Stagnation Traps

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We provide a Keynesian growth theory in which pessimistic expectations can lead to very persistent, or even permanent, slumps characterized by high unemployment and weak growth. We refer to these episodes as *stagnation traps*, because they consist in the joint occurrence of a liquidity and a growth trap. In a stagnation trap, the central bank is unable to restore full employment because weak growth depresses aggregate demand and pushes the policy rate against the zero lower bound, while growth is weak because low aggregate demand results in low profits, limiting firms' investment in innovation. Aggressive policies aiming at restoring growth, such as subsidies to investment, can successfully lead the economy out of a stagnation trap by generating a regime shift in agents' growth expectations.

Key words: Secular stagnation, Liquidity traps, Endogenous growth, Monetary policy, Multiple equilibria, Hysteresis.

JEL Codes: E32, E43, E52, O31, O42

1. INTRODUCTION

Can insufficient aggregate demand lead to economic stagnation, *i.e.* a protracted period of high unemployment and low growth? Economists have been concerned with this question at least since the Great Depression,[1] but interest in this topic has recently reemerged motivated by the two decades-long slump affecting Japan since the early 1990s, as well as by the slow recoveries experienced by the U.S. and the Euro area in the aftermath of the 2008 financial crisis. In fact, all these long-lasting slump episodes have been characterized by the typical symptoms of liquidity traps: high unemployment, low real interest rates and nominal policy rates close to their zero lower bound (Table 1). Moreover, these episodes have been marked by slowdowns in investment, including investment in productivity enhancing activities such as R&D,[2] resulting in weak labour productivity growth and in large deviations of output from pre-slump trends (Figure 1).

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1. See Hansen (1939) for an early discussion of the relationship between aggregate demand, unemployment and technical progress.

2. As shown in Table 1, all the three slump episodes exhibit a fall in R&D intensity, defined as the ratio of investment in R&D to the R&D stock. As discussed in chapter 10 of Barro and Sala-i Martin (2004), R&D intensity is the key determinant of productivity growth in the seminal frameworks developed by Aghion and Howitt (1992), Grossman and Helpman (1991) and Romer (1990), as well as in the model that we present below. Kung and Schmid (2015) provide empirical evidence on the link between R&D intensity and productivity growth. Using US data for

TABLE 1
Japan, U.S., Euro area: before/during slump

	Japan		U.S.		Euro area	
	1982–1991	1992–2014	1998–2007	2008-2014	1999–2007	2008–2014
Policy rate	4.25	0.65	3.68	0.35	2.91	1.15
Unemployment rate	2.49	4.10	4.90	7.88	8.67	10.37
Labour productivity growth	4.03	1.94	2.61	1.24	1.27	0.66
R&D intensity	24.19	18.15	18.56	17.50	18.64	17.92
Real rate - short	2.64	0.23	1.44	−1.57	1.46	−0.38
Real rate - long	4.57	1.43	2.50	0.87	2.78	0.79

Notes: All the values are subsample averages expressed in percentage points. Labour productivity is real GDP/hours worked. R&D intensity is the ratio of investment in R&D to R&D stock for the business sector. The real rate short and long are, respectively, the policy rate and the rate on ten years government bonds deflated by expected inflation. Appendix G contains a detailed description of data sources.

In this article we present a theory in which very persistent, or even permanent, slumps characterized by high unemployment and low growth are possible. Our idea is that the connection between depressed demand, high unemployment and weak growth, far from being casual, can be the result of a two-way interaction. On the one hand, unemployment and weak aggregate demand can have a negative impact on firms' investment in innovation, and result in low growth. On the other hand, low growth can depress aggregate demand, pushing real interest rates down and nominal rates close to their zero lower bound, thus undermining the central bank's ability to maintain full employment by cutting policy rates.

To formalize this insight, and explore its policy implications, we propose a *Keynesian growth* framework that sheds light on the interactions between endogenous growth, unemployment and monetary policy. The backbone of our framework is a standard model of vertical innovation, in the spirit of Aghion and Howitt (1992). We modify this classic endogenous growth framework in two directions. First, we introduce nominal wage rigidities, which create the possibility of involuntary unemployment, and give rise to a channel through which monetary policy can affect the real economy. Secondly, we take into account the zero lower bound on the nominal interest rate, which limits the central bank's ability to stabilize the economy with conventional monetary policy. Our theory thus combines the Keynesian insight that unemployment can arise due to weak aggregate demand, with the notion, developed by the endogenous growth literature, that productivity growth is the result of investment in innovation by profit-maximizing agents. We show that the interaction between these two forces can give rise to prolonged periods of high unemployment and low growth. We refer to these episodes as *stagnation traps*, because they consist in the joint occurrence of a liquidity and a growth trap.

In our economy there are two types of agents: firms and households. Firms' investment in innovation determines endogenously the growth rate of productivity and potential output of our economy. As in the standard models of vertical innovation, firms invest in innovation to gain a monopoly position, and so their investment in innovation is positively related to profits.[3] Through

the period 1953-2008, these authors show that R&D intensity predicts productivity, output and consumption growth at horizons of one to five years.

3. This is the market size effect highlighted by the endogenous growth literature. See Acemoglu and Linn (2004) and Bustos (2011) for empirical evidence on the positive response of firms' investment in productivity enhancing activities to exogenous increases in market size. Relatedly, Comin and Gertler (2006) and Barlevy (2007) show that in the U.S. aggregate business investment in R&D is pro-cyclical. Moreover, Barlevy (2007) provides evidence in favour of the notion that firms investment in R&D is pro-cyclical because the profits linked to innovation are higher in booms than in busts.
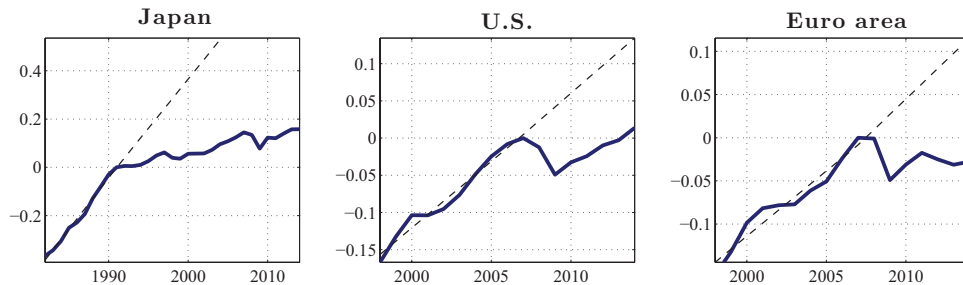
FIGURE 1

Real GDP per capita

Notes: Series shown in logs, undetrended, centered around 1991 for Japan, and 2007 for U.S. and Euro area. Gross domestic product, constant prices, from IMF World Economic Outlook, divided by total population from World Bank World Development Indicators. The linear trend is computed over the period 1982-1991 for Japan, and 1998-2007 for United States and Euro area.

this channel, a slowdown in aggregate demand that leads to a fall in profits, also reduces investment in innovation and the growth rate of the economy. Households supply labour and consume, and their intertemporal consumption pattern is characterized by the traditional Euler equation. The key aspect is that households' current demand for consumption is affected by the growth rate of potential output, because productivity growth is one of the determinants of households' future income. In particular, a fall in the expected growth rate of potential output is associated with lower future income and a reduction in current aggregate demand.[4]

This two-way interaction between productivity growth and aggregate demand results in two steady states. First, there is a full employment steady state, in which the economy operates at potential and productivity growth is robust. However, our economy can also find itself in an unemployment steady state. In the unemployment steady state aggregate demand and firms' profits are low, resulting in low investment in innovation and weak productivity growth. Moreover, monetary policy is not able to bring the economy at full employment, because the low growth of potential output pushes the interest rate against its zero lower bound. Hence, the unemployment steady state can be thought of as a stagnation trap.

Expectations, or animal spirits, are crucial in determining which equilibrium will be selected. For instance, when agents expect growth to be low, expectations of low future income reduce aggregate demand, lowering firms' profits and their investment, thus validating the low growth expectations. Through this mechanism, pessimistic expectations can generate a permanent liquidity trap with involuntary unemployment and stagnation. We also show that, aside from permanent traps, pessimistic expectations can give rise to stagnation traps of finite, but arbitrarily long, expected duration. This does not mean, however, that the fundamentals of the economy are unimportant. In fact, we find that expectations-driven permanent stagnation traps can arise only if fundamentals are such that the real interest rate in the full employment steady state is low enough. This result suggests that structural factors that put downward pressure on the real interest rate, such as population ageing, might also expose the economy to the risk of self-fulfilling stagnation traps.

In the last part of the article, we derive some policy implications. We start by proving that stagnation traps can take place even when monetary policy is conducted optimally. First, we show

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4. Blanchard *et al*. (2017) provide evidence on the link between expectations of future productivity growth and current aggregate demand. Using U.S. data for the period 1991–2014, they show that downward revisions in forecasts of future productivity growth are associated with unexpected falls in consumption and investment.

that a central bank operating under commitment can design interest rate rules that eliminate the possibility of expectations-driven stagnation traps. However, we then show that if the central bank lacks the ability to commit to its future actions stagnation traps are possible even when interest rates are set optimally. Our framework thus suggests that, due credibility issues, monetary policy alone is not sufficient to prevent the economy from falling into a stagnation trap.

We then turn to policies aiming at sustaining the growth rate of potential output, for instance, by subsidizing investment in productivity-enhancing activities. While these policies have been studied extensively in the context of the endogenous growth literature, here we show that they operate not only through the supply side of the economy, but also by stimulating aggregate demand during a liquidity trap. In fact, we find that an appropriately designed subsidy to innovation can push the economy out of a stagnation trap and restore full employment. However, in order to be effective, the policy intervention has to be sufficiently aggressive, so as to generate a regime shift in agents' expectations about future growth.

This article is related to several strands of the literature. First, our article is related to the recent literature on secular stagnation (Eggertsson and Mehrotra, 2014; Caballero *et al.*, 2015; Michau, 2015; Asriyan *et al.*, 2016; Eggertsson *et al.*, 2016, 2017; Caballero and Farhi, 2017). This literature builds on Hansen's secular stagnation hypothesis (Hansen, 1939), that is the idea that a drop in the natural real interest rate might push the economy in a long-lasting liquidity trap, characterized by the absence of any self-correcting force to restore full employment. Hansen formulated this concept inspired by the U.S. Great Depression, but recently some commentators, most notably Summers (2013) and Krugman (2013), have revived the idea of secular stagnation to rationalize the long duration of the Japanese liquidity trap and the slow recoveries characterizing the U.S. and the Euro area after the 2008 financial crisis. Caballero and Farhi (2017) and Caballero *et al.* (2015) conjecture that the secular decline in the real interest rate in the last decade is the byproduct of a shortage of safe assets. In the overlapping-generations model studied by Eggertsson and Mehrotra (2014) and Eggertsson *et al.* (2016, 2017) permanent liquidity traps are the outcome of shocks that alter households' lifecycle saving decisions. Asriyan *et al.* (2016) find that a permanent liquidity trap can arise after the crash of a bubble that wipes out a large fraction of the collateral present in the economy. Michau (2015) shows how secular stagnation can arise with infinitely-lived agents when households derive utility from wealth. We see our article being complementary to these contributions, with the distinctive feature being that the fall in the real natural interest rate that generates a permanent liquidity trap originates from an endogenous drop in investment in innovation and productivity growth.

More broadly, the article contributes to the large literature studying liquidity traps. A first strand of this literature has focused on liquidity traps driven by fundamental factors, such as preference shocks, as in Krugman (1998), Eggertsson and Woodford (2003) and Eggertsson (2008), or financial shocks leading to tighter access to credit, as in Eggertsson and Krugman (2012) and Guerrieri and Lorenzoni (2017). The other approach in modelling liquidity traps is based on self-fulfilling expectations. Benhabib *et al.* (2001) show that permanent liquidity traps arising from self-fulfilling expectations of future low inflation are possible when monetary policy is conducted according to a Taylor rule. In a similar setting, Mertens and Ravn (2014) study the role of fiscal policy. Our contribution belongs to the self-fulfilling approach in modelling liquidity traps with some key differences. In Benhabib *et al.* (2001) permanent liquidity traps do not entail a drop in the real interest rate, and are characterized by normal growth, positive real rates and substantial deflation. Instead, in our model liquidity traps are accompanied by drops in real rates and in productivity growth, and can be consistent with negative real interest rates, and low, but positive, inflation. Moreover, we show that permanent liquidity traps due to self-fulfilling expectations can take place even when monetary policy is optimally conducted, as long as the central bank operates under discretion.

As in the seminal frameworks developed by Aghion and Howitt (1992), Grossman and Helpman (1991) and Romer (1990), long-run growth in our model is the result of investment in innovation by profit-maximizing agents. A small, but growing, literature has considered the interactions between short-run fluctuation and long run growth in this class of models (Fatas, 2000; Comin and Gertler, 2006; Aghion *et al*., 2009, 2014, 2010; Nuño, 2011; Queraltó, 2013; Anzoategui *et al*., 2015; Bianchi *et al*., 2015). However, to the best of our knowledge, we are the first ones to study monetary policy in an endogenous growth model featuring a zero lower bound constraint on the policy rate, and to show that the interaction between endogenous growth and liquidity traps creates the possibility of long periods of stagnation.

Moreover, our article is linked to the literature on fluctuations driven by confidence shocks and sunspots. Some examples of this vast literature are Geanakoplos and Polemarchakis (1986), Kiyotaki (1988), Benhabib and Farmer (1994) and Farmer (2012). We contribute to this literature by describing a new channel through which pessimistic expectations can give rise to economic stagnation.

Finally, our article is related to the empirical literature on the slump in business investment that has characterized most advanced economies during the liquidity traps arising in the aftermath of the 2008 financial crisis. The studies conducted by Bussière *et al*. (2015) and IMF (2015), using aggregate investment data for a sample of advanced economies, suggest that expectations of low future aggregate demand are the main culprit behind the post-crisis slowdown in investment. Interestingly, the slowdown has also affected investment in innovation activities. Anzoategui *et al*. (2015) and Schmitz (2014) show that R&D investment has declined since the crisis, using data, respectively, from the U.S. and Spain, while Corrado *et al*. (2016) and Garcia-Macia (2015) document a similar trend for investment in intangible capital. Our model rationalizes these facts. Our model is also consistent with the evidence provided by Blanchard *et al*. (2017), who find that expectations about weak future productivity growth have played a key role in depressing aggregate demand during the post-crisis years.

The rest of the article is composed of four sections. Section 2 describes the baseline model. Section 3 shows that pessimistic expectations can generate arbitrarily long-lasting stagnation traps. Section 4 extends the baseline model in several directions. Section 5 discusses some policy implications. Section 6 concludes.

2. BASELINE MODEL

In this section, we lay down our *Keynesian growth* framework. The economy has two key elements. First, the rate of productivity growth is endogenous, and it is the outcome of firms' investment in innovation. Secondly, the presence of nominal rigidities imply that output can deviate from its potential level, and that monetary policy can affect real variables. As we will see, the combination of these two factors opens the door to fluctuations driven by shocks to agents' expectations. To emphasize this striking feature of the economy, in what follows we will abstract from any fundamental shock.[5] Moreover, in order to deliver transparently our key results, in this section we will make some simplifying assumptions that enhance the tractability of the model. These assumptions will be relaxed in Section 4.

We consider an infinite-horizon closed economy. Time is discrete and indexed by $t \in \{0, 1, 2, ...\}$. The economy is inhabited by households, firms, and by a central bank that sets monetary policy.

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5. To be clear, we believe that the study of fluctuations driven by fundamental shocks in the context of our model is a promising research area, that we plan to pursue in the future.

2.1. Households

There is a continuum of measure one of identical households deriving utility from consumption of a homogeneous "final" good. The lifetime utility of the representative household is:

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\left(\frac{C_t^{1-\sigma}-1}{1-\sigma}\right)\right], \tag{2.1}$$

where C_t denotes consumption, $0<\beta<1$ is the subjective discount factor, σ is the inverse of the elasticity of intertemporal substitution, and $E_t[\cdot]$ is the expectation operator conditional on information available at time t.

Each household is endowed with one unit of labour and there is no disutility from working. However, due to the presence of nominal wage rigidities to be described below, a household might be able to sell only $L_t<1$ units of labour on the market. Hence, when $L_t=1$ the economy operates at full employment, while when $L_t<1$ there is involuntary unemployment, and the economy operates below capacity.

Households can trade in one-period, non-state contingent bonds b_t. Bonds are denominated in units of currency and pay the nominal interest rate i_t. Moreover, households own all the firms and each period they receive dividends d_t from them.[6]

The intertemporal problem of the representative household consists in choosing C_t and b_{t+1} to maximize expected utility, subject to a no-Ponzi constraint and the budget constraint:

$$P_tC_t+\frac{b_{t+1}}{1+i_t}=W_tL_t+b_t+d_t,$$

where P_t is the nominal price of the final good, b_{t+1} is the stock of bonds purchased by the household in period t, and b_t is the payment received from its past investment in bonds. W_t denotes the nominal wage, so that W_tL_t is the household's labour income.

The optimality conditions are:

$$\lambda_t=\frac{C_t^{-\sigma}}{P_t} \tag{2.2}$$

$$\lambda_t=\beta(1+i_t)E_t\left[\lambda_{t+1}\right], \tag{2.3}$$

where λ_t denotes the Lagrange multiplier on the budget constraint, and the transversality condition $\lim_{s\to\infty}E_t\left[\frac{b_{t+s}}{(1+i_t)...(1+i_{t+s})}\right]=0$.

2.2. Final good production

The final good is produced by competitive firms using labour and a continuum of measure one of intermediate inputs x_j, indexed by $j\in[0,1]$. Denoting by Y_t the output of final good, the production function is:

$$Y_t=L_t^{1-\alpha}\int_0^1 A_{jt}^{1-\alpha}x_{jt}^{\alpha}dj, \tag{2.4}$$

where $0<\alpha<1$, and A_{jt} is the productivity, or quality, of input j.[7]

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6. To streamline the exposition, in the main text we consider a cashless economy. In Appendix B we show that introducing money does not affect our results.

7. More precisely, for every good j, A_{jt} represents the highest quality available. In principle, firms could produce using a lower quality of good j. However, as in Aghion and Howitt (1992) and Grossman and Helpman (1991), the structure of the economy is such that in equilibrium only the highest quality version of each good is used in production.

Profit maximization implies the demand functions:

$$P_t(1-\alpha)L_t^{-\alpha}\int_0^1 A_{jt}^{1-\alpha}x_{jt}^{\alpha}dj=W_t \tag{2.5}$$

$$P_t\alpha L_t^{1-\alpha}A_{jt}^{1-\alpha}x_{jt}^{\alpha-1}=P_{jt}, \tag{2.6}$$

where P_{jt} is the nominal price of intermediate input j. Due to perfect competition, firms in the final good sector do not make any profit in equilibrium.

2.3.   *Intermediate goods production and profits*

In every industry j producers compete as price-setting oligopolists. One unit of final output is needed to manufacture one unit of intermediate good, regardless of quality, and hence every producer faces the same marginal cost P_t. Our assumptions about the innovation process will ensure that in every industry there is a single leader able to produce good j of quality A_{jt}, and a fringe of competitors which are able to produce a version of good j of quality A_{jt}/γ. The parameter $\gamma > 1$ captures the distance in quality between the leader and the followers. Given this market structure, it is optimal for the leader to capture the whole market for good j by charging the price:[8]

$$P_{jt}=\xi P_t, \quad \text{where} \quad \xi\equiv\min\left(\gamma^{1-\alpha},\frac{1}{\alpha}\right)>1. \tag{2.7}$$

This expression implies that the leader charges a constant markup ξ over its marginal cost. Intuitively, $1/\alpha$ is the markup that the leader would choose in absence of the threat of entry from the fringe of competitors. Instead, $\gamma^{1-\alpha}$ is the highest markup that the leader can charge without losing the market to its competitors. It follows that if $1/\alpha \leq \gamma^{1-\alpha}$ then the leader will charge the unconstrained markup $1/\alpha$, otherwise it will set a markup equal to $\gamma^{1-\alpha}$ to deter entry.[9] In any case, the leader ends up satisfying all the demand for good j from final good producers.

Equations (2.6) and (2.7) imply that the quantity produced of a generic intermediate good j is:

$$x_{jt}=\left(\frac{\alpha}{\xi}\right)^{\frac{1}{1-\alpha}}A_{jt}L_t. \tag{2.8}$$

Combining equations (2.4) and (2.8) gives:

$$Y_t=\left(\frac{\alpha}{\xi}\right)^{\frac{\alpha}{1-\alpha}}A_tL_t, \tag{2.9}$$

where $A_t\equiv\int_0^1 A_{jt}dj$ is an index of average productivity of the intermediate inputs. Hence, production of the final good is increasing in the average productivity of intermediate goods and in aggregate employment. Moreover, the profits earned by the leader in sector j are given by:

$$P_{jt}x_{jt}-P_tx_{jt}=P_t\varpi A_{jt}L_t,$$

where $\varpi\equiv(\xi-1)(\alpha/\xi)^{1/(1-\alpha)}$. According to this expression, a leader's profits are increasing in the productivity of its intermediate input and on aggregate employment. The dependence of

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8. For a detailed derivation see, for instance, the appendix to chapter 7 of Barro and Sala-i Martin (2004).

9. To be clear, the results the we present below do not depend at all on whether the leader charges the unconstrained or the constrained markup.

profits from aggregate employment is due to the presence of a market size effect. Intuitively, high employment is associated with high production of the final good and high demand for intermediate inputs, leading to high profits in the intermediate sector.

2.4.　*Research and innovation*

There is a large number of entrepreneurs that can attempt to innovate upon the existing products. A successful entrepreneur researching in sector j discovers a new version of good j of quality γ times greater than the best existing version, and becomes the leader in the production of good j.[10]

Entrepreneurs can freely target their research efforts at any of the continuum of intermediate goods. An entrepreneur that invests I_{jt} units of the final good to discover an improved version of product j innovates with probability:

$$\mu_{jt} = \min\left(\frac{\chi I_{jt}}{A_{jt}}, 1\right),$$

where the parameter $\chi > 0$ determines the productivity of research.[11] The presence of the term A_{jt} captures the idea that innovating upon more advanced and complex products requires a higher investment, and ensures stationarity in the growth process. We consider time periods small enough so that the probability that two or more entrepreneurs discover contemporaneously an improved version of the same product is negligible. This assumption implies, mimicking the structure of equilibrium in continuous-time models of vertical innovation such as Aghion and Howitt (1992) and Grossman and Helpman (1991), that the probability that a product is improved is the sum of the success probabilities of all the entrepreneurs targeting that product.[12] With a slight abuse of notation, we then denote by μ_{jt} the probability that an improved version of good j is discovered at time t.

We now turn to the reward from research. A successful entrepreneur obtains a patent and becomes the monopolist during the following period. For simplicity, in our baseline model we assume that the monopoly position of an innovator lasts a single period, after which the patent is allocated randomly to another entrepreneur.[13] The value $V_t(\gamma A_{jt})$ of becoming a leader in sector j and attaining productivity γA_{jt} is given by:

$$V_t(\gamma A_{jt}) = \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t} P_{t+1} \varpi \gamma A_{jt} L_{t+1}\right]. \tag{2.10}$$

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10. As in Aghion and Howitt (1992) and Grossman and Helpman (1991), all the research activities are conducted by entrants. Incumbents do not perform any research because the value of improving over their own product is smaller than the profits that they would earn from developing a leadership position in a second market. We present a version of the model with innovation by incumbents in Section 4.3.

11. Our formulation of the innovation process follows closely chapter 7 of Barro and Sala-i Martin (2004) and Howitt and Aghion (1998). An alternative is to assume, as in Grossman and Helpman (1991), that labour is used as input into research. This alternative assumption would lead to identical results, since ultimately output in our model is fully determined by the stock of knowledge and aggregate labour.

12. Following Aghion and Howitt (2009), we could have assumed that every period only a single entrepreneur can invest in research in a given sector. This alternative assumption would lead to identical equilibrium conditions.

13. This assumption, which is drawn from Aghion and Howitt (2009) and Acemoglu *et al*. (2012), simplifies considerably the analysis. In Section 4.3, we show that our results extend to a setting in which, more conventionally, the innovator's monopoly position is terminated when a new version of the product is discovered.

$V_t(\gamma A_{jt})$ is equal to the expected profits to be gained in period $t+1$, $P_{t+1}\varpi\gamma A_{jt}L_{t+1}$, discounted using the households' discount factor $\beta\lambda_{t+1}/\lambda_t$. Profits are discounted using the households' discount factor because entrepreneurs finance their investment in innovation by selling equity claims on their future profits to the households. Competition for households' funds leads entrepreneurs to maximize the value to the households of their expected profits. Hence, the expected returns from investing in research are increasing in future profits and decreasing in the cost of funds, captured by the households' discount factor.

Free entry into research implies that expected profits from researching cannot be positive, so that for every good j:[14]

$$P_t \geq \frac{\chi}{A_{jt}}V_t(\gamma A_{jt}),$$

holding with equality if some research is conducted aiming at improving product j.[15] Combining this condition with expression (2.10) gives:

$$\frac{P_t}{\chi} \geq \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}P_{t+1}\gamma\varpi L_{t+1}\right].$$

Notice that this condition does not depend on any variable specific to sector j, because the higher profits associated with more advanced sectors are exactly offset by the higher research costs. As is standard in the literature, we then focus on symmetric equilibria in which the probability of innovation is the same in every sector, so that $\mu_{jt}=\chi I_{jt}/A_{jt}=\mu_t$ for every j. We can then summarize the equilibrium in the research sector with the complementary slackness condition:

$$\mu_t\left(\frac{P_t}{\chi} - \beta E_t\left[\frac{\lambda_{t+1}}{\lambda_t}P_{t+1}\gamma\varpi L_{t+1}\right]\right)=0. \tag{2.11}$$

Intuitively, either some research is conducted, so that $\mu_t > 0$, and free entry drives expected profits in the research sector to zero, or the expected profits from researching are negative and no research is conducted, so that $\mu_t = 0$.

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14. To derive this condition, consider that an entrepreneur that invests I_{jt} in research has a probability $\chi I_{jt}/A_{jt}$ of becoming a leader which carries value $V_t(\gamma A_{jt})$. Hence, the expected return from this investment is $\chi I_{jt}V_t(\gamma A_{jt})/A_{jt}$. Since the investment costs $P_t I_{jt}$, the free entry condition in the research sector implies:

$$P_t I_{jt} \geq \frac{\chi I_{jt}}{A_{jt}}V_t(\gamma A_{jt}).$$

Simplifying we obtain the expression in the main text.

15. It is customary in the endogenous growth literature to restrict attention to equilibria in which in every period a positive amount of research is targeted towards every intermediate good. We take a slightly more general approach, and allow for cases in which expected profits from research are too low to induce entrepreneurs to invest in innovation. This degree of generality will prove important when we will discuss the policy implications of the framework.

2.5. *Aggregation and market clearing*

Market clearing for the final good implies:[16]

$$Y_t - \int_0^1 x_{jt}\,dj = C_t + \int_0^1 I_{jt}\,dj, \tag{2.12}$$

where the left-hand side of this expression is the GDP of the economy, while the right-hand side captures the fact that GDP is either consumed or invested in research. Using equations (2.8) and (2.9) we can write GDP as:

$$Y_t - \int_0^1 x_{jt}\,dj = \Psi A_t L_t, \tag{2.13}$$

where $\Psi \equiv (\alpha/\xi)^{\alpha/(1-\alpha)}(1-\alpha/\xi)$.

   The assumption of a unitary labour endowment implies that $L_t \leq 1$. Since labour is supplied inelastically by the households, $1-L_t$ can be interpreted as the unemployment rate. For future reference, when $L_t = 1$ we say that the economy is operating at full employment, while when $L_t < 1$ the economy operates below capacity.

   Long run growth in this economy takes place through increases in the quality of the intermediate goods, captured by increases in the productivity index A_t. By the law of large numbers, a fraction μ_t of intermediate products is improved every period. Hence, A_t evolves according to:

$$A_{t+1} = \mu_t \gamma A_t + (1-\mu_t)A_t,$$

while the (gross) rate of productivity growth is:

$$g_{t+1} \equiv \frac{A_{t+1}}{A_t} = \mu_t(\gamma - 1) + 1. \tag{2.14}$$

Recalling that $\mu_t = \chi I_{jt}/A_{jt}$, this expression implies that higher investment in research in period t is associated with faster productivity growth between periods t and $t+1$. More precisely, the rate of productivity growth is determined by the ratio of investment in innovation $I_{j,t}$ over the existing stock of knowledge $A_{j,t}$. In turn, the stock of knowledge depends on all past investment in innovation, that is on the R&D stock. Hence, there is a positive link between R&D intensity, captured by the ratio I_{jt}/A_{jt}, and future productivity growth.

2.6. *Wages, prices and monetary policy*

We consider an economy with frictions in the adjustment of nominal wages.[17] The presence of nominal wage rigidities plays two roles in our analysis. First, it creates the possibility of

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16. The goods market clearing condition can be derived combining the households' budget constraint with the expression for firms' profits:

$$d_t = \underbrace{P_t Y_t - W_t L_t - \xi P_t \int_0^1 x_{jt}\,dj}_{\text{profits from final sector}} + \underbrace{(\xi - 1)P_t \int_0^1 x_{jt}\,dj - P_t \int_0^1 I_{jt}\,dj}_{\text{profits from intermediate sector}},$$

where profits are net of research expenditure, and the equilibrium condition $b_{t+1} = 0$, deriving from the assumption of identical households.

17. A growing body of evidence emphasizes how nominal wage rigidities represent an important transmission channel through which monetary policy affects the real economy. For instance, this conclusion is reached by Olivei and Tenreyro (2007), who show that monetary policy shocks in the U.S. have a bigger impact on output in the aftermath of the season in which wages are adjusted. Micro-level evidence on the importance of nominal wage rigidities is provided by Fehr and Goette (2005), Gottschalk (2005), Barattieri *et al.* (2014) and Fabiani *et al.* (2010).

involuntary unemployment, by ensuring that nominal wages remain positive even in presence of unemployment. Secondly, it opens the door to a stabilization role for monetary policy. Indeed, as we will see, prices inherit part of wage stickiness, so that the central bank can affect the real interest rate of the economy through movements in the nominal interest rate.

In our baseline model, we consider the simplest possible form of nominal wage rigidities and assume that wages evolve according to:

$$W_t = \bar{\pi}^w W_{t-1}. \tag{2.15}$$

This expression implies that nominal wage inflation is constant and equal to $\bar{\pi}^w$, and could be derived from the presence of large menu costs from deviating from the constant wage inflation path.

To be clear, our results do not rely at all on this extreme form of wage stickiness. Indeed, in Section 4.2 we generalize our results to an economy in which wages are allowed to respond to fluctuations in employment, giving rise to a wage Phillips curve. However, considering an economy with constant wage inflation simplifies considerably the analysis, and allows us to characterize transparently the key economic forces at the heart of the model.

Turning to prices, combining equations (2.5) and (2.8) gives:

$$P_t = \frac{1}{1-\alpha}\left(\frac{\xi}{\alpha}\right)^{\frac{\alpha}{1-\alpha}} \frac{W_t}{A_t}.$$

Intuitively, prices are increasing in the marginal cost of firms producing the final good. An increase in wages puts upward pressure on marginal costs and leads to a rise in prices, while a rise in productivity reduces marginal costs and prices. This expression, combined with the law of motion for wages, can be used to derive an equation for price inflation:

$$\pi_t \equiv \frac{P_t}{P_{t-1}} = \frac{\bar{\pi}^w}{g_t}, \tag{2.16}$$

which implies that price inflation is increasing in wage inflation and decreasing in productivity growth.

The central bank implements its monetary policy stance by setting the nominal interest rate according to the truncated interest rate rule:

$$1+i_t = \max\left((1+\bar{i})L_t^\phi, 1\right),$$

where $\bar{i} \geq 0$ and $\phi > 0$. Under this rule the central bank aims at stabilizing output around its potential level by cutting the interest rate in response to falls in employment.[18] The nominal interest rate is subject to a zero lower bound constraint, which, as we show in Appendix B, can be derived from standard arbitrage between money and bonds.

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18. To clarify, this particular form of interest rate rule is by no means essential for the results of the article. For instance, following the work of Erceg *et al.* (2000), it is often assumed that in presence of flexible prices and rigid wages the central bank aims at stabilizing wage inflation. We consider this possibility in Section 4.2. Instead, in Section 5.1 we derive the optimal interest rate policy.

2.7. *Equilibrium*

The equilibrium of our economy can be described by four simple equations. The first one is the *Euler* equation, which captures households' consumption decisions. Combining households' optimality conditions (2.2) and (2.3) gives:

$$C_t^{-\sigma} = \beta(1+i_t)E_t\left[\frac{C_{t+1}^{-\sigma}}{\pi_{t+1}}\right].$$

According to this standard Euler equation, demand for consumption is increasing in expected future consumption and decreasing in the real interest rate, $(1+i_t)/\pi_{t+1}$.

To understand how productivity growth relates to demand for consumption, it is useful to combine the previous expression with $A_{t+1}/A_t = g_{t+1}$ and $\pi_{t+1} = \bar{\pi}^w/g_{t+1}$ to obtain:

$$c_t^\sigma = \frac{g_{t+1}^{\sigma-1}\bar{\pi}^w}{\beta(1+i_t)E_t\left[c_{t+1}^{-\sigma}\right]}, \tag{2.17}$$

where we have defined $c_t \equiv C_t/A_t$ as consumption normalized by the productivity index. This equation shows that the relationship between productivity growth and present demand for consumption can be positive or negative, depending on the elasticity of intertemporal substitution, $1/\sigma$. There are, in fact, two contrasting effects. On the one hand, faster productivity growth is associated with higher future wealth. This wealth effect leads households to increase their demand for current consumption in response to a rise in productivity growth. On the other hand, faster productivity growth is associated with a fall in expected inflation. Given i_t, lower expected inflation increases the real interest rate inducing households to postpone consumption. This substitution effect points towards a negative relationship between productivity growth and current demand for consumption. For low levels of intertemporal substitution, *i.e.* for $\sigma > 1$, the wealth effect dominates and the relationship between productivity growth and demand for consumption is positive. Instead, for high levels of intertemporal substitution, *i.e.* for $\sigma < 1$, the substitution effect dominates and the relationship between productivity growth and demand for consumption is negative. Finally, for the special case of log utility, $\sigma = 1$, the two effects cancel out and productivity growth does not affect present demand for consumption.[19]

Empirical estimates point towards an elasticity of intertemporal substitution smaller than one.[20] Hence, in the main text we will focus attention on the case $\sigma > 1$, while we provide an analysis of the cases $\sigma < 1$ and $\sigma = 1$ in Appendix C.

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19. To clarify, different assumptions about wage or price setting can lead to a positive relationship between productivity growth and present demand for consumption even in presence of an elasticity of intertemporal substitution larger than one. For instance, a plausible assumption is that wages are partly indexed to productivity growth, so that:

$$W_t = \bar{\pi}^w g_t^\omega W_{t-1},$$

where $\omega > 0$. In this case, the Euler equation becomes:

$$c_t^\sigma = \frac{g_{t+1}^{\sigma-1+\omega}\bar{\pi}^w}{\beta(1+i_t)E_t\left[c_{t+1}^{-\sigma}\right]},$$

so that a positive relationship between demand for consumption and productivity growth arises as long as $\sigma > 1-\omega$.

20. Havránek (2015) performs a meta-analysis of the literature and finds that, though substantial uncertainty about the exact value of the elasticity of intertemporal substitution exists, most estimates lie well below one. Examples of papers estimating an elasticity smaller than one are Hall (1988) and Ogaki and Reinhart (1998), who use macro data, and Vissing-Jørgensen (2002) and Best *et al*. (2015), who use micro data.

Assumption 1. *The parameter σ satisfies:*

$$\sigma > 1.$$

Under this assumption, the Euler equation implies a positive relationship between the pace of innovation and demand for present consumption.

The second key relationship in our model is the *growth* equation, which is obtained by combining equation (2.2) with the optimality condition for investment in research (2.11):

$$(g_{t+1}-1)\left(1-\beta E_t\left[\left(\frac{c_t}{c_{t+1}}\right)^\sigma g_{t+1}^{-\sigma}\chi\gamma\varpi L_{t+1}\right]\right)=0. \tag{2.18}$$

This equation captures the optimal investment in research by entrepreneurs. For values of profits sufficiently high so that some research is conducted in equilibrium and $g_{t+1}>1$, this equation implies a positive relationship between growth and expected future employment. Intuitively, a rise in employment, and consequently in aggregate demand, is associated with higher monopoly profits. In turn, higher expected profits induce entrepreneurs to invest more in research, leading to a positive impact on the growth rate of the economy. This is the classic market size effect emphasized by the endogenous growth literature. But higher expected growth reduces households' desire to save, leading to an increase in the cost of funds for entrepreneurs investing in research. In fact, in the new equilibrium the rise in growth and in the cost of funds will be exactly enough to offset the impact of the rise in expected profits on the return from investing in research. This ensures that the zero profit condition on the market for research is restored.

The third equation combines the goods market clearing condition (2.12), the GDP equation (2.13) and the fact that $\int_0^1 I_{jt}dj = A_t(g_{t+1}-1)/(\chi(\gamma-1))$:[21]

$$c_t = \Psi L_t - \frac{g_{t+1}-1}{\chi(\gamma-1)}. \tag{2.19}$$

Keeping output constant, this equation implies a negative relationship between productivity-adjusted consumption and growth, because to generate faster growth the economy has to devote a larger fraction of output to innovation activities, reducing the resources available for consumption.

Finally, the fourth equation is the monetary policy rule:

$$1+i_t = \max\left(\left(1+\bar{i}\right)L_t^\phi, 1\right). \tag{2.20}$$

We are now ready to define an equilibrium as a set of processes $\{g_{t+1}, L_t, c_t, i_t\}_{t=0}^{+\infty}$ satisfying equations (2.17)−(2.20) and $L_t \leq 1$ for all $t \geq 0$.

3. STAGNATION TRAPS

In this section, we show that the interaction between aggregate demand and productivity growth can give rise to prolonged periods of low growth, low interest rates and high unemployment, which we call *stagnation traps*. We start by considering non-stochastic steady states, and we derive conditions on the parameters under which two steady states coexist, one of which is a stagnation trap. We then show that stagnation traps of finite expected duration are also possible.

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21. To derive this condition, consider that:

$$\int_0^1 I_{jt}dj = \int_0^1 A_{jt}I_{jt}/A_{jt}dj = I_{jt}/A_{jt}\int_0^1 A_{jt}dj = \mu_t/\chi\int_0^1 A_{jt}dj = A_t\mu_t/\chi = A_t(g_{t+1}-1)/(\chi(\gamma-1)),$$

where we have used the fact that I_{jt}/A_{jt} is the same across all the j sectors.

3.1. *Non-stochastic steady states*

Non-stochastic steady state equilibria are characterized by constant values for productivity growth g, employment L, normalized consumption c and the nominal interest rate i satisfying:

$$g^{\sigma-1} = \frac{\beta(1+i)}{\bar{\pi}^w} \tag{3.21}$$

$$g^{\sigma} = \max(\beta \chi \gamma \varpi L, 1) \tag{3.22}$$

$$c = \Psi L - \frac{g-1}{\chi(\gamma-1)} \tag{3.23}$$

$$1+i = \max\left((1+\bar{i})L^{\phi}, 1\right), \tag{3.24}$$

where the absence of a time subscript denotes the value of a variable in a non-stochastic steady state. We now show that two steady state equilibria can coexist: one characterized by full employment, and one by involuntary unemployment.

3.1.1. Full employment steady state.   We start by describing the full employment steady state, which we denote by the superscripts f. In the full employment steady state the economy operates at full capacity, and hence $L^f = 1$. The growth rate associated with the full employment steady state, g^f, is found by setting $L = 1$ in equation (3.22):

$$g^f = \max\left((\beta \chi \gamma \varpi)^{\frac{1}{\sigma}}, 1\right). \tag{3.25}$$

The nominal interest rate that supports the full employment steady state, i^f, is obtained by setting $g = g^f$ in equation (3.21):

$$i^f = \frac{\left(g^f\right)^{\sigma-1}\bar{\pi}^w}{\beta} - 1. \tag{3.26}$$

The monetary policy rule (3.24) then implies that for a full employment steady state to exist the central bank must set $\bar{i} = i^f$. Finally, steady state (normalized) consumption, c^f, is obtained by setting $L = 1$ and $g = g^f$ in equation (3.23):

$$c^f = \Psi - \frac{g^f - 1}{\chi(\gamma-1)}.$$

We summarize our results about the full employment steady state in a proposition.

Assumption 2.   *The parameters satisfy:*

$$\bar{i} = \frac{(\beta \chi \gamma \varpi)^{1-\frac{1}{\sigma}}\bar{\pi}^w}{\beta} - 1 > 0 \tag{3.27}$$

$$\phi > \max\left(1 - \frac{1}{\sigma}, \frac{1}{(\beta \chi \gamma \varpi)^{\frac{1}{\sigma}}}\right) \tag{3.28}$$

$$1 < (\beta \chi \gamma \varpi)^{\frac{1}{\sigma}} < \min(1 + \Psi \chi(\gamma-1), \gamma). \tag{3.29}$$

Proposition 1. *Suppose assumptions 1 and 2 hold. Then, there exists a unique full employment steady state with $L^f = 1$. The full employment steady state is characterized by positive growth ($g^f > 1$) and by a positive nominal interest rate ($i^f > 0$). Moreover, the full employment steady state is locally determinate.*[22]

Intuitively, assumptions (3.27) and (3.28) guarantee that monetary policy and wage inflation are consistent with the existence of a, locally determinate, full employment steady state. Condition (3.27) ensures that the intercept of the interest rate rule is consistent with existence of a full employment steady state, and that inflation and productivity growth in the full employment steady state are sufficiently high so that the zero lower bound constraint on the nominal interest rate is not binding. Instead, condition (3.28), which requires the central bank to respond sufficiently strongly to fluctuations in employment, ensures that the full employment steady state is locally determinate.[23]

Assumption (3.29) has a dual role. First, it makes sure that consumption in the full employment steady state is positive. Secondly, it implies that in the full employment steady state the innovation probability lies between zero and one ($0 < \mu^f < 1$), an assumption often made in the endogenous growth literature.

Summing up, the full employment steady state can be thought as the normal state of affairs of the economy. In fact, in this steady state, which closely resembles the steady state commonly considered both in New Keynesian and endogenous growth models, the economy operates at its full potential, growth is robust, and monetary policy is not constrained by the zero lower bound.

3.1.2. Unemployment steady state.
Aside from the full employment steady state, the economy can find itself in a permanent liquidity trap with low growth and involuntary unemployment. We denote this unemployment steady state with superscripts *u*. To derive the unemployment steady state, consider that with $i = 0$ equation (3.21) implies:

$$g^u = \left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma - 1}}.$$

Since $\bar{i} > 0$ it follows immediately from equation (3.21) that $g^u < g^f$. Moreover, notice that equation (3.21) can be written as $(1+i)/\pi = g^\sigma/\beta$. Hence, $g^u < g^f$ implies that the real interest rate $(1+i)/\pi$ in the unemployment steady state is lower than in the full employment steady state. To see that the liquidity trap steady state is characterized by unemployment, consider that by equation (3.22) $(g^u)^\sigma = \max(\beta \chi \gamma \varpi L^u, 1)$. Now use $\beta \chi \gamma \varpi = g^f$ to rewrite this expression as:

$$L^u \leq \left(\frac{g^u}{g^f}\right)^\sigma < 1,$$

where the second inequality derives from $g^u < g^f$. Productivity-adjusted steady state consumption, c^u, is then obtained by setting $L = L^u$ and $g = g^u$ in equation (3.23):

$$c^u = \Psi L^u - \frac{g^u - 1}{\chi(\gamma - 1)}.$$

22. All the proofs are collected in Appendix A.
23. Similar assumptions are commonly made in the literature studying monetary policy in New Keynesian models (Galí, 2009). In fact, analyses based on the New Keynesian framework typically focus on fluctuations around a steady state in which output is equal to its natural level, that is the value that would prevail in absence of nominal rigidities, and the nominal interest rate is positive. Moreover, local determinacy is typically ensured by assuming that the central bank follows an interest rate rule that reacts sufficiently strongly to fluctuations in inflation or output.

The following proposition summarizes our results about the unemployment steady state.

Proposition 2.   *Suppose assumptions 1 and 2 hold, and that*

$$1 < \left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}} \tag{3.30}$$

$$\left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}} < 1 + \frac{\frac{\xi}{\alpha}-1}{\xi-1}\left(\frac{\beta}{(\bar{\pi}^w)^\sigma}\right)^{\frac{1}{\sigma-1}}\frac{\gamma-1}{\gamma}. \tag{3.31}$$

Then, there exists a unique unemployment steady state. At the unemployment steady state the economy is in a liquidity trap ($i^u = 0$), there is involuntary unemployment ($L^u < 1$), and both growth and the real interest rate are lower than in the full employment steady state ($g^u < g^f$ and $1/\pi^u < (1+i^f)/\pi^f$). Moreover, the unemployment steady state is locally indeterminate.

Assumption (3.30) implies that $g^u > 1$, and its role is to ensure existence and uniqueness of the unemployment steady state. To gain intuition, consider a case in which assumption (3.30) is violated. Then, it is easy to check that a liquidity trap steady state would feature a negative real interest rate and negative productivity growth. However, since the quality of intermediate inputs does not depreciate, a steady state with negative productivity growth cannot exist.[24] We will go back to this point in Section 4.1, where we introduce the possibility of a steady state with a negative real rate. Instead, assumption (3.31) makes sure that $c^u > 0$. Uniqueness is ensured by the fact that by equation (3.22) there exists a unique value of L consistent with $g = g^u > 1$.[25] Finally, assumption (3.28) guarantees that the zero lower bound on the nominal interest rate binds in the unemployment steady state.

Proposition 2 states that the unemployment steady state is locally indeterminate, so that animal spirits and sunspots can generate local fluctuations around its neighbourhood. This result is not surprising, given that in the unemployment steady state the central bank is constrained by the zero lower bound, and hence monetary policy cannot respond to changes in aggregate demand driven by self-fulfilling expectations.

We think of this second steady state as a stagnation trap, that is the combination of a liquidity and a growth trap. In a liquidity trap the economy operates below capacity because the central bank is constrained by the zero lower bound on the nominal interest rate. In a growth trap, lack of demand for firms' products depresses investment in innovation and prevents the economy from developing its full growth potential. In a stagnation trap these two events are tightly connected. We illustrate this point with the help of a diagram.

Figure 2 depicts the two key relationships that characterize the steady states of our model in the $L-g$ space. The first one is the growth equation (3.22), which corresponds to the *GG* schedule. For sufficiently high L, the *GG* schedule is upward sloped. The positive relationship between L

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24. Since $\beta < 1$ and $g^u > 1$, in our baseline model an unemployment steady state exists only if $\bar{\pi}^w < 1$, that is if wage inflation is negative. This happens because in a representative agent economy with positive productivity growth the steady state real interest rate must be positive. In turn, when the nominal interest rate is equal to zero, deflation is needed to ensure that the real interest rate is positive. However, this is not a deep feature of our framework, and it is not hard to modify the model to allow for positive wage inflation and a negative real interest rate in the unemployment steady state. For instance, in Section 4.1 we show that the presence of precautionary savings due to idiosyncratic shocks creates the conditions for an unemployment steady state with positive inflation and negative real rate to exist.

25. Notice that this assumption rules out the case $g^u = 1$. Under this knife-edged case an unemployment steady state might exist, but it will not be unique, since by equation (3.22) multiple values of L are consistent with $g = 1$.
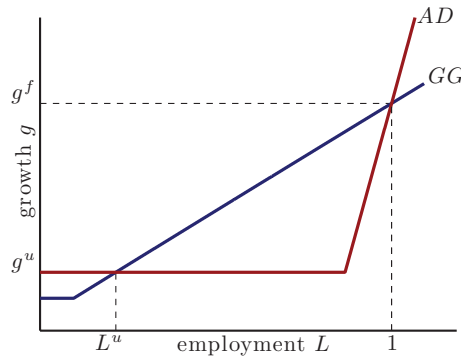
FIGURE 2

Non-stochastic steady states

and G can be explained with the fact that, for L high enough, an increase in employment and production is associated with a rise in firms' profits, while higher profits generate an increase in investment in innovation and productivity growth. Instead, for low values of L the GG schedule is horizontal. These are the values of employment for which investing in research is not profitable, and hence they are associated with zero growth.

The second key relationship combines the Euler equation (3.21) and the policy rule (3.24):

$$g^{\sigma-1} = \frac{\beta}{\bar{\pi}^w} \max\left((1+\bar{i})L^\phi, 1\right).$$

Graphically, this relationship is captured by the AD, *i.e.* aggregate demand, curve. The upward-sloped portion of the AD curve corresponds to cases in which the zero lower bound constraint on the nominal interest rate is not binding.[26] In this part of the state space, the central bank responds to a rise in employment by increasing the nominal rate. In turn, to be consistent with households' Euler equation, a higher interest rate must be coupled with faster productivity growth.[27] Hence, when monetary policy is not constrained by the zero lower bound the AD curve generates a positive relationship between L and g. Instead, the horizontal portion of the AD curve corresponds to values of L for which the zero lower bound constraint binds. In this case, the central bank sets $i=0$ and steady state growth is independent of L and equal to $(\beta/\bar{\pi}^w)^{1/(\sigma-1)}$. As long as the conditions specified in propositions 1 and 2 hold, the two curves cross twice and two steady states are possible.

Importantly, both the presence of the zero lower bound and the procyclicality of investment in innovation are needed to generate steady state multiplicity. Suppose that the central bank is not constrained by the zero lower bound, and hence that liquidity traps are not possible. As illustrated by the left panel of Figure 3, in this case the AD curve reduces to an upward sloped curve, steeper than the GG curve, and the unemployment steady state disappears. Intuitively, the assumptions about monetary policy ensure that, in absence of the zero lower bound, the central bank's reaction to unemployment is always sufficiently strong so as to offset the negative impact of pessimistic expectations on consumption and investment in research.[28] The presence of the

26. Precisely, the zero lower bound constraint does not bind when $L \geq (1+\bar{i})^{-1/\phi}$.

27. Recall that we are focusing on the case $\sigma > 1$.

28. To see this point, consider that ignoring the zero lower bound and using $1+\bar{i} = \bar{\pi}^w \beta^{-\frac{1}{\sigma}} (\chi\gamma\varpi)^{\frac{\sigma-1}{\sigma}}$ the AD curve can be written as

$$g = (\beta\chi\gamma\varpi)^{\frac{1}{\sigma}} L^{\frac{\phi}{\sigma-1}}.$$
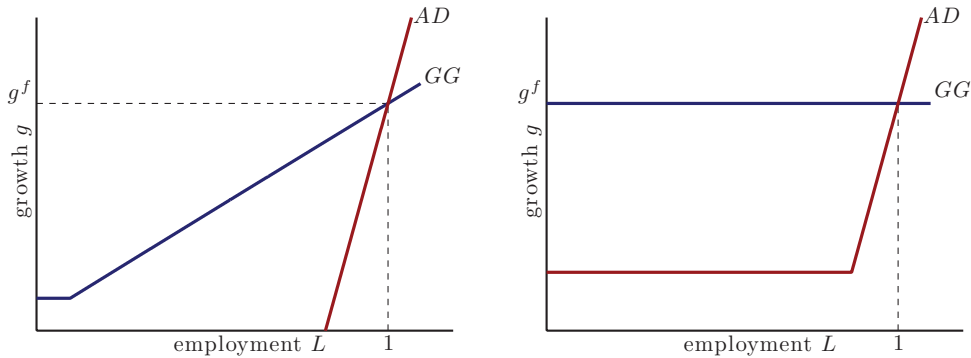
FIGURE 3

Understanding stagnation traps

Notes: Left panel: economy without zero lower bound. Right panel: economy with exogenous growth.

zero lower bound, however, mutes the response of monetary policy to changes in expectations, creating the conditions for multiple equilibria to appear. This is why self-fulfilling stagnation traps are possible only when the central bank ends up being constrained by the zero lower bound.

Now suppose instead that productivity growth is constant and equal to g^f. In this case, as shown by the right panel of Figure 3, the GG curve reduces to a horizontal line at $g = g^f$, and again the full employment steady state is the only possible one. Indeed, if growth is not affected by variations in employment, then condition (3.27) guarantees that aggregate demand and inflation are sufficiently high so that in steady state the zero lower bound constraint on the nominal interest rate does not bind, ensuring that the economy operates at full employment.

We are left with determining what makes the economy settle in one of the two steady states. This role is fulfilled by expectations. Suppose that agents expect that the economy will permanently fluctuate around the full employment steady state. Then, their expectations of high future growth sustain aggregate demand, so that a positive nominal interest rate is consistent with full employment. In turn, if the economy operates at full employment then firms' profits are high, inducing high investment in innovation and productivity growth. Conversely, suppose that agents expect that the economy will permanently remain in a liquidity trap. In this case, low expectations about growth and future income depress aggregate demand, making it impossible for the central bank to sustain full employment due to the zero lower bound constraint on the interest rate. As a result the economy operates below capacity and firms' profits are low, so that investment in innovation is also low, justifying the initial expectations of weak growth. Hence, in our model expectations can be self-fulfilling, and sunspots, that is confidence shocks unrelated to fundamentals, can determine real outcomes.

Interestingly, the transition from one steady state to the other need not take place in a single period. In fact, there are multiple perfect foresight paths, on which agents' expectations can coordinate, that lead the economy to the unemployment steady state. Figure 4 shows one of these paths.[29] The economy starts in the full employment steady state. In period 5, the economy is

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Recalling that the GG curve is $g = \max\left((\beta \chi \gamma \varpi L)^{1/\sigma}, 1\right)$, we have that the assumption $\phi > 1 - 1/\sigma$ implies that for any $0 < L < 1$ the AD curve lies below the GG curve. The result is that the two curves cannot cross at any $0 < L < 1$, unless the zero lower bound binds.

    29. Appendix D provides the numerical algorithm used to find perfect foresight paths that lead to the unemployment steady state. The parameters used to construct the figure are $\beta = .99$, $\sigma = 2$, $\gamma = 1.05$, $\chi = 89.14$, $\alpha = .5$, $\bar{\pi}^w = .98$ and $\phi = 1$. This parameterization is purely illustrative. Section 4.3 presents a calibration exercise.
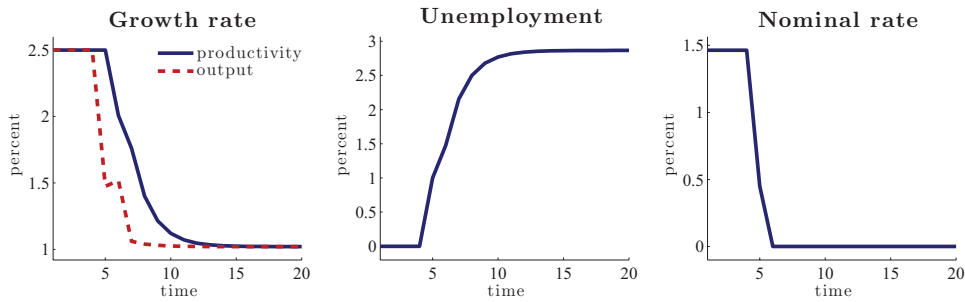
FIGURE 4

An example of transition toward the unemployment steady state

hit by a previously unexpected shock to expectations, which leads agents to revise downward their expectations of future productivity growth. From then on, the economy embarks in a perfect foresight transition towards the unemployment steady state. Initially, pessimism about future productivity triggers a fall in aggregate demand, leading to a rise in unemployment, to which the central bank responds by lowering the policy rate. In period 6, there is a further drop in expected productivity growth, causing a further rise in unemployment which pushes the economy in a liquidity trap. This initiates a long-lasting liquidity trap, during which the economy converges smoothly to the unemployment steady state.

Summarizing, the combination of growth driven by investment in innovation from profit-maximizing firms and the zero lower bound constraint on monetary policy can produce stagnation traps, that is permanent, or very long lasting, liquidity traps characterized by unemployment and low growth. All it takes is a sunspot that coordinates agents' expectations on a path that leads to the unemployment steady state.

Before moving on, it is useful to compare our notion of stagnation traps with the permanent liquidity traps that can arise in the New Keynesian model. In the standard New Keynesian model productivity growth is exogenous, and there is a unique real interest rate consistent with a steady state. As shown by Benhabib *et al*. (2001), permanent liquidity traps can occur in these frameworks if agents coordinate their expectations on an inflation rate equal to the inverse of the steady state real interest rate. Because of this, the New Keynesian model typically feature two steady states, one of which is a permanent liquidity trap. These two steady states are characterized by the same real interest rate, but by different inflation and nominal interest rates, with the liquidity trap steady state being associated with inflation below the central bank's target.

In contrast, in our framework endogenous growth is key in opening the door to steady state multiplicity and permanent liquidity traps. Crucially, in our model the two steady states feature different growth and real interest rates, with the liquidity trap steady state being associated with low growth and low real interest rate. Instead, inflation expectations do not play a major role. In fact, once a wage Phillips curve is introduced in the model, it might very well be the case that inflation in the unemployment steady state is the same, or even higher, than in the full employment one. We will go back to this point in Section 4.2.

3.2. *Temporary stagnation traps*

Though our model can allow for economies which are permanently in a liquidity trap, it is not difficult to construct equilibria in which the expected duration of a trap is finite. To construct an equilibrium featuring a temporary liquidity trap we have to put some structure on the sunspot

process. Let us start by denoting a sunspot by ξ_t. In a sunspot equilibrium agents form their expectations about the future after observing ξ_t, so that the sunspot acts as a coordination device for agents' expectations. Following Mertens and Ravn (2014), let us consider a two-state discrete Markov process, $\xi_t \in (\xi^o, \xi^p)$, with transition probabilities $Pr(\xi_{t+1} = \xi^o | \xi_t = \xi^o) = 1$ and $Pr(\xi_{t+1} = \xi^p | \xi_t = \xi^p) = q < 1$. The first state is an absorbing optimistic equilibrium, in which agents expect to remain forever around the full employment steady state. Hence, once $\xi_t = \xi^o$ the economy settles on the full employment steady state, characterized by $L = 1$ and $g = g^f$. The second state ξ^p is a pessimistic equilibrium with finite expected duration $1/(1-q)$. In this state the economy is in a liquidity trap with unemployment. We consider an economy that starts in the pessimistic equilibrium.

Under these assumptions, as long as the pessimistic sunspot shock persists the equilibrium is described by equations (2.17), (2.18) and (2.19), which, using the fact that in the pessimistic state $i = 0$, can be written as:

$$(g^p)^{\sigma-1} = \frac{\beta}{\bar{\pi}^w}\left(q + (1-q)\left(\frac{c^p}{c^f}\right)^\sigma\right) \tag{3.32}$$

$$(g^p - 1)\left((g^p)^\sigma - \beta\chi\gamma\varpi\left(qL^p + (1-q)\left(\frac{c^p}{c^f}\right)^\sigma\right)\right) = 0. \tag{3.33}$$

$$c^p = \Psi L^p - \frac{g^p - 1}{\chi(\gamma - 1)}, \tag{3.34}$$

where the superscripts p denote the equilibrium while pessimistic expectations prevail. Similar to the case of the unemployment steady state, in the pessimistic equilibrium the zero lower bound constraint on the interest rate binds, there is involuntary unemployment and growth is lower than in the optimistic state.

Characterizing analytically the equilibrium described by equations (3.32)−(3.34) is challenging, but some results can be obtained by using $g^u = (\beta/\bar{\pi}^w)^{1/(\sigma-1)}$ to write equation (3.32) as:

$$(g^p)^{\sigma-1} = (g^u)^{\sigma-1}\left(q^p + (1-q^p)\left(\frac{c^p}{c^f}\right)^\sigma\right).$$

It can be shown that c^p/c^f is smaller than one, *i.e.* switching to the optimistic steady state entails an increase in productivity-adjusted consumption. Hence, the equation above implies that temporary liquidity traps feature slower growth compared to permanent ones.

Figure 5 displays the expected path of productivity growth, unemployment and the nominal interest rate during a temporary stagnation trap.[30] The economy starts in the pessimistic equilibrium, characterized by low growth, high unemployment and a nominal interest rate equal to zero. From the second period on, each period agents expect that the economy will leave the trap and go back to the full employment steady state with a constant probability. Hence, the probability that the economy remains in the trap decreases with time, explaining the upward path for expected productivity growth, employment and the nominal interest rate. However, even though the economy eventually goes back to the full employment steady state, the post-trap increase in the growth rate is not sufficiently strong to make up for the low growth during the trap, so that the trap generates a permanent loss in output.

This example shows that pessimistic expectations can plunge the economy into a temporary stagnation trap with unemployment and low growth. Eventually the economy will recover, but

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30. To construct this figure we have used the same parameters used to construct Figure 4, and set $1/(1-q) = 10$.
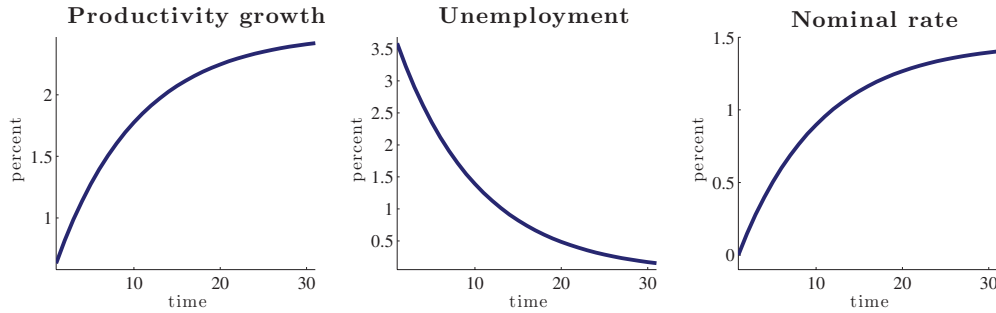
FIGURE 5

A temporary stagnation trap: expected dynamics

the trap lasts as long as pessimistic beliefs persist. Hence, long lasting stagnation traps driven by pessimistic expectations can coexist with the possibility of a future recovery.

4. EXTENSIONS AND NUMERICAL EXERCISE

In this section, we extend the model in three directions. We first show that the introduction of precautionary savings can give rise to stagnation traps characterized by positive inflation and a negative real interest rate. We then show that our key results do not rely on the assumption of a constant wage inflation rate. Lastly, we perform a simple calibration exercise to examine a setting in which, consistent with standard models of vertical innovation, the duration of innovators' monopoly rents is endogenous.

4.1. *Negative real rates and the role of fundamentals*

In our baseline framework positive growth and positive inflation cannot coexist during a permanent liquidity trap. Intuitively, if the economy is at the zero lower bound with positive inflation, then the real interest rate must be negative. But then, to satisfy households' Euler equation, the steady state growth rate of the economy must also be negative. Conversely, to be consistent with positive steady state growth the real interest rate must be positive, and when the nominal interest rate is equal to zero this requires deflation.

However, it is not hard to think about mechanisms that could make positive growth and positive steady state inflation coexist in an unemployment steady state. One possibility is to introduce precautionary savings. In Appendix E, we lay down a simple model in which every period a household faces a probability p of becoming unemployed. An unemployed household receives an unemployment benefit, such that its income is equal to a fraction $b < 1$ of the income of an employed household. Unemployment benefits are financed with taxes on the employed households. We also assume that unemployed households cannot borrow and that trade in firms' share is not possible.

As showed in Appendix E, under these assumptions the Euler equation (2.17) is replaced by:

$$c_t^\sigma = \frac{\bar{\pi}^w g_{t+1}^{\sigma-1}}{\beta(1+i_t)\rho E_t\left[c_{t+1}^{-\sigma}\right]},$$

where:

$$\rho \equiv 1 - p + p/b^{\sigma} > 1.[31]$$

The unemployment steady state is now characterized by:

$$g^u = \left(\frac{\rho \beta}{\bar{\pi}^w} \right)^{\frac{1}{\sigma - 1}}. \tag{4.35}$$

Since $\rho > 1$, an unemployment steady state in which both inflation and growth are positive, and the real interest rate is negative, is now possible.

The key intuition behind this result is that the presence of uninsurable idiosyncratic risk depresses the real interest rate (Huggett, 1993). Indeed, the presence of uninsurable idiosyncratic risk drives up the demand for precautionary savings. Since the supply of saving instruments is fixed, higher demand for precautionary savings leads to a lower equilibrium interest rate. This is the reason why an economy with uninsurable unemployment risk can reconcile positive steady state growth with a negative real interest rate. Hence, once the possibility of uninsurable unemployment risk is taken into account, it is not hard to imagine a permanent liquidity trap with positive growth, positive inflation, and negative real interest rate.

This is a good moment to discuss the relationship between the structural determinants of the propensity to save and the possibility of self-fulfilling stagnation traps. Looking at equation (4.35), and recalling that in our model productivity growth cannot be negative, one can see that an unemployment steady state is possible only if $\rho \beta \geq \pi^w$. For lower values of $\rho \beta$, in fact, households' propensity to save would be too low to make a real interest rate of $1/\pi^w$ consistent with non-negative steady state growth.[32] This suggests that self-fulfilling stagnation traps are possible only in economies in which the fundamentals are such that the desire to save, captured in our model by the parameters β and ρ, is sufficiently high.

It has been argued that since the early 1980s several factors, such as population ageing and higher inequality, have increased the supply of savings in several advanced economies (Rachel and Smith, 2015; Eggertsson et al., 2017). In this respect, our framework suggests that the same structural factors that have increased the supply of savings might have also exposed advanced economies to the risk of self-fulfilling stagnation traps.

4.2. Introducing a wage Phillips curve

Our baseline model features a constant wage inflation rate. Here we introduce a wage Phillips curve, and discuss the implications of our model for inflation and the role of wage flexibility.

To make things simple let us assume, in the spirit of Akerlof et al. (1996), that nominal wages are downwardly rigid:

$$W_t \geq \psi(L_t) W_{t-1},$$

with $\psi' > 0$, $\psi(1) = \bar{\pi}^w$. This formulation allows wages to fall at a rate which depends on unemployment. Capturing some non-monetary costs from adjusting wages downward, here wages

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31. The other equilibrium conditions are the same as in baseline model, except for the growth equation (2.18) which becomes:

$$(g_{t+1} - 1) \left(1 - \rho \beta E_t \left[\left(\frac{c_t}{c_{t+1}} \right)^{\sigma} g_{t+1}^{-\sigma} \chi \gamma \varpi L_{t+1} \right] \right) = 0.$$

32. Graphically, this corresponds to a case in which the horizontal portion of the *AD* curve is lower than the horizontal portion of the *GG* curve.

are more downwardly flexible the more employment is below potential. This form of wage rigidity gives rise to a nonlinear wage Phillips curve. For levels of wage inflation greater than $\bar{\pi}^w$ output is at potential. Instead, if wage inflation is less than $\bar{\pi}^w$ there is a positive relationship between inflation and the output gap.

Similar to the baseline model, monetary policy follows a truncated interest rate rule in which the nominal interest rate responds to deviations of wage inflation from a target π^*:

$$1+i_t = \max\left((1+\bar{i})\left(\frac{\pi_t^w}{\pi^*}\right)^\phi, 1 \right). \tag{4.36}$$

We assume that $\pi^* \geq \bar{\pi}^w$, so that when wage inflation is on target the economy operates at full employment. We also assume that $1+\bar{i}=\pi^*(\beta\chi\gamma\varpi)^{1/\sigma}$ and that ϕ is sufficiently large so that $L^{1/\sigma} > \left(\psi(L)/\pi^*\right)^{\phi/(\sigma-1)}$ for any $0 \leq L < 1$. This assumption, similar to assumption (3.28) of the baseline model, ensures local real determinacy of the full employment steady state and that, in the absence of the zero lower bound, there are no steady states other than the full employment one.

A steady state of the economy is now described by (3.22), (3.23), (4.36) and:

$$g^{\sigma-1} = \frac{\beta(1+i)}{\pi^w} \tag{4.37}$$

$$\pi^w \geq \psi(L). \tag{4.38}$$

It is easy to check that there exists a unique full employment steady state with $L=1$, $g=(\beta\chi\gamma\varpi)^{1/\sigma}$ and $\pi^w=\pi^*$. Hence, the presence of the wage Phillips curve does not affect employment and growth in the full employment steady state.

Let us now turn to the unemployment steady state. Combining equations (4.36)−(4.38) and using $i=0$, gives:

$$g^u = \left(\frac{\beta}{\psi(L^u)}\right)^{\frac{1}{\sigma-1}}. \tag{4.39}$$

This expression implies a negative relationship between growth and employment. To understand this relationship, consider that in a liquidity trap the real interest rate is just the inverse of expected inflation. Due to the wage Phillips curve, as employment increases wage inflation rises generating higher price inflation. Hence, in a liquidity trap a higher employment is associated with a lower real interest rate. The consequence is that during a permanent liquidity trap a rise in employment must be associated with lower productivity growth, to be consistent with the lower real interest rate. As illustrated by Figure 6, graphically this is captured by the fact that the *AD* curve, obtained by combining equations (4.36)−(4.38), is downward sloped for values of L low enough so that the zero lower bound constraint binds.[33]

To solve for the equilibrium unemployment steady state, combine equations (3.22) and (4.39) to obtain:

$$L^u = \frac{1}{\beta\chi\gamma\varpi}\left(\frac{\beta}{\psi(L^u)}\right)^{\frac{\sigma}{\sigma-1}}. \tag{4.40}$$

Since the left-hand side of this expression is increasing in L^u, while the right hand-side is decreasing in L^u, there is a unique L^u that characterizes the unemployment steady state. Moreover,

33. Moreover, the *AD* curve has now a vertical portion to capture the fact that the wage Phillips curve becomes vertical at $L=1$.
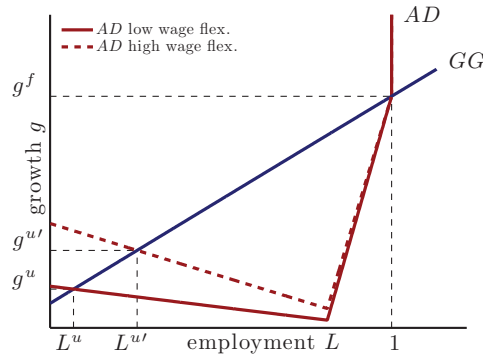
FIGURE 6

Steady states with variable inflation

since $L^u < 1$, the presence of a Phillips curve implies that the unemployment steady state is now characterized by lower wage inflation than the full employment steady state. In sum, the presence of a wage Phillips curve does not alter the key properties of the unemployment steady state, while adding the realistic feature that in the unemployment steady state the central bank undershoots its wage inflation target.

Turning to price inflation, recalling that $\pi_t = \pi_t^w / g_{t+1}$, we have that:

$$\frac{\pi^u}{\pi^f} = \frac{\psi(L^u)}{\pi^*} \frac{g^f}{g^u}.$$

Since $\psi(L^u) < \pi^*$ and $g^f > g^u$, depending on parameter values price inflation in the unemployment steady state can be above, below, or even equal to price inflation in the full employment steady state. This result is due to the fact that in the unemployment steady state the depressive impact on firms' marginal costs and price inflation originating from low-wage inflation is counteracted by the upward pressure exerted by low productivity growth.[34]

To conclude this section, we note that higher wage flexibility, captured by a steeper wage Phillips curve, is associated with better outcomes in the unemployment steady state. For instance, this result can be seen by considering that expression (4.40) implies that the endogenous fall in wage inflation, captured by the term $\psi(L^u)$, sustains employment in the unemployment steady state. Figure 6 illustrates graphically the impact of higher wage flexibility on the determination of the unemployment steady state. The figure shows that higher wage flexibility steepens the downward portion of the *AD* curve, leading to higher growth and employment in the unemployment steady state. This is an interesting result, in light of the fact that analyses based on the standard New Keynesian framework suggest that higher price or wage flexibility typically lead to worse outcomes in terms of output during liquidity traps (Eggertsson, 2010).

4.3. *Numerical exercise*

In this section, we explore further the properties of the model by performing a simple calibration exercise. To be clear, the objective of this exercise is not to provide a careful quantitative evaluation

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34. Interestingly, during the Great Recession several countries have experienced sharp recessions without substantial deflation. The model suggests that the absence of deflation might be explained with the fact that the aftermath of the 2008 financial crisis was also characterized by a slowdown in productivity growth.

of the framework or to replicate any particular historical event. In fact, both of these tasks would require a much richer model. Rather, our aim is to show that the magnitudes implied by the model are quantitatively relevant and reasonable.

For our numerical exercise we enrich the baseline framework in three dimensions. First, we introduce the simple form of household heterogeneity and uninsurable idiosyncratic risk discussed in Section 4.1. This allows the model to reproduce the low interest rate environment that has characterized Japan, the U.S. and the Euro area around their liquidity trap episodes. Secondly, along the lines of Section 4.2, we consider an economy with downward wage rigidities, to capture the fact that liquidity traps are typically accompanied by slowdowns in price and wage inflation. Thirdly, we relax the assumption of one period monopoly rents for innovators in favour of a, more conventional, setting in which innovators' rents can last for several periods.

4.3.1. Endogenous duration of rents from innovation.

In our baseline model we assume that the rents from innovation last a single period, after which the innovator's patent expires. Here we consider a setting in which every period the innovator retains its patent with probability $1-\eta$.[35] As in the baseline mode, after the patent is lost the monopoly position is allocated randomly to another entrepreneur. Under these assumptions, the value of becoming a leader in sector j is:

$$V_t(\gamma A_{jt}) = \beta\rho E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(P_{t+1}\varpi\gamma A_{jt}L_{t+1} + \left(1-\mu_{jt+1}-\eta\right)V_{t+1}(\gamma A_{jt})\right)\right], \tag{4.41}$$

where $\mu_{j,t+1}+\eta\leq 1$. The first term inside the parenthesis on the right-hand side, also present in the baseline model, captures the expected profits to be gained in period $t+1$. In addition, the value of a successful innovation includes the value of being a leader in period $t+1$, $V_{t+1}(\gamma A_{jt})$, times the probability that the entrepreneur remains the leader in period $t+1$, $1-\mu_{jt+1}-\eta$. Notice that the probability of maintaining the leadership is decreasing in μ_{jt+1}, due to the fact that the discovery of a better version of product j terminates the monopoly rents for the incumbent. Future payoffs are discounted using the households' discount factor $\beta\rho\lambda_{t+1}/\lambda_t$, which is now adjusted for the presence of idiosyncratic risk.

To streamline the exposition, in this section we restrict attention to equilibria in which in every period a positive amount of research is targeted towards every intermediate good.[36] In this case, free entry into research implies that expected profits from researching are zero for every product. This zero profit condition implies that:[37]

$$P_t = \frac{\chi}{A_{jt}}V_t(\gamma A_{jt}),$$

for every good j. Moreover, we focus on symmetric equilibria in which the probability of innovation is the same in every sector, so that $\mu_{jt}=\chi I_{jt}/A_{jt}=\mu_t$ for every j. In this case,

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35. Standard models of vertical innovation typically assume that $\eta=0$. Instead, we follow Acemoglu and Akcigit (2012) and consider the case $\eta>0$. This assumption encapsulates all the factors that might lead to the termination of the rents from innovation besides innovation by entrants. These include patent expiration and imitation by competitors.

36. We verify that this condition holds in all the simulations presented below.

37. To derive this condition, consider that an entrepreneur that invests I_{jt} in research has a probability $\chi I_{jt}/A_{jt}$ of becoming a leader which carries value $V_t(\gamma A_{jt})$. Hence, the expected return from this investment is $\chi I_{jt}V_t(\gamma A_{jt})/A_{jt}$. Since the investment costs $P_t I_{jt}$, the zero expected profits condition in the research sector implies:

$$P_t I_{jt} = \frac{\chi I_{jt}}{A_{jt}}V_t(\gamma A_{jt}).$$

Simplifying we obtain the expression in the main text.

$V_t(\gamma A_{jt}) = \bar{V}_t \gamma A_{jt}$ for every j, while free entry into the research sector in period $t+1$ implies $\bar{V}_{t+1} = P_{t+1}/(\gamma \chi)$. Combining these conditions with expression (4.41) gives:

$$\frac{P_t}{\chi} = \beta \rho E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(P_{t+1} \varpi \gamma L_{t+1} + \left(1 - \mu_{t+1} - \eta \right) \frac{P_{t+1}}{\chi} \right) \right].$$

This expression summarizes the equilibrium in the research sector. Combining this expression with equation (2.2) and using $\mu_t = (g_{t+1} - 1)/(\chi(\gamma - 1))$ gives:

$$g_{t+1}^\sigma = \beta \rho E_t \left[\left(\frac{c_t}{c_{t+1}} \right)^\sigma \left(\chi \gamma \varpi L_{t+1} + 1 - \frac{g_{t+2} - 1}{\gamma - 1} - \eta \right) \right], \tag{4.42}$$

which replaces equation (2.18) of the baseline model.

4.3.2. Parameters.

We choose the length of a period to correspond to a year. We set the inverse of the elasticity of intertemporal substitution to $\sigma = 2$ and the discount factor to $\beta = 0.99^4 = 0.96$, in the range of the values commonly considered by the business-cycle literature. We target a real interest rate in the full employment steady state of 1.5 percent, in line with the low interest rates characterizing the U.S. and the Euro area in the run-up to the 2008 financial crisis, and a growth rate in the full employment steady state of 2%. Using $1 + r^f = (g^f)^\sigma / (\rho \beta)$, these targets imply $\rho = 1.067$.[38] The central bank's wage inflation target is set to $\pi^* = 1.04$, so that price inflation in the full employment steady state is 2%.

Turning to the parameters determining the growth process, we set the labour share in gross output to $1 - \alpha = 0.83$, to match a ratio of spending in R&D-to-GDP in the full employment steady state of 2%, close to the long-run average of the business spending in R&D-to-GDP ratio in the U.S.[39] The step size of innovations is set to $\gamma = 1.55$ so that the probability that an innovation occurs in a given sector is 3.6% per year, as in Howitt (2000). We choose the value of χ, the parameter determining the productivity of research, to match our 2% growth target in the full employment steady state. To set the parameter η we follow the same strategy adopted by Kung and Schmid (2015), and assume that the probability that the rents from innovation expire in the full employment steady state, given by $\mu^f + \eta$, is equal to 15%, the rate of depreciation of the R&D stock estimated by the Bureau of Labor Statistics.[40]

We perform the following experiment. We target a value for the output gap in the unemployment steady state, and derive the implications for growth, the real interest rate, and price and wage inflation.[41] Estimates of the output gap for the Japanese slump lie between 5

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38. To gain intuition about the plausibility of this value of ρ, consider a case in which $p = 0.05$, so that every household faces a constant 5% probability of becoming unemployed. Then $\rho = 1.067$ implies a value for unemployment benefits of $b = 0.65$, in the range of the replacement rates typically assumed by the search and matching literature (Hall and Milgrom, 2008).

39. More precisely, in the U.S. during the period between 1953 and 2014 the average ratio of business spending in R&D over GDP has been 1.7%. Here we follow a large part of the endogenous growth literature that uses spending in R&D as the data counterpart of the model's investment in innovation. It is also possible, and in our judgement plausible, to take a broader interpretation of investment in innovation that includes other types of investment, often difficult to measure, that contribute to firms' productivity growth. One example is investment in implementation of existing technologies.

40. To be clear, in our model the stock of knowledge does not depreciate, since when a firm exits due to patent expiration it is replaced by another equally productive firm. Alternatively, one could assume that once a firm exits, for reasons unrelated to creative destruction, its knowledge is lost. This would introduce a positive rate of depreciation in the aggregate stock of knowledge. We have experimented with this alternative, and found that the results are not significantly affected.

41. Thus, throughout our simulations we adjust the shape of the wage Phillips curve so that the unemployment steady state features the targeted output gap. This is the same approach adopted by Eggertsson et al. (2017).

TABLE 2
Parameters

	Value	Source/Target
Elas. intertemporal substitution	$1/\sigma = 0.5$	Standard value
Discount factor	$\beta = 0.96$	Standard value
Idiosyncratic risk	$\rho = 1.067$	$(1+i^f)/\pi^f = 1.015$
Wage inflation at full emp.	$\pi^* = 1.04$	$\pi^f = 1.02$
Share of labor in gross output	$1-\alpha = 0.83$	$I^f/GDP^f = 2\%$
Innovation step	$\gamma = 1.55$	$\mu^f = 3.6\%$
Productivity of research	$\chi = 3.15$	$g^f = 1.02$
Prob. patent expires	$\eta = 0.114$	$\mu^f + \eta = 15\%$

and 10% (Hausman and Wieland, 2014). This is roughly in line with the range of estimates for the output gap during the post-crisis liquidity traps in the U.S. (Del Negro *et al.*, 2015; CBO, 2017) and in the Euro area (Jarocinski and Lenza, 2016). As our baseline, we thus assume that in the unemployment steady state output is 7.5% below potential. Since in our model deviations of output from potential are purely due to unemployment, this implies setting $1-L^u = 0.075$.[42] We later show how the results change as the distance of output from potential varies between 5 and 10%.

Notice that across our simulations we keep ρ, the parameter capturing the impact of idiosyncratic risk on savings, constant. This means that we are abstracting from the impact that changes in employment L might have on idiosyncratic unemployment risk.[43] Introducing these effects, for example by assuming that idiosyncratic unemployment risk is decreasing in aggregate employment ($\rho'(L) < 0$), would amplify the fall in the real interest rate during stagnation traps.

4.3.3.   Results.   Table 3 displays several statistics from this calibrated version of the model. The first column refers to the full employment steady state. As targeted in the calibration, productivity growth is 2%, while, by definition of the full employment steady state, the output gap is equal to zero. The real interest rate is equal to its calibration target of 1.5%, which, coupled with the 2% price inflation target, implies a nominal interest rate of about 3.5%. Wage inflation is 4.04 percent, approximately equal to the sum of price inflation and productivity growth.

The second column shows the statistics for the unemployment steady state in which, as targeted, output is 7.5% below potential. The key result is that both productivity growth and the real interest rate are significantly lower than in the full employment steady state. In fact, moving from the full employment to the unemployment steady state is associated with a drop in productivity growth of 0.33%, and with a fall in the real interest rate of sixty-five basis points. In terms of prices, the unemployment steady state is characterized by low wage inflation and mild price deflation. Intuitively, given the zero nominal rate, mild price deflation is needed to generate a low but positive real interest rate. This does not mean, however, that inflation needs to be negative in the unemployment steady state. Indeed, as we will see shortly, different assumptions

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42. In richer models other factors, such as variable markups or capital utilization, can drive a wedge between potential and actual output. Introducing these elements would reduce the amount of unemployment needed to generate a given amount of output gap.

43. In practice, we are assuming that the rise in unemployment during a stagnation trap manifests itself through a reduction in hours worked per person employed, rather than in a fall in the number of persons employed. This assumption is not made on the ground of realism, but rather to focus on the relationship between growth and interest rates which is at the heart of our framework.

TABLE 3

Calibrated examples

	Full employment steady state	Unemployment steady state		
		Baseline	Incumbents	Spillovers
Productivity growth	2.00	1.67	1.37	1.21
Output gap $(1-L)$	0.00	7.50	7.50	7.50
Nominal interest rate	3.53	0.00	0.00	0.00
Real interest rate	1.50	0.85	0.26	−0.06
Price inflation	2.00	−0.84	−0.26	0.06
Wage inflation	4.04	0.81	1.11	1.28
R&D intensity	17.00	16.67	16.37	16.21

Note: Second column refers to the baseline model with innovation by entrants, third column refers to the model with innovation by incumbents and fourth column refers to the model with innovation by incumbents with productivity spillovers. All the values are expressed in percentage points. The output gap is defined as the distance of actual output from potential, measured by $1-L$. R&D intensity is computed as its empirical counterpart, using the law of motion for the R&D stock described in Appendix G.

about the growth process can generate even more dramatic drops in productivity growth and in the real interest rate, creating the conditions for positive inflation to occur in the unemployment steady state.

4.3.4. Innovation by incumbents.

In our baseline model all the investment in innovation is performed by entrants. In reality, however, incumbent firms carry out the majority of investment in R&D and productivity improvements (Bartelsman and Doms, 2000). We now consider a version of the model consistent with this fact. To preview the main result, not only stagnation traps arise also in this alternative setting, but they tend to be associated with larger drops in growth and real rates compared to the baseline model.

As we detail in Appendix F, we introduce innovation by incumbents by assuming that incumbents have a cost advantage in performing R&D compared to entrants. In particular, following Barro and Sala-i Martin (2004), we assume that incumbents' advantage in performing R&D is so large that all the innovation is carried out by existing firms, and no entry occurs in equilibrium. We parametrize this version of the model to hit the same targets as in the model with innovation by entrants.[44] Hence, the statistics shown in the first column of Table 3 also describe the full employment steady state of the model with innovation by incumbents.

The third column of Table 3 shows the statistics relative to the unemployment steady state in the model with innovation by incumbents. The results are qualitatively in line with the baseline model, but the fall in growth and in the real interest rate are about twice as large. In fact, growth is 0.63% lower than in the full employment steady state, while the real interest rate is 124 basis points lower. To understand this result, consider that in the baseline model with innovation by entrants the prospect of future stagnation has two effects on current incentives to invest in innovation. On the one hand, expectations of future weak demand depress the return from innovation. On the other hand, expectations of low future investment in innovation by competitors, and thus of low firms turnover and high expected duration of monopoly positions, sustain the return from innovation in the present. The presence of the second effect mitigates the drop in investment and growth during a stagnation trap. Instead, when innovation is performed by incumbents the

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44. We thus set $\alpha = 0.197$, $\eta = 0.15$, and χ^I, the parameter capturing incumbents' productivity in research, to 4.2. All the other parameters are as in Table 1.
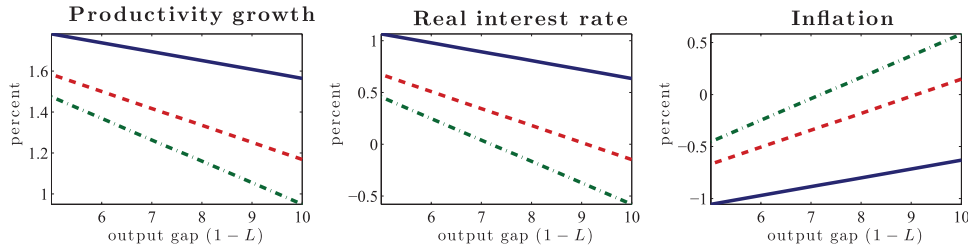
FIGURE 7

Unemployment steady state

Notes: Innovation by entrants (solid lines), innovation by incumbents (dashed lines) and innovation by incumbents with productivity spillovers (dash-dotted lines).

duration of monopoly position is independent of investment in innovation by the other firms in the economy. Hence, only the first effect is present, and the drops in investment and growth are larger.

4.3.5. Productivity spillovers across firms.

Some models of endogenous growth feature productivity spillovers across firms. In this case, innovation by one firm increases productivity and profits for all the other firms in the economy. This happens, for instance, in Grossman and Helpman (1991) and Aghion *et al*. (2001). We introduce productivity spillovers across firms by assuming that the final good is produced according to:

$$Y_t = L_t^{1-\alpha} \int_0^1 \left(A_t^{1-\theta} A_{jt}^{\theta}\right)^{1-\alpha} x_{jt}^{\alpha} dj, \quad \text{where} \quad A_t \equiv \left(\int_0^1 A_{jt}^{\theta} dj\right)^{\frac{1}{\theta}},$$

and $0 < \theta \leq 1$. When $\theta < 1$, this production function implies that the productivity of an intermediate input is increasing in aggregate productivity, capturing the notion of productivity spillovers across firms. Instead, the case $\theta = 1$ corresponds to our baseline model in which spillovers are absent. As we show in Appendix F, in this version of the model the profits of a generic firm j are given by $P_t \varpi A_t^{1-\theta} A_{jt}^{\theta} L_t$. Hence, a lower θ is associated with a stronger link between profits and aggregate productivity.

We embed this production function in the version of the model with innovation by incumbents. Again, we choose parameter values so as to hit the targets listed in Table 2,[45] and we set the parameter θ to the illustrative value of 0.5.[46] The results are shown in the fourth column of Table 3. The key aspect is that the presence of productivity spillovers across firms magnifies the drop in growth and in the real interest rate during stagnation traps. In fact, the unemployment steady state is now associated with a negative real interest rate and positive inflation. This result is due to the fact that with productivity spillovers expectations of low future investment in innovation reduce the expected profits from investing in innovation in the present. This channel amplifies the drop in the return from investing in innovation during a stagnation trap, leading to lower productivity growth and a lower real rate.

45. Hence we set $\alpha = 0.45$, $\chi^I = 9.47$ and $\eta = 0.15$, while leaving the rest of the parameters as in Table 2.
46. For comparison, our baseline model corresponds to the case $\theta = 1$, while setting $\theta \approx 0$ would bring the model close to Grossman and Helpman (1991) and Aghion *et al*. (2001).

4.3.6.   Robustness to different values of the output gap.   We conclude this section by showing how the results vary as a function of the output gap in the unemployment steady state. Figure 7 shows productivity growth, the real interest rate and price inflation in the unemployment steady state as a function of the output gap. The key takeaway is that for plausible values of the output gap the unemployment steady state is associated with substantial drops in growth and interest rates. For instance, when innovation is conducted by incumbents an output gap of 10%, on the high end of the estimates for Japan and the U.S., is associated with a significantly negative real rate and positive inflation, especially if productivity spillovers across firms are present.

Overall, this simple calibration exercise paints the picture of stagnation traps as persistent periods of weak productivity growth and low real rates, coupled with high unemployment and output substantially below potential. As we have seen, the exact quantitative results depend on how the supply side of the economy is specified. This suggests that an interesting area for future research is to explore the notion of stagnation traps in modern quantitive models of endogenous growth, in the spirit of Acemoglu and Akcigit (2012), featuring a more detailed firm side of the economy.

5.   POLICY IMPLICATIONS

Which policy interventions can push the economy out of a stagnation trap? To address this question, we first consider monetary policy interventions. We then turn to growth policies, aiming at sustaining investment in productivity enhancing activities. For simplicity, we discuss the role of these policies in the context of the baseline model described in Section 2.

5.1.   *Monetary policy*

In Section 3, we have shown that stagnation traps can arise if the central bank follows an interest rate rule, in which the interest rate responds monotonically to employment or wage inflation. In this section, we examine the robustness of this result to alternative interest rate policies. The key lesson that we derive is that monetary policy interventions during a stagnation trap are hindered by credibility issues. In fact, as we will see, under commitment a central bank can implement interest rate policies that rule out stagnation traps. Instead, if the central bank operates under discretion monetary policy, even when conducted optimally, is not enough to lift the economy out of a stagnation trap.

5.1.1.   Commitment.   Let us start by examining a central bank that operates under commitment, and seeks to rule out the stagnation traps examined in Section 3, while preserving determinacy of the full employment steady state. We now show that the central bank can attain these goals by adopting a nonlinear interest rate rule.[47] This approach combines a standard interest rate rule, that operates in "normal" times, with an interest rate peg, adopted by the central bank when expectations turn pessimistic. To see how this approach works, define s_t as a binary value that follows:

$$s_t = \begin{cases} 1 & \text{if } i_{t-1}=0 \\ 0 & \text{if } g_t \geq g^f \\ s_{t-1} & \text{otherwise,} \end{cases}$$

47.   This approach follows the spirit of the one proposed by Schmitt-Grohé and Uribe (2017).

for $t \geq 0$ with $s_{-1} = 0$. Now consider a central bank that follows the rule:

$$i_t = \begin{cases} \max\left((1+\bar{i}) L_t^\phi - 1, 0 \right), & \text{if } s_t = 0 \\ i^f & \text{otherwise,} \end{cases}$$

where $\bar{i}$ and ϕ satisfy the conditions stated in assumption 2. Under this rule, the central bank switches to an interest rate peg the period after the nominal interest rate hits the zero lower bound. The peg is maintained for one period, after which the central bank returns to the interest rate rule considered in Section 3.

This policy eliminates the unemployment steady state and the persistent stagnation traps of Section 3.2. In fact, for stagnation traps to occur agents should coordinate their expectations on a protracted period of zero interest rates, a possibility ruled out by the central bank commitment to maintain the interest rate equal to zero for one period at most. Hence, with this rule in place monetary policy acts as an equilibrium selection device, preventing agents' expectations from coordinating on the stagnation traps equilibria. At the same time, since condition (3.28) holds, this policy rule eliminates sunspot fluctuations around the full employment steady state.

5.1.2. Discretion.
The picture changes dramatically if the central bank does not have the ability to commit to its future actions. The following proposition characterizes the behaviour of a benevolent central bank that operates under discretion.

Proposition 3. *Consider a central bank that operates under discretion and maximizes households' expected utility, subject to (2.17), (2.18), (2.19), $L_t \leq 1$ and $i_t \geq 0$. The solution to this problem satisfies:*

$$i_t (L_t - 1) = 0.$$

The intuition behind this result is straightforward. The discretionary central bank seeks to maximize current employment.[48] From the goods market clearing condition, employment is increasing in consumption and investment in research (both normalized by productivity):

$$\Psi L_t = c_t + \frac{I_t}{A_t}.$$

In turn, equations (2.17) and (2.18) imply that, holding expectations about the future constant, both consumption and investment in research are decreasing in the nominal interest rate. In fact, when the nominal interest rate falls also the real interest rate decreases, inducing households to frontload their consumption and entrepreneurs to increase investment in research, thus stimulating output and employment. It follows that, as long as the zero lower bound constraint does not bind, the central bank is able to set the nominal interest rate low enough so that the economy operates

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48. To be precise, the economy is subject to three sources of inefficiency. First, involuntary unemployment is possible. Secondly, due to monopolistic competition production of intermediate goods is inefficiently low. Thirdly, investment in research is subject to the intertemporal spillover and business stealing effects studied by Aghion and Howitt (1992) and Grossman and Helpman (1991), implying that in the *laissez faire* equilibrium the fraction of output invested in research can be higher or lower than in the social planner allocation. However, as shown in the proof to proposition 3, interest rate policy can only seek to correct the first distortion. The economic intuition for this result is that, since consumption and investment in research are both decreasing in the interest rate, interest rate policy cannot affect the allocation of output between consumption and investment, and hence cannot correct for the inefficiencies due to the intertemporal spillover and business stealing effects.

at full employment and $L_t = 1$. However, if a negative nominal interest rate is needed to reach full employment then the best that the discretionary central bank can do is to set $i_t = 0$. Hence, the economy can be in one of two regimes. Either the economy operates at full employment and the zero lower bound constraint on the interest rate does not bind, or the economy is in a liquidity trap with unemployment.[49]

We now show that under a discretionary central bank the economy can experience the same kind of stagnation traps described in Section 3. Let us take the perspective of a discretionary central bank operating in period $t = 0$. Consider a case in which expectations coordinate on the unemployment steady state, so that $E_0[i_t] = 0$, $E_0[L_{t+1}] = L^u$ and $E_0[c_{t+1}] = c^u$ for every future date $t > 0$. From Section 3 we know that if the central bank sets $i_0 = 0$, then $L_0 = L^u < 1$ so the economy will experience unemployment. Can the central bank do better by setting a positive nominal interest rate? The answer is no, because by raising the nominal interest rate above zero the central bank would further depress demand for consumption and investment, thus pushing employment below L^u. Hence, if expectations coordinate on the unemployment steady state the best response of a central bank under discretion is to set $i_0 = 0$, implying that $L_0 = L^u$, $g_1 = g^u$ and $c_0 = c^u$. A similar reasoning holds in any date $t \geq 0$, meaning that the central bank's actions validate agents' expectations and push the economy in the unemployment steady state. Moreover, a similar reasoning implies that if expectations coordinate on the full employment steady state the central bank will set $i = i^f$ and validate them. Hence, under discretionary monetary policy the two steady states analysed in Section 2 are possible equilibria.[50] Moreover, one can show that under discretion the temporary stagnations traps described in Section 3.2 are also possible.

These results highlight the key role that the ability to commit plays in avoiding stagnation traps through interest rate policy. Under commitment, the central bank can design interest rate policies that make expectations of a prolonged liquidity trap inconsistent with equilibrium, thus ruling out the possibility of long periods of stagnation. Instead, under discretion the central bank inability to commit to its future actions leaves the door open to stagnation episodes.[51]

5.2. Growth Policy

One of the root causes of a stagnation trap is the weak growth performance of the economy, which is in turn caused by entrepreneurs' limited incentives to innovate in a low demand environment. This suggests that subsidies to investment in innovation might be a helpful tool in the management of stagnation traps. In fact, these policies have been extensively studied in the context of endogenous growth models as a tool to overcome inefficiencies in the innovation process. However, here we show how policies that foster productivity growth can also play a role in stimulating aggregate demand and employment during a stagnation trap. In fact, we find that an aggressive subsidy to investment can push the economy out of a stagnation trap, by generating a regime shift in agents' expectations about future growth.

The most promising form of growth policies to exit a stagnation trap are those that loosen the link between profits and investment in innovation. For instance, suppose that the government

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49. In fact, the optimal policy under discretion is equivalent to the truncated interest rate rule considered in the baseline model with $\phi \to +\infty$.

50. This result can also be derived using the graphical approach of Figure 2. In fact, the only difference is that in the case of a discretionary central bank the upward portion of the AD curve becomes a vertical line at $L = 1$.

51. The result that discretionary monetary policy opens the door to multiple equilibria is reminiscent of the findings of King and Wolman (2004). As in their work, in our framework the gains from commitment arise because under commitment monetary policy can act as an equilibrium selection device, ruling out undesirable equilibria. This is different from the traditional gains from commitment highlighted by the literature studying liquidity traps driven by fundamental shocks (Eggertsson and Woodford, 2003).

provides a subsidy to innovation, in the form of a lump-sum transfer s_{jt} given to entrepreneurs in sector j to finance investment in innovation.[52] The subsidy can be state contingent and sector specific, and it is financed with lump-sum taxes on households. Under these assumptions, the zero profit condition for research in sector j becomes:[53]

$$V_t(\gamma A_{jt}) = \frac{P_t A_{jt}}{\chi}\left(1 - \frac{s_{jt}}{I_{jt}}\right),$$

where $V_t(\gamma A_{jt})$ is defined as in (2.10). The presence of the term s_{jt}/I_{jt} is due to the fact that entrepreneurs have to finance only a fraction $1 - s_{jt}/I_{jt}$ of the investment in research, while the rest is financed by the government. This expression implies that entrepreneurs are willing to invest in innovation even when the value of becoming a leader is zero, since if $V_t(\gamma A_{jt}) = 0$ then $I_{jt} = s_{jt}$. Hence, assuming that the government can ensure that entrepreneurs cannot divert the subsidy away from innovation activities, investment in innovation will be always at least equal to the subsidy s_{jt}, so $I_{jt} \geq s_{jt}$.

Let us now consider the macroeconomic implications of the subsidy. For simplicity, we keep on focusing on symmetric equilibria in which every sector j has the same innovation probability, and hence we consider subsidies of the form $s_{jt} = s_t A_{jt}$. Assuming a positive subsidy $s_t > 0$, the growth equation (2.18) is replaced by:

$$\left(1 - \frac{s_t \chi(\gamma - 1)}{g_{t+1} - 1}\right) = \beta E_t\left[\left(\frac{c_t}{c_{t+1}}\right)^\sigma g_{t+1}^{-\sigma} \chi \gamma \varpi L_{t+1}\right], \tag{5.43}$$

where to derive this expression we have followed the same steps taken in Section 3 and used $I_{jt}/A_{jt} = \mu_t/\chi = (g_{t+1} - 1)/(\gamma - 1)$. Notice that the expression above implies that $g_{t+1} > 1$, since with the subsidy in place investment in innovation is always positive.

We now show that an appropriately chosen subsidy can eliminate the unemployment steady state. Consider a subsidy of the form $s_t = s(g_{t+1})$ with $s'(\cdot) < 0$ and $s(g^f) = 0$, where g^f is the productivity growth in the full employment steady state under laissez faire. According to this policy, the government responds to a fall in productivity growth by increasing the subsidy to investment in innovation. With the subsidy in place, in steady state the growth equation becomes:

$$g^\sigma \left(1 - \frac{s(g)\chi(\gamma - 1)}{g - 1}\right) = \beta \chi \gamma \varpi L. \tag{5.44}$$

Notice that the term in round brackets on the left-hand side of expression (5.44) is smaller than one, because $s(g_{t+1})A_{jt} < I_{jt}$. Hence, given L, steady state growth is increasing in the subsidy.

It is easy to see that, since $s(g^f) = 0$, the economy features a full employment steady state identical to the one described in Section 3.1. Let us now turn to the unemployment steady state. In the unemployment steady state productivity growth must be equal to $g^u = (\beta/\bar{\pi}^w)^{(1/(\sigma - 1))}$, to satisfy households' Euler equation. However, a sufficiently high subsidy can guarantee that

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52. More precisely, we assume that in sector j the government devotes an aggregate amount of resources $s_t A_{jt}$ to sustain innovation. These resources are equally divided among all the entrepreneurs operating in innovation in that sector.

53. With the subsidy, the cost of investing I_{jt} in research is $P_t(I_{jt} - s_{jt})$, which gives an expected gain of $\chi I_{jt} V_t(\gamma A_{jt})/A_{jt}$. The zero expected profits condition for research in sector j then implies:

$$P_t(I_{jt} - s_{jt}) = \frac{\chi I_{jt}}{A_{jt}} V_t(\gamma A_{jt}).$$

Rearranging this expression we obtain the expression in the main text.

investment in innovation will always be sufficiently strong so that the growth rate of the economy will always be higher than $(\beta/\bar{\pi}^w)^{(1/(\sigma-1))}$. It follows that by setting a sufficiently high subsidy the government can rule out the possibility that the economy might fall in a permanent stagnation trap.

Proposition 4. *Suppose that there is a subsidy to innovation $s(g_{t+1})$ satisfying $s'(\cdot) < 0$, $s(g^f) = 0$ and:*

$$1 + s\left(\left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}}\right)\chi(\gamma-1) > \left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}}, \tag{5.45}$$

and that the conditions stated in Proposition 1 hold. Then there exists a unique steady state. The unique steady state is characterized by full employment.

Intuitively, the subsidy to innovation guarantees that even if firms' profits were to fall substantially, investment in innovation would still be relatively high. In turn, a high investment in innovation stimulates growth and aggregate demand, since a high future income is associated with a high present demand for consumption. By implementing a sufficiently high subsidy, the government can eliminate the possibility that aggregate demand will be low enough to make the zero lower bound constraint on the nominal interest rate bind. It is in this sense that growth policies can be thought as a tool to manage aggregate demand in our framework. Importantly, to lift the economy out of a stagnation trap the subsidy to investment has to be large enough, so as to engineer a radical regime shift in agents' expectations about future growth.[54]

Graphically, the impact of the subsidy is illustrated by Figure 8. The blue solid line corresponds to the *GG* curve of the economy without subsidy, while the blue dashed line represents the *GG* curve of an economy with a subsidy to investment in innovation. The subsidy makes the *GG* curve rotate up, because for a given level of employment and aggregate demand the subsidy increases the growth rate of the economy. If the subsidy is sufficiently high, as it is the case in the left panel of the figure, the unemployment steady state disappears and the only possible steady state is the full employment one. By contrast, the right panel of Figure 8 shows the impact of a small subsidy. A small subsidy makes the *GG* curve rotate up, but not enough to eliminate the unemployment steady state. In fact, the small subsidy leads to lower employment in the unemployment steady state compared to the *laissez faire* economy.

Summarizing, there is a role for well-designed subsidies to growth-enhancing investment, a typical supply side policy, in stimulating aggregate demand so as to rule out liquidity traps driven

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54. One might wonder how important it is that the subsidy is contingent on productivity growth. It turns out that totally analogous results can be obtained with a subsidy contingent on employment. Moreover, even a sufficiently large non-contingent subsidy can rule out the unemployment steady state. However, a non-contingent subsidy will also increase growth, perhaps to an inefficiently high value, in the full employment steady state.

The results summarized in Proposition 4 readily extend, under a caveat, to the case of the economy with a wage Phillips curve described in Section 4.2. The caveat is that wage inflation should not fall too much as employment approaches zero. Using the notation of Section 4.2, the function $\psi(\cdot)$ must be such that:

$$\lim_{L \to 0} \psi(L) \geq \frac{\beta}{\left(g^f\right)^{\sigma-1}}.$$

Intuitively, if this condition does not hold, even when the subsidy is sufficiently high so that productivity growth cannot fall below its value in the full employment steady state there still exists an $0 < L < 1$ consistent with a steady state in which $i = 0$. In this case, it is not possible to rule out the unemployment steady state with a subsidy that is a continuous function of productivity growth or employment. However, even when the condition above fails, it is still possible to rule out the unemployment steady state by designing subsidy schemes that are discontinuous functions of productivity growth or employment.
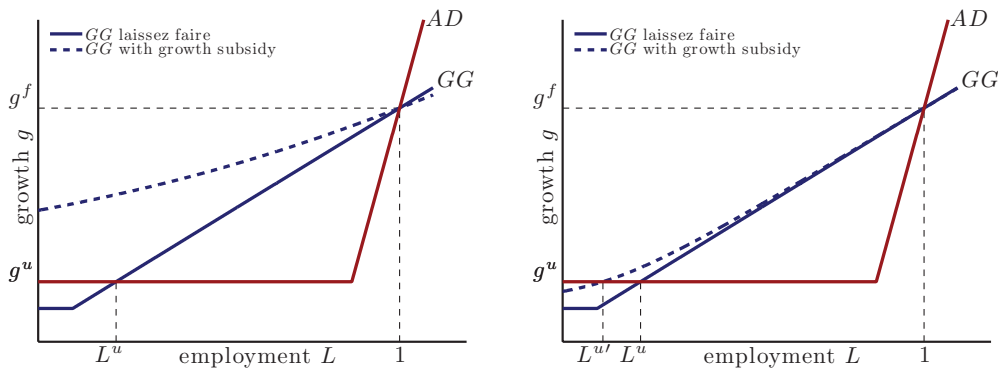
FIGURE 8

Steady state with growth subsidy

Notes: Left panel: large subsidy. Right panel: small subsidy.

by expectations of weak future growth. In turn, the stimulus to aggregate demand has a positive impact on employment. In this sense, our model helps to rationalize the notion of job creating growth.

6. CONCLUSION

In this article, we have presented a Keynesian growth model in which endogenous growth interacts with the possibility of involuntary unemployment due to weak aggregate demand. The combination of these two factors can give rise to stagnation traps, that is persistent liquidity traps characterized by unemployment and weak growth. All it takes for the economy to fall into a stagnation trap is a wave of pessimism about future growth. We show that policy interventions to support growth can lead the economy out of a stagnation trap, as long as they are aggressive enough to generate a regime shift in agents' expectations about future growth .

Our analysis represents a first step towards understanding the interactions between business cycles, growth, and stagnation, and it leaves open several exciting avenues for future research. We conclude the article by mentioning three of them. First, in order to focus the analysis on fluctuations driven by shocks to expectations, in this article we have abstracted from fundamental shocks. Can fundamental shocks, such as productivity, monetary or news shocks, lead to prolonged periods of stagnation? Does the impact of fundamental shocks on the economy depend on whether the economy is undergoing a period of stagnation? These are examples of questions that our model can help to answer. Secondly, in this article we have focused on a subset of possible policy interventions, but it would be interesting to study other policy tools, such as fiscal policy. Thirdly, in this article we have abstracted from financial frictions. However, it is easy to imagine a channel through which growth and aggregate demand can interact when firms' access to financing is limited. In fact, if access to credit is tight, firms' investment in innovation will be constrained by their internal funds. In turn, a period of low aggregate demand and weak profits will erode firms' internal funds, and thus their ability to invest in productivity-enhancing activities. Through this channel, a period of low aggregate demand will lead to low productivity growth. Indeed, we conjecture that economies undergoing a period of financial stress might be particularly exposed to the risk of expectation-driven stagnation traps. This might contribute to explain why the episodes of stagnation affecting Japan, the U.S. and the Euro area coincided, at least in their beginning, with periods of tight access to credit.

APPENDIX

A. PROOFS

A.1. *Proof of Proposition* 1 *(existence, uniqueness, and local determinacy of full employment steady state)*

Proof. We start by proving existence. A steady state is described by the system (3.21)–(3.24). Setting $L = 1$ and using the first inequality in condition (3.29), equation (3.22) implies:

$$g^f = (\beta \chi \gamma \varpi)^{\frac{1}{\sigma}} . \tag{A46}$$

Condition (3.29) implies $1 < g^f < \gamma$. Then equation (3.21) implies:

$$1 + i^f = \frac{\bar{\pi}^w \left(g^f\right)^{\sigma-1}}{\beta} . \tag{A47}$$

Assumption (3.27) guarantees that $i^f > 0$ and $\bar{i} = i^f$, so that the interest rate rule (3.24) is compatible with the existence of a full employment steady state. Moreover, combining equations (3.22) and (3.23) evaluated at $L = 1$ and $g = g^f$ gives:

$$c^f = \Psi - \frac{g^f - 1}{\chi(\gamma - 1)} . \tag{A48}$$

One can check that the second inequality in condition (3.29) ensures that $c^f > 0$. Hence, a full employment steady state exists.

To prove uniqueness, consider that equation (A46) implies that there is only one value of g consistent with the full employment steady state, while equation (A47) establishes that there is a unique value of i consistent with $g = g^f$. Hence, the full employment steady state is unique.

We now show that the full employment steady state is locally determinate. A loglinear approximation of equations (2.17)–(2.20) around the full employment steady state gives:

$$(\sigma - 1)\hat{g}_{t+1} = \hat{i}_t + \sigma(\hat{c}_t - E_t[\hat{c}_{t+1}]) \tag{A49}$$

$$\hat{L}_t = \frac{c^f}{\Psi}\hat{c}_t + \left(1 - \frac{c^f}{\Psi}\right)\frac{g^f}{g^f - 1}\hat{g}_{t+1} \tag{A50}$$

$$\sigma \hat{g}_{t+1} = \sigma(\hat{c}_t - E_t[\hat{c}_{t+1}]) + E_t\left[\hat{L}_{t+1}\right] \tag{A51}$$

$$\hat{i}_t = \phi \hat{L}_t, \tag{A52}$$

where $\hat{x} \equiv \log(x_t) - \log(x^f)$ for every variable x, except for $\hat{g}_t \equiv g_t - g^f$ and $\hat{i} \equiv i_t - \bar{i}$, while Ψ is GDP normalized by the productivity index. This system can be written as:

$$\hat{L}_t = \xi_1 E_t[\hat{L}_{t+1}] + \xi_2 E_t[g_{t+2}] \tag{A53}$$

$$\hat{g}_{t+1} = \xi_3 E_t[\hat{L}_{t+1}] + \xi_4 E_t[g_{t+2}], \tag{A54}$$

where

$$\xi_1 \equiv \frac{\frac{\Psi}{c^f} - \frac{1}{\sigma} + \left(1 + \frac{\Psi - c^f}{c^f}\frac{g^f}{g^f - 1}\right)}{\frac{\Psi}{c^f} + \phi\left(1 + \frac{\Psi - c^f}{c^f}\frac{g^f}{g^f - 1}\right)}$$

$$\xi_2 \equiv -\frac{\frac{\Psi - c^f}{c^f}\frac{g^f}{g^f - 1}}{\frac{\Psi}{c^f} + \phi\left(1 + \frac{\Psi - c^f}{c^f}\frac{g^f}{g^f - 1}\right)}$$

$$\xi_3 \equiv 1 - \phi\xi_1$$

$$\xi_4 \equiv -\phi\xi_2.$$

The system is determinate if and only if:[55]

$$|\xi_1\xi_4 - \xi_2\xi_3| < 1 \tag{A55}$$

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55. See Bullard and Mitra (2002).

$$|\xi_1 + \xi_4| < 1 + \xi_1\xi_4 - \xi_2\xi_3. \tag{A56}$$

Condition (A55) holds if:

$$\phi > \frac{\frac{\Psi - c^f}{\Psi}\frac{g^f}{g^f - 1} - 1}{\frac{c^f}{\Psi} + \frac{\Psi - c^f}{\Psi}\frac{g^f}{g^f - 1}}, \tag{A57}$$

while condition (A56) holds if:

$$\phi > 1 - \frac{1}{\sigma}. \tag{A58}$$

Assumption (3.28) guarantees that (A56) holds. Moreover, using (A48), after some algebra, condition (A57) can be written as:

$$(1 + \phi)\Psi + \frac{\phi - g^f}{\chi(\gamma - 1)} > 0.$$

By condition (3.29) the inequality above is true if:

$$\phi > 1/g^f = 1/(\beta\chi\gamma\varpi)^{1/\sigma},$$

which holds by assumption (3.28). Hence, (A55) holds and the full employment steady state is locally determinate. ‖

A.2. *Proof of Proposition 2 (existence, uniqueness and local indeterminacy of unemployment steady state)*

Proof. We start by showing that it is not possible to have an unemployment steady state with a positive nominal interest rate. Suppose that this is not the case, and that there is a steady state with $1 + i = (1 + \bar{i})L^\phi$ and $0 \leq L < 1$. Then combining equations (3.21) and (3.22), and using $\beta\chi\gamma\varpi = g^f$ and $1 + \bar{i} = (g^f)^{\sigma - 1}\bar{\pi}^w/\beta$ gives:

$$g^f L^{\frac{\phi}{\sigma - 1}} = \left(\max\left(\left(g^f\right)^\sigma L, 1\right)\right)^{\frac{1}{\sigma}}. \tag{A59}$$

Assumption (3.28) implies that the left-hand side of this expression is smaller than the right-hand side for any $0 \leq L < 1$. Hence, we have found a contradiction and an unemployment steady state with $i > 0$ is not possible.

We now prove that an unemployment steady state with $i = 0$ exists and is unique. Setting $i = 0$, equation (3.21) implies that there is a unique value of $g = (\beta/\bar{\pi}^w)^{1/(\sigma - 1)} = g^u$ consistent with the unemployment steady state. Moreover, since $i^f > 0$ equation (3.21) also implies that $g^u < g^f$, while the first inequality in condition (3.30) implies $g^u > 1$. Evaluating equation (3.22) at $g = g^u$ we have:

$$L^u = \frac{(g^u)^\sigma}{\beta\chi\gamma\varpi},$$

ensuring that there is a unique value of $L = L^u > 0$ consistent with $g = g^u$. Moreover, using $g^u < g^f$ and equation (3.22) gives $L^u < 1$. Combining equations (3.22) and (3.23) evaluated at $L = L^u$ and $g = g^u$, one can check that the second inequality in condition (3.30) ensures that $c^u > 0$. Hence, the unemployment steady state exists and is unique. Finally, using $i^f > 0$ and $g^u < g^f$ one can see that $1/\pi^u = g^u/\bar{\pi}^w < (1 + i^f)g^f/\bar{\pi}^w = (1 + i^f)/\pi^f$, so that the real interest rate in the unemployment steady state is lower than the one in the full employment steady state.

To prove local indeterminacy one can follow the steps of the proof to proposition one. Since in the neighbourhood of the unemployment steady state $\hat{i}_t = 0$, it is easy to show that condition (A56) cannot be satisfied. ‖

A.3. *Proof of Proposition 3 (optimal discretionary monetary policy)*

Proof. Under discretion, every period the central bank maximizes the representative household expected utility subject to (2.17), (2.18), (2.19), $L_t \leq 1$ and $i_t \geq 0$, taking future variables as given. The central bank's problem can be written as:

$$\max_{L_t, c_t, g_{t+1}, i_t} E_t\left[\sum_{\tau = t}^\infty \beta^\tau \left(\frac{(C_\tau^{1-\sigma} - 1)}{1 - \sigma}\right)\right] = E_t\left[\beta^t A_t^{1-\sigma}\left(\frac{c_t^{1-\sigma}}{1 - \sigma} + g_{t+1}v_t^1\right)\right] - \frac{1}{(1 - \beta)(1 - \sigma)},$$

subject to:

$$L_t = \frac{1}{\Psi}\left(c_t + \frac{g_{t+1} - 1}{\chi(\gamma - 1)}\right)$$

$$c_t = \left(\frac{g_{t+1}^{\sigma - 1}}{1 + i_t}\right)^{\frac{1}{\sigma}}v_t^2$$

$$g_{t+1} = \max\left(1, \frac{v_t^3}{1+i_t}\right)$$

$$L_t \leq 1$$

$$i_t \geq 0,$$

where the third constraint is obtained by combining (2.17) and (2.18), and:

$$v_t^1 = E_t\left[\sum_{\tau=t+1}^{\infty} \beta^\tau \frac{\left(c_\tau \Pi_{\hat{\tau}=t+2}^\tau g_{\hat{\tau}}\right)^{1-\sigma}}{1-\sigma}\right]$$

$$v_t^2 = \left(\frac{\bar{\pi}^w}{\beta E_t\left[c_{t+1}^{-\sigma}\right]}\right)^{\frac{1}{\sigma}}$$

$$v_t^3 = \bar{\pi}^w \chi \gamma \varpi \frac{E_t[L_{t+1} c_{t+1}^{-\sigma}]}{E_t[c_{t+1}^{-\sigma}]}.$$

v_t^1, v_t^2 and v_t^3 are taken as given by the central bank, because they are function of parameters and expectations about future variables only.

Notice that the objective function is strictly increasing in c_t and g_{t+1}. Also notice that from the second and third constraints we can write $c_t = c(i_t)$ with $c'(i_t) < 0$ and $g_{t+1} = g(i_t)$ with $g'(i_t) \leq 0$. We can then rewrite the problem of a central bank under discretion as

$$\min i_t,$$

subject to:

$$L_t = \frac{1}{\Psi}\left(c(i_t) + \frac{g(i_t)-1}{\chi(\gamma-1)}\right)$$

$$L_t \leq 1$$

$$i_t \geq 0.$$

The solution to this problem can be expressed by the complementary slackness condition:

$$i_t(L_t - 1) = 0.$$

$\parallel$

A.4. *Proof of Proposition 4 (uniqueness of steady state with subsidy to innovation)*

The proof that a full employment steady state exists and is unique follows the steps of the proof to Proposition 1.

We now prove that there is no steady state with unemployment. Following the proof to proposition 2, one can check that if another steady state exists, it must be characterized by $i=0$. Equation (2.17) implies that in this steady state growth must be equal to $(\beta/\bar{\pi}^w)^{(1/(\sigma-1))}$. Suppose that a steady state with $g = (\beta/\bar{\pi}^w)^{(1/(\sigma-1))}$ exists. Then equation (5.44) implies that there must be an $0 \leq \tilde{L} \leq 1$ such that:

$$\tilde{L} = \left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{\sigma}{\sigma-1}}\left(1 - \frac{s\left(\left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}}\right)\chi(\gamma-1)}{\left(\frac{\beta}{\bar{\pi}^w}\right)^{\frac{1}{\sigma-1}} - 1}\right)(\beta\chi\gamma\varpi)^{-1}.$$

But condition (5.45) implies $\tilde{L} < 0$, a contradiction. Hence, an unemployment steady state does not exist. ∎

B. MODEL WITH MONEY

In this appendix, we explicitly introduce money in the model. The presence of money rationalizes the zero lower bound constraint on the nominal interest rate, but does not alter the equilibrium conditions of the model.

Following Krugman (1998) and Eggertsson (2008) we assume that households need to hold at least a fraction $v > 0$ of production in money balances M:

$$M_t \geq v P_t Y_t. \tag{A60}$$

The household's budget constraint is now:

$$P_t C_t + \frac{b_{t+1}}{1+i_t} + M_t = W_t L_t + b_t + M_{t-1} + d_t - T_t, \tag{A61}$$

where M_t denotes money holdings at time t to be carried over at time $t+1$, and T are lump-sum taxes paid to the government.

The optimality condition with respect to M_t is:

$$\lambda_t = \beta E_t[\lambda_{t+1}] + \mu_t, \tag{A62}$$

where $\mu_t > 0$ is the Lagrange multiplier on constraint (A60). Combining optimality conditions (2.3) and (A62) it is easy to see that the presence of money implies $i_t \geq 0$. Intuitively, households will always prefer to hold money, rather than an asset which pays a non-contingent negative nominal return. It is also easy to see that constraint (A60) binds if $i_t > 0$, but it is slack if $i_t = 0$. Hence, households' money demand is captured by the complementary slackness condition:

$$i_t \left(M_t - \nu P_t \left(\frac{\alpha}{\xi} \right)^{\frac{\alpha}{1-\alpha}} A_t L_t \right) = 0, \tag{A63}$$

with $i_t \geq 0$ and $M_t \geq \nu P_t \left(\frac{\alpha}{\xi} \right)^{\frac{\alpha}{1-\alpha}} A_t L_t$, where we have substituted Y_t using equation (2.9).

To close the model, we assume that the government runs a balanced budget:

$$T_t = M_t - M_{t-1},$$

so that seignorage revenue is rebated to households via lump-sum taxes.

We can now define an equilibrium as a set of processes $\{L_t, c_t, g_{t+1}, i_t, M_t, P_t\}_{t=0}^{+\infty}$ satisfying equations (2.17)−(2.20), (A63) and $P_t = \bar{\pi}^w P_{t-1}/g_t$, given initial values P_{-1}, A_0.[56]

Notice that to solve for the path of L_t, c_t, g_{t+1} and i_t only equations (2.17)−(2.20) are needed. Given values for L_t, A_t and P_t, the only use of the money demand equation (A63) is to define the money supply M_t consistent with the central bank's interest rate rule. Specifically, when $i_t > 0$ the equilibrium money supply is $M_t = \nu P_t (\alpha/\xi)^{\frac{\alpha}{1-\alpha}} A_t L_t$, while when $i_t = 0$ any money supply $M_t \geq \nu P_t (\alpha/\xi)^{\frac{\alpha}{1-\alpha}} A_t L_t$ is consistent with equilibrium.

These results do not rest on the specific source of money demand assumed. For instance, similar results can be derived in a setting in which households derive utility from holding real money balances, as long as a cashless limit is considered (Eggertsson and Woodford, 2003).

C. THE CASES OF $\sigma = 1$ AND $\sigma < 1$

In the main text we have focused attention on the empirically relevant case of low elasticity of intertemporal substitution, by assuming that $\sigma > 1$. In this appendix, we consider the alternative cases $\sigma = 1$ and $\sigma < 1$. The key result is that under these cases the steady state is unique.

We start by analyzing the case of $\sigma = 1$. In steady state, equation (3.21) can be written as:

$$1 = \frac{\beta(1+i)}{\bar{\pi}^w}.$$

Intuitively, under this case changes in the growth rate of the economy have no impact on the equilibrium nominal interest rate. Hence, if there exists a full employment equilibrium featuring a positive nominal interest rate, it is easy to check that no unemployment equilibrium can exist.

We now turn to the case $\sigma < 1$. Under this case, equation (3.21) implies a negative relationship between growth and the nominal interest rate. Supposing that a full employment equilibrium featuring a positive nominal interest rate exists, if a liquidity trap equilibrium exists, it must feature a higher growth rate than the full employment one, $i.e.$ $g^u > g^f$. Since $L^f = 1$, it must be the case that $L^u \leq L^f$. But equation (3.22) implies a non-negative steady state relationship between g and L. Then we cannot have a steady state in which $g^u > g^f$ and $L^u \leq L^f$, so that, if the economy features a full employment steady state, an unemployment steady state cannot exist.

D. NUMERICAL SOLUTION METHOD TO COMPUTE PERFECT FORESIGHT TRANSITIONS

The objective is to compute a sequence $\{g_{t+1}, L_t, c_t, i_t\}_{t=0}^{T}$ given an initial condition L_0 for T large such that:

$$c_t^\sigma = \frac{c_{t+1}^\sigma g_{t+1}^{\sigma-1} \bar{\pi}^w}{\beta(1+i_t)} \tag{D64}$$

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56. To derive the law of motion for P_t we have used the equilibrium condition $\pi_t = \pi_t^w/g_t$ and the law of motion for W_t.

$$(g_{t+1}-1)\left(1-\beta\left[\left(\frac{c_t}{c_{t+1}}\right)^{\sigma}g_{t+1}^{-\sigma}\chi\gamma\varpi L_{t+1}\right]\right)=0 \tag{D65}$$

$$c_t=\Psi L_t-\frac{g_{t+1}-1}{\chi(\gamma-1)} \tag{D66}$$

$$1+i_t=\max\left(\left(1+\bar{i}\right)L_t^{\phi},1\right), \tag{D67}$$

and $L_t\leq 1$ hold for all $t\in\{0,...,T\}$. We restrict attention to sequences that converge to the unemployment steady state. The algorithm follows these steps:

1 Guess a sequence $\{L_t\}_{t=1}^{T+1}$. Set $L_{T+1}=L^u$.

2 Use (D64), (D65), (D66) and (D67) evaluated at $t=0$, the initial condition L_0 and the guess for L_1 to solve for g_1 and c_0.

3 For any $t\in\{1,...,T\}$, use equations (D64) and (D67) evaluated at $t-1$ and values for L_{t-1}, g_t and c_{t-1} to solve for c_t. Use (D64), (D65), (D66) and (D67) evaluated at t, and L_{t+1} to solve for g_{t+1}.

4 Use equations (D64) and (D67) evaluated at T and values for L_T, g_{T+1} and c_T to solve for c_{T+1}.

5 Evaluate convergence by checking that the market clearing condition (D66) holds for any $t\in\{1,...,T\}$ and that the sequence has converged to the unemployment steady state. If $\sup||\Psi L_t-c_t-(g_{t+1}-1)(\chi(\gamma-1))||$ for all $t\in\{1,...,T\}$ and $||c_{T+1}-c^u||$ are sufficiently small we have found a solution. Otherwise compute:

$$\hat{L}_t=L_t-\epsilon\left(\Psi L_t-c_t-\frac{g_{t+1}-1}{\chi(\gamma-1)}\right),$$

where ϵ is a small positive number. Update the guess by setting $L_t=\min\left(\hat{L}_t,1\right)$ for any $t\in\{1,...,T\}$ and go to step 2.

E. MODEL WITH UNEMPLOYMENT RISK

In this appendix, we lay down the model with idiosyncratic unemployment risk described in Section 4.1. In this model, each household faces in every period a constant probability p of being unemployed. The employment status is revealed to the household at the start of the period. An unemployed household receives an unemployment benefit, such that its income is equal to a fraction $b<1$ of the income received by employed households. Unemployment benefits are financed with taxes on employed households.

The budget constraint of a household now becomes:

$$P_tC_t+\frac{b_{t+1}}{1+i_t}=v_tW_tL_t+b_t+d_t+T_t.$$

The only change with respect to the benchmark model is the presence of the variables v and T, which summarize the impact of the employment status on a household's budget. v is an indicator variable that takes value 1 if the household is employed, and 0 if the household is unemployed. T represents a lump-sum transfer for unemployed households, and a tax for employed households. T is set such that the income of an unemployed household is equal to a fraction b of the income of an employed household.[57]

Moreover, here we assume that households cannot borrow:

$$b_{t+1}\geq 0,$$

and that trade in firms' shares is not possible, so that every household receives the same dividends d.

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57. More precisely, an unemployed household receives a transfer:

$$T=\frac{bW_tL_t+(b-1)d_t}{1+\frac{bp}{1-p}},$$

while an employed household pays a tax

$$T=-\frac{p}{1-p}\frac{bW_tL_t+(b-1)d_t}{1+\frac{bp}{1-p}}.$$

The Euler equation is now:

$$c_t^{-\sigma} = \frac{\beta(1+i_t)E_t[c_{t+1}^{-\sigma}]}{g_{t+1}^{\sigma-1}\bar{\pi}^w} + \mu_t,$$

where μ_t is the Lagrange multiplier on the borrowing constraint and, as in the main text, $c_t \equiv C_t/A_t$.

We start by showing that the borrowing constraint binds only for unemployed households. Since neither households nor firms can borrow, in equilibrium every period every household consumes her entire income. Denoting, by c_t^e and c_t^{ne} the consumption of respectively an employed and an unemployed household, we have $c_t^{ne} = bc_t^e < c_t^e$. Moreover, due to the assumption of i.i.d. idiosyncratic shocks, $E_t[c_{t+1}^{-\sigma}]$ is independent of the employment status. Hence, from the Euler equation it follows that $\mu_t > 0$ only for the unemployed, and so the borrowing constraint does not bind for employed households.

The Euler equation of the employed households is:

$$(c_t^e)^\sigma = \frac{\bar{\pi}^w g_{t+1}^{\sigma-1}}{\beta(1+i_t)\rho E_t\left[(c_{t+1}^e)^{-\sigma}\right]},$$

where $\rho \equiv 1 - p + p/b^\sigma > 1$, and we have used the fact that the probability of becoming unemployed is independent of aggregate shocks. Moreover, using $c_t = pc_t^{ne} + (1-p)c_t^e = c_t^e(bp+1-p)$, we obtain:

$$c_t^\sigma = \frac{\bar{\pi} g_{t+1}^{\sigma-1}}{\beta(1+i_t)\rho E_t[c_{t+1}^{-\sigma}]}.$$

This equation, which is the equivalent of equation (2.17) in the baseline model, determines the demand for consumption in the model with idiosyncratic unemployment risk.

F. ALTERNATIVE GROWTH PROCESSES

F.1. *Innovation by incumbents*

In our baseline model, as in Aghion and Howitt (1992) and Grossman and Helpman (1991), all the innovation is performed by entrants. However, in the data most quality improvements of existing products seem the result of investment by incumbent firms, and the contribution of incumbents to productivity growth is substantial (Bartelsman and Doms, 2000). Motivated by these facts, in this appendix we describe a model in which innovation is carried out by incumbent firms. As in chapter 7 of Barro and Sala-i Martin (2004), incumbents end up investing in innovation because they enjoy a cost advantage in research with respect to entrants.

In line with the baseline model, we assume that an incumbent in a generic sector j that invests I_{jt} units of the final good to discover an improved version of product j innovates with probability:

$$\mu_{jt} = \min\left(\frac{\chi^I I_{jt}}{A_{jt}}, 1\right).$$

We follow Barro and Sala-i Martin (2004) and assume that χ^I, the parameter capturing incumbents' productivity in research, is sufficiently larger than χ, the parameter capturing outsiders' productivity in research, so that all the innovation activities are carried out by incumbents.

The flow of monopoly profit enjoyed by an incumbent with productivity A_{jt} is still given by $P_t \varpi A_{jt} L_t$.[58] The end-of-period value associated with being an incumbent with productivity A_{jt} is thus given by:

$$V_t^I(A_{jt}) = \beta\rho E_t\left[\frac{\lambda_{t+1}}{\lambda_t}\left(P_{t+1}\varpi A_{jt}L_{t+1} + (1-\eta)\left(\mu_{j,t+1}V_{t+1}^I(\gamma A_{jt}) + (1-\mu_{j,t+1})V_{t+1}^I(A_{jt}) - P_{t+1}I_{j,t+1}\right)\right)\right]. \quad \text{(F68)}$$

The first term inside the parenthesis on the right-hand side, also present in the baseline model, captures the expected profits to be gained in period $t+1$. The second term captures the incumbent's continuation value in period $t+1$. With probability $1-\eta$ the firm survives.[59] In period $t+1$ the firm makes a new discovery with probability $\mu_{j,t+1}$, in which case its value is given by $V_{t+1}^I(\gamma A_{jt})$. Instead, with probability $1-\mu_{j,t+1}$ no innovation occurs in period $t+1$, and the continuation value is $V_{t+1}^I(A_{jt})$. The term $P_{t+1}I_{j,t+1}$ captures the cost of the investment in innovation performed by the firm in period $t+1$. The expected profits are discounted using the households' discount factor $\beta\rho E_t[\lambda_{t+1}/\lambda_t]$.

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58. Since there are no competitors, however, each incumbent sets the unconstrained markup and so $\xi = 1/\alpha$.

59. As in the model with innovation by entrants, we assume that every period there is a constant probability η that the incumbent firm dies and is replaced by another firm that inherits its technology.

We now derive the equilibrium in the market for innovation. The expected return that an incumbent receives from investing I_{jt} in research is given by

$$\frac{\chi^I I_{jt}}{A_{jt}} \left(V_t^I(\gamma A_{jt}) - V_t^I(A_{j,t}) \right) - P_t I_{jt}. \tag{F69}$$

This expression captures the fact that the incumbent captures only the increment in present value from innovation, $V_t^I(\gamma A_{jt}) - V_t^I(A_{j,t})$, because the incumbent does not value the expropriation of its own monopoly profits. We focus on equilibria in which investment in innovation is positive and finite. Differentiating the previous expression with respect to I_{jt}, this requires that

$$P_t = \frac{\chi^I}{A_{jt}} \left(V_t^I(\gamma A_{jt}) - V_t^I(A_{j,t}) \right),$$

so that in equilibrium the innovation costs are equal to the expected gains. Moreover, we focus on symmetric equilibria in which $V_t^I(A_{jt}) = \bar{V}_t^I A_{jt}$ for every j. Plugging this condition in expression (F69) implies that in equilibrium $\bar{V}_t^I = P_t/((\gamma - 1)\chi^I)$. We can thus write expression (F68) as:

$$\frac{P_t}{\chi^I} = \beta \rho E_t \left[\frac{\lambda_{t+1}}{\lambda_t} \left(P_{t+1} \varpi (\gamma - 1) L_{t+1} + (1 - \eta) \frac{P_{t+1}}{\chi^I} \right) \right].$$

This expression summarizes the equilibrium in the research sector. Combining this expression with equation (2.2) gives:

$$g_{t+1}^\sigma = \beta \rho E_t \left[\left(\frac{c_t}{c_{t+1}} \right)^\sigma \left(\chi^I (\gamma - 1) \varpi L_{t+1} + 1 - \eta \right) \right],$$

which replaces equation (2.18) of the baseline model.

F.2. *Productivity spillovers across firms*

In this appendix, we lay down a version of the model in which the profits of non-innovating firm partly depend on aggregate productivity growth. For instance, this is the case in Grossman and Helpman (1991), in which profits are the same across all the firms and grow at the rate of aggregate productivity growth.

To implement this idea, we assume that the production function of the final good is:

$$Y_t = L_t^{1-\alpha} \int_0^1 \left(A_t^{1-\theta} A_{jt}^\theta \right)^{1-\alpha} x_{jt}^\alpha dj, \quad \text{where} \quad A_t \equiv \left(\int_0^1 A_{jt}^\theta dj \right)^{\frac{1}{\theta}}, \tag{F70}$$

where $0 < \theta \leq 1$. When $\theta < 1$, this production function implies that the productivity of an intermediate input is increasing in aggregate productivity, capturing the notion of productivity spillovers across firms. Instead, the case $\theta = 1$ corresponds to our baseline model in which spillovers are absent.

Following the same steps used to solve the baseline model, it is easy to show that under this production function aggregate output is given by:

$$Y_t = \left(\frac{\alpha}{\xi} \right)^{\frac{\alpha}{1-\alpha}} A_t L_t, \tag{F71}$$

while the profits of an incumbent firm in a generic sector j are equal to:

$$P_t \varpi A_t^{1-\theta} A_{jt}^\theta L_t. \tag{F72}$$

Hence, when $\theta < 1$ an increase in aggregate productivity generates a rise in profits for all firms. This implies that the profits of non-innovating firms grow at rate $g_t^{1-\theta}$. It is also easy to check that aggregate productivity growth is given by:

$$g_{t+1} = \left(\mu_t(\gamma^\theta - 1) + 1 \right)^{\frac{1}{\theta}}, \tag{F73}$$

where μ_t is the fraction of innovating sectors.

It is possible to embed this production function both in the model with innovation by entrants or incumbents. For concreteness, we focus on the latter case. As in the baseline model, we assume that the cost of innovation is proportional to the productivity of the intermediate input $A_t^{1-\theta} A_{jt}^\theta$:

$$\mu_{jt} = \min \left(\frac{\chi^I I_{jt}}{A_t^{1-\theta} A_{jt}^\theta}, 1 \right),$$

The growth equation in this version of the model, which can be derived following the steps outlined in Section F.1, is given by:

$$g_{t+1}^{\sigma - (1-\theta)} = \beta \rho E_t \left[\left(\frac{c_t}{c_{t+1}} \right)^\sigma \left(\chi^I (\gamma - 1) \varpi L_{t+1} + 1 - \eta \right) \right],$$

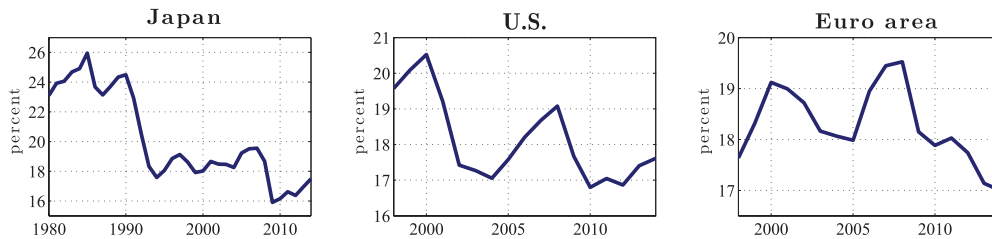which replaces equation (2.18) of the baseline model.

FIGURE 9

R&D intensity

G. DATA APPENDIX

This appendix provides details on the construction of the series underlying Table 1.

1. **Policy rate.** Central bank discount rate. Monthly series from the IMF International Financial Statistics database.

2. **Unemployment rate.** Annual series from the IMF International Financial Statistics database.

3. **Labour productivity.** Real GDP divided by total hours worked. Annual data from the Conference Board Total Economy Database. The Euro area is defined as the aggregate of Austria, Belgium, Finland, Germany, Greece, Ireland, Italy, Netherlands, Portugal, and Spain.

4. **R&D intensity.** R&D intensity is the ratio of investment in R&D to R&D stock for the business sector. We use this measure of investment in R&D for two reasons. First, this is the variable that determines the growth rate of productivity in our model, as well as in the seminal growth frameworks developed by Aghion and Howitt (1992), Grossman and Helpman (1991) and Romer (1990). Secondly, Kung and Schmid (2015), using U.S. data for the period 1953–2008, show that this variable predicts productivity, output and consumption growth at horizons of one to five years. Figure 9 shows the research intensity series underlying the numbers in Table 1.
Formally, R&D intensity is defined as $RD_{inv,t}/RD_{stock,t}$, where $RD_{inv,t}$ is investment in R&D by the business sector during period t, while $RD_{stock,t}$ is the stock of business R&D at the start of period t. We construct a measure of the business sector R&D stock in accordance with the methodology used by the U.S. Bureau of Labor Statistics. In particular, we assume that the R&D stock evolves according to

$$RD_{stock,t+1} = RD_{inv,t} + (1-\delta)RD_{stock,t},$$

where δ is the depreciation rate of the R&D stock. We set $\delta = 0.15$, which is consistent with the annualized depreciation rate of 15% assumed by the BLS in the R&D stock calculations. The initial value for the R&D stock is set to $RD_{stock,0} = RD_{inv,0}/\delta$.[60]
In the data, we measure $RD_{inv,t}$ as the nominal investment in R&D by the business sector deflated by the GDP deflator. The data on business investment in R&D are annual. For Japan, the data sources are the Statistical Survey Department, Statistics Bureau, Ministry of Internal Affairs and Communications, and the OECD. The U.S. data are from the National Science Foundation. Data for the Euro area come from the OECD. Due to the lack of data on business investment in R&D for Austria and Greece, the Euro area sample is composed of Belgium, Finland, Germany, Ireland, Italy, the Netherlands, Portugal, and Spain.[61] For each country in the sample, we computed country-level series for research intensity. In every period, the Euro area research intensity is defined as the sum of the individual country research intensities weighted by the country's population share.

5. **Real rate - long.** For the U.S. and Euro area we follow Rachel and Smith (2015), and proxy the long real rate by the nominal interest rate on ten years government bonds deflated by one-year ahead inflation expectations from Consensus Economics.[62] To control for changes in default risk, we use the return on German government bonds as our measure of return on government bonds in the Euro area.

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60. Setting $RD_{stock,0} = 0$ does not affect our results.

61. For the Netherlands, there is a break in the business R&D investment series between 2010 and 2011, due to a change in the sample used to compute R&D spending statistics. The OECD estimates that this change in data collection has led to an increase in the estimates of R&D spending by businesses of around 15%. To adjust for this break, we multiply the values of business R&D spending in the Netherlands during the period 2011–2014 by 0.85.

62. We thank Lukasz Rachel for sharing their data.

For Japan, since data on ten years government bonds going back to 1982 are not available, we use the rate on nine years government bonds as our measure of the long nominal rate. Since the series for inflation expectations from Consensus Economics go back only to 1990, we constructed a proxy for inflation expectations using the following procedure. We regressed the one-year ahead inflation expectations from Consensus Economics on a constant, the contemporaneous value of CPI inflation, and the CPI inflation lags at 12 and 24 months. We use the predicted values from this regression, which from 1990 on track closely the Consensus Economics series, as our measure of inflation expectations. Data for the return on government bonds are provided by the Japanese Ministry of Finance, while data on CPI inflation are from the IMF International Financial Statistics database.

6. **Real rate - short.** Policy rate deflated by one-year ahead inflation expectations. For the U.S. and the Euro area we use the one-year ahead inflation expectations from Consensus Economics. For Japan, we use the same proxy for inflation expectations used to deflate the long nominal rates.

Supplementary Data

Supplementary data are available at *Review of Economic Studies* online.

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