Designing a Simple Loss Function for Central Banks: Does a Dual Mandate Make Sense?*

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July 12, 2018

Abstract

Yes, it makes a lot of sense. This paper studies how to design simple loss functions for central banks, as parsimonious approximations to social welfare. We show, both analytically and quantitatively, that simple loss functions should feature a high weight on measures of economic activity, sometimes even larger than the weight on inflation. Two main factors drive our result. First, stabilising economic activity also stabilises other welfare-relevant variables. Second, the estimated model features mitigated inflation distortions due to a low elasticity of substitution between monopolistic goods and a low interest rate sensitivity of demand. The result holds up in the presence of measurement errors, with large shocks that generate a trade-off between stabilising inflation and resource utilisation, and also when imposing a moderate degree of interest rate volatility.

The recent global financial crisis and the sovereign debt crisis in Europe have revived a classical debate about the role of central banks and what their objectives should be. A simple answer is that central banks should enhance social welfare. But in practice, the design of central banks' objectives is a much more complicated task. After the inflationary episodes of the 1970's and the 1980's, there has been broad consensus on the need to separate the conduct of monetary policy from direct political influence. At the same time, basic democratic principles require that central banks are assigned with a clear set of goals and held accountable for their actions. For this reason,

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We are grateful to the editor Morten Ravn and two anonymous referees for very helpful suggestions. Jordi Galí, Marc Giannoni, Lars Svensson, Andrea Tambalotti, and our discussants Tom Tallarini, Julio Andres Blanco, and David Berger also provided very helpful comments. We also thank seminar and conference participants at the Bank of England, Bank of Japan, Banque de France, Banco de Portugal, Central Bank of Brazil, ECB, the Federal Reserve Banks of Atlanta, New York and Richmond, Federal Reserve Board, IMF, National Bank of Belgium, National University of Singapore, Norges Bank, Sveriges Riksbank, Universidade Nova de Lisboa, Universitat Pompeu Fabra, University London Queen Mary, University of Surrey, University of Virginia, Society of Economic Dynamics 2014, and Southern Economic Association Conference 2014 for very useful feedback. Part of the work on this paper was carried out while some of the authors were employees (Lindé and Nunes) and consultant (Kim) of the International Finance Division at the Federal Reserve Board, Boston Fed (Nunes) and the IMF (Lindé). Jinill Kim acknowledges the support by Korea Research Foundation Grant funded of the Korean Government (NRF-2013S1A5A2A03044693) and Debortoli acknowledges support from the Marie Curie FP7-PEOPLE-2013-IIF grant 628855 - MONFISCPOL and the Spanish Ministry of Economy and Competitiveness grant ECO2014-55555-P. The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of Sveriges Riksbank or of any person associated with the Federal Reserve System.

many central banks are mandated to pursue "simple" objectives, which involve only a few target variables (see e.g. the surveys of Svensson, 2010, and Reis, 2013). The simplicity of the objectives makes monetary policy more transparent, facilitates accountability, and simplifies communication with the public.[1]

There is an open debate over the formulation of simple mandates. Should central banks be mainly responsible for maintaining price stability, for stabilising economic activity, or for both? The goal of this paper is to contribute to this important debate.

Advances in academic research, notably the seminal work of Rogoff (1985) and Walsh (1995), supported a strong focus on price stability as a means to enhance the independence and credibility of central banks. As discussed in further detail in Svensson (2010), an overwhelming majority of central banks also adopted an explicit inflation target. For example, the mandate of the European Central Bank is to maintain price stability, without any explicit reference to economic activity.[2]

One exception to the reigning central banking practice is the U.S. Federal Reserve, which since 1977 has been assigned the so-called "dual mandate" which requires it to "promote maximum employment in a context of price stability".[3] Only as recently as January 2012, the Fed announced an explicit long-run inflation target of 2 percent but also made clear its intention to keep a balanced approach between mitigating deviations of both inflation and employment from target levels.

Our reading of the academic literature to date, perhaps most importantly the seminal work by Woodford (2003), is that a welfare-maximising central bank should put a high weight on inflation relative to other variables.[4] In a similar framework, Blanchard and Galí (2007) established that stabilising inflation allows the central bank to simultaneously stabilise all the welfare-relevant measures of economic activity—a property known as the "divine coincidence". Taken together, these findings suggest that the strong focus on inflation stabilisation by many central banks is sufficient for macroeconomic stabilisation, and that the focus on resource utilisation in the Fed's mandate is redundant or even harmful.

But is a dual mandate really harmful, or could it in fact benefit societies? In this paper we

[1] Prominent scholars like Svensson (2010) also argue that a simple mandate is more robust to model and parameter uncertainty than a more complicated objective. As an alternative to simple mandates, Taylor and Williams (2010) argue in favor of simple and robust policy rules.

[2] The primary objective of the European Central Bank, set out in Article 127(1) of the Treaty on the Functioning of the European Union, is to maintain price stability within the Eurozone. The ECB Governing Council in October 1998 defined price stability as headline HICP inflation year-on-year increase of close but under 2 percent.

[3] The dual mandate was codified only in the Federal Reserve Reform Act of 1977. See Bernanke (2013) for a summary of the Federal Reserve's one hundred years.

[4] Rotemberg and Woodford (1998) and Woodford (2003) showed that the objective function of households in a basic New Keynesian sticky-price model could be approximated as a (purely) quadratic function in inflation and the output gap, with the weights determined by the specific features of the economy.

revisit this question. For comparability to the earlier literature, most of our analysis focuses on a simple mandate which includes only two variables: price inflation and a measure of economic activity (e.g. the output gap). We then investigate to what extent placing a high weight on economic activity is desirable from a social-welfare perspective. The main novelty of our work is to study economies with several sources of inefficiencies—both nominal and real rigidities. As we argue below, assigning a high weight on standard measures of economic activity could be strongly beneficial, as economic activity serves as an overall proxy for welfare-relevant variables not included in the simple mandate.[5] To illustrate this result, we start with a simplified model which permits an analytical solution, and then perform numerical experiments in a rich quantitative model.

The simple model we use is the canonical New Keynesian sticky-price and sticky-wage model of Erceg, Henderson and Levin (2000), EHL henceforth. Using this model, we show how the desirability of a dual mandate depends on specific features of the economy, like the relative magnitude of the different distortions. Contrary to the conventional wisdom, we find that the optimal simple mandate features a high weight on the output gap. This is because as long as the inefficiencies due to price and wage rigidities are of similar magnitudes, a property which seems to hold empirically (see e.g. Christiano et al., 2010, and Smets and Wouters, 2007), the output gap summarises all the welfare-relevant frictions in the goods and labour markets.

Next, we perform a quantitative analysis of simple mandates within the estimated medium-scale model of Smets and Wouters (2007) model, SW henceforth. That model can be viewed as the backbone of the models used for policy analysis at many central banks and policy institutions, and thus constitutes a natural laboratory for our purposes.[6] On the contrary, many of the previous studies have focused on simple calibrated models. Even though policy recommendations are model consistent, their relevance may be questioned given the simplicity of those models and the fact that they have not been estimated.

In line with the analytical results in the simple EHL model, our numerical analysis shows that a large weight on any of the typical measures of resource utilisation (e.g. the output gap, detrended output, or output growth) improves welfare significantly. Specifically, we find that the optimised

[5] In a medium-scale model as in Smets and Wouters (2007) the welfare criterion includes a large number of target variables. For that reason, it would infeasible, from any practical purposes, to include all the targets in a central bank mandate. See also Edge (2003), who derives analytically the welfare criterion for a model with capital accumulation.

[6] The SW model, being an empirical model consistent with optimising behavior, should be less prone to the Lucas (1976) critique than other studies on optimal monetary policy that are based on backward-looking models (see e.g. Rudebusch and Svensson, 1999, and Svensson, 1997). Consistent with this argument, several papers estimating dynamic general-equilibrium models have found that the deep parameters are largely invariant to alternative assumptions about the conduct of monetary policy. For example, see Adolfson, Laséen, Lindé and Svensson (2011), Ilbas (2012), and Chen, Kirsanova and Leith (2013).

weight on the output gap is about 1 in a simple loss function with the weight on annualised inflation normalised to unity. This value is considerably higher than the reference value of 0.048 derived in Woodford (2003) and the value of 0.25 assumed by Yellen (2012).[7] The high weight on the output gap stems from several empirically relevant characteristics of the estimated model which reduce the importance of inflation relative to the output gap. These include a low elasticity of substitution between monopolistic goods, price indexation to lagged inflation by non-optimising firms, and a low interest rate sensitivity of demand. In addition, significant real rigidities in goods and labour markets as modelled in Kimball (1995) tempers the degree of price stickiness (enabling the model to fit the microevidence on price setting) and thereby makes inflation fluctuations less costly relative to output fluctuations.

The sizeable welfare gains attained with a large weight on output gap originate from the presence of a significant trade-off between stabilising inflation and the output gap. At first glance, this result may appear to be contradictory to Justiniano, Primiceri and Tambalotti (2013), who estimated a similar model, and argued that there is no important trade-off between those two objectives. However, the different findings can be reconciled by recognising that the key drivers behind the trade-off in the SW model—the price- and wage-markup shocks—are absent in the baseline model analyzed by Justiniano et al. (2013).[8] While considerable uncertainty remains about the role of these inefficient shocks as drivers of business cycles, we want to stress that our results hold regardless. In particular, if inefficient shocks are irrelevant for business cycle fluctuations, then stabilising inflation or output is approximately equivalent, and attaching a high weight to output is still optimal. And as long as inefficient shocks do play some role—as in the SW model—a high weight on output stabilisation is imperative.[9]

Our results remain valid in a variety of alternative environments. For instance, following Orphanides and Williams (2002), we explore a realistic situation in which both the actual and the potential level of output are measured with significant errors in real time. In addition, and in line with Levin et al. (2005), we show that alternative variables like wage inflation and employment may improve welfare relative to a standard mandate with inflation and the output gap.[10] Finally, we imposed limits to the volatility of interest rates, to ensure a low probability of hitting the zero

[7] Yellen (2012) assumed a value of unity for the unemployment gap, which by the Okun's law translates into a value of 0.25 for the output gap.

[8] The alternative model of Justiniano et al. (2013) includes wage-markup shocks and is closer to the model in this paper.

[9] Our basic finding that the central bank should respond vigorously to resource utilisation is consistent with the arguments in Reifschneider, Wascher and Wilcox (2013) and English, López-Salido and Tetlow (2013).

[10] Levin et al. (2005) suggest that nominal wage inflation suffices to approximate Ramsey policy well. Our analysis corroborates this finding conditional on an important role for the hours gap.

lower bound. In all these cases, the optimal weight on measures of economic activity remains high.

The remainder of the paper is structured as follows. We start in Section 1 by describing how to compute the optimal (Ramsey) policy and evaluate the alternative simple mandates. In Section 2, we present the analytical results with the EHL model. Section 3 turns to the numerical analysis with the SW model. The robustness of the results in the SW model along some key dimensions is subsequently discussed in Section 4. Finally, Section 5 provides some concluding remarks and suggestions for further research.

1 The Utility-Based Welfare Criterion

We begin our analysis by defining a welfare criterion to evaluate the performance of alternative simple mandates. As shown in Rotemberg and Woodford (1998) and Benigno and Woodford (2012), households' welfare can be approximated by a (purely) quadratic function:

$$\sum_{t=0}^{\infty} \mathrm{E}_0 \left[\beta^t U(X_t)\right] \simeq \text{constant } - \sum_{t=0}^{\infty} \mathrm{E}_0 \left[\beta^t X_t' W^H X_t\right], \tag{1}$$

where X_t is a $N \times 1$ vector with the model variables measured as their deviation from the steady state; therefore, $X_t' W^H X_t$ is referred to as the quadratic approximation of the household utility function $U(X_t)$.

A welfare criterion like (1) can be obtained in a large class of economic models, including models with sizeable frictions and an inefficient steady state.[11] In general, the matrix W^H in eq. (1) is obtained through a second-order approximation of the utility function $U(.)$ and of all the equations describing the private sector behavior. As a result, deriving the welfare criterion requires making specific assumptions about the functional forms of technology, demand, adjustment functions, etc. For example, in the SW model, we make assumptions about the capital utilisation cost function, the investment adjustment cost function, and the Kimball aggregators, so that the functions are consistent with the linearised equations in the original paper. More details about the derivation of the welfare criterion are described in online Appendix A.

As a benchmark for our analysis, we first solve for the optimal Ramsey policy. We define Ramsey policy as a policy that maximises (1) subject to the $N-1$ constraints of the economy (the N^{th} equation is provided by the evolution of the monetary policy instrument). Since the constant term in (1) depends only on the deterministic steady state of the model, which is invariant across

[11] Many theoretical studies assume the presence of subsidies that eliminate steady-state distortions. Instead, most prominent empirically oriented papers including Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003, 2007) consider economies with inefficient steady states.

different policies considered in this paper, the optimal policy implemented by a Ramsey planner can be solved as

$$\tilde{X}_t^*\left(W^H;\tilde{X}_{t-1}\right)\equiv\underset{X_t}{\arg\min}\mathrm{E}_0\left[\sum_{t=0}^{\infty}\beta^t X_t' W^H X_t\right], \tag{2}$$

where following Marcet and Marimon (2012), the Lagrange multipliers associated with the constraints become state variables. Accordingly $\tilde{X}_t'\equiv[X_t',\ \varpi_t']$ now includes the Lagrange multipliers ϖ_t. For expositional ease, we denote these laws of motion more compactly as $\tilde{X}_t^*\left(W^H\right)$.

To calculate welfare using eq. (1) requires taking a stance on the initial conditions. Doing so is particularly challenging when Lagrange multipliers are part of the vector of state variables because these are not readily available. We therefore adopt the unconditional expectations operator as a basis for welfare evaluation.[12] The loss under Ramsey optimal policy is then defined by

$$Loss^R=\mathrm{E}\left[\left(X_t^*\left(W^H\right)\right)' W^H\left(X_t^*\left(W^H\right)\right)\right]. \tag{3}$$

Our choice of an unconditional expectation as the welfare measure is standard in the literature (see for instance Woodford, 2003). Furthermore, when the discount factor is close to unity—as is the case in our calibration—unconditional and conditional welfare are also quite similar.[13]

The Ramsey policy is a useful benchmark. But in practice societies provide central banks with a simple loss function with only few target variables. Such a simple loss function can be represented by

$$\mathrm{E}_0\left[\sum_{t=0}^{\infty}\beta^t X_t' W^{CB} X_t\right], \tag{4}$$

where W^{CB} is a sparse matrix with only a few non-zero entries. Given a simple mandate, the optimal behavior of the central bank is

$$\tilde{X}_t^*\left(W^{CB};\tilde{X}_{t-1}\right)=\underset{X_t}{\arg\min}\mathrm{E}_0\left[\sum_{t=0}^{\infty}\beta^t X_t' W^{CB} X_t\right]. \tag{5}$$

When the simple mandate does not coincide with the Ramsey policy, we have that $W^{CB}\neq W^H$ and therefore that $\tilde{X}_t^*\left(W^{CB}\right)\neq\tilde{X}_t^*\left(W^H\right)$.[14] To compute the extent to which the simple mandate

[12] See Jensen and McCallum (2010) for a detailed discussion about this criterion—with a comparison to the timeless perspective. They motivate the optimal unconditional continuation policy based on the presence of time inconsistency, since the policy would reap the credibility gains successfully. We note, however, that our approach does not exactly follow theirs in that their optimal steady state could be different from the steady state under the Ramsey policy in a model with steady-state distortions.

[13] The unconditional criterion is equivalent to maximising the conditional welfare when the society's discount factor, $\tilde{\beta}$ in the expression $\left(1-\tilde{\beta}\right)\mathrm{E}_0\left[\sum\tilde{\beta}^t\left[\tilde{X}_t^{CB}\left(W^{CB};\tilde{X}_{t-1}\right)\right]' W^H\left[\tilde{X}_t^{CB}\left(W^{CB};\tilde{X}_{t-1}\right)\right]\right]$, is approaching unity. In our case, we have that $\beta\gamma^{-\sigma_c}=0.993$ based on the parameter values in Table C.1.

[14] One can only obtain that $W^{CB}\neq W^H$ and $\tilde{X}_t^*\left(W^{CB}\right)=\tilde{X}_t^*\left(W^H\right)$ in special circumstances. Related to this point, we will show in the analysis that if certain trade-offs are not salient in the model then changing certain coefficients in W^{CB} will not affect welfare much.

approximates optimal policy, one can calculate its associated loss according to the formula:

$$Loss^{CB}\left(W^{CB}\right) = \mathrm{E}\left[\left(X_t^*\left(W^{CB}\right)\right)' W^H \left(X_t^*\left(W^{CB}\right)\right)\right]. \tag{6}$$

The performance of the simple mandate can then be assessed by taking the difference between $Loss^{CB}$ in eq. (6) and $Loss^{R}$ in eq. (3). Throughout the paper, we express this welfare difference in consumption equivalent variation (CEV) units as follows:

$$CEV = 100\left(\frac{Loss^{CB} - Loss^{R}}{\bar{C}\left(\frac{\partial U}{\partial C}|_{s.s.}\right)}\right), \tag{7}$$

where $\bar{C}\left(\frac{\partial U}{\partial C}|_{s.s.}\right)$ can be interpreted as how much welfare increases when consumption in the steady state is increased by one percent. That is, CEV represents the percentage point increase in households' consumption, in every period and state of the world, that makes them in expectation equally well-off under the simple mandate as they would be under Ramsey policy.[15]

So far we have proceeded under the assumption that the law governing the behavior of the central bank specifies both the variables and the weights in the quadratic objective, i.e. W^{CB} in (4). But in practice, the mandates of central banks are only indicative and not entirely specific on the weights that should be attached to each of the target variables. A straightforward way to model this is to assume that society designs a law Ω that constrains the weights on some variables to be equal to zero, without imposing any restriction on the exact weight to be assigned to the remaining variables. When determining the simple mandate consistent with the law Ω, we assume the central bank is benevolent and selects a weighting matrix, W^{CB^*}, which minimises the expected loss of the society. Formally,

$$W^{CB^*} = \underset{W \in \Omega}{\arg\min} \mathrm{E}\left[\left(X_t^*\left(W\right)\right)' W^H \left(X_t^*\left(W\right)\right)\right], \tag{8}$$

where the weighting matrix W^H is defined by (1).

Throughout the analysis, we assume that the central bank operates under commitment. We believe this is a good starting point for two reasons. First, the evidence in Debortoli, Maih and Nunes (2014) and Debortoli and Lakdawala (2016), suggests that the Federal Reserve operates with a high degree of commitment, at least before the zero lower bound became binding.[16] Second, this

[15] Given the presence of habits, there are two ways to compute CEV. One can choose whether the additional consumption units do or do not affect the habit component (lagged consumption in each period). Consistent with the convention (see e.g. Lucas, 1987, and Otrok, 2001) of increasing the steady-state consumption in all periods, our chosen measure is calibrated to the case where both current and lagged consumption are increased. It is imperative to understand that the ranking of the mandates is invariant with respect to which measure is used. The only difference between the two measures is that the other measure is 3.4125 times smaller, reflecting that accounting for the habit component requires a larger steady-state compensation. In the limit when the habit coefficient $\varkappa$ is set to unity, households would need to be compensated in terms of consumption growth.

[16] Bodenstein, Hebden and Nunes (2012) report evidence of a lower degree of commitment at the zero lower bound.

assumption makes our analysis more comparable with the literature on simple interest rate rules, which also imply some form of central bank commitment.

Our approach is similar in spirit to the extensive literature that has studied the design of optimal simple rules [see e.g. Collard and Dellas (2006), Juillard et al. (2006), Levin et al. (2005), Kim and Henderson (2005), and Schmitt-Grohé and Uribe (2007)]. The key difference with respect to this literature is that we focus on simple mandates. Also, the variables that should be included in the simple mandate are not necessarily those that make simple rules mimic the Ramsey policy.

2 Analytical Results in a Canonical New Keynesian Model

This section considers the canonical sticky-price and sticky-wage model with constant aggregate capital by Erceg, Henderson and Levin (2000) to build intuition for the analysis with the workhorse SW model with endogenous aggregate capital. The key point we want to make is that stabilising the output gap also helps stabilise additional welfare-relevant variables. For this reason, it is desirable to attach a significant weight to the output gap in simple mandates that do not include all the welfare-relevant targets. We show analytically that under certain conditions the weight on the output gap should be infinite. More generally, we derive an approximate expression for the weight on the output gap, which can be easily calculated using a few simple statistics. Below, we first outline the model environment, and then establish our results in a log-linearised version.

2.1 The EHL Model

2.1.1 Firms and price setting

Final Goods Production: Final output Y_t is produced by competitive firms, combining a continuum of intermediated goods $Y_t(f)$ purchased at a price $P_t(f)$, according to a Dixit-Stiglitz aggregator

$$Y_t = \left[\int_0^1 Y_t(f)^{\frac{1}{1+\theta_p}} df \right]^{1+\theta_p}, \tag{9}$$

where the net markup $\theta_p > 0$. Cost minimisation leads to the following expression for the aggregate price index P_t:

$$P_t = \left[\int_0^1 P_t(f)^{\frac{-1}{\theta_p}} df \right]^{-\theta_p}. \tag{10}$$

Intermediate Goods Production: A continuum of intermediate goods $Y_t(f)$ for $f \in [0,1]$ is produced by monopolistically competitive firms, each producing a single differentiated good. From eqs. (9) and (10), it follows that each intermediate goods producer faces the following demand function:

$$Y_t(f) = \left[\frac{P_t(f)}{P_t}\right]^{\frac{-(1+\theta_p)}{\theta_p}} Y_t. \tag{11}$$

Each intermediate goods producer utilises capital services $K_t(f)$ and a labour index $L_t(f)$ (defined below) to produce an output good according to a Cobb-Douglas production function:

$$Y_t(f) = Z_t K_t(f)^{\alpha} L_t(f)^{1-\alpha}, \tag{12}$$

where Z_t is total factor productivity. Firms face perfectly competitive factor markets for hiring capital (which is constant in the aggregate at $\bar{K}$, but shares of it can be freely allocated among the f intermediate producers) and the labour index. Thus, each firm chooses $K_t(f)$ and $L_t(f)$, taking as given both the rental price of capital R_t^K and the aggregate wage index W_t (defined below), and can costlessly adjust either factor of production. Thus, the standard static first-order conditions for cost minimisation imply that all firms have identical marginal cost per unit of output.

The prices of the intermediate goods are determined by Calvo (1983) and Yun (1996) style staggered nominal contracts. In each period, each firm f faces a constant probability, $1-\xi_p$, of being able to re-optimise its price $P_t(f)$. If a firm is not allowed to re-optimise its price in a given period, it is assumed that it adjusts its price by the steady-state rate of inflation, i.e., $P_t(f) = (1+\pi) P_{t-1}(f)$. This leads to the following first-order condition for the optimal price

$$\mathrm{E}_t \sum_{j=0}^{\infty} \xi_p^j \psi_{t,t+j} \left[\frac{(1+\tau_p)(1+\pi)^j P_t^{opt}(f)}{1+\theta_p} - MC_{t+j}\right] Y_{t+j}(f) = 0, \tag{13}$$

where τ_p is a subsidy which undoes the steady-state distortion of monopolistic competition. By implication of eq. (10) and the updating formulae for the non-optimising firms, the evolution of the final goods price is given by

$$P_t = \left[(1-\xi_p)\left(P_t^{opt}\right)^{\frac{-1}{\theta_p}} + \xi_p\left((1+\pi) P_{t-1}\right)^{\frac{-1}{\theta_p}}\right]^{-\theta_p}, \tag{14}$$

where we have used the fact that all firms that re-optimise will set the same price (because they face the same marginal costs), and that the updating price for the non-optimising firms equals the past aggregate price (as we consider a continuum of firms which does not re-optimise).

2.1.2 Households and wage setting

There is a continuum of identical households $h \in [0,1]$, with preferences given by

$$\mathbb{E}_t \sum_{j=0}^{\infty} \beta^j \{ \frac{1}{1-\sigma_c} \left[C_{t+j}\left(h\right) \right]^{1-\sigma_c} - \frac{1}{1+\sigma_l} \left[N_{t+j}\left(h\right) \right]^{1+\sigma_l} \}, \tag{15}$$

where the discount factor β satisfies $0 < \beta < 1$, σ_c denotes the intertemporal elasticity of substitution of consumption, and σ_l the inverse of the Frisch elasticity of labour supply. The period utility function depends positively on household h's current consumption $C_t\left(h\right)$, and inversely on hours worked $N_t\left(h\right)$. EHL also included real money balances and two preference shocks, but we omit them for expositional simplicity.

Household h's budget constraint in period t states that its expenditure on goods and net purchases of financial assets must equal its disposable income:

$$P_t C_t\left(h\right) + \delta_{t+1,t} B_t(h) - B_{t-1}(h) = \left(1+\tau_w\right) W_t\left(h\right) N_t\left(h\right) + \Gamma_t\left(h\right) - T_t(h). \tag{16}$$

Asset accumulation consists of net acquisition of state-contingent claims. Each element of the vector $\delta_{t+1,t}$ represents the price of an asset that will pay one unit of currency in a particular state of nature in the subsequent period, while the corresponding element of the vector $B_t(h)$ represents the quantity of such claims purchased by the household. $B_{t-1}(h)$ indicates the value of the household's claims given the current realisation of the state of nature. Labour income $W_t\left(h\right) N_t\left(h\right)$ is subsidised at a fixed rate τ_w. Each household owns an equal share of all firms and of the aggregate fixed capital stock, and receives an aliquot share $\Gamma_t\left(h\right)$ of aggregate profits and rental income. The government's budget is balanced every period, so that lump-sum transfers $T_t(h)$ to the household equal the net of output and labour subsidies.

Following EHL, we assume that each household supplies a differentiated labour service $N_t\left(h\right)$ to the production sector. It is convenient to assume that a representative labour aggregator (union) combines households' labour hours in the same proportions as firms would choose. The union minimises the cost of producing a given amount of the aggregate labour index, taking each household's wage rate $W_t\left(h\right)$ as given, and then sells units of the labour index to the production sector at their unit cost W_t. This leads to the following well-known Dixit-Stiglitz relationships for the aggregate labour and real wage indexes:

$$L_t = \left[\int_0^1 N_t\left(h\right)^{\frac{1}{1+\theta_w}} dh \right]^{1+\theta_w}, W_t = \left[\int_0^1 W_t\left(h\right)^{\frac{-1}{\theta_w}} dh \right]^{-\theta_w}, \tag{17}$$

where the net wage markup $\theta_w > 0$. The remainder of the wage and labour decisions are modelled analogously with the goods pricing and production decisions, in which ξ_w is the degree of nominal wage stickiness.

2.2 Log-linearised Solution

The log-linearized solution of the model is characterized by the equations

$$\pi_t^p = \beta \mathrm{E}_t \pi_{t+1}^p + \kappa_p y_t^{gap} + \vartheta_p \omega_t^{gap} \tag{18}$$

$$\pi_t^\omega = \beta \mathrm{E}_t \pi_{t+1}^\omega + \kappa_w y_t^{gap} - \vartheta_w \omega_t^{gap} \tag{19}$$

$$\omega_t^{gap} \equiv \omega_{t-1}^{gap} + \pi_t^\omega - \pi_t^p - \Delta\omega_t^n, \tag{20}$$

where $\vartheta_p \equiv \left(\left(1-\beta\xi_p\right)\left(1-\xi_p\right)/\xi_p\right)\left(\left(1-\alpha\right)/\left(1-\alpha+\alpha(1+\theta_p)/\theta_p\right)\right)$, $\kappa_p = \frac{\alpha}{1-\alpha}\vartheta_p$, $\vartheta_w \equiv (1-\beta\xi_w)$ $(1-\xi_w)\ /\ [\xi_w\left(1+\sigma_l\left(1+\theta_w\right)/\theta_w\right)]$ and $\kappa_w \equiv \left(\sigma_c + \frac{\sigma_l}{1-\alpha}\right)\vartheta_w$. Eqs. (18) and (19) are the New-Keynesian Phillips curves describing the evolution of price inflation π_t^p and wage inflation π_t^ω as a function of the output gap y_t^{gap}, and the real wage gap ω_t^{gap}. The latter variable, defined in eq. (20), measures the deviation of the actual real wage ω_t from its frictionless counterpart ω_t^n. Similarly, y_t^{gap} is the deviation of actual output from its efficient level. The composite parameters κ_p and ϑ_p (κ_w and ϑ_w) are both inversely related to the probability of the firm (household) not being able to re-optimise its price (nominal wage), implying that their values fall when the degree of price (wage) stickiness increases.

The quadratic approximation to the household utility around a non-distorted steady state gives rise to the following loss function

$$L_t^R = \frac{1}{2}\left[\left(\pi_t^p\right)^2 + \lambda_w^{opt}\left(\pi_t^w\right)^2 + \lambda_y^{opt}\left(y_t^{gap}\right)^2\right], \tag{21}$$

where $\lambda_w^{opt} \equiv \frac{1+\theta_w}{\theta_w}\frac{\theta_p}{1+\theta_p}\frac{\vartheta_p}{\vartheta_w}\left(1-\alpha\right)$ and $\lambda_y^{opt} \equiv \frac{\theta_p}{1+\theta_p}\vartheta_p\left(\sigma_c + \frac{\sigma_l+\alpha}{1-\alpha}\right)$ denote the weights on wage inflation and output gap relative to price inflation.

For our purposes, we consider that the central bank is assigned the following simple mandate,

$$L_t^{CB} = \frac{1}{2}\left[\left(\pi_t^p\right)^2 + \lambda_y\left(y_t^{gap}\right)^2\right], \tag{22}$$

which does not include one of the target variables in the social loss function L_t^R, namely wage inflation.[17] Next, we study how to select the appropriate weight λ_y so that the actual central bank policy under this simple but suboptimal mandate is as close as possible to the optimal policy (i.e. minimise $L_t^{CB} - L_t^R$).

A critical feature of this economy is that it is not possible to simultaneously stabilise the output gap and the two inflation rates. For example, in response to changes in the natural real-wage—e.g.

[17] This is without loss of generality. The same considerations would apply as long as only one inflation rate is included.

due to changes in productivity—perfectly stabilising the output gap requires a change in the real wage, and thus a change in either prices or nominal wages (or both). As a result, as it can be seen from eqs. (18)-(20), it is not feasible to achieve simultaneously $y_t^{gap} = 0$, $\pi_t^p = 0$, and $\pi_t^w = 0$.

Nevertheless, combining eqs. (18) and (19) gives that the composite inflation index $\vartheta_w \pi_t^p + \vartheta_p \pi_t^\omega$evolves according to

$$\vartheta_w \pi_t^p + \vartheta_p \pi_t^w = \beta \mathrm{E}_t \left[\vartheta_w \pi_{t+1}^p + \vartheta_p \pi_{t+1}^w \right] + (\vartheta_w \kappa_p + \vartheta_p \kappa_\omega) \, y_t^{gap}. \tag{23}$$

This equation implies that perfectly stabilising the output gap leads to perfect stabilisation of the composite inflation index $\vartheta_w \pi_t^p + \vartheta_p \pi_t^\omega$, where a higher weight is attached to the inflation rate of the sector of the economy where nominal rigidities are more severe. Thus, stabilising the output gap also mitigates the costs of nominal rigidities both in the goods and in the labour markets.

In what follows, we study under which circumstances such a policy is actually desirable. Because a complete analytical solution for the optimal simple mandate is infeasible, we present our results in two exercises. First, we solve the dynamic model in a case with equal slope of the price and wage Phillips curves. Second, we solve a static version of the model, with arbitrary slopes of the two Phillips curves.

2.3 A Dynamic Model with Equal Slope of Price and Wage Phillips Curves

Let's first consider a benchmark case of equal slope of the price and wage Phillips curves, that is $\kappa_p = \kappa_w \equiv \kappa$. According to the findings in Smets and Wouters (2007) for the U.S. and Christiano, Motto and Rostagno (2010) for the euro area (and the U.S.), this case is arguably empirically relevant and has the virtue that the model admits an analytical solution with $\lambda_w^{opt} = \vartheta_p / \vartheta_w$. As shown in online Appendix B, the optimal Ramsey policy can in this case be described by the targeting rule

$$\vartheta_w \pi_t^p + \vartheta_p \pi_t^w = -\frac{\lambda_y^{opt}}{\kappa} \vartheta_w \left(y_t^{gap} - y_{t-1}^{gap} \right), \tag{24}$$

which combined with eq. (23) implies that in equilibrium $y_t^{gap} = 0$ and $\vartheta_w \pi_t^p + \vartheta_p \pi_t^\omega = 0$ in all periods $t \geq 0$.[18]

The intuition for this result is as follows. In principle, tolerating some output gap may require smaller adjustments of prices and wages, and thus reduce the costs associated with nominal rigidities. However, as it can be seen from eqs. (18) and (19), when the output gap is fully stabilised,

[18] Throughout our examples, we use the initial condition $y_{-1}^{gap} = 0$, which is consistent with the optimal policy under commitment.

price and wage inflation move in opposite directions, and the ratio between the two movements is $-\vartheta_p/\vartheta_\omega$. If this ratio coincides with the weight on the variance of nominal wages in the loss function, λ_w^{opt}, there is no incentive to change the relative volatility of the two inflation rates. In addition, since $\kappa_p = \kappa_w = \kappa$, a unitary change in the output gap changes the two inflation rates by the same amount κ. Even though the volatility of one of the inflation rates may decrease, the welfare costs of nominal rigidities—$(\pi_t^p)^2 + \lambda_w^{opt}(\pi_t^w)^2$—would necessarily increase. As a result, the central bank does not have any incentive to allow for fluctuations in the output gap, and strict output gap targeting is optimal.

This reasoning and the conditions in eqs. (23) and (24) allow us to derive analytically the value of λ_y that maximises households' welfare in the simple mandate given by eq. (22). When doing so, we find that it is optimal to assign an infinite weight to output gap stabilisation, i.e. $\lambda_y = \infty$. Moreover, it turns out that the simple mandate in this case also replicates the optimal policy, so $L_t^R = L_t^{CB}$ in equations (21) and (22).[19] Any other weight λ_y in the simple mandate implies a welfare loss for households. In particular, there is a welfare loss if the central bank exclusively focuses on price stability or assigns a low/negligible weight on the output gap.

2.4 A Static Model with Arbitrary Slopes of Price and Wage Phillips Curve

When the sensitivity of price and wage inflation to the output gap differs, full stabilisation of the output gap is generally not optimal. An analytical expression for the optimal weight λ_y is not available in such a general case. However, it is still possible to get some insights about the factors affecting the magnitude of λ_y within a static version of the model.

In particular, suppose there is only one period ($t = 0$), and there is no uncertainty. Also, assume the initial conditions are $\omega_{-1}^{gap} = \omega_{-1}^n = 0$, and the terminal conditions are $\pi_1^p = \pi_1^w = 0$. Since the economy is back to steady state in period 1, there is no scope for managing inflation expectations and hence there is no distinction between commitment and discretionary policies.

Under these assumptions, eq. (20) can be used to substitute for ω_t, so that eqs. (18) and (19) simplify to

$$\pi_0^p = \tilde{\kappa}_p y_0^{gap} + \tilde{\vartheta}_p \omega_0^n \tag{25}$$

$$\pi_0^w = \tilde{\kappa}_w y_0^{gap} - \tilde{\vartheta}_w \omega_0^n, \tag{26}$$

[19] Note that optimal policy can also be implemented with a simple mandate that includes both price and wage inflation, with weight $\lambda_{\pi^\omega} = \vartheta_p/\vartheta_\omega$. This case is a knife-edge case because it requires the central bank to have a perfect estimate of $\vartheta_\omega/\vartheta_p$. Even in this case any non-negative λ_y continues to be optimal, including $\lambda_y = \infty$.

where $\tilde{\kappa}_p \equiv \frac{\vartheta_p\kappa_w+\kappa_p(1+\vartheta_w)}{1+\vartheta_p+\vartheta_w}$, $\tilde{\vartheta}_p \equiv \frac{-\vartheta_p}{1+\vartheta_p+\vartheta_w}$, $\tilde{\kappa}_w \equiv \frac{\vartheta_w\kappa_p+\kappa_w(1+\vartheta_p)}{1+\vartheta_p+\vartheta_w}$, and $\tilde{\vartheta}_w \equiv \frac{-\vartheta_w}{1+\vartheta_p+\vartheta_w}$. Minimising (21) subject to the latter two equations implies that under the optimal Ramsey policy

$$y_t^{gap} = -\psi^{opt}\omega_t^n, \tag{27}$$

with

$$\psi^{opt} \equiv \frac{\tilde{\kappa}_p\tilde{\vartheta}_p - \tilde{\kappa}_w\tilde{\vartheta}_w\lambda_w^{opt}}{\lambda_y^{opt} + \tilde{\kappa}_p^2 + \lambda_w^{opt}\tilde{\kappa}_w^2}. \tag{28}$$

In this case, it is easy to show that a central bank which follows the simple mandate (22) implements the optimal equilibrium if (and only if) the weight on the output gap

$$\lambda_y = \frac{\tilde{\vartheta}_p\tilde{\kappa}_p}{\psi^{opt}} - \tilde{\kappa}_p^2. \tag{29}$$

The last equation indicates that an approximate measure for λ_y could be inferred from simple statistics. In particular, λ_y is inversely related to the parameter ψ^{opt} in eq. (28) which determines the volatility of output gap according to eq. (27) under the optimal policy. ψ^{opt}, in turn, crucially depends on the differences between the parameters of the price inflation and wage inflation Phillips curve, i.e. $\tilde{\kappa}_p\tilde{\vartheta}_p - \tilde{\kappa}_w\tilde{\vartheta}_w\lambda_w^{opt}$ as can be seen from eq. (27).[20] Intuitively, in economies where wage and price inflation have similar impacts on real activity, stabilising the output gap helps achieve the optimal balance between the volatility of the two inflation rates. However, if prices are much more rigid than wages (or if the price elasticity of output demand is higher than the wage elasticity of labour demand) so that $\tilde{\vartheta}_p\tilde{\kappa}_p$ is low, the optimal weight on the output gap should be small.

For instance, under the baseline calibration in Galí (2008), where wages are more rigid than prices, the parameters of the Phillips curve are $\tilde{\kappa}_p = 0.02$ and $\tilde{\vartheta}_p = 0.04$, while $\tilde{\kappa}_w = 0.03$ and $\tilde{\vartheta}_w = 0.01$. Those values imply that $\psi^{opt} = 0.0021$, and that the output gap should receive a weight that is about 6.5 times the weight on (annualised) inflation—arguably a much larger weight than under the conventional wisdom. For the full dynamic model we find that the optimal weight under commitment is even higher (38.5). Hence, the simplifying assumptions we made in order to deduce an analytical solution are not the driver of the high weight on the output gap.

2.5 Additional Considerations

In more complex models, with several welfare-relevant targets, it is often not possible to replicate the optimal policy by following a simple mandate. Nevertheless, the basic result that targeting

[20] Consistently with the previous analysis, in the special case with $\lambda_w^{opt} = \vartheta_p/\vartheta_w$ and $\kappa_p = \kappa_w \equiv \kappa$, then $\tilde{\kappa}_p\tilde{\vartheta}_p = \tilde{\kappa}_w\tilde{\vartheta}_w\lambda_w^{opt}$ and the optimal policy prescribes to fully stabilise the output-gap, i.e. $\psi^{opt} = 0$. Also, in the limiting case where either prices or wages are flexible, any value of λ_y would replicate the optimal policy, including $\lambda_y = \infty$.

the output gap helps to stabilise additional welfare-relevant variables remains valid. Consider for instance the standard NK model with sticky prices and partial indexation (ι_p) to past inflation for the non-optimising firms. In this case it is well known that the true welfare loss function is given by $L^R = (\pi_t - \iota_p \pi_{t-1})^2 + \lambda_y^{opt} (y_t^{gap})^2$. However, suppose now that following common practice the central bank does not target the quasi-difference in inflation, but simply just inflation π_t. In this case, it can easily be shown that the Ramsey policy is replicated only when $\lambda_y = \infty$ in the simple mandate (22). The intuition for this result is that even though the central bank is not targeting the welfare correct quasi-change in inflation $(\pi_t - \iota_p \pi_{t-1})$, the central bank effectively stabilises the correct inflation variable by stabilising the output gap.

Similar findings arise in models in which production sectors are heterogeneous in the degree of price stickiness or in the elasticities of substitution across various goods. For instance, if we consider the model of Aoki (2001) but make the central bank to target headline rather than core inflation, then stabilising the output gap in the simple mandate is optimal.[21] Moreover, Bodenstein, Erceg and Guerrieri (2008) and Natal (2012) argue that energy price fluctuations is yet another reason why a large weight on the output gap approximates optimal policy well.

Notably, Woodford (2003) acknowledges that output gap stabilisation can deliver results very close to welfare optimal policies and has the advantage of producing very robust results under different calibrations. Erceg, Henderson and Levin (2000) also advocated the robustness and efficiency of output gap stabilisation in the context of simple rules. However, even if our and their analyses have shown that there are several convincing theoretical arguments why the output gap deserves a large weight in simple mandates, there are also some key arguments not considered thus far that may limit the desirability of stabilising measures of economic activity. One of them is the presence of inefficient price- and wage-markup shocks. As is shown in online Appendix B, the introduction of shocks which creates a substantial trade-off between stabilising inflation and the output gap makes it non-optimal to fully stabilise output gap fluctuations because doing so will create unwarranted excessive movements in price and wage inflation. An additional important consideration is the presence of measurement errors, which also may limit considerably the benefits of targeting the output gap. These and other issues are explicitly analyzed next in the context of the estimated workhorse SW model.

[21] This insight is empirically relevant because, as noted in the introduction, the European Central Bank has a mandate in terms of headline rather than core inflation.

3 Quantitative Analysis

We now turn to the quantitative analysis within the workhorse model of Smets and Wouters (2007). Following the EHL model described in Section 2, this model includes monopolistic competition in the goods and labour markets and nominal frictions in the form of sticky prices and wages, but it allows non-optimizing firms (households) to index prices (wages) to a composite of steady-state and lagged inflation. Households can also save in physical capital, with a one-period time to build before new investments turns into productive capital. In addition to the EHL model, the Smets and Wouters model also features several real rigidities in the form of habit formation in consumption, investment adjustment costs, variable capital utilisation, and fixed costs in production. The model dynamics are driven by six structural shocks. Two inefficient shocks—variations in price- and wage-markups—follow an ARMA(1,1) process. Four efficient shocks (total factor productivity, risk premium, investment-specific technology, and government spending shocks) follow AR(1) processes. The exact specification of the model is described in detail in online Appendix C.

The model parameters are fixed at the posterior mode of the SW original estimates.[22] An alternative approach would be to allow for both parameter and model uncertainty (see e.g. Walsh, 2005). However, we believe it is instructive to start out by performing our exercise in a specific model, under specific parameter values. Throughout the analysis, we discuss the sensitivity of our results to alternative parameterisations.

3.1 Benchmark Results

Table 1 reports our benchmark results. The benchmark simple mandate we consider reflects the standard practice of monetary policy, and is what Svensson (2010) refers to as "flexible inflation targeting." Specifically, we use the framework in Woodford (2003) and assume that the simple mandate can be captured by the following period loss function

$$L_t^a = (\pi_t^a - \pi^a)^2 + \lambda^a x_t^2, \tag{30}$$

where π_t^a denotes the annualised rate of quarterly inflation and x_t is a measure of economic activity with λ^a denoting its corresponding weight.

We consider three different measures of economic activity. Our first measure is the model-

[22] Adolfson et al. (2012) find that the estimated deep parameters are invariant to assuming that the central bank follows a Taylor-type interest rate rule or assuming that it minimises a standard loss function.

consistent output gap,

$$y_t^{gap} = y_t - y_t^{pot}, \tag{31}$$

i.e. the difference between actual and potential output where the latter is defined as the level of output that would prevail if prices and wages were fully flexible and inefficient markup shocks were excluded.[23] The second measure we consider is simply the level of output (as deviation from the deterministic labour-augmented trend, i.e. $y_t - \bar{y}_t$). Finally, we also consider annualised output growth in the spirit of the work on "speed-limit" policies by Walsh (2003).

Table 1: *Benchmark Results for "Flexible Inflation Targeting" Mandate in eq.* (30).

	x_t: Output gap		x_t: Output (dev from trend)		x_t: Output growth (Ann.)	
Simple Mandate	λ^a	CEV (%)	λ^a	CEV (%)	λ^a	CEV (%)
Woodford (2003)	0.048	0.471	0.048	0.554	0.048	0.611
Dual Mandate	0.250	0.140	0.250	0.276	0.250	0.404
Optimised Weight	1.042	0.044	0.542	0.244	2.943	0.302

Notes. CEV denotes the consumption equivalent variation (in percentage points) needed to make households indifferent between the Ramsey policy and the simple mandate under consideration according to eq. (7). The "Dual Mandate" refers to a weight of unity for the unemployment gap in the loss function (30), which translates into $\lambda^a = 0.25$ when applying a variant of Okun's law. Finally, "Optimised Weight" refers to minimisation of eq. (6) w.r.t. λ^a in eq. (30).

The first two rows of Table 1 contain a comparison between two benchmark values of λ^a. In the first row of Table 1 we set $\lambda^a = 0.048$, corresponding to the welfare-maximising weight on output-gap in Woodford (2003).[24] The second row of Table 1 examines instead the dual mandate. In a recent speech, Yellen (2012) describes the dual mandate through a simple loss function that assigns equal weights for annualised inflation and the unemployment gap (i.e. actual unemployment minus the NAIRU).[25] In addition, Yellen stipulates that the Federal Reserve converts the unemployment gap into an output gap according to a value of roughly 0.5—and such a value is based on the widely spread empirical specification of the Okun's law $u_t - u_t^{pot} = (y_t - y_t^{pot})/2$. Accordingly, the unit weight on the unemployment gap converts into a weight of $\lambda^a = 0.25$ on the output gap.[26]

As we can see from the second row in Table 1, increasing the weight on real activity from the value of Woodford (2003) to the value consistent with the dual mandate significantly reduces welfare

[23] This measure of potential output is below the efficient level (roughly by a constant amount) because we do not assume that steady-state subsidies remove the distortions due to habits externalities and monopolistic competition. An alternative definition of potential output—e.g. the measure of the U.S. Congressional Budget Office—is based on the noninflationary maximum level of output. See Plosser (2014) for a discussion about these two measures from a policy perspective.

[24] More precisely, Woodford's (2003) quarterly weight of $\lambda^q = 0.003$ translates into an annualised weight of $\lambda^a = 16\lambda^q = 0.048$. Throughout this paper, we will report annualised values.

[25] A similar description of the dual mandate is also present in Svensson (2011), where the weight placed on economic activity is substantially higher than in Woodford (2003). See also Reifschneider, Wascher and Wilcox (2013) and English, López-Salido and Tetlow (2013).

[26] Moreover, Gali, Smets and Wouters (2011) argue within a variant of the SW model with unemployment that fluctuations in their estimated output gap closely mirror those experienced by the unemployment rate. Therefore, the Okun's law we apply can also find support empirically in a structural modelling framework.

losses, namely by a factor of three for our benchmark measure of economic activity (the output gap), and by about a factor of two for alternative measures (output level and output growth). In all cases, the welfare gains are large compared to similar studies in the monetary policy literature—e.g. larger than the threshold value of 0.05% used by Schmitt-Grohe and Uribe (2007).

The last row in Table 1 displays the results when the weight λ^a is optimised. The key finding is that the optimal value of λ^a is much higher than the values considered so far, for all measures of economic activity. For example, the optimised coefficient for the output gap is 1.042. Coincidentally, this is very similar to the unit weight on the unemployment gap as used in Yellen (2012). For the level of output (as deviation from trend), the optimised coefficient is lower (0.5) but still twice as high as implied by the dual mandate. In the case of output growth, the optimised coefficient is even higher (around 2.9), which essentially is a so-called speed-limit regime (see Walsh, 2003). Notably, our analysis shows that adopting a simple mandate with a high weight on any of the resource utilisation measures improves welfare with respect to considering the model-based output gap but assigning to it a low weight—e.g. as in Woodford (2003).[27] This is since assigning a high weight to detrended output or output growth in the loss function helps reducing considerably the volatility of output gap, albeit not to the same extent as when targeting it directly.

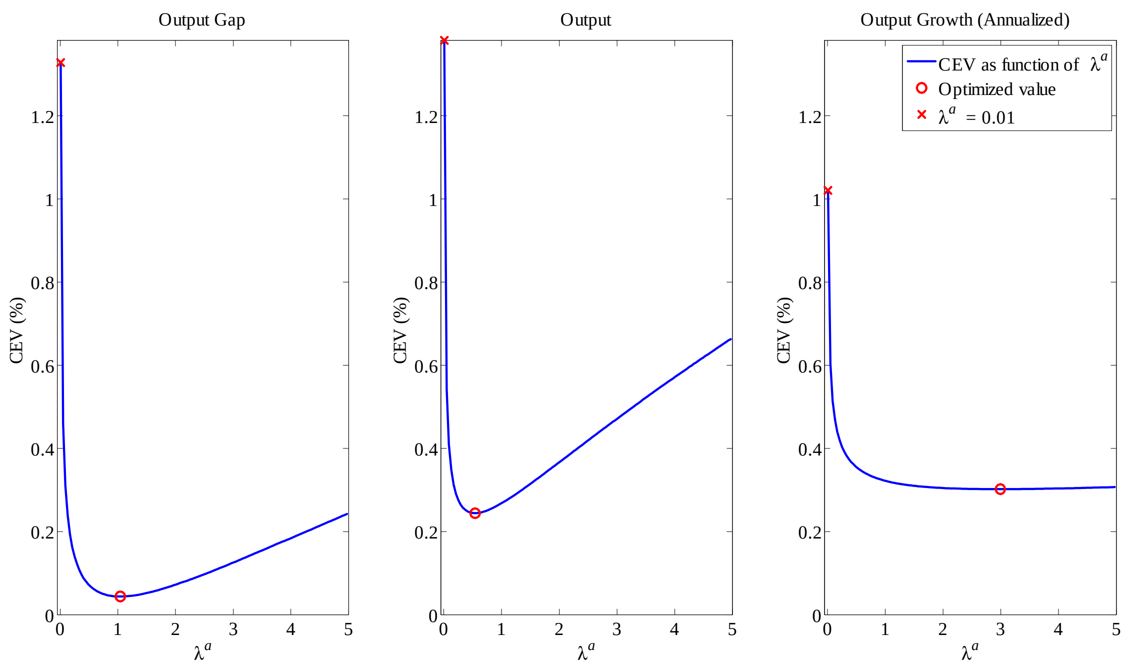
To gauge the sensitivity of the CEV with respect to the weight assigned to resource utilisation, Figure 1 plots the CEV as a function of λ^a for the three resource measures. Consistent with the results in Table 1, we see that there is quite some curvature of the CEV function for small values of λ^a for all three measures. Moreover, for the output gap we see that values of λ^a between 0.5 and 1.5 perform about equally well, whereas the mandate with detrended output has a higher curvature near the optimum. For output growth, the figure shows that any value above unity yields virtually the same CEV.

To clarify the mechanism behind our results, we follow Taylor (1979), Erceg, Henderson and Levin (1998), and Clarida et al. (1999) and study the main trade-offs involved in stabilising measures of inflation vs. measures of economic activity through variance frontiers. Figure 2 plots the variance of price or wage inflation (horizontal axis), together with measures of economic activity (vertical axis), while letting the weight λ^a vary from a small (0.01) to a large value (5.00). The slope of the resulting curve is referred to as the trade-off between the two variances. The upper panels refer to the benchmark loss function with price inflation and output gap. Panel A shows that there

[27] We have also analyzed loss functions with a yearly inflation rate, i.e. $\ln(p_t/p_{t-4})$, instead of the annualised quarterly inflation rate in eq. (30). Our findings change little for this alternative inflation measure. For the output gap, for example, the optimised λ^a is equal to 0.95 with a CEV of 0.044. These results are very close to our benchmark findings of $\lambda^a = 1.04$ and CEV$= 0.044$.

is a clear trade-off between stabilising price inflation and the output gap. Indeed, a lower volatility of output gap is always associated with an increase in the volatility of price inflation. Instead, Panel B shows that there is not necessarily a trade-off between stabilising output gap and wage inflation. For example, as long as $\lambda^a < 0.1$, reducing the volatility of the output gap also reduces the volatility of wage inflation, consistent with our theoretical results of Section 2. Figure 2 shows that increasing the weight λ^a on the output gap up to a value of 0.1 also stabilises wage inflation—a welfare-relevant variable not explicitly targeted by the central bank in its loss function. In fact, the volatility of nominal wage inflation remains lower relative to a benchmark strict inflation targeting loss function ($\lambda^a = 0.01$) for values of λ^a up to 0.4 (not shown in Panel B). This explains why in this economy measures of economic activity should receive a relatively high weight in a central bank's simple mandate that does not include all the welfare-relevant targets.

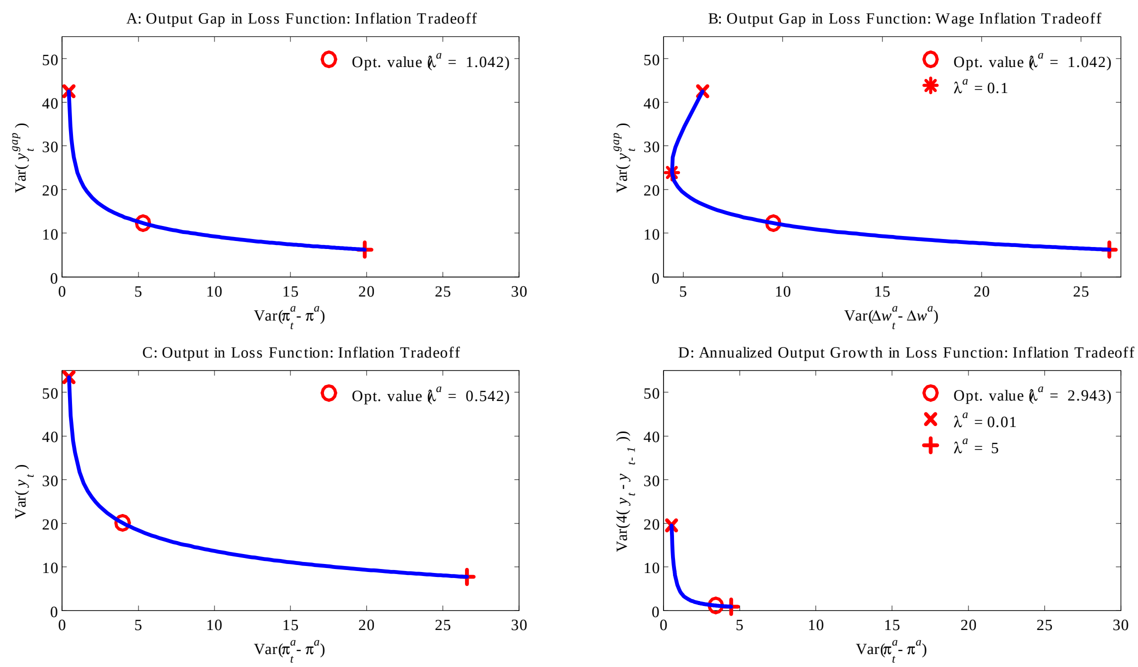
Figure 1: *Consumption Equivalent Variation (percentage points) as Function of the Weight (λ^a) on Economic Activity.*



Notes. The figure plots the CEV (in %) for the simple mandate with inflation and: output gap (left panel), output level (middle panel), output growth (right panel) The coordinate with an '×' mark shows the CEV for $\lambda^a = 0.01$, the 'o' mark shows the CEV for the optimised weight.

The lower panels of Figure 2 plot variance frontiers when the measure of economic activity is given by the output level and output growth both in the loss function and the frontier itself. Panel D in the figure shows that the trade-off between stabilising inflation and economic activity is most favorable when the resource utilisation measure is output growth; the variance of annualised output growth can be reduced to nearly 1 percent without $\text{Var}(\pi_t^a)$ increasing by much. Moreover, the flatness of the CEV witnessed in the right panel of Figure 1 for values of λ^a higher than optimal can be readily explained by the fact that panel D in Figure 2 shows that such values induce only small changes in the volatilities of inflation and output growth. For detrended output shown in panel C, the figure shows that the trade-off is most pronounced. Accordingly, values of λ^a higher than optimal translate into a higher curvature of the CEV function in Figure 1.

Figure 2: *Variance Frontier for Alternative Resource Utilisation Measures.*



Notes. The figure plots the variance frontier for the simple mandate with inflation and: output gap (Panel A), output level (Panel C), output growth (Panel D). Panel B shows the variance combination of the output gap and the annualised nominal wage inflation when varying λ^a for the price inflation-output gap loss function (i.e. same loss function as in Panel A). The coordinate with an '×' mark shows the volatility for $\lambda^a = 0.01$, the 'o' mark shows the volatility for the optimised weight, and the '+' mark shows the volatility for $\lambda^a = 5$.

As noted in Section 1, a strength of the methodology used in this paper is that it can handle a non-efficient steady state. The results in Table 1 and Figure 1, however, are robust to allowing for subsidies to undo the steady-state distortions stemming from the presence of external habits, as well as firms' and households' monopoly power in price and wage setting. For detrended output and the output gap, the optimised weights are even larger when considering the efficient steady state; for example, λ^a equals 2.34 with an associated CEV of 0.0119 for the output gap when the steady state is efficient. For output growth, however, the optimised λ^a is notably lower (0.43). But given the flatness of the CEV function in Figure 1, it is not surprising that the exact weight for output growth can be somewhat sensitive to the specific assumptions. Even so, the optimised weight remains relatively large, reflecting the larger curvature for smaller values of λ^a. In principle, moving from a distorted to an efficient steady state could make a big difference when we consider a model with relatively large distortions in both goods and labour markets. However, in our model, the surge in steady-state output when removing these distortions are to a large extent offset by removing external habit formation, so the efficient steady-state level for output is only about 6 percent higher than our distorted steady state.[28]

3.2 The Importance of Real Activity

In this section, we seek to clarify further why the model suggests that a high weight on real economic volatility improves household welfare. We begin the analysis by using the parameter estimates of SW (see Table A.1) to recompute λ^a according to the analytic formula provided in the sticky-price model by Woodford (2003):

$$\lambda^a \equiv \frac{16\kappa_p}{\left(\frac{\phi_p}{\phi_p - 1}\right)}, \tag{32}$$

where κ_p is the coefficient for the output gap in the linearised pricing schedule (i.e. in the New Keynesian Phillips curve, see eq. 18), and $\frac{\phi_p}{\phi_p-1}$ is the elasticity of demand of intermediate goods ($\phi_p \equiv 1 + \theta_p$). In the SW model, the NKPC is given by

$$\pi_t - \iota_p \pi_{t-1} = \beta\gamma^{1-\sigma_c} \left(\mathrm{E}_t \pi_{t+1} - \iota_p \pi_t\right) + \frac{\left(1 - \beta\gamma^{1-\sigma_c}\xi_p\right)\left(1 - \xi_p\right)}{\xi_p\left(\left(\phi_p - 1\right)\epsilon_p + 1\right)} mc_t + \varepsilon_t^p, \tag{33}$$

where the parameters ι_p, β, σ_c and ξ_p have been defined in Section 2, γ is the gross steady-state growth rate of output and ϵ_p is the Kimball elasticity (calibrated 10 in SW, a value of nil provides Dixit-Stiglitz calibration).

Unfortunately, in the fully fledged SW model, there is no analytical expression available for the direct sensitivity of inflation to the output gap (κ_p). This parameter depends on the mapping between the output gap and real marginal cost, which is not available in a model with endogenous aggregate capital and sticky wages. But abstracting from the latter two features, one would get a value of $\kappa_p = 0.264$, which combined with the estimated average markup ϕ_p results into a value of $\lambda^a = 1.62$.

A value of 1.62 is considerably higher than Woodford's (2003) value of 0.048, mainly for four reasons. First, the estimated gross markup in SW ($\phi_p = 1.61$) implies a substantially lower substi-

[28] See Levine, McAdam and Pearlman (2008) for a more detailed discussion of why the inefficient and efficient steady states are not too different.

tution elasticity ($\frac{\phi_p}{\phi_p - 1} = 2.64$) compared to Woodford's value (7.88). If we replace Woodford's value with the one estimated by SW, λ^a in eq. (32) rises to 0.30. Second, if we replace Woodford's value of the intertemporal substitution elasticity (6.25) with the value estimated by SW ($1/\sigma_c = 0.72$), λ^a increases further to 0.92. Third, if we relax the assumption of firm-specific labour (the Yeoman-farmer model of Rotemberg and Woodford, 1997), we have that λ^a equals 1.50. The remaining small difference to the SW value (1.62) can largely be explained by the slightly higher degree of price stickiness in Woodford's calibration. Fourth and finally, real rigidities in the form of the Kimball aggregators for prices and wages play an important role as they enable the SW model to fit both the macroevidence of a low sensitivity of price (wage) inflation to marginal costs (labour wedge) and the microevidence suggesting frequent price (and wage) re-optimisation every 3-4 quarters (see e.g. Klenow and Malin, 2010, and Nakamura and Steinsson, 2013). Had the estimated Smets and Wouters (2007) model not included this feature, the price stickiness parameter would have been considerably higher (about 0.9 as in Smets and Wouters, 2003), and the optimal weight on the output gap considerably lower (about 0.10 according to eq. 32). But again, such a high degree of price stickiness is at odds with the microevidence, and the very reason why the SW model features Kimball aggregators.

Table 2: *Perturbations of the Benchmark Model.*

	x_t: Output gap		x_t: Output (dev from trend)		x_t: Output growth (Ann.)	
Simple Mandate	λ^a	CEV (%)	λ^a	CEV (%)	λ^a	CEV (%)
Benchmark	1.042	0.044	0.542	0.244	2.943	0.302
No Indexation	0.318	0.042	0.179	0.220	0.817	0.285
No ε_t^p Shocks	0.914	0.039	0.343	0.220	1.235	0.278
No ε_t^w Shocks	2.094	0.020	0.355	0.213	1.267	0.226
Small ε_t^p and ε_t^w Shocks	1.268	0.024	0.112	0.167	0.157	0.180
No ε_t^p and ε_t^w Shocks	Large	0.016	0.161	0.150	0.025	0.134

Notes. "No Indexation" refers to setting $\iota_p = \iota_w = 0$; "No ε_t^p (ε_t^w) Shocks" refers to setting the variance of the price markup shock (wage markup shock) to zero; "Small ε_t^w and ε_t^p Shocks" means that the std. of these shocks are set to a 1/3 of their baseline values; and "No ε_t^w and ε_t^p Shocks" refers to setting the variance of both shocks to zero. "Large" means that the optimised value is equal or greater than 5.

The previous analysis is only suggestive, as it omits some of the key features of the SW model—wage stickiness and capital accumulation. As a consequence, the corresponding values of λ^a only partially reflect the full model structure. We now therefore turn to explore what are the key mechanisms within the fully fledged SW model. Our approach is to turn off one friction or shock at a time to isolate its impact on the optimal weight λ^a. The findings are summarised in Table 2.

Row 2 of Table 2 considers a case with no indexation in price- and wage-setting ($\iota_p = \iota_w = 0$) while keeping the other parameters at their baseline values. In this case, the optimised weight is about a third of the benchmark value for all the measures of economic activity considered. For example, the weight on the output gap (column 2) falls from 1.04 to 0.318.

To understand the role played by indexation, it is instructive to go back to the simple New Keynesian model with indexation and sticky prices. As discussed in Section 2.3, in that model the welfare-based loss function is given by

$$(\pi_t - \iota_p \pi_{t-1})^2 + \lambda \left(y_t^{gap}\right)^2 . \tag{34}$$

Suppose further, for simplicity, that inflation dynamics in equilibrium can be represented by an AR(1) process $\pi_t = \rho\pi_{t-1} + \varepsilon_t$. In that case, the welfare metric can be expressed as

$$\mathrm{E}_0 \left[(\rho - \iota_p)^2 (\pi_{t-1})^2 + \lambda_y^{opt} \left(y_t^{gap}\right)^2 \right], \tag{35}$$

where the term ε_t is assumed to be independent of policy, and can thus be ignored. The relative weight on output gap is then given by $\lambda_y^{opt}/(\rho - \iota_p)^2$. In several estimated models like SW, inflation persistence (ρ) is in large part explained by the indexation parameters (ι_p). Therefore, these two parameter tend to take similar values, and thus the relative weight on output gap is large. Intuitively, in economies where prices have a component indexed to their lags, the distortions arising from inflation are not as severe. Consequently, there is less need to stabilise inflation.

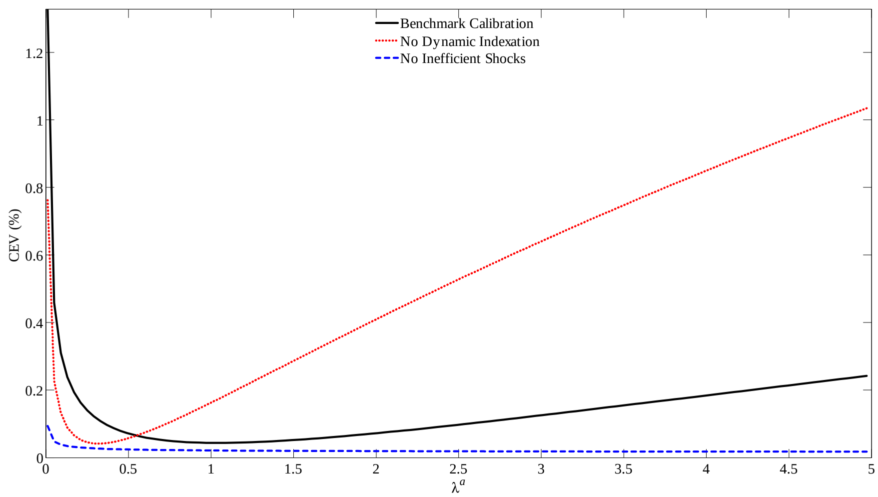
In spite of these considerations, our results confirm the importance of targeting economic activity, even in economies with no indexation in prices and wages. In fact, the optimal λ^a is still higher than the value implied by the dual mandate.[29] In addition, as shown in Figure 3, low values of λ^a (below 0.2) would lead to a rather sharp decline in welfare. This is a reassuring result, given the weak support of indexation found in micro-data.

Rows 3–6 in Table 2 examine the role of the inefficient markup shocks. The key point is that even when one of these shocks is taken out of the model, the central bank should still respond vigorously to economic activity. For the simple mandate with the output gap (column 2), λ^a is large regardless of the size of markup shocks. Instead, for the simple mandates with output and output growth (columns 4 and 5), λ^a falls only when both shocks are reduced or taken out completely. As an alternative to reducing the size of the markup shocks, we also reduced the steady-state gross markups from 1.61 (ϕ_p) and 1.5 (ϕ_w), respectively, to 1.20 following the evidence in Christiano,

[29] Indexation to lagged inflation matters more in wage-setting (ι_w) than in price-setting (ι_p). Still, setting $\iota_p = 0$ but keeping ι_w unchanged at 0.65 results in an optimised $\lambda^a = 0.82$, close to our benchmark optimised value. If in addition to setting $\iota_p = \iota_w = 0$ we set the inflation target to zero ($\pi^a = 0$), then the model does not feature indexation to either lagged inflation or to a positive steady-state inflation rate. In this case the results are essentially identical to those reported in the no indexation case of Table 2.

Eichenbaum and Evans (2005). Also under this parametrisation, we find that a large weight on economic activity is optimal. For instance, the optimised λ^a for the output gap equals 1.01.

Figure 3: *CEV (in percentage points) as Function of* λ^a *for Alternative Calibrations.*

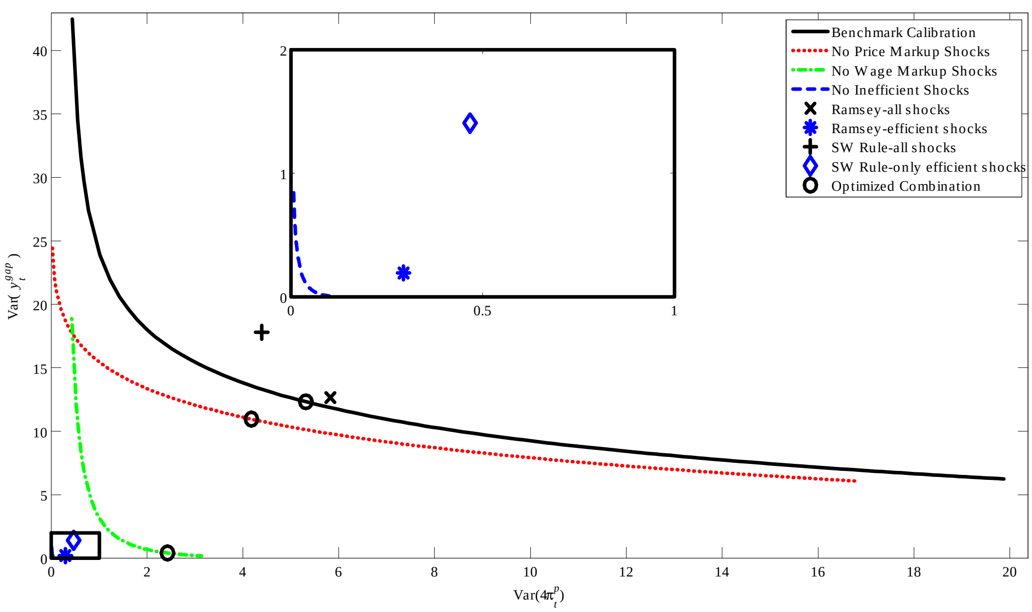


Notes. The figure plots the CEV (in %) as a function of λ^a for three different calibrations. The solid line refers to the benchmark calibration. The dotted line refers to the calibration in which $\iota_p = \iota_w = 0$. The dashed line refers to the calibration in which $var\left(\varepsilon_t^w\right) = var\left(\varepsilon_t^p\right) = 0$.

In terms of welfare, Figure 3 shows that when both markup shocks are set to nil, any $\lambda^a > 0.1$ produces roughly the same CEV of about 0.016, although a $\lambda^a \geq 5$ generates the lowest welfare losses. This finding is supported by our analytical results in Section 2, which established that the weight on the output gap should be very high in a simple mandate like eq. (30) when the distortions in goods and labour markets are of similar magnitude. Even so, the flatness of the CEV as a function of λ^a in Figure 3 shows that in the absence of price- and wage-markup shocks there is only a weak trade-off between inflation and output gap stabilisation. This suggests that the *divine coincidence* property holds approximately in this case, implying that the weight on the output gap is largely inconsequential.

The policy trade-offs for alternative calibrations of the model are illustrated in Figure 4. The figure shows variance frontiers when varying λ^a from 0.01 to 5. It also includes the implied variances of inflation and output gap under the Ramsey policy and the estimated SW policy rule with all shocks (marked by black 'x' and '+' marks, respectively) and without markup shocks (the blue '*' and '$\diamond$' marks). As expected, the variances implied by the estimated SW rule and the Ramsey policy lie outside the frontier associated with the simple mandate (solid black line). However, when λ^a is set at its optimal value of 1.042, the variances of inflation and the output gap are very close to those implied by the Ramsey policy. We interpret this finding as providing a strong indication that the simple mandate approximates the Ramsey policy well in terms of the equilibrium output gap and inflation, and not just in terms of welfare.[30] The estimated SW rule is instead associated with a lower variance of price inflation, but a higher variance of the output gap. Turning to the role of the markups shocks, Figure 4 shows that the trade-off between inflation and output gap remains sizeable both in the absence of wage markup shocks (dash-dotted green line), and in the absence of price markup shocks (red dotted line). Only when both the inefficient shocks are excluded, the trade-off is relatively small (dashed blue line in Figure 4, shown in more detail in the small inset box).

Figure 4: *Variance Frontiers for Alternative Calibrations.*



Notes. The figure plots the variance frontier for several calibrations: benchmark (solid line), $var\left(\varepsilon_t^p\right) = 0$ (dotted line), $var\left(\varepsilon_t^w\right) = 0$ (dashed-dotted line), and $var\left(\varepsilon_t^w\right) = var\left(\varepsilon_t^p\right) = 0$ (dashed line). The 'o' mark shows the volatility for the optimised weight both for the benchmark and $var\left(\varepsilon_t^p\right) = 0$ calibrations. The coordinates with an '×' and the '+' mark denote the Ramsey and SW policy rule, respectively, with all shocks. The box in the graph zooms in the case with $var\left(\varepsilon_t^w\right) = var\left(\varepsilon_t^p\right) = 0$, in which coordinates with '*' and '⋄' marks denote the Ramsey and SW policy rule, respectively.

Since substantial uncertainty remains about the importance of markup shocks over the business cycle, we also consider a case where at least a small proportion of the observed variation in inflation and wages is in fact driven by inefficient price- and wage-markup shocks. The fifth row in Table 2 reports results in which the standard deviations of both the inefficient shocks have been set to

[30] Although the Ramsey policy is associated with higher inflation and output gap volatility relative to the loss-function frontier, simple mandates are nevertheless inferior in terms of households' welfare.

a third of their baseline values. For the wage-markup shock, this alternative calibration can be motivated by the empirical work by Galí, Smets and Wouters (2011), who can distinguish between labour supply and wage markup shocks by including the unemployment rate as an observable when estimating a model similar to the SW model. For the price markup shock, our choice is more arbitrary and follows Justiniano et al. (2013) by assuming that almost 90 percent of the markup shock variances are in fact variations in the inflation target.[31] Even in this case, the table shows that the optimal weight on the output gap remains high. The reason is that if all shocks are efficient then a high λ^a is still optimal (recall Figure 3), and if some shocks are indeed inefficient then a high λ^a is required. Therefore, a high weight λ^a is a robust choice if there is uncertainty about the inefficiency of the shocks.

4 Robustness Analysis

In this section, we explore the robustness of our results along some key dimensions. First, we examine to what extent adding labour market variables, such as hours worked and wage inflation, to the loss function improves welfare. Second, we consider the extent to which the implied interest rate movements for the simple mandates under consideration are reasonable, and if our results hold up when augmenting the loss function with an interest rate term. Third and finally, we examine the robustness of the high output gap weight when assuming that the gap is measured with considerable errors in real time.[32]

4.1 Should Labour Market Variables be considered?

One of the reasons for the popularity of inflation targeting comes from the results in the New Keynesian literature—importantly Clarida et al. (1999) and Woodford (2003)—that inflation in the general price level is costly to the economy. The old Keynesian literature, however, emphasised the importance of wage inflation.[33] In online Appendix D we establish that it makes sense within the SW model—which features substantial frictions in the labour market—to target wage inflation and a labour market gap instead. Doing so would reduce the welfare costs of the simple mandate even further. Moreover, we show that this conclusion is robust even if one considers the level of

[31] To account for inflation persistence without correlated price markup shocks, Justiniano et al. (2013) allow for serially correlated shocks to the Fed's inflation target which are subsequently excluded in their optimal policy exercises.

[32] In online Appendix E, we consider the merits of speed limit policies analyzed by Walsh (2003) and price- and wage-level targeting following Vestin (2006) and others. We find that they perform worse than the standard inflation-output objectives in Section 3; see the online appendix for further details.

[33] See Kim and Henderson (2005) for a more detailed discussion and references.

output and hours worked instead of their deviations from potential.

Still, because the SW model does not incorporate several realistic frictions in the labour market—such as imperfect risk sharing due to unemployment risk or search frictions—it would be interesting to extend the analysis into models that are more realistic along those dimensions, such as the models by Gali, Smets and Wouters (2011), Ravenna and Walsh (2012a,b) among others. It is conceivable that the optimal weight on economic activity and labour variables would be even higher if we had considered these additional frictions in labour markets. Even so, we acknowledge the political difficulties of targeting certain labour market variables (like the rate of increase in nominal wages), which in practice likely means that the most important aspect of these results is that we find a robust and important role for economic activity in the central bank's objective (may it be output or hours worked) even without additional labour market frictions, in line with our benchmark results in Table 1.

4.2 Volatility of Interest Rates

In addition to inflation and some measure of resource utilisation, simple objectives often include a term involving the volatility of interest rates; see e.g. Rudebusch and Svensson (1999). In practice, this term is often motivated by reference to "aversion to interest-rate variability" and financial stability concerns. From a theoretical perspective, Woodford (2003) derives an extended version of (30) augmented with an interest rate gap term $\lambda_r \left(r_t^a - r^a\right)^2$ when allowing for monetary transactions frictions ($r_t^a - r^a$ is the deviation of the annualised nominal policy rate r_t^a around the steady-state annualised policy rate r^a).

As an alternative, some researchers (e.g. Rudebusch and Svensson, 1999) and policymakers (e.g. Yellen, 2012) instead consider augmenting the objective function with the variance of the change in the short-run interest rate, $\lambda_r \left(\Delta r_t^a\right)^2$.[34] By allowing for a lag of the interest rate in the loss function, the specification introduces interest rate smoothing, as the reduced-form solution will feature the lagged interest rate in the central bank's reaction function. Although the inclusion of $r_t^a - r^a$ or Δr_t^a does not affect welfare much, this offers a simple way to examine the extent to which these interest rate terms mitigate any excessive volatility.

A related issue to interest rate volatility is the zero lower bound. In this vein, high volatility of interest rates could be problematic if the probability distribution of nominal rates for the mandates

[34] Another alternative is to augment the objective function with the variance of the surprise in the interest rate, $E_{t-1}r_t^a - r_t^a$, as in Rudebusch (2006).

under consideration covers the negative range in a nontrivial way. To address the zero lower bound problem, we use a standard approach to limit the standard deviation of the nominal interest rate: Rotemberg and Woodford (1998) adopted the rule of thumb that the steady-state nominal rate minus two standard deviations (std) for the rate should be non-negative. Others, like Adolfson et al. (2011) adopted a three std non-negativity constraint. Since our parameterisation of the SW model implies an annualised nominal interest rate of 6.25 percent, the allowable std is 3.125 under the Rotemberg and Woodford's rule of thumb and slightly below 2.1 under the stricter three-std criterion adopted by Adolfson et al. (2011).[35]

Table 3: *Interest Rate Volatility for Output Gap in Loss Function.*

Loss Function	λ^a	λ_r	CEV (%)	std(r_t^a)
Woodford	0.048	–	0.471	8.92
Dual Mandate	0.250	–	0.140	8.76
Optimised	1.042	–	0.044	9.00
Woodford: $r_t^a - r^a$	0.048	0.0770	0.462	0.98
Yellen: Δr_t^a	0.250	1.0000	0.186	1.24
Optimised*: $r_t^a - r^a$	1.161	0.0770*	0.076	2.24
Optimised*: Δr_t^a	1.110	1.0000*	0.084	2.04

Notes. std(r_t^a) denotes the standard deviation for the annualised nominal interest rate. y_t^{gap} is used as the measure of x_t in the loss function. The * in the last two rows denote that these values have been fixed, and are hence not optimised.

Table 3 reports the result of our exercise. For brevity of exposition we focus on the output gap only, but the results are very similar for output level and output growth. As seen from the first three rows in the table, the objective functions in Table 1 that involve only inflation and the output gap are indeed associated with high interest rate volatility. The std's are all around 9 percentage points—a few times bigger than our thresholds. Hence, these loss functions are contingent on unrealistically large movements in the short-term policy rate. Turning to the fourth and fifth rows, which report results for the Woodford and Yellen loss functions augmented with interest rate terms, we see that the std's for the policy rate shrink by almost a factor of ten; these specifications are hence clearly consistent with reasonable movements in the stance of monetary policy.

The last two rows in the table report results when we re-optimise the weight on the output gap (λ^a), given a weight of 0.077 for $(r_t^a - r^a)^2$ (next-to-last row) and 1 for $(\Delta r_t^a)^2$ (last row) in the loss function. As seen from the last column, these policies generate considerably lower interest rate volatility relative to the optimised loss function which excludes any interest rate terms, and the obtained std's are in line with even the three-std threshold applied by Adolfson et al. (2011). To

[35] Another option to circumvent the zero lower bound problem is to increase the inflation target. However, given that raising the inflation target appears difficult to implement in practice, we preferred to conduct this analysis through the more traditional inclusion of interest rate smoothing terms.

compensate for the interest rate terms, the optimisation generates a slightly higher λ^a compared with the simple loss function with the output gap only. Overall, the lower flexibility to adjust policy rates is associated with lower welfare; the CEV roughly doubles in both cases. But it is notable that the CEV does not increase to the same extent as $\text{std}(r_t^a)$ is reduced, reflecting that the central bank—which is assumed to operate under commitment—can still influence the long-term interest rate effectively by smaller but persistent movements of the short-term policy rate. Therefore, we can conclude that our benchmark result of a large weight on the real activity term holds for a plausible degree of interest rate volatility.

4.3 Robustness to Measurement Errors

A common counterargument for assigning a prominent role to the output gap is that it is measured with considerable error in real time (see e.g. McCallum, 2001). Indeed, the output gap is given by the difference between the actual level of output from its potential counterpart, and both are measured with errors in real time. We therefore examine the robustness of our main findings to the presence of significant measurement errors.[36]

To that end, we consider a case where the central bank has available imperfect measures of output and potential output in real time, so that it observes

$$y_t^{gap,obs} \equiv y_{t|t} - y_{t|t}^{pot,obs}, \tag{36}$$

where the notation $t|t$ reflects the real time dimension in the measurement of actual and potential output. Following Orphanides and Williams (2002), we assume that the difference between the observed $y_t^{gap,obs}$ and the true output gap y_t^{gap} (see eq. 31) evolves according to an AR(1) process

$$y_t^{gap,obs} - y_t^{gap} = \rho \left(y_{t-1}^{gap,obs} - y_{t-1}^{gap} \right) + \varepsilon_t, \tag{37}$$

where $0 < \rho < 1$ and $\varepsilon_t \sim N(0, \sigma_\varepsilon)$ is an exogenous error term. We then calculate the optimal weight λ^a in a loss eq. (30), but now considering that the central bank responds to the observed output gap $y^{gap,obs}$ rather than to y_t^{gap}.

We consider three alternative calibrations for the parameters ρ and σ_ε. First, we set $\rho = 0.95$ and $\sigma_\varepsilon = 0.36$, consistently with the estimates obtained by Orphanides and Williams (2002) for the period 1969Q1-2002Q2. Second, we consider the values obtained in Rudebusch (2001) using official

[36] Rudebusch (2001) studied how measurement errors impact the coefficients of the output gap and inflation in optimised interest rate rules; our focus is how measurement errors affect the weight of the output gap in the simple mandate.

real-time estimates of the output gap, namely $\rho = 0.75$ and $\sigma_\varepsilon = 0.84$. Finally, we re-estimate eq. (37) for the Smets-Wouters sample period (1965-2004) using real time data from the Philadelphia Fed to incorporate revisions in data vintages that may lead to an additional source of measurement errors. Specifically, we compute a series for $y_t^{gap,obs}$ using the real-time HP-filtered observation (one-sided filter) in each vintage of the GDP releases (for the first vintage covering period t, actual output in period t is our estimate of $y_{t|t}$ and the HP-trend value for this vintage in period t is the estimate of $y_{t|t}^{pot,obs}$). Then, y_t^{gap} is simply measured as the HP-filtered GDP series available today (i.e. two-sided filter). The resulting estimates are $\rho = 0.92$ and $\sigma_\varepsilon = 0.63$. More details are described in online Appendix F.

Clearly, this procedure captures well the errors associated with filtering in real-time (i.e. one-sided vs. two-sided filtering) as well as the errors related to revisions in actual GDP which in turn compound the filtering problem. However, this procedure has limitations. The model consistent and welfare relevant y_t^{pot} does not correspond to the potential level of output approximated by a two-sided HP-filter.[37] Moreover, in deriving a model consistent measure one could also account for model uncertainty and mispecification. Nevertheless, our crude approach of measuring y_t^{gap} provides a higher unconditional volatility of the measurement error than the ones in the literature ($\sigma_\varepsilon^2 / \left(1 - \rho^2\right) = 2.58$ compared to Rudebusch's 1.61), and thus provides a more stringent test for the usefulness of the output gap in simple mandates.[38]

Table 4: *Results When the Output Gap Is Measured with Errors.*

Measurement of Output Gap	λ^a	CEV (%)
No measurement errors	1.042	0.044
Orphanides and Williams	0.969	0.084
Rudebusch	1.024	0.209
HP-filtered Real Time Data	0.918	0.157

Notes. The table reports optimised weights on the output gap in the loss function (30) under alternative assumptions about the influence of measurement errors. The first row assumes that the output gap is measured without errors, the second uses the Orphanides and Williams (2002) calibration with $\rho = 0.95$ and $\sigma_\varepsilon = 0.36$ in eq. (37), the third uses Rudebusch (2001) estimates $\rho = 0.75$ and $\sigma_\varepsilon = 0.84$, and the fourth uses our approach with HP-filtered real-time data which gives $\rho = 0.92$ and $\sigma_\varepsilon = 0.63$.

Results are summarised in Table 4. For all the calibrations considered, the optimal weight λ^a is large, and always remains above 0.9. Interestingly, Table 4 also shows that the CEV is still lower when the gap is measured with errors compared to when either detrended output or output growth replaces the gap as a target variable in the objective. In a "worst case" scenario, CEV

[37] Despite this issue, Galí, Smets and Wouters (2011) and Justiniano, Primiceri and Tambalotti (2011) find that their model concepts of potential output behaves similarly to HP-filtered estimates of potential output, suggesting that our procedure is reasonable.

[38] Moreover, we have also verified that λ^a in Table 4 is above 0.78 if the measurement errors (σ_ε^2) are doubled.

equals about 0.21 (Rudebusch's estimates). For output as deviation from trend and output growth, Table 1 shows that CEV equals 0.24 and 0.30, respectively. Consequently, our results suggest that attaching a high weight to the observed output gap, even though it is measured with significant errors, enhances welfare, and could be a better alternative than targeting more directly observable measures of economic activity.

5 Conclusions

There appears to be broad consensus among academics that central banks should primarily focus on price stability and devote only modest effort to stabilise measures of real economic activity. Many influential studies in the monetary policy literature show that such a behavior would deliver the best possible policy from a social welfare perspective. Given this, it is not surprising that essentially all instrument-independent central banks have been asked to focus on price stability with little or no role for stabilising some measure of resource utilisation; the outlier is the U.S. Federal Reserve that has a strong focus on economic activity through its dual mandate. The question is then: Is a dual mandate redundant or even welfare deteriorating?

This paper examined this question within the context of an estimated medium-scale model for the US economy, and showed that the prevailing consensus may not be right. Looking at measures of economic activity seems to be more important than previously recognised in academia and in policy circles. And although our analysis is based on a model estimated for the U.S. economy, our result is relevant to all economies affected by non-trivial real rigidities and inefficient shocks, thus displaying a relevant trade-off between stabilising inflation and economic activity. For instance, both VAR evidence (see e.g. Angeloni et al., 2003) and estimated New Keynesian models (see e.g. Adolfson et al. 2005 and Christiano et al. 2010) suggest that the transmission of monetary policy, the structure of the economy, and shocks are very similar in the European economies.

In practice, it is of course difficult to assess the importance of real rigidities and the role inefficient shocks may play in magnifying policy trade-offs. But that argument does not invalidate our main conclusion. A central bank that assigns a high weight to measures of economic activity would deliver good economic outcomes even in the absence of relevant policy trade-offs.[39]

During the recent financial crisis many central banks, including the Federal Reserve and the

[39] If we follow Nekarda and Ramey (2013) and define the markup as the inverse of the labour share, we find that shocks to the markup exert a significant influence on output using a medium-sized VAR similar to Christiano, Eichenbaum and Evans (2005). These results, available upon request, suggest that price markup shocks may indeed be relevant for business cycle fluctuations.

Bank of England, cut policy rates aggressively to prevent further declines in resource utilisation although the fall in inflation and inflation expectations were modest. By traditional metrics, such as the Taylor (1993) rule, these aggressive and persistent cuts may be interpreted as a shift of focus from price stability to resource utilisation by central banks during and in the aftermath of the recession. Our results make the case for a stronger response to measures of economic activity even during normal times. In our model, the policy trade-offs mainly arise from imperfections in goods and labour markets. Considering an economy where inefficiencies are primarily associated with frictions in the financial markets would be an interesting extension to address some of the recent debates. Recent work by Laureys, Meeks, and Wanengkirtyo (2016) suggests that including financial variables in the central bank's loss function improves welfare, but that the weight on financial variables is low and the weight on the output gap remains very high. This is supportive of the central tenet in our paper, but further work in this important area is needed before one can draw firmer conclusions.

Using a calibrated open-economy model, Benigno and Benigno (2008) studied how international monetary cooperative allocations could be implemented through inflation targeting aimed at minimising a quadratic loss function consisting of only domestic variables such as GDP, inflation, and the output gap. It would thus be interesting to extend our investigation to an open economy framework with an estimated two-country model of, for example, the United States and the euro area. Another interesting extension would be to examine our results in models with additional labour market dynamics and frictions.

Throughout the paper, we have assumed that non-optimising firms and households index prices and nominal wages completely to the convex combination of lagged and steady-state inflation rates. Hence, indexation is complete and there is no price and wage dispersion in the steady state. However, microeconomic evidence suggests that non-optimising firms often do not change prices. Ascari (2004) and Yun (2005) have shown that the costs of inflation and inflation fluctuations may rise notably when indexation is incomplete even under modestly positive trend inflation rates. Yet, the attention to measures of economic activity is likely to be the reason why central banks might choose a positive inflation target in the first place, e.g. due to downard nominal wage rigidities, or to the possibility of hitting the zero-lower bound. Therefore, it would be of interest to extend our analysis to a framework with incomplete or no indexation for non-optimising firms and households in an environment with moderate yet positive steady-state inflation rates.

Finally, our analysis postulated that central banks operate in an almost ideal situation, with the exception of not being able to measure the output gap accurately in real time. In this respect our approach could be extended to study the design of simple policy objectives in even more realistic situations, in which the central bank faces uncertainty about the structure of the underlying economy or cannot implement their desired policies because of implementation lags or credibility problems.

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