# The Effects of Conventional and Unconventional Monetary Policy: A New Approach

Atsushi Inoue<sup>†</sup>

Barbara Rossi\*

Vanderbilt University

ICREA-Univ. Pompeu Fabra, Barcelona GSE, and CREI

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Abstract: We propose a new approach to analyze economic shocks. Our new procedure identifies economic shocks as exogenous shifts in a function; hence, we call them "functional shocks". We show how to identify such shocks and how to trace their effects in the economy via VARs using a procedure that we call "VARs with functional shocks". Using our new procedure, we address the crucial question of studying the effects of monetary policy by identifying monetary policy shocks as shifts in the whole term structure of government bond yields in a narrow window of time around monetary policy shocks, both in conventional and unconventional periods, and shows that traditional identification procedures may miss important effects. We find that, overall, unconventional monetary policy has similar effects to conventional expansionary monetary policy, leading to an increase in both output growth and inflation; the response is hump-shaped, peaking around one year to one year and a half after the shock. The new procedure has the advantage of identifying monetary policy shocks during both conventional and unconventional monetary policy periods in a unified manner and can be applied more generally to other economic shocks.

\*Corresponding author: Barbara Rossi, CREI, Univ. Pompeu Fabra, c. Ramon Trias Fargas 25-27, 08005 Barcelona, Spain. E-mail: barbara.rossi@upf.edu

<sup>†</sup>Department of Economics, Vanderbilt University, VU Station B, Box #351819, 2301 Vanderbilt Place, Nashville, TN 37235, USA. E-mail: atsushi.inoue@vanderbilt.edu

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## 1 Introduction

What is a monetary policy shock? And how large and pervasive are the effects of monetary policy? Such questions are of fundamental importance in economics, and have spurred countless and lively debates. In this paper, we propose a novel procedure to analyze economic shocks; then, we use our procedure to shed new light on the important question of identifying monetary policy shocks, questioning the traditional approach and showing that it might have missed important aspects.

Our new procedure identifies economic shocks as exogenous shifts in a function; hence, we refer to these shocks as "functional shocks". There are several important examples where shocks can be identified in this way. An important example is the identification of monetary policy shocks. Our new definition of a monetary policy shock is a shift in the entire term structure of interest rates in a short window of time around Central banks' monetary policy announcement dates. Clearly, the entire term structure contains important information on the length of the zero lower bound and on the expected effects of monetary policy (see Gürkaynak and Wright, 2012, for a survey of the relationship between the term structure and the macroeconomy). Hence, our definition of monetary policy shocks is broader than the one used in the existing literature, where monetary policy shocks are identified as exogenous changes in the short term interest rate alone, and has the potential to encompass more broadly other changes that monetary policy has on both short- and long-term interest rates, such as announcement effects associated with forward guidance or quantitative easing. While a lot is known about the effects of monetary policy during conventional times – that is, at times in which the monetary authority can freely change the short-term interest rate or money supply (see e.g. Christiano, Eichenbaum and Evans, 1999) – much less is known about the effects of monetary policy during zero-lower bound periods, where Central banks have to resort to unconventional monetary policy since the short-term interest rate is close to zero and it cannot be lowered further. In recent years, a consensus has emerged regarding the effects of unconventional monetary policy on the term structure of interest rates (Wright, 2012; Gürkaynak, Sack and Swanson, 2005a,b, 2007); however, the overall effects on macroeconomic aggregates have been challenging to estimate, delivering sometimes estimates that are different from those expected from theory (Wu and Xia, 2014). Understanding how unconventional monetary policy affects the economy is a crucial task that provides important guidance to policymakers.

Other examples of "functional" shocks include: (i) the identification of demand or supply shocks, which shift the whole demand or supply function. In fact, demand and supply shocks may affect a multivariate demand function in different ways, by shifting the demand of a product towards other products or simply by shifting the demand of all products in a similar way; (ii) the identification of tax policy shocks, in cases where tax policy shocks are exogenous changes in the tax schedule; (iii) the identification of productivity shocks, where productivity shocks are interpreted as exogenous shifts in the production function; (iv) shocks to income or wage distributions, where the entire change in the distribution function is of interest.

We identify economic shocks as a shift in a function. In our leading example on the identification of a monetary policy shock, where the function of interest is the term structure, we use the Nelson and Siegel (1987) and Diebold and Li (2006) approach to model yields as a

function of their maturity. The approach provides a widely-used and parsimonious model of the term structure based on three factors: level, curvature and slope. The factors naturally capture different aspects of monetary policy. In particular, they allow us to distinguish between conventional monetary policy, which typically operates by affecting the short-term factor, and monetary policy that affects the medium- and long-term, captured by the level and curvature factors; the latter include unconventional monetary policy, such as forward guidance, as well as monetary policy announcements that shift people's expectations about the future path of interest rates or about risk premia without actually changing the shortterm interest rate.<sup>1</sup> Our approach also provides interesting insights on the curvature factor, which so far has eluded an economic interpretation.

As we show, the monetary policy shock that we define is substantially different from the monetary policy shock traditionally defined as an exogenous change in short-term interest rates during conventional monetary policy periods. As we show, for example, both monetary policy shocks in 5/16/2000 and 11/6/2001 decreased the three-month rate by a similar magnitude, and would be considered similar monetary policy shocks in the traditional literature. In our approach, instead, it is clear that the shocks are very different: the former decreased proportionally all the yields, while the latter decreased short-term yields, increased medium-term yields and left unchanged long-term ones. Similarly, the shock on 1/28/2004 led to no change in the short term rate and would be ignored by the traditional literature, while in fact it did have large effects on medium- and long-term interest rates. Thus, our monetary policy shock is a more comprehensive measure of monetary policy than traditional measures.

Within our framework, we illustrate how monetary policy considerably changed its behavior over time: on average during the conventional period, monetary policy affected mostly the short end of the yield curve while leaving the long end unaffected; in the unconventional period, short-term interest rates were stuck at the zero-lower bound, yet monetary policy successfully shifted the long end of the yield curve. Such changes are mainly explained by changes in the way the monetary policy has affected both short- and long-term financial market's expectations of interest rates and risk premia. Our results, overall, suggest that, notwithstanding these changes, monetary policy has not lost its effectiveness during the zero lower bound period.

Another appealing feature of our framework is that the shock can be multi-dimensional – that is, could involve several "functions". We offer such an example in Section 6, where we define a monetary policy shock as the shift in both the term structure of interest rates as well as mortgage rates at maturities of either 15 or 30 months.

Using our framework, we quantitatively estimate the effects of monetary policy shocks during both conventional and unconventional monetary policy periods in a unified manner. In fact, it is important to merge information on both normal and exceptional times to have a large enough sample to estimate the effects of monetary policy: our approach is appropriate in this case, as the change over time in the shape of the term structure (described by, e.g., level, slope and curvature) has the potential to capture both conventional and unconventional monetary policy shocks. We revisit the empirical evidence on the effects of monetary policy

<sup>&</sup>lt;sup>1</sup>Note that, in this paper, we do not disentangle changes in the term structure due to expectations about the future path of interest from those due to risk premia. See Rogers, Scotti and Wright (2015) for an approach to do so.

shocks using our framework to answer the following questions: How big of a change in the term structure should the monetary policy authority aim at achieving when the economy is at the zero lower bound if they would like to stimulate output growth by, e.g., 1%? How long will it take to affect the economy? Our empirical results, based on US data and Jorda's (2005) local projections, show that an unexpected unconventional monetary policy easing typically decreases the term structure; the effects on slope and curvature depend on the episode, although they typically decrease the slope and increase the curvature. This means that both short- and long-term interest rates decrease after a quantitative easing, but the effect is stronger on the long than at the short end of the term structure. As a result, output typically increases, reaching a peak of about 1% one year after the initial shock. The bigger the decrease in the long end of the yield curve, the more protracted the effects on output: a monetary policy shock that has half the effect on long-term yields as another shock typically has effects on output that are both smaller in magnitude (between 0.5 and 1 percent, depending on the episode) as well as more short-lived (the effects start to disappear about six months earlier). The effects on inflation differ in magnitude in a similar way and by similar amounts, but the persistence is not affected at all. Importantly, we show that the traditional approach to the identification of monetary policy shocks may have either missed important shocks or been unable to differentiate between shocks that were very different from one another. There are only two "monetary policy easing" episodes that markets interpreted as increases in the term structure: 1/28/2009 and 9/13/2012; in one case, the easing was considered "disappointing" relative to market expectations, which might explain why the reaction was contrary to what one would expect based on theory; in the other case, while the short end of the term structure increased, the long term level did decrease, so the effects were perceived more in the long than in the short run.

On the one hand, one of the contributions of our paper is to propose a new approach to the identification of economic shocks. In this regard, our paper is related to the large literature on shock identification, in particular in VAR settings – see Kilian and Lütkepohl (2017) for a recent review of the literature. While we broadly build on existing approaches to shock identification, our approach is very different, as, unlike the traditional approach, it identifies shocks as shifts in a function rather than being summarized by a scalar. One limitation of the existing approaches is that they yield identical impulse responses up to scale for different policy announcements. In contrast, our approach yields different impulse responses for different policy announcements unless two changes in the yield curve are exact scalar multiples of each other (which is highly unlikely). This allows us to analyze and understand the effects of monetary policy at a deeper level. In particular, Gürkaynak et al. (2005a) have highlighted the importance of considering alternative "dimensions" in which monetary policy affects stock prices. Our framework is inspired by their work and allows researchers to directly evaluate and quantify the importance of these additional "dimensions".

On the other hand, the empirical analysis in our paper is related to the large literature that estimates the effects of monetary policy shocks. Traditionally, the VAR-based identification of monetary policy shocks has frequently relied on a recursive identification approach, although other approaches have been considered as well (Christiano et al., 1999). In the recursive identification approach, a monetary policy shock is identified as a change in the short-term interest rate (the Fed Funds rate, hereafter FFR) that is not an endogenous reaction to the state of the economy. Typically, recursive approaches lead to similar estimates of macroeconomic effects of monetary policy shocks as narrative approaches (Romer and Romer, 2004). More recently, as new and unconventional types of monetary policies have been implemented, such as quantitative easing and forward guidance, the literature has taken advantage of alternative identification schemes, including heteroskedasticity-based and high frequency identification (Wright, 2012; Gürkaynak et al., 2005a, Swanson, 2017). While we use high frequency data to extract the exogenous component of monetary policy, our approach results in a shock identified differently from that in the existing literature: namely, the shift in the entire term structure of interest rates (as opposed to a shift in short-term interest rates, or in interest rates at ad-hoc maturities).<sup>2</sup>

Our work is also related to the literature on the effects of unconventional monetary policy on the macroeconomy. For example, Kulish, Morley and Robinson (2016), Baumeister and Benati (2013) and Wu and Zhang (2017) argue, like we do, that it is important to have methodologies that can provide estimates of monetary policy effects during both periods of conventional monetary policy and the zero lower bound, and do so by estimating structural DSGE models or time-varying VARs. Alternatively, Wu and Xia (2014) and Krippner (2015) propose a "shadow rate" estimated from a finance model of the term structure to measure the stance of monetary policy during unconventional times. As previously discussed, the difference between these approaches and ours is that our shock is a function rather than a scalar, and it can capture multiple dimensions of monetary policy at the same time.<sup>3</sup>

Our paper is more generally related to the literature that measures the effects of unconventional monetary policy on the yield curve, and, in particular, the literature on the effects of news on the yield curve, such as Kuttner (2001), Wright (2012), Gürkaynak, Sack and Swanson (2005b, 2007), Baumeister and Benati (2013) and Altavilla and Giannone (2017). While our work builds on these contributions, it markedly differs from them: unlike these papers, which focus on the effects of monetary policy on the yield curve, we use shifts in the yield curve themselves to identify monetary policy shocks and then study their effects on key macroeconomic variables. Another key aspect that differentiates our work from theirs is that existing papers estimate impulse responses to shocks to either level or slope and not to the response to the functional shocks. In other words, we define an impulse response to the joint change in the whole shape of the yield curve.<sup>4</sup>

Finally, the model we use to fit the term structure is a dynamic Nelson and Siegel model augmented with macroeconomic data, although we explore results based on a non-parametric model for the term structure in the Appendix. Alternatively, one could rely on more general parametric models that allow for measurement error in the extracted yield curve factors (see

<sup>&</sup>lt;sup>2</sup>Note that our analysis is not confined to high frequency data, and it can be applied more generally to other well-known identification procedures, such as a Cholesky approach.

<sup>&</sup>lt;sup>3</sup>For example, Baumeister and Benati (2013) identify monetary policy shocks as exogenous movements in the spread between the 10 years and the 3 month rates. In our case, it is the whole profile of yields as a function of maturity.

<sup>&</sup>lt;sup>4</sup>Other papers have identified the effects of unconventional monetary policy using external instruments. For example, Gertler and Karadi (2014) identify unconventional monetary policy shocks using high frequency changes in interest rates around the date of the announcements as external instruments, and study the effects of the policies on key macroeconomic aggregates. Our work differs from theirs since we identify the unconventional monetary policy shock as the shift in the whole term structure.

Diebold et al., 2005, Diebold and Rudebusch, 2012, Moench, 2012, Altavilla et al., 2017), although these models do not rule out arbitrage, or, more generally, macro-yield models with no-arbitrage restrictions, as in Ang and Piazzesi (2003) and Moench (2012).

Section 2 presents our novel framework; Section 3 presents the monetary policy shock analysis, while Section 4 highlights the differences between our approach and those existing in the literature. Section 5 discusses the empirical results on the effects of monetary policy shocks on the macroeconomy in both conventional and unconventional times, Section 6 discusses the longer term effects of monetary policy, and Section 7 and concludes.

## 2 The "VAR with Functional Shocks" Approach

We propose to construct impulse responses to a shock which is defined as a function (not simply as a scalar); this requires a new and more general methodological approach. Appendix A provides some general definitions. In this section, we define the VAR approach that we utilize and show that it has a functional AR interpretation. Hence, we will refer to our proposed methodology as the "VAR with functional shocks".

For a given  $\lambda > 0$ , consider a class of possibly time-varying functions of the form:

$$f_t(\tau;\lambda) = \sum_{j=1}^q \beta_{j,t} g_j(\tau;\lambda), \qquad (1)$$

where the function is a linear combination of q time-varying factors ( $\beta_{j,t}$ , where t denotes time) with coefficients that are functions of a scalar  $\tau$  and depend on tuning parameters  $\lambda$ . The special type of function we consider is inspired by the Nelson and Siegel (1987)/Diebold and Li (2006) model, which we will describe in detail in the next Section.<sup>5</sup>

For a given weight function  $w(\cdot)$ , let  $I_j = \int w(\tau)g_j(\tau;\lambda)d\tau$ , j = 1, ..., q, and assume that they exist and are finite. Consider a stationary first-order linear AR model that consists of a scalar random variable and a random function:

$$X_t = c_1 + \phi_{1,1} X_{t-1} + \phi_{1,2} \int w(\tau) f_{t-1}(\tau; \lambda) d\tau + u_{X,t}, \qquad (2)$$

$$f_t(\cdot;\lambda) = c_2(\cdot;\lambda) + \phi_{2,1}(\cdot;\lambda)X_{t-1} + \phi_{2,2}f_{t-1}(\cdot;\lambda) + u_{f,t}(\cdot;\lambda), \qquad (3)$$

where  $c_2(\cdot)$ ,  $\phi_{2,1}(\cdot)$  and  $u_{f,t}(\cdot)$  belong to the above class of functions and are linear:

$$c_2(\tau;\lambda) = \sum_{j=1}^{q} \tilde{c}_j g_j(\tau;\lambda), \qquad (4)$$

$$\phi_{2,1}(\tau;\lambda) = \sum_{j=1}^{q} \tilde{\phi}_j g_j(\tau;\lambda), \qquad (5)$$

$$u_{f,t}(\tau;\lambda) = \sum_{j=1}^{q} \tilde{u}_{j,t}g_j(\tau;\lambda).$$
(6)

<sup>&</sup>lt;sup>5</sup>In the Nelson and Siegel (1987) model, q = 3,  $\tau$  is the maturity,  $g_1(\tau; \lambda) = 1$ ,  $g_2(\tau; \lambda) = [1 - \exp(\tau/\lambda)]/(\tau/\lambda)$  and  $g_3(\tau; \lambda) = [1 - \exp(\tau/\lambda)]/(\tau/\lambda) - \exp(-\tau/\lambda)$ .

Appendix A shows that, applying repeated substitutions to eqs. (2) and (3), and ignoring irrelevant constants, we have:

$$X_t = \sum_{i=0}^{\infty} \theta_{1,i} u_{X,t-i} + \sum_{i=1}^{\infty} \psi_{1,i} \left( \int w(\tau) u_{f,t-i}(\tau;\lambda) d\tau \right)$$
(7)

where the coefficients  $\theta_{1,i}$  and  $\psi_{1,i}$  are defined in the Appendix and  $\theta_{1,0} = 1$ . Then, using eqs. (6) and (7), the differential of  $X_{t+h}$  in the direction

$$u_{f,t}^*(\tau;\lambda) = \sum_{j=1}^q \tilde{u}_{j,t}^* g_j(\tau;\lambda)$$
(8)

is:6

$$\psi_{1,h} \int w(\tau) u_{f,t}^*(\tau;\lambda) \ d\tau = \psi_{1,h} \sum_{j=1}^q I_j \tilde{u}_{j,t}^*.$$
(9)

As shown in Appendix A, this model can be written as a (q+1)-variable VAR model:

$$\begin{bmatrix} X_t \\ \beta_{1,t} \\ \cdots \\ \beta_{q,t} \end{bmatrix} = \begin{bmatrix} \phi_{1,1} & \phi_{1,2}I_1 & \cdots & \phi_{1,2}I_q \\ \tilde{\phi}_1 & \phi_{2,2} & 0 & 0 \\ \cdots & \cdots & \cdots \\ \tilde{\phi}_q & 0 & 0 & \phi_{2,2} \end{bmatrix} \begin{bmatrix} X_{t-1} \\ \beta_{1,t-1} \\ \cdots \\ \beta_{q,t-1} \end{bmatrix} + \begin{bmatrix} u_{X,t} \\ \tilde{u}_{1,t} \\ \cdots \\ \tilde{u}_{q,t} \end{bmatrix}.$$
(10)

Similarly, Appendix A shows that the VAR has a vector moving average representation:

$$X_{t} = \sum_{i=0}^{\infty} \theta_{1,i} u_{X,t-i} + \psi_{1,1} \left( \sum_{i=1}^{q} I_{i} \tilde{u}_{i,t-1} \right) + \psi_{1,2} \left( \sum_{i=1}^{q} I_{i} \tilde{u}_{i,t-2} \right) + \dots$$
(11)

$$\beta_{1,t} = \theta_{2,1} u_{X,t-1} + \theta_{2,2} u_{X,t-2} + \psi_{2,1} \tilde{u}_{1,t} + \psi_{2,2} \tilde{u}_{1,t-1} + \cdots$$
(12)

$$\beta_{q,t} = \theta_{q+1,1} u_{X,t-1} + \theta_{q+1,2} u_{X,t-2} + \psi_{q+1,1} \tilde{u}_{q,t} + \psi_{q+1,2} \tilde{u}_{q,t-1} + \cdots$$
(13)

where constant terms are omitted for notational simplicity. It turns out that this moving average representation is identical to that of the (q + 1)-variable VAR model (10) as the integration is a linear operator and the space of functions is finite-dimensional. This allows us to focus on the conventional VAR model to calculate the moving average representation to obtain the impulse responses without having to estimate equations (11)-(13) directly. The differential of  $X_{t+h}$  is the inner product of the moving average coefficient of  $X_{t+h}$  on  $\tilde{u}_{1,t}, ...,$  $\tilde{u}_{q,t}$  in (11) and  $\tilde{u}_{1,t}^*$ . Note that the results generalize to  $X_t$  being a vector of variables (rather than a scalar).

Importantly, note that eq. (10) is a reduced-form VAR. The structural interpretation could be achieved by recursive, sign-restrictions, high-frequency or heteroskedasticity approaches (see Kilian and Lütkepohl, 2017, for a review). However, note that such approaches

<sup>&</sup>lt;sup>6</sup>As we discuss in Appendix A, the differential we define here is a Gateaux differential. Because of the linearity, the Frechet differential of  $X_{t+h}$  in the direction of  $u_{t,t}^*(\tau; \lambda)$  is also given by (9).

are to be applied to the whole function, and that is where our identification differs from the literature. In fact, while we broadly build on existing approaches to shock identification, our approach is very different as it identifies shocks as shifts in a function, rather than being summarized by a scalar. To see the differences more clearly, consider the VAR in eqs. (11)-(13) and consider identifying the shocks using a Cholesky (recursive) approach. The standard Cholesky approach would impose a triangularity assumption on the vector  $[X'_t, \beta_{1,t} \dots \beta_{q,t}]'$ , thus separately identifying the shocks to the  $\beta's$ . In our approach, the shock is instead identified by a contemporaneous change in all the  $\beta's$  without separately identifying them. Our approach is really about identifying shifts in a function which is summarized by a specific combination of the  $\beta's$ . Thus, it is very different from identifying the VAR in eqs. (11)-(13) simply using a recursive identification on the  $\beta's$ .

We now discuss detailed examples of identification restrictions within our general framework. Define the covariance matrix of the vector of the reduced-form shocks in eqs. (2)-(3), namely  $[u_{X,t}, u_{f,t}(\cdot; \lambda)]' = \left[u_{X,t}, \sum_{j=1}^{q} \tilde{u}_{j,t}g_j(\tau; \lambda)\right]'$ , by

$$\Sigma_{u}(\tau;\lambda) = \begin{bmatrix} \sigma_{XX} & \sigma_{X1} & \cdots & \sigma_{Xq} \\ \sigma_{1X} & \sigma_{11} & \cdots & \sigma_{1q} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{qX} & \sigma_{q1} & \cdots & \sigma_{qq} \end{bmatrix}^{\prime} \begin{bmatrix} \sigma_{XX} & \sigma_{X1} & \cdots & \sigma_{Xq} \\ \sigma_{1X} & \sigma_{11} & \cdots & \sigma_{1q} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{qX} & \sigma_{q1} & \cdots & \sigma_{qq} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & g_{1}(\lambda;\tau) \\ \vdots & \vdots \\ 0 & g_{q}(\lambda;\tau) \end{bmatrix}^{\prime},$$

$$(14)$$

where  $\sigma_{Xk} = Cov(X, \tilde{u}_k)$  for k = 1, ..., q and  $\sigma_{jk} = Cov(\tilde{u}_j, \tilde{u}_k)$  for j, k = 1, ..., q. Similarly, denote the covariance matrix of the structural shocks in eq.(10), namely,  $[\varepsilon_{X,t}, \varepsilon_{f,t}(\cdot; \lambda)]' = [\varepsilon_{X,t}, \sum_{j=1}^{q} \tilde{\varepsilon}_{j,t} g_j(\tau; \lambda)]'$ , by

$$\Sigma_{\varepsilon} = \begin{bmatrix} 1 & 0 \\ 0 & g_{1}(\lambda;\tau) \\ \vdots & \vdots \\ 0 & g_{q}(\lambda;\tau) \end{bmatrix}' \begin{bmatrix} \omega_{X} & 0 & 0 & 0 & 0 \\ 0 & \omega_{1} & 0 & \cdots & 0 \\ 0 & 0 & \omega_{2} & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \omega_{q} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & g_{1}(\lambda;\tau) \\ \vdots & \vdots \\ 0 & g_{q}(\lambda;\tau) \end{bmatrix},$$
(15)

where  $\omega_X = \sigma_{XX}$  and  $\omega_j = Var(\tilde{\varepsilon}_j)$  for j = 1, ..., q.

Comparing (14) and (15), the problem of identification boils down to identifying the  $(q+1) \times (q+1)$  matrix A whose diagonal elements are ones, such that

$$\begin{bmatrix} \sigma_{XX} & \sigma_{X1} & \cdots & \sigma_{Xq} \\ \sigma_{1X} & \sigma_{11} & \cdots & \sigma_{1q} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{qX} & \sigma_{q1} & \cdots & \sigma_{qq} \end{bmatrix} = A \begin{bmatrix} \omega_X & 0 & 0 & \cdots & 0 \\ 0 & \omega_1 & 0 & \cdots & 0 \\ 0 & 0 & \omega_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & \omega_q \end{bmatrix} A' = \tilde{A}\tilde{A}'$$

Although  $\Sigma_u$  is identified,  $\omega_X$ ,  $\omega_1$ , ...,  $\omega_q$  and A are not identified from  $\Sigma_u$  unless an identification condition is imposed.

- Short-Run Identification. Although it may be difficult to justify a recursive ordering among the functional structural shocks  $\tilde{\varepsilon}_1, ..., \tilde{\varepsilon}_q$ , one can argue that a macroeconomic aggregate does not contemporaneously respond to the monetary policy shock but not vice versa, for example. In that case, one can impose a block recursive structure on the impact matrix A:

$$\tilde{A} = \begin{bmatrix} X & 0 & \cdots & 0 \\ X & X & \cdots & X \\ \vdots & \vdots & \cdots & \vdots \\ X & X & \cdots & X \end{bmatrix},$$

where X denotes an element that is not necessarily zero. With this identification condition, one can identify the structural impulse responses of the macroeconomic variable to the monetary policy shock. Similarly, an oil price shock can be identified by including a similar block recursive structure using the term structure of oil price futures.

- Long-Run Identification. Let  $X_t = \Delta Y_t$  and rewrite eq. (10) as  $\Psi(L) [X'_t, \beta_{1,t} \dots \beta_{q,t}]' = \begin{bmatrix} u_{X,t} & \tilde{u}_{1,t} & \dots & \tilde{u}_{q,t} \end{bmatrix}', \Psi(L) = I - \Psi L$ . Then  $\Psi^{-1}(1)\tilde{A}$  is the long-run effect of the structural shocks on X. If the monetary policy shock does not have X, then one would impose

$$\Psi^{-1}(1)\tilde{A} = \begin{bmatrix} X & 0 & \cdots & 0 \\ X & X & \cdots & X \\ \vdots & \vdots & \cdots & \vdots \\ X & X & \cdots & X \end{bmatrix},$$

- Sign Restrictions. Sign restrictions can be imposed directly on the matrix A. For example, a typical restriction in the context of monetary policy is that an unexpected monetary policy contraction is associated with an increase in the short-term interest rate, a decrease in non-borrowed reserves and a decrease in prices for a few months after the shock (see Uhlig, 2005). Similarly, an oil price shock can be identified by imposing the relevant sign restrictions on the matrix A in a VAR that includes the term structure of oil futures.

- Heteroskedasticity-based Restrictions. Let the variance of the structural shocks change at time t from  $\Lambda_{\varepsilon,A} = \text{diag}(\omega_{x,A}, \omega_{1,A}, ..., \omega_{q,A})$  to  $\Lambda_{\varepsilon,B}$ . Because  $\Sigma_{u,A} = A\Lambda_{\varepsilon,A}A'$  and  $\Sigma_{u,B} = A\Lambda_{\varepsilon,B}A'$ , together with a normalization restriction, yield (q+1)(q+2) equations with  $(q+1)^2 - (q+1)$  unknowns, the structural together with a normalization restriction, the structural parameters of interest are identified. In the case of monetary policy, one could impose that the volatility of interest rates is higher on a day of a monetary policy shock (e.g., Nakamura and Steinsson, 2018). In the case of oil prices, one could impose that the variance of oil prices is larger than that of other financial variables on a day of an oil price shock.

Note that our framework can be implemented in a framework where other parameters in the model are time-varying. For example, one could allow the coefficients in eq. (10) to be time-varying, with a pre-specified law of motion. In the empirical analysis of this paper, we split the sample in two (the conventional and the unconventional monetary policy regimes) to allow for changes in the parameters in eq. (10), which reflect changes in the transmission mechanism.

Note that the theory applies to any impulse response, whether it is estimated by local projections or VAR procedures. While our approach is general, in this paper it turns out to be convenient to use a high-frequency identification approach and to estimate impulse responses via local projections. The weight function is set to one for the rest of the paper.

## 3 A New Approach to the Identification of Monetary Policy Shocks

We illustrate our approach in the leading case of the identification of monetary policy shocks. It is well-known that monetary policy operates (directly or indirectly) by affecting interest rates, which we plot in Figure 1. Panel A depicts daily US zero-coupon bond yields over time between January 1995 and June 2016.<sup>7</sup> The data are from Gürkaynak, Sack and Wright (2007). At every point in time, we have data on yields at different maturities, from 3 months to 10 years.<sup>8</sup> The top panel shows clearly the zero lower bound period, which we date starting in 2008:11 in our analysis, following the beginning of the first large-scale asset purchase program (LSAP-I). The yield curve as a function of maturity is depicted in Panel B of Figure 1. As the figure shows, the term structure of yields changed considerably over time in terms of its intercept, slope and curvature; we are interested, in particular, in exploring episodes of such shifts to identify monetary policy shocks in a more comprehensive manner.

### **INSERT FIGURE 1 HERE**

We define a monetary policy shock as the shift in the entire term structure due to an exogenous monetary policy move. To illustrate how our functional shock can capture monetary policy within a theoretical macroeconomic model, we rely on a simple rule monetary policy rule a' la Taylor augmented with forward guidance shocks (Campbell et al., 2012, and Del Negro et al., 2015). Let the interest rate at time t,  $i_t$ , obey the following monetary policy rule (up to a constant, which we ignore):

$$i_{t} = \mu + \rho i_{t-1} + (1 - \rho) \left[ \phi_{\pi} \pi_{t} + \phi_{u} u_{t}^{gap} \right] + \sum_{j=0}^{\tau_{\max}} \varepsilon_{t-j,j},$$
(16)

<sup>&</sup>lt;sup>7</sup>We start the sample in 1995 as the Fed did not release statements of monetary policy decision after its FOMC meetings before 1994. Also, importantly, Gürkaynak et al. (2005a) show that, after 1995, daily data provide an accurate identification of monetary policy shocks, which provides another rationale for using daily yields from 1995 onward in our analysis. Appendix B describes the data in detail.

<sup>&</sup>lt;sup>8</sup>The analysis of longer maturities requires a more general model and will be carried out in Section 7.

where  $\pi_t$  and  $u_t^{gap}$  are the inflation rate and the unemployment gap,<sup>9</sup> the parameter  $\rho$  describes the degree of interest rate smoothing and the parameters  $\phi_{\pi}$ ,  $\phi_u$  describe the inflation and unemployment gap aversion of the Central bank, respectively. The monetary policy shock,  $\sum_{j=0}^{\tau_{\max}} \varepsilon_{t-j,j}$ , is a convolution of shocks at different maturities in the future  $(\tau = 0, 1, ..., \tau_{\max} - 1)$ :  $\varepsilon_{t,0}, \varepsilon_{t-1,1}, ..., \varepsilon_{t-\tau,\tau}, ...$  We refer to  $\varepsilon_{t,0}$  as the conventional monetary policy shock, that is, the monetary policy shock that appears in the conventional monetary policy rules.<sup>10</sup> The remaining shocks are forward guidance shocks, revealed to the public earlier than the time in which they are implemented in practice. For example,  $\varepsilon_{t-1,1}$  is the monetary policy shock announced at time (t-1) to be applied by the Central bank one period hence, that is at time t. Each of these announcements affects the expected path of interest rates at the time the announcement is made.

Let the expectations of the interest rate given information at the start of period tmade at time t for  $\tau$  periods ahead be denoted by  $i_{t+\tau}^{\tau}$ . Note that, from eq. (16):

$$i_{t+\tau}^{\tau} = \mu + \rho i_{t+\tau-1}^{\tau-1} + (1-\rho) \left[ \phi_{\pi} \pi_{t+\tau}^{\tau} + \phi_{u} u_{t+\tau}^{gap,\tau} \right] + \sum_{j=\tau+1}^{\tau_{\max}} \varepsilon_{t+\tau-j,j}$$

Hence, the monetary policy shocks announced at time t for  $\tau = 0, 1, ..., \tau_{\max} - 1$  periods into the future will affect the whole term-structure at those maturities. The sequence of shocks  $\varepsilon^f = (\varepsilon_{t,0}, \varepsilon_{t,1}, ..., \varepsilon_{t,\tau_{\max}})'$  is the shock that we are capturing with our functional approach, where we will define  $\varepsilon^f(\tau)$  to be the  $(\tau + 1) - th$  element of the vector.

We use a high frequency identification inspired by Gürkaynak et al. (2005a,b, 2007), where the shock is identified as the shift in the term structure in a short window of time around monetary policy announcements. The novelty in our paper relative to Gürkaynak et al. (2005a,b, 2007) is that we identify the whole change in the term structure at a given point in time as the monetary policy shock. There is nothing special about using a high frequency identification within our approach: we could have alternatively used a Cholesky identification approach, for example, as discussed in the previous section. The dates of unconventional monetary policy announcements are from Wright (2012),<sup>11</sup> which we extend ourselves to a longer sample up to 2016:6, while those of conventional monetary policy are from Nakamura and Steinsson (2017). Note that, in principle, it is possible to control for cuncurrent news, such as macroeconomic releases, although for simplicity we do not.

Panel A in Figure 2 shows how the monetary policy shock is identified in some representative episodes of conventional monetary policy in US history. Each sub-panel in the figure depicts the shift in the term structure at the time of a monetary policy announcement, reported on top of the panel. Each circle represents the value of a yield at a given maturity (in months) before an exogenous monetary policy move, while the asterisk denotes its value afterwards. We define the monetary policy shock as the *joint* shift in yields *at all maturities* caused by the exogenous monetary policy move.

 $<sup>^{9}\</sup>mathrm{The}$  unemployment gap is the difference between the unemployment rate and the natural rate of unemployment.

<sup>&</sup>lt;sup>10</sup>We will refer to  $\varepsilon_{t,0}$  later as  $\varepsilon_t^{trad}$ .

<sup>&</sup>lt;sup>11</sup>The unconventional monetary policy dates are reported in the Not-for-Publication Appendix.

### **INSERT FIGURE 2 HERE**

The monetary policy shock that we define is substantially different from the monetary policy shock traditionally defined as the change in short-term (3-month) interest rates during conventional monetary policy periods. Such difference can be appreciated by looking closely at Figure 2. In the figure, the traditional monetary policy shock can be viewed as the shift in the interest rate at the 3-month maturity, that is the difference between the circle and the square at the shortest maturities, hence closest to the origin. Clearly, shifts of the same magnitude in short-term interest rates are interpreted in the traditional monetary policy literature as carrying the same information about monetary policy. For example, both monetary policy shocks in 5/16/2000 and 11/6/2001 decreased the three-month rate by a similar magnitude, and would be considered similar monetary policy shocks in the traditional literature. In our approach, instead, it is clear that the shocks are very different: the former decreased proportionally all the yields, while the latter decreased short-term yields, increased medium-term yields and left unchanged long-term ones. Similarly, the shock on 1/28/2004led to no change in the short-term rate and would be ignored by the traditional literature, while in fact it did have large effects on medium- and long-term interest rates. The difference between the monetary policy shock that we identify and that traditionally identified in the literature, thus, is that the latter is typically measured by a scalar (e.g. exogenous changes in the short-term interest rate) while our shock is a function: it is the whole shift in the term structure. Thus, each monetary policy shock can be different not only because it changes the short-term interest rate, but also because, at the same time, it changes the medium- and the long-term ones, and each of them in a potentially different way. In addition, it also matters how the whole term structure shifts, as opposed to how the short- or the long-term rates separately shift, as it is the joint combination of changes in the intercept, slope or curvature of the term structure that matters, as opposed to shifts in a specific maturity of the term structure.

We identify the economic shock as a shift in a function using two approaches. The first approach is parametric while the second uses raw yield data directly. The parametric approach estimates the shock using a parametric model. In particular, we use the Nelson and Siegel (1987)/Diebold and Li (2005) approach to model yields as a function of their maturity. The approach provides a widely-used and parsimonious model for the term structure. Alternatively, one could use raw yield data directly, which does not require any model: we consider this approach in the Not-for-Publication Appendix. Notice, however, that even if one uses raw yield data, our approach is very different from that in the existing literature as the shock is a simultaneous change in all the yields.

In the Nelson and Siegel (1987) framework, the yield curve at any point in time is summarized by a time-varying three dimensional parameter vector ( $\beta_{1t}$ ,  $\beta_{2t}$  and  $\beta_{3t}$ ) capturing latent level, slope and curvature factors. The model for the yield curve is the following:

$$y_t(\tau) = \beta_{1,t} + \beta_{2,t} \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau}\right) + \beta_{3,t} \left(\frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)$$
(17)

where  $y_t(\tau)$  is the yield to maturity,  $\tau$  is the maturity and  $\lambda$  is a tuning parameter.

The continuous lines in Figure 2 plot the monetary policy shock identified parametrically as a shift in  $y_t(\tau)$  in eq. (17).<sup>12</sup> The solid line depicts the term structure before the exogenous monetary policy move, while the dashed line depicts it afterwards. Clearly, monetary policy shocks (i.e., the difference between the solid and the dashed lines) come in many diverse shapes. Salient episodes of conventional and unconventional monetary policy are depicted in Panels A and B, respectively. Note how monetary policy shocks differ between the two periods: in the unconventional period, the shocks mainly affect medium and long-term maturities while leaving short-term maturities unaffected. For example, consider the shock on November 25, 2008 (depicted in Figure 2, Panel B), when the Fed announced the purchase of mortgage backed securities and agency bonds and the start of the LSAP-I program, and compare it with the shock on November 6, 2001, after the terrorist attacks of 9/11, depicted in Figure 2, Panel A. The figure illustrates how different the shocks are: even if they are both expansionary, the first shock tilts the function (as the short-term rates were fixed at the zero lower bound) while the second is a parallel shift in the function. Thus, each monetary policy shock can be different due to a variety of factors (how it affects short-term expectations and how it affects long term expectations or risk premia) as well as their combination (how it affects short-term expectations *versus* how it affects long term expectations or risk premia).

The functional monetary policy shocks themselves are depicted in Figure 3. They are defined as:

$$\varepsilon_t^f(\tau) \equiv \Delta y_t(\tau) \cdot d_t, \tag{18}$$

where  $d_t$  is a dummy variable equal to one if there is a monetary policy shock at time t and  $\Delta$  denotes time differences:  $\Delta y_t(\tau) \equiv y_t(\tau) - y_{t-1}(\tau)$ . Not only the shocks have different shapes in the conventional and unconventional periods, which can be appreciated by comparing Panels A and B in Figure 3, but they also differ from each other even in the conventional monetary policy period, as Figure 3 shows. For example, notice again how the change in the short-end of the yield curve is similar for both the 11/6/2001 and the 5/16/2000 shocks, while their shape is very different. The shocks of 1/28/2004 and 2/3/1999 are instead an example of similar effects on long-term yields but very different effects on short- and medium-term ones: no effects on short-term yields and large effects on medium-term yields for the 1/28/2004 shock and negative effects on short-term yields but very yields but very yields but very yields but very yields but yelds but yelds but yelds but yelds yelds but yelds y

#### **INSERT FIGURE 3 HERE**

The Nelson and Siegel (1987) model that we use to describe our monetary policy shock has several advantages. In particular, the model is quite flexible and the factors in eq. (17) have an economically interesting interpretation. Since  $\beta_{1,t}$  does not vanish as  $\tau$  approaches infinity, it can be interpreted as the long-term factor (or level factor, since it equally increases all yields independently of their maturity  $\tau$ );  $\beta_{2,t}$  is the factor with a coefficient  $\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau}\right)$ 

 $<sup>1^{12}</sup>$  The  $R^2$  of the estimates for the yield curves are very high, and equal to 0.9981, 0.9995, 0.9977, 0.9993, 0.9999, 0.9991, 0.9986, 0.9989, 0.9996, 1.0000, 0.9992, 0.9971 for the maturities that we consider, that is 3, 6, 12, 24, 36, 48, 60, 72, 84, 96 and 120 months.

that equals unity at  $\tau = 0$  but then decays to zero as  $\tau$  increases; thus, it reflects a factor that is important in the short-term (this factor can also be interpreted as the slope, as it equals  $y_t(\infty) - y_t(0)$ ); finally,  $\beta_{3,t}$  is the factor with a coefficient  $\left(\frac{1-e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}\right)$  that equals zero at  $\tau = 0$ , increases and subsequently decreases as a function of  $\tau$ , thus reflects neither short-term nor long-run factors but a factor that is important in the medium-term, where the medium-term definition depends on the value chosen for  $\lambda$  (this factor is also known as the curvature). The estimation follows Diebold and Li (2006) by calibrating  $\lambda$  to 0.0609, which is the value that maximizes the loading on the medium term factor at 30 months.

Importantly, note that  $\beta_{1,t}$ ,  $\beta_{2,t}$  and  $\beta_{3,t}$  capture different aspects of monetary policy. In particular,  $\beta_{2,t}$  describes conventional monetary policy, which typically operates by affecting short-term interest rates.  $\beta_{3,t}$ , instead, captures monetary policy shocks that affect the medium-term; these include unconventional monetary policy shocks, such as forward guidance, where the short-term is at the zero lower bound, as well as monetary policy announcements that shift people's expectations of future interest rates or risk premia without actually changing the short-term interest rate (such as, for example, the FOMC announcement of January 28, 2004, depicted in Figure 2).<sup>13</sup> Finally,  $\beta_{1,t}$  captures any effects of monetary policies that simultaneously shift all interest rates, and derives from the Central Bank's ability to shift proportionally both short- and long-term expectations at the same time.

Certain linear combinations of the factors may also carry valuable information. For example, the instantaneous yield equals  $(\beta_{1,t} + \beta_{2,t})^{14}$  while  $(\beta_{3,t} - \beta_{1,t})$  captures changes in long-run expectations or risk premia that do not result in parallel shifts in the term structure. That is, the latter captures additional information that monetary policy shocks contain exclusively about the future path of monetary policy not already contained in shifts in the short-term policy instrument, i.e. additional and potentially important "dimensions" of monetary policy. For example, Panel A in Figure 2 shows several interesting patterns arising from these linear combinations, whose values are reported in Table 1. The top panels depict a parallel downward shift in the term structure, which corresponds to a decrease in  $(\beta_{1,t} + \beta_{2,t})$  due mostly to a decrease in  $\beta_{1,t}$ . The two figures in the middle depict a change in short-term interest rates associated with an increase in medium-term rates, and with an increase in the long-term rates in the panel on the left but unchanged long-term rates in the panel on the right. These changes correspond to a small change in  $(\beta_{1,t} + \beta_{2,t})$  in the first and a large and negative change in  $(\beta_{1,t} + \beta_{2,t})$  in the second, combined with relatively larger increase in both  $\beta_{1,t}$  and  $\beta_{3,t}$  for the former, and no change in  $\beta_{1,t}$  for the latter. The bottom panels depict situations in which the instantaneous interest rate is unchanged  $(\beta_{1,t} + \beta_{2,t} = 0)$ yet monetary policy affects medium- and long-term interest rates by increasing  $(\beta_{3,t} - \beta_{1,t})$ , especially in the latter episode.

#### **INSERT TABLE 1 HERE**

Our analysis is thus related, although distinct, from that in Gürkaynak et al. (2005a) and Rogers, Scotti and Wright (2014). In their work, Gürkaynak et al. (2005a) extract

<sup>&</sup>lt;sup>13</sup>See Gurkaynak et al. (2005, p. 56) for a discussion of the FOMC announcement of January 28, 2004.

<sup>&</sup>lt;sup>14</sup>Note that  $y_t(0) = \beta_{1,t} + \beta_{2,t}$ .

factors from changes in bond yields and stock prices around the time of monetary policy announcements and find that two factors are important.<sup>15</sup> To give factors an economic interpretation, they rotate the second factor so that it is independent of changes in the Federal Funds rate (FFR) in the current month. Thus, the first factor is labeled the "current FFR factor", which corresponds to a surprise change in the current FFR target, and the second factor is labeled the "future path of policy factor", which corresponds to changes in future one-year-ahead rates independent of changes in the first factor. They find that both monetary policy actions and statements affect asset prices, and the latter have more effects on long-term yields. They show that monetary policy announcements affect asset prices primarily via changing financial markets' expectations of future monetary policy (rather than changing their expectations on the current FFR). Swanson (2017) extends Gürkaynak et al.'s (2005) methodology to include the zero-lower bound period, and aims at separately identifying changes in the FFR, forward guidance and LSAP by extracting three factors from a dataset of asset prices that includes the FFR, exchange rates, Treasury bond yields and the stock market. Differently from Gürkaynak et al.'s (2005) and Swanson (2017), in our identification, instead, we do not separately identify shocks, as the entire change in the yield curve is the shock itself. While these works have inspired ours, the differences between our approach and theirs are several. First, and most importantly, we define a monetary policy shock as a specific and time-varying combination of changes in the various factors that we identify: each monetary policy shock is potentially different from another; previous works, instead, are interested mainly in determining whether how many factors provide a good description of the movements in asset prices at the time of a monetary policy shock and how the factors evolve over time.<sup>16</sup> Second, our factors are derived directly from the Nelson and Siegel (1987)/ Diebold and Li (2006) model. While the first two principal components in the yield curve are typically level and slope, and thus may correspond to our first two factors, in our work, we find that a third factor, the curvature, is potentially important in selected monetary policy episodes. On the other hand, Gürkaynak et al. (2005a) and Swanson (2007) extract factors from a joint panel of Treasury yields and stock prices, while we only use the former as our goal is to identify a monetary policy shock. A third, substantial difference is that, unlike them, we study the effects of monetary policy on macroeconomic variables rather than asset prices. Rogers, Scotti and Wright (2014), like Gürkaynak et al. (2005a), extract two principal components; they notice that the first principal component is correlated with an increase in all the yields, and interpret it as an LSAP shock, while the second seems to rotate the yield curve (pushing short rates down and long rates up), and interpret this as a forward guidance shock. By arguing that forward guidance cannot be credible at long horizons, they can also distinguish between forward guidance and risk premia: they interpret changes in yields that are concentrated in forward rates five years and beyond as caused by shifts in risk premia. Our approach, instead, allows us to directly estimate the various dimensions of monetary policy shocks. The next section provides a more formal analysis of the empirical importance of alternative dimensions of monetary policy.

 $<sup>^{15}</sup>$ The importance of the factors is tested by the Cragg and Donald (1997) test.

<sup>&</sup>lt;sup>16</sup>The fact that Swanson (2017) finds three factors is not inconsistent with our findings, as his dataset includes not only yields but other asset prices as well.

## 4 A More Comprehensive Measure of Monetary Policy Shocks

More formally, how do traditional monetary policy shocks identified in the existing literature compare with the monetary policy shock that we identify as the change in the whole yield curve over time? If their correlation is high, then they are measuring the same unobserved shock and researchers can use either one of them; however, if their correlation is low, the existing literature may have missed important information on the identification of the shock. Gürkaynak et al. (2005a) have argued that the information extracted by conventional monetary policy shocks is incomplete and our empirical results can shed light on this important issue. Let  $\varepsilon_t^{trad}$  denote a traditional measure of monetary policy shocks, e.g. a narrative measure. We consider the following regression:

$$\varepsilon_t^f(\tau) = \alpha(\tau) + \gamma(\tau)\varepsilon_t^{trad} + \eta_t, \tag{19}$$

which we estimate separately in the conventional and unconventional monetary policy periods. Panel A in Figure 4 plots the estimates of  $\gamma(\tau)$  as a function of the maturity  $\tau$ using the traditional Romer and Romer (2004) monetary policy shock as a proxy for the traditional monetary policy shock,  $\varepsilon_t^{trad,17}$  Interestingly, the correlation during the conventional monetary policy sample, depicted on the left, is the highest for short-term maturities, while the correlation is the highest for the longest-term maturities in the unconventional monetary policy portion of the sample, depicted on the right. This means that monetary policy considerably changed its behavior: on average, during the conventional monetary policy period, monetary policy affected mostly the short end of the yield curve while leaving the long end unaffected; in the unconventional period, short-term interest rates were stuck at the zero-lower bound, yet monetary policy successfully shifted the long end of the yield curve, although short term rates were unaffected. Indeed, the data show strong evidence of a structural change: we filtered the daily yields by a VAR(1) model and then tested the equality of the means between the two sub-samples. The p-values of the Wald tests are all zero. Thus, the mean of the yields has indeed changed over time. Panel B in Figure 4 repeats the analysis using a monetary policy shock based on Wu and Xia's (2014) shadow rate as the proxy for the traditional monetary policy shock,  $\varepsilon_t^{trad}$ .<sup>18</sup> The latter is estimated in a VAR with inflation, output and the shadow rate, and identified using a Cholesky identification with the variables in the same order. The figure shows that the results are qualitatively similar.<sup>19</sup>

### **INSERT FIGURE 4 HERE**

<sup>&</sup>lt;sup>17</sup>We use the traditional Romer and Romer shock up to 2007:12 and we proxy the traditional monetary policy shock after that by the change in the 3-month Treasury yield in a one-day window around monetary policy announcement dates.

<sup>&</sup>lt;sup>18</sup>The data are available at: https://www.frbatlanta.org/cqer/research/shadow\_rate.aspx?panel=1.

<sup>&</sup>lt;sup>19</sup>The Not-for-Publication Appendix repeats the analysis using Nakamura and Steinsson's (2017) shock and shows that the results are qualitatively similar.

In order to understand the difference between our identified monetary policy shock and the traditional shock, we investigate which components of our shock are more correlated with the conventional monetary policy shock. Note that we can decompose our functional shock in eq. (18) as:

$$\varepsilon_t^f(\tau) = \Delta \widetilde{\beta}_{1,t} + \Delta \widetilde{\beta}_{2,t} \left( \frac{1 - e^{-\lambda \tau}}{\lambda \tau} \right) + \Delta \widetilde{\beta}_{3,t} \left( \frac{1 - e^{-\lambda \tau}}{\lambda \tau} - e^{-\lambda \tau} \right)$$

$$\equiv \Delta y_t^{(1)}(\tau) + \Delta y_t^{(2)}(\tau) + \Delta y_t^{(3)}(\tau) .$$
(20)

where  $\Delta \widetilde{\beta}_{j,t} \equiv d_t \cdot \Delta \beta_{j,t}$ . Consider the following regressions:

$$\Delta y_t^{(1)}(\tau) = \alpha_1(\tau) + \gamma_1(\tau) \varepsilon_t^{trad} + \eta_{1,t}$$
(21)

$$\Delta y_t^{(2)}(\tau) = \alpha_2(\tau) + \gamma_2(\tau) \varepsilon_t^{trad} + \eta_{2,t}$$
(22)

$$\Delta y_t^{(3)}(\tau) = \alpha_3(\tau) + \gamma_3(\tau) \varepsilon_t^{trad} + \eta_{3,t}, \qquad (23)$$

which we separately estimate in the conventional and unconventional monetary policy subsamples, respectively. To evaluate the instantaneous effects, which are captured by  $\Delta y_t^{(1)}(0) + \Delta y_t^{(2)}(0)$ , we also estimate the regression:

$$\Delta y_t^{(1)}(\tau) + \Delta y_t^{(2)}(\tau) = \alpha_4(\tau) + \gamma_4(\tau)\varepsilon_t^{trad} + \eta_{4t}.$$
(24)

Figures 5 and 6 report the estimates of  $\gamma_i(\tau)$  for the two traditional monetary policy shocks that we consider: Romer and Romer (2004) and Wu and Xia (2014), respectively. In each figure, the top panel (A) shows the values of  $\gamma_i(\tau)$  for the conventional monetary policy period (1995:2-2008:10) while the bottom panel (B) shows those for the unconventional monetary policy period (2008:11-2014:4).

#### **INSERT FIGURES 5 AND 6 HERE**

Clearly, the figures show drastic changes in the regression coefficients. While in the conventional period the largest correlation between  $\Delta y_t^{(1)}(\tau) + \Delta y_t^{(2)}(\tau)$  and the monetary policy shock is the highest at short maturities, it is the highest at the long maturities in the unconventional period. This suggests that the conventional shock is measuring only the short-term effects of monetary policy and does not contain much information regarding its medium to long-term effects, which instead our shock can capture. Furthermore, the relationship between  $\Delta y_t^{(1)}(\tau)$  and the monetary policy shock, which is constant by construction across maturities, changes from very small and negative in the conventional monetary policy period to positive and much larger in the unconventional period. In addition, with our identification procedure, we find that the regression coefficient between the curvature ( $\Delta y_t^{(3)}(\tau)$ ) and the monetary policy shock changes from negligible to negative values between the two periods, with a hump-shape in the unconventional period peaking around 30 months. Thus, our analysis can identify important channels describing how monetary policy has changed

over time when moving to the unconventional period. Figure 6 shows that the results are similar for Wu and Xia's (2014) shock. Krippner (2015) provides an alternative measure of shadow rates. Figure 6(b) shows that the empirical results are qualitatively the same if we use Krippner's (2015) shadow rate shock.

Figure 7 plots the components of the estimated functional monetary policy shocks over time. Note how the nature of the monetary policy shock changes over time. The behavior of  $\Delta\beta_{1,t}$  is somewhat constant over time, suggesting that the effectiveness of monetary policy in affecting all the yields overall has not decreased over time: if anything, monetary policy shocks in the unconventional period (in particular, in 2008) had much larger effects (in magnitude) than before. This has important implications, as it suggests that monetary policy did not lose its effectiveness during the zero lower bound period. The behavior of  $\Delta\beta_{2,t}$ and  $\Delta\beta_{3,t}$  also changed, becoming larger in magnitude in 2008-2009, suggesting important changes in the short-run and medium-run components of the monetary policy as well. The fact that the nature of the shocks has changed over time is confirmed by a test of outlier detection based on Tukey's range test.<sup>20</sup>

### **INSERT FIGURE 7 HERE**

Can monetary policy be fully summarized by movements in short-term interest rates (a situation which we refer to as "one-dimensional monetary policy", following Gürkaynak et al., 2005a), or is monetary policy operating in other ways as well? We investigate this issue by plotting the monetary policy shocks in the top left graph in Figure 8. If monetary policy shocks were "one-dimensional" then all the shocks should line up along one dimension, that is, they should belong to the same line. The figure visually suggests that this is not the case. To control for the possible asymmetry of monetary policy shocks, we consider expansionary and contractionary shocks separately, and we also distinguish between conventional and unconventional monetary policy shocks, depicted in the graphs on the right, seem scattered around along more than two dimensions. The contractionary shocks instead, depicted on the bottom left graph, visually appear to be lying on a plane.

### **INSERT FIGURE 8 HERE**

To investigate the issue more formally, we implement a modification of Robin and Smith's (2000) rank test proposed by Donald, Fortuna and Pipiras (2014). We focus on testing the rank of the matrix  $E\left(\Delta \tilde{\beta}_t \Delta \tilde{\beta}'_t\right)$ , where  $\Delta \tilde{\beta}_t \equiv \left(\Delta \tilde{\beta}_{1,t}, \Delta \tilde{\beta}_{2,t}, \Delta \tilde{\beta}_{3,t}\right)'$ . If the space of the monetary policy shocks is spanned by just one shock, then the rank of the matrix is one.<sup>21</sup>

 $<sup>^{20}\</sup>mathrm{The}$  outliers are in the last months of 2008.

<sup>&</sup>lt;sup>21</sup>Robin and Smith's (2000) rank test requires some modifications in order to be applied to symmetric and positive semi-definite matrices, such as the matrix we are interested in. In particular, Donald, Fortuna and Pipiras (2014, Sections 4.2-4.3) describe how to implement Robin and Smith's (2000) tests for symmetric and semi-definite matrices. We implement the test using a HAC variance estimator with one lag to control for serial correlation.

The results of the rank test are reported in Table 2. The test shows that the monetary policy shocks in the term structure were not well-described just by changes in the one of the  $\tilde{\beta}'_t s$  over the sub-sample up to 2007:12. Thus, conventional monetary policy cannot be summarized only by the information contained in changes in short-term interest rates. However, we cannot strongly reject that monetary policy can be summarized by one dimension after 2008:1, although the p-value is close to 0.10 and the result may be driven by the small sample that we consider.

### **INSERT TABLE 2 HERE**

### 5 The Effects of Monetary Policy Shocks

What is the effect of an increase in interest rates on output and inflation after one year? How much do quantitative easing and forward guidance policies contribute to future growth in output? We answer these questions by using our functional shocks as the measure of monetary policy shocks.

We estimate the effects of monetary policy using local projections (Jorda', 2005). Ideally, Vector Autoregressions (VARs) allow comparisons between our empirical results and those of existing methods during the conventional period, where the VAR is a frequently used approach. This would require including monetary policy shocks as variables in the VAR; however, since the monetary policy shocks can be zero at times when there is no monetary policy shock, this is not possible. Therefore, we estimate the responses using local projections. We estimate the responses directly from following regression:

$$X_{t+h} = \Gamma_{0,h} + \Gamma_{1,h}(L) \Delta \beta_{1,t} + \Gamma_{2,h}(L) \Delta \beta_{2,t} + \Gamma_{3,h}(L) \Delta \beta_{3,t} + A(L) X_{t-1} + u_{t+h}, \ h = 1, 2, ..., H$$
(25)

where  $X_t$  contains inflation and industrial production; h = 1, 2, ..., H is the horizon of the response and the lag length is 2. The coefficients  $\Gamma_{j,h}$  are the responses at time t + h to a shock in  $\beta_{j,t}$  at time  $t, j = 1, 2, 3.^{22}$  Since  $\Delta \tilde{\beta}_{1,t}$  and  $\Delta \tilde{\beta}_{2,t}$  appear to be collinear, two factors may be sufficient to describe changes in the term structure during the conventional period. Thus, in practice, we include only  $\Delta \tilde{\beta}_{2,t}$  and  $\Delta \tilde{\beta}_{3,t}$  in eq. (25).

To allow for changes in the transmission mechanism in different monetary policy periods, we estimate eq. (25) in two sub-samples: the conventional monetary policy period (1995:1-2008:10) and the unconventional period (2008:11-2016:6). Note that the second sub-sample starts in November 2008, given that November 25 2008 marked the start of the first large scale asset purchasing program, LSAP-I.

Since we are working with data estimated at different frequencies (the term structure is daily, while inflation and industrial production are monthly), we need to attribute the shock (i.e., the daily change in the term structure at the time of a monetary policy announcement) to a given month. We attribute the shock to the month in which it took place.

We assume that, on monetary policy announcements dates, unexpected changes in monetary policy shift the entire yield curve by simultaneously changing the  $\beta'_t s$ . We then use

 $<sup>^{22}</sup>A(L) = \sum_{s=1}^{p} A_s L^s$ , where L is the lag operator and p is the lag length.

the chain rule to identify the response of macroeconomic variables to the unconventional monetary policy shock as follows:

$$\frac{\partial X_{t+h}}{\partial \varepsilon_t^f(.)} = \frac{\partial X_{t+h}}{\partial \Delta \widetilde{\beta}'_t} \frac{\partial \Delta \beta_t}{\partial \varepsilon_t^f(.)} = \Lambda_h \delta_t, \tag{26}$$

where the first component on the right hand side,  $\frac{\partial X_{t+h}}{\partial \Delta \tilde{\beta}_t}$ , is estimated in the eq. (25), and the second component,  $\delta_t \equiv \Delta \tilde{\beta}_t = \Delta \beta_t \cdot d_t$ , is the change in the term structure (proxied by  $\Delta \beta_t$ ) times a dummy variable ( $d_t$ ) equal to unity if there is a monetary policy announcement at time t.<sup>23</sup>

We use a high frequency identification that relies on the following set of identification conditions:

#### Assumption I.

(a) *Shock identification condition*: Inflation and output are not contemporaneously affected by yield curve shocks.

(b) *Relevance condition*: A change in the yield curve on an announcement date is only due to the monetary policy shock.

(c) *Exogeneity condition*: The change in the yield curve after an announcement date in the sampling period is not due to the monetary policy shock.

Under Assumption I, the method described in the paper correctly identifies the effects of monetary policy shocks.

The particular type of identification that we choose (the high frequency identification in Assumption I) follows Gürkaynak et al. (2005a). However, note that our "functional shock" approach does not necessarily rely on a high frequency identification: recursive, signrestrictions or other typical restrictions can be used as well, as highlighted in Section 2. Assumption I(a) is frequently used in the VAR literature, where monetary policy shocks are commonly identified via a recursive approach, for example. Importantly, note that we do not need to separately identify shocks to each of the different components in the yield curve (i.e. each of the  $\beta'_t s$ ): the monetary policy shock is a simultaneous change in the whole yield curve. Note that Assumption I(a) could be removed, as one might leave the coefficient unrestricted under the assumption that the shock is strictly exogenous contemporaneously; we prefer to be robust and impose this assumption in our estimation.

Our method is an IV-based method, hence the instrument needs to be both relevant and exogenous, that is, satisfy Assumption I(b-c). Assumption I(b) is not as restrictive as it may seem. The assumption is still empirically valid if, on announcement days, the magnitude of the monetary policy shock is significantly bigger than that of any other shock. In principle, it is possible to improve the likelihood that this assumption holds by shortening the window of time in which the shock is identified. In the empirical application in this paper, we

<sup>&</sup>lt;sup>23</sup>In the model we consider here, the Fréchet derivatives of the macroeconomic variables with respect to the yield curve, defined in eq. (30) in Appendix A, are simply linear combinations of the Gateaux derivatives, eq. (29).

assume a one-day window, consistently with the finding in Gürkaynak et al. (2005a) that a window of one day is sufficient to describe monetary policy behavior. Assumption I(c) requires that, for example, there is only one monetary policy shock in any given month in a monthly dataset. In practice, there are a handful of months with more than one shock, in which case we take the average of the shocks. Finally, one should interpret the empirical results as if the monetary policy shock realizes at the end of the month. Note that this is the implicit assumption underlying VARs estimated at the monthly frequency for the conventional period.<sup>24</sup>

At the time of the monetary policy announcement, the term structure changes. Recall that each monetary policy shock can be potentially different: it could either result in a parallel shift in the term structure (thus affecting only  $\beta_{1,t}$ ) or it could shift the slope by affecting more (less) the long-term interest rates than the short-term ones (thus affecting  $\beta_{2,t}$ ), or it could affect the curvature by affecting the medium-term rates more than the rest of the maturities (thus affecting mainly  $\beta_{3,t}$ ) – or, it could be a combination of all these components with different degrees. That is, the monetary policy shock is described as  $\left\{\Delta \tilde{\beta}_{1,t}, \Delta \tilde{\beta}_{2,t}, \Delta \tilde{\beta}_{3,t}\right\}$ .

At any point in time, the response of the macroeconomic variables  $(X_{t+h})$  to the monetary policy shock  $(\varepsilon_t^f(.))$  is a combination of changes in each of these components:

$$\frac{\partial X_{t+h}}{\partial \varepsilon_t^f\left(.\right)} = \sum_j \Lambda_{j,h} \left(\Delta \beta_{j,t} d_t\right).$$
(27)

The estimation of eq. (25) provides  $\Lambda_{j,h} \equiv \frac{\partial X_{t+h}}{\partial \Delta \tilde{\beta}'_{j,t}}$ , while  $\Delta \beta_{j,t} d_t$  are estimated by the change in the term structure in a short window of time around the monetary policy announcement.

Equation (27) shows that each monetary policy announcement has a different impulse response, which is realistic and enhances our understanding of monetary policy. In contrast, the conventional analysis imposes that impulse responses are identical up to scale across different announcements.

### 5.1 Empirical Results on the Effects of Conventional Monetary Policy

Traditional VAR approaches typically identify monetary policy shocks during conventional times as changes in the short-term interest rate that are not caused by an endogenous reaction to the current state of the economy. In those approaches, the effects of monetary policy are estimated as the reaction to, say, an exogenous unitary increase in the short-term interest rate.<sup>25</sup> Thus, there is one impulse-response, and the effects of monetary policy proportionally depend on the magnitude of the increase (or decrease) in the short-term interest rate. Let

<sup>&</sup>lt;sup>24</sup>Alternatively, one could design alternative weighting schemes to take into account the day of the month in which the shock realized, to adjust for the length of time in which output could have responded to the shock. In practice, such an adjustment would require ad-hoc assumptions.

<sup>&</sup>lt;sup>25</sup>Alternatively, the response can be measured as the reaction to a one standard deviation increase in the short-term interest rate. The logic of the argument that follows is unaffected by choice of the unit or measure.

us proxy changes in the short-term interest rate with changes at the short-end of the term structure around monetary policy announcement dates,  $\Delta \tilde{\beta}_{2,t}$ . We estimate a traditional structural VAR that includes inflation, output and  $\Delta \tilde{\beta}_{2,t}$ , using this ordering. The responses to the monetary policy shock from the traditional VAR are depicted in Figure 9.<sup>26</sup> The figure replicates the well-known empirical finding that output and inflation decrease after an unexpected monetary policy tightening, a stylized fact typically encountered in the VAR literature (e.g. see Stock and Watson, 2001, p. 107). The effects of a monetary tightening are qualitatively similar to those surveyed in Stock and Watson (2001): they are hump-shaped, reaching their largest effects on output after about one year, while peaking after one quarter (four months) and quickly disappearing after one year for inflation. The effects are also similar in magnitude for output, while a bit smaller in our sample for inflation (our sample includes a longer period of very low inflation).

#### **INSERT FIGURE 9 HERE**

In our framework, instead, the response of the macroeconomic variables to the shock depends on the combination of  $\Delta \tilde{\beta}_{1,t}$ ,  $\Delta \tilde{\beta}_{2,t}$ ,  $\Delta \tilde{\beta}_{3,t}$ , and can, in principle, differ depending on the way the term structure changes beyond just the short-run effect. We depict responses for selected episodes in Figures 10 and 11. For each episode, the figures depict the change in the term structure (panel on the right) and the corresponding response of the macroeconomic variable (panel on the left). Notice how a similar decrease in the short-run interest rate may result in different output responses by comparing the 11/6/2001 and the 9/29/1998 announcements (depicted in the top two panels in Figure 10). Both announcements resulted in a decrease in short-term interest rates of similar magnitude ( $\Delta \tilde{\beta}_{1,t} + \Delta \tilde{\beta}_{2,t}$  around -0.2 from Table 1); yet, the former resulted in a short-run decrease in output while output increased in the latter. The reason is the very different behavior of  $\Delta \tilde{\beta}_{2,t}$  and  $\Delta \tilde{\beta}_{3,t}$ : in the former, one decreased and the other increased, while in the latter both increased. Their opposite behavior resulted in a proportionally larger decrease in long-term interest rates in the latter episode. A similar result holds for the response of inflation in these episodes: inflation decreases in the former and increases in the latter.

### INSERT FIGURES 10 AND 11 HERE

### 5.2 Empirical Results on the Effects of Unconventional Monetary Policy

Our results in Section 3 show that, typically, after a quantitative easing, the term structure moves towards the origin, implying a decrease in both the short-term and the longer-term

 $<sup>^{26}</sup>$ To facilitate the comparison with the existing literature, we estimated the VAR in an iterated, rather than direct, way.

interest rates (cfr. Figure 2, Panel B), except two episodes: 1/28/2009 and 9/13/2012. In most cases, the decrease in the level of the term structure is associated with an increase in the slope and an increase in the curvature, whose combined action results in stronger effects of monetary policy at the long end of the term structure.

### **INSERT FIGURE 12 HERE**

Figures 12 and 13 plot the responses of macroeconomic aggregates to selected unconventional monetary policy shocks. Figure 12 shows that quantitative easing typically increases output after a few months (about six), as one would expect from theory; the response is hump-shaped, with the largest effects after about one to one and a half year after the shock, and starting to disappear after two years. The magnitude of the effect varies depending on the episode: the maximum effect is typically between one and two percent. Some of the largest output responses (peaking around one percent) are on 11/25/2008 and 12/16/2008: the first is associated with the announcement that started LSAP-I, and the second with the reduction of the FFR to its effective zero lower bound. Hence, indeed, we find that the announcement of the large scale asset purchases did change the yield curve substantially. There are two occasions where the monetary policy easing decreased subsequent industrial production, and are the two dates where the term structure moved in the opposite direction, that is 1/28/2009 and 9/13/2012. The first is in line with well-known fact that the Federal Open Market Committee (FOMC) statement of 1/28/2009 was considered disappointing by financial markets, as it did not contain concrete language regarding the purchase and timing of long-term Treasuries in the secondary markets (Gilchrist et al., 2013); the second episode is the announcement of LSAP-III. In both cases, however, the level increased while both the slope and the curvature decreased and long term interest rates actually decreased (see Table 1).

#### **INSERT FIGURE 13 HERE**

The effects on inflation are also similar to what would be expected by theory – see Figure 13. In particular, one would expect inflation to increase after a monetary policy easing; this is what we find in most cases, again except 1/28/2009 and 9/13/2012. In general, we find that the response of inflation is hump-shaped; the timing of its peak is about 6 to 10 months, similar to that of industrial production. However, the effects on inflation die away more slowly than those on output, and are still different from zero even after 20 months.

Note that the confidence bands are large. This is potentially due to the local projection approach: on the one hand, the approach is useful to guard against nonlinearities, since it gives the best linear approximation, which is important in our analysis; on the other hand, it leads to less precise estimates of the responses since it does not impose the constraints associated with a parametric VAR structure. Overall, our main conclusion is that the effects of unconventional monetary policy shocks are very similar to those of conventional monetary policy when the financial markets interpret the monetary policy easing as a decrease in interest rates in the medium to long run. However, their overall effects in terms of magnitude differ across episodes.

### 5.3 Which Features of Monetary Policy Shocks Matter The Most To Explain Macroeconomic Fluctuations?

How much of the response of output and inflation to monetary policy shocks are associated with changes in specific features of the shape of the term structure of interest rates? Or, in other words, what are the effects of the various dimensions of monetary policy on output and inflation over time? Figures 14-15 report such a decomposition for the conventional period while Figures 16-17 do the same for the unconventional period.

By comparing Figures 10 and 14, it is clear that, in the conventional period, the response of output is mainly explained by changes in the slope  $(\Delta \tilde{\beta}_{2,t})$  – that is, how monetary policy affects long-term versus short-term expectations. By comparing Figures 11 and 15, instead, it becomes apparent that the response of inflation is instead explained mostly by the curvature  $(\Delta \tilde{\beta}_{3,t})$  – that is, how monetary policy affects medium-term expectations.

### INSERT FIGURE 14 AND 15 HERE

Turning to the unconventional period, a comparison of Figures 12 and 16 similarly reveals that the way monetary policy affects future output is mainly explained by the effect that the monetary policy shock has on the slope. There are a few exceptions, however, where the curvature becomes an important factor, such as during 2012. In the latter, the information regarding the medium-term contained in the monetary policy is the one having real effects on future output. However, a close comparison between Figures 13 and 17 reveals that the way monetary policy affects future inflation in the unconventional period is rather different than in the conventional period. In several cases, the behavior of the inflation response is explained by the slope rather than the curvature; the curvature seems to matter only around 2012.

While the level factor is typically related to expected inflation and the slope is typically related to expected real activity, the curvature factor has so far eluded an economic interpretation in the literature. Our results suggest an interesting interpretation of the elusive curvature factor in several monetary policy episodes: the curvature is correlated with the unanticipated effects of monetary policy; in particular, with how inflation responds to unexpected changes in monetary policy in the conventional period. Thus, forward guidance on inflation is captured by the curvature factor in the conventional period and by the slope factor in the unconventional period.

### INSERT FIGURE 16 AND 17 HERE

### 6 The Longer-Term Effects of Monetary Policy

The data we used so far are suitable to study the effects of monetary policy up to 10 years. But what are the longer-term effects of monetary policy? To answer this question, we consider longer-term zero-coupon bond yields from the Gürkaynak, Sack and Wright (2007) dataset. The dataset has the advantage of fitting a long time series of zero-coupon yields at very long maturities, up to 30 years. Gürkaynak, Sack and Wright (2006, p. 14) note that the traditional Nelson and Siegel model finds it challenging to fit the term structure if it includes maturities of or above twenty years. As they note, the reason is that the convexity shape of the curve, while fitting well short-term maturities, asymptotes too quickly in the long run and is unable to capture additional convexities in long-term maturities. Gürkaynak, Sack and Wright (2007) fit an extension of the Nelson and Siegel model due to Svensson (1994), which allows for two humps to fit both short- and long-term convexity effects. In their generalization, the yield curve at any point in time is summarized by four time-varying factors ( $\beta_{1,t}$ ,  $\beta_{2t}$ ,  $\beta_{3t}$  and  $\beta_{4t}$ ) describing the level, slope and two curvature factors, one to fit short-term maturities and one to fit long-term ones. The model for the yield curve is the following:

$$y_{t}(\tau) = \beta_{1,t} + \beta_{2,t} \frac{1 - e^{-\lambda_{1,t}\tau}}{\lambda_{1,t}\tau} + \beta_{3,t} \left(\frac{1 - e^{-\lambda_{1,t}\tau}}{\lambda_{1,t}\tau} - e^{-\lambda_{1,t}\tau}\right) + \beta_{4,t} \left(\frac{1 - e^{-\lambda_{2,t}\tau}}{\lambda_{2,t}\tau} - e^{-\lambda_{2,t}\tau}\right)$$
$$\equiv y_{t}^{(1)} + y_{t}^{(2)} + y_{t}^{(3)} + y_{t}^{(4)}, \tag{28}$$

where  $y_t(\tau)$  is the yield to maturity,  $\tau$  is the maturity (expressed in years in this section) and  $\lambda_{1,t}, \lambda_{2,t}$  are tuning parameters. Note that in the original Nelson and Siegel's (1987) specification, the shock depends on t only via changes in the factors (represented by  $\Delta\beta's$ ); thus, the change over time of the yield curve can be summarized by a linear combination of changes in the factors and constant maturity-specific coefficients. In contrast, in the Gürkaynak, Sack and Wright's (2007) and Svensson's (1994) specification, also  $\lambda's$  depend on time and, therefore, the coefficients of the linear combination of the factors depend on time and maturity. Thus, the shock is a non-linear function of both time and maturity.

Figure 18 plots the monetary policy shocks as a function of maturity (in years) for the same selected episodes that we considered earlier in the paper in conventional and unconventional times (Panel A and B, respectively). The figure shows that the results are broadly similar, except for a small number of cases, and confirm the existence of shocks with a wide variety of shapes in conventional times, and more pronounced medium- and longer-term effects in unconventional times.

### **INSERT FIGURE 18 HERE**

Figure 19 plots the correlation between our identified shock and traditional monetary policy shocks, proxied by either the Romer and Romer (2004) or the Wu and Xia (2014) shock. The correlations are estimated from eq. (19). The pattern again points to a high and positive correlation mostly at short-term maturities in the conventional period, while the correlation becomes the largest at medium- and long-term maturities in the unconventional period. Note the difference between the narrative shock a' la Romer and Romer (2004) and the shock based on Wu and Xia's (2014) shadow rate: in the former, the correlation peaks around ten years while in the latter it peaks at the longest maturities (around thirty years). Thus, measuring unconventional monetary policy shocks using the shadow rate gives more prominence to very long-term changes in expectations due to unexpected monetary policy, while narrative measures capture medium-term changes.

### **INSERT FIGURE 19 HERE**

We depict the effects of monetary policy shocks on macroeconomic variables in Figures 20-23. For simplicity, and in parallel with the analysis in the previous section, we proxy the shock by changes in  $\beta_{2,t}$ ,  $\beta_{3,t}$  and  $\beta_{4,t}$ . Again, in most cases, a monetary policy tightening results in decreases in output and inflation, as expected by economic theory.

### **INSERT FIGURES 20-23 HERE**

Another important aspect of our methodology is that it is possible to easily include other variables in the monetary policy shock definition. We explore including other asset prices from the analysis. In particular, one could include private-sector borrowing rates, such as mortgage rates (see Krishnamurthy and Vissing-Jorgensen, 2011), as that could represent an additional dimension of monetary policy. In particular, mortgage rates are available for maturities of 15 or 30 years. Figures 24 and 25 plot the estimated effects of unconventional monetary policy on output when monetary policy includes the 15- and the 30-year mortgage rates, respectively.<sup>27</sup> That is,  $\varepsilon_t^f(\tau) \equiv \Delta y_t(\tau) \cdot d_t$ , where  $y_t(\tau)$  includes not only eq. (17) but also the mortgate rate.

The figures show that the results are qualitatively similar to those we estimated in Section 5.2.

#### **INSERT FIGURES 24-25 HERE**

## 7 Conclusions

This paper proposes a novel approach to identify economic shocks. We view shocks as exogenous shifts in a function – as opposed to changes in a variable. In our empirical analysis, in particular, we define monetary policy shocks as shifts in the whole term structure in a short window of time around monetary policy announcements – as opposed to exogenous changes in just short-term interest rates. This allows us to capture more broadly the effects

 $<sup>^{27}</sup>$ We do not report results for inflation for brevity, although they are similar to what we previously found in Section 5.2.

that monetary policy has, including the information that it transmits to financial markets regarding the medium and long run path of interest rates. In addition, by being more comprehensive, our identification procedure allows us to estimate unconventional monetary policy shocks in a way similar to that in the conventional monetary policy period.

We find that, like conventional monetary policy shocks, unconventional ones have expansionary effects: they typically lead to an increase in output and inflation, peaking about one year to one year and a half after the initial shock. The effects of monetary policy during the zero lower bound are, therefore, very similar to those in normal periods – just the instrument of monetary policy is different. However, it is interesting to note that monetary policy cannot be described just by just shifts in short-term interest rates. Monetary policy has other dimensions as well, which we show are statistically significant in particular episodes.

More generally, our "functional shocks" approach is amenable to being used more widely: it can be applied to many other contexts where the shock is a shift in a function, such as demand, supply, fiscal policy or productivity shocks, which we are currently investigating.

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## Appendix A

### A.1 Technical Definitions

Yield curves can be viewed as functions that map  $\Re_+$  to  $\Re$ , which we will denote by  $y_t(\cdot)$ . Define a space of such yield curves by  $\mathcal{B}$  with norm  $\|\cdot\|$ . Also, let

$$f_t(y_t(\cdot)) \equiv E(z_{t+h}|y_t(\cdot), I_t)$$

where  $z_t$  is a variable of interest, such as inflation and output. To simplify the notation, we drop the subscript t from this point on.

The *h*-step-ahead impulse response of a variable is the "derivative" of its expected value with respect to a yield curve. Let  $y(\cdot) \in \mathcal{B}$  and  $y^*(\cdot) \in \mathcal{B}$ . If

$$\partial f(y(\cdot); y^*(\cdot)) = \lim_{\alpha \to 0} \frac{f(y(\cdot) + \alpha y^*(\cdot)) - f(y(\cdot))}{\alpha}$$
(29)

exists, it is called the *Gateaux* differential of f at  $y(\cdot)$  with direction (or increment)  $y^*(\cdot)$ . If the limit exists for each  $y^*(\cdot) \in \mathcal{B}$ , it is said to be *Gateaux* differentiable.

If there exists  $\partial f(y(\cdot); y^*(\cdot))$  which is linear and continuous with respect to  $y^*(\cdot)$  for each  $y(\cdot) \in \mathcal{B}$  and each  $y^*(\cdot) \in \mathcal{B}$  such that

$$\lim_{\|y^*(\cdot)\|\to 0} \frac{\|f(y(\cdot) + y^*(\cdot)) - f(y(\cdot)) - \partial f(y(\cdot); y^*(\cdot))\|}{\|y^*(\cdot)\|} = 0,$$
(30)

then f is said to be *Fréchet* differentiable at  $y(\cdot)$ , and  $\partial f(y(\cdot); y^*(\cdot))$  is said to be the *Fréchet* differential of f at  $y(\cdot)$  with increment  $y^*(\cdot)$ .

**A.2 Finite-dimensional representation.** Suppose that  $g_1(\cdot)$ , ...,  $g_q(\cdot)$  are known functions that map the set of maturities, T, to  $\Re$ , where q is a known positive integer. Define a class of functions of the form:<sup>28</sup>

$$\{f: f(\tau) = \sum_{j=1}^{q} c_j g_j(\tau), \text{ for some } c_1, c_2, ..., c_q\}.$$
(31)

For example, q = 3,  $g_1(\tau) = 1$ ,  $g_2(\tau) = (1 - e^{-\lambda\tau})/(\lambda\tau)$  and  $g_3(\tau) = (1 - e^{-\lambda\tau})/(\lambda\tau) - e^{-\lambda\tau}$ in the Nelson and Siegel (1987) model, where, for simplicity, we ignore the dependence of the function g(.) on nuisance parameters. It should be noted that the linear specification is not necessary for local projections, however.

Let  $d_f(\cdot)$ ,  $f_t(\cdot)$ ,  $\Phi_{21,1}(\cdot),...,\Phi_{21,p}(\cdot)$  and  $u_{f,t}(\cdot)$  belong to the class of functions described in eq. (31) and let  $\beta_{1,t},...,\beta_{q,t}$ ,  $\phi_{i,1},...,\phi_{i,q}$ , and  $\widetilde{u}_{1,t},...,\widetilde{u}_{q,t}$  denote the constants  $c_1,...,c_q$  of  $f_t(\cdot)$ ,  $\Phi_{21,i}(\cdot)$  and  $u_{f,t}(\cdot)$ , respectively.<sup>29</sup> Consider a *p*th-order VAR model

$$\Phi_{11}(L)X_t + \Phi_{12}(L) \int_{T} w(\tau) f_t(\tau) d\tau = d_X + u_{X,t}, \qquad (32)$$

$$\Phi_{21}(L,\cdot)X_t + \Phi_{22}(L)f_t(\cdot) = d_f(.) + u_{f,t}(\cdot), \qquad (33)$$

<sup>&</sup>lt;sup>28</sup>The class of functions define functions to be a linear combination of q basis functions. A function at time t is an element of this set and so is a function at time t.

<sup>&</sup>lt;sup>29</sup>Note that  $\beta_{1,t}, ..., \beta_{q,t}$  and  $\widetilde{u}_{1,t}, ..., \widetilde{u}_{q,t}$  are scalars while  $\phi_{i,1}, ..., \phi_{i,q}$  are column vectors.

where  $X_t$  is an  $(n \times 1)$  vector of variables (for simplicity n = 1 in the discussion in the main paper),  $\Phi_{11}(L) = \Phi_{11,0} - \Phi_{11,1}L - \dots - \Phi_{11,p}L^p$ ,  $\Phi_{12}(L) = -\Phi_{12,1}L - \dots - \Phi_{12,p}L^p$ ,  $\Phi_{21}(L; \cdot) = -\Phi_{21,1}(\cdot)L - \dots - \Phi_{21,p}(\cdot)L^p$ ,  $\Phi_{22}(L) = \Phi_{22,0} - \Phi_{22,1}L - \dots - \Phi_{22,p}L^p$ ,  $\Phi_{22,0}$  and  $\Phi_{11,0}$  are the identity matrix, and  $w : T \to \Re$  is some weight function such that  $I_j = \int_T w(\tau) g_j(\tau) d\tau$ exists for j = 1, 2, ..., q.

Then, omitting the intercept terms  $d_X$  and  $d_f(.)$  for notational simplicity, eqs. (32-33) can be written as:

$$\Phi_{11}(L)X_t + \Phi_{12}(L)\sum_{j=1}^q \beta_{j,t}I_j = u_{X,t}, \qquad (34)$$

$$-\sum_{i=1}^{p}\sum_{j=1}^{q}\phi_{i,j}'g_{j}(.)X_{t-i} + \sum_{j=1}^{q}\beta_{j,t}g_{j}(.) - \sum_{i=1}^{p}\Phi_{22,i}\sum_{j=1}^{q}\beta_{j,t-i}g_{j}(.) = \sum_{j=1}^{q}g_{j}(.)\widetilde{u}_{j,t}.$$
 (35)

Because the last equation must hold at each  $\tau \in T$ , it can be written as a finitedimensional VAR model:

$$\begin{bmatrix} X_{t} \\ \beta_{1,t} \\ \vdots \\ \beta_{q,t} \end{bmatrix} = \begin{bmatrix} \Phi_{11,1} & \Phi_{12,1}I_{1} & \Phi_{12,1}I_{2} & \cdots & \Phi_{12,1}I_{q} \\ \phi_{1,1}' & \Phi_{22,1} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \phi_{1,q}' & 0 & 0 & \cdots & \Phi_{22,1} \end{bmatrix} \begin{bmatrix} X_{t-1} \\ \beta_{1,t-1} \\ \vdots \\ \beta_{q,t-1} \end{bmatrix} + \cdots$$

$$+ \begin{bmatrix} \Phi_{11,p} & \Phi_{12,p}I_{1} & \Phi_{12,p}I_{2} & \cdots & \Phi_{12,p}I_{q} \\ \phi_{p,1}' & \Phi_{22,p} & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ \phi_{p,q}' & 0 & 0 & \cdots & \Phi_{22,p} \end{bmatrix} \begin{bmatrix} X_{t-p} \\ \beta_{1,t-p} \\ \vdots \\ \beta_{q,t-p} \end{bmatrix} + \begin{bmatrix} u_{X,t} \\ \widetilde{u}_{1,t} \\ \vdots \\ \widetilde{u}_{q,t} \end{bmatrix},$$
(36)

where the intercept terms are omitted for notational simplicity and  $I_i = \int w(\tau) g_i(\tau) d\tau$ .

A.3 Proof of Equations (10, 11, 12, 13). We consider the equation for the case of the Nelson and Siegel (1987) model. In the Nelson and Siegel (1987) model, q = 3 and  $g_1(\tau; \lambda) = 1$ , where

$$c_{2}(\tau;\lambda) = \tilde{c}_{1} + \tilde{c}_{2}g_{2}(\tau;\lambda) + \tilde{c}_{3}g_{3}(\tau;\lambda),$$
  

$$\phi_{21}(\tau;\lambda) = \tilde{\phi}_{1} + \tilde{\phi}_{2}g_{2}(\tau;\lambda) + \tilde{\phi}_{3}g_{3}(\tau;\lambda),$$
  

$$u_{f,t}(\tau;\lambda) = \tilde{u}_{1,t} + \tilde{u}_{2,t}g_{2}(\tau;\lambda) + \tilde{u}_{3,t}g_{3}(\tau;\lambda).$$
(37)

Repeated substitutions of (2) and (3) into themselves yield:

$$\begin{aligned} X_t &= c_1 + \phi_{1,1}(c_1 + \phi_{1,1}X_{t-2} + \phi_{1,2} \int w(\tau)f_{t-2}(\tau;\lambda)d\tau + u_{X,t-1}) \\ &+ \phi_{1,2} \int w(\tau)(c_2(\tau;\lambda) + \phi_{2,1}(\tau;\lambda)X_{t-2} + \phi_{2,2}f_{t-2}(\tau;\lambda) + u_{f,t-1}(\tau;\lambda))d\tau + u_{Xt} \\ &= (1 + \phi_{1,1})c_1 + \phi_{1,2} \int w(\tau)c_2(\tau;\lambda)d\tau + u_{X,t} + \phi_{1,1}u_{X,t-1} + \phi_{1,2} \int w(\tau)u_{f,t-1}(\tau;\lambda)d\tau \\ &+ (\phi_{1,1}^2 + \phi_{1,2} \int w(\tau)\phi_{2,1}(\tau;\lambda)d\tau)X_{t-2} + \phi_{1,2}(\phi_{1,1} + \phi_{2,2}) \int w(\tau)f_{t-2}(\tau;\lambda)d\tau \\ &= (1 + \phi_{1,1} + \phi_{1,2}^2 + \phi_{1,2} \int w(\tau)\phi_{2,1}(\tau;\lambda)d\tau)c_1 \\ &+ (1 + \phi_{1,1} + \phi_{2,2})\phi_{1,2} \int w(\tau)c_2(\tau;\lambda)d\tau \\ &+ u_{X,t} + \phi_{1,1}u_{X,t-1} + (\phi_{1,1}^2 + \phi_{1,2} \int w(\tau)\phi_{2,1}(\tau;\lambda)d\tau)u_{X,t-2} \\ &+ \underbrace{\phi_{1,2}}_{=\psi_{1,1}} \int w(\tau)u_{f,t-1}(\tau;\lambda)d\tau + \underbrace{\phi_{1,2}(\phi_{1,1} + \phi_{2,2})}_{=\psi_{1,2}} \int w(\tau)u_{f,t-2}(\tau;\lambda)d\tau + \cdots, \qquad (38) \end{aligned}$$

$$f_t(\cdot;\lambda) = c_2(\cdot;\lambda) + \phi_{2,1}(\cdot;\lambda)(c_1 + \phi_{1,1}X_{t-2} + \phi_{1,2} \int w(\tau)f_{t-2}(\tau;\lambda)d\tau + u_{X,t-1}) \\ &+ \phi_{2,2}(c_2(\cdot;\lambda) + \phi_{2,1}(\cdot;\lambda)X_{t-2} + \phi_{2,2}f_{t-2}(\cdot;\lambda) + u_{f,t-1}(\cdot;\lambda)) + u_{f,t}(\cdot;\lambda), \\ &= (1 + \phi_{2,2})c_2(\cdot;\lambda) + \phi_{2,1}(\cdot;\lambda)C_1 + \phi_{2,1}(\cdot;\lambda)u_{X,t-1} + u_{f,t}(\cdot;\lambda) + \phi_{2,2}u_{f,t-1}(\cdot;\lambda) \\ &+ (\phi_{1,1} + \phi_{2,2})\phi_{2,1}(\cdot;\lambda)X_{t-2} + \phi_{2,1}(\cdot;\lambda)\phi_{1,2} \int w(\tau)f_{t-2}(\tau;\lambda)d\tau + \phi_{2,2}^2f_{t-2}(\cdot;\lambda) \\ &+ (\psi_{1,1} + \phi_{2,2})\phi_{2,1}(\cdot;\lambda)X_{t-2} + \phi_{2,1}(\cdot;\lambda)\phi_{1,2} \int w(\tau)f_{t-2}(\tau;\lambda)d\tau + \phi_{2,2}^2f_{t-2}(\cdot;\lambda) \\ &+ \cdots . \end{aligned}$$

Then, using eqs. (37) and (38), the differential<sup>30</sup> of  $X_{t+h}$  in the direction

 $u_{f,t}^{*}(\tau;\lambda) = \tilde{u}_{1,t}^{*} + \tilde{u}_{2,t}^{*}g_{2}(\tau;\lambda) + \tilde{u}_{3,t}^{*}g_{3}(\tau;\lambda)$ 

 $\mathbf{is}$ 

$$\psi_{1,h} \int w(\tau) u_{f,t}^* d\tau = \psi_{1,h} (I_1 \tilde{u}_{1,t}^* + I_2 \tilde{u}_{2,t}^* + I_3 \tilde{u}_{3,t}^*).$$
(40)

Because of the linearity, the Frechet differential of  $X_{t+h}$  in the direction of  $u_{f,t}^*(\tau; \lambda)$  is also given by (40).

Because (3) holds for every  $\tau$ , this model can be written as a four-variable VAR model:

$$\begin{bmatrix} X_t \\ \beta_{1,t} \\ \beta_{2,t} \\ \beta_{3,t} \end{bmatrix} = \begin{bmatrix} \phi_{1,1} & \phi_{1,2}I_1 & \phi_{1,2}I_2 & \phi_{1,3}I_3 \\ \tilde{\phi}_1 & \phi_{2,2} & 0 & 0 \\ \tilde{\phi}_2 & 0 & \phi_{2,2} & 0 \\ \tilde{\phi}_3 & 0 & 0 & \phi_{2,2} \end{bmatrix} \begin{bmatrix} X_{t-1} \\ \beta_{1,t-1} \\ \beta_{2,t-1} \\ \beta_{3,t-1} \end{bmatrix} + \begin{bmatrix} u_{X,t} \\ \tilde{u}_{1,t} \\ \tilde{u}_{2,t} \\ \tilde{u}_{3,t} \end{bmatrix},$$
(41)

 $3^{0}$  As we discuss in the Not-for-Publication Appendix, the differential we define here is a Gateaux differential.

where the intercept terms are omitted for simplicity. Similarly, because (39) holds for each  $\tau$ , we have a vector moving average representation:

$$X_{t} = u_{X,t} + \underbrace{\phi_{1,1}}_{\equiv \theta_{1,1}} u_{X,t-1} + \underbrace{(\phi_{1,1}^{2} + \phi_{1,2} \int w(\tau)\phi_{2,1}(\tau)d\tau)}_{\equiv \theta_{1,2}} u_{X,t-2} + \underbrace{\phi_{1,2}}_{\equiv \psi_{1,1}} (I_{1}\tilde{u}_{1,t-1} + I_{2}\tilde{u}_{2,t-1} + I_{3}\tilde{u}_{3,t-1})$$

$$(12)$$

$$+\underbrace{\phi_{1,2}(\phi_{1,1}+\phi_{2,2})}_{\equiv\psi_{1,2}}(I_1\tilde{u}_{1,t-2}+I_2\tilde{u}_{2,t-2}+I_3\tilde{u}_{3,t-2})+\dots$$
(42)

$$\beta_{1t} = \tilde{\phi}_1 u_{X,t-1} + (\phi_{1,1} + \phi_{2,2}) \tilde{\phi}_1 u_{X,t-2} + \tilde{u}_{1t} + \phi_{22} \tilde{u}_{1,t-1} + (\phi_{1,2} \tilde{\phi}_1 I_1 + \phi_{22}^2) \beta_{1,t-2} + \cdots$$

$$(43)$$

$$\beta_{2t} = \phi_2 u_{X,t-1} + (\phi_{1,1} + \phi_{2,2})\phi_2 u_{X,t-2} + \tilde{u}_{2t} + \phi_{22}\tilde{u}_{2,t-1} + (\phi_{1,2}\phi_2 I_1 + \phi_{22}^2)\beta_{2,t-2} + \cdots$$

$$(44)$$

$$\beta_{3t} = \phi_3 u_{X,t-1} + (\phi_{1,1} + \phi_{2,2})\phi_3 u_{X,t-2} + \tilde{u}_{3t} + \phi_{22}\tilde{u}_{3,t-1} + (\phi_{1,2}\phi_3 I_1 + \phi_{22}^2)\beta_{3,t-2} + \cdots$$
(45)

i.e., using a more general notation:

 $\beta$ 

. . .

$$X_{t} = u_{X,t} + \theta_{1,1}u_{X,t-1} + \theta_{1,2}u_{X,t-2} + \psi_{1,1}(\sum_{j=1}^{q} I_{j}\tilde{u}_{j,t-1})$$

$$(46)$$

$$+\psi_{12}(\sum_{j=1}^{L} I_j u_{j,t-2}) + \dots$$

$$= \theta_{2,1} u_{X,t-1} + \theta_{2,2} u_{X,t-2} + \tilde{u}_{1,t} + \psi_{2,2} \tilde{u}_{1,t-1} + \cdots$$
(47)

$$\beta_{q,t} = \theta_{q+1,1} u_{X,t-1} + \theta_{q+1,2} u_{X,t-2} + \tilde{u}_{q,t} + \psi_{q+1,2} \tilde{u}_{q,t-1} + \cdots, \qquad (48)$$

where  $\theta_{1,1} = \phi_{1,1}, \ \theta_{1,2} = (\phi_{1,1}^2 + \phi_{1,2} \int w(\tau) \phi_{2,1}(\tau) d\tau), \ \theta_{2,1} = \tilde{\phi}_1, \ \theta_{2,2} = (\phi_{1,1} + \phi_{2,2}) \tilde{\phi}_1, \ \psi_{1,1} = \phi_{1,2}, \ \psi_{1,2} = \phi_{1,2}(\phi_{1,1} + \phi_{2,2}), \ \text{etc.}$ 

Note that, if the data follow the VAR(p) model in equation (37), equation (38) provides a basis for local projections in equation (22). Omitting the intercept terms, and expressing the local projection in terms of the reduced form shocks, it follows from equation (38) that

$$\begin{aligned} X_{t+1} &= \Gamma_{1,1}\widetilde{u}_{1,t} + \Gamma_{2,1}\widetilde{u}_{2,t} + \dots \Gamma_{q,1}\widetilde{u}_{q,t} + A_1(L)[X'_{t-1}\ \widetilde{u}'_{1,t-1}\ \widetilde{u}'_{2,t-1}\ \dots \widetilde{u}'_{q,t-1}]' + e_{t+1}, \\ X_{t+2} &= \Gamma_{1,2}\widetilde{u}_{1,t} + \Gamma_{2,2}\widetilde{u}_{2,t} + \dots \Gamma_{q,2}\widetilde{u}_{q,t} + A_2(L)[X'_{t-1}\ \widetilde{u}'_{1,t-1}\ \widetilde{u}'_{2,t-1}\ \dots \widetilde{u}'_{q,t-1}]' + e_{t+2}, \\ \vdots & \vdots \end{aligned}$$

where A(L) is a lag polynomial,  $e_{t+h}$  is an error term,  $h = 1, 2, ..., \Gamma_{1,1} = \psi_{1,1}I_1, \Gamma_{2,1} = \psi_{1,1}I_2,$  $\Gamma_{3,1} = \psi_{1,1}I_3, \Gamma_{1,2} = \psi_{1,2}I_1, \Gamma_{2,2} = \psi_{1,2}I_2, \Gamma_{3,2} = \psi_{1,2}I_3$ , etc. The local projections are valid even if the data do not follow a VAR process, however. Also, a similar reasoning holds for local projections expressed in terms of the structural shocks.

## Appendix B

### **Data Description**

We collect data from January 1995 to June 2016 on the term structure of yields, industrial production and inflation. We start the sample in 1995 as the Fed did not release statements of monetary policy decision after its FOMC meetings before 1994. Also, importantly, Gürkaynak et al. (2005a) show that, after 1995, daily data provide an accurate identification of monetary policy shocks, which provides another rationale for using daily yields from 1995 onward in our analysis. We end the sample at the end of the zero lower bound period.

### Term structure

The term structure data used in Sections 3-5 are daily zero-coupon yields (mnemonics "SVENY") from Gürkaynak, Sack and Wright (2007) and include yields at 1 to 30 years maturities. The daily frequency is dictated by the availability of data: the highest frequency at which the term structure of yields is available is daily. While one might be interested in investigating the identification at a higher frequency, Gürkaynak, Sack and Swanson (2007a) show that daily data are sufficient for extracting monetary policy shocks using a high-frequency identification if the sample is limited to post-1995 data, which is our case. The 3- and 6-month zero-coupon yields are from the Federal Reserve Board H-15 release.

### Inflation

Data on inflation is from the Federal Reserve Bank of St. Louis' FRED. Inflation is measured as the annual percentage change in the Consumer Price Index for All Urban Consumers – All Items; it is a monthly, seasonally adjusted time series. The mnemonics for the price definition we use is CPIAUCSL.

### Output

Data on industrial production are. Output is measured by the industrial production index also transformed in an annual percent change. The data is from the Federal Reserve Bank of St. Louis' FRED. This series is monthly and seasonally adjusted as well, and the mnemonics of industrial production is INDPRO.

## Tables

in Selected Conventional Episodes								
Date				Summary Statistics				
Month	Day	Year	$\Delta\beta_{1t}$	$\Delta\beta_{2t}$	$\Delta \boldsymbol{\beta}_{3t}$	$\Delta \left( \beta_{1t} + \beta_{2t} \right)$	$\Delta \left(\beta_{3t} - \beta_{1t}\right)$	
11	6	2001	-0.141	-0.092	0.153	-0.234	0.295	
9	29	1998	-0.196	0.036	0.125	-0.159	0.322	
2	3	1999	0.116	-0.195	0.222	-0.079	0.106	
5	16	2000	-0.060	-0.157	0.574	-0.217	0.634	
1	31	2007	-0.051	0.047	0.025	-0.004	0.076	
1	28	2004	0.041	-0.074	0.547	-0.033	0.506	

### Table 1, Panel A. Monetary Policy Shocks in Selected Conventional Episodes

Table 1, Panel B. Monetary Policy Shocks
in Selected Unconventional Episodes

Date				Summary Statistics				
Month	Day	Year	$\Delta \beta_{1t}$	$\Delta \beta_{2t}$	$\Delta \beta_{3t}$	$\Delta \left(\beta_{1t} + \beta_{2t}\right)$	$\Delta \left(\beta_{3t} - \beta_{1t}\right)$	
11	25	2008	-0.392	0.360	0.063	-0.032	0.455	
12	1	2008	-0.308	0.391	-0.133	0.083	0.174	
12	16	2008	-0.609	0.504	1.182	-0.104	1.792	
1	28	2009	0.347	-0.315	-0.131	0.032	-0.478	
3	18	2009	-0.673	0.662	0.399	-0.010	1.072	
8	10	2010	-0.219	0.218	0.246	-0.001	0.466	
9	21	2010	-0.249	0.233	0.309	-0.015	0.558	
11	3	2010	-0.217	0.235	0.168	0.017	0.386	
8	9	2011	-0.334	0.318	0.314	-0.015	0.648	
9	21	2011	-0.447	0.350	1.109	-0.096	1.556	
1	25	2012	-0.196	0.237	0.030	0.040	0.226	
6	20	2012	-0.023	0.014	0.143	-0.009	0.167	
9	13	2012	0.162	-0.134	-0.321	0.027	-0.483	

Note to the table. The table reports the estimated value of the shocks to the factors (or linear combinations thereof) at dates of selected monetary policy announcements.

Table 2	. Rank	Test
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Sample	Test Statistic	5% Critical Value	10% Critical Value	Sample Size
1995:1-2007:12	0.065	0.039	0.025	106
2008:1-2016:7	0.501	0.777	0.526	33

Notes to the table. The table reports the modified version of Robin and Smith's (2000) rank test proposed by Donald, Fortuna and Pipiras (2014) calculated over the full sample as well as in sub-samples: conventional (1995:1-2007:12) and unconventional (2008:1-2016:7).

## Figures

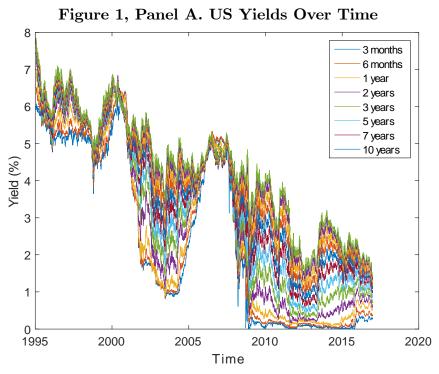
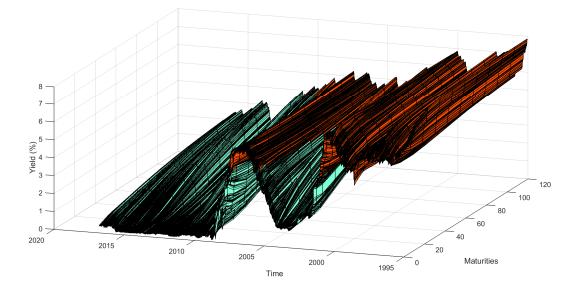


Figure 1, Panel B. The US Term Structure



Notes to the Figure. Panel A plots daily US Treasury yields over time; panel B plots the term structure of daily Treasury yields as a function of time and maturity.

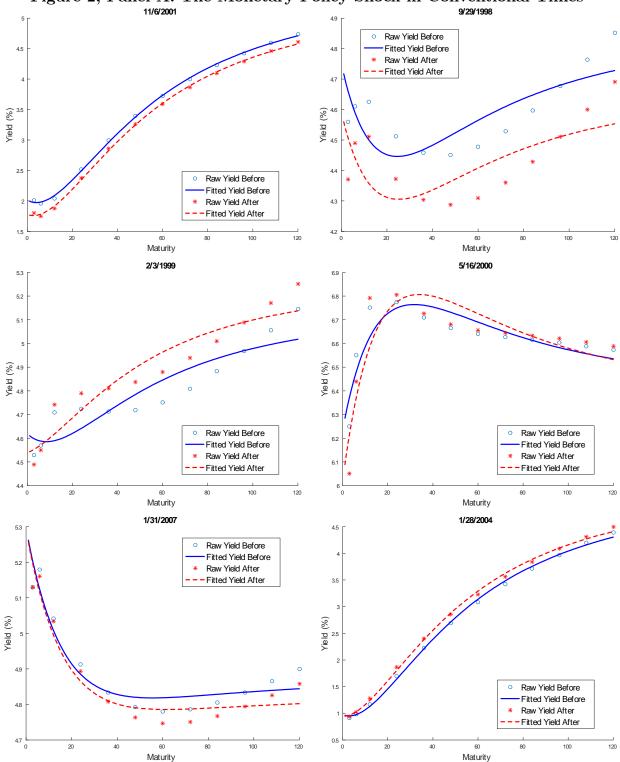


Figure 2, Panel A. The Monetary Policy Shock in Conventional Times

Notes. The figure depicts six representative examples of our newly defined monetary policy shock during the conventional monetary policy period. The date is reported in the title.

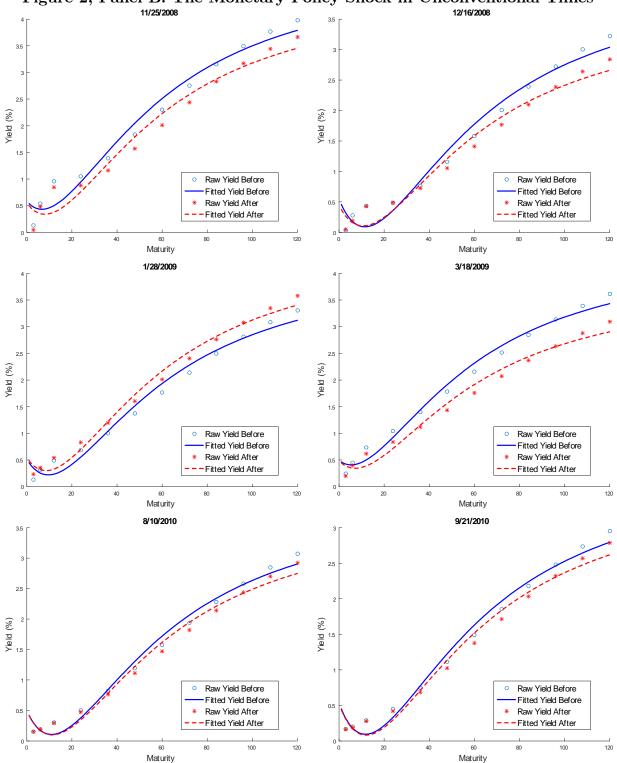
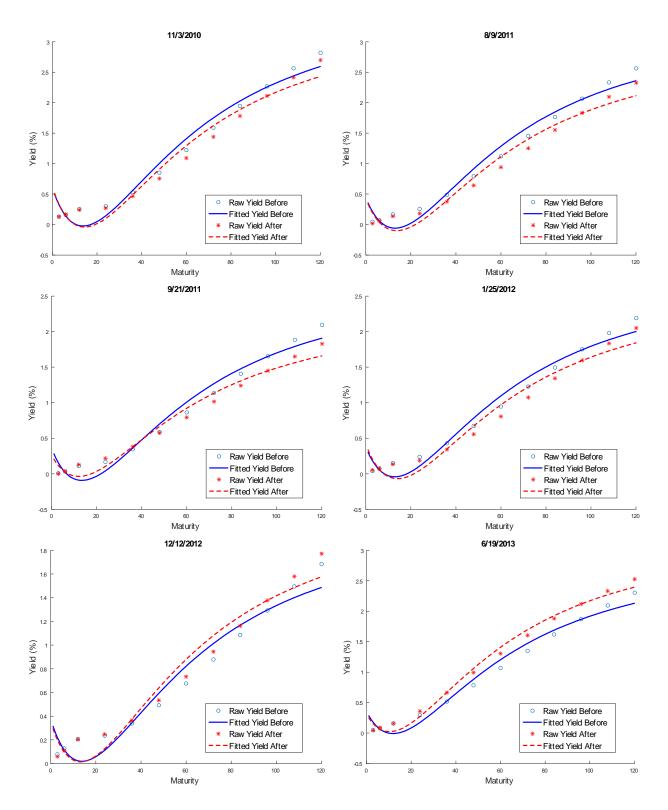


Figure 2, Panel B. The Monetary Policy Shock in Unconventional Times



Notes. The figure depicts our newly defined monetary policy shock during the unconventional monetary policy period. The date is reported in the title.

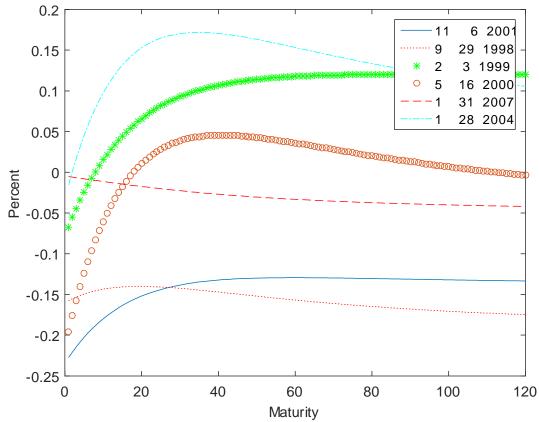


Figure 3, Panel A. The Monetary Policy Shock in Conventional Times

Notes to the figure. The figure depicts six representative examples of our newly defined monetary policy shock during the conventional monetary policy period. The shock date is reported in the legend.

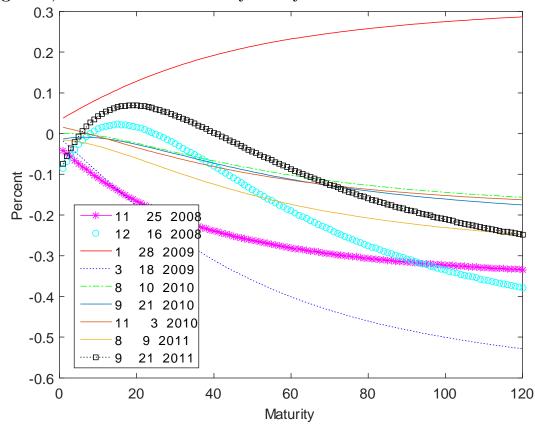
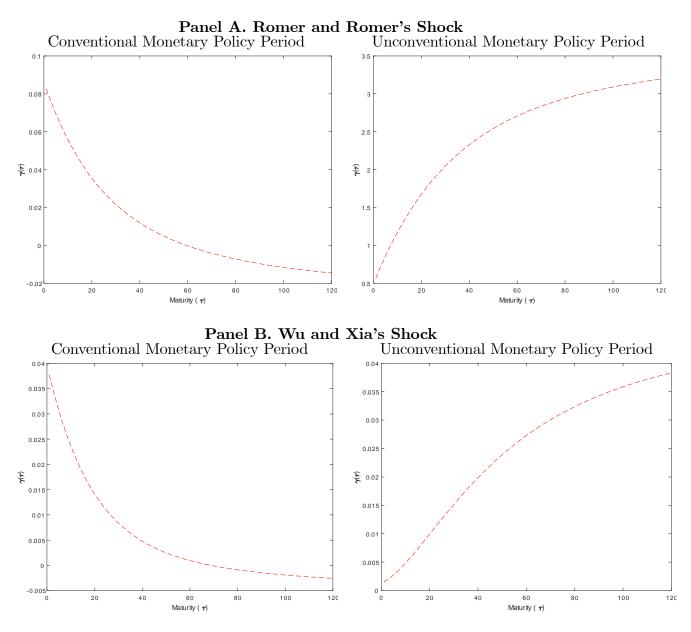


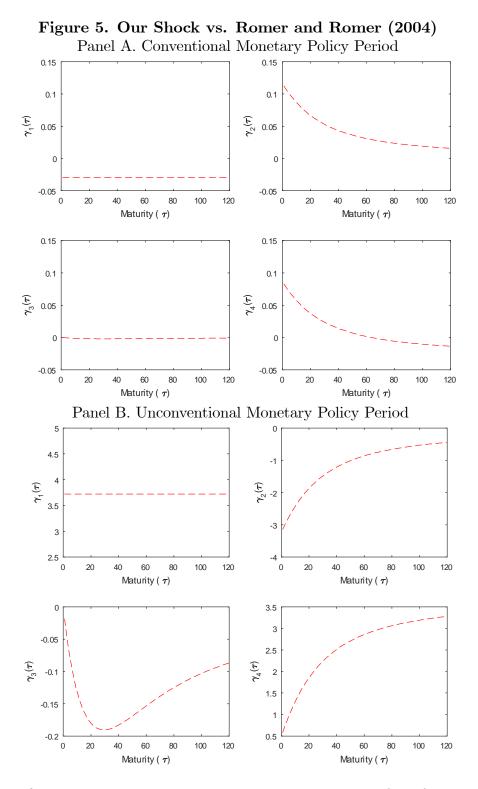
Figure 3, Panel B. The Monetary Policy Shock in Unconventional Times

Notes to the figure. The figure depicts representative examples of our newly defined monetary policy shock during the unconventional monetary policy period. The shock date is reported in the legend.

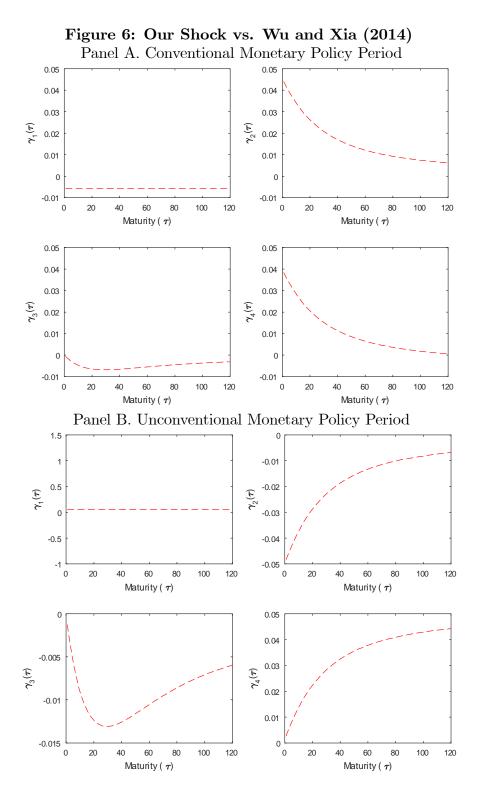
## Figure 4: Relationship Between Our Monetary Policy Shock and Traditional Monetary Policy Shocks



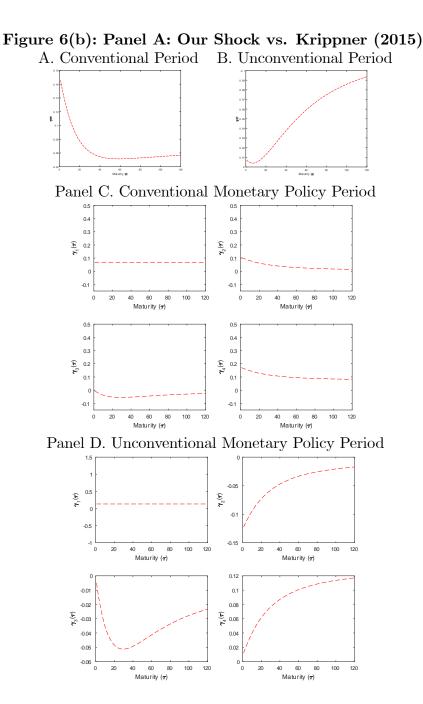
Notes. The figure depicts the correlation between our functional monetary policy shock,  $\varepsilon_t^f(\tau)$ , and a traditional (narrative) monetary policy shock: Romer and Romer (2004) in the top panels and Wu and Xia (2014) in the bottom panels.



Notes. The figure depicts the correlation between the components of our functional monetary policy shock in eq. (20) and Romer and Romer's (2004) traditional (narrative) monetary policy shock.



Notes. The figure depicts the correlation between the components of our functional monetary policy shock in eq. (20) and Wu and Xia's (2014) monetary policy shock.



Notes. Panels A-B in the figure depicts the correlation between our functional monetary policy shock,  $\varepsilon_t^f(\tau)$ , and Krippner (2015) in the bottom panels. Panels C-D depicts the correlation between the components of our functional monetary policy shock in eq. (20) and Krippner's (2015) monetary policy shock.

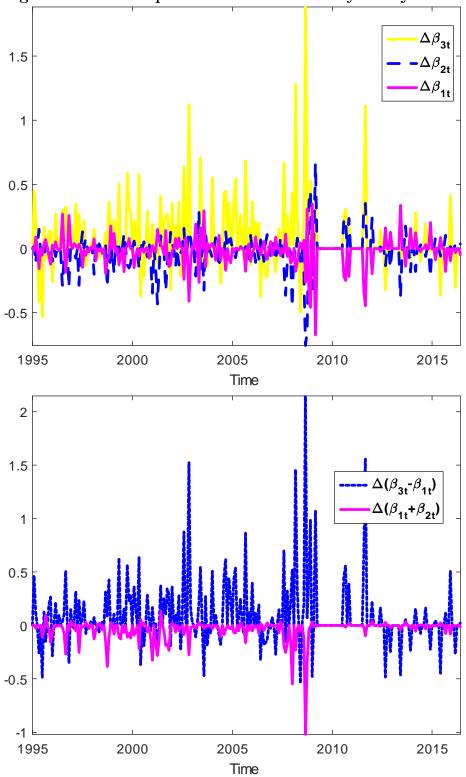


Figure 7. The Components of Our Monetary Policy Shock

Notes. The figure depicts the various factors in our functional monetary policy shock.

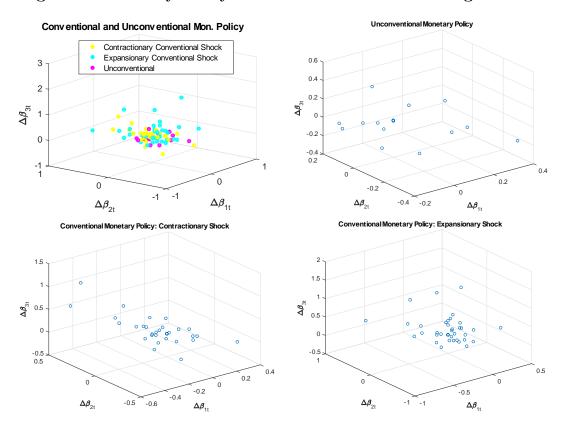
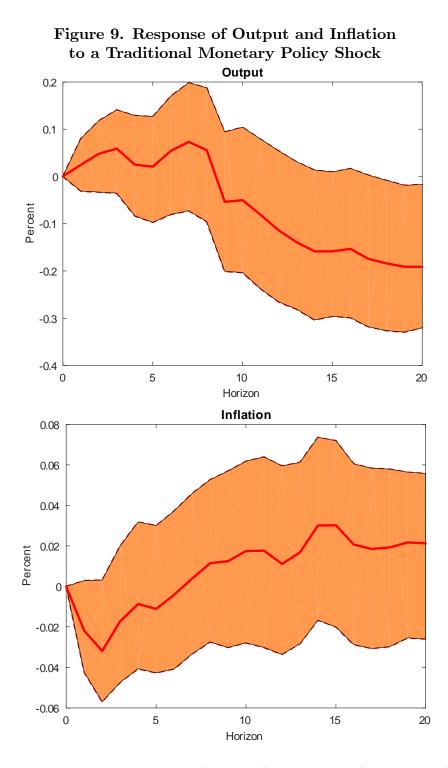
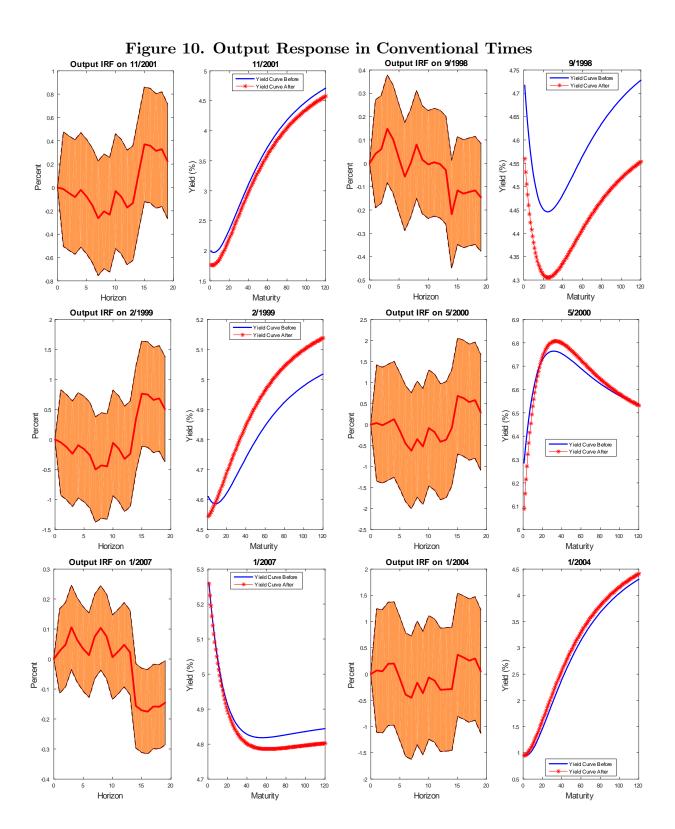


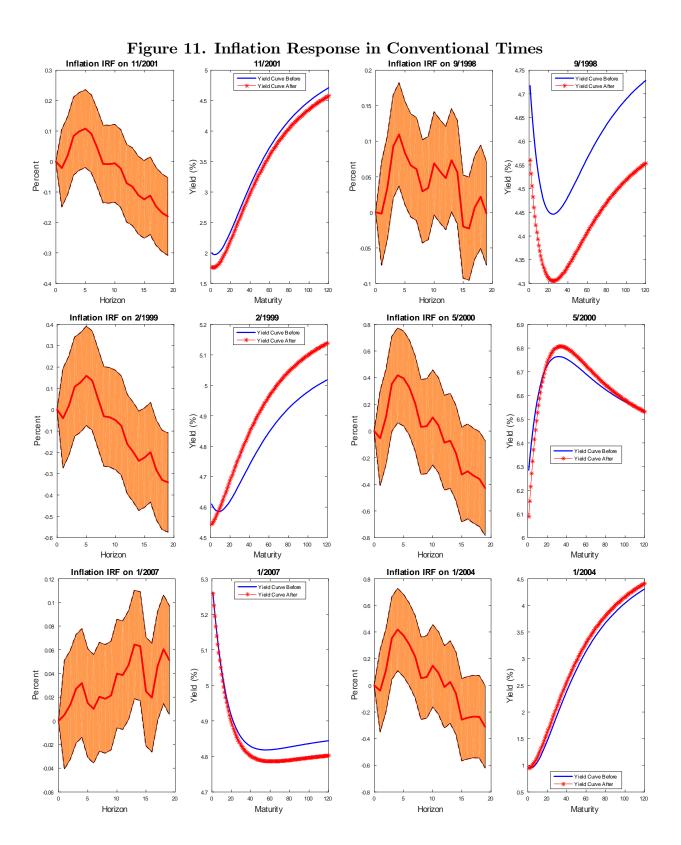
Figure 8. Monetary Policy Shocks in the Nelson and Siegel Model

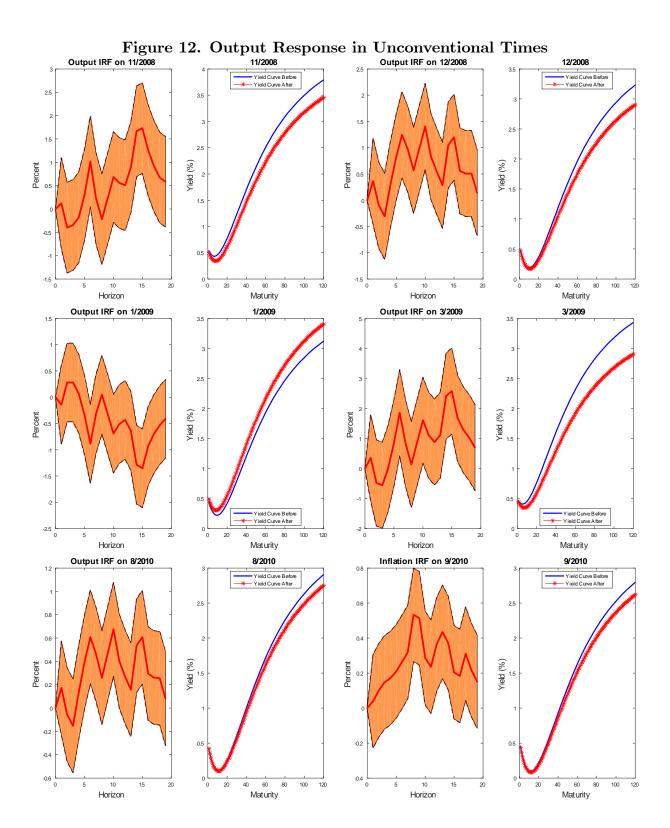
Notes to Figure 8. The scatter plots depict the monetary policy shocks as a function of the factors  $\beta_{1t},\,\beta_{2t}$  and  $\beta_{3t}.$ 

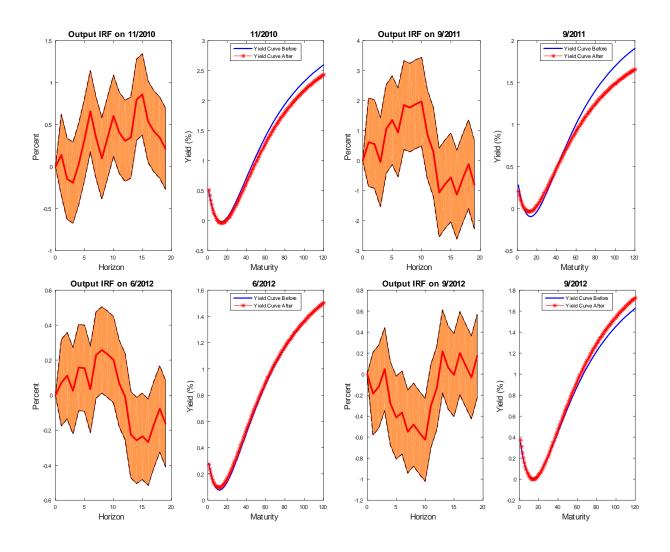


The figures depict the response of output (top panel) and inflation (bottom panel) to a one unit increase in the short-term interest rate, proxied by an increase in  $\beta_{2t}$  at the time of the monetary policy announcement  $(\Delta \beta_{2t} \cdot d_t)$ .

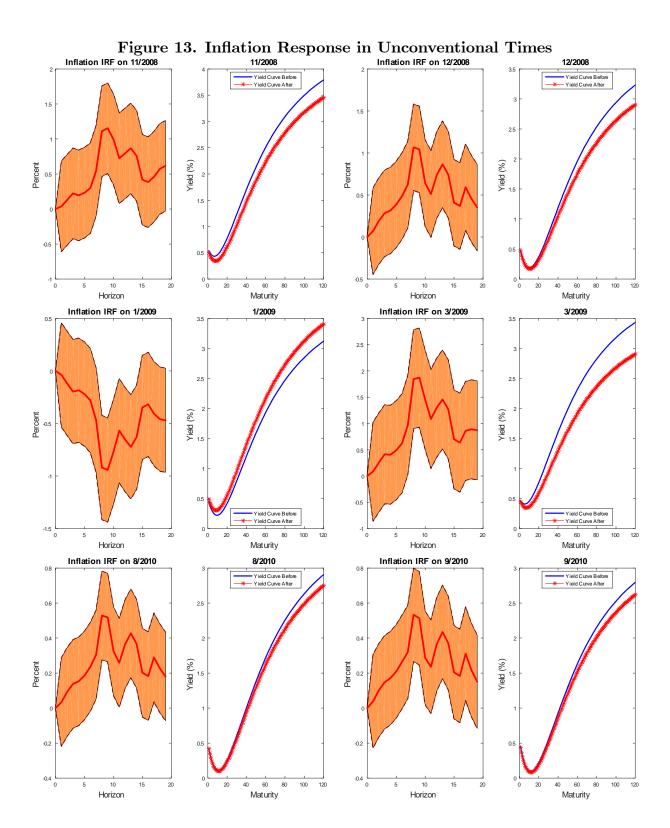


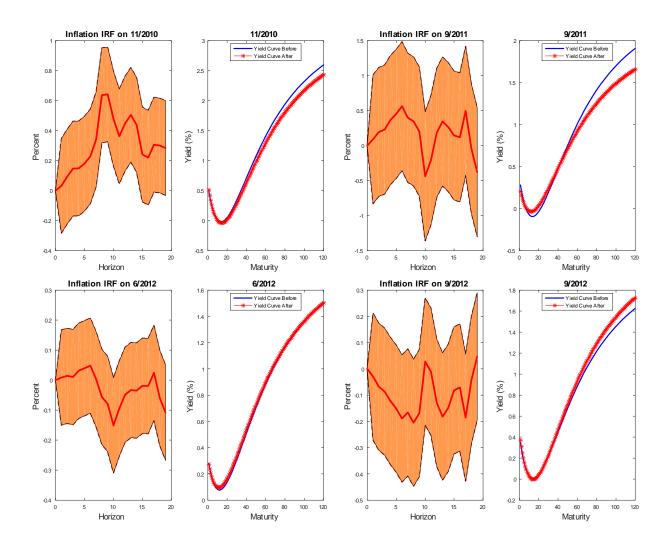




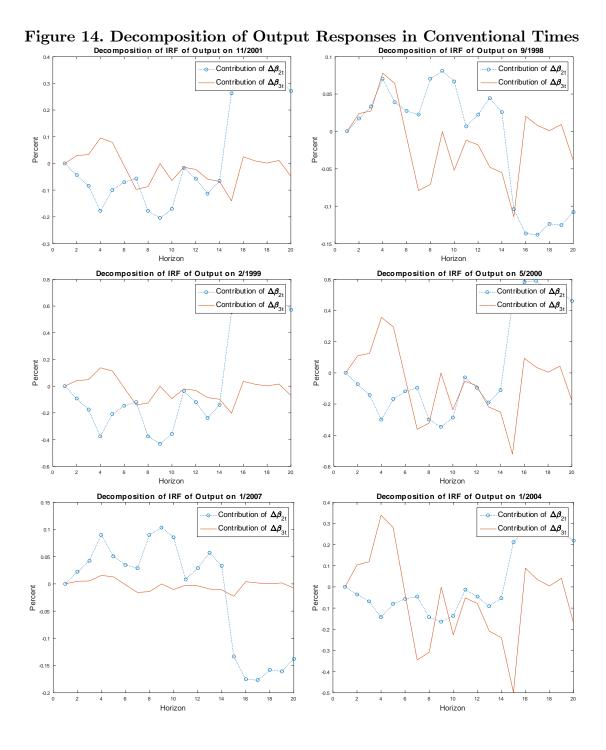


Notes to the figure. The figure plots impulse response functions of industrial production to the monetary policy shock together with 68% confidence bands.





Notes to Figures 10-13. The figures plot impulse response functions of output (Figures 10 and 12) and inflation (Figures 11 and 13) to the monetary policy shock together with 68% confidence bands.



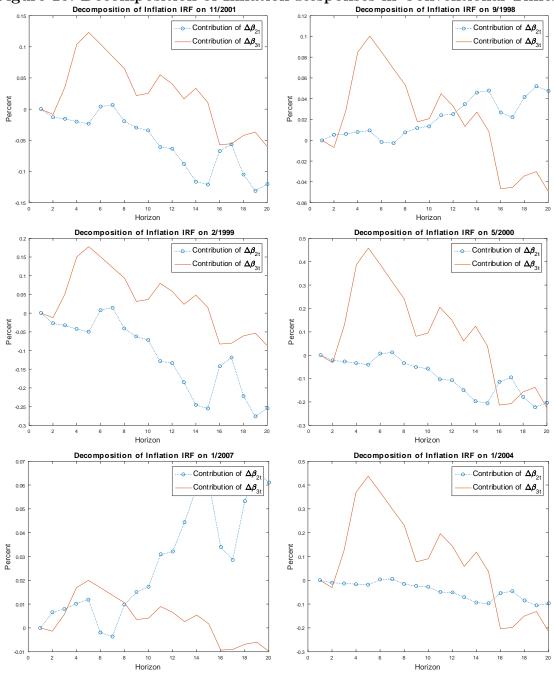


Figure 15. Decomposition of Inflation Responses in Conventional Times Decomposition of Inflation IRF on 11/2001 Decomposition of Inflation IRF on 9/1998

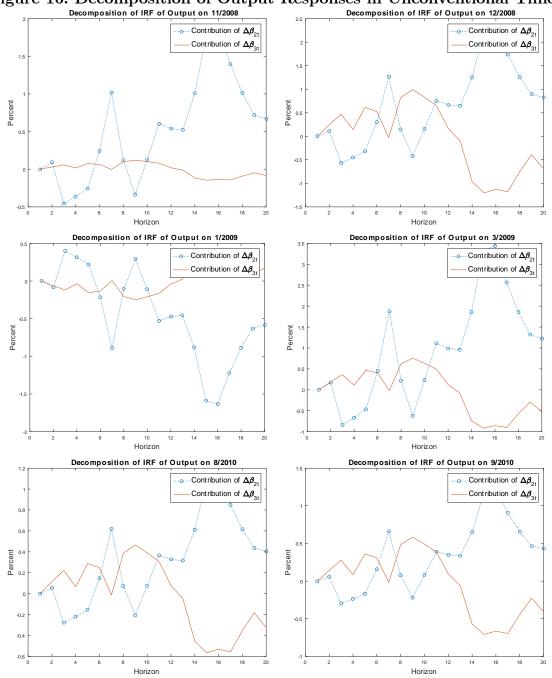
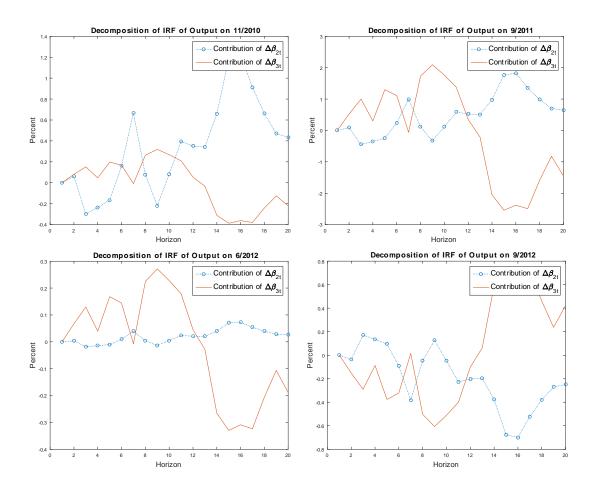


Figure 16. Decomposition of Output Responses in Unconventional Times



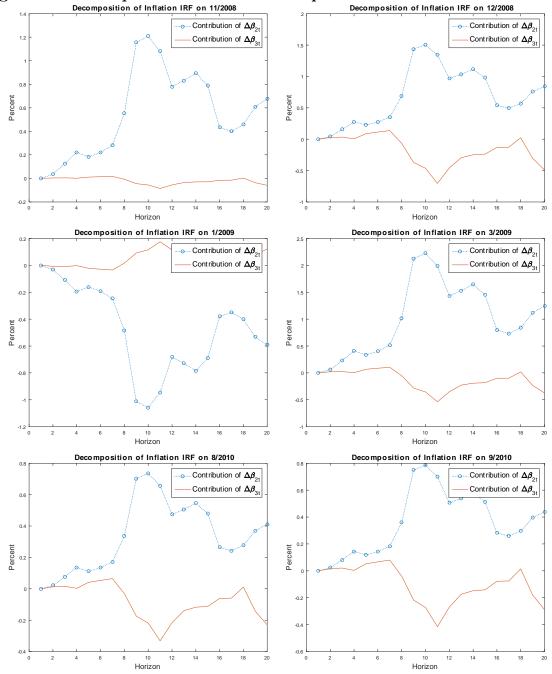
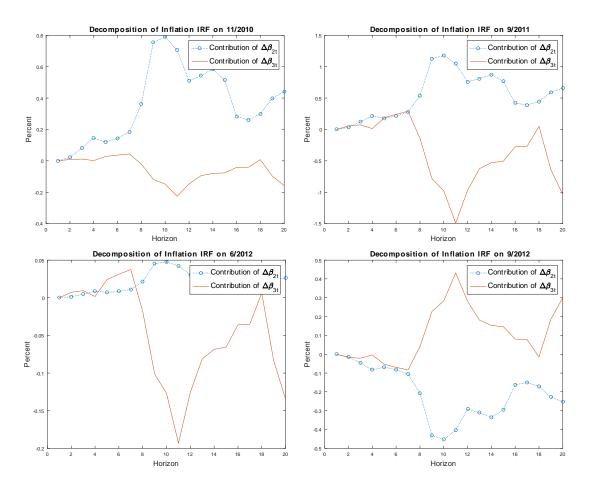


Figure 17. Decomposition of Inflation Responses in Unconventional Times



Note to Figures 14-17. The figures plots the decomposition of the responses of output and inflation in the parts related to shocks associated with curvature and slope of the term structure, respectively.

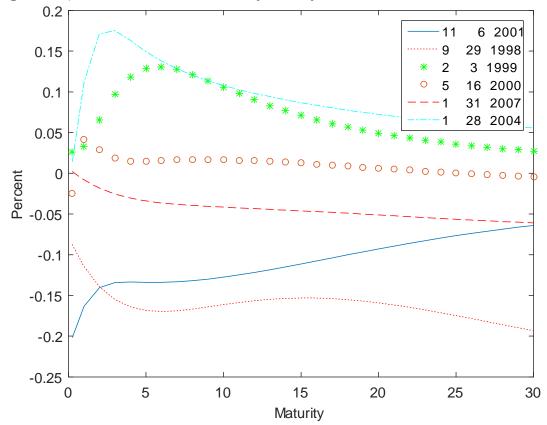


Figure 18, Panel A. The Monetary Policy Shock in Conventional Times

Notes to the figure. The figure depicts six representative examples of our newly defined monetary policy shock during the conventional monetary policy period. The shock date is reported in the legend.

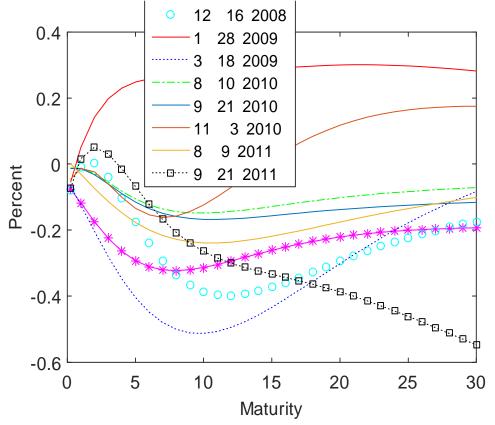
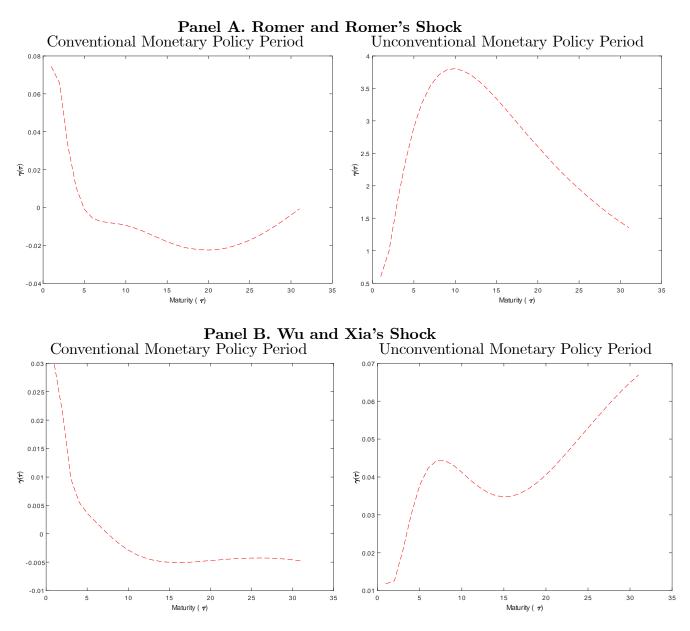


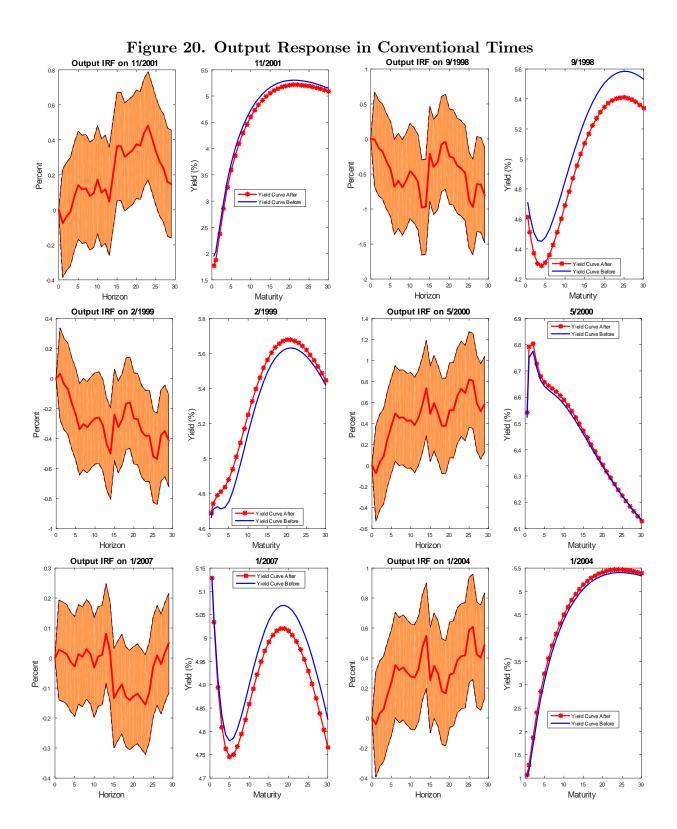
Figure 18, Panel B. The Monetary Policy Shock in Unconventional Times

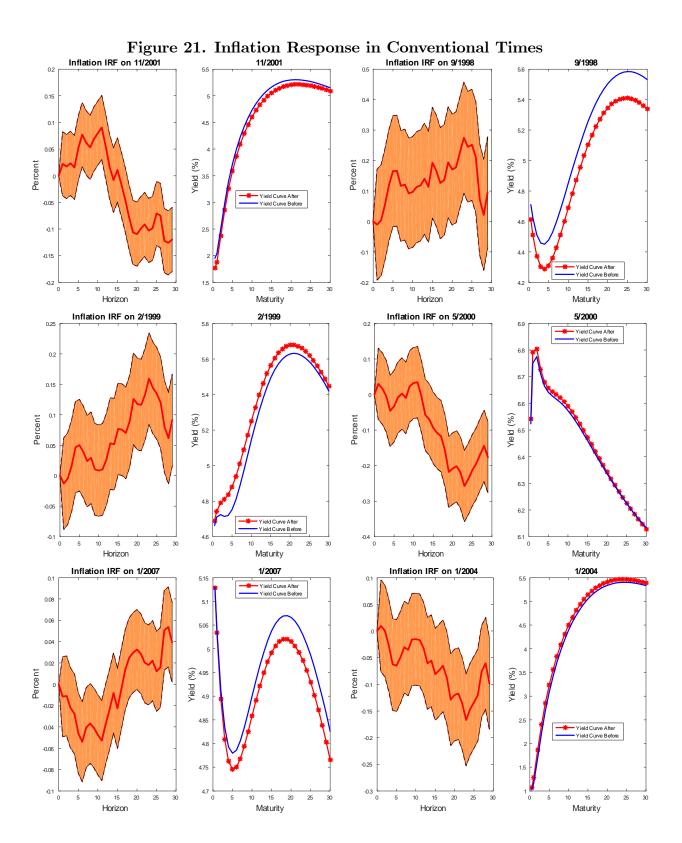
Notes to the figure. The figure depicts representative examples of our newly defined monetary policy shock during the unconventional monetary policy period. The shock date is reported in the legend.

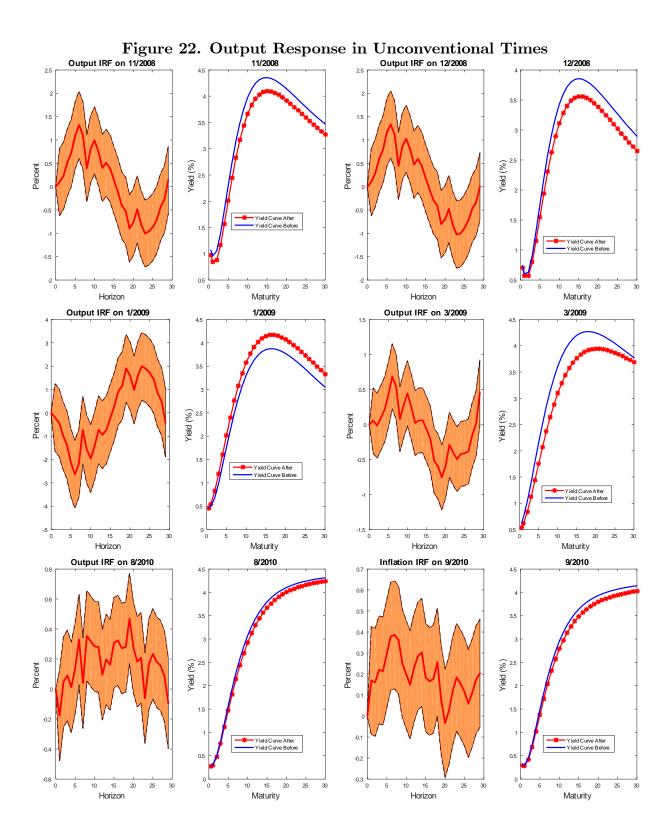
## Figure 19: Correlation Between Our Monetary Policy Shock and Traditional Monetary Policy Shocks

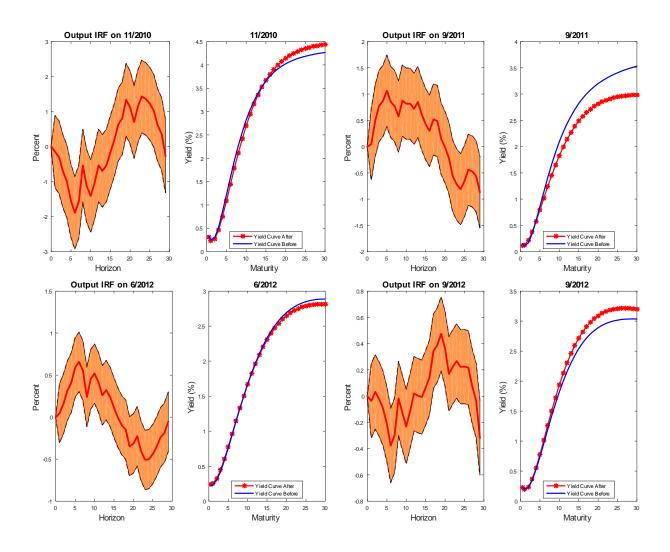


Notes. The figure depicts the correlation between our functional monetary policy shock,  $\varepsilon_t^f(\tau)$ , and a traditional (narrative) monetary policy shock: Romer and Romer (2004) in the top panels and Wu and Xia (2014) in the bottom panels.

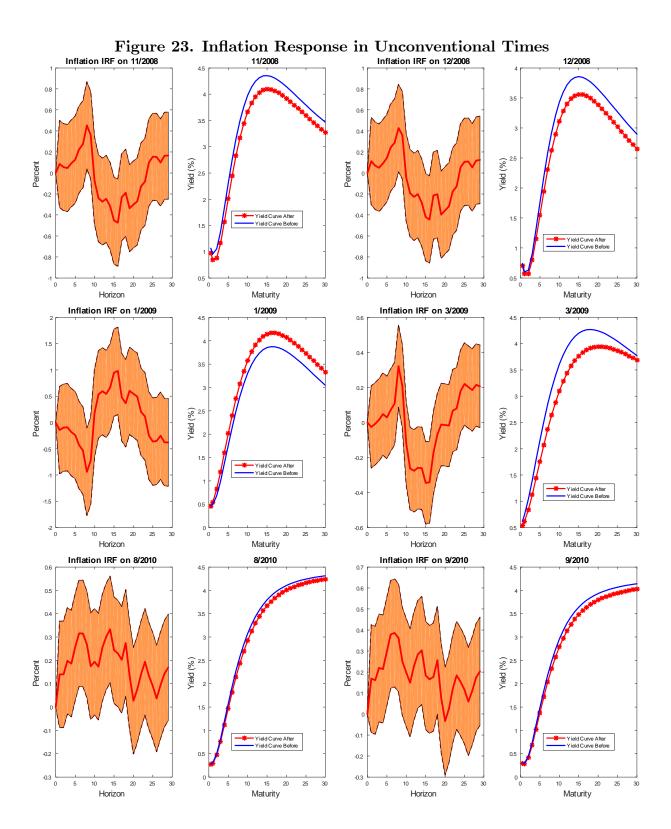


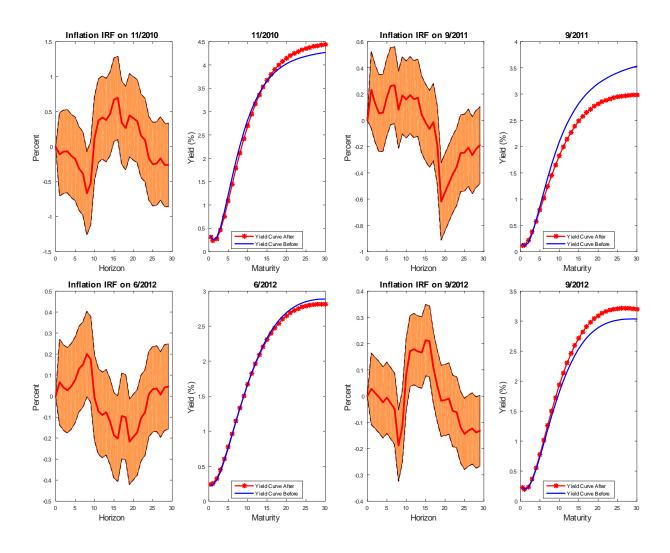






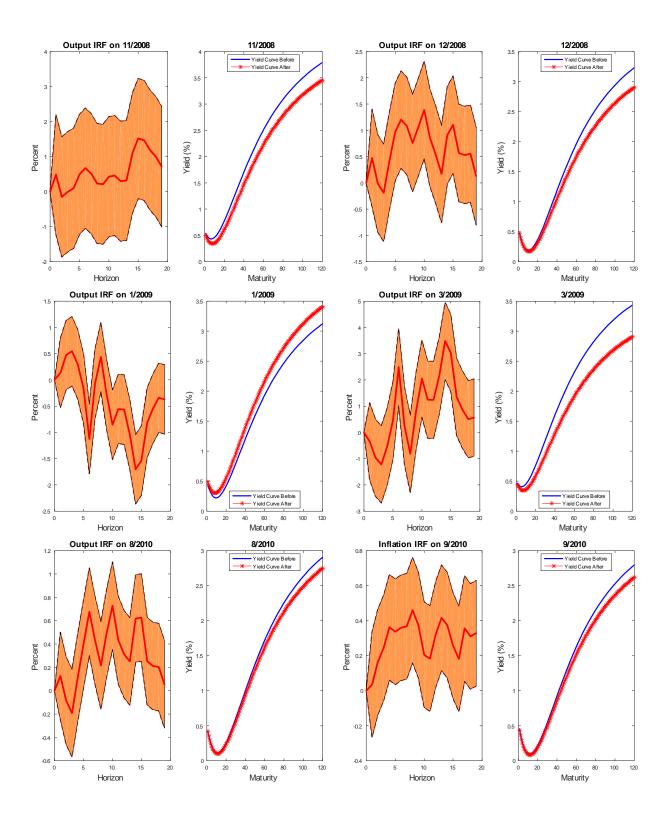
Notes to the figure. The figure plots impulse response functions of industrial production to the monetary policy shock together with 68% confidence bands.

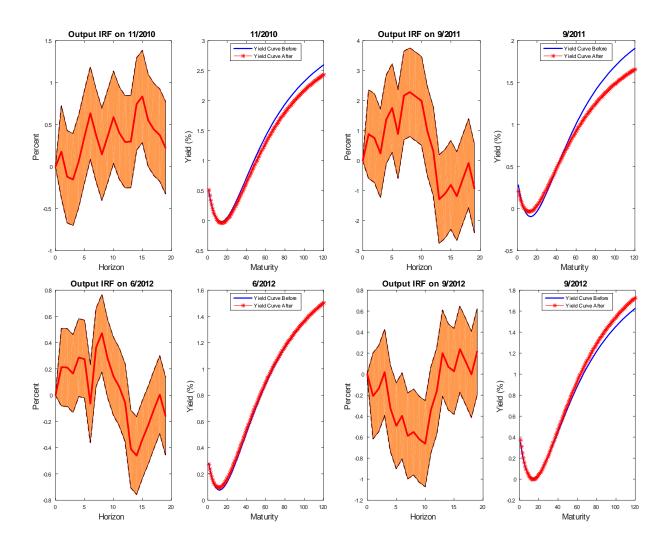




Notes to Figures 20-23. The figures plot impulse response functions of output (Figures 20 and 22) and inflation (Figures 21 and 23) to the monetary policy shock together with 68% confidence bands.

Figure 24. Output Response in Unconventional Times (VAR with 15-yrs. Mortgate Rates)





Notes to the figure. The figure plots impulse response functions of industrial production to the monetary policy shock together with 68% confidence bands.

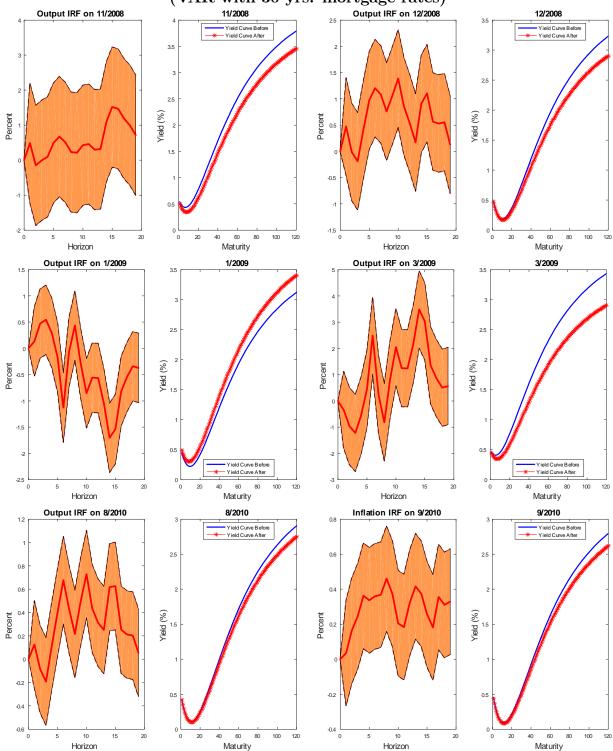
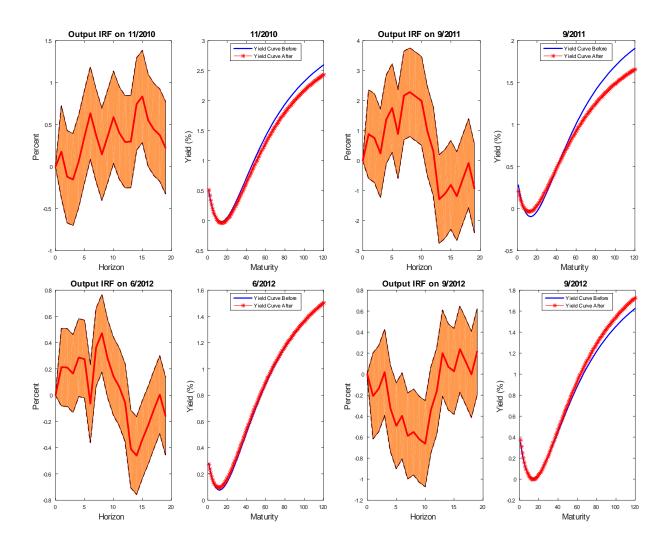


Figure 25. Output Response in Unconventional Times (VAR with 30 yrs. mortgage rates)



Notes. The figure plots impulse response functions of output to the monetary policy shock together with 68% confidence bands.