Forward Guidance and the Exchange Rate

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Abstract

I analyze the effectiveness of forward guidance policies in open economies, focusing on the role played by the exchange rate in their transmission. An open economy version of the "forward guidance puzzle" is shown to emerge. In partial equilibrium, the effect on the current exchange rate of an anticipated change in the interest rate does not decline with the horizon of implementation. In general equilibrium, the size of the effect is larger the longer is that horizon. Empirical evidence using U.S. and euro area data euro-dollar points to the presence of a *forward guidance exchange rate puzzle*: expectations of interest rate differentials in the near (distant) future have much larger (smaller) effects on the euro-dollar exchange rate than is implied by the theory.

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1 Introduction

The challenges posed by the global financial crisis to central bankers and the latter's increasing reliance on unconventional monetary policies has triggered an explosion of theoretical and empirical research on the effectiveness of such policies, i.e. policies that seek to substitute for changes in the short-term nominal rate –the instrument of monetary policy in normal times– when the latter attains its zero lower bound (ZLB). A prominent example of an unconventional policy adopted by several central banks in recent years is given by *forward guidance*, i.e. the attempt to influence current macroeconomic outcomes by managing expectations about the future path of the policy rate once the ZLB is no longer binding.

In the present paper I analyze the effectiveness of forward guidance policies in an *open economy*, focusing on the role played by the exchange rate in their transmission. As I discuss below, that transmission hinges to an important extent on the dependence of the exchange rate on the *undiscounted* sum of expected future interest rate differentials, as implied by the theory. Importantly, that relation relies only a (relatively) weak assumption: the existence at each point in time of some investors with access to both domestic and foreign bonds.

In the first part of the paper I analyze the effects of forward guidance on the exchange rate, under the assumption of constant prices (or, equivalently, when the induced effects of the interest rates and the exchange rate on output and prices are ignored). In that environment, the combination of uncovered interest parity with the long run neutrality of monetary policy yields a strong implication: the impact on the current exchange rate of an announcement of a future adjustment of the nominal rate is *invariant* to the timing of that adjustment.

Next I turn to the analysis of forward guidance policies in general equilibrium, i.e. allowing for feedback effects on output and prices, using a simple New Keynesian model of a small open economy. In general equilibrium, the size of the effect of forward guidance policies on the exchange rate is shown to be *larger* the *longer* is the horizon of implementation of a given adjustment in the nominal interest rate. A similar prediction applies to the effect on output and inflation. Both results are closely connected to the findings in the closed economy literature on the forward guidance puzzle, as discussed below.[1]

The same framework can be used to analyze the relation between the effectiveness of forward guidance policies and openness. I start by showing a simple condition under which the size of the effects of forward guidance policies on the exchange rate and other macro variables is invariant to the economy's openness. When that condition does not apply, the sign of that relation between openness and the size of the effects of forward looking policies can no longer be pinned down analytically. As an illustration, I show that under my baseline calibration the impact of forward guidance on some variables (output, the nominal exchange rate) increases with the degree of openness, whereas the opposite is true for some other variables (e.g., the real exchange rate).

Finally, I turn to the data, and provide some empirical evidence on the role of current and expected future interest rate differentials as a source of exchange rate fluctuations. Using data on euro-dollar exchange rate and market-based forecasts of interest rate differentials between the U.S. and the euro area, I provide evidence suggesting that expectations of interest rate differentials in the near (distant) future have much larger (smaller) effects than is implied

[1] See Del Negro et al. (2015), and McKay et al. (2016, 2017), among others,

by the theory. I refer to the apparent disconnect between theory and empirics on this issue as the *forward guidance exchange rate puzzle*, and discuss why the solutions to the forward guidance puzzle found in the closed economy literature are unlikely to apply in the presence of an exchange rate channel.

The remainder of the paper is organized a follows. Section 2 describes the related literature. Section 3 discusses the effects of forward guidance on the exchange rate in a partial equilibrium framework. Section 4 revisits that analysis in general equilibrium, using a small open economy New Keynesian model as a reference framework. Section 5 presents the empirical evidence. Section 6 summarizes and concludes.

2 Background: The Forward Guidance Puzzle

The effectiveness of forward guidance and its role in the design of the optimal monetary policy under a binding ZLB was analyzed in Eggertsson and Woodford (2003) and Jung et al. (2005), using a standard New Keynesian model. Those papers emphasized the high effectiveness of forward guidance as a stabilizing instrument implied by the theory, at least under the maintained assumption of credible commitment.

More recently, the contributions of Carlstrom et al. (2015), Del Negro et al. (2015), and McKay et al. (2016, 2017), among others, have traced the strong theoretical effectiveness of forward guidance to a "questionable" property of one of the key blocks of the New Keynesian model, the Euler equation, which in its conventional form implies that future interest rates are not "discounted" when determining current consumption. Formally, the dynamic IS equation (DIS) of the New Keynesian Model can be solved forward and written as:

$$\widehat{y}_t = -\frac{1}{\sigma} \sum_{k=0}^{\infty} \mathbb{E}_t\{\widehat{r}_{t+k}\}$$

where y_t is (log) output and $r_t \equiv i_t - \mathbb{E}_t\{\pi_{t+1}\}$ is the real interest rate. The $\widehat{}$ denotes deviations from steady state. Note that the predicted effect on output of a given anticipated change in the real interest rate is invariant to the horizon of implementation of that change. Furthermore, when combined with the forward-looking nature of inflation inherent to the New Keynesian Phillips curve, the previous property implies that the announcement of a future *nominal* rate adjustment of a given size and persistence is predicted to have a stronger effect on current output and inflation the longer the horizon of implementation, given the positive relation between that horizon and the size of the inflation response, determined by a forward-looking New Keynesian Phillips curve. That prediction, at odds with conventional wisdom, has been labeled the *forward guidance puzzle.*

Several potential "solutions" to the forward guidance puzzle have been proposed in the literature, in the form of modifications of the benchmark model that may generate some kind of discounting in the Euler equation, including the introduction of finite lives (Del Negro et al. (2015)), incomplete markets (McKay et al. (2016, 2017)), lack of common knowledge (Angeletos and Lian (2017)), and behavioral discounting (Gabaix (2017)). The proposed solutions typically generate a "discounted" DIS equation of the form

$$\widehat{y}_t = \alpha \mathbb{E}_t\{\widehat{y}_{t+1}\} - \frac{1}{\sigma}\mathbb{E}_t\{\widehat{r}_t\}$$

where $\alpha \in (0,1)$, leading to the forward-looking representation

$$\widehat{y}_t = -\frac{1}{\sigma}\sum_{k=0}^{\infty}\alpha^k \mathbb{E}_t\{\widehat{r}_{t+k}\}$$

which implies that the effect of future interest rate changes on current output is more muted the longer is the horizon of their implementation.

Interestingly, and as discussed below, many of those solutions would not seem to be relevant in the presence of the exchange rate channel introduced below.

Next I show that a phenomenon analogous to the forward guidance puzzle applies to the real exchange rate in an open economy.

3 Forward Guidance and the Exchange Rate in Partial Equilibrium

Consider the asset pricing equations

$$1 = (1+i_t)\mathbb{E}_t\{\Lambda_{t,t+1}(P_t/P_{t+1})\} \tag{1}$$

$$1 = (1+i_t^*)\mathbb{E}_t\{\Lambda_{t,t+1}(\mathcal{E}_{t+1}/\mathcal{E}_t)(P_t/P_{t+1})\} \tag{2}$$

for all t, where i_t denotes the yield on a nominally riskless one-period bond denominated in domestic currency purchased in period t (and maturing in period $t+1$). i_t^* is the corresponding yield on an analogous bond denominated in foreign currency. $\mathcal{E}_t$ is the exchange rate, expressed as the price of foreign currency in terms of domestic currency. $\Lambda_{t,t+1}$ is the stochastic discount factor for an investor with access to the two bonds in period t.

Combining (1) and (2) we have

$$\mathbb{E}_t\{\Lambda_{t,t+1}(P_t/P_{t+1})\left[(1+i_t)-(1+i_t^*)(\mathcal{E}_{t+1}/\mathcal{E}_t)\right]\} = 0 \tag{3}$$

In a neighborhood of a perfect foresight steady state, and to a first-order approximation, we can rewrite the previous equation as:

$$i_t = i_t^* + \mathbb{E}_t\{\Delta e_{t+1}\} \tag{4}$$

for all t, where $e_t \equiv \log \mathcal{E}_t$. This is the familiar uncovered interest parity condition.

Letting $q_t \equiv p_t^* + e_t - p_t$ denote the (log) real exchange rate, one can write the "real" version of (4) as:

$$q_t = r_t^* - r_t + \mathbb{E}_t\{q_{t+1}\} \tag{5}$$

where $r_t \equiv i_t - \mathbb{E}_t\{\pi_{t+1}\}$ is the real interest rate and $\pi_t \equiv p_t - p_{t-1}$ denotes (CPI) inflation, both referring to the home economy. r_t^* and with π_t^* are defined analogously for the foreign economy. Under the assumption that $\lim_{T\to\infty}\mathbb{E}_t\{q_T\}$ is well defined and bounded, (5) can be solved forward and, after taking the limit as $T\to\infty$, rewritten as:

$$q_t = \sum_{k=0}^{\infty}\mathbb{E}_t\{r_{t+k}^* - r_{t+k}\} + \lim_{T\to\infty}\mathbb{E}_t\{q_T\} \tag{6}$$

Equation (6) can be generalized to the case of a deterministic trend in q_t, which would generally make $\lim_{T\to\infty} \mathbb{E}_t\{q_T\}$ unbounded. The existence of a different trends in productivity growth between the home and foreign economy is a possible source of that underlying trend. Let $q_t = f_t + \widehat{q}_t$, where f_t is a deterministic trend such that $\lim_{T\to\infty} \mathbb{E}_t\{\widehat{q}_T\}$ is bounded. Combining the previous assumptions with (5) we can derive:

$$\widehat{q}_t = \sum_{k=0}^{\infty} \mathbb{E}_t\{r^*_{t+k} - r_{t+k} - d_{t+k}\} + \lim_{T\to\infty} \mathbb{E}_t\{\widehat{q}_T\} \tag{7}$$

where $d_t \equiv -\Delta f_{t+1}$ is the real interest rate differential along the deterministic trend path.

Equation (6) (and its generalization (7)) determines the real exchange rate as a function of (i) current and expected real interest rate differentials and (ii) the long run expectation of the real exchange rate. Forward-looking real exchange rate equations similar to (7) have often been used in the empirical exchange rate literature, though not in connection to forward guidance.[2] For the purposes of the present paper a key property of (6) (and (7)) must be highlighted, namely, the lack of discounting of future real interest rate differentials. As discussed in the introduction, an analogous property can be found in the dynamic IS equation of the New Keynesian model, which provides the source for the forward guidance puzzle. In what follows I discuss some of the implications of that property for the real exchange rate and its connection to forward guidance policies, and explore its empirical support.

3.1 A Forward Guidance Experiment

Assume that at time t the home central bank credibly announces an increase of the nominal interest rate of size δ, starting T periods from now and of duration D (i.e., from period $t+T$ to $t+T+D-1$), with no reaction expected from the foreign central bank. Furthermore, assume that the path of domestic and foreign prices remains unchanged (this assumption is relaxed below). Both the transitory nature of the intervention, as well as the assumption of long run neutrality of monetary policy, imply that $\lim_{T\to\infty} \mathbb{E}_t\{q_T\}$ should not change in response to the previous announcement. It follows from (6) that the real exchange rate will vary in response to the announcement by an amount given by

$$\widehat{q}_t = -D\delta$$

i.e. the exchange rate appreciation at the time of the announcement is proportional to the *duration* and the *size* of the announced interest rate increase, but is *independent of its planned timing* (T). Thus, a D-period increase of the real interest rate 10 years from now is predicted to have the same effect on today's real exchange rate as an increase of equal size and duration to be implemented immediately.

Once the interest rate increase is effectively implemented in period $t+T$, the exchange rate depreciates at a constant rate δ per period, i.e. $\Delta q_{t+T+k} = \delta$ for $k = 1, 2, ..D$ and stabilizes at its initial level once the intervention concludes, i.e. $q_{t+T+k} = q_t$ for $k = D+1, D+2, ...$

Figure 1 illustrates that prediction by displaying the implied path of the interest rate and the exchange rate when an interest rate rise of 1% (in annual terms) is announced at $t = 0$, to be implemented at $T = 4$ and lasting for $D = 4$ periods.

[2] See, e.g., Engel and West (2004, 2006), Chinn (2008), Mark (2009), Clarida and Walmann (2008), among many others.

4 Forward Guidance and the Exchange Rate in General Equilibrium

Consider the (log-linearized) equilibrium conditions of a standard small open economy model with Calvo staggered price-setting, law of one price (producer pricing), and complete markets.[3]

$$\pi_{H,t} = \beta \mathbb{E}_t\{\pi_{H,t+1}\} + \kappa y_t - \omega q_t \tag{8}$$

$$y_t = (1-\upsilon)c_t + \vartheta q_t \tag{9}$$

$$c_t = \mathbb{E}_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - \mathbb{E}_t\{\pi_{t+1}\}) \tag{10}$$

$$c_t = \frac{1}{\sigma} q_t \tag{11}$$

where $\pi_{H,t} \equiv p_{H,t} - p_{H,t-1}$ denotes domestic inflation, y_t is (log) output and c_t is (log) consumption. Equation (8) is a New Keynesian Phillips curve for the small open economy. Coefficients κ and ω are defined as $\kappa \equiv \lambda(\sigma + \varphi)$ and $\omega \equiv \frac{\lambda(\sigma\eta - 1)\upsilon(2-\upsilon)}{1-\upsilon}$ where $\upsilon \in [0,1]$ is an index of openness (equal the share of imported goods in domestic consumption in the steady state), $\sigma > 0$ is the (inverse) elasticity of intertemporal substitution, $\eta > 0$ is the elasticity of substitution between domestic and foreign goods, and $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} > 0$ is inversely related to the Calvo price stickiness parameter θ. (9) is the goods market clearing condition, with $\vartheta \equiv \eta\upsilon\left(1 + \frac{1}{1-\upsilon}\right) > 0$. (10) is the consumption Euler equation, with $\pi_t \equiv p_t - p_{t-1}$ denoting CPI inflation. (11) is the international risk sharing condition, derived under the assumption of complete markets. The above specification of the equilibrium conditions assumes constant prices and real interest rates in the rest of the world, normalized to zero for notational ease (i.e. $r_t^* = p_t^* = 0$ all t). Also for simplicity I abstract from any non-policy shocks, with the analysis focusing instead on the effects of exogenous monetary policy changes.

Note that (10) and (11) imply the real version uncovered interest parity analyzed in the previous section:[4]

$$q_t = \mathbb{E}_t\{q_{t+1}\} - (i_t - \mathbb{E}_t\{\pi_{t+1}\}) \tag{12}$$

Furthermore, under the maintained assumption of full pass through, CPI inflation and domestic inflation are linked by

$$\begin{aligned} \pi_t &\equiv (1-\upsilon)\pi_{H,t} + \upsilon \Delta e_t \\ &= \pi_{H,t} + \frac{\upsilon}{1-\upsilon}\Delta q_t \end{aligned} \tag{13}$$

As emphasized in Galí and Monacelli (2005) the previous equilibrium conditions can be combined to obtain a system of two difference equations for domestic inflation $\pi_{H,t}$ and output y_t that is isomorphic to that of the closed economy, namely:

$$\pi_{H,t} = \beta \mathbb{E}_t\{\pi_{H,t+1}\} + \kappa_\upsilon y_t \tag{14}$$

[3] Detailed derivations of the equilibrium conditions can be found in Galí and Monacelli (2005) and Galí (2015, chapter 8) With little loss of generality I assume an underlying technology that is linear in labor input.

[4] The assumption of complete markets at the international level is sufficient (though not necessary) to derive the uncovered interest parity equation. As discussed in section 2 above that equation can be derived as long as there are some investors each period with access to both domestic and foreign one-period bonds.

$$y_t = \mathbb{E}_t\{y_{t+1}\} - \frac{1}{\sigma_\upsilon}(i_t - \mathbb{E}_t\{\pi_{H,t+1}\}) \tag{15}$$

where $\sigma_\upsilon \equiv \frac{\sigma}{1+(\sigma\eta-1)\upsilon(2-\upsilon)} > 0$ and $\kappa_\upsilon \equiv \lambda(\sigma_\upsilon + \varphi) > 0$ are now both functions of the open economy parameters (υ, η).

In addition, combining (9) and (11) we can derive the following simple relation between the real exchange rate and output:

$$q_t = \sigma_\upsilon(1-\upsilon)y_t \tag{16}$$

In order to close the model, a description of monetary policy is required. I assume the simple rule

$$i_t = \phi_\pi \pi_{H,t} \tag{17}$$

where $\phi_\pi > 1$. It can be easily checked that in the absence of exogenous shocks the equilibrium in the above economy is (locally) unique and given by $\pi_{H,t} = y_t = q_t = i_t = 0$ for all t.

Consider next a forward guidance experiment analogous to the one analyzed in the previous section, but allowing for an endogenous response of inflation to the anticipated change in the interest rate. More specifically, assume that at time 0, the home central bank credibly announces a *nominal* interest rate one-period increase of 0.25 (i.e. one percentage point in annualized terms) in period T . Furthermore, the central bank commits to keeping the nominal interest rate at its initial level (normalized to zero in the impulse responses) until period $T-1$, independently of the evolution of inflation. At time $T+1$ it restores the interest rate rule (17) and, with it, the initial equilibrium. I use (14), (15) and (16) to determine the response of output, domestic inflation and the real exchange rate to that forward guidance experiment. Given the response of $\pi_{H,t}$ and q_t, (13) can be used to back out the response of CPI inflation, π_t. The latter can then be used to derive the response of the (consumption) price level, which combined with the relation $e_t = q_t + p_t$ allows one to derive the response of the nominal exchange rate.

Figure 2 displays the response of interest rates, the exchange rate, output, and inflation, to the above experiment under three alternative time horizons for implementation: $T = \{1, 2, 4\}$. The parameters of the model are calibrated as follows: $\beta = 0.99$, $\upsilon = 0.4$, $\sigma = \varphi = 1$, $\eta = 2$, and $\theta = 0.75$. Note that a version of the "forward guidance puzzle" for the open economy emerges: the longer is the horizon of implementation, the larger is the impact of the announcement on the real and nominal exchange rates as well as on output and inflation. As emphasized by McKay el at. (2016), the reason for the amplification has to do with the fact that inflation depends on current and expected future output, combined with the property that the longer is the implementation of a given interest rise the more persistent the output response. It follows that the longer is the implementation horizon of a given change in the *nominal* rate the larger will be the response of the *real* rate –and hence of output and the real exchange rate– between the time of the announcement and that of policy implementation.

Figure 3 illustrates more explicitly the forward guidance puzzle as applies to the nominal and real exchange rates. It displays the percent response of those two variables on impact when a one-period increase in the nominal rate is announced, to be implemented at alternative horizons represented by the horizontal axis. As the Figure makes clear the percent appreciation of the home currency, both in real and nominal terms, increases exponentially with the horizon of implementation. Note also that the appreciation of the nominal exchange rate is substantially larger than that of the real exchange rate, with the gap between the two increasing with the horizon of implementation. That gap, which corresponds to the percent decrease in the CPI in response to the forward guidance announcement, is also increasing in the horizon due to the forward-lookingness of the New Keynesian Phillips curve. The fall in inflation, in turn, leads to a further rise in current and future real interest rates, thus generating an additional appreciation of the real exchange rate.

An alternative perspective on the previous experiment can be obtained by focusing on the determination of the nominal exchange rate. Consider an announcement of an interest rate increase of size δ and duration D, to be implemented T periods ahead. Iterating forward equation (4) we can express the nominal exchange rate at the time of the policy announcement as:

$$\begin{aligned} e_t &= \delta D + \mathbb{E}_t\{e_{t+T+D}\} \\ &= \delta D + \mathbb{E}_t\{p_{t+T+D}\} \end{aligned} \tag{18}$$

The first term on the right hand side of (18) captures the dependence of the nominal exchange rate on anticipated changes in nominal interest rate differentials. As discussed in section 2 that effect is a function of the size (δ) and duration (D) of the anticipated policy intervention, but *not* of its timing. This captures the partial equilibrium dimension of the forward guidance exchange rate puzzle. The second term, $\mathbb{E}_t\{p_{t+T+D}\}$, which reinforces the effect of the first term, is the result of general equilibrium effects working through (i) the impact on aggregate demand and output of the changes in consumption and the real exchange rate induced by the anticipation of higher future nominal interest rates (given prices), and (ii) their subsequent effects on inflation and the price level, which depend on the duration of the output effects (as implied by (14)) and, hence, on the timing of the policy implementation.

The strength of some the general equilibrium effects pointed out above is, from an empirical perspective, a controversial subject. This is true, in particular, with regard to the forward-lookingness of inflation, i.e. that variable's sensitivity to expected future output developments. An empirical analysis of those general equilibrium channels in the determination of exchange rates is clearly beyond the scope of the present paper.[5] Instead, in the remainder of the paper I turn to an empirical exploration of the (partial equilibrium) link between the exchange rate and future interest rate differentials, with a focus on the role played by the horizon of anticipated interest rate changes.

5 Interest Rate Expectations and the Exchange Rate: Does the Horizon Matter?

In the present section I examine the evidence on the extent to which fluctuations in the euro-dollar real exchange rate can be accounted for by variations in expected interest rate differentials at different horizons. I start by defining the following two variables measuring anticipated real

[5] See, e.g. Mavroeidis et al. (2014), Rudd and Whelan (2005) and Galí et al. (20015), as well as other contributions to the special issue of the *Journal of Monetary Economics* (vol. 52, issue 6) on the empirics of the New Keynesian Phillips curve for a discussion of some the issues in that controversy.

interest rate differentials at short and long horizons:

$$q_t^S(M) \equiv \sum_{k=0}^{M-1} \mathbb{E}_t\{r_{t+k}^* - r_{t+k}\}$$

$$q_t^L(M) \equiv \sum_{k=M}^{T-1} \mathbb{E}_t\{r_{t+k}^* - r_{t+k}\}$$

for any $0 < M < T$ and given a "large" long-term horizon T. Note that $q_t^S(M)$ is a measure of the expected interest rate differentials over the *short run* (i.e. over the next M periods), while $q_t^L(M)$ captures the corresponding effect of expected interest rate differentials at a longer horizon (i.e. beyond the next M periods, and up to T). Below I report OLS estimates of the regression equation

$$q_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \gamma_S q_t^S(M) + \gamma_L q_t^L(M) + \varepsilon_t \tag{19}$$

using the empirical counterparts to $q_t^S(M)$ and $q_t^L(M)$ described below for the US and the euro area. Note that coefficients γ_S and γ_L measure the reduced form (semi) elasticities of the real exchange rate with respect to the short term and long term component of expected real interest rate differentials.

As a benchmark, consider the implications of the theoretical model introduced in section 2. Equilibrium condition (5) combined with rational expectations implies:

$$q_t = q_t^S(M) + q_t^L(M) + \mathbb{E}_t\{q_{t+T}\} \tag{20}$$

Again, let $q_t = f_t + \widehat{q}_t$ with f_t a deterministic trend such that $\lim_{T\to\infty} \mathbb{E}_t\{\widehat{q}_{t+T}\}$ is bounded. Assuming that the stochastic component of the real exchange rate, $\widehat{q}_t$, is a stationary process with zero mean, so that, for large T, $\mathbb{E}_t\{\widehat{q}_{t+T}\} \simeq \lim_{T\to\infty} \mathbb{E}_t\{\widehat{q}_{t+T}\} = 0$, one can rewrite (20) as:

$$q_t = q_t^S(M) + q_t^L(M) + f_{t+T} \tag{21}$$

Under the assumption (confirmed by the evidence below) that the long run trend in the (log) real exchange rate can be approximated by a quadratic function of time, equation (21) is consistent with empirical equation (19) with $\gamma_S = \gamma_L = 1$, thus providing a useful theoretical benchmark. As noted above, the absence of discounting in (21) implies that, ceteris paribus, a change in $q_t^S(M)$ (given $q_t^L(M)$) should have the same effect on the real exchange rate as a commensurate change in $q_t^L(M)$ (given $q_t^S(M)$). Furthermore, that effect should be "one-for-one" in both cases.

In order to estimate (19) I need to construct empirical counterparts to $q_t^S(M)$ and $q_t^L(M)$. This requires some assumptions, which I take as reasonable approximations, given the purpose at hand. Thus, and given that the empirical analysis makes use of monthly data, the annualized nominal yield on a M-period bond is assumed to satisfy

$$i_t(M) = \frac{12}{M} \sum_{k=0}^{M-1} \mathbb{E}_t\{i_{t+k}\}$$

where $i_t(1) \equiv i_t$, i.e. the interest rate on a one-month nominally riskless bond. Subtracting (annualized) expected inflation between t and $t+M$ from both sides of the previous equation we can write:

$$r_t(M) = \frac{12}{M} \sum_{k=0}^{M-1} \mathbb{E}_t\{r_{t+k}\}$$

An analogous expression holds for foreign bonds. Thus, it follows that,

$$q_t^S(M) = \frac{M}{12}[r_t^*(M) - r_t(M)] \tag{22}$$

I construct measures of $r_t^*(M)$ and $r_t(M)$ using monthly data on German and US government bond zero coupon yields with 2, 5, 10 and 30 year maturity (thus corresponding to $M \in \{24, 60, 120, 360\}$), combined with monthly measures of expected inflation over the same four horizons derived from inflation swaps. Constraints on data availability for the latter variable force me to start the sample period in 2004:8. Given the time series for real interest rates at different horizons thus obtained, I use (22) to construct a time series for $q_t^S(M)$ for different M values.

In order to obtain an empirical counterpart to $q_t^L(M)$ I set $T = 360$ (corresponding to a 30 year horizon) and then use the relation:

$$q_t^L(M) \equiv q_t^S(T) - q_t^S(M)$$

I construct a monthly time series for the (log) real exchange rate q_t, using data on the (log) euro-dollar nominal exchange rate, and the (log) CPI indexes for the US and the euro area.

5.1 Findings: Level Specification

Table 1 reports the OLS estimates of γ_S and γ_L in (19), for $M \in \{24, 60, 120\}$. Note that all the estimated coefficients are positive and highly significant. Thus, the evidence confirms the link between the real exchange rate and current and expected real interest rate differentials, with the sign of the relation consistent with the theory. On the other hand, the null $\gamma_S = \gamma_L = 1$ is easily rejected for all specifications (as reflected in the reported p value corresponding to a test of that null), with the estimates of γ_S being in all cases an order of magnitude larger than those of γ_L. In words: changes in expected real interest rate differentials in the near future are associated with much larger variations in the real exchange rate than commensurate changes anticipated to take place in the more distant future. A look at the pattern of γ_S estimates across specifications suggests that the exchange rate elasticity with respect to expected interest rate differentials diminishes with the horizon, with the elasticity with respect to $q_t^L(24)$ being by far the largest one, more than twice the size implied by the benchmark theoretical model. On the other hand, the elasticity of the real exchange rate with respect to expected real interest rate differentials at long horizons is systematically less than one, and significantly so. I refer to this apparent disconnect between theory and empirics as the *forward guidance exchange rate puzzle.*

Based on the R^2 value, reported on the last column of Table 1, the specification for $M = 24$ appears to provide the best fit. The goodness-of-fit is illustrated in Figure 4a which displays q_t together with its corresponding fitted value, based on the estimated regression with $M = 24$. The correlation between the two is 0.91.[6] In order to confirm that such good fit is not just driven by the deterministic trend, Figure 4b shows the estimated non-deterministic component of the real exchange rate, $\widehat{q}_t \equiv q_t - (\alpha_0 + \alpha_1 t + \alpha_2 t^2)$, together with its fitted value $\widehat{q}_t^* \equiv$

[6]Note however that the fit worsens considerably in 2017, with the real exchange rate and its fitted value moving in opposite directions.

$\gamma_S q_t^S(M) + \gamma_L q_t^L(M)$. Their correlation is, again, very high: 0.85. Finally, Figure 5 further decomposes $\widehat{q}_t^*$ into $\widehat{q}_t^{*,S} \equiv \gamma_S q_t^S(M)$ and $\widehat{q}_t^{*,L} \equiv \gamma_L q_t^L(M)$, and points to the dominant role of expected interest rate differentials less than two years ahead in accounting for fluctuations in $\widehat{q}_t$, with the correlation between $\widehat{q}_t$ and $\widehat{q}_t^{*,S}$ being equal to 0.58. By contrast, expected interest rate differentials beyond the two-year horizon seem to play a small role in accounting for short term fluctuations in the real exchange rate, possibly with the exception of the 2015-2017 period. On the other hand, $\widehat{q}_t^{*,L}$ and $\widehat{q}_t$ appear to display a much higher correlation at lower frequencies, with the gradual decline in expected real interest rate differentials beyond two years seemingly driving the gradual real appreciation of the US dollar until 2016. The correlation between $\widehat{q}_t$ and $\widehat{q}_t^{*,L}$ over the full sample period is equal to 0.64.

5.2 Findings: First-Difference Specification

A possible concern with the estimation of (19) is the potential "spurious regression" problem if q_t, $q_t^S(M)$ and $q_t^L(M)$ had a non-stationary component not fully captured by the deterministic function f_t (e.g. a unit root).[7] To get around this problem I re-estimate the equation in first differences, that is:

$$\Delta q_t = \alpha_0 + \alpha_1 t + \gamma_S \Delta q_t^S(M) + \gamma_L \Delta q_t^L(M) + \xi_t \tag{23}$$

The benchmark theoretical model above is consistent with that approach if one treats the stochastic component of the (log) real exchange rate as having a unit root, thus allowing for permanent effects of (at least some) shocks on that variable. In that case, $\{\Delta \widehat{q}_t\}$ is stationary with zero mean, satisfying $\lim_{T\to\infty} \mathbb{E}_t\{\Delta \widehat{q}_{t+T}\} = 0$. Thus, taking first differences on both sides of (20), one obtains the approximate relation

$$\Delta q_t = \Delta q_t^S(M) + \Delta q_t^L(M) + \Delta f_t + \xi_t \tag{24}$$

where $\xi_t \equiv \mathbb{E}_t\{\widehat{q}_{t+T}\} - \mathbb{E}_{t-1}\{\widehat{q}_{t+T}\}$ and where I have used the approximation $\mathbb{E}_{t-1}\{\Delta \widehat{q}_{t+T}\} \simeq 0$ for large T.

Table 2 reports the OLS estimates for γ_S and γ_L in (23) for $M \in \{24, 60, 120\}$. Note that some of the key findings obtained using the level specification re-emerge here. Thus, the null $\gamma_S = \gamma_L = 1$ associated with the benchmark model is systematically rejected, with the γ_S estimates being larger than those of γ_L for the three alternative cutoff horizons considered. Note also that the estimates of γ_S display a declining pattern, and larger than one (though not significantly so) in the case of $M = 24$, suggesting that the real exchange rate is particularly sensitive (possibly overly so) to variations in forecasts of real interest rate differentials over a relatively short horizon. On the other hand, the estimated elasticity with respect to expected real interest rate differentials at longer horizons is close to zero for the three horizons considered (though statistically insignificant only for a horizon beyond ten years, corresponding to $M = 120$).

6 Possible Explanations

In the present section I discuss possible explanations for the forward guidance exchange rate puzzle uncovered above.

[7] I am indebted to Jón Steinsson and Shogo Sakabe for suggesting that alternative approach.

It is worth noting at the outset that some of the solutions to the closed economy forward guidance puzzle found in the literature are unlikely to apply to the case at hand. Those solutions involve a "downward adjustment" in the relevant stochastic discount factor $\Lambda_{t,t+1}$ as a result of a variety of assumptions, including the risk of death (Del Negro et al. (2015) or the risk of lower future consumption in the presence of borrowing constraints (McKay et al. (2016,2017)). The interest parity condition (5), on the other hand, holds independently of the size of the discount factor $\Lambda_{t,t+1}$. Intuitively, the reason is that (5) involves a "contemporaneous arbitrage" between two assets (whose payoffs are subject to the same discounting), as opposed to the "intertemporal arbitrage" associated with the consumer's Euler equation.

On the other hand, deviations from rational expectations that implied some discounting of subjective expectations, $\mathbb{E}_t^*\{q_{t+1}\}$, relative to rational expectations, i.e. $\mathbb{E}_t^*\{q_{t+1}\} = \alpha \mathbb{E}_t\{q_{t+1}\}$, as in the behavioral model of Gabaix (2017) would imply that (6) could be rewritten

$$q_t = \sum_{k=0}^{\infty} \alpha^k \mathbb{E}_t\{r_{t+k}^* - r_{t+k}\}$$

with anticipated changes in the interest rate to be implemented in the distant future predicted to have a more muted effect on the real exchange rate than those implemented at a shorter horizon. Note, however, that such an assumption would not be able to account for the seeming overreaction of the real exchange rate to anticipated changes in interest rate differentials in the near future, as uncovered by some of the estimates reported above.

A possible limitation of (5) is that it is derived as a first order approximation of (3) and hence ignores the potential presence of a (possibly time-varying) risk premium. The latter has often been put forward as a potential explanation for empirical violations of uncovered interest parity, even in the context of an economy with rational expectations and no frictions (e.g. Engel (2016)). Letting $z_{t+1} \equiv i_t^* - i_t + \Delta e_{t+1}$ denote the excess return on foreign versus home one-period bonds, we *define* the associated (foreign exchange) risk premium as $\mathbb{E}_t\{z_{t+1}\}$. Thus, we can generalize (5) and write it as:

$$q_t = r_t^* - r_t + \mathbb{E}_t\{q_{t+1}\} - \mathbb{E}_t\{z_{t+1}\}$$

Accordingly, the real exchange rate equation can now be written as

$$\begin{aligned} q_t &= \sum_{k=0}^{T-1} \mathbb{E}_t\{r_{t+k}^* - r_{t+k}\} - v_t + \mathbb{E}_t\{q_{t+T}\} \\ &\simeq q_t^S(M) + q_t^L(M) - v_t + f_{t+T} \end{aligned}$$

where $v_t \equiv \sum_{k=1}^{T} \mathbb{E}_t\{z_{t+k}\}$ is the expected cumulative risk premium. Under the assumption that the risk premium is orthogonal to interest rate differentials at all leads and lags, OLS estimates of γ_S and γ_L in (19) and (23) should be consistent and thus converge to one asymptotically under the above model. The failure to do so, as shown above, suggests that the joint hypothesis of uncovered interest parity, rational expectations and uncorrelated fluctuations in the risk premium is at odds with the evidence shown above. I view as a challenge for future research to come up with a model of risk premium determination which can be reconciled with the evidence reported above, while preserving the assumption of rational expectations.[8]

While coming up with a structural model that could account for the estimates of (19) and (23) shown above is beyond the scope of the present paper. Nevertheless, next I put forward a simple, reduced form "behavioral" model which may potentially capture the main qualitative features of the evidence, namely, the overreaction (underreaction) to expected interest rate differentials in the near (far) future. The proposed model involves two key modifications from its standard rational expectations counterpart. Firstly, uncovered interest parity is assumed to take the form:

$$i_t = i_t^* + \gamma \mathbb{E}_t^* \{\Delta e_{t+1}\}$$

where $\gamma \in [0, 1)$ captures a downweighting of exchange rate component in the expected return on the foreign bond, while $\mathbb{E}_t^*$ denotes the subjective expectational operator. Note that we can rewrite the previous condition in real terms

$$r_t = r_t^* + \gamma \mathbb{E}_t^* \{\Delta q_{t+1}\}$$

The second assumption requires that $\mathbb{E}_t^*\{q_{t+1}\} = \alpha \mathbb{E}_t\{q_{t+1}\}$ where $\alpha \in [0, 1)$. In words, agents expect the real exchange rate to revert faster than implied by rational expectations to its trend value, which here I normalize to zero for convenience. Combining the two previous assumptions yields the real exchange equation

$$\begin{aligned} q_t &= \frac{1}{\gamma}(r_t^* - r_t) + \alpha \mathbb{E}_t\{q_{t+1}\} \\ &= \frac{1}{\gamma} \sum_{k=0}^{\infty} \alpha^k \mathbb{E}_t\{r_{t+k}^* - r_{t+k}\} \end{aligned}$$

with $\gamma < 1$ thus accounting for the overreaction to near term (rationally) expected interest rate differentials and $\alpha < 1$ implying an eventual underreaction to the same variable for sufficiently long horizons. Providing possible micro-foundations for the above reduced form model, based on either information frictions or behavioral assumptions, is left for future research.

7 Concluding Comments

The present paper has analyzed the effectiveness of forward guidance policies in open economies, focusing on the role played by the exchange rate in their transmission. Uncovered interest parity implies that the current exchange rate is determined by current and expected future interest rate differentials, *undiscounted*. Accordingly, in partial equilibrium (i.e. ignoring the feedback effects on inflation) the effect on the current exchange rate of a given anticipated change in the interest rate does not decline with the horizon of its implementation.

In general equilibrium, and using a simple New Keynesian model of a small open economy as a reference framework, I show that the size of the effect of forward guidance policies on the current exchange rate, as well as on output and inflation, is larger the longer is the horizon of

[8] See Bacchetta and van Wincoop (2018) for an attempt to provide such microfoundations using a model with portfolio adjustment costs.

implementation of the announced policies. Under my baseline calibration, the size of the effects of forward guidance policies on some variables (output, nominal exchange rate) is increasing in the degree of openness, but it is decreasing for some other variables (e.g. real exchange rate).

Using data on the euro-dollar real exchange rate and market-based forecasts of real interest rate differentials between the U.S. and the euro area, I provide evidence that conflicts with the prediction of undiscounted effects of anticipated real interest rate differentials. In particular, expectations of interest rate differentials in the near (distant) future appear to have much larger (smaller) effects than is implied by the theory, an observation which I refer to as the *forward guidance exchange rate puzzle*. Further research to provide a theoretical explanation to that puzzle seems warranted.

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Table 1. Empirical Exchange Rate Equation

Level specification

	$\widehat{\gamma}_S$	$\widehat{\gamma}_L$	p	R^2
M=24	2.74^{**} (0.24)	0.26^{**} (0.03)	0.00	0.83
M=60	1.70^{**} (0.16)	0.18^{**} (0.03)	0.00	0.81
M=120	0.98^{**} (0.13)	0.12^{**} (0.04)	0.00	0.77

Note: The table reports the OLS estimates of γ_S and γ_L in the regression equation

$$q_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \gamma_S q_t^S(M) + \gamma_L q_t^L(M) + \varepsilon_t$$

using monthly data spanning the sample period 2004:8-2017:12. The p statistic corresponds to the null hypothesis $\gamma_S = \gamma_L = 1$. Two asterisks indicate significance at 5 percent level.

Table 2. Empirical Exchange Rate Equation				
First difference specification				
	$\widehat{\gamma}_S$	$\widehat{\gamma}_L$	p	R^2
M=24	1.40^{**} (0.28)	0.10^{**} (0.03)	0.00	0.14
M=60	0.82^{**} (0.18)	0.08^{*} (0.03)	0.00	0.13
M=120	0.65^{**} (0.13)	0.01 (0.04)	0.00	0.14

Note: The table reports the OLS estimates of γ_S and γ_L in the regression equation

$$\Delta q_t = \alpha_0 + \alpha_1 t + \gamma_S \Delta q_t^S(M) + \gamma_L \Delta q_t^L(M) + \varepsilon_t$$

using monthly data spanning the sample period 2004:8-2017:12. The p statistic corresponds to the null hypothesis $\gamma_S = \gamma_L = 1$. Two asterisks indicate significance at 5 percent level.

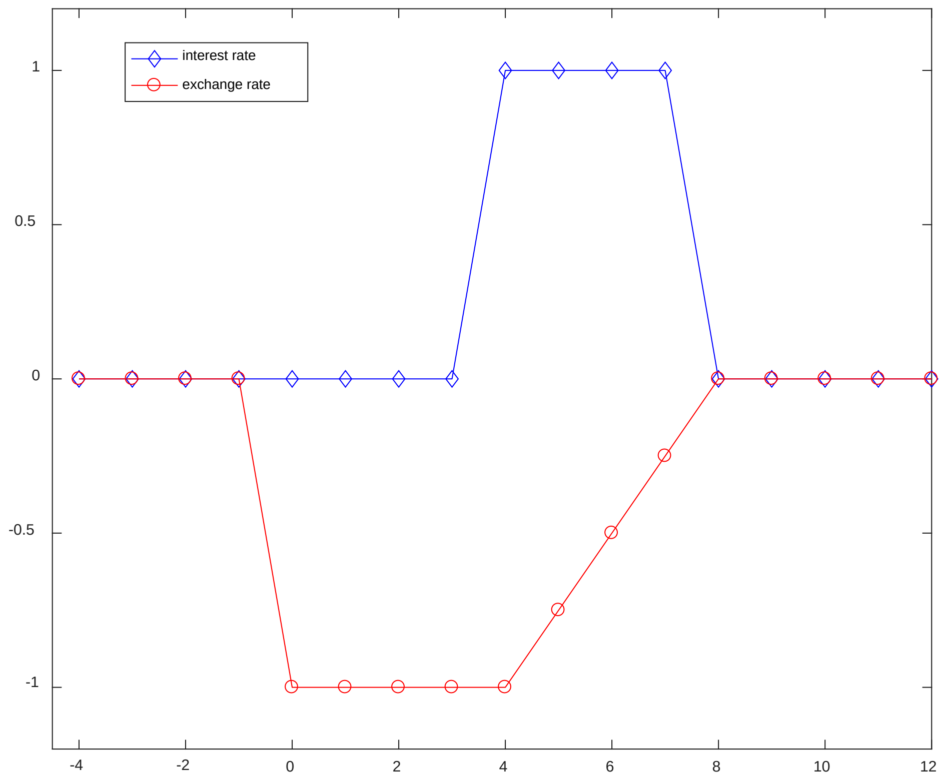

Figure 1. Forward Guidance and the Exchange Rate: Partial Equilibrium

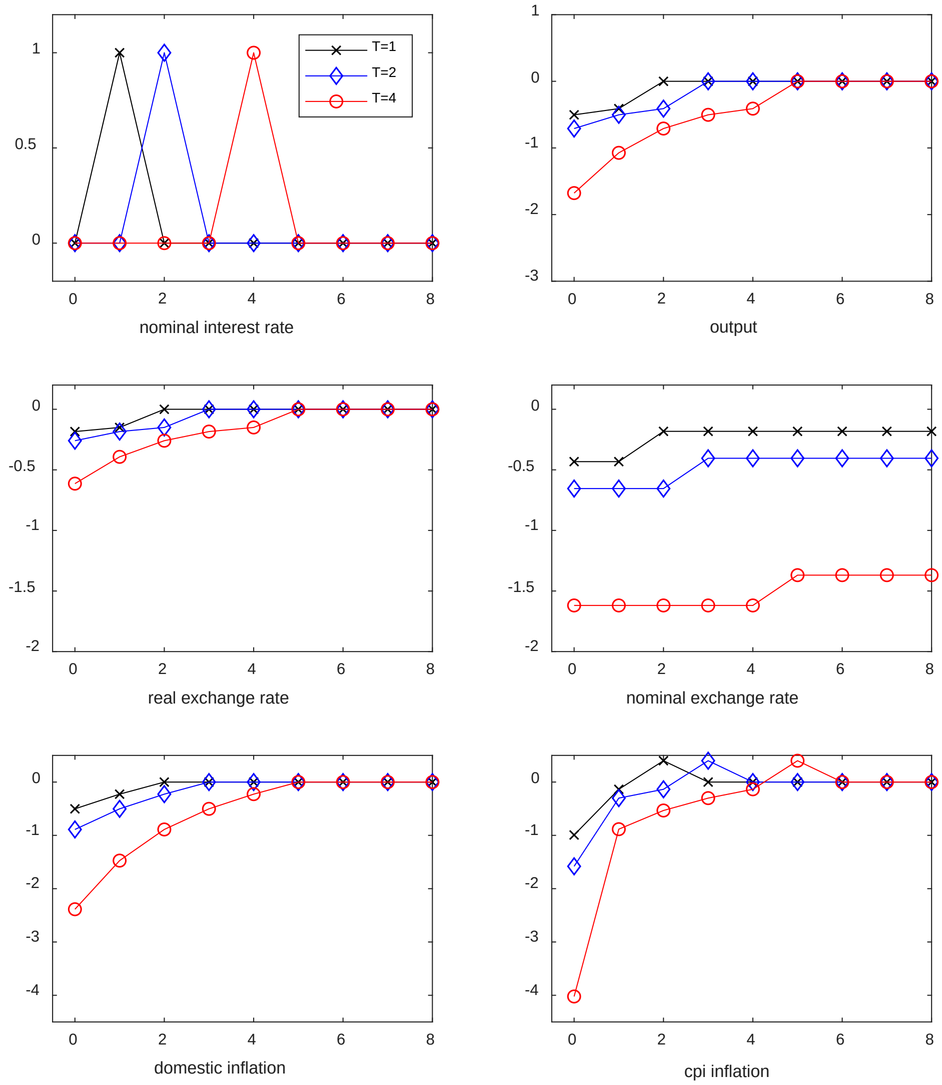

Figure 2. Forward Guidance in the Open Economy: The Role of the Horizon

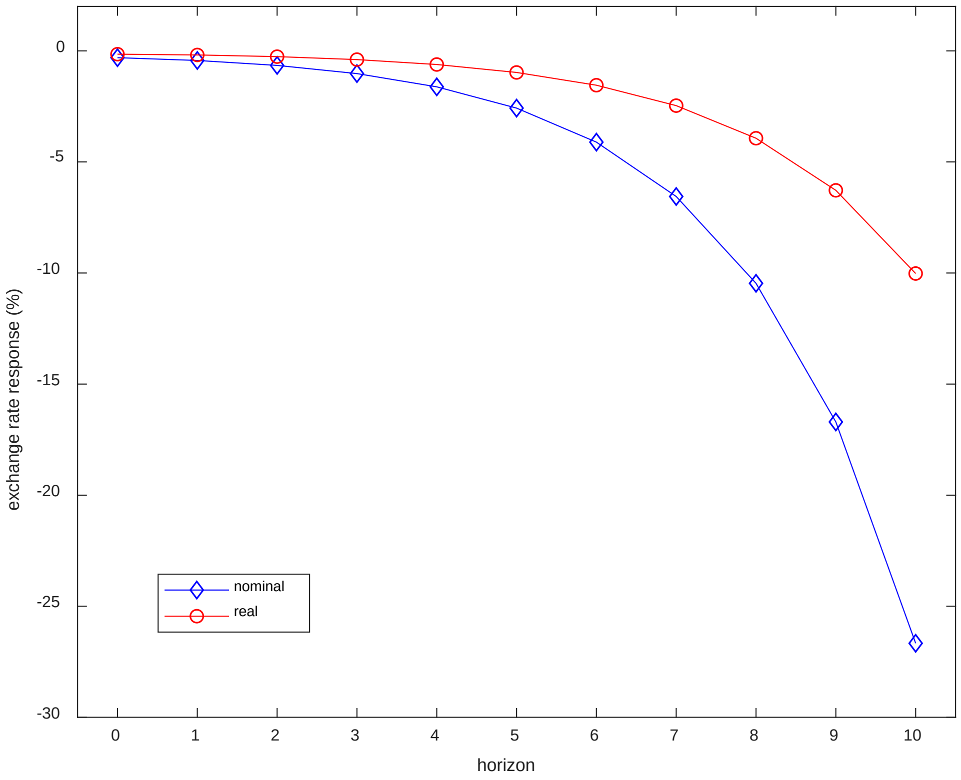

Figure 3. The Forward Guidance Exchange Rate Puzzle

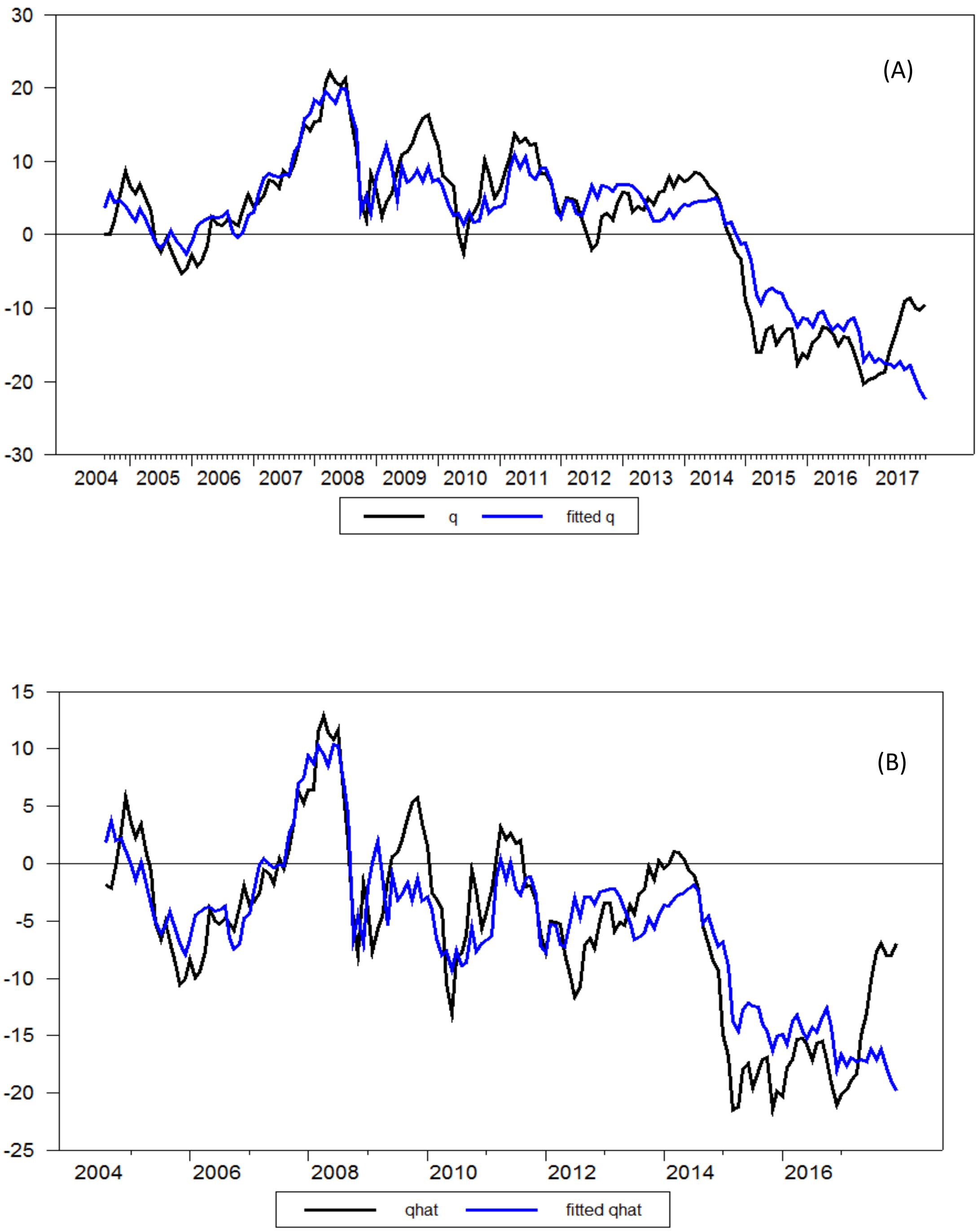

Figure 4. Expected Real Interest Rate Differentials and the Real Exchange Rate (M=24)

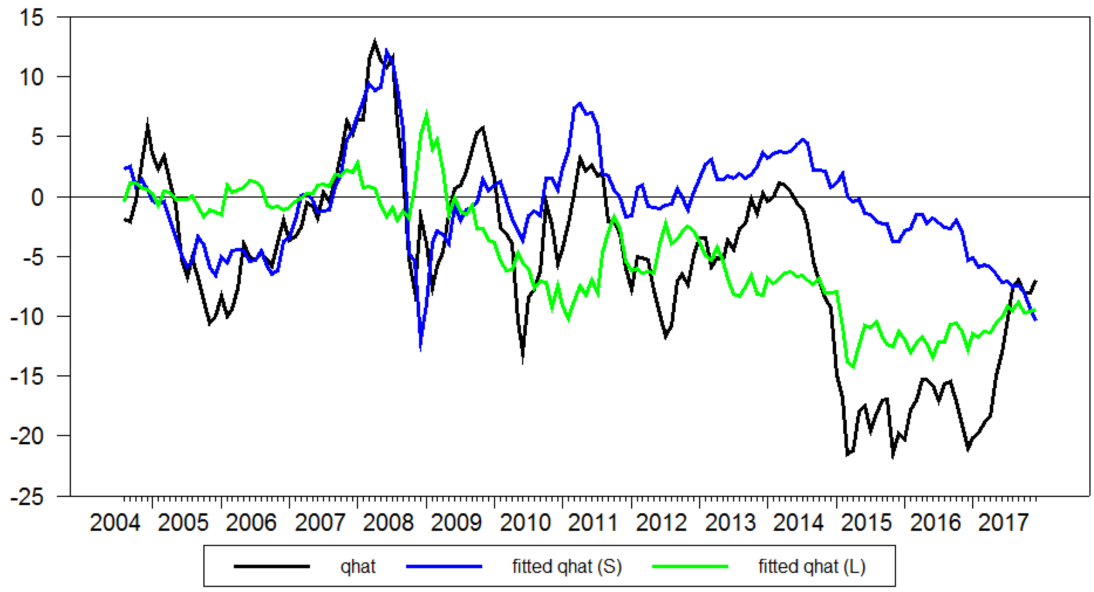

Figure 5. Short-run and Long-run Components of Expected Real Interest Rate Differentials and the Real Exchange Rate (M=24)