

Strategic Complexity

Vladimir Asriyan, Dana Foarta and Victoria Vanasco*

September 29, 2018

Abstract

This paper explores the incentives of product designers to produce complex products, and the resulting implications for overall product quality. In our model, there is a consumer who can accept or reject a product proposed by a designer, who jointly chooses the quality and the complexity of the product. While the product's quality determines the direct benefits of the product to the consumer, the product's complexity primarily affects the information she can extract about the product's quality. Examples include banks that design financial products that they later offer to retail investors, or policymakers who propose policies for approval by voters. We find that complexity is not necessarily a feature of bad quality products. For example, while an increase in alignment between the consumer and the designer leads to more complex but better quality products, higher demand or lower competition among designers leads to more complex and worse quality products. We discuss how our findings can rationalize the observed trends in complexity of financial products and of regulation.

Keywords: strategic complexity, information frictions, financial products, product design, regulation.

JEL Codes: D82, D83, G18, P16, D78.

*Asriyan: CREi, UPF and Barcelona GSE, vasriyan@crei.cat; Foarta: Stanford Graduate School of Business, ofoarta@stanford.edu; Vanasco: CREi, UPF and Barcelona GSE, vvanasco@crei.cat. We thank Anat Admati, Larbi Alaoui, Fernando Broner, Steve Callander, Peter DeMarzo, Ben Hébert, Jason Roderick Donaldson, Deeksha Gupta (discussant), Andrey Malenko (discussant), Alberto Martin, Giorgia Piacentino, Giacomo Ponzetto, Andy Srkzypacz, Jaume Ventura, Jeffrey Zwiebel and seminar/conference participants at CREi-UPF, EIEF Rome, Stanford University, 11th Annual Paul Woolley Centre Conference at LSE, Sixth Oxford Financial Intermediation Theory Conference, and IX Workshop on Institutions, Warwick/Princeton/Utah Political Economy Conference in Venice, and Individual Behavior and Economic Outcomes in Sardinia for helpful comments and feedback.

1 Introduction

In recent decades, the issue of rapidly increasing complexity has been raised by both financial market participants and policymakers. In the financial industry, for instance, it is argued that the products sold to retail investors have become increasingly complex – with product descriptions that contain jargon and complicated or vague explanations (Carlin, 2009; Carlin and Manso, 2011; Célérier and Vallée, 2015). Similarly, in the regulatory or the legislative process, it has become more common to encounter policy proposals lacking specificity, with broadly worded or ambiguous provisions (Davis, 2017). Such increase in complexity could be a concern if it prevents consumers from evaluating the quality of the financial products that they buy, or of the policies that they support. This, in turn, could foster the proliferation of bad quality products and policies. Consistent with this argument, Célérier and Vallée (2015) find that issuers of lower quality financial products deliberately increase their complexity in order to hide risks from investors.¹

In many situations, consumers evaluate a product before deciding whether to accept it or not. Some examples include retail investors who evaluate financial products, the median-voter who evaluates a policy proposal, the Editor of a journal who evaluates a paper. To make a decision, a consumer gathers information from several sources, such as the description of the product attributes, reviews, media reports, etc. The product designer, in turn, can influence the quality of the information the consumer receives by designing a more or less complex product.² For example, a product can be made more complex by adding unnecessary attributes and contingencies, or by opting for complicated jargon and lengthy and ambiguous descriptions. In such an environment, however, if all (and only) bad products were complex, it would be very easy for a consumer to reject all products whose attributes she doesn't understand well. This logic seems to contradict the increasing and prevalent provision of complex financial products and complex regulation witnessed in the last two decades. Motivated by this puzzle, we develop a framework to understand the drivers of product complexity, and its implications for the production and proliferation of bad quality products.

In the model, there is one consumer who wants a product that can only be produced by a product designer. When designing a product, the designer privately and separately chooses the product's output and complexity. While a product's output determines the direct payoff to the consumer, which can be good or bad, a product's complexity influences the

¹Also related, Christoffersen and Musto (2002) show that financial institutions use distortions in transparency in order to price discriminate among investors.

²We think of financial intermediaries as the designers of financial products, and of policymakers as the designers of legislation and policy proposals.

consumer's ability to learn the product's output. We model this by assuming that when a product is more complex, the consumer is more likely to extract noisy information about the product's output. We focus on cases in which the objective of the product designer is to have his product *accepted*, while the objective of the consumer is to accept a good product. For example, the objective of a bank is to convince a retail investor to accept a given savings account; while a policymaker wants voters' approval for his tax reform proposal.³ Finally, we suppose that product designers can be of two types, aligned or misaligned with the consumer, where aligned (misaligned) designers receive a high payoff from having a good (bad) product accepted. The (mis-)alignment captures in reduced form differences between the interest of the consumer and that of the designer stemming from career concerns, ideological preferences, or privately negotiated sales commission incentives. Importantly, the designer's type is his private information. The timing of the game is as follows. First, a designer chooses a product's output and complexity, and proposes the designed product to the consumer. Then, the consumer obtains information about product output and decides whether to accept or reject the proposed product. If the consumer accepts, product payoffs are realized; otherwise, everyone gets their outside options.

An important contribution of our paper is to model the joint decision of choosing a product's quality and complexity, which we view as two attributes that a designer can control separately.⁴ For example, the quality (i.e., output) of a financial product should be determined by the net present value (NPV) that it generates to an investor. There are, however, many financial contracts that generate the same NPV. Thus, for a given NPV, a financial product can be made more complex by adding contingencies that generate zero NPV to the investor, by using ambiguous words in the product's description, by linking payments to financial indeces that the consumer is unlikely to know, etc. A similar argument can be made for policymakers in charge of writing policy proposals. By studying both attributes separately, we are able to gain a better understanding of the incentives to produce good/bad quality products vs. complex/simple products. For simplicity, we have also assume that a product's complexity does not directly affect the consumer's payoff from the product. We show that our qualitative results do not depend on this assumption in Section 5, where we allow for deviations from some "natural level of complexity" to be costly for the consumer.

Our framework delivers several powerful insights. We show that complexity is not always

³Our objective as paper designers is to have the Editor accept our proposed paper for publication.

⁴In a different context, [Bar-Isaac et al. \(2010\)](#) study incentives to produce different product attributes by exploring firms integrated strategy for marketing, pricing, and investment in quality, where marketing affects the information consumers' receive about the product's other attributes.

a feature of bad quality products. In fact, designers of good quality products will sometimes choose to complexify them, and designers of bad quality products will sometimes choose to simplify them. The model generates novel implications for the relationship between product quality and complexity. In particular, as product designers become more aligned with the consumer, both product quality and complexity increase. On the other hand, as the demand for a product increases and/or competition among product designers decreases, product quality falls and complexity increases. With these results, we are able to discuss possible (and novel) drivers of the recent trend towards complexification in finance and regulation, and to better understand the resulting implications for product quality.

A key insight of our model is that incentives to design complex or simple products depend crucially on the consumer’s acceptance strategy in the absence of information. In particular, all designers have incentives to design a complex product when the consumer would accept the product in the absence of information –in this case, we say the consumer is *optimistic*. This result, though surprising at first, is intuitive: information can only increase the chances of a product being rejected, since it is already accepted with probability one in the absence of information. In contrast, when the consumer would reject a product in the absence of information –i.e., the consumer is *pessimistic*,– the designers have incentives to design simple products. Now, information weakly increases the chances of a product being accepted, since it is rejected with probability one in the absence of information. A related finding is present in [Perez-Richet and Prady \(2011\)](#), who consider a setting with a privately informed sender that can “complicate to persuade” a receiver to obtain a certification, where complication increases the cost of the receiver to acquire information.

For the previous result, it is essential that the product designer cannot perfectly control the precision of the information that the consumer receives. By choosing a complex (or simple) product, a designer increases (decreases) the likelihood that the consumer receives noisy information, but he cannot guarantee that she receives no (or perfect) information.⁵ This is natural, since in many settings product designers cannot fully control the information set of the consumer. The latter may feel comfortable with the words used in the description due to her education, or may have access to reviews from other consumers or media articles that further simplify or confuse her understanding of the product’s quality. Thus, our model suggests that in some situations, when designers of good quality products cannot perfectly reveal their quality to consumers, they may be better off by making their products complex.

Our model predicts that all designers have incentives to complexity their products when

⁵Otherwise, in our model, designers of good products would choose to perfectly reveal their quality, and thus designers of bad products would be perfectly identified as well.

the consumer is optimistic, i.e., when (i) alignment between the consumer and the designer is high, or (ii) the consumer’s outside option is sufficiently low, which we show can reflect high product demand or low competition among designers. At the other extreme, all designers have incentives to simplify, while in intermediate cases bad products are made complex and good products are made simple.

We then examine the implications for the design of bad quality products. Since aligned designers always produce good products (simple or complex), average product quality depends on the incentives of the misaligned designers to produce good products as well. A misaligned designer trades-off a higher probability of acceptance (with a good product) with a higher payoff conditional on acceptance (with a bad product). As a result, when the consumer’s acceptance strategy is very strict, misaligned designers produce more good products. In equilibrium, of course, the strictness of the consumer’s acceptance strategy depends also on how complex products are. Thus, products are more likely to be better quality when (i) the alignment between the consumer and the designer is high, (ii) demand for the product is low, or (iii) competition among designers is high.

Finally, we combine our previous results to obtain new predictions about the relationship between product quality and complexity in equilibrium. First, as the alignment between the consumer and the designer increases, products will tend to be better quality but more complex, suggesting a positive relation between quality and complexity. Second, as the consumer’s demand for the product increases, or competition among designers decreases, products will tend to be worse quality and more complex, suggesting a negative relation between quality and complexity. Therefore, in order to understand the relationship between quality and complexity of products, it is essential to understand the underlying drivers of product heterogeneity.

We show that our results are robust to several extensions. We first suppose that the consumer can observe perfectly the designer’s choice of complexity (as in [Perez-Richet and Prady \(2011\)](#)). We show that in this scenario, two pooling equilibria co-exist: all designers simplify or they all complexity. Furthermore, when the consumer is sufficiently optimistic (pessimistic), all designers are better off in the complex (simple) equilibrium. An advantage of our approach is that it eliminates the multiplicity of equilibria that is common to signaling games, due to the freedom in setting off-equilibrium beliefs; this in turn allows us to obtain rich comparative statics. Second, we consider direct costs of complexity by supposing that deviations from some “natural level” of complexity are costly to the consumer. We show that our main results remain qualitatively unchanged, with the intuitive new prediction that complexity decreases (increases) as it becomes more (less) costly to the consumer. Finally,

we show that our model can be re-stated, though at a loss in tractability, as an optimal information extraction problem, where more complex products tend to tighten the consumer’s capacity constraint on entropy.

Even though our model is stylized and abstracts from many institutional details of real-world settings, we nevertheless discuss our model’s predictions within the context of our two concrete applications: financial products and regulatory policy. In financial markets, intermediaries design financial products to offer to retail investors, such as savings account and asset-backed securities. The design of a product consists of determining a set of cash flows for different states of the world (price, future payments, fees, contingencies, etc.) and of writing these contract terms down. The investor evaluates a product and decides whether to invest/borrow or not. If she does not, her outside option is to either search for another financial advisor, or to do nothing. In this context, our model suggests that the increase in the complexity of financial products documented by [Célérier and Vallée \(2015\)](#) could be driven by (i) an increase in investor’s trust in financial advisors, or (ii) an increased demand for financial products. Both of these features were characteristics of financial markets prior to the 2008/09 crisis, and while the trust in the financial system may have fallen in response to the crisis, the high demand for relatively safe financial products still persists. This suggests that the observed proliferation of worse and more complex products could be an endogenous response of product designers to an increasing demand for relatively safe financial products. These results are in contrast, and complementary, to those in [Pagano and Volpin \(2012\)](#), who show that securitizers may have incentives to increase the opacity of their products when the release of information creates a winner’s curse problem to unsophisticated investors, a mechanism that is not present in our setting.

In the political sphere, politicians are the designers in charge of proposing policies, such as plans for taxation or regulation. We can interpret the consumer in such a setup as the median voter, from whom the politician must obtain approval for policy proposals. A more complex policy proposal by a politician may, for instance, take the form of less specific promises and hazy details on the exact implementation of the policy goal. This idea is present in the literature of strategic ambiguity, deliberate vagueness or noise by politicians ([Alesina and Cukierman, 1990](#); [Aragones and Postlewaite, 2002](#); [Espinosa and Ray, 2018](#)). Our model suggests that policy proposals are more likely to be complex when (i) public opinion about the politician’s alignment with the median voter is high; and (ii) there is urgency to pass a given policy; that is, the status-quo is costly. Both of these features were present when complex policies, such as the Affordable Care Act and the Dodd-Frank Act were passed. Moreover, in

policy areas where public opinion of politicians is low, more policy proposals tend to be simple and to delegate subsequent details and lawmaking authority to federal agencies, for example to the FDA in the case of the pharmaceutical industry.

Our paper is closely related to the literature on obfuscation ([Carlin, 2009](#); [Ellison and Ellison, 2009](#); [Carlin and Manso, 2011](#); [Ellison and Wolitzky, 2012](#)), and shrouding attributes ([Gabaix and Laibson, 2006](#)) or limited awareness ([Auster and Pavoni, 2018](#)). These papers study the incentives of sellers of homogeneous goods to obfuscate by limiting the buyers' ability to observe certain product attributes. In common with these papers, in our model the producer can take an action to affect the information that the consumer receives. Our approach, however, differs in several respects. First, in our setting, the designer jointly chooses the quality and the complexity of the product, resulting in heterogeneous goods being offered in equilibrium. This allows us to study the incentives to produce complex *and* bad products. Second, the consumer in our setting is Bayesian, and she rationally processes all of the information she receives; i.e., there are no hidden-attributes or information search costs. The interaction of complexity added by the sender and the receiver's learning from a noisy signal relates our model to [Dewatripont and Tirole \(2005\)](#); however, their focus is on costly communication in a setting with moral hazard in teams.

Finally, our paper relates to the broader literature on strategic information transmission ([Crawford and Sobel, 1982](#); [Grossman, 1981](#); [Milgrom, 1981](#); [Kartik, 2009](#)), and on Bayesian persuasion ([Green and Stokey, 2007](#); [Kamenica and Gentzkow, 2011](#)). These papers focus on the optimal information structure with private information (ex-post design) or with commitment (ex-ante design), which is not the focus of our paper. Although in our setting, information transmission (i.e. complexity) is strategic and chosen ex-post, the designer does not optimally choose an information structure. In particular, he cannot perfectly transmit, nor perfectly obfuscate, information about the quality of the product he has chosen, which as we have argued is essential for our results. Instead, we view our approach as closer to the literature on rational inattention, as in [Sims \(2003\)](#), if we interpret product complexity as an attribute that limits the consumer's ability to process information.⁶

The rest of the paper is organized as follows. In Section 2, we present the setup of the model. In Section 3, we illustrate our main results and in Section 4, we present the model comparative statics, which we discuss in the context of our leading applications in Section 5. In Section 6 we show that our model is robust to several extensions. Section 7 concludes. All of the proofs are relegated to the Appendix.

⁶For a recent survey of the literature see [Wiederholt et al. \(2010\)](#).

2 The Model

We consider the following interaction between a consumer and a product designer. The consumer needs a product, that only the designer can produce. The designer takes two actions $\{y, \kappa\}$, where $y \in \{\text{Good}, \text{Bad}\}$ determines the product's output, and $\kappa \in \{\underline{\kappa}, \bar{\kappa}\}$ determines the product's complexity, which is defined in detail below. After this, the designer proposes the product to the consumer, who evaluates it and decides whether to accept it ($a = 1$) or take an outside option ($a = 0$). The designer can be one of two types, $t \in \{\text{High}, \text{Low}\}$, with probabilities p and $1 - p$ respectively, where different types differ in their incentives to design a given product. Most importantly, the designer's type, t , is his private information and his actions $\{y, \kappa\}$ are unobservable to the consumer.

A Product's Output. By taking action y , the designer affects the output of the proposed product. We assume that the payoff to the consumer from accepting a product with output y (which we refer to as a y -product) is $w(y)$, and her outside option if no product is accepted is w_0 . A t -type designer receives payoff $v_t(y)$ from having a y -product being accepted, and zero otherwise. We make the following assumptions on the payoffs:

Assumption 1 *The payoffs satisfy the following properties:*

1. $w(G) > w_0 > w(B) \geq 0$.
2. $v_H(G) = v_L(B) = \bar{v} > v_L(G) = v_H(B) = \underline{v}$, with $\bar{v}, \underline{v} > 0$.

The first assumption states that the consumer wants to accept a G -product but reject a B -product. The second assumption states that an H -type (L -type) designer receives a higher payoff when the G -product (B -product) is accepted. Thus, the L -type designer, in contrast to the H -type, has preferences that are *misaligned* with those of the consumer.⁷

A Product's Complexity. The complexity of a product determines how difficult it is for the consumer to understand the product's output y . Formally, we suppose that after the designer proposes product (y, κ) , the consumer is able to extract a binary signal $S \in \{b, g\}$ about the product's output with some noise $z \equiv \mathbb{P}(y = G|S = b) = P(y = B|S = g)$, where $z \sim f(\cdot|\kappa)$ with full support on $[0, \frac{1}{2}]$ for all κ with the property that $\frac{f(z|\bar{\kappa})}{f(z|\underline{\kappa})}$ increasing in z (MLRP). That is, the signal that the consumer extracts is more likely to be noisy when the product is

⁷In the financial products industry, misalignment can arise due to financial advisors receiving higher fees for selling products that are not necessarily the best fit for their clients (i.e., fixed vs. adjustable-rate mortgages). In the policy sphere, misalignment of policymakers vis-à-vis the public may arise due to ideological differences, lobbying, or career concerns.

complex.⁸ That complexity, κ , is not perfectly observed by consumers is convenient but not essential for our main results: it rules out multiplicity of equilibria, typical of signaling games, that arise from the freedom in specifying off-equilibrium beliefs. This facilitates comparative statics, and it also has the natural interpretation that consumers are not able to perfectly observe the underlying actions of the designers towards complexification. Nevertheless, we show that our main results do not depend on this assumption in Section 6.1.

Remark 1 *We have assumed that complexity does not directly affect the consumer's payoffs. This is convenient, as it allows us to isolate the strategic role of complexity in deterring information acquisition or learning by consumers. For example, a product could be made "more" complex by the use of complicated words and jargon in its description, without necessarily affecting the product's attributes and consumers' payoffs. Nevertheless, we incorporate direct costs of complexity (or simplicity) to the consumer in Section 5.2 and Appendix D.*

The Consumer's Problem. The consumer has to decide whether to accept the designer's product or not. Before making her decision, the consumer observes the signal realization s with noise z , and forms her posterior beliefs about the output y , denoted by $\mu(s, z) \equiv \mathbb{P}(y = G|s, z)$. The consumer's acceptance strategy maximizes her expected payoff, given by

$$W(a|s, z) \equiv a \cdot [\mu(s, z) \cdot w(G) + (1 - \mu(s, z)) \cdot w(B)] + (1 - a) \cdot w_0. \quad (1)$$

The Designer's Problem. A t -type designer's expected payoff is given by

$$V_t(y, \kappa) \equiv \mathbb{P}(a = 1|y, \kappa) \cdot v_t(y) \quad (2)$$

where $\mathbb{P}(a = 1|y, \kappa)$ denotes the probability that product $\{y, \kappa\}$ is accepted by the consumer. The designer chooses $y \in \{G, B\}$ and $\kappa \in \{\underline{\kappa}, \bar{\kappa}\}$ to maximize (2). We denote t -type designer's strategy by $\{m_t, \sigma_{t,G}, \sigma_{t,B}\}$, where $m_t = \mathbb{P}(y_t = B)$ is the probability with which the t -type designer chooses the B -product, and $\sigma_{t,y} = \mathbb{P}(\kappa_t = \bar{\kappa}|y_t = y)$ is the probability with which he chooses high complexity, conditional on him also choosing the y -product.

Equilibrium Concept. We use Perfect Bayesian Equilibrium (PBE) as our equilibrium concept. This has the following implications. First, given her beliefs, the consumer's acceptance strategy must maximize her expected payoff (*Consumer Optimality*). Second, the designer's

⁸The restriction to binary-symmetric signals facilitates tractability but is not crucial for our main results.

strategy must maximize his expected payoff, given the consumer's strategy (*Designer Optimality*). Finally, the consumer's beliefs must be consistent with the designer's strategy and updated using Bayes' rule when possible (*Belief Consistency*).

2.1 Benchmarks

Before we proceed to the equilibrium analysis, we find it useful to establish two benchmarks against which our results can be contrasted. First, we consider the allocations that would arise in the absence of asymmetric information (*Perfect information*). Second, we consider the allocations that would arise if the designers were able to perfectly communicate their product's attributes to the consumer (*Perfect communication*).

The findings of this section are summarized in the following proposition.

Proposition 1 *In both the perfect information and perfect communication benchmarks, in equilibrium, only G-products are produced by the designers.*

Perfect information. Suppose that the consumer perfectly observes the output and the complexity, $\{y, \kappa\}$, chosen by the designer. Since the consumer's outside option, w_0 , is greater than the payoff she obtains from a B -product, $w(B)$, and smaller than the payoff she obtains from a G -product (Assumption 1), it follows that she will accept a proposed product if and only if its output is $y = G$. Given this acceptance strategy, it is clear that it is optimal for both designer types to only choose G -products.

Perfect communication. Suppose that the designer can perfectly communicate the output of her product to the consumer. Formally, assume the designer can freely choose the $z \in [0, \frac{1}{2}]$ of the signal, independently of his choice of complexity κ . First, note that the designer of a G -product maximizes his payoff by choosing to perfectly reveal the product's output, i.e., $z = 0$, since this would imply it is accepted with probability one.⁹ It follows that the interim belief after observing noise z is $\mu(z) = 0$ for $z \neq 0$; that is, the consumer infers that she has been proposed a B -product when the designer does not perfectly communicate his product's output. Thus, the consumer rejects any product with $z \neq 0$, which implies that it is optimal for both designer types to produce G -products and to perfectly communicate this to the consumer, $z = 0$. Note that this argument does not depend on the information structure as long as perfect information transmission is a costless and available option to the designers.

⁹We assume that the consumer's posterior belief after observing a perfectly informative signal is $\mu(g, 0) = 1$ and $\mu(b, 0) = 0$ for all prior beliefs $\mu \in [0, 1]$.

In our baseline model, complexity κ does not directly affect the payoff to the consumer. In addition, with perfect information or communication, κ does not affect the payoff to the designer either. As a result, any choice of complexity is consistent with equilibrium in both benchmarks. However, if κ imposed direct costs to the consumer, as we consider in Section 5.2, then the designers would always choose the complexity level preferred to the consumer.

3 Equilibrium

In this section, we characterize the equilibria of our game. First, we characterize the consumer's acceptance strategy, given her beliefs about the product proposed by the designer (Section 3.1). Second, we analyze the designer's strategy: his choice of output (Section 3.3) and of complexity (Section 3.2), given the consumer's optimal acceptance strategy. Finally, we impose belief consistency to obtain the equilibria of our model (Section 3.4).

3.1 Consumer's Acceptance Strategy

From the consumer's problem, as given in (1), we see that her acceptance strategy follows a threshold strategy. In particular, she accepts the product, $a(s, z) = 1$, if and only if her posterior belief about the product having good output is sufficiently high, $\mu(s, z) \geq \omega$, where $\omega \equiv \frac{w_0 - w(B)}{w(G) - w(B)}$ captures the relative value of the consumer's outside option.¹⁰

Thus, to understand the consumer's acceptance strategy, we need to analyze the determinants of her posterior belief. Let $\mu \equiv \mathbb{P}(y = G)$ denote the consumer's prior belief. After the designer proposes his product, the consumer observes signal s with noise z about the product's output. Since z is informative about the choice of κ , it may contain information about output y . Let $\mu(z)$ denote the consumer's interim belief upon observing z ,

$$\mu(z) \equiv \mathbb{P}(y = G|z) = \frac{\mu}{\mu + (1 - \mu)\ell(z)} \quad (3)$$

where prior belief μ and likelihood ratio $\ell(z) \equiv \frac{\mathbb{P}(z|y=B)}{\mathbb{P}(z|y=G)}$ are computed using the designer's equilibrium strategies, which the consumer takes as given. As a result, the consumer's posterior belief, $\mu(s, z)$, after observing signal z and s , is as follows

$$\mu(s, z) = \frac{\mathbb{P}(S = s|y = G) \cdot \mu(z)}{\mathbb{P}(S = s|y = G) \cdot \mu(z) + \mathbb{P}(S = s|y = B) \cdot (1 - \mu(z))}. \quad (4)$$

¹⁰If indifferent, we assume without loss that the consumer accepts the product.

The consumer's acceptance strategy is contingent on the observed signal only if she approves the product when the signal is good, $S = g$, and rejects it when the signal is bad, $S = b$. For this to be optimal, the signal has to be informative enough so that:

$$\mu(b, z) \leq \omega \leq \mu(g, z) \quad (5)$$

Now, consider the threshold noise level \bar{z} at which $\mu(s, \bar{z}) = \omega$ for some $s \in \{b, g\}$. This threshold determines the maximum noise level at which the consumer makes her acceptance strategy contingent on the signal.¹¹ The following definition will be useful in what follows.

Definition 1 *We say that the consumer is **optimistic** when the threshold \bar{z} is given by condition $\omega = \mu(b, \bar{z})$, and that the consumer is **pessimistic** when it is given by $\omega = \mu(g, \bar{z})$.*

That is, the consumer is optimistic when in the absence of information she accepts the proposed product. Intuitively, the consumer is more likely to be optimistic when “trust” in the designer, captured by her prior belief μ , is high, or when her relative outside option, ω , is low. Consistent with this, when the noise of the signal is relatively high ($z > \bar{z}$) the consumer disregards her signal: she always accepts the product if she is *optimistic*, and rejects it if she is *pessimistic*. Conversely, when the signal is sufficiently informative ($z \leq \bar{z}$), the consumer makes her acceptance decision contingent on the signal: she approves the product after observing a good signal, and rejects it after a bad signal. These results are formalized in the following lemma.

Lemma 1 *When the consumer is optimistic, her acceptance strategy is:*

$$a = \begin{cases} \mathcal{I}_{\{S=g\}} & \text{if } z \leq \bar{z} \\ 1 & \text{if } z > \bar{z} \end{cases}, \quad (6)$$

whereas when the consumer is pessimistic, her acceptance strategy is:

$$a = \begin{cases} \mathcal{I}_{\{S=g\}} & \text{if } z \leq \bar{z} \\ 0 & \text{if } z > \bar{z} \end{cases} \quad (7)$$

¹¹In principle, \bar{z} could take multiple values. In the Appendix we impose a condition on the distribution of z to ensure that in equilibrium $\mu(b, z)$ is increasing in z . This guarantees the existence of a unique \bar{z} , since $\mu(g, z)$ can be shown to always be decreasing in z , $\mu(g, 0) = 1$, $\mu(b, 0) = 0$, and $\mu(g, 0.5) = \mu(b, 0.5)$. Intuitively, the condition boils down to assuming that the information content of the signal, S , is greater than the information content of observing the realization of the noise itself, z . Though not essential for our main results, it considerably facilitates the analysis.

where $\mathcal{I}_{\{S=s\}}$ is the indicator equal to one when the signal is equal to s .

It is important to highlight the role of information in each scenario. When the consumer is optimistic, more precise information (weakly) increases the chances that any product is rejected, because a product is always accepted when information is sufficiently noisy ($z > \bar{z}$). The opposite holds when the consumer is pessimistic, where more precise information (weakly) increases the chances that any product is accepted.

3.2 The Designer's Choice of Complexity

We next consider the designer's choice of complexity, given his choice of output, y , and the consumer's acceptance strategy as described in the previous section. From the designer's objective in (2), it follows that a t -type designer who chooses a y -product also chooses low complexity, $\underline{\kappa}$, whenever

$$\mathbb{P}(a = 1|y, \underline{\kappa}) \geq \mathbb{P}(a = 1|y, \bar{\kappa}). \quad (8)$$

Otherwise, the designer chooses high complexity, $\bar{\kappa}$. Note that, once we condition on the designer's choice of output, the designer's choice of complexity is independent of his type, a feature that greatly facilitates our analysis. From now on, we denote by σ_y the (common) probability with which the designer chooses to complexify, given that he has chosen output y .

Using the results from Lemma 1, we can compute the probability of acceptance of a y -product conditional on a signal noise level, z , which we denote by $\pi(y, z)$. In the optimistic consumer case (o),

$$\pi^o(G, z) = \begin{cases} 1 - z & \text{if } z < \bar{z} \\ 1 & \text{if } z \geq \bar{z} \end{cases} \quad \text{and} \quad \pi^o(B, z) = \begin{cases} z & \text{if } z < \bar{z} \\ 1 & \text{if } z \geq \bar{z}, \end{cases} \quad (9)$$

whereas in the pessimistic consumer case (p),

$$\pi^p(G, z) = \begin{cases} 1 - z & \text{if } z \leq \bar{z} \\ 0 & \text{if } z > \bar{z} \end{cases} \quad \text{and} \quad \pi^p(B, z) = \begin{cases} z & \text{if } z \leq \bar{z} \\ 0 & \text{if } z > \bar{z}. \end{cases} \quad (10)$$

Thus, the designer's expected probability of a (y, κ) product being accepted depends on whether the consumer is optimistic ($j = o$) or pessimistic ($j = p$) and is given by:

$$\mathbb{P}(a = 1|y, \kappa) = \int_0^{\frac{1}{2}} \pi^j(y, z) \cdot f(z|\kappa) \cdot dz. \quad (11)$$

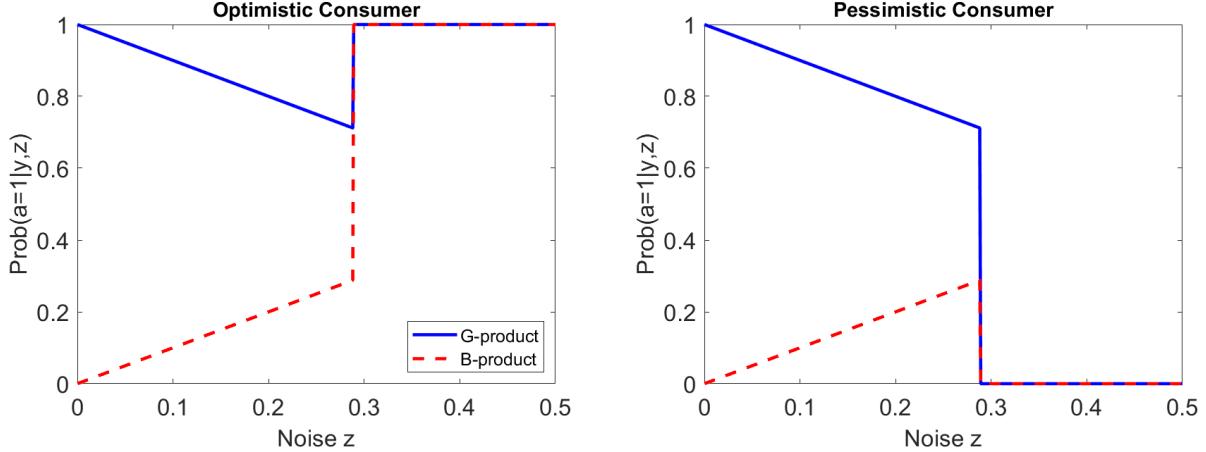


Figure 1: The probability of acceptance of a y -product as a function of the signal's noise, z . The left panel depicts the acceptance strategy of an optimistic consumer. The right panel depicts the acceptance strategy of a pessimistic consumer

The following proposition provides the optimal choice of complexity of a designer that has produced a y -product. We say that the designer *simplifies* when he chooses $\underline{\kappa}$ ($\sigma_y = 0$), and that he *complexifies* when he chooses $\bar{\kappa}$ ($\sigma_y = 1$).

Proposition 2 *Let \hat{z} denote the unique solution to $\int_0^{\hat{z}} z \cdot f(z|\underline{\kappa})dz = \int_0^{\hat{z}} z \cdot f(z|\bar{\kappa})dz$. Then, when the consumer is optimistic,*

$$\sigma_B = 1 \text{ and } \sigma_G \begin{cases} = 1 & \text{if } \bar{z} < \hat{z} \\ \in [0, 1] & \text{if } \bar{z} = \hat{z} \\ = 0 & \text{if } \bar{z} > \hat{z}, \end{cases} \quad (12)$$

whereas, when the consumer is pessimistic,

$$\sigma_B \begin{cases} = 0 & \text{if } \bar{z} < \hat{z} \\ \in [0, 1] & \text{if } \bar{z} = \hat{z} \text{ and } \sigma_G = 0. \\ = 1 & \text{if } \bar{z} > \hat{z} \end{cases} \quad (13)$$

The proposition shows that when the consumer is optimistic (pessimistic), there is a tendency to complexify (simplify). The intuition for this result can be obtained from Figure 1, which illustrates the acceptance probabilities $\pi^j(y, z)$ as a function of z , in each region $j \in \{o, p\}$. First, a B -product's probability of acceptance increases in the noise of the signal when the consumer is optimistic. As a result, the designer of a B -product chooses to complex-

ify in such scenario. Second, a G -product's probability of acceptance decreases in the noise of the signal when the consumer is pessimistic. As a result, the designer of a G -product chooses to simplify in such scenario. The effect of z on the probability of acceptance, however, is non-monotonic for a B -product when the consumer is optimistic and for a G -product when the consumer is pessimistic. Here, the choice of complexity depends critically on the consumer's acceptance strategy, summarized by the threshold \bar{z} , which will be determined in equilibrium.

We have shown that a product's complexity, σ_y , depends solely on the product's output, y , and is independent of the designer's type, t . As we show next, however, the designer's choice of product output is affected by his type.

3.3 The Designer's Choice of Output

We now study the designer's problem of choosing product output. Recall from (2) that a t -type designer receives payoff \bar{v} when choosing his preferred output, and \underline{v} otherwise. From (9) and (10), it follows that, for a given κ , the probability of acceptance of a G -product is higher than that of a B -product. Thus, the H -type designer always chooses a G -product: this increases both his probability of acceptance and his payoff conditional on acceptance.

The problem of the L -type designer is more subtle, since he faces a trade-off between increasing the acceptance probability (by choosing $y = G$) or increasing his payoff conditional on acceptance (by choosing $y = B$). For a given acceptance strategy of the consumer, the net expected payoff to the L -type from choosing the G -product over the B -product is:

$$\gamma \equiv \max_{\kappa} \mathbb{P}(a = 1|G, \kappa) \cdot \underline{v} - \max_{\kappa} \mathbb{P}(a = 1|B, \kappa) \cdot \bar{v}. \quad (14)$$

The first term is the expected payoff to the L -type from choosing the G -product given the corresponding best choice of complexity, as characterized in Proposition 2. The second term is the expected payoff to the L -type from choosing the B -product given the corresponding best choice of complexity. The probabilities in each scenario are computed as in equation (11), given the consumer's acceptance strategy as characterized by the optimism/pessimism region and the threshold \bar{z} described in Lemma 1. The next result then follows immediately.

Proposition 3 *In any equilibrium, the H -type chooses the G -product, i.e. $m_H = 0$, whereas*

the L -type chooses the B -product with probability

$$m_L \begin{cases} = 0 & \text{if } \gamma > 0 \\ \in [0, 1] & \text{if } \gamma = 0 \\ = 1 & \text{if } \gamma < 0, \end{cases}$$

where γ is given by (14).¹²

3.4 Characterization of Equilibria

In Section 3.1, we characterized the consumer's acceptance strategy given her prior belief μ and interim belief $\mu(z)$. In Sections 3.2 and 3.3, we characterized the designer's choice of output and complexity, given the consumer's acceptance strategy. We now impose belief consistency to characterize the equilibria of our model.

We find it instructive to proceed in two steps. In the first step, we take the equilibrium distribution of product output (i.e., μ) as given, and we require that the consumer's interim belief, $\mu(z)$, be consistent with the designer's choice of complexity. This allows us to characterize the choices of complexity that are consistent with an equilibrium belief of μ . In the second step, we require that belief μ be also consistent with the designer's choice of output. This two-step procedure will help clearly isolate the determinants of product output and complexity, and how these two product attributes are related in equilibrium.

3.4.1 Consistency of Interim Beliefs

For a given equilibrium belief, μ , the consumer's interim beliefs are computed using the designer's strategies and Bayes rule, as given in equation (3). Since we have shown that complexity may be informative about the product's output (Proposition 2), the consumer's interim belief, $\mu(z)$ depends on $\{\sigma_y\}$ through the likelihood ratio as follows

$$\ell(z) = \frac{\mathbb{P}(z|y=B)}{\mathbb{P}(z|y=G)} = \frac{\sigma_B f(z|\bar{\kappa}) + (1 - \sigma_B)f(z|\underline{\kappa})}{\sigma_G f(z|\bar{\kappa}) + (1 - \sigma_G)f(z|\underline{\kappa})}. \quad (15)$$

Interim belief consistency requires that a y -product designer's choice of complexity, $\{\sigma_y\}$, be consistent with the consumer's acceptance strategy, computed using interim beliefs, which in turn depend on $\{\sigma_y\}$. Note that the consumer's acceptance strategy only depends on $\{\sigma_y\}$

¹²If the consumer's strategy is to reject all products, we assume that the t -type designer chooses his preferred action (e.g., H -type chooses the G -product). This rules out the uninteresting equilibrium in which all types propose B -products and the consumer always rejects them.

when the equilibrium features separation in complexity levels. Otherwise, when $\sigma_G = \sigma_B$, noise z does not provide information about product output and thus $\mu(z) = \mu, \forall z$. This is consistent with an equilibrium if the acceptance strategy induced by such belief results in all product designers optimally choosing the same level of complexity. When $\sigma_B > \sigma_G$ (the other inequality never arises in equilibrium), the implied acceptance strategy of the consumer, which now does depend on $\{\sigma_y\}$, needs to be consistent with the designers choosing different complexity levels. The following proposition characterizes the choices of complexity that are consistent with an equilibrium belief $\mu \in (0, 1)$.

Proposition 4 *For a given equilibrium belief, $\mu \in (0, 1)$, there exist belief thresholds $\mu_1 - \mu_4$, which are given by (35)-(44), such that:*

1. *If $\mu \leq \mu_1$, all designers simplify, $\sigma_B = \sigma_G = 0$.*
2. *If $\mu \in (\mu_1, \mu_2]$, the G-product designer simplifies, $\sigma_G = 0$, and the B-product designer complexifies with probability*

$$\sigma_B = \left(\frac{f(\hat{z}|\bar{\kappa})}{f(\hat{z}|\underline{\kappa})} - 1 \right)^{-1} \left(\frac{1-\hat{z}}{\hat{z}} \frac{\mu}{1-\mu} \frac{1-\omega}{\omega} - 1 \right).$$

3. *If $\mu \in (\mu_2, \mu_3]$, the G-product designer simplifies, $\sigma_G = 0$, and the B-product designer complexifies, $\sigma_G = 1$.*
4. *If $\mu \in (\mu_3, \mu_4)$, the G-product designer complexifies with probability*

$$\sigma_G \in \left\{ 0, 1 - \left(1 - \frac{f(\hat{z}|\underline{\kappa})}{f(\hat{z}|\bar{\kappa})} \right)^{-1} \left(1 - \frac{1-\hat{z}}{\hat{z}} \frac{1-\mu}{\mu} \frac{\omega}{1-\omega} \right), 1 \right\},$$

and the B-product designer complexifies, $\sigma_B = 1$.

5. *If $\mu \geq \mu_4$, all designers complexify, $\sigma_B = \sigma_G = 1$.*

These results are illustrated in Figure 2. When μ is relatively low, all designers simplify their products. This is because a pessimistic consumer is very likely to reject a product after observing noisy information, and thus all designers benefit from reducing noise. At the opposite end, when μ is relatively high, all designers complexify their products. Since an optimistic consumer is more likely to accept the product after observing noisy information, all designers benefit from keeping the consumer uninformed. In addition, since complexity is relatively

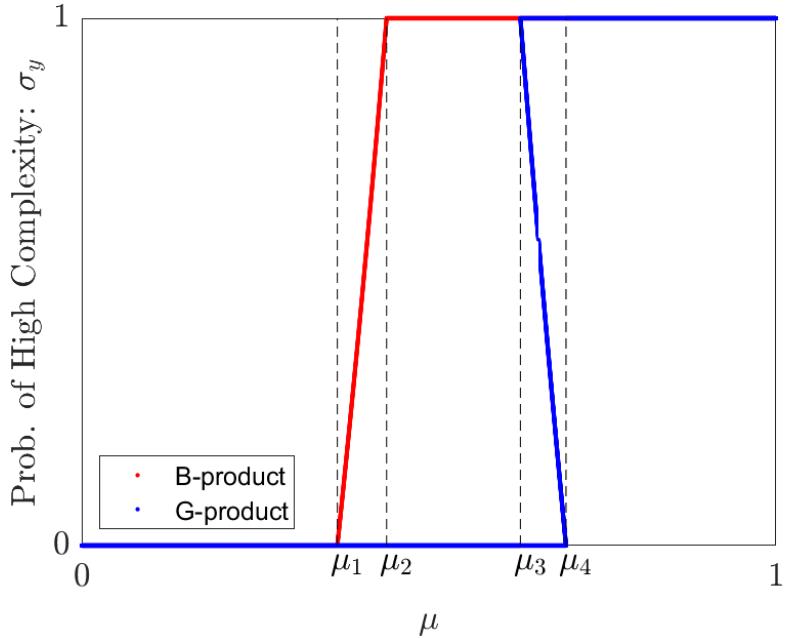


Figure 2: The choice of complexity of an designer that has chosen the y -product as a function of consumer's belief, μ .

more beneficial for a B -product designer, separation may occur as well for intermediate levels of μ , where $\sigma_B > \sigma_G$. Thus, the degree of product complexity depends in important ways on the environment as characterized by (ω, \hat{z}) , which jointly determine thresholds $\mu_1 - \mu_4$ in Proposition 4.

Finally, it is worth noting that there is a region of beliefs, $\mu \in (\mu_3, \mu_4)$, where multiple choices of complexity are consistent with equilibrium. We will see next that this feature may give rise to multiple equilibria.

3.4.2 Equilibrium

We now find the PBE of our model by requiring that belief μ be consistent with the designer's choice of output. Since H -type designers always produce G -products (Proposition 3), we are left to determine the output choice of the L -type, m_L , which implies equilibrium beliefs, $\mu = p + (1 - p)(1 - m_L) \geq p$, where recall p is the probability of a designer being an H -type.

We begin the analysis by making explicit the dependence of the L -type's net payoff from choosing the G -product, $\gamma(\mu)$ in (14), on the consumer's belief μ . The latter determines complexity strategies, $\{\sigma_y\}$ (Proposition 4), and the consumer's acceptance strategy. We define a correspondence $\Gamma : [0, 1] \rightarrow 2^{\mathbb{R}}$, where $\Gamma(\mu)$ is the set of $\gamma(\mu)$ implied by all the complexity choices consistent with an equilibrium with belief μ . The next lemma establishes

some properties of this correspondence, which will be used to determine the set of all PBE.

Lemma 2 *The set of prior beliefs $\{\mu > 0 : 0 \in \Gamma(\mu)\}$ is non-empty and generically has only one element, denoted by ψ . Furthermore, $\gamma(\mu) \geqslant 0$ if and only if $\mu \leqslant \psi$.*

Figure 3 illustrates the behavior of this correspondence, which is single-valued for $\mu \notin (\mu_3, \mu_4)$. When μ is low (i.e., $\mu < \psi$), $\gamma(\mu) > 0$. This is because a pessimistic consumer tends to reject products with high probability. Furthermore, this is the region where products are simple so it is relatively easy for the consumer to distinguish them. Thus, the L -type prefers to increase his probability of acceptance by proposing a G -product, at the expense of losing $\bar{v} - \underline{v}$. However, belief $\mu < 1$ such that $\gamma(\mu) > 0$ cannot be consistent with an equilibrium, since then all designers would choose G -product, contradicting belief consistency. Therefore, if $\gamma(p) > 0$, the equilibrium is in mixed strategies with $m_L \in (0, 1)$ such that $\mu = \psi$.

Conversely, when μ is high (i.e., $\mu > \psi$), $\gamma(\mu) < 0$. An optimistic consumer is very likely to accept a proposed product. Furthermore, this is the region in which products are complex and thus difficult for the consumer to distinguish. Thus, the L -type designer is better off producing a B -product. Thus, $\mu = p$ such that $\gamma(p) < 0$ is consistent with an equilibrium with $m_L = 1$.¹³ These arguments are formalized in the following proposition.

Proposition 5 *An equilibrium always exists, there are generically at most three equilibria, and any equilibrium has the following properties:*

1. *If $p \geq \psi$, then the L -type designer chooses B -product w.p. $m_L = 1$, which implies a prior belief $\mu = p$.*
2. *If $p \leq \psi$, then:*
 - (i) *There is always an equilibrium in which the L -type designer chooses B -product w.p. $m_L = \frac{1-\psi}{1-p}$, which implies a prior belief $\mu = \psi$.*
 - (ii) *If $\inf \Gamma(\mu_3) \leq 0$ and $p \geq \mu_3$, then there is also an equilibrium in which the L -type designer chooses B -product w.p. $m_L = 1$, which implies a prior $\mu = p$.*

Equilibrium complexity is given by Proposition 4 for the corresponding belief μ .

¹³For $\mu \in (\mu_3, \mu_4)$, there may be several $\gamma(\mu) \leq 0$ that are consistent with equilibrium. In Figure 3, for example, the payoff $\gamma(\mu)$ for all such μ is negative, and so yield the same prediction with regards to L -type's choice of output, but there is multiplicity in terms of equilibrium complexity. This, however, need not be the case in general and is not the focus of our paper.

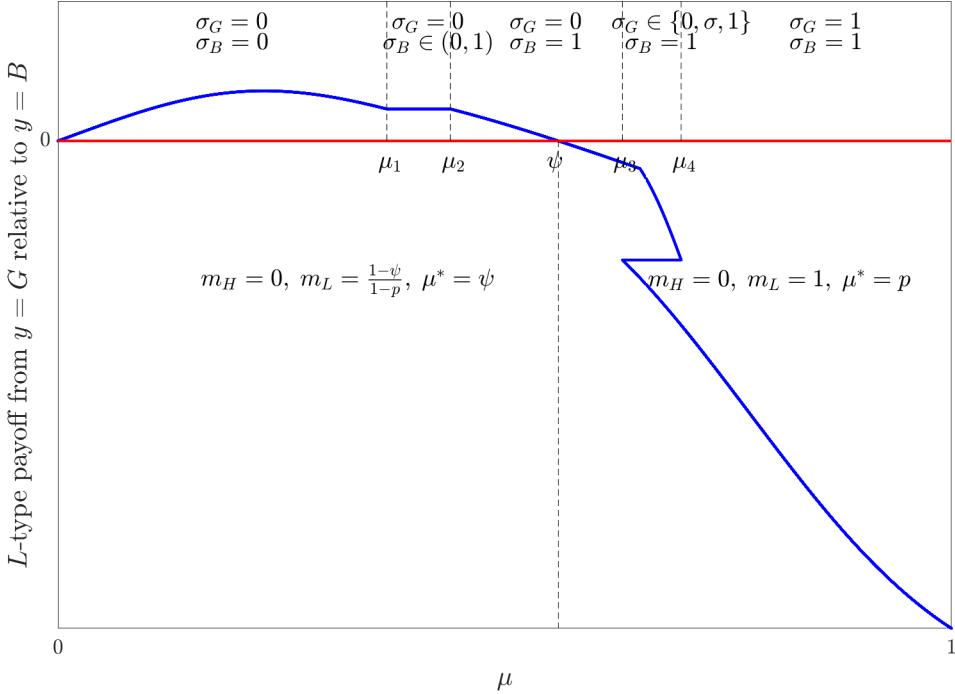


Figure 3: The L -type designer's net benefit of choosing the G -product, for a given consumer belief μ .

The results in Propositions 4 and 5 provide a full characterization of the equilibrium of our model. Figure 3 summarizes these results by indicating both the choices of output (m_y) and complexity (σ_y) consistent with PBE, as a function of $p \in (0, 1)$.

4 Comparative Statics

In this section, we study the comparative statics properties of our model, which we later interpret in the context of two applications (Section 5). We will say that product quality increases when the probability of a product having a good output, $\mathbb{P}[y = G] = \mu$, increases; and that overall complexity increases when the probability of a product being complex, $\mathbb{E}[\sigma_y] = \mu\sigma_G + (1 - \mu)\sigma_B$, increases. We also discuss the quality and complexity of accepted products by conditioning these expectations on a product being accepted.¹⁴

We begin by considering the effect of a decrease in the consumer's relative outside option. A decrease in ω could result from a decrease in the consumer's payoff when the product is

¹⁴Throughout this section, if multiple equilibria arise (i.e. only if $\mu \in (\mu_3, \mu_4)$), we focus on the equilibrium with the lowest level of complexity. Our qualitative results, however, remain unchanged if we focus on any of the other possible equilibria.

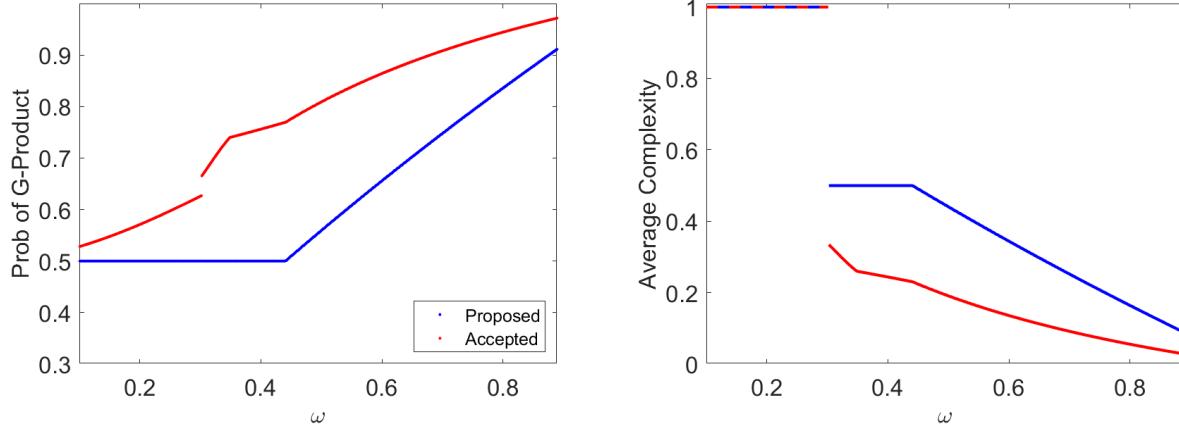


Figure 4: Product quality and complexity of proposed and of approved products as a function of the consumer’s relative outside option $\omega \in [0, 1]$.

rejected, w_0 , or by an increase in the consumer’s payoffs when the product is accepted, i.e., higher $w(G)$ and/or $w(B)$. That is, a decrease in ω reflects that the net value of the product to the consumer increases, making the product more attractive.

Proposition 6 *As the consumer’s relative outside option, ω , decreases, product quality falls, while product complexity increases. Moreover, all designers complexify if ω is sufficiently low.*

Intuitively, when ω is sufficiently large, the consumer is very selective in accepting products. This gives all designers an incentive to produce simple products, and the L -type designer an incentive to produce a G -product (reflected in $\mu = \psi$ increasing). As ω decreases, however, incentives to design simpler and better products fall, since the consumer’s acceptance strategy becomes less strict. When ω is sufficiently low, the consumer accepts almost all products. Thus, all designers complexify and L -type designers produce only B -products (reflected in $\mu = p$ constant). Figure 4 presents these results graphically, and shows that the quality and complexity of accepted products follow the same pattern.

Next, we consider the effect of an increase in the measure of H -type designers, p .

Proposition 7 *As the measure of H -type designers, p , increases, product quality increases, while complexity decreases for μ sufficiently low, and increases otherwise. Moreover, all designers complexity if p is sufficiently large.*

Not surprisingly, as the measure of H -type designers increases, so does product quality, since

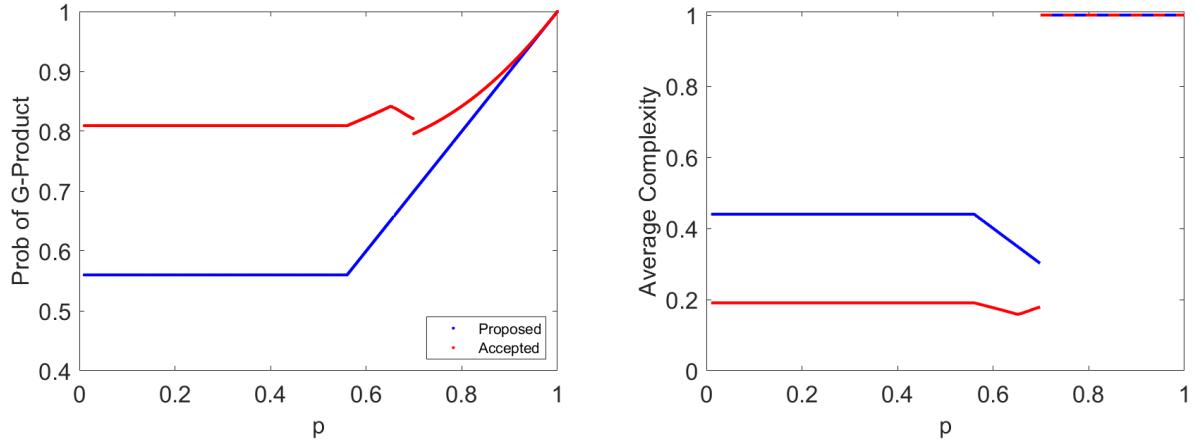


Figure 5: Quality of approved products: output and complexity as a function of p

they always produce G -products.¹⁵ The downside, however, is that the increase in average quality relaxes the acceptance strategy of the consumer, which increases designers' incentives to produce complex products. Consistent with this, when p is sufficiently large, the consumer will accept almost all products, and thus all designers complexify. Even though the quality of proposed products increases in p , the resulting increase in complexity worsens the consumers' ability to identify a G -product, which can result in a decrease in the quality of accepted products, as shown in Figure 5.

Proposition 8 *As \underline{v} increases, product quality (generically) increases, while complexity decreases for μ sufficiently low and increases otherwise. Moreover, all designers complexify if \underline{v} is sufficiently close to \bar{v} .*

As \underline{v} increases, the net payoff to an L -type designer from producing a G -product increases, resulting in an improvement in overall product quality, captured by an increase in ψ . The effect on complexity, however, depends on the nature of the initial equilibrium. In particular, if the initial equilibrium features $\sigma_G < \sigma_B$, then complexity decreases as more L -types propose simple G -products rather than complex B -products. As the expected quality of products increases, however, so do σ_G and σ_B , and eventually complexity increases with overall product quality. This non-monotonicity can be seen graphically in Figure 6.

In the proposition we use the caveat *generically* because, in the region of multiple equilibria, an increase in \underline{v} could generate a jump from the separating equilibrium to the complex one when the former ceases to exist. As a result, ψ jumps down, and so does average quality whenever $p < \psi$. In this scenario, average quality is increasing in \underline{v} before and after the jump.

¹⁵For $p < \psi$ such initial increase in quality improves the incentives of the L -type to produce B -products,

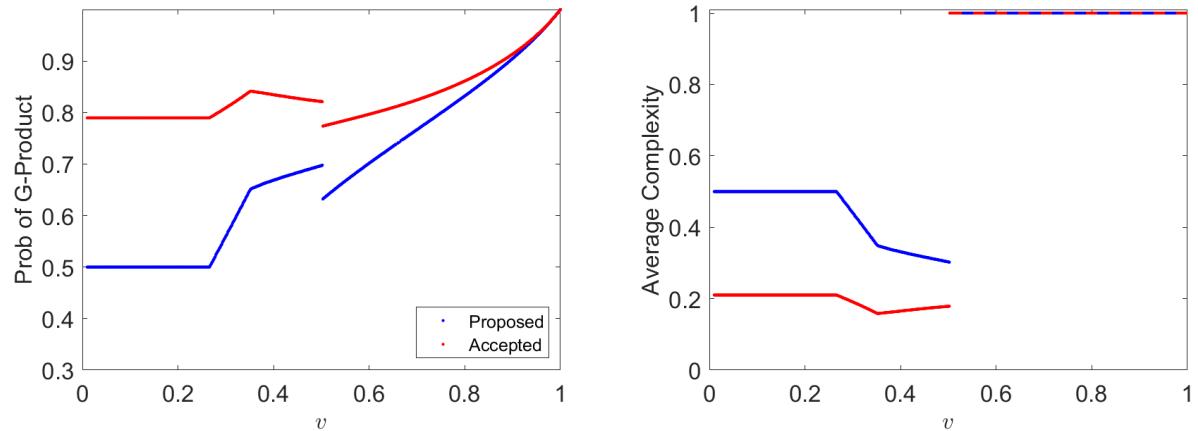


Figure 6: Quality of approved products: output and complexity as a function of v

The results in Propositions 6 to 7 have implications for the relationship between product quality and complexity, and highlight the importance of understanding the underlying drivers of product heterogeneity. We have shown that while more alignment between the consumer and the designer (captured by an increase in the measure of aligned designers, p , or by an increase in the payoff to L -types from taking the aligned action, v), results in better and more complex products, higher demand for the products results in worse and more complex products. In what follows, we interpret the results of this section in the context of two applications.

5 Applications

We focus on two main applications that are at the center of the policy and academic discussion on issues related to complexity, and that have motivated our work: the design of financial products and of regulatory policies. In what follows, we describe in detail how to map our model to these applications, and the resulting implications. Even though our model is stylized and not able to capture the richness of the institutional details of these environments, we present simple extensions to address particular issues of interest that arise within a particular application.

resulting in an increase in m_L and a constant overall quality, $\mu = \psi$.

5.1 Financial products

Banks and other financial intermediaries design financial products that they offer retail investors, such as savings and retirement accounts, mortgages, credit lines, etc. When a retail investor (i.e., the consumer in our model) approaches a financial advisor (i.e., the product designer), the latter chooses which product to offer to the consumer. In practice, financial advisors may receive different payments from selling a given financial product to investors, and those products that give the financial advisor a higher commission need not be the best ones for the retail investor. When incentives are not aligned, the financial advisor faces a trade-off as the one faced by the designer in our model: to increase the probability of acceptance by offering the product that best suits the needs of the investor, or to increase his payoff conditional on acceptance by choosing the product with a higher commission. In turn, the choice of product design within financial institutions will crucially depend on the financial advisor's ability to sell different types of financial products, such as more or less complex. In what follows, we examine the drivers of product quality (i.e. the net present value of a financial product to the investor) and of the complexity in financial products (i.e., the multidimensionality of the contract) through the lenses of our model.¹⁶

Trust in financial advisors. Proposition 7 states that as the measure of aligned financial advisors, p , increases (which we view as an increase in the fraction of advisors whom retail investors trust, or that are “honest advisors”) the overall quality of financial product increases, but so does their complexity. In our model, retail investors are rational and they do not misperceive the distribution of financial advisors. However, if we allowed for such deviations, an unjustified increase in the trust in financial advisors would generate a decrease in product quality accompanied by an increase in complexity. This observation is consistent with the behavior of several financial intermediaries that during the 2000s were allegedly designing and selling increasingly worse and more complex financial products to overly optimistic retail investors, a behavior that contributed to the financial crisis and resulted in multiple lawsuits.¹⁷

Motivated by this conflict of interest, the Securities and Exchange Comission (SEC) has increased its efforts to forbid the use of the term “financial adviser” for those managing brokerage accounts (particularly retirement funds) unless the broker has formally accepted a

¹⁶While the net present value of a financial product is related to its dimensionality, it is also possible that two products provide the same net present value to an investor but vary in their dimensionality, as we have modeled it. We consider the case of complexity affecting the consumer's payoff directly in Section 5.2.

¹⁷In 2011, the Federal Housing Finance Agency filed lawsuits against some of the largest US financial institutions, involving allegations of securities law violations and fraud in the packaging and sale of mortage-backed-securities. For a detailed description, see <https://www.fhfa.gov/SupervisionRegulation/LegalDocuments/Pages/Litigation.aspx>.

fiduciary duty to act in the investor’s best interest.¹⁸ If we interpret such policy as increasing the fraction of aligned financial advisors, our model predicts that it would not only increase the quality of financial products (as intended by the SEC), but also their complexity.

Competition in financial markets. To capture competition in financial markets, we extend our model to a dynamic search setting to capture the effects of competition. In particular, we now suppose that, if the consumer rejects a product, she searches for a new designer, whom she finds with probability $\beta \in (0, 1)$. The new designer proposes a product to the consumer, and the game repeats until the consumer accepts an offered product. In this setting, a higher β implies lower search frictions and, hence, more competition in the market for designers.

In a stationary equilibrium, in which U denotes the consumer’s equilibrium value, we have:

$$U = \mathbb{E} \left[\max_{a \in \{0,1\}} \{a \cdot [\mu(s, z) \cdot w(G) + (1 - \mu(s, z)) \cdot w(B)] + (1 - a) \cdot \beta U\} \right]. \quad (16)$$

If we set $w_0 = \beta U$, then the static equilibrium is fully characterized in Section 3.4.

Proposition 9 *An equilibrium exists, and in it $\beta U \in (w(B), w(G))$ provided that β is not too low. Furthermore, βU is increasing in β .*

Comparative statics with respect to β are qualitatively similar to the comparative statics with respect to ω in Proposition 6. This is because, in a search environment, βU is the consumer’s effective outside option, which we have shown increases in β . Thus, just as in our baseline model, an increase in β leads to an increase in average quality and a decrease in complexity of products.

Demand for financial products. An increase in the relative payoff of given financial product to the consumer is captured in our model by a decrease in the consumer’s outside option (w_0), or by an increase in the payoffs associated with a given product ($w(G)$ and $w(B)$). As shown in Proposition 6, our model suggests that as retail investors’ demand for a given financial product increases, the quality of such products falls while their complexity increases. These predictions are consistent with the observed trend in financial products that were perceived as “safe” before the financial crisis. The increase in investor demand for these products, sometimes blamed on the so-called *savings glut*, could have been an important driver of their worsening quality and increased complexity, as exemplified by mortgage-backed-securities.¹⁹

¹⁸ “Fiduciary Rule” Poised for Second Life Under Trump Administration, article by Dave Michaels on the Wall Street Journal, January 10th, 2018: <https://www.wsj.com/articles/fiduciary-rule-poised-for-second-life-under-trump-administration-1515580200>.

¹⁹For evidence on the increasing demand for safe products; see Bernanke (2005), on the worsening quality

Compensation structures. Our model can also be used to examine the role played by the financial advisors' compensation structure. If the compensation of financial advisors is linked to the volume or the characteristics of financial products that they sell, then the advisors are more likely to be misaligned with retail investors. In our model, we interpret this friction as the difference between the payoff associated with selling good vs. bad product: $\bar{v} - \underline{v}$. As shown in Proposition 8, as the difference in payments across products is reduced and the incentives of the financial advisor become more aligned with those of the investor, there is an increase provision of good financial products. Interestingly, such a change in compensation structures could result in more complexity when the effect on overall quality is strong enough. Note, however, that complexity is not too detrimental when the overall quality of financial products is very high.

5.2 Regulatory Complexity

Politicians design and propose policies to achieve a policy agenda. In doing so, they balance their own preferences (determined by ideology, informational lobbying etc.) and the need to obtain voter approval for that policy. In this environment, politicians (i.e., product designers) propose policies to voters (i.e., the consumer) who may accept or reject them. For instance, if the policy in question is a tax reform, the policy's quality is given by whether it implies "higher or lower taxes" or "more or less redistribution." In contrast, the policy is more complex if it contains unnecessarily complicated wording, several unlikely contingencies etc. An illustrative example of such complexity comes from the regulatory framework proposed by the Basel Committee on Banking Supervision. An analysis of its text has shown that an average sentence in the Basel documents consists of 25.7 words, significantly longer than the average 21 words in a sentence of the British National Corpus.²⁰ Moreover, the second sentence of the very first document published by the Basel Committee on Banking supervision spans over 72 words.²¹

Public opinion. In most cases, politicians do not have to wait until election time to learn about voters' support. Public opinion data provides politician's with real time information about voters' perceptions. Public opinion in our model would be captured by the voters' belief about the politicians being aligned with their interests in a certain policy area, captured by μ . Our

of securitized products see Jaffee et al. (2009); and on the increasing complexity of financial products see Célérier and Vallée (2015).

²⁰The British National Corpus is a collection of texts covering a broad range of modern British English.

²¹Analysis performed by Neue Zürcher Zeitung, as cited by Marie-Jose Kolly and Jurg Muller, <https://www.endofbanking.org/2018/05/22/how-banking-regulation-has-grown-out-of-all-proportions>

model suggests that when public opinion is high, politicians have incentives to complexify policies in that area, while simplicity should arise when public opinion is low. A variant of this implication has been used by legal scholars to explain why policy proposals coming out of the US Congress in domains in which public opinion of politicians is low tend to be simpler, leaving it to federal agencies to propose additional policies and to draft rules for these industries.²² For instance, voters have higher distrust of politicians when it comes to policies pertaining to industries that are major lobbyists and contributors to campaigns, such as the financial services or pharmaceutical industries, and politicians tend to delegate more of that lawmaking to regulators rather than proposing complex policies themselves.

In terms of the quality of proposed policies, our model predicts that the quality of policies should be high when the improvement in public opinion is rational, but it may low if voters are being overly optimistic about the alignment between their interests and those of the politicians.

Urgency. The pressure or urgency to pass a given policy may vary depending on the reform under discussion. For example, there was a strong sense of urgency to pass financial regulation reform after the 2008-09 crisis. One possible reason being that the public did not feel they could trust the financial system otherwise. Other type of policies, such as environmental policies, do not seem to be considered with the urgency that they maybe deserve. Through the lenses of our model, urgency could be captured as the voters' outside option, a measure of the status-quo: i.e., the payoff to voters if the reform does not pass. In light of this, our model suggests that when there is urgency to pass a given reform, policies will tend to be of worse quality and more complex; while lack of urgency would result in better quality and simpler policies. This prediction is consistent with the mere observation that in the U.S., reforms passed in times of urgency, such as the Dodd-Frank Act or the Affordable Care Act, have been described as overly complex, while those passed in normal times, such as the Clean Air Act, seem to be seen as much simpler.

Direct costs of complex or simple rules. In the context of regulation, it is natural to think that the level of complexity may have a direct impact on the voter. On the one hand, complex policies may be worse if they imply a higher costs of compliance, e.g. hiring accountants and lawyers. On the other hand, more complex policies may be better when regulating complex systems, such as the financial system. In view of this, in Appendix D we extend our model to introduce what we refer to as a “natural level of complexity,” which we denote by κ^n , where deviations from this natural level are costly to the consumer. Specifically, we suppose that

²²See [Stiglitz \(2017\)](#).

the consumer pays a cost $c(\kappa) > 0$ when a product with complexity $\kappa \neq \kappa^n$ is accepted, and zero otherwise, where $\kappa^n \in \{\underline{\kappa}, \bar{\kappa}\}$. Then, given information (s, z) and acceptance decision $a \in \{0, 1\}$, the consumer's payoff is:

$$W(a|s, z) \equiv a \cdot E [w(y) - c(\kappa)|s, z] + (1 - a) \cdot w_0 \quad (17)$$

In contrast to our baseline model, the consumer's acceptance strategy is now modified to incorporate the direct cost of complexity (or simplicity). The consumer's acceptance rule is again contingent on the signal only when the signal is sufficiently informative, i.e., $z < \bar{z}$, but with an adjusted threshold \bar{z} . The equilibrium analysis is analogous to the baseline model, with the not surprising prediction that the equilibrium level of complexity will be lower (higher) if complexity (simplicity) is costlier. Hence, our model helps us understand the policymakers' or regulators' strategic motives for designing more or less complex products relative to what would otherwise be natural, i.e. optimal for the voters.²³

6 Extension

6.1 Observable Complexity

We now consider the case in which the designer's choice of $\kappa \in \{\underline{\kappa}, \bar{\kappa}\}$ is observable to the consumer. The rest of the model is unchanged from the setup presented in the main text. When κ is observed by the consumer, it can potentially operate as a signal of the product's output. Note that the noise z , in turn, no longer provides any information since κ is directly observed and thus there is no need to infer it from z . Thus, given prior belief μ , the consumer does a first belief update after observing κ to interim belief

$$\mu(\kappa) = \Pr(y = G|\kappa) = \frac{\mu}{\mu + (1 - \mu) \frac{\Pr(\kappa|y=B)}{\Pr(\kappa|y=G)}}, \quad (18)$$

and a second belief update after observing the signal to posterior beliefs $\mu(s, z)$, given by (4) where interim beliefs $\mu(z)$ should be replaced by $\mu(\kappa)$. The first important result is that, since a B -product is never accepted by the consumer, i.e., there are no gains from trade for the B -product, then separation cannot be obtained in equilibrium.

Lemma 3 *In any equilibrium, there is pooling on complexity; that is, $\sigma_G = \sigma_B \in \{0, 1\}$.*

²³It is straightforward to show that under full information, i.e. if (y, κ) were observable to the consumer, then all designers would produce (G, κ^n) products.

Thus, even though complexity can in principle be used as a signaling device by designers, those designing B -products choose to always mimic the complexity choice of G -product designers. Intuitively, any equilibrium in which different product types come with different complexities can be ruled out by requiring belief consistency from the consumer. However, the freedom of setting off-equilibrium beliefs gives rise to multiple equilibria in this setting. For example, if the consumer assigns a belief of $\mu = 0$ to deviations from equilibrium complexity, κ , then no designer would find it profitable to deviate. As a result, all complexity levels can be supported in equilibrium. This is formalized in the following proposition.

Proposition 10 *There are two equilibria, which always co-exists, with the following features:*

- *Simple equilibrium (SE):* $\sigma_G^S = \sigma_B^S = 1$, with $m_H^S = 0$ and $m_L^S = \begin{cases} \frac{1-\psi^S}{1-p} & \text{if } p \leq \psi^S \\ 1 & \text{o.w.} \end{cases}$.
- *Complex equilibrium (CE):* $\sigma_G^C = \sigma_B^C = 0$, with $m_H^C = 0$ and $m_L^C = \begin{cases} \frac{1-\psi^C}{1-p} & \text{if } p \leq \psi^C \\ 1 & \text{o.w.} \end{cases}$.

where $\psi^C < \psi^S$ are given by equations (66)-(67) respectively.

Therefore, even when complexity is observable, there exists an equilibrium in which G -product designers choose to complexify their products, and an equilibrium in which B -product designers choose to simplify them. To provide a sharper characterization, we select the equilibrium that is preferred by the high type.²⁴

Proposition 11 *The equilibrium that provides the H -type designer the highest payoff has the following properties:*

- If $\max\{p, \psi^S\} \leq \bar{\mu}$, then $\mu = \max\{p, \psi^S\}$ and all designers simplify $\sigma_G = \sigma_B = 0$.
- If $\max\{p, \psi^S\} > \bar{\mu}$, then $\mu = \max\{p, \psi^C\}$ and all designers complexity $\sigma_G = \sigma_B = 1$.

where $\bar{\mu} = \frac{\omega(1-\hat{z})}{\omega(1-\hat{z}) + \hat{z}(1-\omega)}$, and \hat{z} is defined in Proposition 2.

As in our baseline model, as the fraction of H -type designers increases, products are more likely to be more complex and of better quality. The results in Proposition 11 show that the main dynamics of our model survive when the choices of κ are observable when we select the

²⁴This selection mechanism is also used in Perez-Richet and Prady (2011), and it is reasonable in our setting given the leading role the G -product designer play in determining equilibrium product complexity levels. That is, B -product designers always mimic the choice of G -product designers.

equilibrium preferred by the H -type.²⁵ The comparative statics on designer's alignment, as measured by p and \underline{v} , and relative outside option, ω , are qualitatively as those in the main model as well.

7 Conclusion

We presented a model of complexity, in which complexity is a strategic choice made by product designers. The model sheds light on the incentives of designers to add complexity to products in order to increase their chance of acceptance by a consumer. The model delivers two powerful insights. First, complexity is not necessarily a feature of worse quality products. All designers may complexify their products or they may all simplify their products. We show how this choice depends crucially on the consumer's beliefs about the designers and the consumer's outside option. Second, the relationship between average product quality and complexity depends on the underlying drivers of product heterogeneity. More alignment between the consumer and the designer increases average quality as well as average complexity. More competition between designers, however, increases average product quality and decreases average complexity.

Our model provides a tractable framework for analyzing the joint choice of a product's quality and complexity, and it can be extended along several dimensions. In particular, future work may examine the evolution of quality and complexity over time for financial products and policies which are subject to amendments or renegotiations.

²⁵If we were to select the equilibrium preferred by the L -type, the results in Proposition 11 continue to hold for a higher threshold $\bar{\mu}$.

References

- Alesina, A. and A. Cukierman (1990). The politics of ambiguity. *The Quarterly Journal of Economics* 105(4), 829–850.
- Aragones, E. and A. Postlewaite (2002). Ambiguity in election games. *Review of Economic Design* 7(3), 233–255.
- Auster, S. and N. Pavoni (2018). Optimal delegation and limited awareness, with an application to financial intermediation.
- Bar-Isaac, H., G. Caruana, and V. Cuñat (2010). Information gathering and marketing. *Journal of Economics & Management Strategy* 19(2), 375–401.
- Bernanke, B. (2005). The global saving glut and the u.s. current account deficit. Speech 77, Board of Governors of the Federal Reserve System (U.S.).
- Carlin, B. I. (2009). Strategic price complexity in retail financial markets. *Journal of financial Economics* 91(3), 278–287.
- Carlin, B. I. and G. Manso (2011). Obfuscation, learning, and the evolution of investor sophistication. *Review of Financial Studies* 24(3), 754–785.
- Célérier, C. and B. Vallée (2015). Catering to investors through product complexity.
- Christoffersen, S. E. and D. K. Musto (2002). Demand curves and the pricing of money management. *The Review of Financial Studies* 15(5), 1499–1524.
- Crawford, V. P. and J. Sobel (1982). Strategic information transmission. *Econometrica*, 1431–1451.
- Davis, S. (2017). Regulatory complexity and policy uncertainty: headwinds of our own making. *Becker Friedman Institute for Research in Economics Working Paper No. 2723980*.
- Dewatripont, M. and J. Tirole (2005). Modes of communication. *Journal of Political Economy* 113(6), 1217–1238.
- Ellison, G. and S. F. Ellison (2009). Search, obfuscation, and price elasticities on the internet. *Econometrica* 77(2), 427–452.
- Ellison, G. and A. Wolitzky (2012). A search cost model of obfuscation. *The RAND Journal of Economics* 43(3), 417–441.

- Espinosa, F. and D. Ray (2018, May). Noisy agents. Working Paper 24627, National Bureau of Economic Research.
- Gabaix, X. and D. Laibson (2006). Shrouded attributes, consumer myopia, and information suppression in competitive markets*. *The Quarterly Journal of Economics* 121(2), 505–540.
- Green, J. R. and N. L. Stokey (2007). A two-person game of information transmission. *Journal of economic theory* 135(1), 90–104.
- Grossman, S. J. (1981). The informational role of warranties and private disclosure about product quality. *The Journal of Law and Economics* 24(3), 461–483.
- Jaffee, D., A. Lynch, M. Richardson, and S. Van Nieuwerburgh (2009). Mortgage origination and securitization in the financial crisis. *Financial Markets, Institutions & Instruments* 18(2), 141–143.
- Kamenica, E. and M. Gentzkow (2011). Bayesian persuasion. *The American Economic Review* 101(6), 2590–2615.
- Kartik, N. (2009). Strategic communication with lying costs. *The Review of Economic Studies* 76(4), 1359–1395.
- Milgrom, P. R. (1981). Good news and bad news: Representation theorems and applications. *The Bell Journal of Economics*, 380–391.
- Pagano, M. and P. Volpin (2012). Securitization, transparency, and liquidity. *The Review of Financial Studies* 25(8), 2417–2453.
- Perez-Richet, E. and D. Prady (2011). Complicating to persuade. *SSRN Working Paper*.
- Sims, C. A. (2003). Implications of rational inattention. *Journal of monetary Economics* 50(3), 665–690.
- Stiglitz, E. H. (2017). Delegating for trust. *U. Pa. L. Rev.* 166, 633.
- Wiederholt, M. et al. (2010). Rational inattention. *The New Palgrave Dictionary of Economics (Online Edition ed.)*.

A Appendix: Proofs and Complementary Lemmas

A.1 Proofs for Section 3

Proof of Lemma 1. See text. ■

Proof of Proposition 2. We begin by studying the designer's optimal choice of κ in the optimistic region, as in Definition 1.

Case 1 (consumer is optimistic). In this region, the designer is accepted w.p.1 when information is noisy enough, $z > \bar{z}$. So, his optimal choice of κ solves:

$$\max_{\kappa \in \{\underline{\kappa}, \bar{\kappa}\}} \int_0^{\bar{z}} P(s = g|y) \cdot f(z|\kappa) dz + \int_{\bar{z}}^{1/2} f(z|\kappa) dz. \quad (19)$$

Thus, it is optimal for the designer of B -product to choose $\bar{\kappa}$ if

$$\int_0^{\bar{z}} z \cdot f(z|\bar{\kappa}) dz + \int_{\bar{z}}^{1/2} f(z|\bar{\kappa}) dz \geq \int_0^{\bar{z}} z \cdot f(z|\underline{\kappa}) dz + \int_{\bar{z}}^{1/2} f(z|\underline{\kappa}) dz, \quad (20)$$

and it is uniquely optimal if the inequality is strict. This is equivalent to:

$$\int_0^{\bar{z}} (1 - z) \cdot (f(z|\underline{\kappa}) - f(z|\bar{\kappa})) dz \geq 0. \quad (21)$$

But, note that we have:

$$\int_0^{\bar{z}} (1 - z) \cdot (f(z|\underline{\kappa}) - f(z|\bar{\kappa})) dz > (1 - \bar{z})(F(\bar{z}|\underline{\kappa}) - F(\bar{z}|\bar{\kappa})) > 0. \quad (22)$$

for $\bar{z} > 0$, as will be the case in equilibrium. Thus, condition (20) is satisfied with strict inequality, and it is uniquely optimal for the designer of the B -product to choose $\bar{\kappa}$.

On the other hand, it is optimal for the designer of G -product to choose $\bar{\kappa}$ if

$$\int_0^{\bar{z}} (1 - z) \cdot f(z|\bar{\kappa}) dz + \int_{\bar{z}}^{1/2} f(z|\bar{\kappa}) dz \geq \int_0^{\bar{z}} (1 - z) \cdot f(z|\underline{\kappa}) dz + \int_{\bar{z}}^{1/2} f(z|\underline{\kappa}) dz, \quad (23)$$

and it is uniquely optimal if the inequality is strict. This is equivalent to:

$$\int_0^{\bar{z}} z \cdot (f(z|\underline{\kappa}) - f(z|\bar{\kappa})) dz \geq 0. \quad (24)$$

Condition (23) is satisfied if $\bar{z} \leq \hat{z}$, and holds with strictly inequality if $\bar{z} < \hat{z}$. Thus, if $\bar{z} < \hat{z}$,

it is uniquely optimal for the designer of G -product to choose $\bar{\kappa}$. Otherwise, if $\bar{z} = \hat{z}$, the designer is indifferent to the choice of κ , and if $\bar{z} > \hat{z}$, it is uniquely optimal to choose $\underline{\kappa}$.

Next, we study the designer's optimal choice of κ in the pessimistic region.

Case 2 (consumer is pessimistic). In the region, the designer is rejected if information is too noisy, $z > \bar{z}$. So, his optimal choice of κ solves:

$$\max_{\kappa \in \{\underline{\kappa}, \bar{\kappa}\}} \int_0^{\bar{z}} P(s = g|y) \cdot f(z|\kappa) dz. \quad (25)$$

Thus, it is optimal for the designer of B -product to choose $\underline{\kappa}$ if

$$\int_0^{\bar{z}} z \cdot f(z|\bar{\kappa}) dz \leq \int_0^{\bar{z}} z \cdot f(z|\underline{\kappa}) dz, \quad (26)$$

and it is uniquely optimal if the inequality is strict. This is equivalent to:

$$\int_0^{\bar{z}} z \cdot (f(z|\underline{\kappa}) - f(z|\bar{\kappa})) dz \geq 0. \quad (27)$$

Condition (26) is satisfied if $\bar{z} \leq \hat{z}$, and holds with strict inequality if $\bar{z} < \hat{z}$. Thus, if $\bar{z} < \hat{z}$, it is uniquely optimal for the designer of B -product to choose $\underline{\kappa}$. Otherwise, if $\bar{z} = \hat{z}$, the designer is indifferent to the choice of κ , and if $\bar{z} > \hat{z}$, it is uniquely optimal to choose $\bar{\kappa}$.

On the other hand, it is optimal for the designer of the G -product to choose $\underline{\kappa}$ if

$$\int_0^{\bar{z}} (1-z) \cdot f(z|\bar{\kappa}) dz \leq \int_0^{\bar{z}} (1-z) \cdot f(z|\underline{\kappa}) dz, \quad (28)$$

and it is uniquely optimal if the inequality is strict. This is equivalent to:

$$\int_0^{\bar{z}} (1-z) \cdot (f(z|\underline{\kappa}) - f(z|\bar{\kappa})) dz \geq 0. \quad (29)$$

Re-writing the above condition, we have

$$\int_0^{\bar{z}} (f(z|\underline{\kappa}) - f(z|\bar{\kappa})) dz > \int_0^{\bar{z}} z \cdot (f(z|\underline{\kappa}) - f(z|\bar{\kappa})) dz, \quad (30)$$

which immediately implies that condition (28) is satisfied for all $\bar{z} > 0$, as will be the case in equilibrium, and it is uniquely optimal for the designer of the G -product to choose $\underline{\kappa}$. ■

Proof of Proposition 3. The H -type's net benefit from choosing the G -product is:

$$\gamma(\mu) = \max_{\kappa} \mathbb{P}_{\mu}(a = 1|G, \kappa) \cdot \bar{v} - \max_{\kappa} \mathbb{P}_{\mu}(a = 1|B, \kappa) \cdot \underline{v}. \quad (31)$$

Since $\mathbb{P}_{\mu}(a = 1|G, \kappa) \geq \mathbb{P}_{\mu}(a = 1|B, \kappa) > 0$ for all κ and $\bar{v} > \underline{v}$, it follows that the net benefit for the H -type is positive, and it is optimal for him to choose the G -product.

On the other hand, the L -type's net benefit from choosing the G -product is:

$$\gamma(\mu) = \max_{\kappa} \mathbb{P}_{\mu}(a = 1|G, \kappa) \cdot \underline{v} - \max_{\kappa} \mathbb{P}_{\mu}(a = 1|B, \kappa) \cdot \bar{v}. \quad (32)$$

Since again $\mathbb{P}_{\mu}(a = 1|G, \kappa) \geq \mathbb{P}_{\mu}(a = 1|B, \kappa) > 0$ and $\bar{v} > \underline{v}$, we can have $\gamma(\mu) \leq 0$. The L -type chooses the G -product whenever $\gamma(\mu) > 0$, the B -product whenever $\gamma(\mu) < 0$, and he is indifferent whenever $\gamma(\mu) = 0$. ■

Proof of Proposition 4. Suppose that, in equilibrium, the consumer's belief that the designer has produced G -product is $\mu \in (0, 1)$.

Pooling equilibria. Consider first the candidate pooling equilibrium in which $\sigma_B = \sigma_G = 0$. By Proposition 2, this requires that $\mu \leq \omega$ and $\bar{z} \leq \hat{z}$. On equilibrium path, the consumer does not update from observation of z and, thus, threshold \bar{z} is given by

$$\mu(g, \bar{z}) = \omega, \quad (33)$$

which is equivalent to:

$$\bar{z} = \frac{(1 - \omega) \cdot \mu}{(1 - \omega) \cdot \mu + \omega \cdot (1 - \mu)}. \quad (34)$$

This is an equilibrium if and only if $\bar{z} \leq \hat{z}$, which is equivalent to:

$$\mu \leq \frac{\omega \cdot \frac{\hat{z}}{1 - \hat{z}}}{\omega \cdot \frac{\hat{z}}{1 - \hat{z}} + 1 - \omega} \equiv \mu_1. \quad (35)$$

Consider next the candidate pooling equilibrium in which $\sigma_B = \sigma_G = 1$. By Proposition 2, this requires that $\mu \geq \omega$ and $\bar{z} \leq \hat{z}$. On equilibrium path, the consumer does not update from observation of z and, thus, threshold \bar{z} is given by

$$\mu(b, \bar{z}) = \omega, \quad (36)$$

which is equivalent to:

$$\bar{z} = \frac{\omega \cdot (1 - \mu)}{(1 - \omega) \cdot \mu + \omega \cdot (1 - \mu)}. \quad (37)$$

This is an equilibrium if and only if $\bar{z} \leq \hat{z}$, which is equivalent to:

$$\mu \geq \frac{\omega \cdot \frac{1-\hat{z}}{\hat{z}}}{\omega \cdot \frac{1-\hat{z}}{\hat{z}} + 1 - \omega} \equiv \mu_3. \quad (38)$$

Therefore, $\sigma_B = \sigma_G = 0$ is an equilibrium if and only if $\mu \in (0, \mu_1]$, whereas $\sigma_B = \sigma_G = 1$ is an equilibrium if and only if $\mu \in [\mu_3, 1)$.

Separating equilibria. Consider the candidate *separating equilibrium* in which $\sigma_B = 1$ and $\sigma_G = 0$. There are two cases to consider, depending on whether the consumer is optimistic or pessimistic.

First, suppose that

$$\mu \left(g, \frac{1}{2} \right) = \mu \left(b, \frac{1}{2} \right) = \frac{\mu}{\mu + (1 - \mu) \cdot \ell \left(\frac{1}{2} \right)} \leq \omega, \quad (39)$$

where $\ell(\cdot) = \frac{f(\cdot | \bar{\kappa})}{f(\cdot | \underline{\kappa})}$. Then, the consumer must be pessimistic. On equilibrium path, there is updating from observation of z , and thus threshold \bar{z} is given by

$$\mu(g, \bar{z}) = \frac{\mu}{\mu + (1 - \mu) \cdot \ell(\bar{z}) \cdot \frac{\bar{z}}{1 - \bar{z}}} = \omega. \quad (40)$$

This is an equilibrium if and only if also $\bar{z} \geq \hat{z}$, i.e.

$$\mu_2 \equiv \frac{\omega \cdot \ell(\hat{z}) \cdot \frac{\hat{z}}{1 - \hat{z}}}{\omega \cdot \ell(\hat{z}) \cdot \frac{\hat{z}}{1 - \hat{z}} + 1 - \omega} \leq \mu \leq \frac{\omega \cdot \ell \left(\frac{1}{2} \right)}{\omega \cdot \ell \left(\frac{1}{2} \right) + 1 - \omega} \equiv \tilde{\mu}. \quad (41)$$

Second, suppose that

$$\mu \left(g, \frac{1}{2} \right) = \mu \left(b, \frac{1}{2} \right) = \frac{\mu}{\mu + (1 - \mu) \cdot \ell \left(\frac{1}{2} \right)} > \omega, \quad (42)$$

Then, the consumer must be optimistic. The threshold \bar{z} is now given by

$$\mu(b, \bar{z}) = \frac{\mu}{\mu + (1 - \mu) \cdot \ell(\bar{z}) \cdot \frac{1 - \bar{z}}{\bar{z}}} = \omega. \quad (43)$$

This is an equilibrium if and only if also $\bar{z} \geq \hat{z}$, i.e.

$$\tilde{\mu} = \frac{\omega \cdot \ell\left(\frac{1}{2}\right)}{\omega \cdot \ell\left(\frac{1}{2}\right) + 1 - \omega} < \mu \leq \frac{\omega \cdot \ell(\hat{z}) \cdot \frac{1-\hat{z}}{\hat{z}}}{\omega \cdot \ell(\hat{z}) \cdot \frac{1-\hat{z}}{\hat{z}} + 1 - \omega} \equiv \mu_4. \quad (44)$$

Therefore, $\sigma_B = 0$ and $\sigma_G = 1$ is an equilibrium if and only if $\mu \in [\mu_2, \mu_4]$.

Semi-separating equilibria. Consider the candidate semi-separating equilibrium, in which $\sigma_B \in (0, 1)$. By Proposition 2, such an equilibrium requires that the consumer be pessimistic and so $\sigma_G = 0$. On equilibrium path, there is updating from observation of z , and threshold \bar{z} must exactly equal \hat{z} so that the designer of B -product is indifferent to the choice of κ (Proposition 2) and is willing to mix:

$$\mu(g, \hat{z}) = \frac{\mu}{\mu + (1 - \mu) \cdot (\sigma_B \cdot \ell(\hat{z}) + 1 - \sigma_B) \cdot \frac{\hat{z}}{1 - \hat{z}}} = \omega, \quad (45)$$

which in turn implies that:

$$\sigma_B = \frac{\frac{1-\hat{z}}{\hat{z}} \cdot \frac{\mu}{1-\mu} \cdot \frac{1-\omega}{\omega} - 1}{\frac{f(\hat{z}|\bar{\kappa})}{f(\hat{z}|\underline{\kappa})} - 1}. \quad (46)$$

Since the posterior belief $\mu(g, \hat{z})$ is continuous and decreasing in σ_B (MLRP implies that $\ell(\hat{z}) > 1$), this equilibrium exists if and only if:

$$\mu(g, \hat{z})|_{\sigma_B=1} < \omega < \mu(g, \hat{z})|_{\sigma_B=0}, \quad (47)$$

which is equivalent to:

$$\mu_1 = \frac{\omega \cdot \frac{\hat{z}}{1-\hat{z}}}{\omega \cdot \frac{\hat{z}}{1-\hat{z}} + 1 - \omega} < \mu < \frac{\omega \cdot \ell(\hat{z}) \cdot \frac{\hat{z}}{1-\hat{z}}}{\omega \cdot \ell(\hat{z}) \cdot \frac{\hat{z}}{1-\hat{z}} + 1 - \omega} = \mu_2. \quad (48)$$

Therefore, $\sigma_G = 0$ and $\sigma_B \in (0, 1)$ is an equilibrium if and only if $\mu \in (\mu_1, \mu_2)$.

Consider the candidate semi-separating equilibrium in which $\sigma_G \in (0, 1)$. By Proposition 2, such an equilibrium requires that the consumer be optimistic and so $\sigma_B = 1$. On equilibrium path, there is updating from observation of z , and threshold \bar{z} must exactly equal \hat{z} so that the designer of G -product is indifferent to the choice of κ and is willing to mix::

$$\mu(b, \hat{z}) = \frac{\mu}{\mu + (1 - \mu) \cdot \frac{1-\hat{z}}{\hat{z}} \cdot \frac{1}{\sigma_G + (1 - \sigma_G) \cdot \ell(\hat{z})^{-1}}} = \omega. \quad (49)$$

which in turn implies that

$$\sigma_G = 1 - \frac{1 - \frac{1-\hat{z}}{\hat{z}} \cdot \frac{1-\mu}{\mu} \cdot \frac{\omega}{1-\omega}}{1 - \frac{f(\hat{z}|\kappa)}{f(\hat{z}|\bar{\kappa})}}. \quad (50)$$

Since the posterior belief $\mu(b, \hat{z})$ is continuous and increasing in σ_G , this equilibrium exists if and only if:

$$\mu(b, \hat{z})|_{\sigma_G=0} < \omega < \mu(b, \hat{z})|_{\sigma_G=1}, \quad (51)$$

which is equivalent to:

$$\mu_3 = \frac{\omega \cdot \frac{1-\hat{z}}{\hat{z}}}{\omega \cdot \frac{1-\hat{z}}{\hat{z}} + 1 - \omega} < \mu < \frac{\omega \cdot \ell(\hat{z}) \cdot \frac{1-\hat{z}}{\hat{z}}}{\omega \cdot \ell(\hat{z}) \cdot \frac{1-\hat{z}}{\hat{z}} + 1 - \omega} = \mu_4. \quad (52)$$

Therefore, $\sigma_G \in (0, 1)$ and $\sigma_B = 1$ is an equilibrium if and only if $\mu \in (\mu_3, \mu_4)$.

We have thus characterized all the possible equilibrium $\{\sigma_y\}$, as a function of equilibrium belief μ :

1. If $\mu \leq \mu_1$, the equilibrium is $\sigma_B = \sigma_G = 0$.
2. If $\mu \in (\mu_1, \mu_2)$, then $\sigma_G = 0$ and $\sigma_B = \frac{\frac{1-\hat{z}}{\hat{z}} \cdot \frac{\mu}{\mu} \cdot \frac{1-\omega}{\omega} - 1}{\frac{f(\hat{z}|\bar{\kappa})}{f(\hat{z}|\kappa)} - 1}$.
3. If $\mu \in [\mu_2, \mu_4]$, then $\sigma_G = 0$ and $\sigma_B = 1$.
4. If $\mu \in (\mu_3, \mu_4)$, $\sigma_B = 1$ and $\sigma_G \in \left\{ 0, 1 - \frac{1 - \frac{1-\hat{z}}{\hat{z}} \cdot \frac{1-\mu}{\mu} \cdot \frac{\omega}{1-\omega}}{1 - \frac{f(\hat{z}|\bar{\kappa})}{f(\hat{z}|\kappa)}} , 1 \right\}$.
5. If $\mu \geq \mu_4$, then $\sigma_B = \sigma_G = 1$.

This establishes the stated result. ■

Proof of Lemma 2. The L -type's expected net payoff from choosing G -product is:

$$\gamma(\mu) = \max_{\kappa} \mathbb{P}_{\mu}(a = 1|G, \kappa) \cdot \underline{v} - \max_{\kappa} \mathbb{P}_{\mu}(a = 1|B, \kappa) \cdot \bar{v}. \quad (53)$$

Consider the correspondence $\Gamma(\mu)$ as defined in text. Let us look at $\mu \in (0, 1]$; recall from Proposition 3 that the H -type chooses G -product w.p.1 and thus $\mu \geq p > 0$.

The values $\max_{\kappa} \mathbb{P}_{\mu}(a = 1|G, \kappa)$ and $\max_{\kappa} \mathbb{P}_{\mu}(a = 1|B, \kappa)$ depend on the corresponding equilibrium $\{\sigma_y\}$, given in Proposition 4. Note that $\Gamma(\mu)$ is a singleton for $\mu \notin (\mu_3, \mu_4)$, since equilibrium $\{\sigma_y\}$ corresponding to such μ are unique. On the other hand, $\Gamma(\mu)$ consists of three elements when $\mu \in (\mu_3, \mu_4)$, since then the equilibrium can feature either ($\sigma_G = 0$ and $\sigma_B = 1$), ($\sigma_G \in (0, 1)$ and $\sigma_B = 1$), or ($\sigma_G = 1$ and $\sigma_B = 1$).

In this proof, we will heavily rely on the results in Proposition 4 and the thresholds $\mu_1 - \mu_4$ defined in its proof, without referencing them. To follow this proof, please read the proposition and its proof in advance.

Case $\mu \leq \mu_1$. In this region, the equilibrium has $\sigma_G = \sigma_B = 0$, and the consumer is pessimistic, since $\mu_1 < \tilde{\mu}$. Furthermore, $\Gamma(\mu)$ is single-valued and given by:

$$\Gamma(\mu) = \underline{v} \cdot \int_0^{\bar{z}(\mu)} (1-z) f(z|\underline{\kappa}) dz - \bar{v} \cdot \int_0^{\bar{z}(\mu)} z f(z|\underline{\kappa}) dz. \quad (54)$$

Therefore:

$$\Gamma'(\mu) = [\underline{v} - (\underline{v} + \bar{v}) \cdot \bar{z}(\mu)] \cdot f(z|\underline{\kappa}) \cdot \frac{d\bar{z}}{d\mu}. \quad (55)$$

where $\bar{z}(\mu)$ is given by (34). It is easy to check that $\frac{d\bar{z}}{d\mu} > 0$, $\bar{z}(0) = 0$, and $\bar{z}(\mu_1) = \hat{z}$. As a result, for μ sufficiently small, $\Gamma'(\mu) > 0$ and thus $\Gamma(\mu) > 0$. Next, consider μ_v such that:

$$\bar{z}(\mu_v) = \frac{\underline{v}}{\underline{v} + \bar{v}} \implies \mu_v \equiv \frac{\underline{v}\omega}{\underline{v}\omega + \bar{v}(1-\omega)}. \quad (56)$$

Observe that, if $\mu_v > \mu_1$, then $\Gamma'(\mu) > 0 \forall \mu \in (0, \mu_1)$. Otherwise, $\Gamma'(\mu) > 0$ for $\mu \in (0, \mu_v)$ and $\Gamma'(\mu) < 0$ for $\mu \in (\mu_v, \mu_1)$.

Case $\mu \in (\mu_1, \mu_2]$. In this region, the equilibrium has $\sigma_G = 0$ and $\sigma_B \in (0, 1)$, and the consumer is pessimistic, i.e. $\mu_2 < \tilde{\mu}$. Most importantly, in this case $\bar{z} = \hat{z}$ and, thus, $\Gamma(\mu)$ is single-valued and given by:

$$\Gamma(\mu) = \underline{v} \cdot \int_0^{\hat{z}} (1-z) f(z|\underline{\kappa}) dz - \bar{v} \cdot \int_0^{\hat{z}} z f(z|\underline{\kappa}) dz \quad (57)$$

Since $\bar{z}(\mu_1) = \hat{z}$, it is immediate that Γ is continuous at μ_1 and constant on interval (μ_1, μ_2) .

Case $\mu \in (\mu_2, \mu_3]$. In this region, the equilibrium has $\sigma_G = 0$ and $\sigma_B = 1$. The consumer is pessimistic if $\mu < \tilde{\mu}$, and she is optimistic otherwise.

Suppose that $\mu < \tilde{\mu}$. Then, $\Gamma(\mu)$ is single-valued and given by:

$$\Gamma(\mu) = \underline{v} \cdot \int_0^{\bar{z}(\mu)} (1-z) f(z|\underline{\kappa}) dz - \bar{v} \cdot \int_0^{\bar{z}(\mu)} z f(z|\bar{\kappa}) dz. \quad (58)$$

Therefore:

$$\Gamma'(\mu) = [\underline{v} \cdot (1 - \bar{z}(\mu)) \cdot f(\bar{z}(\mu)|\underline{\kappa}) - \bar{v} \cdot \bar{z}(\mu) \cdot f(\bar{z}(\mu)|\bar{\kappa})] \frac{d\bar{z}}{d\mu}. \quad (59)$$

where $\bar{z}(\mu)$ is given by (40). Since $\bar{z}(\mu_2) = \hat{z}$, Γ is continuous at μ_2 . Furthermore, $\Gamma'(\mu) \geq 0$ iff

$$\bar{z}(\mu) \leq \frac{1}{1 + \frac{\bar{v}}{v} \cdot \ell(\bar{z}(\mu))} \iff \frac{(1-\mu) \cdot \omega}{\mu \cdot (1-\omega)} \geq \frac{\bar{v}}{v} \iff \mu \leq \mu_v, \quad (60)$$

and $\Gamma'(\mu) > 0$ if $\mu < \mu_v$. Furthermore, since $\frac{(1-\mu) \cdot \omega}{\mu \cdot (1-\omega)} \geq \frac{\bar{v}}{v}$ is decreasing in μ and equal to $\ell(\frac{1}{2}) < 1$ when $\mu = \tilde{\mu}$, it follows that $\mu_v < \tilde{\mu}$ and thus $\Gamma'(\tilde{\mu}) < 0$.

Next, suppose that $\mu > \tilde{\mu}$. Now, $\bar{z}(\mu) \geq \hat{z}$ and is given by (43), and note that $\frac{d\bar{z}}{d\mu} < 0$. Therefore:

$$\begin{aligned} \Gamma(\mu) &= \underline{v} \cdot \left[\int_0^{\bar{z}(\mu)} (1-z) f(z|\underline{\kappa}) dz + (1 - F(\bar{z}|\underline{\kappa})) \right] - \bar{v} \cdot \left[\int_0^{\bar{z}(\mu)} z f(z|\bar{\kappa}) dz + (1 - F(\bar{z}|\bar{\kappa})) \right] \\ &= \underline{v} - \underline{v} \cdot \int_0^{\bar{z}(\mu)} z f(z|\underline{\kappa}) dz - \bar{v} \cdot \left[\int_0^{\bar{z}(\mu)} z f(z|\bar{\kappa}) dz + (1 - F(\bar{z}|\bar{\kappa})) \right], \end{aligned} \quad (61)$$

and thus:

$$\Gamma'(\mu) = [\bar{v} \cdot (1 - \bar{z}(\mu)) \cdot f(\bar{z}(\mu)|\bar{\kappa}) - \underline{v} \cdot \bar{z}(\mu) \cdot f(\bar{z}(\mu)|\underline{\kappa})] \frac{d\bar{z}}{d\mu} < 0, \quad (62)$$

where we use the fact that $\ell(\bar{z}(\mu)) \geq \ell(\hat{z}) > 1$. Recall that $\tilde{\mu}$ is the threshold between the region where the consumer is pessimistic and the region where she is optimistic. Since $\bar{z}(\tilde{\mu}) = \frac{1}{2}$, it is easy to check that Γ is continuous at $\tilde{\mu}$.

To summarize, we have shown that $\Gamma(\mu)$ is positive and (weakly) increasing for $\mu \in (0, \mu_v]$, and (weakly) decreasing for $\mu \in (\mu_v, \mu_3]$, with $\mu_v \in (0, \tilde{\mu})$. We are left to consider regions $\mu \in (\mu_3, \mu_4)$ and $\mu \in [\mu_4, 1]$

Case $\mu \geq \mu_4$. In this region, the equilibrium has $\sigma_G = \sigma_B = 1$, and the consumer is optimistic. Moreover, $\bar{z}(\mu)$ given by (37). We have that Γ is single-valued and given by

$$\Gamma(\mu) = \underline{v} \cdot \left[\int_0^{\bar{z}(\mu)} (1-z) f(z|\bar{\kappa}) dz + (1 - F(\bar{z}(\mu)|\bar{\kappa})) \right] - \bar{v} \cdot \left[\int_0^{\bar{z}(\mu)} z f(z|\bar{\kappa}) dz + (1 - F(\bar{z}(\mu)|\bar{\kappa})) \right].$$

Then, Γ decrease in μ for $\mu > \mu_4$.

$$\Gamma'(\mu) = [\bar{v} \cdot (1 - \bar{z}) - \underline{v} \cdot \bar{z}] \cdot f(z|\bar{\kappa}) \cdot \frac{d\bar{z}}{d\mu} < 0.$$

Finally, note that $\Gamma(1) = \underline{v} - \bar{v} < 0$.

Case $\mu \in (\mu_3, \mu_4)$. In this region, $\Gamma(\mu)$ is not single-valued. For $j \in \{S, M, P\}$, let $\gamma^j(\mu) \in \Gamma(\mu)$ denotes the net expected payoff to the L -type from choosing G -product, when the equilibrium

complexity is given respectively by (i) Separating on complexity $\sigma_G = 0, \sigma_B = 1$, (ii) Mixing of the H -type designer with $\sigma_G \in (0, 1), \sigma_B = 1$, and (iii) Pooling on complexity $\sigma_G = \sigma_B = 1$.

We have already shown that the functions $\gamma^S(\mu)$ (i.e., given by $\Gamma(\mu)$ analyzed for $\mu \in (\tilde{\mu}, \mu_3)$ above) and $\gamma^P(\mu)$ (i.e., given by $\Gamma(\mu)$ analyzed for $\mu > \mu_4$ above) are decreasing in μ . We are therefore left to consider $\gamma^M(\mu)$. For the H -type to mix, it must be that $\bar{z}(\mu) = \hat{z}$ for all $\mu \in (\mu_3, \mu_4)$. Since the consumer is optimistic in this region, we have that:

$$\gamma^M(\mu) = \underline{v} - \underline{v} \cdot \int_0^{\hat{z}} z f(z|\bar{\kappa}) dz - \bar{v} \cdot \int_0^{\hat{z}} z f(z|\bar{\kappa}) dz - \bar{v} \cdot (1 - F(\hat{z}|\bar{\kappa})), \quad (63)$$

which is independent of μ . Furthermore, since $\bar{z}(\mu_3) = \hat{z}$ when pooling at complexity and $\bar{z}(\mu_4) = \hat{z}$ when separating at complexity, it follows that $\gamma^M(\mu) = \gamma^P(\mu_3) = \gamma^S(\mu_4)$. Furthermore, it is straightforward that $\lim_{\mu \uparrow \mu_3} \Gamma(\mu) = \gamma^S(\mu_3)$ and $\lim_{\mu \downarrow \mu_4} \Gamma(\mu) = \gamma^P(\mu_4)$.

To summarize, we have that $\Gamma(\mu)$ is single-valued for $\mu < \mu_3$, with $\lim_{\mu \rightarrow 0} \Gamma(\mu) > 0$ and $\Gamma(\mu)$ increasing for $\mu \leq \mu_v$ and decreasing for $\mu \in (\mu_v, \mu_3)$. We have also shown that (i) $\lim_{\mu \uparrow \mu_3} \Gamma(\mu) = \gamma^S(\mu_3)$, with $\gamma^S(\mu)$ decreasing in μ , (ii) $\gamma^S(\mu_4) = \gamma^M(\mu) = \gamma^P(\mu_3)$, and (iii) $\gamma^P(\mu)$ decreasing for $\mu > \mu_3$, and (iv) $\Gamma(\mu) = \gamma^P(\mu)$ for $\mu > \mu_4$, and thus single-valued and decreasing until $\Gamma(1) < 0$. These results are depicted in Figure 3.

Since Γ is single-valued and positive for μ small, then increases until it reaches a maximum, and decreases (i.e. does not jump up in the region where it is not single-valued) and becomes single-valued and negative for μ relatively large, it follows that Γ intersects 0 generically once. ■

Proof of Proposition 5. As shown in Proposition 3, the H -type designer always chooses $y = G$. Thus, $\mu \geq p$.

If $p > \psi$, from Lemma 2 we have that there exists a $\gamma(p) \in \Gamma(p) < 0$. Thus, the L -type designer produces a B -product with probability $m_L = 1$, consistent with an equilibrium belief of $\mu = p$.

If $p < \mu_3$, then $\gamma(p) \in \Gamma(p) > 0$. Thus, if $\mu = p$, the L -type produces a G -product with probability one, i.e., $m_L = 0$, which is consistent with a belief of $\mu = 1$, reaching a contradiction. Thus, there cannot be an equilibrium with $\mu < \mu_3$. Thus, for $p < \mu_3$ the equilibrium requires the L -type designer to follow a mixed strategy $m_L \in (0, 1)$ with $\mu = p + (1 - p)(1 - m_L)$, where m_L is such that the L -type is indifferent between producing a B - or a G -product: $\mu = \psi$, which indicates there exists a $\gamma(\psi) \in \Gamma(\psi) = 0$.

If $p \in [\mu_3, \psi]$ and $\min\{\Gamma(\mu_3)\} > 0$, then the equilibrium is as the one described above for $p < \mu_3$. Otherwise, if $\min\{\Gamma(\mu_3)\} < 0$, we have that (i) $\mu = \psi$ is consistent with an

equilibrium in which the L -type follows a mixed strategy, $m_L \in (0, 1)$ since there exists a j such that $\gamma^j(\psi) \in \Gamma(\psi) = 0$, (ii) $\mu = p$ is consistent with an equilibrium since there exists $\gamma^j(p) \in \Gamma(p) < 0$.

Furthermore, if $\mu \in (\mu_3, \mu_4)$, there could be multiple complexity levels consistent with an equilibrium belief of μ if $\gamma^j(\mu) \in \Gamma(\mu) \leq 0$ for more than one $j \in \{S, M, P\}$. ■

A.2 Proofs for Section 4

Proof of Proposition 6. For this comparative static we focus on the simplest equilibrium. It is easy to check that an increase in ω affects thresholds $\mu_1 - \mu_4$ as follows

$$\frac{d\mu_1}{d\omega} > 0; \frac{d\mu_2}{d\omega} > 0; \frac{d\tilde{\mu}}{d\omega} > 0; \frac{d\mu_3}{d\omega} > 0; \frac{d\mu_4}{d\omega} > 0.$$

but has no effect effect on Γ otherwise; that is, $\Gamma(\mu)$ is unchanged if we condition on the complexity choices. Thus, it follows that ψ also weakly increase in ω , but note that it will always stay in the same region; that is, at $\mu = \psi$ the complexity choices of the designers remain unchanged (even as ψ increases with ω).

If $p > \psi$, an increase in ω does not affect average quality which stays constant at $\mu = p$. It may, however, result in a decrease in average complexity, as μ stays constant but thresholds μ_1, μ_4 , and thus $\sigma_y(\mu)$, increase.

As ψ increases, eventually $p \leq \psi$. In this case, an increase in ω generates an increase in average quality as $\mu = \psi$ increases. In this scenario, however, σ_y do not change since ψ has not changed complexity regions (e.g. if it is between (μ_2, μ_3) it continues to be in this region after the change in ω , even though thresholds have changed). Thus, average complexity increases as μ increases only when $\sigma_G < \sigma_B$; that is, for $\mu = \psi \in (\mu_1, \mu_4)$. ■

Proof of Proposition 7. For this comparative statics we focus on the simplest equilibrium. We consider an increase from p to p' .

If $p' \leq \psi$, equilibrium average quality stays constant at $\mu = \psi$, with complexity given by Proposition 4. As the fraction of H -types increases, so does the fraction of L -types producing B -products, m_L , and thus $\mu = \psi$, and complexity, are constant.

If $p' > \psi$, then the equilibrium features $m_L = 1$, and average quality $\mu = p' > \max\{p, \psi\}$. Thus, average quality has increased. As for the effect on average complexity,

$$E[\sigma] = \mu\sigma_G(\mu) + (1 - \mu)\sigma_B(\mu),$$

note that σ_y (weakly) increases in μ . Thus, the increase in μ has no effect on complexity if $\mu < \mu_1$, as both designers simplify in this region, average complexity decreases if $\sigma_G(\mu) < \sigma_B(\mu)$, i.e., for $\mu \in (\mu_2, \mu_4)$, and eventually increases as all designers produce complex products when μ is sufficiently high, i.e., $\mu > \mu_4$. ■

Proof of Proposition 8. For this comparative static we focus on the simplest equilibrium. Note that an increase in \underline{v} does not affect the choices of complexity for a given μ . That is, thresholds $\mu_1 - \mu_4$ and $\sigma_y(\mu)$ remain unchanged (where $\sigma_y(\mu)$ denotes the mapping from beliefs, μ , to complexity choices given in Proposition 4, where simplicity is the chosen equilibrium for $\mu \in (\mu_3, \mu_4)$). We can re-state Γ as follows

$$\Gamma(\mu) = P(a = 1 | \sigma_G(\mu))\underline{v} - P(a = 1 | \sigma_B(\mu))\bar{v} \quad (64)$$

It is immediate that since σ_y is independent of \underline{v} , $\Gamma(\mu)$ increases in \underline{v} for all $\mu \in (0, 1)$. Thus, ψ given by $\Gamma(\psi) = 0$ must increase as well*.

Thus, if $p > \psi$, an increase in \underline{v} does not affect average quality which stays constant at $\mu = p$. Once ψ is sufficiently high as to have $p \leq \psi$, however, an increase in \underline{v} generates an increase in average quality, since $\mu = \psi$ has increased.

Since the thresholds $\mu_1 - \mu_4$ remain unchanged, when average quality increases, σ_y (weakly) increase as a result (otherwise complexity remains unchanged). As in the comparative static with respect to p , the effect on average complexity may be non-monotonic. In particular, the increase in μ has no effect on complexity if $\mu < \mu_1$, as both designers simplify in this region, average complexity decreases if $\sigma_G(\mu) < \sigma_B(\mu)$, which occurs for $\mu \in (\mu_2, \mu_4)$, but eventually increases as all designers produce complex products when μ is sufficiently high, i.e., $\mu > \mu_4$.

*Caveat: when $p < \psi$ and the equilibrium lies in the region of multiple equilibria; that is, $\psi \in (\mu_3, \mu_4)$ with $\min\{\Gamma(\mu_3)\} < 0$, an increase in \underline{v} can generate a jump from the separating equilibrium to the complex one when the former ceases to exist (that is, we are out of the multiplicity region). In this case, ψ jumps down, and so does average quality. Since average quality is increasing in \underline{v} before and after the jump, we state our results by saying that average quality generically increases in ψ . ■

B Proofs of Section 5

Proof of Proposition 9. For the existence result, it is without loss of generality to focus on the most simple equilibrium (proof analogous for the most complex). For each

$U \in [w(B), w(G)]$, consider the map $T_\beta : U \mapsto \mathbb{R}$ defined by:

$$T_\beta(U) = \mathbb{E} \left\{ \max_{a \in \{0,1\}} \{a \cdot (\mu(s, z) \cdot w(G) + (1 - \mu(s, z)) \cdot w(B)) + (1 - a) \cdot \beta U\} \right\}, \quad (65)$$

where $\mu(s, z)$ is the consumer's equilibrium belief that the proposed product has output G , given signal s with noise z . For an exogenously given consumer value U , which pins down the consumer's outside option $w_0 = \beta U$, this map gives us a new ex-ante welfare $T_\beta(U)$. An equilibrium is a fixed point of this map, and we denote it by U^* . Since we have selected the simplest equilibrium, $T_\beta(\cdot)$ is single-valued. Since we have selected the simplest equilibrium, $T_\beta(\cdot)$ is single-valued.

Since, in equilibrium, the consumer has some information, it must be that $T_\beta(w(B)) > pw(G) + (1 - p)w(B)$. Also, in equilibrium, B -products are produced with positive probability and the consumer's information is imperfect, it must be that $T_\beta(w(G)) < w(G)$. Thus, if $\beta(pw(G) + (1 - p)w(B)) > w(B)$, then the outside option βU^* will satisfy Assumption 1, i.e. $\beta U^* \in (w(B), w(G))$. We assume that β is not too low, so that this condition holds.²⁶

Therefore, to show that an equilibrium exists, it suffices to show that $T_\beta(\cdot)$ is increasing. But, the consumer's ex-ante welfare increases in the outside option βU , and for three reasons. First, there is the direct effect of the outside option. Second, in equilibrium, σ_B and σ_G are decreasing in the outside option. To see this, recall that the thresholds $\mu_1 - \mu_4$ are increasing in the outside option (see proof of Proposition 6) and, when $\mu \in (\mu_1, \mu_2]$, which is the only case where complexity is interior in the most simple equilibrium, σ_B is decreasing in the outside option and $\sigma_G = 0$. Finally, also by Proposition 6, the probability that the designers propose G -product, μ , is increasing in the outside option.

For comparative statics, note that, for a given U , an increase from β to some β' is equivalent to an increase in the consumer's outside option. But then again, it must be that $T_\beta(U) < T_{\beta'}(U)$. If there is a unique solution to $T_{\beta'}(U) = U$, then the solution must be higher at β' than any solution at β , since $T_\beta(\cdot)$ is increasing. If there are multiple solutions to $T_{\beta'}(U) = U$, then the statement holds for the maximal solution. ■

C Proofs of Section 6

Proof of Lemma 3. First, note that $\mu(\kappa) = 1$ is not consistent with an equilibrium for any $\kappa \in \{\underline{\kappa}, \bar{\kappa}\}$. If that was the case, the L -type designer would produce a B -product with

²⁶Otherwise, when β is small enough, then the consumer will accept all products and the equilibrium strategies and payoffs will be independent of β in that region.

complexity κ , since it would be accepted with probability one, which implies a contradiction. Second, note that $\mu(\kappa) = 0$ is not consistent for any equilibrium complexity level $\kappa \in \{\underline{\kappa}, \bar{\kappa}\}$. Suppose WLOG that $\mu(\underline{\kappa}) = 0$, that is, only the B -type designer chooses $\underline{\kappa}$ in equilibrium. If this was the case, the B -product designer would be better off by choosing complexity $\bar{\kappa}$, since it would give him a positive expected payoff (note that $\mu(\bar{\kappa}) > 0$), which is higher than the payoff of zero associated with the choice of $\underline{\kappa}$, which reaches a contradiction. Thus, it must be that in any equilibrium, both types choose the equilibrium complexity levels with positive probability: that is, either (1) $\sigma_G \in (0, 1)$ and $\sigma_B \in (0, 1)$, or (2) $\sigma_G = \sigma_B \in \{0, 1\}$. Suppose (1) holds. Since both types are mixing, it must be that they are indifferent between choosing a complex or a simple product. However, it is easy to check that for any interim belief function, if the B -producer is indifferent, then the G -product designer strictly prefers simple products; while if the G -product designer is indifferent, the B -product designer strictly prefers complex products, which implies a contradiction. Thus, we are left with option (1). ■

Proof of Proposition 10. Lemma 3 has established that there are only two candidates for complexity levels in equilibrium. Suppose off-equilibrium beliefs are such that they assign a deviation to off-equilibrium complexity levels to the B -product designer. Then, any deviations from equilibrium complexity have a zero payoff, which deters deviations. Finally, it remains to show that with such beliefs, both equilibria exist. The proof is analogous to the proof of Proposition 5. Consider the equilibrium with pooling at $\bar{\kappa}$. Following the same steps as in the proof to this proposition, we have that there exists a unique $\psi^C > 0$ such that

$$\gamma(\psi^C) = \mathbb{P}_{\psi^C}(a = 1|G, \bar{\kappa})\underline{v} - \mathbb{P}_{\psi^C}(a = 1|B, \bar{\kappa})\bar{v} = 0 \quad (66)$$

with $\gamma(\mu) > 0$ for $\mu < \psi^C$ and $\gamma(\mu) < 0$ for $\mu > \psi^S$. Similarly, in the equilibrium with pooling at simplicity, $\underline{\kappa}$, there exists a unique $\psi_S = 0$ such that

$$\gamma(\psi^S) = \mathbb{P}_{\psi^S}(a = 1|G, \underline{\kappa})\underline{v} - \mathbb{P}_{\psi^S}(a = 1|B, \underline{\kappa})\bar{v} = 0 \quad (67)$$

The construction of the equilibrium is as the one in Proposition 5 for the respective ψ threshold.

Finally, it remains to show that $\psi^C < \psi^S$. This follows given the MLRP property of the function $f(z|\kappa)$. Specifically, there are two cases to consider.

Case 1. If $\mu > \omega$, the consumer is optimistic (notice this is true regardless of the equilibrium

complexity choice). In the equilibrium with least complexity, at ψ^S , we have

$$\gamma(\psi^S) = \underline{v} - \bar{v} - \int_0^{\bar{z}(\psi^S)} (\underline{v} \cdot z - \bar{v} \cdot (1-z)) \cdot f(z|\underline{\kappa}) dz = 0. \quad (68)$$

Similarly, in the equilibrium with most complexity, at ψ^C , we have

$$\gamma(\psi^C) = \underline{v} - \bar{v} - \int_0^{\bar{z}(\psi^C)} (\underline{v} \cdot z - \bar{v} \cdot (1-z)) \cdot f(z|\bar{\kappa}) dz = 0 \quad (69)$$

Then, consider the case when $\bar{z}(\psi^S) = \bar{z}(\psi^C)$. By the MLRP property, the left-hand side of (69) would then be higher than the left-hand side of (68), unless $\bar{z}(\psi^C) = \hat{z}$. Thus, generically, given $(\underline{v} \cdot z - \bar{v} \cdot (1-z)) < 0$, it must be the case that $\bar{z}(\psi^C) > \bar{z}(\psi^S)$. In the optimistic region, $\frac{d\bar{z}}{d\mu} < 0$, hence $\psi^C < \psi^S$.

Case 2. If $\mu < \omega$, the consumer pessimistic, and so at $\psi \in \{\psi^S, \psi^C\}$,

$$\gamma(\psi) = \int_0^{\bar{z}} (\underline{v} \cdot (1-z) - \bar{v} \cdot z) \cdot f(z|\kappa) dz = 0. \quad (70)$$

Thus, if $\int_0^{\bar{z}(\psi^C)} (\underline{v} \cdot (1-z) - \bar{v} \cdot z) \cdot f(z|\bar{\kappa}) dz = 0$, and $\bar{z}(\psi^C) = \bar{z}(\psi^S)$, then by the MLRP property, it must be that $\int_0^{\bar{z}(\psi^S)} (\underline{v} \cdot (1-z) - \bar{v} \cdot z) \cdot f(z|\underline{\kappa}) dz > 0$ (unless $\bar{z}(\psi^C) = \hat{z}$). Thus, for (70) to hold, we must have $\bar{z}(\psi^C) < \bar{z}(\psi^S)$. In the pessimistic region, $\frac{d\bar{z}}{d\mu} > 0$, hence $\psi^C < \psi^S$. If $\bar{z}(\psi^C) = \hat{z}$, then $\psi^C = \psi^S$. ■

Proof of Proposition 11. Consider the expected payoff of the H -type designer in the equilibrium with pooling on $\bar{\kappa}$ ($\sigma_G = \sigma_B = 1$) relative to the equilibrium with pooling on $\underline{\kappa}$ ($\sigma_G = \sigma_B = 0$).

If $\mu \geq \omega$, the consumer is optimistic and therefore

$$\frac{\gamma^H(\mu)}{\bar{v}} = \int_0^{\bar{z}(\mu)} z \cdot [f(z|\underline{\kappa}) - f(z|\bar{\kappa})] dz. \quad (71)$$

Then, $\frac{\gamma^H(\mu)}{\bar{v}} > 0$ if and only if $\bar{z}(\mu) < \hat{z}$. This condition reduces to

$$\mu > \frac{\omega(1-\hat{z})}{\omega(1-\hat{z}) + \hat{z}(1-\omega)} \equiv \bar{\mu}. \quad (72)$$

If $\mu < \omega$,

$$\frac{\gamma^H(\mu)}{\bar{v}} = \int_0^{\bar{z}(\mu)} (1-z) \cdot [f(z|\bar{\kappa}) - f(z|\underline{\kappa})] dz < 0. \quad (73)$$

Summing up, the H -type designer derives the highest payoff in the equilibrium with $\sigma_G = \sigma_B = 0$ if $\mu < \bar{\mu}$, and he obtains the highest payoff in the equilibrium with $\sigma_G = \sigma_B = 1$ if $\mu \geq \bar{\mu}$.

If $p > \bar{\mu}$, then the best equilibrium for the H -type is $\sigma_G = \sigma_B = 1$. Otherwise, if $p < \bar{\mu}$, there are three possible cases. First, if $\psi^S < \bar{\mu}$, then $\mu \leq \bar{\mu}$. Thus, the best equilibrium for the H -type is $\sigma_G = \sigma_B = 0$. Second, if $\psi^C > \bar{\mu}$, given $\psi^S > \psi^C$, it must be that $\mu \geq \bar{\mu}$. Thus, the best equilibrium for the H -type is $\sigma_G = \sigma_B = 1$. Third, if $\psi^S > \bar{\mu} > \psi^C$, we have

$$\frac{\gamma^H(\mu)}{\bar{v}} = \int_0^{\bar{z}(\psi^C)} (-z) \cdot [f(z|\bar{\kappa}) - f(z|\underline{\kappa})] dz + \int_{\bar{z}(\psi^C)}^{\bar{z}(\psi^S)} z \cdot f(z|\underline{\kappa}) dz.$$

Since $\bar{z}(\psi^C) < \hat{z}$, it follows that $\frac{\gamma^H(\mu)}{\bar{v}} > 0$. Hence, the best equilibrium for the H -type in this case is $\sigma_G = \sigma_B = 1$. ■

D Costly Complexity

In this extension, we incorporate a direct cost for the consumer if the product deviates from a ‘natural level of complexity.’ We suppose that the consumer pays a cost $c(\kappa)$ when a product with complexity κ is approved. The cost is $c(\kappa) = 0$ if $\kappa = \kappa_n$ and $c(\kappa) = \bar{c} > 0$ otherwise. Thus, the consumer’s utility becomes:

$$W(a|s, z) \equiv a \cdot E[w(y) - c(\kappa)|s, z] + (1 - a) \cdot w_0 \quad (74)$$

We continue to study the case of binary complexity, and we make the following assumptions for payoffs.

Assumption 2 *The payoffs satisfy the following property: $w(G) - \bar{c} > w_0$.*

The assumption states that the cost of complexity is not too large, so that a product is approved if the consumer is sufficiently confident that it is a G -product. We begin by considering the case in which $\kappa_n = \underline{\kappa}$. Later, we discuss the converse case when $\kappa_n = \bar{\kappa}$. The only difference with our model described in Section 3 is that the consumer’s acceptance strategy is now modified to incorporate this direct cost of complexity. In particular, the consumer’s expected payoff can be re-written as follows,

$$a \cdot \{\mu(s, z) \cdot [w(G) - \sigma_G(z) \cdot \bar{c}] + (1 - \mu(s, z)) \cdot [w(B) - \sigma_B(z) \cdot \bar{c}]\} + (1 - a) \cdot w_0, \quad (75)$$

where

$$\sigma_y(z) \equiv \mathbb{P}(\kappa = \bar{\kappa}|y, z) = \frac{\sigma_y f(z|\bar{\kappa})}{\sigma_y f(z|\bar{\kappa}) + (1 - \sigma_y) f(z|\underline{\kappa})}. \quad (76)$$

In contrast to our baseline model, the consumer now also updates her beliefs about the complexity of the product after observing noise z , since $z \sim f(z|\kappa)$. Note that $\sigma_y(z)$ weakly increases in z for all y .

By inspection of (75), we see that the consumer approves the product if:

$$\mu(s, z) \geq \frac{w_0 - w(B) + \sigma_B(z) \cdot \bar{c}}{w(G) - w(B) + (\sigma_B(z) - \sigma_G(z)) \cdot \bar{c}} \equiv \omega(z). \quad (77)$$

The left-hand side of condition (77) is the consumer's posterior belief about being offered a G -product, and it is the same as in (4). The right-hand side incorporates the fact that complexity is costly to the consumer. The new relative outside option, $\omega(z)$, differs from ω in Section 3 for two reasons. First, it is higher for all z , to reflect the fact that complex products are more costly to the consumer. Second, its value weakly increases in the noise of the signal.

As in Section 3, we let \bar{z} be defined as the solution to (77) with equality for some s . Then, the solution to $\omega(\bar{z}) = \mu(b, \bar{z})$ gives the threshold \bar{z} when the consumer is optimistic, and the solution to $\omega(\bar{z}) = \mu(g, \bar{z})$ gives the threshold \bar{z} when the consumer is pessimistic.

The following Lemma then characterizes the consumer's acceptance strategy.

Lemma 4 *The threshold \bar{z} is unique provided that \bar{c} is not too large, and the consumer's acceptance strategy is as in Lemma 1. Furthermore, threshold \bar{z} is increasing in the cost of complexity, \bar{c} , when the consumer is optimistic, and decreasing when the consumer is pessimistic.*

The acceptance strategy depends, through beliefs $\mu(z)$ and $\sigma_y(z)$, on the t -designer's equilibrium strategies $\{m_t, \sigma_t\}$, which the consumer takes as given. As in the no cost model, the consumer's acceptance strategy rule is contingent on the signal only when the signal is sufficiently informative, i.e. $z < \bar{z}$. However, when complexity is costly, the consumer's decision rule becomes more tight as she tends to reject products more often. The optimistic consumer relies more on the signal than in the no cost case making acceptance less likely to occur, while the pessimistic consumer relies less on the signal making rejection more likely to occur.

The equilibrium characterization follows analogously to the no cost case, and it is summarized in the following proposition.

Proposition 12 *The equilibrium characterization is as in Propositions 4 and 5, but with modified thresholds $\mu_1-\mu_4$ and ψ are given in (81) – (83). In particular, the equilibrium level of product complexity is decreasing in the cost \bar{c} .*

Thus, the main qualitative results of our model carry over to the case with a direct cost to complexity for the consumer, with the not very surprising prediction that the equilibrium level of complexity will be lower if it is costlier.

To complete the analysis, we now turn to the case when $\kappa_n = \bar{\kappa}$. The cost of complexity then becomes $c(\underline{\kappa}) = \bar{c}$ and $c(\bar{\kappa}) = 0$. The analysis is analogous to the above. Lemma 4 holds as before, and, perhaps surprisingly, the qualitative result of Proposition 12 continues to hold. An increase in \bar{c} still results in lower equilibrium complexity. A higher cost of owning low complexity products increases the consumer's relative outside option, since buying the product yields a lower expected payoff. This in turn raises the threshold on the informativeness of the product that is needed for acceptance. Hence, in equilibrium, designers who want their products accepted simplify them in order make them more informative for the consumer, even if doing so is costlier for the consumer. We conclude, therefore, that having a cost to complexity implies a lower equilibrium level of complexity, regardless of the value of κ_n .

Proofs of Lemmas and Propositions in this Appendix.

Proof of Lemma 4. To drive the value of \bar{z} , we must consider the possible equilibria in each region. Consider first the pessimistic consumer.

Case 1: Equilibrium with $\sigma_G = \sigma_B = 0$. In this case, \bar{z} is given by (34).

Case 2: Equilibrium with $\sigma_G = 0$ and $\sigma_B \in (0, 1)$. In this case, for the designer of the B -product to mix, it must be that $\bar{z} = \hat{z}$.

Case 3: Equilibrium with $\sigma_G = 0$ and $\sigma_B = 1$. For this to be an equilibrium, it requires that

$$\frac{f\left(\frac{1}{2}|\underline{\kappa}\right) \cdot \mu}{f\left(\frac{1}{2}|\underline{\kappa}\right) \cdot \mu + f\left(\frac{1}{2}|\bar{\kappa}\right) \cdot (1 - \mu)} < \frac{w_0 - w(B) + \bar{c}}{w(G) - w(B) + \bar{c}}.$$

Then, \bar{z} is given by

$$\frac{(1 - \bar{z}) \cdot \mu \cdot \ell(\bar{z})}{(1 - \bar{z}) \cdot \mu \cdot \ell(\bar{z}) + \bar{z} \cdot (1 - \mu)} = \frac{w_0 - w(B) + \bar{c}}{w(G) - w(B) + \bar{c}}, \quad (78)$$

where $\ell(\bar{z})$ is defined in (15). In (78), the solution \bar{z} is unique under Assumption 2 about the magnitude of \bar{c} . Notice that the threshold $\omega(z)$ is increasing in \bar{c} . The left-hand side of equation (78) is decreasing in \bar{z} . Thus, as \bar{c} increases, \bar{z} decreases.

Consider next the optimistic consumer.

Case 1: Equilibrium with $\sigma_G = \sigma_B = 1$. In this case, \bar{z} is given by

$$\frac{\bar{z} \cdot \mu}{\bar{z} \cdot \mu + (1 - \bar{z}) \cdot (1 - \mu)} = \frac{w_0 - w(B) + \bar{c}}{w(G) - w(B)}. \quad (79)$$

Case 2: Equilibrium with $\sigma_B = 1$ and $\sigma_G \in (0, 1)$. In this case, for the holder of the G -product to mix, it must be that $\bar{z} = \hat{z}$.

Case 3: Equilibrium with $\sigma_G = 0$ and $\sigma_B = 1$. For this to be an equilibrium, it requires that

$$\frac{f(\frac{1}{2}|\kappa) \cdot \mu}{f(\frac{1}{2}|\kappa) \cdot \mu + f(\frac{1}{2}|\bar{\kappa}) \cdot (1 - \mu)} \geq \frac{w_0 - w(B) + \bar{c}}{w(G) - w(B) + \bar{c}}.$$

Then, \bar{z} is given by

$$\frac{\bar{z} \cdot \mu \cdot \ell(\bar{z})}{\bar{z} \cdot \mu \cdot \ell(\bar{z}) + (1 - \bar{z}) \cdot (1 - \mu)} = \frac{w_0 - w(B) + \bar{c}}{w(G) - w(B) + \bar{c}}. \quad (80)$$

In both (79) and (80), the solution \bar{z} is unique under Assumption 2. The left-hand side of equations (79) and (80) is increasing in \bar{z} . Thus, as \bar{c} increases, $\omega(z)$ increases, and thus \bar{z} increases. ■

Proof of Proposition 12. The proof is analogous to the proof from the case with no cost. We highlight here the differences in the values of the thresholds $\mu_1 - \mu_4$ given the cost parameter. For the equilibrium with $\sigma_G = \sigma_B = 0$, the threshold for acceptance is the same as before, so μ_1 is given by (35). For the equilibrium with $\sigma_G = 0$ and $\sigma_B \in (0, 1)$, the threshold for acceptance is given by:

$$\mu(G; \hat{z}) = \frac{(1 - \hat{z}) \cdot \mu}{(1 - \hat{z}) \cdot \mu + \hat{z} \cdot (1 - \mu) \cdot (\sigma_B + (1 - \sigma_B) \cdot \ell(\hat{z}))}.$$

This leads to threshold μ_2 :

$$\mu_2 = \frac{w_c^c \cdot \hat{z}}{w_c^c \cdot \hat{z} + (1 - w_c^c) \cdot \ell(\hat{z}) \cdot (1 - \hat{z})}, \quad (81)$$

where

$$w_c^c \equiv \frac{\omega_0 - \omega(B) + \bar{c}}{\omega(G) - \omega(B) + \bar{c}}.$$

For the equilibrium with $\sigma_G = 0$ and $\sigma_B = 1$, the threshold at which the pessimistic

consumer region ends is given by

$$\tilde{\mu} = \frac{w_c^c}{w_c^c + (1 - w_c^c) \frac{f(0.5|\kappa)}{f(0.5|\bar{\kappa})}}. \quad (82)$$

An equilibrium with $\sigma_G = 0$ and $\sigma_B = 1$, exists in the optimistic consumer region for $\mu > \tilde{\mu}$ and $\mu < \mu_4$, with

$$\mu_4 = \frac{w^c}{w^c + (1 - w^c) \frac{\hat{z}}{1 - \hat{z}}}, \quad (83)$$

where

$$w^c \equiv \frac{\omega_0 - \omega(B) + \bar{c}}{\omega(G) - \omega(B)}.$$

The equilibrium with $\sigma_B = 1$ and $\sigma_G = 0$ exists in the optimistic consumer region between thresholds μ_3 and μ_4 , with

$$\mu_3 = \frac{w_c^c}{w_c^c + (1 - w_c^c) \frac{f(\hat{z}|\kappa)}{f(\hat{z}|\bar{\kappa})} \frac{\hat{z}}{1 - \hat{z}}}. \quad (84)$$

As shown in the model with no cost of complexity, the equilibrium with $\sigma_B = 1$ and $\sigma_G \in (0, 1)$, exists between two thresholds μ_3 and μ_4 .

From (81)-(83),

$$\frac{d\mu_2}{dw_c^c} > 0, \quad \frac{d\mu_3}{dw_c^c} > 0, \quad \frac{d\mu_4}{dw_c^c} > 0.$$

Also

$$\frac{dw_c^c}{d\bar{c}} > 0, \quad \frac{dw^c}{d\bar{c}} > 0.$$

Then, a marginal increase in c weakly increases thresholds $\mu_2 - \mu_4$, which implies a decrease in the size of the interval (μ_2, μ_4) over which $\sigma_B > 0$. Moreover, since μ_3 increases as well, the interval $[\mu_3, 1]$ over which $\sigma_G > 0$ decreases. Thus, an increase in \bar{c} reduces expected equilibrium complexity. ■