Informed Intermediation over the Cycle

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Abstract

We construct a dynamic model of financial intermediation in which changes in the information held by financial intermediaries generate asymmetric credit cycles as the ones documented by Reinhart and Reinhart (2010). We model financial intermediaries as "expert" agents who have a unique ability to acquire information about firm fundamentals. While the level of "expertize" in the economy grows in tandem with information that the "experts" possess, the gains from intermediation are hindered by informational asymmetries. We find the optimal financial contracts and show that the economy inherits not only the dynamic nature of information flow, but also the interaction of information with the contractual setting. We introduce a cyclical component to information by supposing that the fundamentals about which experts acquire information are stochastic. While persistence of fundamentals is essential for information to be valuable, their randomness acts as an opposing force and diminishes the value of expert learning. Our setting then features economic fluctuations due to waves of "confidence" in the intermediaries' ability to allocate funds profitably.

Keywords: financial intermediation, asymmetric information, portfolio choice, learning by lending

1 Introduction

Credit cycles are a pervasive feature of all modern economies. As documented by Reinhart and Reinhart (2010), financial crises are preceded by long periods of credit expansion and rising leverage, and followed by slow recoveries initiated by flight to quality episodes and strong cuts on new lending that can take up to a decade to recover. Another well documented feature of these crises is the asymmetric behavior of credit and investment: strong booms are followed by sharp declines and gradual rebounds. Since the financial sector plays a prominent role in the intermediation of funds between savers and borrowers, understanding the behaviour of financial intermediaries should shed some light on the forces that underpin these credit cycles. Financial intermediaries such as banks, hedge funds, and pension funds manage the major bulk of household financial holdings and are the dominant source of external finance for non-financial businesses in all economies. As Gorton and Winton (2002) state, "the savings-investment process, the workings of capital markets, corporate finance decisions, and consumer portfolio choices cannot be understood without studying financial intermediation." We adopt this view here and present a simple framework that puts a central emphasis on financial intermediaries.

We construct a dynamic model of financial intermediation in which changes in the information held by financial intermediaries allow us to rationalize key features of the documented credit cycles, but also of the financial contracts observed in practice. We suppose that some agents in the economy, whom we call "experts", have a unique ability to acquire information about firm and sector fundamentals. Better information allows for better allocation of resources, and this informational advantage makes these experts the natural contenders to intermediate funds between households and businesses. The level of "expertize" in the economy and the potential gains from intermediation grow in tandem with the information that these experts possess; these gains, however, are hindered since experts' information is inherently private. Financial contracts must be strained to balance allocational efficiency with the provision of appropriate incentives for these experts. The economy therefore inherits not only the dynamic nature of information flow, but also the interaction of information with the contractual setting. We introduce a cyclical component to information by supposing that the fundamentals about which experts acquire information are stochastic. While persistence of fundamentals is essential for information to be valuable, their randomness acts as an opposing force and diminishes the value of expert learning.

The combination of a model of financial intermediation and a dynamic model of private information not only allows us to study credit cycles from a new perspective, but it also provides new testable predictions about the connection between confidence in the financial sector, intermediation fees, and financial players' portfolios. We provide a novel mechanism that connects the severity of credit contractions with structural changes in an economy, understood as changes in the underlying productivities of different economic sectors.

In our model, the financial system amplifies and propagates real shocks to fundamentals by contracting credit to productive sectors, and by slowing down the access of these sectors to credit in the years to follow. The intuition of the mechanism is as follows. During stable times, financial intermediaries raise funds, lend, and acquire information. Over time their expertise increases, their perceived uncertainty about the investment set is reduced, and this is reflected in higher credit to risky sectors. On top of this, households' confidence in experts increases, and so do intermediation fees in response. Given the nature of our learning process, unexpected changes in underlying fundamentals act as a volatility shock for financial intermediaries, since their accumulated expertize becomes obsolete. This generates a loss of confidence in financial intermediaries, a reduction in their fees, and a contraction of credit to risky sectors. As time passes, intermediaries accumulate information again, and credit slowly recovers, together with the confidence in the financial sector and its fees.

We find that economic fluctuations can be rationalized by waves of "confidence" in the experts' ability to allocate funds profitably. The asymmetry of credit cycles arises from the asymmetric nature of information acquisition: even though it takes time to acquire information through a learning-by-lending process, expertize can be lost the moment a shock to fundamentals is perceived. Credit to risky sectors responds one to one to these changes in expertize, and so do intermediation fees. These results arise in a framework in which optimal intermediation contracts match those observed in reality: a portfolio manager receives an intermediation fee that is proportional to assets under management, and a percentage of the total portfolio. In the literature on intermediation fees the fraction of the portfolio is referred to as an incentive fee, since in most of the literature moral hazard is the prevalent friction. In our paper, as there is no moral hazard, sharing portfolio returns is the result of risk-sharing between households and experts rather than incentives. We believe both arguments are reasonable, reality is probably in between. Given the optimal contracts that we find, we analyze how allocations evolve as information, and thus financial expertize, is accumulated.

We consider an overlapping generations model with heterogeneous agents. Each generation is exogenously divided between households and experts. There is a storage technology and risky projects available to all agents. We assume agents do not know the true underlying distribution of project returns, but they are born with a prior about it. After investing in a particular project, however, an agent receives a signal about the mean of the distribution that can be used to compute a more precise posterior of project returns. We assume experts have the ability to process these signals in a more sophisticated way, and are thus able to get more precise information than households. This can be rationalized by thinking that experts have access to "soft" information that only they are able to interpret.[1] In this context, we think of financial intermediaries as experts that intermediate

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[1] Soft information is essentially qualitative in nature and consists of ideas, suggestions, opinions, rumours, gossip, feedback, anecdotes, speculation, and tips. Is information it cannot be transmitted easily accross individuals, as oposed to hard information that can be summarized with indexes, balance sheets, etc.

funds between households and projects, i.e. they manage households' portfolios. We model the dynamics of information by assuming that information can be freely transmitted to those that can interpret it, i.e. from old to young.

In this context, experts have private information about investment opportunities, and this is an obstacle when they need to raise funds from uninformed households. We solve a signaling model in which experts with superior, private, information are able to signal uninformed households their private information through their own investment choices. The idea that investment choices can signal private information was first introduced by Leland and Pyle (1977). As in their setting, the presence of asymmetric information introduces an inefficiency. Experts overinvest in risky assets in an attempt to make households overly optimistic and extract higher intermediation fees. In equilibrium households understand this and information is perfectly transmitted, but overinvestment in risky assets cannot be avoided.

Literature Review

There is a large strand of literature that tries to rationalize the role of financial intermediaries. Diamond and Dybvig (1983) argue that intermediaries allow households to smooth their uncertain consumption needs, while Holmstrom and Tirole (1997) show that intermediaries can help firms to pool their liquidity more efficiently. Another view posits that the role of intermediaries is to reduce monitoring costs in the presence of agency problems (Diamond (1984), Williamson (1984b, 1987)). In this paper, however, we model financial intermediaries as information producers. The main proponents of this theory are Leland and Pyle (1977), Campbell and Kracaw (1980), and Boyd and Prescott (1986). These papers focus on the ability of an intermediary to solve the classic "reliability" and "apropriability" problems that arise when there are costs to acquiring information. In these models, financial intermediaries are coallision of agents that acquire information on behalf of others and their existance is endogenous. In our paper, however, the presence of agents with the ability to learn more than households is exogenously given. We view this as a simplyfying assumption that can be microfunded by the previously mentioned papers.

In particular, a recent paper about trust in financial advisers by Gennaioli, Vishny, and Shleifer (2012), connects very closely to our work. They construct a model of money management in which investors delegate their portfolio decision to managers based on "trust," and not only on performance. Their main idea is that a good manager is able to reduce an investor's uncertainty exogenously, by reducing their anxiety about taking risks. Mullainathan at al. (2008) conduct an audit of financial advisers and find evidence to support the fact that investors choose their financial advisers based on factors other than past performance. In particular, they argue that many financial advisers do not advertise based on past performance, but rather on experience

and dependability. This is a key feature of our paper, where households choose to delegate their portfolio decisions to those agents with better quality information that they have acquired with experience. In contrast to Gennaioli, Vishny, and Shleifer (2012), the trust households put on our intermediaries is rational, since households understand that experts have more precise information. In both papers, the decision to delegate does not depend on past performance, but on "confidence" in the knowledge of the portfolio manager.

A number of studies have found evidence supporting the theory that intermediaries do possess superior information about borrowers with whom they have established a relationship. The explanation often given is that in the process of establishing a relation with a firm, the lender obtains "soft" information about the firm. Slovin, Sushka, and Polonchek (1993) examine the stock price of bank borrowers after the announcement of the failure of their main bank, Continental Illinois. They find that Continental borrowers incurred negative abnormal returns of 4.2% on average after the failure announcement. If bank loans where indistinguishable from corporate bonds, borrowers could borrow directly from the market when their bank disappeared; this, however, was not the case, leading the authors to the conclusion that the intermediary had some information about the borrowers that the market did not. Gibson (1995) reaches a similar conclusion by studying the effect of Japanese banks' health on borrowing firms. Petersen and Rajan (1994) and Berger and Udell (1995) both show in independent studies that a longer bank relationship (controlling for firms' age), implies better access to credit in the form of lower interest rates or less collateral requirement.

Finally, recent works by Veldkamp (2005), Ordoñez (2010), and Kurlat (2011) emphasize the role of information over the cycle. These papers point to cyclical asymmetries that arise from the naturally asymmetric flow of information over the cycle. In contrast, we study the role of financial intermediaries in generating and amplifying these informational cycles.

The paper is organized as follows. In Section 2, the model setup is described. In section 3, we solve the static problem and characterize optimal intermediation contracts. In section 4, we introduce dynamics and characterize how intermediation activity evolves over time; we also incorporate aggregate shocks and study how intermediaries propagate and amplify shocks to the real economy. We discuss two interesting extensions in Section 5. Section 6 concludes.

2   The Model

We construct an overlapping generations model (OLG) with two-period lived agents. Each generation is of unit mass and is exogenously divided between an equal number of experts (e) and households (h). There is a single consumption good that can be stored at an exogenous gross risk-free rate $R_f \geq 1$.

Preferences and Endowments. Agents are born with an endowment w^j $(j = e, h)$ units of the consumption good. They consume only when old and have preferences $u(c) = -e^{-\gamma^j c}$ with $\gamma^j > 0$ for $j = e, h$. The objective of agent j born at date t is to maximize expected utility $U_t^j = E_t^j \left\{ u \left(c_{t+1}^j \right) \right\}$.

Technology. There are N risky projects and investment in these projects is costly: an agent who makes investments in risky projects experiences a non-pecuniary cost $\chi > 0$. The payoff structure of these projects is summarized by the vector of project returns $R_{t+1} \equiv [R_{1,t+1}, ..., R_{N,t+1}]$ which follows the stochastic process given by

$$R_{t+1} = \theta_t + \varepsilon_{t+1}$$

$$\theta_t = (1 - X_t)\theta_{t-1} + X_t \tilde{\theta}_t \quad \forall t > 0$$

$$\theta_0 = \tilde{\theta}_0$$

where we suppose that $\varepsilon_{t+1} \sim^{iid} N(0, \Sigma_\varepsilon)$ and $\tilde{\theta}_t \sim^{iid} N(\bar{\theta}, \Sigma_\theta)$, and where $X_t \sim Bernoulli(p)$ for all $t \geq 0$. The project returns at date $t+1$ are thus decomposed into a transitory component given by ε_{t+1} and a persistent component given by θ_t. The parameter p captures the persistence of returns and thus the degree to which historic data is useful to understanding future investment returns.

Information and Expertize. Experts and households understand the model of the economy but do not know the realization of θ_t. They are born with prior beliefs $\theta_t \sim N(\bar{\theta}, \Sigma_\theta^e)$ and $\theta_t \sim N(\bar{\theta}, \Sigma_\theta^h)$ respectively, where $\Sigma_\theta^e = \Sigma_\theta$ and $\Sigma_\theta^h = \Sigma_\theta + \Sigma_N$, i.e. we assume experts know the underlying distribution of θ_t, while households have a more dispersed prior.

Learning. Agents are born with a prior $\theta \sim N(\bar{\theta}, \Sigma_\theta)$ and after receiving a signal $s \sim N(\theta, \Sigma_s)$, they update their beliefs using Bayes' rule to $\theta \sim N(\hat{\theta}, \hat{\Sigma}_\theta)$ where

$$\hat{\theta} = E[\theta|s] = \left[\Sigma_\theta^{-1} + \Sigma_s^{-1} \right]^{-1} \left[\Sigma_\theta^{-1} \bar{\theta} + \Sigma_s^{-1} s \right]$$

$$\hat{\Sigma}_\theta = V\left[\theta | \Sigma_s^{-1} \right] = \left[\Sigma_\theta^{-1} + \Sigma_s^{-1} \right]^{-1}$$

The arrival of signal s reduces the perceived volatility of mean project returns and, in the absence of intermediation, this reduction in volatility is larger for experts than for households. This asymmetry alone is sufficient for the households to want to delegate their investment decisions to the experts, since we will assume that the experts' informational advantage is common knowledge.

Intermediation. Experts have a comparative advantage over households in investment activity since they face a more precise distribution of project returns than households. Therefore, households may want to delegate their

portfolio decisions to the experts. We define intermediation as the investment activity that the experts conducts on behalf of the households; that is, experts intermediate funds between the households and the investment projects. In our setting, there are three potential sources of gains from such intermediation. Firstly, there is a fixed non-pecuniary cost of investing in risky assets, and it can be split among agents if investment occurs jointly. Second, the risks from investment activity can be spread more widely across agents. And finally, and more importantly, the funds in the economy can be allocated more efficiently due to the presence of expert information. This last channel is the focus of our paper; the two other motives severely simplify the problem by fixing the outside option of households. To be consistent with Leland and Pyle (1977), we make the following two assumptions to motivate the contractual setting:

Assumption 1. [Complex Information]

    i) experts' posterior beliefs are not observable by households, and

    ii) households know the distribution of expert's private signals.

Assumption 2. [Contractable Information]

    i) portfolios chosen by experts are ex-post verifiable by the participating households, and

    ii) contractual terms between households and experts are not publicly observable.

By ex-post verifiable, we mean that portfolio weights can only be verified after contracts have been accepted, i.e. portfolio weights can only be observed when the investment of funds is actually made in a given portfolio. This ex-post assumption is not only realistic, but desirable, since it allows the experts to exploit their informational advantage when offering the contract.

Assumption 3. [Costly Investment] *The non-pecuniary cost of investment $\chi > 0$ satisfies:*

$$\frac{1}{2} \left[\mu_0^h - R_f 1_N \right]' \Sigma_0^{h-1} \left[\mu_0^h - R_f 1_N \right] < \chi < \frac{1}{2} \left[\mu_0^e - R_f 1_N \right]' \Sigma_0^{e-1} \left[\mu_0^e - R_f 1_N \right]$$

We now discuss the implications of the above assumptions for the contractual setting. First, note that Assumption 2 implies that the only information that cannot be communicated between experts and households is the mean of the experts' posterior distribution $\hat{\theta}_t$. The reason for this is that the experts' posterior can be fully characterized by its mean and variance, and that the variance of the experts' posterior is common knowledge by Assumption 2 (i). Second, since project returns are public information, Assumption 3 implies that portfolio returns are verifiable. Therefore, Assumption 2 and 3 imply that experts and households can contract upon the precision of the expert information, the expert portfolio choice, and the realized portfolio returns. Finally, assumption 4 ensures that households are not be willing to invest in risky assets on their own, but would do so through an expert.

3 Optimal Intermediation Contracts

At each date t, an expert and a household get randomly matched. After the match is realized, the expert offers the household a take it or leave it *intermediation contract* that the household can accept or reject.[2] We define an *intermediation contract* as a contract in which an expert asks the household to deposit its funds in return for payoffs contingent on verifiable outcomes. If the household rejects the contract, then both the expert and the household invest on their own (these are their outside options). If the household accepts the contract, contractual terms are executed.

As Leland and Pyle (1977) have shown, total funds that are invested in the risky asset by the expert are a signal about the expert's private information. We solve a signaling problem, in which experts offer contracts taking into account that their portfolio weights signal to households their private information. Given Assumption 3, contracts can (and will) be contingent on portfolio weights. The timing of the per period problem is as follows. First, the expert offers the household to pull their funds together in exchange for a payoff contingent on the constructed portfolio and on the realized return of the chosen portfolio. To ensure that the household participates, the expert chooses the payoff functions so that the household gets its outside option [3] Second, once the funds have been raised, the expert chooses her preferred portfolio and commits to the per-specified return-contingent payoff function for that particular portfolio choice.

The presence of overlapping generations of experts and households that are randomly matched to enter an intermediation contract allows us to isolate the per period problem, given the state variables: i) the state of the economy $\left\{X_t, \theta_t, \tilde{\theta}_t\right\}$, and ii) the private information of the experts $\{R^t\}$ summarized in their posterior $\{\mu_t, \Sigma_t\}$. First, we focus on the problem of an expert that enters period t with a posterior distribution $\theta \sim N(\mu_t, \Sigma_t)$, and we solve for the optimal intermediation contract, and optimal consumption and investment allocations. Second, we introduce dynamics to characterize the evolution of key variables over time.

For the analyzes of the per period problem, we drop the t subscripts when characterizing the stage problem. Let $\mu = \hat{\theta}_t$ and $\Sigma = \hat{\Sigma}_t$ denote the mean and precision of the expert's information, c^e, c^h denote the consumption allocations of experts and households determined by the contract, and α denote the expert's chosen portfolio. The solve the expert's problem, we formulate the following conjecture.

Conjecture 1. *Portfolio weights chosen by the expert (fully) reveal his private information.*

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[2]Our qualitative results do not depend on the distribution of bargaining power.

[3]WLOG, this can modeled as the expert offering a menu to the households, given by $\left\{c^e(R, \alpha(\mu)), c^e(R, \alpha(\mu)), \alpha(\mu)\right\}_{\forall \mu}$ where $c^e(R, \alpha(\mu)), c^h(R, \alpha(\mu))$ are the consumption allocations of the expert and the household, contingent on realized returns and on portfolio weights, and μ is the expert's private information (mean of its posterior distribution). In equilibrium, after the contract is accepted, the expert has to choose from that menu the triple consistent with its real $\mu = \hat{\theta}_t$. We show in the Appendix that these two problems are equivalent, and that the mechanism is optimal.

Let $\tilde{\mu}(\alpha)$ denote the signal about underlying beliefs embedded in the portfolio choice, an expert with posterior beliefs characterized by μ solves the following problem:

$$\max_{c^e, c^h, \alpha} E\left[u^e(c^e)|\mu\right] \tag{1}$$

$$E\left[u^h(c^h)|\tilde{\mu}(\alpha)\right] \geq \bar{U}^h \quad (\lambda_{pc})$$

$$c^e(R) + c^h(R) \leq \left[\alpha'(R - R_f 1_N) + R_f\right]w \quad (\lambda_{fc1}(R))$$

where w denotes the total funds of an intermediary, and it is given by the sum of experts and households initial endowments, $w = w^h + w^e$; and $E[x|\mu]$ denotes the expected value of x conditional on beliefs characterized by μ (experts), and $\tilde{\mu}(\alpha)$ (households). The experts problem is to maximize its expected utility subject to the participation constraint of the household (multiplier λ_{pc}, and the feasibility constraint (multiplier λ_{fc}).)[4] Since we focus on separating equilibria, we impose the following "truth revelation" condition: $\tilde{\mu}(\alpha(\mu)) = \mu$.

Proposition 1. *Under the optimal intermediation contract the expert receives a fixed payment and a fraction of the returns of the portfolio where all funds are invested. Consumption allocations under the optimal contract are given by*

$$c^e(R, \alpha) = \frac{\gamma^h}{\gamma^h + \gamma^e} R_p w + Z(\alpha)$$

$$c^h(R, \alpha) = \frac{\gamma^h}{\gamma^h + \gamma^e} R_p w - Z(\alpha)$$

where $R_p \equiv R_f + [R - R_f 1_N]' \alpha$ are the total portfolio returns and $Z(\alpha)$ is a transfer contingent on portfolio weights.

The optimal contract presented in Proposition 1 has a straight-forward interpretation. The first term of the contract is a variable payoff and it is given by the the corresponding fraction of the total portfolio returns that each agent is receives ($\frac{\gamma^e}{\gamma^h + \gamma^e} R_p$ for households and $\frac{\gamma^h}{\gamma^h + \gamma^e} R_p$ for experts). This fraction is chosen to smooth marginal utilities across states between households and experts, to attain full-risk sharing. To see this, note that for a given portfolio α, consumption allocations presented in Proposition **??** guarantee that:

$$u^{e'}(c^e(R, \alpha)) = \lambda u^{h'}(c^h(R, \alpha)) \quad \forall R$$

for $u^h(c) = -\exp\left[-\gamma^h c\right]$ and $u^e(c) = -\exp\left[-\gamma^e c\right]$.

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[4] Households outside option $\bar{U}^h$ is given by the utility households derive from investing their endowments in the risk-free firms, i.e. $\bar{U}^h \equiv u^h\left(w^h R_f\right)$.

The last term of the contract is a transfer made from the household to the expert. For exposition purposes, we decompose the transfer into a fixed payment plus a payment contingent on portfolio weights: $Z(\alpha) = \bar{f} + f(\alpha)$. The first term of this decomposition, $\bar{f}$, is a constant transfer that ensures that the participation constraint of the household binds. The second term, $f(\alpha)$, is what we refer to as an intermediation fee, since it reflects the value of acquiring the expert's intermediation services. One interesting result is that this fee is contingent on the portfolio weights chosen by the expert. This is because, from Conjecture 1, portfolio weights affect households' beliefs (and thus the value of intermediation) by signaling the expert's private information.

Proposition 2. *For a given portfolio α chosen by the expert, the optimal transfer that the expert receives from the household is given by $Z(\alpha) = \bar{f} + f(\alpha)$ where*

$$\bar{f} = R_f \left[\frac{\gamma^e}{\gamma^h + \gamma^e} w^e - \frac{\gamma^h}{\gamma^h + \gamma^e} w^h \right]$$

$$f(\alpha) = \frac{1}{\gamma^h} \left[\bar{\gamma} \left[\mu(\alpha) - R_f 1_N \right]' \alpha w - \frac{\bar{\gamma}^2}{2} (\alpha w)' \Sigma (\alpha w) \right]$$

Proof. Using Conjecture 1, we take the optimal consumption allocations and make the participation constraint of the households bind. $\square$

The previous results shows that by choosing portfolio weights, α, experts do not only affect the expected returns that arise from the portfolio, but also the fee charged to the households by manipulating their beliefs. This result relies on the fact that the funds invested in risky assets signal the expert's private information about the return of these assets (see also Leland and Pyle (1977)). Using the consumption allocations presented in Proposition 2, we solve for the expert's optimal portfolio choice. We proceed as follows. First, we conjecture the functional relationship between α and μ. Second, given our conjecture, we find the expert's optimal portfolio choice α when both the portfolio and the manipulation of beliefs effects are considered. Finally, we verify our conjecture.

Conjecture 2. *Given expert's private information μ, portfolio weights are given by $\alpha(\mu) = \kappa \left[w \bar{\gamma} \Sigma \right]^{-1} \left[\mu - R_f 1_N \right]$ with $\kappa > 1$.*

In Conjecture 2 we claim that when portfolio weights signal the expert's private information, there is a multiplicative distortion from optimal portfolios. In the absence of private information, optimal portfolios are given by the standard Sharpe ratio $\left[w \bar{\gamma} \Sigma \right]^{-1} \left[\mu - R_f 1_N \right]$. When portfolios signal private information, by investing more in the risky assets the experts can make households more optimistic about portfolio returns and thus increase the fee charged for intermediation. The experts distort their portfolio choice towards riskier positions to increase intermediation fees.

Proposition 3. *The total funds invested in risky projects are given by*

$$\alpha w = \kappa \left(\bar{\gamma} \Sigma \right)^{-1} \left[\mu - R_f 1_N \right]$$

where $\kappa = \frac{\gamma^h + 2\gamma^e}{\gamma^h + \gamma^e}$ *and* $\bar{\gamma} = \frac{\gamma^h \gamma^e}{\gamma^h + \gamma^e}$.

Proof. Using the FOC with respect to α of the expert's problem (1), plugging in the conjecture, and using the method of undetermined coefficients yields result. (See Appendix for details). $\square$

Corollary 1. *The expert with posterior beliefs* $\{\mu^e, \Sigma^e\}$ *charges the following portfolio-contingent intermediation fee:*

$$f = \frac{1}{2} \frac{\gamma^h + 2\gamma^e}{\left(\gamma^h + \gamma^e \right)^2} \left[\mu^e - R_f 1_N \right]' \left(\Sigma^e \right)^{-1} \left[\mu^e - R_f 1_N \right]$$

When the investment in risky assets signals private information about the underlying quality of these assets, experts overinvest. The distortion in portfolios is generated by the expert's incentives to inflate the household's beliefs about portfolios returns, and thus raise a higher fee from intermediation. This overinvestment occurs despite the fact that in equilibrium the expert's strategy is inferred by households. The distortion to the expert's portfolio choice, $\kappa - 1$, is equal to the percentage of risk that the household is exposed to, $\gamma^e / \left(\gamma^e + \gamma^h \right)$, and is thus increasing (decreasing) in expert's (household's) risk aversion. The more the household is exposed to risk, the larger the expert's gains from convincing the household that risky returns are favorable.

Optimal contracts in our model match qualitatively the contracts offered by many hedge funds and portfolio managers. This is popularly referred to as the 2/20 fee structure, where managers charge a fee of 2% of assets under management, and receive 20% of the returns of the chosen portfolio. As Deuskar et al. (2011) show, however, this type of contracts are a generalization, since when looking at the data on hedge fund fees, there are significant cross-section and time series variations in these amounts. What this means is that in reality, even though the contracts do look like a manager's fee and a fraction of the portfolio, the level of these two components is variable. This is consistent with our model, where fees can vary with the level of expertize of financial intermediaries, and with the set of investment opportunities experts can offer to households. In most of the literature on portfolio managers' fees, the variable component is referred to as the incentive fee. In our model, there is no moral hazard and thus the reasons why experts hold a fraction of the portfolio are: i) that they are actually investing their own funds in these portfolio, and (most importantly), ii) they share risks optimally with households. If moral hazard was introduced into the model, the variable component would not only be a function of the risk aversions, but some distortion might arise to provide incentives to experts. We choose to avoid adding the moral hazard friction to be able to fully focus on the asymmetric information problem that arises when portfolio managers possess superior information about the quality of assets they invest households

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funds in, since we believe this is an interesting problem on its own.

Finally, we would like to end this section with an interesting fact documented in Mullainathan et al. (2008) in their audit to financial advisers. They find that "some advisers refused to offer any specific advice as long as the potential client has not transferred the account to the company of the adviser," and they argue that this happens because no useful information wants to be revealed before the contract is accepted. This supports the view that there is not only a moral hazard problem present in intermediation, but an asymmetric information problem at the moment of contracting as well. They say: "it makes sense that advisers want to protect their time and insights so that clients do not replicate the advice for free." Even though it makes sense, they find this result puzzling, since investors need to make decisions without knowing what the adviser knows. In our model this is exactly the case. We also present a solution to this puzzle: investors (households in our model) are conceptually being offered a contract that is contingent on portfolio weights, and these weights signal the expert's private information ex-post. As we have shown, the problem of apropriability and reliability of private information is solved when fees are made contingent on the investment choices the advisers make after the contract is accepted.

In the following section we introduce dynamics to understand how private information, and thus portfolios and intermediation fees, evolve over time. The dynamic model provides interesting testable predictions for the correlation between the riskiness of portfolios and intermediation fees, and for the evolution of the overall income of financial intermediaries as a function of confidence in their expertize.

4   The Dynamic Economy

The dynamic economy is a straight-forward extension of the static problem presented in the previous section. Due to their short horizon, the contracts between experts and households are still short-term, but the information that the expert possess evolves over time. Before setting up the dynamic problem, we discuss the evolution of learning and how it responds to structural shocks. We have assumed that the mean of project returns follows the process given by:

$$\theta_t = (1 - X_t)\,\theta_{t-1} + X_t \tilde{\theta}_t \quad \forall t > 0$$

where $X_t \sim Binomial\,\{1, p\} \quad \forall t > 0$ and $X_0 = 1$. Thus, the mean θ_t remains unchanged as long as $X_t = 0$, but is redrawn anew from distribution $N\left(\bar{\theta}, \Sigma_\theta\right)$ whenever $X_t = 1$. This process allows to add a cyclical component to information by supposing that the fundamentals about which experts acquire information are stochastic. In particular, while persistence of fundamentals is essential for information to be valuable ($X_t = 0$), their randomness acts as an opposing force and diminishes the value of expert learning (when $X_t = 1$). Since X_t

is public information for all t, when computing posterior distributions for θ_t, agents only incorporate signals received after the change of state, i.e. signals received after date T given by $T = \sup \{\tau < t : X_\tau = 1\}$. The experts' posterior mean and variance are therefore given by

$$\mu_t = \left[\Sigma_\theta^{-1} + \Sigma_{s_t^R}^{-1} \right]^{-1} \left[\Sigma_\theta^{-1} \bar{\theta} + \Sigma_{s_t^R}^{-1} s_t^R \right] \tag{2}$$

$$\hat{\Sigma}_{\theta,t} = \left[\Sigma_\theta^{-1} + \Sigma_{s_t^R}^{-1} \right]^{-1} \tag{3}$$

where the updating is conditional on signals $s_t^R = (t - T)^{-1} \sum_{\tau=T}^t R_\tau$ with precision $\Sigma_{s_t^R}^{-1} = (t - T) \Sigma_\epsilon^{-1}$. The expert's problem at time $t \geq 0$, with public information $\left\{ \hat{\Sigma}_{\theta,t}, X^t \right\}$, is given by

$$V^e \left(\mu_t, \hat{\Sigma}_{\theta,t}, X_t \right) = \max_{c_{t+1}^e, c_{t+1}^h, \alpha_t} E \left[u^e \left(c_{t+1}^e \right) | \mu_t, \hat{\Sigma}_{\theta,t} \right] \tag{4}$$

$$E \left[u^h \left(c_{t+1}^h \right) | \tilde{\mu}_t \left(\alpha_t \right), \hat{\Sigma}_{\theta,t} \right] \geq \bar{U}_t^h \quad (\lambda_{pc})$$

$$c_{t+1}^e \left(R_{t+1} \right) + c_{t+1}^h \left(R_{t+1} \right) \leq \left(\left[R_{t+1} - R_f 1_N \right]' \alpha_t + R_f \right) w \quad (\lambda_{fc1} \left(R_{t+1} \right)) \quad \forall R_{t+1}$$

$$R_{t+1} = \theta_t + \varepsilon_{t+1} \quad \varepsilon_{t+1} \sim^{iid} N \left(0, \Sigma_\varepsilon \right)$$

$$\theta_t = \left(1 - X_t \right) \theta_{t-1} + X_t \tilde{\theta}_t \quad \tilde{\theta}_t \sim N \left(\bar{\theta}, \Sigma_\theta \right)$$

where μ_t and $\hat{\Sigma}_{\theta,t}$ are given by equations (2) and (3) respectively and θ_t is not observed by agents.

It is straightforward to verify that the solution to the dynamic problem matches the solution to the static model, given the prevalent state variables. This is because we have chosen an overlapping generations framework, where agents have short-horizons. The qualitative results of the model would not change if both agents had an infinite horizon, as long as some relevant level of informational asymmetry persists. We chose to avoid long horizons to avoid situations in which after long periods of stability households learn through their own experience with the expert, and thus intermediation is no longer motivated by expertize (information asymmetries become irrelevant when both agents have precise posteriors). We believe that the assumption that experts hold consistently more precise information than households is a realistic one, and this is why we have choose a simple framework where the entrance of new uninformed households allows the informational asymmetry to persist over time. Finally, note that in this model there would be no gains from allowing agents to write long-term contracts, or contracts contingent on past information, as portfolio weights are a sufficient statistic for the expert's private information. Using results from the previous section, we characterize the equilibrium of the dynamic economy.

Proposition 4. *In the dynamic economy, with public information $\left\{ \hat{\Sigma}_{\theta,t}, X^t \right\}$, and state variables $\left\{ \theta_t, \tilde{\theta}_t, X_t \right\}$,*
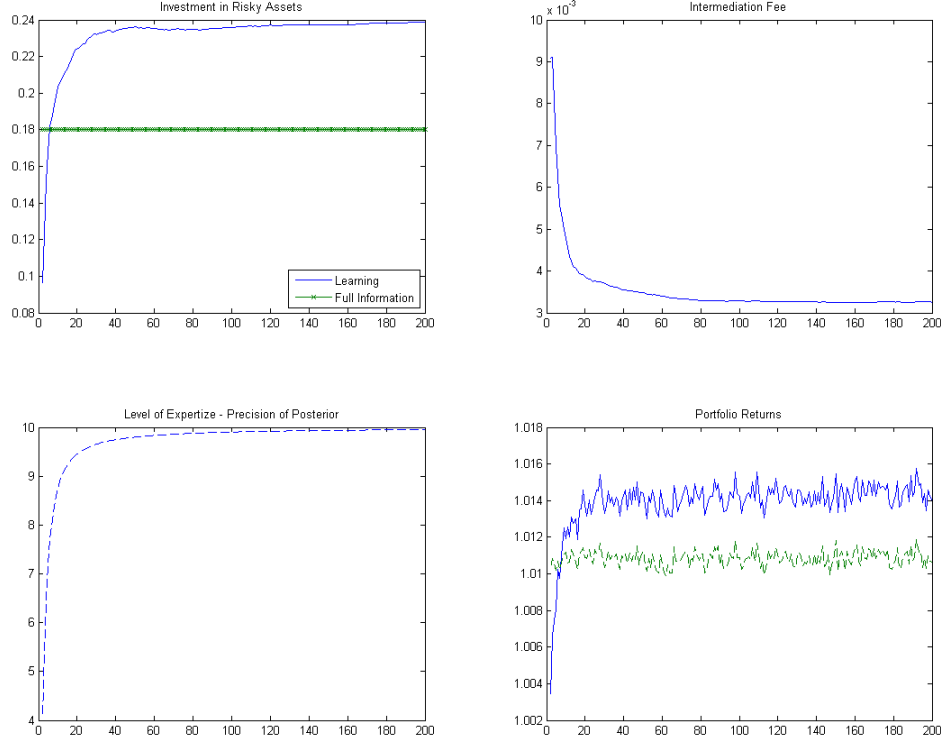
Figure 1: **Learning by Lending**

Number of simulations is 1000. Economy consists of one safe asset and two risky assets with time horizon $T = 200$. Parameter values are: $\gamma^e = 5, \gamma^h = 10$, $\omega^e = \omega^h = 1$, $R_f = 1$, $p = 0.01$, $\sigma_0 = 0.1$, $\sigma^e = 0.3$, $\sigma^\eta = 1$, $\theta = \theta_0 = 1.03$.

the consumption allocations are given by

$$c^e (R_{t+1}, \alpha_t) = \frac{\gamma^h}{\gamma^h + \gamma^e} \left([R_{t+1} - R_f 1_N]' \alpha_t + R_f \right) w + \bar{f} + f(\alpha_t)$$

$$c^h (R_{t+1}, \alpha_t) = \frac{\gamma^e}{\gamma^h + \gamma^e} \left([R_{t+1} - R_f 1_N]' \alpha_t + R_f \right) w - \bar{f} - f(\alpha_t)$$

$\forall t > 0$; the expert's portfolio choice is given by

$$\alpha_t = \kappa \left[w \bar{\gamma} \hat{\Sigma}_{\theta,t} \right]^{-1} [\mu_t - R_f 1_N]$$

where $\kappa = \frac{\gamma^h + 2\gamma^e}{\gamma^h + \gamma^e}$, and $\left(\mu_t, \hat{\Sigma}_{\theta,t} \right)$ are given by equations (2) and (3). Finally, the intermediation fee charged to households is:

$$f(\mu_t) = \left(\gamma^h \right)^{-1} \left(\kappa - \frac{1}{2} \kappa^2 \right) [\mu_t - R_f 1_N]' \hat{\Sigma}_{\theta,t}^{-1} [\mu_t - R_f 1_N]$$
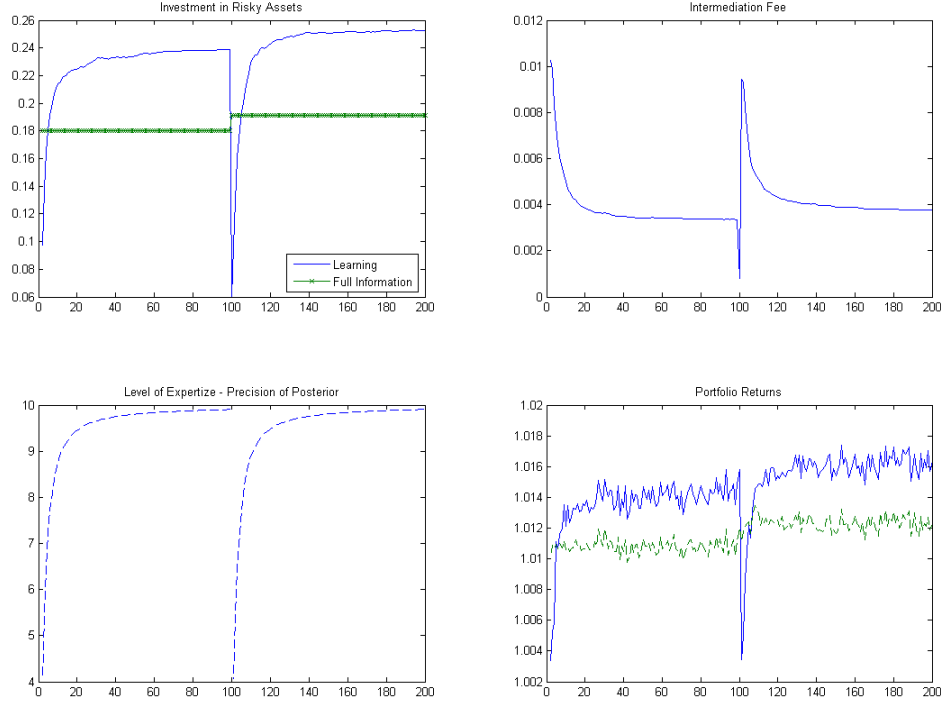
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Figure 2: **Structural Shocks**

Same calibration as before. At $T = 100$, there is a structural shock, i.e. $X_T = 1$.

Proof. See Propositions 1-3. $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\square$

We now illustrate the dynamics of our economy graphically. First, in Figure 1, we use a simple parametrization of our model to simulate a period of economic stability following an initial draw of the fundamental state θ. Then, in Figure 2, we show the effects of a shock to mean of project returns θ. We use the economy with full information, where all agents know the true value of θ, as a useful benchmark against which to compare our results. Figure 1 shows that the economy with learning is more volatile relative to the economy with full information, but that volatility diminishes over time as the level of expertize (precision of posterior beliefs) in the economy grows. Thus, periods of continued economic stability are associated with periods of gradually declining volatility. Volatility of investment between the two economies is a good case in point. In the full information economy, risky investment is fixed by a sharp ratio that is constant - all agents know the true mean of project returns and the returns are iid. When there is learning, however, both the perceived mean of project returns and their perceived variance change over time.

The mechanism we aim to highlight is the detrimental effect that the loss of "inside" information has when the economy experiences changes in fundamentals. A shock to fundamentals (positive or negative) generates an

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endogenous volatility shock for experts due to the loss of their expertize, reflected in a decrease in the precision of their private information. As a response, experts contract credit to risky sectors, while they start learning about the new economy. This amplifies and propagates shocks to the real economy, that in a standard model with full information would only generate a once and for all change in allocations at the moment of impact, and no change at all in uncertainty. We do not interpret these type of shocks as drivers of business cycles, these are permanent shocks that alter the distribution of productivities across firms or sectors in an economy. They should be interpreted as technological shocks that change the relative productivity of sectors within an economy. For example, the introduction of the internet affected some sectors positively and other negatively, this is one example of a shock that has altered relative productivities. The dot.com boom is a good example to describe the mechanism we have in mind: while at the beginning it was difficult to raise funding for internet related activities, once credit started going into this sector, optimism increased over time, possibly generating the so called dot.com bubble. Finally, it became clear that the sector was not as profitable as expected, credit to the private sector contracted, the U.S. had a recession, but eventually funds were allocated to new sectors (real state related investment, for example).

5   Discussion

Re-Allocation of Funds

One interesting feature of our model is that it can generate contractions as a response to a sectoral reallocation of productivities. In the previous simulations, the shock that hits the economy has an impact on aggregate productivity, since it changes the mean of project returns. In response to this, total investment to the risky sectors changes not only as response to the loss of expertize, but also as a response to the changes in fundamentals. In this section, we analyze a shock that shuffles productivities across sectors, but has no impact on aggregate returns if portfolios are re-adjusted accordingly. The response to a shock of this nature in a model with full information is to simply reallocate capital across sectors according to the new distribution of productivities. In our model, experts understand that there is a shock but they do not know how productivities have been re-shuffled. The lack of knowledge about the nature of the shock acts as a volatility shock, generating a fall in credit, and a slow recovery. As experts learn the new distribution of productivities and reallocate funds accordingly, the economy converges to its previous levels. Figure 3 shows the results for a simulation in which the mean return of sectors one and three is interchanged. Notice that in this example, the benchmark economy of full information experiences no change in the levels of investment to the risky sectors.
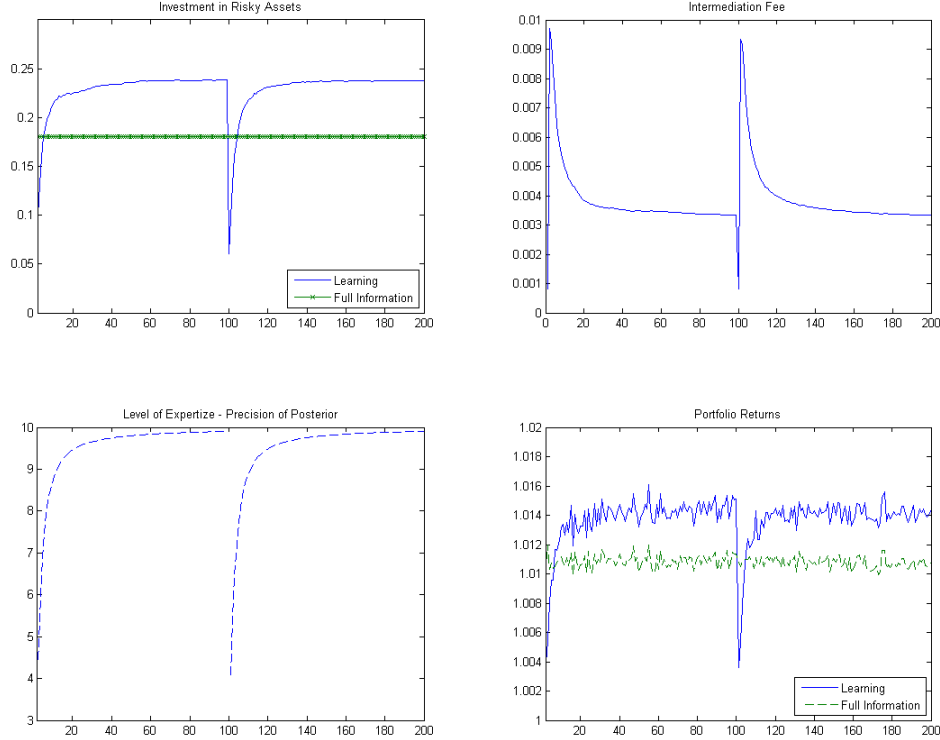
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Figure 3: **Sectoral Reallocation**

Same calibration as before. At $T = 100$, there is a structural shock that has no aggregate effect, $\theta_1' = \theta_2$ and $\theta_2' = \theta_1$.

Unobservable Aggregate States

A natural extension to this model is to make the aggregate state, X_t unobservable. In this scenario, agents should infer the change of state from observed returns. In a model with Bayesian learning, it is very hard to generate strong reactions to bad signals after long periods of stability. In the presence of a negative shock to fundamentals, experts would take a long time to realize that the state has changed, and credit cycles would not be strongly asymmetric as observed in the data. An alternative learning mechanism that allows to generate highly asymmetric responses was introduced by Marcet and Nicolini (2003). Their paper presents a boundedly rational learning model, where agents re-adjust their beliefs very strongly once they observe realizations that are highly unlikely under their prevalent beliefs. The drawback of this learning mechanism is that it is not fully rational. However, it is extremely intuitive and a good candidate to generate abrupt changes in beliefs. If we were to make the aggregate state X_t unobservable, we would model expert's beliefs as follows: when realizations of returns are very far on the tales of the posterior distribution (a threshold is imposed), agents understand there has been a new draw of fundamentals, and the economy behaves as if the shock had been public. There are two

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new implications of using this learning mechanism: first, after a negative shock to fundamentals, it might take time for experts to realize this, and thus for some periods returns are going to be low on average. Second, cycles could be generated without having shocks to fundamentals at all. If an outlier is drawn, experts would interpret this as an aggregate shock and start reacting accordingly by putting more weight on recent observations, and disregarding past data. We believe that the results that could be obtained from an alternative learning as the one described here are very interesting, but we postpone that analyzes to future research.

6   Conclusions

We presented a dynamic model of financial intermediation in which changes in the information held by financial intermediaries generate asymmetric credit cycles as the ones documented by Reinhart and Reinhart (2010). Our model is able to generate long periods of credit expansion, followed by sharp contractions in lending and slow recoveries. We model financial intermediaries as information producers, we assume they are "expert" agents that have a unique ability to acquire information about firm/sector fundamentals. Better information allows for better allocation of resources, and this informational advantage makes these actors be the natural contenders to intermediate funds between households and businesses. The level of "expertize" in the economy and the potential gains from intermediation grow in tandem with the information that these experts possess; these gains, however, are hindered since experts' information is inherently private. We find the optimal financial contracts that balance allocational efficiency with the provision of appropriate incentives. The economy therefore inherits not only the dynamic nature of information flow, but also the interaction of information with the contractual setting. To generate contractions in credit we introduce a cyclical component to information by supposing that the fundamentals about which experts acquire information are stochastic. While persistence of fundamentals is essential for information to be valuable, their randomness acts as an opposing force and diminishes the value of expert learning. Our setting then features economic fluctuations due to waves of "confidence" in the experts' ability to allocate funds profitably.

References

Adam, Klaus, Albert Marcet, and Juan Pablo Nicolini, "Stock Market Volatility and Learning," 2008.

Allen, Franklin, "Diversity of Opinion and Financing of New Technologies," *Journal of Financial Intermediation*, January 1999, *8* (1-2), 68–89.

Berger, Allen N and Gregory F Udell, "A More Complete Conceptual Framework for Financing of Small and Medium Enterprises," 2005.

Bloom, Nicholas, "The Impact of Uncertainty Shocks," *Econometrica*, 2009, *77* (3), 623–685.

Boyd, John and Edward C Prescott, "Financial Intermediary-Coalitions," 1985.

Campbell, Tim S and William A Kracaw, "Information Production, Market Signalling, and the Theory of Financial Intermediation," *The Journal of Finance*, 2012, *35* (4), 863–882.

Diamond, Douglas W., "Financial Intermediarion and Delegated Monitoring," *Economic Analysis*, 1984, *51* (3), 393–414.

_ , "Liquidity, Banks, and Markets," *Journal of Political Economy*, October 1997, *105* (5), 928.

_ **and Philip H. Dybvig**, "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy*, January 1983, *91* (3), 401.

Gennaioli, Nicola, Andrei Schleifer, and Robert W Vishny, "Money Doctors," 2012.

Gorton, Gary and Andrew Metrick, "Securitized banking and the run on repo," *Journal of Financial Economics*, 2011, pp. 1–27.

_ **and Andrew Winton**, "Financial Intermediation," 2002, (May).

Hirshleifer, Jack, "The Private and Social Value of Information and the Reward to Inventive Activity," *American Economic Review*, 2008, *61* (4), 561–574.

Holmström, Bengt, "Coordination of Information," 1977, p. Chapter 2 Dissertation.

_ , "Managerial Inventive Problems: A Dynamic Perspective," *The Review of Economic Studies*, 1999, *66*, 169–182.

Holmstrom, Bengt and Jean Tirole, "Financial Intermediation, Loanable Funds, and The Real Sector," *Quarterly Journal of Economics*, August 1997, *112* (3), 663–691.

Kurlat, Pablo, "Lemons , Market Shutdowns and Learning," 2010.

Leland, Hayne E and David H Pyle, "Informational Asymmetries, Financial Structure, and Financial Intermediation," *The Journal of Finance*, 1977, *32* (2).

Marcet, Albert and Juan Pablo Nicolini, "Recurrent Hyperinflations and Learning," *SSRN Electronic Journal*, 1998.

Maskin, Eric and Jean Tirole, "The Principal-Agent Relationship with an Informed Principal : The Case of Private Values," *Econometrica*, 1990, *58* (2), 379–409.

_ **and** _ , "The Principal-Agent Relationship with an Informed Principal , II : Common Values," *Econometrica*, 1992, *60* (1), 1–42.

Mullainathan, Sendhil, Markus Noth, and Antoinette Schoar, "The Market for Financial Advice : An Audit Study," 2010.

Opp, Christian C G, "Intertemporal Information Acquisition and Asset Market Dynamics," 2008, (September 2007), 1–23.

Petersen, Mitchell A . and Raghuram G Rajan, "The Effect of Credit Market Competition on Lending Relationships," *The Quaterly Journal of Economics1*, 995, *110* (2), 407–443.

Reinhart, Carmen M and Vincent R Reinhart, "After the fall," 2010.

Roger B . Myerson, "Incentive Compatibility and the Bargaining Problem," *Econometrica*, 1979, *47* (1), 61–73.

Stracca, Livio, "Delegated Portfolio Management: A Survey of the Theoretical Literature," 2005.

Williamson, Stephen, "Costly Monitoring , Loan Contracts , and Equilibrium Credit Rationing," *The Quarterly Journal of Economics*, 1987, *102* (1), 135–146.

7 Appendix

Expert's Problem

Let c^e, c^h be the payoff of experts and households receptively, let μ be the mean of the experts' posterior distribution, and let $\tilde{\mu}$ be the households beliefs about the expert's private information (beliefs updated after observing portfolio allocations: α). The expert's problem is given by:

$$\max_{c^e, c^h, \alpha} E\left[u^e\left(c^e\right)|\mu\right]$$

$$E\left[u^h\left(c^h\right)|\tilde{\mu}\right] \geq \bar{U}^h \quad (\lambda_{pc}(\tilde{\mu}))$$

$$c^e(R,\alpha) + c^h(R,\alpha) \leq \left[\alpha'(R - R_f) + R_f\right]w \quad (\lambda_{fc1}(R,\alpha))$$

$$w \leq w^e + w^h \quad (\lambda_{fc2})$$

$$\tilde{\mu}(\alpha(\mu)) = \mu$$

Proof of Proposition 1. Combining the constraints, the problem of expert with private information μ, can

be re-written as follows:

$$\max_{c^e(R,\alpha),\alpha} E\left[u^e\left(\left[\alpha'(R - R_f) + R_f\right]w - c^h(R)\right)|\mu\right]$$

$$E\left[u^h\left(c^h(R,\alpha)\right)|\tilde{\mu}\right] \geq \bar{U}^h \quad (\lambda_{pc}(\tilde{\mu}))$$

$$\tilde{\mu}(\alpha(\mu)) = \mu$$

Given that $u'(c) = \gamma \exp\left[-\gamma c\right]$, the FOC with respect to consumptions evaluated at $\tilde{\mu}(\alpha(\mu)) = \mu$ yield:

$$c^e(R,\alpha) = \frac{\gamma^h}{\gamma^h + \gamma^e}\left[\alpha'(R - R_f) + R_f\right]w + Z(\alpha)$$

$$c^h(R,\alpha) = \frac{\gamma^e}{\gamma^h + \gamma^e}\left[\alpha'(R - R_f) + R_f\right]w - Z(\alpha)$$

where $Z(\alpha) = \frac{1}{\gamma^h + \gamma^e} log\left(\frac{\lambda_{pc}\gamma^e}{\gamma^h}\right)$.

Proof of Proposition 2. To solve for $\bar{f}$ and $f(\alpha)$, we find the fee that makes the participation constraint of the households, for given beliefs $\mu(\alpha)$, bind:

$$E\left[u^h\left(\frac{\gamma^e}{\gamma^h + \gamma^e}\left[\alpha'(R - R_f) + R_f\right]w - Z(\alpha)\right)|\mu(\alpha)\right] = \bar{U}^h$$

and using that $u^h(x) = e^{-\gamma^h x}$ and $E\left\{e^{-\gamma^h x}\right\} = e^{-\gamma^h E\{x\} + \frac{1}{2}\gamma^{h2}V(x)}$ for x that is normally distributed, we have

$$-\gamma^h\left(\frac{\gamma^e}{\gamma^h + \gamma^e}\left[R_f + \left[\mu(\alpha) - R_f 1_N\right]'\alpha\right]w - Z(\alpha)\right) + \frac{1}{2}\gamma^{h2}\left(\frac{\gamma^e}{\gamma^h + \gamma^e}\right)^2 (\alpha w)'\Sigma(\alpha w) = -\gamma^h R_f w^h$$

$$\Longleftrightarrow$$

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$$Z\left(\alpha\right) = \frac{\gamma^e}{\gamma^h + \gamma^e}\left[\mu\left(\alpha\right) - R_f 1_N\right]' \alpha w - \frac{1}{2}\gamma^h\left(\frac{\gamma^e}{\gamma^h + \gamma^e}\right)^2 (\alpha w)' \Sigma\left(\alpha w\right) + R_f\left(\frac{\gamma^e}{\gamma^h + \gamma^e}w^e - \frac{\gamma^e}{\gamma^h + \gamma^e}w^h\right)$$

Decomposing the fee as in the text, $Z\left(\alpha\right) = \overline{f} + f\left(\alpha\right)$, we have that

$$\overline{f} = R_f\left(\frac{\gamma^e}{\gamma^h + \gamma^e}w^e - \frac{\gamma^e}{\gamma^h + \gamma^e}w^h\right)$$

$$f\left(\alpha\right) = \frac{\gamma^e}{\gamma^h + \gamma^e}\left[\mu\left(\alpha\right) - R_f 1_N\right]' \alpha w - \frac{1}{2}\gamma^h\left(\frac{\gamma^e}{\gamma^h + \gamma^e}\right)^2 (\alpha w)' \Sigma\left(\alpha w\right)$$

Proof of Proposition 3. Using the results of Proposition 2, the expert's problem can be expressed as

$$\max_{c^e(R,\alpha),\alpha}\left\{-\gamma^e\left[\frac{\gamma^h}{\gamma^h + \gamma^e}\left[\alpha\left(\mu - R_f\right) + R_f\right]w + \overline{f} + f\left(\alpha\right)\right] + \frac{1}{2}\gamma^{e2}\left(\frac{\gamma^h}{\gamma^h + \gamma^e}\right)^2 (\alpha w)' \Sigma\left(\alpha w\right)\right\}$$

$$s.t.$$

$$\overline{f} = R_f\left(\frac{\gamma^e}{\gamma^h + \gamma^e}w^e - \frac{\gamma^e}{\gamma^h + \gamma^e}w^h\right)$$

$$f\left(\alpha\right) = \frac{\gamma^e}{\gamma^h + \gamma^e}\left[\mu\left(\alpha\right) - R_f 1_N\right]' \alpha w - \frac{1}{2}\gamma^h\left(\frac{\gamma^e}{\gamma^h + \gamma^e}\right)^2 (\alpha w)' \Sigma\left(\alpha w\right)$$

where as before we use that that $u^h\left(x\right) = e^{-\gamma^h x}$ and $E\left\{e^{-\gamma^h x}\right\} = e^{-\gamma^h E\{x\} + \frac{1}{2}\gamma^{h2}V(x)}$ for x that is normally distributed. The first order condition with respect to α yields

$$-\gamma^e\left[\frac{\gamma^h}{\gamma^h + \gamma^e}\left(\mu - R_f\right)w + f'\left(\alpha\right)\right] + \gamma^{e2}\left(\frac{\gamma^h}{\gamma^h + \gamma^e}\right)^2 w\Sigma\alpha w = 0$$

where

$$f'\left(\alpha\right) = \frac{\gamma^e}{\gamma^h + \gamma^e}\left[\mu\left(\alpha\right) - R_f 1_N\right]w + \frac{\gamma^e}{\gamma^h + \gamma^e}\left[\frac{d}{d\alpha}\mu\left(\alpha\right)\right]' \alpha w - \gamma^h\left(\frac{\gamma^e}{\gamma^h + \gamma^e}\right)^2 w\Sigma\alpha w$$

Using Conjecture 1, we have that

$$\frac{d}{d\alpha}\mu\left(\alpha\right) = \kappa^{-1}\Sigma$$

and combining these expressions and solving for α yields

$$\alpha = \kappa\Sigma^{-1}\left[\mu - R_f 1_N\right]$$

with $\kappa = \frac{\gamma^h + 2\gamma^e}{\gamma^h + \gamma^e}$.

Proof of Corollary 1. Plugging the optimal choice of α from Proposition 3 into the expression for $f\left(\alpha\right)$ yields

$$f\left(\alpha\right) = \frac{1}{2}\frac{\gamma^h + 2\gamma^e}{\left(\gamma^h + \gamma^e\right)^2}\left[\mu - R_f 1_N\right]' \Sigma^{-1}\left[\mu - R_f 1_N\right]$$

Optimality of Delegation (implication of Assumption 4). The household's welfare within the contract

is given by

$$U^h = \gamma^h R_f w^h$$

If the household was able to invest in the portfolio with prior beliefs (μ_0, Σ_0), then its welfare would be

$$U^h = \frac{1}{2} \left[\mu_0 - R_f 1_N\right]' \Sigma_0^{-1} \left[\mu_0 - R_f 1_N\right] + \gamma^h R_f w^h$$

and so if the non-pecuniary cost of investment satisfies $\chi > \frac{1}{2} \left[\mu_0 - R_f 1_N\right]' \Sigma_0^{-1} \left[\mu_0 - R_f 1_N\right]$, the household will only invest through the expert.