the logic behind this result. The key problem in this economy is the presence of costly liquidations at $t = 2$, which depresses the returns of the projects and thus investment at $t = 0$ (relative to first-best). The conventional focus in the literature has been on the entrepreneurs' overall borrowing behavior at $t = 0$. The argument goes as follows. Each atomistic entrepreneur, in her borrowing decisions, cannot internalize the 'fire-sale' externality that she exerts on other entrepreneurs: if some entrepreneurs reduce liquidations, asset prices will increase, the constraints of other entrepreneurs will become relaxed, and they will not need to liquidate as much. Hence, to ensure that entrepreneurs do not 'over-liquidate', the literature argues that policy makers should limit entrepreneurs' overall borrowing. Thus, in order to correct the ex-post 'fire-sale' externality, such a policy distorts entrepreneurs' investment even further away from first-best.

The results of Proposition 4, however, imply that the focus on the overall borrowing behavior of economic agents may be misplaced. There are two reasons for this. First, targetting the contingency in contracts is a more direct tool if the goal is to reduce the costly liquidations in the bad state. Second, the cost to contingency in this economy arises due to the illiquidity of contingent contracts in secondary markets. These illiquidity costs, however, are private and not social, since they arise due to mispricing of contingent contracts in secondary markets. Hence, while an atomistic investor tries to protect himself against such mispricing by asking the entrepreneur for a compensation, the planner internalizes that in aggregate these are just transfers among investors. These results thus provide a theoretical backing to the recent proposals to impose or subsidize macro contingencies in private contracts. For practical implementation one should think of these policies as encouraging (through price or quantity interventions) the issuance of claims that allow for write-downs in bad aggregate states of the economy (e.g. contingent convertible debt).

One may wonder whether interventions which would target the secondary market distortions at their source would be preferrable to the ex-ante policies analyzed above. While indeed these type of interventions would be more direct, I have ignored them for two reasons. First, with multi-dimensional heterogeneity and private information, analyzing interventions in secondary markets is a rather complex exercise. Second, in the parametric case analyzed in the paper, the ex-ante policy described in Proposition 4 replicates secondary market interventions that fully eliminate the distortion to risk-sharing; hence, in this case, secondary market interventions would be redundant.

The reason for this stark result is that while secondary markets misprice financial claims, they still lead to their efficient allocation among investors. To correct this mispricing, it then sufficies to intervene with an ex-ante policy alone. However, this would no longer be the case if financial claims were also misallocated.[36] I conjecture that in such an environment an optimal policy would consist of both secondary market interventions and an ex-ante policies; this is an interesting but complex problem to tackle in further research. Finally, the conclusion that policy makers should target the contingency of financial contracts and not leverage relies on the fact that, conditional on the contracts issued by entrepreneurs, mispricing of financial claims creates transfers across investors but does not result in ex-ante welfare loss. If financial claims were also misallocated, then some of these transfers would also be socially costly. An optimal ex-ante policy would then consist of *both* a subsidy to contingent contracts *and* a tax on overall borrowing (leverage); however, a policy that solely targets borrowers' leverage would still be sub-optimal.

6 Conclusions

In this paper, I have proposed a novel theory to rationalize the concentration of macroeconomic risks, that is thought to drive balance sheet recessions, as a result of informational and trading frictions in financial markets.

[36]This would be the case if investors have liquidity needs that are less severe and are willing to not sell their claims when mispricing is large: more formally, investor liquidity need β takes values in $\{\underline{\beta}, 1\}$ with $\underline{\beta}$ sufficiently close to 1.

The key idea was that sharing macroeconomic risks requires borrowers and creditors to introduce macro contingencies in their contracts, but that contingent contracting will be costly when information about the future state of the economy is dispersed and when markets for such contracts are imperfectly competitive. I argued that endogenizing the limits to macro contingencies is insightful not only for understanding the nature of macroeconomic fluctuations but also for studying the role that policy makers can play in mitigating the impact of macroeconomic shocks. First, my theory predicts that, all else equal, the size of macroeconomic fluctuations should be tightly linked to the magnitudes of secondary market frictions. Second, I have shown that policy makers can achieve a Pareto improvement by either imposing (or subsidizing) macro contingencies in private contracts, which is a prediction that would be difficult to obtain from models with exogenously incomplete markets. From a methodological standpoint, the auction-theoretic approach to markets proposed in this paper can be useful to study other general equilibrium environments where asset trade occurs under dispersed information. Although these insights were drawn from a rather stylized framework, I conjecture that the underlying mechanisms that drive the results of the paper should extend more broadly.

7 Appendices

7.1 Appendix A

Proof of Lemma 1. The result follows directly from the fact that buyers agree about the contractual payoffs and because $n \geq 2$ buyers compete a la Bertrand.

Proof of Lemma 2. Suppose that buyers have been matched with a seller with $\beta^S = 1$ and that buyers believe that this investor has received signal $x^S \in X$ where X is some measurable subset of $[\underline{x}, \overline{x}]$. Suppose also that the contract is positively correlated with the state of the economy. The payoff to buyer 1 who receives signal x but bids as if he has received signal z is given by

$$\Pi(z,x) = \int_{\underline{x}}^{z} \left(v^+(x,y,X) - p(z)\right) f_{y_1^+}(y|x,X)dy$$

where for $x, y \in [\underline{x}, \overline{x}]$, $v^+(x,y,X) \equiv \mathbb{E}\{d(s)k|x_1^B = x, y_1^+ = y, x^S \in X\}$ and $F_{y_1^+}(\cdot|x,X)$ is the conditional distribution of the second highest signal among signals $\{x_2^B, ..., x_n^B\}$, conditional on $x_1^B = x$ and $x^S \in X$ (I omit indexation by contract and liquidity needs for brevity). In particular, we have that

$$F_{y_1^+}(y|x,X) = \frac{\sum_s F_s^{n-1}(y) f_s(x) \pi_X(s)}{\sum_s f_s(x) \pi_X(s)}$$

$$f_{y_1^+}(y|x,X) = \frac{\sum_s (n-1) F_s^{n-2}(y) f_s(x) \pi_X(s)}{\sum_s f_s(x) \pi_X(s)}$$

where $\pi_X(s) \equiv \Pr\left(s|x^S \in X\right)$, i.e. all buyers bid as in a standard first-price common value auction only after adjusting their priors from $\pi(l)$ to $\pi_X(l)$. Differentiation with respect to z and evaluation at $x = z$ implies that the equilibrium strategy satisfies the ODE

$$p'(x) = \left(v^+(x,x,X) - p(x)\right) \frac{f_{y_1^+}(x|x,X)}{F_{y_1^+}(x|x,X)}$$

With the boundary condition $p(\underline{x}) = v(\underline{x}, \underline{x}, X)$, the solution to the above ODE is given by

$$p(x) = \int_{\underline{x}}^{x} v^+(z,z,X)dG^+(z|x,X) \tag{10}$$

for $x \in [\underline{x}, \overline{x}]$, and where for $z \leq x$, $G^+(z|x,X) = \exp\left(-\int_z^x \frac{f_{y_1^+}(t|t,X)}{F_{y_1^+}(t|t,X)}dt\right)$. The boundary condition holds since a buyer with signal $\underline{x}$ earns a negative payoff if $p(\underline{x}) > v^+(\underline{x}, \underline{x}, X)$, and if $p(\underline{x}) < v^+(\underline{x}, \underline{x}, X)$ then he can deviate and earn a positive payoff. The function $v^+(x, x, X)$ is increasing in x and $G^+(z|x', X) \geq G^+(z|x, X)$ for $z \leq x$ and $x' > x$; thus the strategy $p(\cdot)$ is indeed increasing and differentiable. To show that p is a maximum, note that MLRP implies that

$$\frac{d\Pi}{dz} = F_{y_1^+}(z|x,X)\left[\left(v^+(x,z,X) - p(z)\right)\frac{f_{y_1^+}(z|x,X)}{F_{y_1^+}(z|x,X)} - p'(z)\right]$$

is positive for $z < x$, negative for $z > x$, and it is zero at $z = x$ by construction. This establishes that the strategy p is optimal for buyers. The derivation of buyers' offers for the case of a negatively correlated contract is analogous. The equilibrium offer strategy in that case is decreasing and is given by

$$p(x) = \int_x^{\overline{x}} v^-(z, z, X)dG^-(z|x, X) \tag{11}$$

for $x \in [\underline{x}, \overline{x}]$, where $v^-(y, x, X) = \mathbb{E}\{b(s) + B(s)|x_1^B = x, y_1^- = y, x^S \in X\}$, and where for $z \geq x$, $G^-(z|x,X) = \exp\left(-\int_x^z \frac{f_{y_1^-}(t|t,X)}{1-F_{y_1^-}(t|t,X)}dt\right)$. To compute the off-equilibrium strategies, set $X = \{\overline{x}\}$ for negatively correlated contracts, and $X = \{\underline{x}\}$ otherwise.

That in equilibrium investors with $\beta = 0$ sell their contracts is clear: contractual prices are always positive and these investors do not value consumption in period 2. Suppose now that investors with $\beta = 1$ and signals in some measurable set $X \subset [\underline{x}, \overline{x}]$ also sell their contracts in equilibrium, and consider buyers who have matched with a seller with $\beta = 1$ and contract $\mathcal{C}$ that is contingent on the state. Note that investor with $\beta = 1$ is indifferent between selling a non-contingent contract and keeping it, and thus he will not sell it by the indifference-breaking assumption. Let $p(\cdot, 1, \mathcal{C})$ denote the equilibrium offer strategy followed by buyers, and let $p^{max}(1, \mathcal{C})$ denote the maximal offer implied by this strategy (as defined in Section 3.2). Then since buyers observe the seller's liquidity need, we have that for non-negatively correlated contracts

$$\begin{aligned} p(x) &= \int_{\underline{x}}^{x} v^{+}(z,z,X)dG^{+}(z|x,X) \\ &\leq \int_{\underline{x}}^{x} v^{+}(x,z,X)dG^{+}(z|x,X) \\ &\leq \int_{\underline{x}}^{x} v^{+}(x,z,X)dF_{y_1^+}(z|x,X) \\ &= \mathbb{E}\{b(s)+B(s)|x_1^B=x, y_1^+<x, x^S\in X\} \end{aligned}$$

Similar reasoning shows that $p(x) \leq \mathbb{E}\{b(s)+B(s)|x_1^B = x, y_1^- > x, x^S \in X\}$ for a negatively correlated contract. Using symmetry and integrating over buyers' signals, we get:

$$\mathbb{E}\{p^{max}(1,\mathcal{C})|x^S\in X\} \leq \mathbb{E}\{b(s)+B(s)|x^S\in X\}$$

But the payoff from posting the contract to a an investor without liquidity need and with signal $x \in X$ is given by $U^S(x,1,\mathcal{C}) = \mathbb{E}\{p^{max}(1,\mathcal{C})|x^S = x\} - \mathbb{E}\{b(s)+B(s)|x^S = x\}$ and must be positive for all $x \in X$ by the indifference-breaking assumption; integration over $x^S \in X$ then implies that

$$\mathbb{E}\{p^{max}(1,\mathcal{C})|x^S\in X\} > \mathbb{E}\{b(s)+B(s)|x^S\in X\}$$

Hence, we have a contradiction, i.e. there is no equilibrium in which investors without liquidity needs post their contracts for sale.

Proof of Proposition 1. Since investors sell their contracts if and only if they experience liquidity needs, the expected present value of contract $\mathcal{C} = \{b(s), B(s)\}$ to an investor is given by

$$\begin{aligned} \mathcal{L}(\{b(s)+B(s)\}) &= (1-\lambda)\mathbb{E}\{b(s)+B(s)\} + \lambda\mathbb{E}\{p^{max}(0,\mathcal{C})\} \\ &= \mathbb{E}\{b(s)+B(s)\} + \lambda\left(\mathbb{E}\{p^{max}(0,\mathcal{C})\} - \mathbb{E}\{b(s)+B(s)\}\right) \end{aligned}$$

where $\mathbb{E}\{p^{max}(0,\mathcal{C})\}$ denotes the expected price at which the investor with $\beta = 0$ sells his contract. From

Proposition 1, we have that

$$\mathbb{E}\{p^{max}(0,\mathcal{C})\} = \begin{cases} \mathbb{E}\{\int_{\underline{x}}^{x_1^B} v^+(z,z)dG^+(z|x_1^B)|\{x_1^B > y_1^+\}\} & \text{if} \quad b(h)+B(h) \geq b(l)+B(l) \\ \mathbb{E}\{\int_{x_1^B}^{\overline{x}} v^-(z,z)dG^-(z|x_1^B)|\{x_1^B < y_1^-\}\} & \text{otherwise} \end{cases}$$

where $v^+(x,y) = \mathbb{E}\{b(s)+B(s)|x_1^B = x, y_1^+ = y\}$ and $v^-(x,y) = \mathbb{E}\{b(s)+B(s)|x_1^B = x, y_1^- = y\}$ because investors with $\beta = 0$ sell their contracts irrespective of signals received. Using the forms of the functions $v^+(\cdot,\cdot)$ and $v^-(\cdot,\cdot)$, we have that

$$\mathcal{L}(\{d(s)k\}) = \mathbb{E}\{b(s)+B(s)\} - \zeta_C|b(h)+B(h)-b(l)-B(l)|k$$

for $\zeta_{\mathcal{C}} = \begin{cases} \zeta^+ & \text{if } b(h)+B(h) \geq b(l)+B(l) \\ \zeta^- & \text{if } b(h)+B(h) < b(l)+B(l) \end{cases}$ with $\zeta^+, \zeta^- > 0$ given by

$$\begin{aligned} \zeta^+ &= \lambda \cdot \left(\mathbb{E}\left\{ \int_{\underline{x}}^{x_1^B} \Pr\left(s = l|x_1^B = z, y_1^+ = z\right) dG^+(z|x_1^B)|\{x_1^B > y_1^+\} \right\} - \pi \right) \\ \zeta^- &= \lambda \cdot \left(\pi - \mathbb{E}\left\{ \int_{x_1^B}^{\overline{x}} \Pr\left(s = l|x_1^B = z, y_1^- = z\right) dG^-(z|x_1^B)|\{x_1^B < y_1^-\} \right\} \right) \end{aligned}$$

That $\zeta^+ > 0$ follows from the fact that $\Pr\left(s = l|x_1^B = z, y_1^+ = z\right) > \Pr\left(s = l|x_1^B = x, y_1^+ = z\right)$ for all $x > z$, and from the fact that $G^+(\cdot|x)$ is fosd dominated by $F_{y_1^+}(\cdot|x)$ for all x:

$$\begin{aligned} \pi &= \mathbb{E}\left\{ \int_{\underline{x}}^{x_1^B} \Pr\left(s = l|x_1^B > y_1^+ = z\right) dF_{y_1^+}(z|x_1^B)|\{x_1^B > y_1^+\} \right\} \\ &< \mathbb{E}\left\{ \int_{\underline{x}}^{x_1^B} \Pr\left(s = l|x_1^B = y_1^+ = z\right) dF_{y_1^+}(z|x_1^B)|\{x_1^B > y_1^+\} \right\} \\ &\leq \mathbb{E}\left\{ \int_{\underline{x}}^{x_1^B} \Pr\left(s = l|x_1^B = y_1^+ = z\right) dG_{y_1^+}(z|x_1^B)|\{x_1^B > y_1^+\} \right\} \end{aligned}$$

The proof that $\zeta^- > 0$ is analogous.

7.2 Appendix B

Proof of Lemma 0. Traditional sector firms set $k^d(s) = 0$ if $q(s) > A$ and $k^d(s) = g'^{-1}((q(s))$ if $q(s) \leq A$. Since entrepreneurs will liquidate all of their capital if $q(s) > A$ and entrepreneurs choose $k > 0$, $q(s) > A$ cannot be an equilibrium; hence, $q(s) \leq A$. Capital goods market clearing then implies that $q(s) = g'(z(s)k)$ for some $z(s) \in [0, 1]$ and $k > 0$. Finally, since $(g'(x) - \theta A)x$ is increasing in x, we have $(p\,(s) - \theta A)z(s)k = (g'(z(s)k) - \theta A)z(s)k > (g'(0) - \theta A)0 = 0$ if $z(s)k > 0$, and as $g'(0) = A$, we have that $\theta A < q(s) \leq A$.

Proof of Lemma 3. Entrepreneurial budget constraint and the consumption non-negativity constraints in period 2 are

$$\begin{aligned} c(s) &= (a(s) - d(s) + z(s)q(s) + (1 - z(s))A)\,k \\ d(s) &\leq a(s) + \theta A + z(s)q(s) \end{aligned}$$

Since by Lemma 0, $q\,(s) \in (\theta A, A]$ with $q\,(s) < A$ if and only if $z\,(s) > 0$, in equilibrium entrepreneurs set $z(s) > 0$ if and only if the consumption non-negativity constraint binds with $z(s) = 0$. Thus, $z(s) > 0$ if and only if $d(s) > a(s) + \theta A$, for $s \in \{l, h\}$.Then we have

$$\begin{aligned} a(h) - d(h) + \widehat{d}(h) &\geq a(h) - (\theta a(h) + q(h)) + \theta A \\ &\geq a(h) - (\theta a(h) + A) + \theta A \\ &= (1 - \theta)(a(h) - A) \\ &\geq 0 \end{aligned}$$

where the first inequality follows from the financial constraint, the second inequality follows from Lemma 0, and the last inequality holds by Assumption 1.3. Thus, we have $z(h) = 0$ and $z(l) = \max\{0, \frac{d(l) - a(l) - \theta A}{q(l) - \theta A}\}$, which by Lemma 0 implies that $\theta A < q(l) \leq q(h) = A$, where $q(l) = g'(z(l)k)$.

Proof of Proposition 2. Because entrepreneurs want to insure against state l drop in the price of capital goods, it suffices to consider contracts with $d(l) \leq d(h)$.

First, I show that Assumption 1.4 implies that $a\,(l) + \theta A \leq d\,(l)$. Suppose to the contrary that at the optimum $d(l) < a(l) + \theta A$. Then entrepreneur's marginal utility of wealth is 1 in both states and, because contingency is costly, it is optimal for the entrepreneurs to set $d(h) = d(l)$. Now, consider increasing d by a small amount ϵk.

For ϵ small, the marginal cost of this increase is ϵk since the contract is still non-contingent, while the marginal benefit is given by $\frac{\sum_s \pi(s)(a(s)+A)-d}{\chi'(k)-d} \cdot \epsilon k$ where k satisfies the budget constraint $\chi(k) = (d+\epsilon)k < (a(l)+\theta A)k$. Assumption 1.4 then implies that $\chi'(k) < \sum_s \pi(s)(a(s) + A)$ and therefore the marginal benefit of such an increase is greater than the marginal cost. Thus, $a\,(l) + \theta A \leq d\,(l)$.

Second, using the above results, the entrepreneurial problem can be re-written as

$$\max_{\{k,d(l),d(h)\}} \left[\pi(l)(a(l) + \theta A - d(l))\frac{(1-\theta)A}{q(l) - \theta A} + (1 - \pi(l))(a(h) + \theta A - d(h)) + (1 - \theta)A\right] k$$

subject to

$$\chi(k) = \left(\sum_s \xi(s)d(s)\right) k$$

$$d(l) \leq \theta a(l) + q(l)$$

$$d(h) \leq \theta a(h) + A$$

$$a(l) + \theta A \leq d(l)$$

$$d(l) \leq d(h)$$

where $\xi\,(l) \equiv \pi\,(l) + \zeta^+$ and $\xi\,(h) \equiv 1 - \xi\,(l)$. Let $\nu, \mu(l), \mu(h), \omega(l), \omega(h) \geq 0$ denote the multipliers on the above constraints in order as they appear. The entrepreneur's first order conditions with respect to $k, d(l), d(h)$ are

$$\begin{aligned}
\nu &= \frac{\pi(l)(a(l) + \theta A - d(l))\frac{(1-\theta)A}{q(l)-\theta A} + (1 - \pi(l))(a(h) + \theta A - d(h)) + (1 - \theta)A}{\chi'(k) - \sum_s \xi(s)d(s)} \\
\nu(1 - \xi(l)) &= 1 - \pi(l) + (\mu(h) - \omega(h))k^{-1} \\
\nu\xi(l) &= \pi(l)\frac{(1-\theta)A}{q(l) - \theta A} + (\mu(l) - \omega(l) + \omega(h))k^{-1}
\end{aligned}$$

and the complementary slackness conditions are

$$\mu(h)(\theta a(h) + A - d(h)) = 0$$

$$\mu(l)(\theta a(l) + q(l) - d(l)) = 0$$

$$\omega(l)(d(l) - a(l) - \theta A) = 0$$

$$\omega(h)(d(h) - d(l)) = 0$$

These conditions together with the period 0 budget constraint and the inequality constraints for the repayments fully characterize the solution to the entrepreneurial problem. We can now derive the three types of the entrepreneurial contract:

- Type I: $q(l) > \underline{q} \Longrightarrow \frac{\pi(l)}{\xi(l)} \frac{(1-\theta)A}{q(l)-\theta A} < \frac{1-\pi(l)}{1-\xi(l)}$. Therefore, from the foc's we have that

$$\frac{\mu(l) - \omega(l) + \omega(h)}{\xi(l)} > \frac{\mu(h) - \omega(h)}{1 - \xi(l)}$$

  If the contract satisfies $d(l) = d(h)$, then we are done. If, on the other hand, $d(l) < d(h)$, then we have $\omega(h) = 0$, and thus $\mu(l) > 0$. Hence, $d(l) < d(h) \Longrightarrow d(l) = \theta a(l) + q(l)$.

- Type II: $q(l) < \underline{q} \Longrightarrow \frac{\pi(l)}{\xi(l)} \frac{(1-\theta)A}{q(l)-\theta A} > \frac{1-\pi(l)}{1-\xi(l)}$. Therefore, from the foc's we have that

$$\frac{\mu(l) - \omega(l) + \omega(h)}{\xi(l)} < \frac{\mu(h) - \omega(h)}{1 - \xi(l)}$$

  If the contract satisfies $a(l) + \theta A = d(l)$, then we are done. On the other hand, if $a(l) + \theta A < d(l)$, then $\omega(l) = 0 \leq \mu(l)$ and thus $\mu(h) > 0$. Thus, $a(l) + \theta A < d(l)$ implies $d(l) < d(h) = \theta a(h) + A$.

- Type III: $q(l) = \underline{q} \Longrightarrow \frac{\pi(l)}{\xi(l)} \frac{(1-\theta)A}{q(l)-\theta A} = \frac{1-\pi(l)}{1-\xi(l)}$. Therefore, the entrepreneur is indifferent about contract type.

Proof of Proposition 3. Part (1) : To show that $d(l) > a(l) + \theta A$ and $q(l) < A$, suppose to the contrary that at the optimum $d(l) = a(l) + \theta A$ and thus $q(l) = A$; note that foc's w.r.t. $d(h)$ and $d(l)$ then imply that

$d(h) = d(l) = d = a(l) + \theta A$. Then we must have

$$\begin{aligned}\nu &= \frac{\pi(l)(a(l)+A-d)+(1-\pi(l))(a(h)+A-d)}{\chi'(\underline{k})-d} \\ &= \frac{\mathbb{E}\{a(s)+A\}-d}{\chi'(\underline{k})-d} \\ &> 1\end{aligned}$$

where the last inequality follows from Assumption 3. Adding the foc's with respect to $d(l)$ and $d(h)$, we have

$$\nu = \pi(l)\frac{(1-\theta)A}{q(l)-\theta A} + 1 - \pi(l) - \omega(l)k^{-1} < 1$$

since $\mu(h) = 0 < \omega(l)$ and $q(l) = A$, a contradiction. Thus, we must have $d(l) > a(l) + \theta A$ and $q(l) < A$. The remaining characterization of the financial contract follows from the financial constraints and Proposition 2. Part (2) : That investment is below first-best follows from the result in Part (1) which implies that $\nu > 1$ and thus

$$\left[\chi'(k) - \sum_s \xi(s)d(s)\right]\nu = \pi(l)(a(l)+q(l)-d(l))\frac{(1-\theta)A}{q(l)-\theta A} + (1-\pi(l))(a(h)+A-d(h))$$

$$\Longrightarrow$$

$$\chi'(k) - \sum_s \xi(s)d(s) < \pi(l)(a(l)+q(l)-d(l))\frac{(1-\theta)A}{q(l)-\theta A} + (1-\pi(l))(a(h)+A-d(h))$$

$$\Longrightarrow$$

$$\begin{aligned}\chi'(k) &< \pi(l)(a(l)+A) + (1-\pi(l))(a(h)+A) + \sum_s (\xi(s)-\pi(s))\, d(s) \\ &\leq \pi(l)(a(l)+A) + (1-\pi(l))(a(h)+A) = \chi'\left(k^{fb}\right)\end{aligned}$$

The expression for liquidations follows from the beginning of period 2 budget constraint, and liquidations are positive by Part (1). Part (3) : That the price of capital in state l is below A follows from Part (2) and Proposition 2. The bounds on asset price fluctuations follow from combining Proposition 1 with the foc's w.r.t. $d(l)$ and $d(h)$ by setting $\mu(h) = 0$.

Proof of Lemma - Non-binding constraint in state h. Let $\overline{k}$ be implicitly defined by $\chi\left(\overline{k}\right) = \xi(l)(a(l)+\theta A)\overline{k}+$

$(1-\xi(l))(\theta a(h)+A)\overline{k}$. Then a sufficient condition for the collateral constraint to be loose in state h is that $\chi'(\overline{k}) \geq \sum \xi(s)(a(s)+A)$. To show this, suppose to the contrary that in equilibrium $d(h) = \theta a(h) + A$ and recall that from Proposition 3 we have $d(l) > a(l) + \theta A$ and $q(l) < A$. Thus, from the entrepreneur's foc's we must have that

$$\begin{aligned}\nu(1-\xi(l)) &\geq 1-\pi(l)\\ \nu\xi(l) &\geq \pi(l)\nu(l)\end{aligned}$$

where $\nu(l) = \frac{(1-\theta)A}{q(l)-\theta A}$ and therefore

$$\begin{aligned}\nu &= \frac{\pi(l)\nu(l)(a(l)+A-d(l)) + (1-\pi(l))(a(h)+A-d(h))}{\chi'(k) - \sum\xi(s)d(s)}\\ &< \frac{\xi(l)\nu(a(l)+A-d(l)) + (1-\xi(l))\nu(a(h)+A-d(h))}{\chi'(k) - \sum\xi(s)d(s)}\end{aligned}$$

Now, note that $d(l) > a(l) + \theta A$ implies that $k > \overline{k}$; hence, we have

$$\begin{aligned}1 &< \frac{\xi(l)\nu(a(l)+A-d(l)) + (1-\xi(l))\nu(a(h)+A-d(h))}{\chi'(k) - \sum\xi(s)d(s)}\\ &< \frac{\sum\xi(s)(a(l)+A) - \sum\xi(s)d(s)}{\chi'(k) - \sum\xi(s)d(s)}\\ &< 1\end{aligned}$$

where the last inequality follows from $\chi'(k) > \chi'(\overline{k}) \geq \sum\xi(s)(a(s)+A)$. Thus, we have a contradiction.

Proof of Proposition 4. The planner's problem is equivalent to:

$$\max_{\{k,d(l),d(h),\tau\}} \left[\pi(l)(a(l)+\theta A - d(l))\frac{(1-\theta)A}{q(l)-\theta A} + (1-\pi(l))(a(h)+\theta A - d(h)) + (1-\theta)A\right]k$$

subject to

$$
\begin{aligned}
\chi(k) &= \left(\sum_s \pi(s)d(s)\right)k - \tau k \\
d(l) &\leq \theta a(l) + q(l) \\
d(h) &\leq \theta a(h) + A \\
d(l) &\leq d(h) \\
\pi(l)\left(\Pi^{DE}(l) - g\left(z(l)k\right) + q(l)z(l)k\right) + \zeta^{+} \cdot (d^{DE}(h) - d^{DE}(l))k^{DE} &\leq \tau k
\end{aligned}
$$

where $q(l) = g'(z(l)k)$, $z(l) = \frac{d(l)-a(l)-\theta A}{q(l)-\theta A}$, and $\tau \equiv \frac{T}{k}$. Let $\widehat{\nu}, \mu(l), \mu(h), \omega(l), \omega(h), \kappa$ denote the multipliers on the constraints of the planner's problem in the order as they appear. The planner's first order condition w.r.t. k is given by

$$
\widehat{\nu}\left[\sum_s \pi(s)d(s) - \tau - \chi'(k)\right] + \left[\sum_s \pi(s)\nu(s)(a(s) + \theta A - d(s)) + (1-\theta)A\right] \\
+\kappa\left[\tau - \pi(l)(z(l)k)q_k(l)\right] + \mu(l)q_k(l) + \pi(l)\nu(l)(z(l)k)q_k(l) = 0
$$

where $\nu(h) = 1 < \frac{(1-\theta)A}{q(l)-\theta A} = \nu(l)$, and where $q_k(l)$ denotes the derivative of the equilibrium price of capital $q(l)$ w.r.t. k. The planner's first order conditions with respect to $d(l)$ and $d(h)$ yield

$$
\begin{aligned}
\widehat{\nu}\pi(l)k - \pi(l)\nu(l)k - \omega(h) - \mu(l) + \pi(l)\nu(l)(z(l)k)q_d(l) + \mu(l)q_d(l) - \kappa\pi(l)(z(l)k)q_d(l) &= 0 \\
\widehat{\nu}(1-\pi(l))k - (1-\pi(l))k + \omega(h) - \mu(h) &= 0
\end{aligned}
$$

where $q_d(l)$ denotes the derivative of the equilibrium price of capital $q(l)$ w.r.t. $d(l)$ and, thus, in equilibrium must satisfy $kq_k(l) = (d(l) - a(l) - \theta A)q_d(l)$. The planner's first-order conditions w.r.t. τ is given by

$$
-\widehat{\nu} + \kappa = 0
$$

Finally, we have the complementary slackness conditions

$$\mu(h)(\theta a(h) + A - d(h)) = 0$$
$$\mu(l)(\theta a(l) + q(l) - d(l)) = 0$$
$$\omega(h)(d(h) - d(l)) = 0$$
$$\kappa\left[\tau k - \pi(l)\left(\Pi^{DE}(l) - g\left(z(l)k\right) + q(l)z(l)k\right) - \zeta^{+} \cdot (d^{DE}(h) - d^{DE}(l))k^{DE}\right] = 0$$

These conditions fully characterize the solution to the planner's problem. For $s \in \{l, h\}$, define $\widehat{\nu}(s)$ by

$$\begin{aligned}\widehat{\nu}(l) &= \nu(l) + (\widehat{\nu} - \nu(l))\, z(l) q_d(l) - \frac{\mu(l) q_d}{\pi(l)k} \\ \widehat{\nu}(h) &= \nu(h) = 1\end{aligned}$$

and note that then the foc w.r.t. $k, d(l), d(h)$ become

$$\begin{aligned}\widehat{\nu}\left(\chi'(k) - \sum_s \pi(s)d(s)\right) &= \left(\sum_s \pi(s)\widehat{\nu}(s)(a(s) + \theta A - d(s)) + (1-\theta)A\right) \\ \widehat{\nu}\pi(l) &= \pi(l)\widehat{\nu}(l) + (\mu(l) + \omega(h))k^{-1} \\ \widehat{\nu}(1 - \pi(l)) &= 1 - \pi(l) + (\mu(h) - \omega(h))k^{-1}\end{aligned}$$

Planner borrows with Contingent contracts. Since $\widehat{\nu} \geq 1$, we immediately conclude from the foc w.r.t. $d(h)$ that at the planner's allocation, $\omega(h) = 0$, i.e., the planner never issues a non-contingent contract that exposes him to liquidations, $a(l) + \theta A < d(l) = d(h)$. By Assumption 1.4, he will also never find it optimal to borrow non-contingent without allowing any liquidations, $d(l) = d(h) \leq a(l) + \theta A$. Hence, the planner always borrows with contingent contracts.

Planner liquidates only if Contingencies are exhausted. Suppose now that the planner's allocation features liquidations. This implies that $\widehat{\nu}(l) > 1$ and thus $\widehat{\nu} > 1$. But then from the foc w.r.t. $d(h)$, we must have that $\mu(h) > 0$, i.e., the planner liquidates only if the financial constraint in the high state binds, i.e., $d(h) = \theta a(h) + A$.

Planner strictly reduces liquidations in the low state. Suppose that at DE, the financing constraint does not bind in the low state, i.e., $d^{DE}(l) < \theta a(l) + q^{DE}(l)$. Recall from the previous result that the planner liquidates only if $d^{SP}(h) = \theta a(h) + A$. So, suppose that $d^{SP}(h) = \theta a(h) + A$ and that the planner liquidates, i.e., $d^{SP}(l) > a(l) + \theta A$. For contradiction, assume that $d^{SP}(l) \geq d^{DE}(l)$, then because the planner liquidates and borrows more in each state than entrepreneurs, we have that $\widehat{\nu}^{SP} < \widehat{\nu}^{DE}$ and $\widehat{\nu}^{SP}(l) > \widehat{\nu}^{DE}(l)$. Since the financing constraint in the low state does not bind at DE, we have $\widehat{\nu}^{DE} < \widehat{\nu}^{DE}(l)$. Combined with the previous inequalities, this implies $\widehat{\nu}^{SP} < \widehat{\nu}^{SP}(l)$, which is sub-optimal for the planner. Hence, we must have $d^{SP}(l) < d^{DE}(l)$.

Since the financing constraint in the low state will not bind for the planner either, we have that $\widehat{\nu}^{SP} = \widehat{\nu}^{SP}(l) = \nu^{SP}(l)$, i.e, the marginal utility of wealth of the planner has to be equalized between date 0 and date 1-state l and, furthermore, the marginal utilities of the planner and the entrepreneurs at $t = 1$ coincide. I will now show that the planner liquidates strictly less in the low state than entrepreneurs. To this end, I need to make a additional parametric assumption.

Assumption 2 *Let* $\varphi \equiv \frac{(1-\theta)(\pi(h)a(h)+\pi(l)A)}{\pi(l)(\theta a(h)+A-a(l)-\theta A)}$*, then I assume that* $\varphi \geq \frac{\xi(l)}{\xi(h)}\frac{\pi(h)}{\pi(l)}$.

This condition is satisfied if $\pi(l)$ is not too large. Using the fact that the planner sets $d^{SP}(h) = \theta a(h) + A$ and $\widehat{\nu}^{SP} = \nu^{SP}(l)$, we can re-write the planner's foc w.r.t. k as:

$$\chi'\left(k^{SP}\right) = \frac{\pi(l)\,\nu^{SP}(l)\left[a(l) + \theta A\right] + (1-\theta)\left[\pi(h)\,a(h) + \pi(l)\,A\right] + \pi(h)\,\nu^{SP}(l)\left(\theta a(h) + A\right)}{\nu^{SP}(l)}$$

Suppose for contradiction that $\nu^{SP}(l) \geq \nu^{DE}(l)$ and note that we must have $k^{SP} > k^{DE}$. There are two cases to consider depending on whether the contract at the decentralized allocation (DE) is contingent or not:

- *The contract at DE is non-contingent.* We thus must have $\nu^{DE}(l) \leq \frac{\xi(l)}{\xi(h)}\frac{\pi(h)}{\pi(l)}$ and $\nu^{DE} = \pi(l)\,\nu^{DE}(l) + \pi(h)$. Then combining foc's w.r.t. k, $d(l)$, and $d(h)$, we have that

$$\chi'\left(k^{DE}\right) = \frac{\pi(l)\,\nu^{DE}(l)\left(a(l) + \theta A\right) + \pi(h)\left(a(h) + \theta A\right) + (1-\theta)\,A}{\pi(l)\,\nu^{DE}(l) + \pi(h)}$$

  Thus, we have that

$$\chi'\left(k^{SP}\right) - \chi'\left(k^{DE}\right) > 0$$

$$\Longleftrightarrow$$

$$\left[\pi(l)\,\nu^{DE}(l)\left(a(l) + \theta A\right) + \pi(h)\left(a(h) + \theta A\right) + (1-\theta)\,A\right]\nu^{SP}(l) -$$

$$\left[\pi(l)\nu^{SP}(l)\left[a(l)+\theta A\right]+(1-\theta)\left[\pi(h)a(h)+\pi(l)A\right]+\pi(h)\nu^{SP}(l)\left(\theta a(h)+A\right)\right]\left[\pi(l)\nu^{DE}(l)+\pi(h)\right]<0$$

$$\Longleftrightarrow$$

$$\frac{\pi(l)\pi(h)\left(\theta a(h)+(1-\theta)A-a(l)\right)}{(1-\theta)\left(\pi(h)a(h)+\pi(l)A\right)}>\frac{\nu^{SP}(l)-\nu^{DE}(l)+\pi(h)\left(\nu^{DE}(l)-1\right)}{\nu^{SP}(l)\left(\nu^{DE}(l)-1\right)}$$

Since the RHS is increasing in $\nu^{SP}(l)$, $\nu^{SP}(l)\geq\nu^{DE}(l)$ implies that $\nu^{DE}(l)>\varphi\geq\frac{\xi(l)}{\xi(h)}\frac{\pi(h)}{\pi(l)}$, which is a contradiction.

- *The contract at DE is contingent.* If $d^{DE}(h)=\theta a(h)+A$, then the result follows since it has been shown above that $d^{SP}(l)<d^{DE}(l)$. Therefore, suppose that $d^{DE}(h)<\theta a(h)+A$. We thus must have $\nu^{DE}(l)=\frac{\xi(l)}{\xi(h)}\frac{\pi(h)}{\pi(l)}$ and $\nu^{DE}=\frac{\pi(l)}{\xi(l)}\nu^{DE}(l)$. Then combining foc's w.r.t. k, $d(l)$, and $d(h)$, we have

$$\chi'\left(k^{DE}\right)=$$

$$\frac{\xi(l)\left\{\pi(l)\nu^{DE}(l)\left(a(l)+\theta A\right)+\pi(h)\left(a(h)+\theta A-d^{DE}(h)\right)+(1-\theta)A\right\}+\xi(h)\pi(l)\nu^{DE}(l)d^{DE}(h)}{\pi(l)\nu^{DE}(l)}$$

Thus, we have that

$$\chi'\left(k^{SP}\right)-\chi'\left(k^{DE}\right)>0$$

$$\Longleftrightarrow$$

$$\xi(l)\nu^{SP}(l)\left\{\pi(l)\nu^{DE}(l)\left(a(l)+\theta A\right)+\pi(h)\left(a(h)+\theta A-d^{DE}(h)\right)+(1-\theta)A\right\}+$$

$$\nu^{SP}(l)\nu^{DE}(l)\xi(h)\pi(l)d^{DE}(h)-$$

$$\pi(l)\nu^{DE}(l)\left\{\pi(l)\nu^{SP}(l)\left[a(l)+\theta A\right]+(1-\theta)\left[\pi(h)a(h)+\pi(l)A\right]+\pi(h)\nu^{SP}(l)\left(\theta a(h)+A\right)\right\}<0$$

$$\Longleftrightarrow$$

$$\frac{\pi(l)\left(\theta a(h)+(1-\theta)A-a(l)\right)}{(1-\theta)\left(\pi(h)a(h)+\pi(l)A\right)}>\frac{\xi(l)\nu^{SP}(l)-\pi(l)\nu^{DE}(l)}{\left(\xi(l)-\pi(l)\right)\nu^{DE}(l)\nu^{SP}(l)}$$

Since the RHS is increasing in $\nu^{SP}(l)$, $\nu^{SP}(l)\geq\nu^{DE}(l)$ implies that $\nu^{DE}(l)>\varphi\geq\frac{\xi(l)}{\xi(h)}\frac{\pi(h)}{\pi(l)}$, which is a contradiction.

Thus, we have established that when the borrowing constraint does not bind in the low state and $\varphi\geq\frac{\xi(l)}{\xi(h)}\frac{\pi(h)}{\pi(l)}$, then liquidations are smaller at SP then at DE.

Implementation. Let ι^{con} be the marginal subsidy that the planner gives to the entrepreneur for each unit of contingent borrowing and set $\iota^{con} = \zeta^{+}$: subsidy received by entrepreneurs is given by $\zeta^{+} \cdot \max\{0, d(h) - d(l)\}k$. Let Ω denote the lump sum transfer from entrepreneurs to investors at $t = 0$ and note that the transfer now needs to reflect not only the transfer in the planner's problem but also the financing of the subsidy:

$$\Omega = \widehat{T} - \zeta^{+} \cdot (d^{DE}(h) - d^{DE}(l))k$$

where the transfer $\widehat{T}$ is given as before. It can be readily checked that the solution to the entrepreneur's problem with the subsidy ι^{con} and transfer Ω is equivalent to the solution of the planner's problem.

7.3 Appendix C

Proof of Lemma 4. The derivation of buyers' optimal offer strategies is analogous to the proof of Lemma 2, except that buyers' beliefs are augmented with the belief that the seller is in the set $H(\widehat{x}) = \{\beta^S = 0\} \cup (\{\beta^S = 1\} \cap \{x^S < \widehat{x}\})$, i.e. buyers' prior is updated to

$$\widehat{\pi}(l) \equiv \Pr(s = l | H(\widehat{x})) = \frac{(\lambda + (1-\lambda) F_l(\widehat{x}))\pi_l}{(\lambda + (1-\lambda) F_l(\widehat{x}))\pi_l + (\lambda + (1-\lambda) F_h(\widehat{x}))(1-\pi_l)}$$

It suffices to show that there is a threshold such that non-liquidity hit investors with signals below that threshold post their contracts for sale and those with signals greater or equal to that threshold do not. Let $\widehat{x}$ be the selling threshold, and let y^{max} be the maximal order statistic among signals $x_1, ..., x_n$ drawn independently from distribution $F_s(\cdot)$. Then for an investor with signal x, the buyers' optimal offer strategy (given in Lemma 2) implies a payoff from selling of

$$\begin{aligned}
\mathrm{E}\{p^{max}(\mathcal{C}, \widehat{x}) | x\} &= \int_{\underline{x}}^{\overline{x}} p(y, \widehat{x}) f_{y^{max}}(y|x) \, dy \\
&= \int_{\underline{x}}^{\overline{x}} p(y, \widehat{x}) \frac{\sum \pi(s) f_s^n(y) f_s(x)}{f(x)} dy \\
&= \int_{\underline{x}}^{\overline{x}} p(y, \widehat{x}) \sum f_s^n(y) \frac{\pi(s) f_s(x)}{f(x)} dy \\
&= \int_{\underline{x}}^{\overline{x}} p(y, \widehat{x}) \sum_s f_s^n(y) \Pr(s|x) \, dy
\end{aligned}$$

First, I show that $U(x,\widehat{x}) = \mathrm{E}\left\{b(s)+B(s)|x\right\} - \mathrm{E}\left\{p^{max}\left(y,\widehat{x}\right)|x\right\}$ is increasing in x, i.e., a more optimistic buyer has a higher payoff to keeping his asset rather than selling it. Let $\widehat{\pi}_l\left(y,\widehat{x}\right)$ denote the implied state l probability corresponding to the offer $p\left(y,\widehat{x}\right)$, then we need to show that

$$\int_{\underline{x}}^{\overline{x}} \widehat{\pi}_l\left(y,\widehat{x}\right) \sum_s f_s^n\left(y\right) \Pr\left(s|x\right) dy - \Pr\left(l|x\right)$$

is decreasing in $\Pr\left(l|x\right)$. Differentiation w.r.t. $\Pr\left(l|x\right)$ yields

$$\begin{aligned}
\int_{\underline{x}}^{\overline{x}} \widehat{\pi}_l\left(y,\widehat{x}\right)\left(f_l^n\left(y\right) - f_h^n\left(y\right)\right) dy - 1 &= \int_{\underline{x}}^{\overline{x}} \widehat{\pi}_l\left(y,\widehat{x}\right) f_l^n(y) \left(1 - \frac{f_h^n(y)}{f_l^n(y)}\right) dy - 1 \\
&< \int_{\underline{x}}^{\overline{x}} \widehat{\pi}_l\left(y,\widehat{x}\right) f_l^n(y) dy - 1 \\
&\leq 0
\end{aligned}$$

Second, for the existence of the threshold $\widehat{x}$, I consider the following two cases:

- Case 1: If $U(\underline{x},\underline{x}) \geq 0$, then we have that $\widehat{x} = \underline{x}$ is an equilibrium because even the most pessimistic investor with $\beta = 1$ does not want to sell when the threshold is $\underline{x}$. Since $U(\cdot,\underline{x}) \geq 0$ is increasing, it follows that more optimistic investors will also not want to sell. Note that this equilibrium coincides with the equilibrium in the economy with observable liquidity needs.

- Case 2: If $U(\underline{x},\underline{x}) < 0$, then threshold $\underline{x}$ cannot be an equilibrium because otherwise, by continuity of $U(\cdot,\underline{x})$, there would be a set of signals above $\underline{x}$ for which investors with $\beta = 1$ would want to sell. Consider the equation $U(x^*,\widehat{x}) = 0$, which defines a map $x^* : [\underline{x},\overline{x}] \to [\underline{x},\overline{x}]$ that, for a given threshold $\widehat{x}$, gives the signal $x^*(\widehat{x})$ of the non-liquidity hit investor who is indifferent between selling his contract and keeping it. Note that $x^*(\underline{x}) > \underline{x}$ by assumption that $U(\underline{x},\underline{x}) < 0$ and $x^*(\overline{x}) < \overline{x}$ since the expected resale price is always strictly lower than the most optimistic valuation ($x^*(\widehat{x})$ is a singleton because $U(\cdot,\widehat{x})$ is increasing). In addition, since $U(x,\widehat{x})$ is continuous in both x and $\widehat{x}$, we have that the map $x^*(\cdot)$ is continuous on $[\underline{x},\overline{x}]$. There thus exists an $\widehat{x}$ such that $x^*(\widehat{x}) = \widehat{x}$.

Thus, we have established that there exists a threshold selling strategy with a threshold given by $\widehat{x} \in [\underline{x},\overline{x})$.

Proof of Proposition 5. Let $\widehat{x}$ be an equilibrium threshold and let $H(\widehat{x})$ denote the event that an investor sells contract $\mathcal{C}$, and note that

$$\Pr(H(\widehat{x})) = \lambda + (1-\lambda)F(\widehat{x})$$

The ex-ante price of contract $\mathcal{C}$ is given by

$$\begin{aligned}
\mathcal{L}(\{b(s)+B(s)\}) &= \mathbb{E}\{\gamma(x,\beta,\mathcal{C})\mathbb{E}\{p^{\max}(\mathcal{C},\widehat{x})|x\} + (1-\gamma(x,\beta,\mathcal{C}))\beta\mathbb{E}\{b(s)+B(s)|x\}\} \\
&= \Pr(H(\widehat{x}))\mathbb{E}\{p^{\max}(\mathcal{C},\widehat{x})|H(\widehat{x})\} + (1-\Pr(H(\widehat{x})))\mathbb{E}\{b(s)+B(s)|H(\widehat{x})^C\} \\
&= \mathbb{E}\{b(s)+B(s)\} + \Pr(H(\widehat{x}))\mathbb{E}\{p^{\max}(\mathcal{C},\widehat{x}) - b(s) - B(s)|H(\widehat{x})\}
\end{aligned}$$

Finally, that $\Pr(\widehat{x})\mathbb{E}\{b(s)+B(s)-p^{\max}(\mathcal{C},\widehat{x})|\widehat{x}\} = \widehat{\zeta}_C \cdot |b(h)+B(h)-b(l)-B(l)|$ follows from the linearity of offer strategies and the fact that buyers earn informational rents. The proof that informational rents are positive is analogous to that of Proposition 1.

Random Matching and Beliefs. In this section, I show that the results of the paper do not depend of the number of sellers that each buyer matches with. Let $\mu^B(s)$ and $\mu^S(s)$ denote the measures of buyers and sellers in state $s \in \{l,h\}$ respectively. As before, each seller is matched with n buyers who are randomly selected from the set of potential buyers. I suppose that the number of sellers that a buyer is matched with is stochastic:

- With probability $\frac{n\cdot\mu^S(s)}{m\cdot\mu^B(s)}$, a buyer is matched with m randomly selected posted contracts, and
- With probability $1-\frac{n\cdot\mu^S(s)}{m\cdot\mu^B(s)}$, a buyer remains unmatched.

Because matching is random, we have that $\frac{n\cdot\mu^S(s)}{m\cdot\mu^B(s)}$ is also the fraction of buyers that are matched to trade in secondary markets in state s. Recall that an investor is a buyer if and only if he has not experienced a liquidity need; hence, $\mu^B(s) = 1-\lambda$ for $s = l,h$ and assuming that $m \geq \frac{n}{1-\lambda}$ is sufficient for the above probabilities to be in $(0,1)$.

Consider a market for contract $\mathcal{C}_i$, and note that it is without loss of generality to consider contracts that are non-negatively correlated with the state. There are n buyers in this market and recall that these buyers have received dispersed signals. If the contract were non-contingent then, as before, because information asymmetry is irrelevant for pricing the contract, it would trade at its expected value. Thus, suppose that contract $\mathcal{C}_i$ is contingent.

It may seem that the degree of contingency of contracts $\mathcal{C}_{-i}$ held by other investors can matter for pricing as it provides some additional information to buyers; however, as I show next, this is not the case. Suppose that buyers believe that an investor posts a contingent contract for sale if and only if either he has a liquidity need or when he has received a signal below $\widehat{x}$.[37] If all contracts are contingent, then upon matching with the seller of contract $\mathcal{C}_i$ a buyer updates his beliefs about the state to

$$\widehat{\pi}\left(l\right)=\frac{\left[\lambda+\left(1-\lambda\right)F_{l}\left(\widehat{x}\right)\right]\pi\left(l\right)}{\left[\lambda+\left(1-\lambda\right)F_{l}\left(\widehat{x}\right)\right]\pi\left(l\right)+\left[\lambda+\left(1-\lambda\right)F_{h}\left(\widehat{x}\right)\right]\left(1-\pi\left(l\right)\right)}$$

The buyer needs to take into account that matching is (potentially) more likely in the low state of the world than in the high state. Note that this expression is identical to the buyers' beliefs in the case where buyers match with at most one seller. Now, if all but one contract are non-contingent, we need to take care of buyers beliefs when a buyer is matched with a contingent contract since the set of contingent contracts is now of Lebesgue measure 0. To this end, I define the beliefs as follows. Suppose hypothetically that fraction ϵ of issued contracts is contingent and the rest are non-contingent. Then if a buyer is matched with 1 contingent contract and $m-1$ non-contingent contracts, he updates his beliefs about the state to

$$\begin{aligned}\widehat{\pi}\left(l\right)|_{\epsilon} &= \frac{\left[\left(\lambda+\left(1-\lambda\right)F_{l}\left(\widehat{x}\right)\right)\cdot\epsilon\right]\cdot\left[\lambda\cdot\left(1-\epsilon\right)\right]^{m-1}\pi\left(l\right)}{\left[\left(\lambda+\left(1-\lambda\right)F_{l}\left(\widehat{x}\right)\right)\cdot\epsilon\right]\cdot\left[\lambda\cdot\left(1-\epsilon\right)\right]^{m-1}\pi\left(l\right)+\left[\left(\lambda+\left(1-\lambda\right)F_{h}\left(\widehat{x}\right)\right)\cdot\epsilon\right]\cdot\left[\lambda\cdot\left(1-\epsilon\right)\right]^{m-1}\left(1-\pi\left(l\right)\right)} \\ &= \widehat{\pi}(l)\end{aligned}$$

Define the buyers' belief upon being matched with the only contingent contract by

$$\widehat{\pi}(l)|_0 \equiv \lim_{\epsilon\to 0}\widehat{\pi}(l)|_{\epsilon} = \widehat{\pi}(l)$$

We thus have that the buyers' belief upon matching with multiple sellers in equilibrium coincides with the beliefs when buyers were matched with at most one seller.

[37] $\widehat{x}=0$ if liquidity needs are observable and $\widehat{x}\geq 0$ otherwise.