On the Empirical (Ir)Relevance of the Zero Lower Bound Constraint*

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March 2019

Abstract

The *zero lower bound (ZLB) irrelevance hypothesis* implies that the economy's performance is not affected by a binding ZLB constraint. We evaluate that hypothesis for the recent ZLB episode experienced by the U.S. economy (2009Q1-2015Q4). We focus on two dimensions of performance that were likely to have experienced the impact of a binding ZLB: (i) the volatility of macro variables and (ii) the economy's response to shocks. Using a variety of empirical methods, we find little evidence against the irrelevance hypothesis, with our estimates suggesting that the responses of output, inflation and the long-term interest rate were hardly affected by the binding ZLB constraint, possibly as a result of the adoption and fine-tuning of unconventional monetary policies. We can reconcile our empirical findings with the predictions of a simple New Keynesian model under the assumption of a shadow interest rate rule.

JEL Classification No.: E44, E52

Keywords: regime changes, liquidity trap, unconventional monetary policies, time-varying structural vector-autoregressive models.

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1 Introduction

The magnitude of the global financial crisis of 2007-2008 and the recession that it triggered, led many central banks to lower their policy rates down to values near zero, their theoretical lower bound.[1] Given the impossibility of further reductions in the short-term nominal rate –the instrument of monetary policy in normal times– central banks increasingly relied on unconventional monetary policies (UMPs) in their attempt to stimulate the economy. Two prominent examples of unconventional policies adopted by several central banks in recent years are (i) *forward guidance* (i.e. the attempt to manage expectations on the future path of the policy rate) and (ii) *quantitative easing* (i.e. central banks' large asset purchase programs). The adoption of such policies sought to stimulate economic activity through a variety of channels: lowering expectations of future policy rates, reducing the term and/or risk premia of longer term debt, increasing the overall liquidity of the financial system, and supporting asset valuation, among other channels.

How effective have UMPs been at getting around the zero lower bound (ZLB) constraint? A growing literature has emerged that aims at answering those questions, using a variety of empirical approaches. The present paper seeks to contribute to that literature. More specifically, our goal is to evaluate the merits of what we refer to as the *ZLB irrelevance hypothesis,* i.e. the hypothesis that the economy's performance has not been affected by a binding ZLB constraint. In particular, we focus on two dimensions of that performance that are likely to have experienced the impact of a binding ZLB: (i) the volatility of macro variables and (ii) the economy's response to shocks.

We start our empirical exploration with an assessment of the possible changes in the volatility of macro variables during the period in which the ZLB constraint was

[1] Several central banks lowered their policy rates down to values below zero, thus proving that the latter should be seen a soft lower bound. From the point of view of our paper what matters is the existence of a "perceived" value below which a given central bank is not willing to lower the policy rate, i.e. an *effective* lower bound. In the case of the U.S. economy, which is the focus of the present paper, zero appears to be the Fed's effective lower bound.

binding. A rise in volatility could have been expected as a result of the Fed's hands being tied due to the Federal Funds Rate having hit the ZLB, since this prevented the "usual" stabilizing policy response to aggregate shocks. Yet, we find little evidence of an increase in the volatility of either real or nominal U.S. macro variables over the period during which the Federal Funds Rate attained its zero lower bound, i.e. from January 2009 through December 2015 (henceforth, the ZLB period). The previous finding is at odds with the predictions of a baseline New Keynesian model, as we show by means of a number of simulations, under the assumption that the central bank follows a simple interest rate rule.

In the second part of our paper we ask ourselves whether the response of different macro variables to a variety of aggregate shocks has been affected by the ZLB. Our empirical approach involves the estimation of a structural vector autoregressive model with time-varying coefficients (TVC-SVAR), driven by four shocks that are identified by means of a combination of long run and sign restrictions. Under the "irrelevance hypothesis" there should not be any significant change in the estimated responses over the binding ZLB period, relative to period before the ZLB was binding. This is indeed what we find. In particular, we show that the estimated response of the long-term interest rate during the ZLB period is very similar to its counterpart for the pre-ZLB period. Furthermore, when we estimate a "rule" for the long-term rate we find little evidence of a break during the ZLB period. We interpret the previous results as suggesting that UMPs may have been highly effective in steering the long rate as desired during the ZLB period, despite the constant policy rate. We complete our analysis by showing how the previous findings can be reconciled with the predictions of our baseline New Keynesian model when we assume an interest rate rule based on a shadow interest rate. That rule can be interpreted as a simple way of capturing the role of forward guidance or other types of unconventional monetary policies in getting around the constraints imposed by the ZLB.

Our findings should not be interpreted as downplaying the significance of the Great Recession and the slowness of the subsequent recovery. Together with the associated deflationary pressures, they were undoubtedly the main factors behind the sharp reduction in the Federal Funds Rate down to its zero lower bound. Our findings suggest, however, that no special role should be attributed to the ZLB constraint as explanation for the depth and persistence of the recession. Instead, the size, persistence and financial nature of the shocks experienced by the U.S. economy (before the start of the ZLB episode) are instead more likely explanations for the severity of the downturn, as had been the case for many other financial crises experienced by different countries in the past, and which did not generally involve a binding ZLB constraint.[2]

The remainder of the paper is organized as follows. Section 2 discusses the related literature. Section 3 provides evidence on the impact of the binding ZLB on macroeconomic volatility. Section 4 contrasts that evidence with the predictions of a baseline New Keynesian model. Section 5 studies how the binding ZLB constraint may have affected the economy's response to a variety of shocks. Section 6 analyzes the ability of a modified interest rate rule to account for the empirical evidence. Section 7 summarizes and concludes.

2 Related Literature

Our paper seeks to contribute to a growing literature that aims at assessing the effectiveness of unconventional monetary policies through alternative approaches. A large number of papers in the literature aim to measure the impact of UMP announcements or their implementation on financial variables. Examples of that work include D'Amico and King (2013), who use security-level data on Treasury prices and quantities to docu-

[2] Many empirical papers have provided evidence on the unusual depth and persistence of downturns caused by financial crises. See e.g. Reinhart and Rogoff (2009), Cerra and Saxena (2008), and IMF (2009).

ment a "local supply" effect along the yield curve during the large-scale asset purchase (LSAP) interventions starting in 2009, documenting both a substantial response of yields to changes in supplies outstanding of a given maturity ("stock effect") as well as to the purchases themselves when they occurred ("flow effect"). The segmentation of the Treasury market suggested by their evidence would make it possible for QE programs to help stabilize the economy in the face of a binding ZLB constraint, thus overcoming the "neutrality" result that emerges in frictionless settings, and providing a possible interpretation of our findings.[3] Krishnamurthy and Vissing-Jorgensen (2011) and Hamilton and Wu (2012) provide related evidence of relative supply effects on the yield curve.

In a more recent paper, D'Amico and King (2017) use a VAR with sign restrictions on survey forecasts and uncover strong and persistent effects on inflation and output of forward guidance policies, i.e. of policy interventions that rely on anticipated changes in future short-term rates. Swanson (2017) provides evidence pointing to large effects on the yield curve, stock prices and exchange rates of both forward guidance and LSAPs during the 2009-2015 ZLB period. Those effects are shown to be comparable in magnitude to the effects of conventional policies in the pre-ZLB period.

Our work is closer in spirit to papers that seek to evaluate, using different approaches, some form of ZLB irrelevance hypothesis. Thus, Swanson and Williams (2014) estimate the time-varying sensitivity of yields to macroeconomic announcements using high-frequency data, and conclude that long-term yields were essentially unconstrained throughout 2008 to 2012, and short-term yields seemed to be constrained only by late 2011. Similarly, Campbell et al. (2012) provide evidence suggesting that forward guidance announcements by the FOMC have been successful in moving interest rates that are relevant for household's' and firms' decisions, despite the binding ZLB constraint.

The recent work of Wu and Xia (2016) and Wu and Zhang (2017) is also closely related to our paper. Thus, Wu and Xia (2016) propose a shadow rate indicator as a

[3]See, e.g., Eggertsson and Woodford (2003) for a rigorous statement of that neutrality result.

measure of the monetary policy stance that also applies to binding ZLB periods. They embed their shadow rate in an identified FAVAR model similar to that in Bernanke et al. (2005), and find that (exogenous) changes in the shadow rate have an impact on the economy during the ZLB period similar to the Federal Funds rate in the pre-ZLB period. A counterfactual simulation, in which the shadow rate is prevented from becoming negative, points to large real effects of the having a persistently negative shadow rate during the ZLB period, which they attribute to the adoption unconventional monetary policies. Wu and Zhang (2017) study a New Keynesian model where aggregate demand is a function of a shadow rate which is not subject to a ZLB constraint and which is determined according to a conventional Taylor-type rule. The equilibrium dynamics are thus equivalent to those of the standard New Keynesian model without a ZLB constraint. Wu and Zhang discuss alternative channels through which the central bank can lower the shadow rate below zero, including purchases of assets by the central bank (combined with a preferred habitat-like assumption), direct lending to firms and/or changes in tax rates on interest income. They conclude that a binding ZLB constraint on the policy rate does not have to alter the responses of aggregate variables to supply and demand shocks relative to periods with a non-binding ZLB, as long as the central bank adjusts the shadow rate suitably.

Christiano et al. (2015) estimate and analyze a DSGE model that incorporates a truncated shadow rule similar to (4)-(5) below. They use the estimated model to interpret the Great Recession. They attribute the bulk of the fall in output to a drop in TFP and a rise in the cost of working capital, with counterfactual simulations without a ZLB constraint suggesting that the latter played a small role in accounting for the drop in output. That finding contrasts with Gust et al. (2017) who carry out a similar counterfactual experiment find that a 30 percent of the output contraction observed during the Great Recession can be attributed to the constraint imposed by the ZLB on the ability of monetary policy to stabilize the economy, with that constraint playing an even larger role in accounting for the slow recovery. Those estimates are, however, subject to large uncertainty.[4]

Our findings above point to the benefits of adopting a shadow rule with sufficient inertia, which we interpret as a shortcut for UMPs. A number of papers have also uncovered a similar result using alternative models and assumptions, including Reifschneider and Roberts (2006), Kiley and Roberts (2017), and Bernanke et al. (2019).

Throughout the paper we have maintained the assumption of an unchanged inflation target, which we have taken as given. A branch of the literature has instead focused on the determination of the optimal inflation rate in the presence of the ZLB constraint, given the trade-off between the distortions associated with a higher average inflation and the benefits from it in the form of a smaller incidence of a binding ZLB. Contributions to that branch of the literature include Coibion et al. (2012), Dordal-i-Carreras et al. (2016), Blanco (2016), Kiley and Roberts (2017) and Andrade et al. (2019).

3 Macroeconomic Volatility and the Zero Lower Bound: Some Evidence

We start with an empirical assessment of the impact of the binding ZLB constraint on U.S. macroeconomic volatility. We report statistics for GDP and total hours in the nonfarm business sector (both in log first differences), as well as three measures of quarterly inflation based on the GDP deflator, the core CPI and the core PCE deflator, respectively. All data are quarterly. The first column of Table 1 reports the standard deviation of several macro variables over the ZLB episode (2009Q1-2015Q4) *relative* to the period 1984Q1-2018Q2 but excluding the binding ZLB episode (henceforth, the no-ZLB period). Note that 1984 is often viewed as the date marking the beginning of the

[4] As the Gust et al. (2017) themselves acknowledge, their estimates "*are subject to considerable uncertainty as the 68 percent credible region does not exclude the possibility that the estimated effects of the ZLB constraint were much smaller or much larger.*"

Great Moderation. The second column reports an analogous statistic, but excluding from both the no-ZLB and ZLB periods the observations corresponding to the Great Recession (2008Q1-2009Q2, according to the NBER chronology).

The previous statistics show little evidence of an increase in macro volatility during the ZLB period. Many of the reported statistics are below one, suggesting if anything a decline in volatility over that period. The previous statistics contrast starkly with the volatility in the pre-1984 period relative to the same benchmark, shown in the last column. In the latter case the estimated relative standard deviation is well above one for all the variables considered.[5]

Table 2 provides additional evidence pertaining to potential changes in volatility during the ZLB episode. It reports the estimates from an OLS regression of the absolute value of the deviation of each variable (i.e. GDP growth, hours growth and each of the three inflation measures) from a period-specific mean, on a constant and a dummy variable for the ZLB episode. Together with each point estimate, we report the corresponding Newey-West standard errors, which are robust to the presence of heteroskedasticity and autocorrelation. As a robustness check we also report estimates from regressions that include a dummy for the Great Recession in addition to the ZLB period. The eventual impact of the binding ZLB on the volatility of each variable should be captured by the estimated coefficient of the ZLB dummy in the corresponding regression. As the estimates reported in Table 2 indicate, there is no evidence of a significant volatility increase during the ZLB period in *any* of the variables considered, for any specification. By contrast, we find two instances of a significant reduction in volatility during that episode: this is the case for core CPI and core PCE inflation.

The evidence reported in Tables 1 and 2 is reflected graphically in Figure 1 which shows the evolution of GDP growth and inflation (based on the GDP deflator) over the

[5] Changes in volatility between the Great Moderation and the pre-Great Moderation periods have been uncovered by many authors (e.g., McConnell and Pérez-Quirós (2000), Stock and Watson (2002)).

period 1984Q1-2018Q2, with the binding ZLB episode marked with a shaded area. It is not obvious at all to the naked eye that the volatility of any of the two variables was affected one way or another during the ZLB episode.

The evidence provided above contrasts with much of the literature on the effects of a binding ZLB constraint. The next section illustrates that contrast.

4 Macroeconomic Volatility and the Zero Lower Bound: Predictions of a Benchmark Model

Next we analyze the predictions of a baseline New Keynesian (NK) model regarding the implications of a binding ZLB constraint for the equilibrium behavior of different macroeconomic variables. Needless to say, we are not the first to carry out an analysis of this kind. Examples of related earlier work include Christiano et al. (2011), Eggertsson (2011), and Wieland (2019), among many others. In contrast with those papers our focus here is on the implications of the binding ZLB constraint for macro volatility, relative to "normal" times.

Our model is fully standard so we restrict ourselves to describing its main elements. We assume an infinitely-lived representative household with expected utility given by

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta_{0,t} \left(\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} \right) \right\}$$

where C_t is a CES function (with elasticity of substitution $\epsilon > 1$) of the quantities consumed of a continuum of differentiated goods and N_t denotes hours worked. The discount factor $\beta_{0,t}$ is defined recursively by $\beta_{0,t} = \beta_{0,t-1} \exp\{-z_t\}$, for $t = 1, 2, 3, ...$with $\beta_{0,0} \equiv 1$, and where $\{z_t\}$ is a stochastic discount rate with two components:

$$z_t = \rho_t + \eta_t$$

The first component, $\{\rho_t\}$, follows a two-state Markov process, switching between a "normal" value $\rho > 0$ and a low value $\rho_L < 0$. The realization of the latter, which we interpret as a large adverse demand shock, will pull the short-term interest rate against the ZLB constraint, given the assumed policy rule. The second component, $\{\eta_t\}$, is meant to capture "regular" or "recurrent" demand shocks, and is assumed to follow an $AR(1)$ process

$$\eta_t = \rho_\eta \eta_{t-1} + \varepsilon_t^\eta$$

where $\rho_\eta \in [0, 1)$, and $\{\varepsilon_t^\eta\}$ is a white noise process with variance σ_η^2. For concreteness, in the simulations below we restrict ourselves to shifts in the discount rate $\{z_t\}$ as a source of aggregate fluctuations.

The supply side is fully standard. We assume a continuum of identical monopolistically competitive firms, each facing a constant Calvo probability θ of not being able to reoptimize its price on any given period. Technology is given by $Y_t = N_t^{1-\alpha}$. All output is consumed. The labor market is perfectly competitive.

The log-linearized equilibrium conditions describing the private sector of this economy are given by the familiar New Keynesian Phillips curve and dynamic IS equations:

$$\widehat{\pi}_t = \beta \mathbb{E}_t\{\widehat{\pi}_{t+1}\} + \kappa \widehat{y}_t$$

$$\widehat{y}_t = \mathbb{E}_t\{\widehat{y}_{t+1}\} - (i_t - \mathbb{E}_t\{\pi_{t+1}\} - z_t)$$

where $\widehat{\pi}_t \equiv \pi_t - \pi$ and $\widehat{y}_t \equiv y_t - y$ respectively denote the deviation of inflation and (log) output from their steady state values, i_t is the short-term (one-period) nominal interest rate, $\beta \equiv \exp\{-\rho\}$ and $\kappa \equiv \frac{(1-\theta)(1-\beta\theta)(1+\varphi)}{\theta(1-\alpha+\alpha\epsilon)}$.[6] It can be easily checked that under our assumptions the natural (flexible price) level of output is constant (and corresponds to the steady state), while z_t has the interpretation of the natural (flexible price) rate of interest.

[6] See, e.g. Galí (2015) for a derivation. By allowing for a nonzero steady state inflation we are implicitly assuming that prices are indexed to that variable. We also solved and simulated a fully nonlinear version of the model with price adjustment cost a-la-Rotemberg, obtaining very similar results to the linearized version considered here.

The model is closed by assuming a monetary policy rule. As an empirically plausible baseline we consider the "truncated" version of a Taylor-type rule with inertia, used in a variety of applications:

$$i_t = \max\left[0,\ \phi_i i_{t-1} + (1-\phi_i)(\rho + \pi + \phi_\pi \widehat{\pi}_t + \phi_y \Delta \widehat{y}_t)\right] \tag{1}$$

Below we also analyze the equilibrium behavior of a long-term interest rate. We define the latter as the yield on a pure discount bond with stochastic maturity. More precisely, the long-term bond is assumed to mature each period with probability $1-\gamma$, in which case it pays one unit of the numéraire (with no coupon payment otherwise). The (normalized) yield of that bond, denoted by i_t^L, can be shown to satisfy the following difference equation in equilibrium (and up to a first order approximation):

$$i_t^L = (1-\beta\gamma)i_t + \beta\gamma E_t\{i_{t+1}^L\}$$

We adopt a quarterly calibration of the model. We assume $\theta = 0.75$, which implies an average price duration of four quarters. The coefficients of the policy rule are set to $\phi_\pi = 1.5$, $\phi_y = 0.5$, and $\phi_i = 0.7$, in line with the empirical evidence. Steady state inflation π is set to 0.005, consistent with an (annual) inflation target of 2 percent. We assume $\alpha = 0.25$ and $\varphi = 1$, both conventional values. We set $\epsilon = 6$ implying a flexible price markup of 20 percent. We assume $\rho_\eta = 0.8$ and $\sigma_\eta = 0.001$. Under the previous settings, the standard deviation of quarterly output growth when the ZLB is not binding is about 0.7 percent, consistent with that of U.S. GDP growth over the 1984Q1-2008Q4 period. We set $\rho = 0.005$, implying an average real rate of 2 percent in "normal" times. We assume $\rho_L = -0.01$, so that a large adverse demand shock implies a natural rate of minus 4 percent in annual terms (and in the absence of other shocks). The probability of remaining in the "normal" regime is 0.994, while the probability of remaining in the low demand regime is 0.66. These values imply that ZLB episodes occurs on average once every 140 quarters, with each episode expected to last 3 quarters. Under the previous settings, the contemporaneous impact on output of a large adverse demand shock (i.e. a Markov transition to ρ_L) is about minus 4 percent, which roughly corresponds to the observed decline in U.S. GDP over the Great Recession. Most importantly for our purposes, the realization of a large adverse demand shock generally brings the policy rate down to zero, where it remains until the "normal" Markov state is restored again. Finally, we set $\gamma = 0.975$, consistent with an expected maturity for the long-term bond of 40 quarters.

Similarly to Fernández-Villaverde et. al. (2015), we solve the model using a global projection method to accurately account for the non-linearities associated with the presence of the occasionally binding ZLB constraint. In particular, we approximate the policy functions for inflation, output, and interest rates with Chebyshev polynomials (or splines) through a collocation method on a discrete grid for the four state variables (lagged output and interest rate, and the two components of the demand shocks).

Figure 2 shows the dynamic responses of several macro variables to a negative η_t shock, under the two possible states of the economy, namely, "normal" times (i.e. when the ZLB is non-binding, shown in blue circled lines) and under a binding ZLB regime (shown in red lines with diamonds). The figure illustrates clearly the destabilizing role of a binding ZLB in the face of an adverse demand shock, relative to "normal" times: the inability to bring down the policy rate leads to much larger decline of output and inflation. Note that the long-term rate responds in opposite directions to the negative demand shock under the two scenarios. It declines in normal times, reflecting the expectations of a persistently lower short-term rate. On the other hand, when the negative demand shock hits the economy during a binding ZLB episode, the long rate increases, reflecting the expectations of a higher path of short-term rates in the future, when the ZLB stops being binding, since in that case the economy would experience higher output growth in its transition to the new steady state, having started from a lower output level due to the current negative demand shock. Note also that the contractionary effect of a expected higher nominal short-term rates (reflected in the higher long-term nominal rate) is further amplified by the deflationary expectations caused by the shock, leading to an even larger increase in the long-term *real* rate (not shown).

Figure 3 displays the time series for output growth and inflation around the time of a large adverse demand shock, based on a simulation of the calibrated model described above. The timing of the large adverse shock and its eventual undoing is indicated with a vertical line. The "tunnel" within which the time series evolves represents, for each period, a 95% confidence interval for the realizations of the plotted variable across 1000 simulations. The figure illustrates the increase in the volatility of output and inflation during the binding ZLB period relative to the earlier and later periods when the ZLB constraint is not binding. The model's predictions in that regard seem to conflict with the patterns of volatility observed in the U.S. economy discussed in the previous section. Tables 3 and 4, discussed below, formalize that visual intuition.

Table 3 reports the mean, across 1000 model simulations, of the ratio of the estimated standard deviation of output growth and inflation over the binding ZLB episodes relative to normal times, together with a 95% confidence interval. In computing that ratio we use simulations of the calibrated model for which the *realized* length of the binding ZLB period is equal to that observed in the recent U.S. episode, namely, 28 quarters. As the values reported in the Table make clear, and in a way consistent with the impulse responses of Figure 2 and the evidence in Figure 3, the binding ZLB episode is characterized by a much larger estimated volatility on average, especially when we exclude the Markov transition observations from the computation of the standard deviation (right column).

Table 4 shows the estimated coefficients, using simulated data, from a regression of the absolute value of (demeaned) output growth and inflation on a ZLB dummy, with and without a control dummy for the two periods when a Markov transition occurs.[7]

[7] As in Table 3 we allow for a specific mean during the binding ZLB episode.

The regression uses time series generated by 1000 simulations of the calibrated model. Each time series has 138 observations, with a binding ZLB episode taking place between periods 101 and 128, i.e. a pattern that corresponds to that observed in the empirical sample period used in the previous section. In addition to the mean coefficient estimates, we report the 95% confidence band across simulations (shown in brackets), as well as the fraction of simulations for which we reject the null of a zero coefficient on the ZLB dummy at the 5 percent significance level, using the Newey-West estimate of the standard error. Note that the mean (as well as the 95 percent confidence band) of the estimated coefficients on the ZLB dummy are systematically positive, reflecting the increase in volatility associated with ZLB episodes. This is true for both output growth and inflation, and independently of whether we control for the periods with "large shocks," corresponding to the Markov transitions. The previous finding contrasts with the estimates of the corresponding regressions using U.S. data and reported in Table 2, and where most of the estimated coefficients on the ZLB dummy were insignificant, with the exception of a few instances in which they were significantly negative (thus pointing to a reduction in volatility). Finally, note that the large fraction of simulations (ranging between 86 and 98 percent, depending on the variable and specification) for which the null of no change in volatility during the ZLB episode is rejected at the 5 percent significance level. The previous finding suggests that the relatively short ZLB period is not an obstacle for the change in volatility test to have a high power, when the data are generated by our baseline calibrated model.

5 Did the Binding ZLB Affect the Economy's Response to Shocks?

In the present section we use a structural vector autoregression model with time-varying coefficients (TVC-SVAR), to estimate the dynamic responses of a number of macro variables to several identified aggregate shocks. The main motivation for using a model with time-varying coefficients lies in our interest in assessing the extent to which the binding ZLB episode implied a change in the way the economy responded to different shocks. In addition, the use of a TVC-SVAR provides a flexible specification which allows for other structural changes that the U.S. economy may have experienced over the sample period used in the estimation.[8]

5.1 Our Empirical Approach

Let $\mathbf{x}_t = [\Delta(y_t - n_t),\ n_t,\ \pi_t,\ i_t^L]'$ where y_t is (log) output in the nonfarm business sector, n_t denotes (log) hours of all persons in the nonfarm business sector, π_t is GDP deflator inflation and i_t^L is the 10-year Treasury bond yield. Both output and hours are normalized by civilian population. All data are for the U.S. economy, at a quarterly frequency, and cover the period $1953Q2$ through $2015Q4$.[9] We assume that the evolution of $\mathbf{x}_t$ can be described by the following TVC-VAR model

$$\mathbf{x}_t = \mathbf{A}_{0,t} + \mathbf{A}_{1,t}\mathbf{x}_{t-1} + \mathbf{A}_{2,t}\mathbf{x}_{t-2} + ... + \mathbf{A}_{p,t}\mathbf{x}_{t-p} + \mathbf{u}_t \tag{2}$$

where $\mathbf{A}_{0,t}$ is a vector of time-varying intercepts, $\mathbf{A}_{i,t}$, for $i = 1, ..., p$ are matrices of time-varying coefficients, and $\mathbf{u}_t$ is a Gaussian white noise vector process with covariance matrix $\mathbf{\Sigma}_t$. We assume the reduced form innovations $\mathbf{u}_t$ are a linear transformation of the underlying structural shocks $\boldsymbol{\varepsilon}_t$ given by

$$\mathbf{u}_t \equiv \mathbf{Q}_t\boldsymbol{\varepsilon}_t$$

[8]These may include the change in the cyclical behavior of productivity emphasized in Galí and Gambetti (2009), as well as the change in monetary policy starting with Paul Volcker's tenure at the Fed uncovered in Clarida, Galí and Gertler (2000)

[9]We construct our dataset using the following time series drawn from the FRED database: real output per hour of all persons (nonfarm business sector) (OPHNFB); hours of all persons (nonfarm business sector) (HOANBS); civilian noninstitutional population (CNP16OV); GDP deflator (GDPDEF); 10-Year Treasury Constant Maturity Rate (GS10).

where $\mathbb{E}\{\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t'\} = I$ and $\mathbb{E}\{\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_{t-k}'\} = 0$ for all t and $k = 1, 2, 3, ...$ It follows that $\mathbf{Q}_t\mathbf{Q}_t' = \boldsymbol{\Sigma}_t$.

Following Primiceri (2005), we assume that the coefficients of the autoregressive matrices $\{\mathbf{A}_{i,t}\}$ and the covariance matrix $\boldsymbol{\Sigma}_t$ follow random walks, as described in detail in the Appendix. The resulting reduced form model is estimated as in Del Negro and Primiceri (2013). Given the estimated reduced form VAR for any given period t we recover the reduced form MA representation

$$\mathbf{x}_t = \boldsymbol{\mu}_t + \mathbf{B}_t(L)\mathbf{u}_t$$

The identifying restrictions assumed below allow us to determine the linear mapping $\mathbf{Q}_t$. Given the latter, we can write the structural MA representation as

$$\mathbf{x}_t = \boldsymbol{\mu}_t + \mathbf{C}_t(L)\boldsymbol{\varepsilon}_t$$

where $\mathbf{C}_t(L) \equiv \mathbf{B}_t(L)\mathbf{Q}_t$ describes the dynamic responses of the economy in period t. The possible changes over time in those responses are the focus of our analysis. Next we turn to the determination of $\mathbf{Q}_t$, i.e. to the issue of identification.

5.2 Identification

We assume fluctuations in $\mathbf{x}_t$ are driven by four structural shocks, represented by the elements of vector $\boldsymbol{\varepsilon}_t$: technology, demand, monetary policy and temporary supply shock. We use a mix of long run and sign restrictions in order to identify those shocks, as follows: (i) the technology shock is the only one with a permanent effect on labor productivity; (ii) a demand shock generates a positive comovement of prices, GDP and the long-term interest rate; (iii) a monetary policy shock generates a positive comovement between prices and GDP, but a negative one between the previous variables and the long-term interest rate; (iv) a transitory supply shock (e.g. a markup shock) implies a negative comovement of inflation and GDP. All the restrictions on the sign of comovements refer to a one-year horizon. The long run restriction used to identify technology was first proposed in Galí (1999). The sign restrictions are consistent with the predictions of a standard New Keynesian model under a plausible policy rule, and are generally satisfied by estimated DSGE models (e.g. Smets and Wouters (2007)). In the Appendix we discuss how the previous identification strategy is implemented in practice.

5.3 Evidence

Figure 4a displays the estimated impulse responses of output, inflation, and the long-rate (in both nominal and real terms to the four shocks considered. The responses plotted correspond to the *averages* of the estimated responses for the pre-ZLB period 2002Q1-2008Q4 (displayed with blue circled lines) and the ZLB period 2009Q1-2015Q4 (red lines with diamonds). Note that both subsamples contain 28 periods. We also display 68 and 95 percent confidence bands for the average impulse responses in the pre-ZLB period, based on 1000 draws from the posterior distribution of the estimated model.

The differences in the estimated responses between the two periods are very small, for all variables and shocks considered. In particular, the estimated responses of output and inflation for the two periods lie almost on top of each other. Most importantly, note that the lack of a significant gap between the responses in the two periods carries over to both the nominal and real long-term yields, which are arguably more relevant in the determination of aggregate demand than the short term nominal rate (which does not adjust during the binding ZLB period). While the response of both the nominal and real long-term rate over the ZLB period appears slightly muted relative to the pre-ZLB period, the difference is quantitatively very small and statistically insignificant. Figure 4b displays the corresponding impulse response *differentials* with their associated confidence bands. One *cannot* reject the null of a zero differential response for any variable, even at a 32 percent significance level.

The finding of similar responses of the long term rate across the two subsample periods is consistent with our "irrelevance hypothesis." It suggests that through the use of unconventional policies like forward guidance and quantitative easing the Fed managed to steer long-term rates "as in normal times," in response to shocks hitting the economy during the ZLB period, thus leading to similar responses of output and inflation.

A caveat that might be raised regarding the evidence reported in Figures 4a and 4b is that the estimated impulse responses may be somehow distorted by the Great Recession, which overlaps with both the pre-ZLB and ZLB periods. Figure 4c reports estimates of average impulse responses for the pre-ZLB and ZLB periods, but excluding from the respective averages the estimated responses for the Great Recession quarters (i.e. those between 2008Q1 and 2009Q2). None of the qualitative findings discussed above seem to be affected by the exclusion of that episode.

An additional potential caveat is that the methodology used may not be able capture changes in dynamics that take place suddenly, as opposed to gradually over time. Figure 4d seeks to dispel that concern by comparing the average impulse responses over the ZLB period with the median estimated average response across 500 draws, with each draw corresponding to a sample period of consecutive 28 observations, drawn randomly from the set {1984Q1:1990Q4,...,2002Q1:2008Q4}.[10] The resulting estimates are once again very similar across the two periods, suggesting a relatively uniform response over the entire post-1984 period.

We conclude this section by providing an alternative perspective on the "irrelevance hypothesis" and the possible role played by the long-term interest rate in getting around

[10] The average irf are computed as follows. The pre-zlb sample has 100 quarters 1984Q1-2008Q4.

1. Draw an integer between 1 and 78 with equal probabilitites over the set of possible starting periods. Call it $t_0(j)$.

2. Take a draw from the distribution of IRFs (at all the horizons) from $t_0(j)$ up to $t_0(j)+27$, call it $x(t,j)$ with $t = t_0(j), ..., t_0(j)+27$ and $j = 1, 2, ..., 500$ (number of draws).

3. Compute the average IRF over the sample period drawn, $\overline{x}(t) = (1/28)\sum_{t=t_0(j)}^{t_0(j)+27} x(t,j)$

4. Repeat 1-3, 500 times. Compute the median and the percentiles of the corresponding $\overline{x}(t)$s.

the ZBL constraint. Consider the following (admittedly ad-hoc) descriptive "rule" for the long-term interest rate,

$$i_t^L = \phi_0 + \phi_i i_{t-1}^L + (1-\phi_i)[\phi_\pi \pi_t + \phi_y \Delta y_t] + \varepsilon_t^m \quad (3)$$

where $\{\varepsilon_t^m\}$ is interpreted as an exogenous monetary policy shock. We estimate (3) and try to uncover changes in the coefficients on inflation and output growth by including as right-hand variables multiplicative dummies for the binding ZLB period and examining their significance. Furthermore, and in order to overcome the likely endogeneity of the regressors with respect to the policy shock $\{\varepsilon_t^m\}$, we estimate (3) using the time series for $\{i_t^L,\ \pi_t,\ \Delta y_t\}$, obtained after subtracting from each of them the corresponding component associated with the monetary policy shock in the estimated TVC-SVAR model described above.[11] Under our assumptions, the resulting "cleansed" variables should be uncorrelated with the monetary policy shock, so that (3) can be estimated consistently using OLS. Table 5 reports the corresponding estimated coefficients, using two alternative specifications (with and without output growth in the rule). The reported estimates of the inflation and output growth coefficients point to a strong and highly significant response to both variables. In particular, the estimate of ϕ_π is well above one, implying that the estimated rule satisfies the so-called Taylor principle. Most interesting, however, is the insignificance of the estimated coefficient on the multiplicative dummies associated with the binding ZLB period, suggesting the absence of a discernable change in the systematic response of the long-term interest rate to inflation and output growth as a consequence of the ZLB constraint becoming binding.

[11]Note that the monetary polcy shock component for the ith element of $\mathbf{x}_t$ element is given by $\mathbf{C}_t^{i3}(L)\varepsilon_t^m$.

6 Reconciling Theory and Evidence

The above empirical findings lend support, overall, to the irrelevance hypothesis. In addition, they suggest that the implementation of unconventional monetary policies during the binding ZLB episode made it possible to steer the long-term interest rate "as in normal times" in response to developments in the economy. The absence of unconventional policies in the baseline New Keynesian model analyzed in section 3 may thus account for the discrepancy between our evidence and the model predictions.

In the present section we study whether modifying the specification of monetary policy in our baseline model can help reconcile theory and evidence. In particular, and following Christiano et al. (2015), Gust et al. (2017) and Andrade et al. (2018), among others, we replace the baseline interest rate rule (1) with the following *shadow rate rule*:

$$i_t = \max[0, i_t^s] \tag{4}$$

where i_t^s is a shadow interest rate which evolves according to

$$i_t^s = \phi_i i_{t-1}^s + (1 - \phi_i)(\rho + \pi + \phi_\pi \widehat{\pi}_t + \phi_y \Delta \widehat{y}_t) \tag{5}$$

Figure 5 displays the dynamic responses to an adverse demand shock (i.e. a negative η_t realization) under the previous rule, with all the parameters of the model (including the coefficients in the rule) calibrated as before. Impulse responses are displayed under the two regimes, characterized by a binding and a nonbinding ZLB. As the figure makes clear the responses of output and inflation under the two regimes are now very similar, despite the large discrepancy in the responses of the short-term rate (also displayed in the figure) due to the central bank's inability to lower that rate when the ZLB is binding. By contrast, the central bank manages to bring down the long-term nominal rate in response to the adverse demand shock, albeit not as much as under the non-binding ZLB regime, and despite the lack of an adjustment in the short-term rule. The channel through which the shadow rate rule (5) manages to stabilize the economy "as in normal times" is one typically associated with *forward guidance* policies: the inertial term in (5), which is not bounded below, implies that the short-term rate will be kept "lower for longer" relative to the baseline rule (1) in the aftermath of a binding ZLB episode, with the expected length of that additional stimulus being commensurate to the size of the decline in inflation and output.

Tables 6 and 7, based on simulations of the calibrated New Keynesian model with the shadow rate rule in place, report evidence on the implied changes in volatility during the binding ZLB period, in a way analogous to Tables 3 and 4 for the baseline rule. Note that the estimates of the standard deviation of output growth and inflation in the binding ZLB period relative to the nonbinding ZLB period, shown in Table 6, are noticeably lower than those in Table 3, reflecting the gains in stability from the adoption of the shadow rate rule. Furthermore the reported relative standard deviations are comparable to some of the estimates in Table 1, obtained using actual U.S. data. Similarly, the estimated volatility regressions reported in Table 7 show very limited evidence of an increase in volatility during the binding ZLB episodes, with the implied fraction of simulations for which the null of no change in volatility is rejected at the 5 percent significance level being very small for most specifications (with a largest value of 49 percent). The previous findings are visually captured by Figure 6, which displays the time series for output growth and inflation around the time of a large adverse demand shock, based on a simulation of our calibrated model under the shadow rate rule. In contrast to Figure 3, which was based on simulations under the baseline rule, neither the volatility of the simulated series nor width of the "tunnel" appear to change in a discernible way during the binding ZLB episode (the period between the two vertical lines).

7 Concluding Comments

The *ZLB irrelevance hypothesis* implies that the economy's performance is not affected by a binding ZLB constraint. The objective of the present paper was to evaluate that hypothesis for the recent ZLB episode experienced by the U.S. economy (2009Q1-2015Q4). We have focused on two dimensions of performance that were likely to have experienced the impact of a binding ZLB: (i) the volatility of macro variables and (ii) the economy's response to shocks. Using a variety of empirical approaches, we find little evidence against the irrelevance hypothesis, with our estimates suggesting that macro volatility did not increase significantly as a result of the binding ZLB constraint. Similarly, the responses of output, inflation and the long-term interest rate to different shocks do not appear to have been much affected by that constraint, possibly as a result of the adoption and fine-tuning of unconventional monetary policies. Our empirical findings can be reconciled with the predictions of a simple New Keynesian model under the assumption of a shadow interest rate rule, which can be viewed as a capturing the effects of unconventional policies.

The present paper contributes, albeit indirectly, to a growing literature that aims to evaluate empirically the effectiveness of unconventional monetary policies during the recent ZLB episode. We interpret our findings as being consistent with (though not a proof of) the hypothesis that the unconventional monetary policies implemented during the ZLB years may have succeeded, at least to some extent, at getting around the constraints imposed by the ZLB on conventional monetary policy.

Under that hypothesis, the unusual magnitude of Great Recession and the slowness of the subsequent recovery would be just the consequence of the large size and high persistence, as well as the financial nature, of the shock(s) that impinged on the U.S. economy during that episode, with no special amplifying role attributed to the binding ZLB constraint.

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APPENDIX

Let $\boldsymbol{\theta}_t = vec(\mathbf{A}_t')$ where $\mathbf{A}_t = [\mathbf{A}_{0,t}, \mathbf{A}_{1,t}..., \mathbf{A}_{p,t}]$ and $vec(\cdot)$ is the column stacking operator. We assume $\boldsymbol{\theta}_t$ evolves over time according to the following equation:

$$\boldsymbol{\theta}_t = \boldsymbol{\theta}_{t-1} + \boldsymbol{\omega}_t \tag{6}$$

where $\boldsymbol{\omega}_t$ is Gaussian white noise vector process with covariance matrix $\boldsymbol{\Omega}$.

Time variation of $\boldsymbol{\Sigma}_t$ is modeled in the standard way. Let $\boldsymbol{\Sigma}_t = \mathbf{F}_t\mathbf{D}_t\mathbf{F}_t'$, where $\mathbf{F}_t$ is lower triangular, with ones on the main diagonal, and $\mathbf{D}_t$ a diagonal matrix. The vector containing the diagonal elements of $\mathbf{D}_t^{1/2}$, denoted by $\boldsymbol{\sigma}_t$, is assumed to evolve according to the process

$$\log \boldsymbol{\sigma}_t = \log \boldsymbol{\sigma}_{t-1} + \boldsymbol{\zeta}_t. \tag{7}$$

Moreover let $\boldsymbol{\phi}_{i,t}$ denote the column vector with the non-zero elements of the $(i+1)$-th row of $\mathbf{F}_t^{-1}$. We assume

$$\boldsymbol{\phi}_{i,t} = \boldsymbol{\phi}_{i,t-1} + \boldsymbol{\nu}_{i,t} \tag{8}$$

where $\boldsymbol{\zeta}_t$ and $\boldsymbol{\nu}_{i,t}$ are Gaussian white noise vector processes with zero mean and (constant) covariance matrices $\boldsymbol{\Xi}$ and $\boldsymbol{\Psi}_i$, respectively. We further assume that $\boldsymbol{\nu}_{i,t}$ is independent of $\boldsymbol{\nu}_{j,t}$, for all $j \neq i$, and that $\boldsymbol{\omega}_t$, $\boldsymbol{\varepsilon}_t$, $\boldsymbol{\zeta}_t$ and $\boldsymbol{\nu}_{i,t}$ (for all i) are mutually independent. Estimation is carried out as in Del Negro and Primiceri (2013).[12]

Reduced form impulse response functions can be derived from the local moving average (MA) representation of the model. First let us consider the companion form representation of eq. (2):

$$\tilde{\mathbf{x}}_t = \tilde{\boldsymbol{\mu}}_t + \tilde{\mathbf{A}}_t\tilde{\mathbf{x}}_{t-1} + \tilde{\mathbf{u}}_t$$

where $\tilde{\mathbf{x}}_t \equiv [\mathbf{x}_t', \mathbf{x}_{t-1}', ..., \mathbf{x}_{t-p+1}']'$, $\tilde{\mathbf{u}}_t \equiv [\mathbf{u}_t', 0, ..., 0]'$, $\tilde{\boldsymbol{\mu}}_t \equiv [\mathbf{A}_{0,t}', 0, ..., 0]'$ and $\tilde{\mathbf{A}}_t$ is the corresponding companion matrix. The (local) time-varying reduced form MA represen-

[12] We refer the reader to Galí and Gambetti (2016) for details.

tation of the model is given by

$$\mathbf{x}_t = \mathbf{b}_t + \sum_{j=0}^{\infty} \mathbf{B}_{t,j}\mathbf{u}_{t-j}$$

where $\mathbf{B}_{t,j} = [\tilde{\mathbf{A}}_t^j]_{n,n}$, for $j = 1, 2, ..,$ where $[M]_{n,n}$ represents the first n rows and n columns of any matrix M, and where $\mathbf{B}_{t,0} = \mathbf{I}$.

The identification is implemented as follows. Let

$$\mathbf{H}_t^n = \begin{pmatrix} 1 & \mathbf{0}' \\ \mathbf{0} & \mathbf{H}_t^{n-1} \end{pmatrix}$$

where $\mathbf{0}$ is n-dimensional column vector of zeros, $\mathbf{H}_t^n$ and $\mathbf{H}_t^{n-1}$ are orthogonal matrices of dimension $n \times n$ and $n-1 \times n-1$ respectively. To impose the restrictions we follow the standard algorithm of Rubio-Ramirez, Waggoner and Zha (2010). We draw $\mathbf{H}_t^{n-1}$ using the QR decomposition and compute the potential structural impulse response functions as in (**??**) with $\mathbf{Q}_t = \mathbf{B}_t(1)^{-1}\mathbf{S}_t\mathbf{H}_t^n$, where $\mathbf{S}_t$ is the Cholesky factor of $\mathbf{B}_t\mathbf{\Sigma}_t\mathbf{B}_t'$. We retain the draw if the sign restrictions are satisfied. We collect a total of 500 draws at each point in time. With no loss of generality we order the shocks as follows: technology, demand, monetary policy and supply.

Table 1
Relative Volatility

	ZLB		*Pre-84*
GDP	0.92	0.89	2.19
Hours	1.32	0.74	1.60
GDP Deflator	1.02	0.88	3.11
Core CPI	0.52	0.54	3.03
Core PCE	0.52	0.50	2.52
Great Recession?	yes	no	no

Standard deviations are computed relative to the NO-ZLB period given by 1984Q1-2008Q4 and 2016Q1-2018Q2. The ZLB period is 2009Q1-2015Q4. When the Great Recession is excluded the pre-ZLB sample period ends in 2007Q4 and the ZLB period starts in 2009Q3. The pre-84 period starts in 1960Q1 and ends in 1983Q4.

Table 2
Volatility Regressions

	CONST	*ZLB*	*GR*
GDP	0.41* (0.04)	0.01 (0.05)	
	0.37* (0.03)	−0.01 (0.05)	0.94* (0.19)
Hours	0.47* (0.05)	0.05 (0.16)	
	0.42* (0.04)	−0.00 (0.09)	1.39* (0.42)
GDP Deflator	0.70* (0.07)	0.03 (0.12)	
	0.69* (0.07)	0.02 (0.11)	0.37 (0.26)
Core CPI	0.91* (0.10)	−0.47* (0.13)	
	0.91* (0.10)	−0.47* (0.13)	−0.05 (0.13)
Core PCE	0.83* (0.08)	−0.41* (0.10)	
	0.83* (0.09)	−0.42* (0.10)	0.13 (0.23)

The Table reports the estimated coefficients from an OLS regression of the absolute value of the deviation of each variable's growth rate from its mean, on a constant and a dummy for the ZLB period (2009Q1-2015Q4), with and without a control dummy for the Great Recession period (2008Q1-2009Q2). The sample period is 1984Q1-2018Q2. Standard errors obtained using the Newey-West estimator (4 lags).

Table 3
Relative Volatility: Simulations
Baseline Interest Rate Rule

Output	1.49 [0.86,2.37]	2.29 [1.69,2.95]
Inflation	1.94 [0.91,3.38]	2.39 [1.02,3.86]
Markov transitions?	yes	no

For each variable the Table reports the mean of the standard deviation in the ZLB period relative to the no-ZLB period over 1000 model simulations under the baseline interest rate rule. The no-ZLB period is given by the first 100 observations and the last 8 observations in the simulation. The ZLB period corresponds to the intermediate 28 observations. 95% confidence intervals reported in brackets.

Table 4
Volatility Regressions: Simulations
Baseline Interest Rate Rule

	$CONST$	ZLB	MT	$\%REJ$
Output	0.32* [0.27,0.36]	0.35* (0.16,0.56)		0.86
	0.26* [0.23,0.3]	0.34* [0.19,0.50]	4.15* [3.34,4.92]	0.98
Inflation	0.27* [0.23,0.32]	0.47* [0.21,0.79]		0.98
	0.26* [0.22,0.30]	0.47* [0.22,0.79]	0.61* [0.02,1.31]	0.98

For each variable the Table reports the mean, over 1000 model simulations under the baseline interest rate rule, of the estimated coefficients from an OLS regression of the absolute value of the demeaned growth rate of each variable on a constant, a dummy indicating the ZLB period and, when it applies, a dummy for the two periods when a Markov transition occurs (MT). 95% confidence bands reported in brackets. $\%REJ$ is the fraction of simulations for which the estimated coefficient on the ZLB dummy is positive and statistically significant using the Newey-West estimate of the standard error (4 lags).

Table 5
Estimated Long-Term Interest Rate Rule

π_t	2.42^* (0.61)	2.82^* (0.82)	2.26^* (0.23)	2.61^* (0.32)
$\pi_t * ZLB_t$	-0.08 (0.08)	-0.01 (0.06)	-0.17^* (0.06)	-0.45 (0.50)
Δy_t			3.52^* (0.42)	4.43^* (0.58)
$\Delta y_t * ZLB_t$			-0.16 (0.08)	-0.60 (0.89)
ϕ_0 and ϕ_i dummies?	Yes	No	Yes	No

The Table reports the OLS estimates of the long term rate rule described in the text, with multiplicative dummies for the binding ZLB period, and using the non-monetary component of the long-term interest rate, output growth and inflation obtained from the estimated TVC-SVAR model.

Table 6
Relative Volatility: Simulations
Shadow Rate Rule

Output	1.01 [0.65,1.9]	1.50* [1.03,1.94]
Inflation	0.82 [0.50,1.38]	1.0 [0.59,1.41]
Markov transitions?	yes	no

For each variable the Table reports the mean of the standard deviation in the ZLB period relative to the pre-ZLB period over 1000 model simulations under the baseline interest rate rule. The no-ZLB period is given by the first 100 observations and the last 8 observations in the simulation. The ZLB period corresponds to the intermediate 28 observations.. 95% confidence intervals reported in brackets.

Table 7
Volatility Regressions: Simulations
Shadow Rate Rule

	CONST	*ZLB*	*MT*	*%REJ*
Output	0.31^{*} [0.28,0.35]	0.1 (−0.03,0.27)		0.15
	0.26^{*} [0.23,0.3]	0.14^{*} [0.02,0.26]	3.11^{*} [2.66,3.6]	0.49
Inflation	0.28^{*} [0.24,0.32]	0.03 [−0.06,0.14]		0.07
	0.26^{*} [0.22,0.29]	0.05 [−0.04,0.14]	1.37^{*} [1.07,1.69]	0.16

For each variable the Table reports the mean over 1000 simulations under the shadow rate rule of the estimated coefficients from an OLS regression of the absolute value of the demeaned growth rate of the variable on a constant, a dummy indicating the ZLB period and, when it applies, a dummy for the period of a Markov transition. 95% confidence bands reported in brackets. $\%REJ$ is the fraction of simulations for which the estimated coefficient on the ZLB dummy is positive and statistically significant using the Newey-West estimate of the standard error (4 lags).

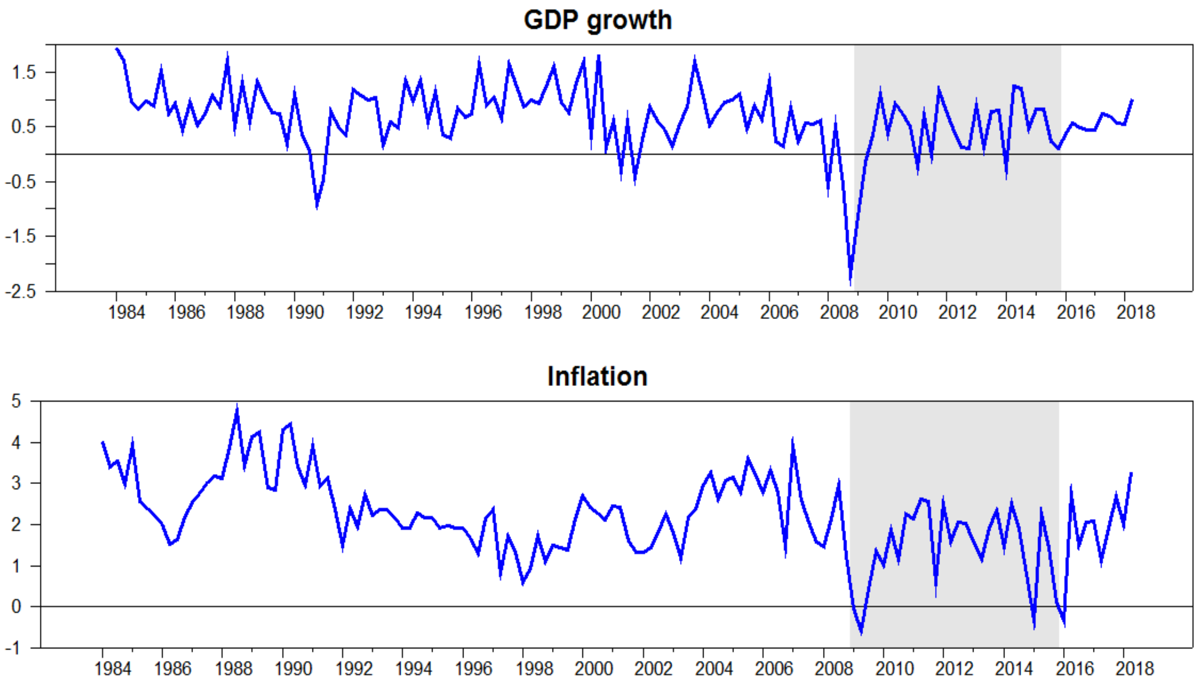


Figure 1. Macroeconomic Volatility and the ZLB

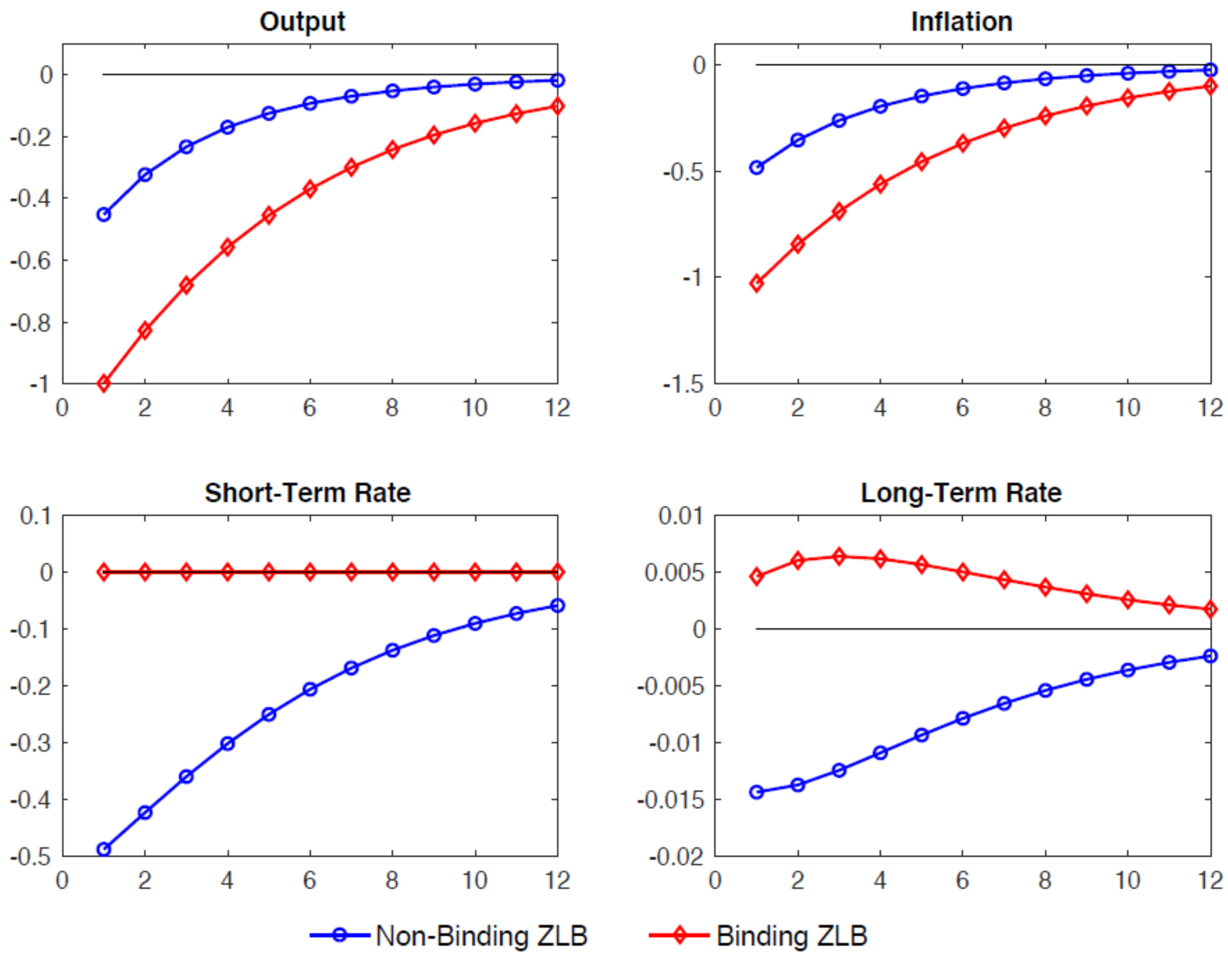


Figure 2. The Impact of a Binding ZLB on the Dynamic Effects of a Demand Shock
Baseline Interest Rate Rule

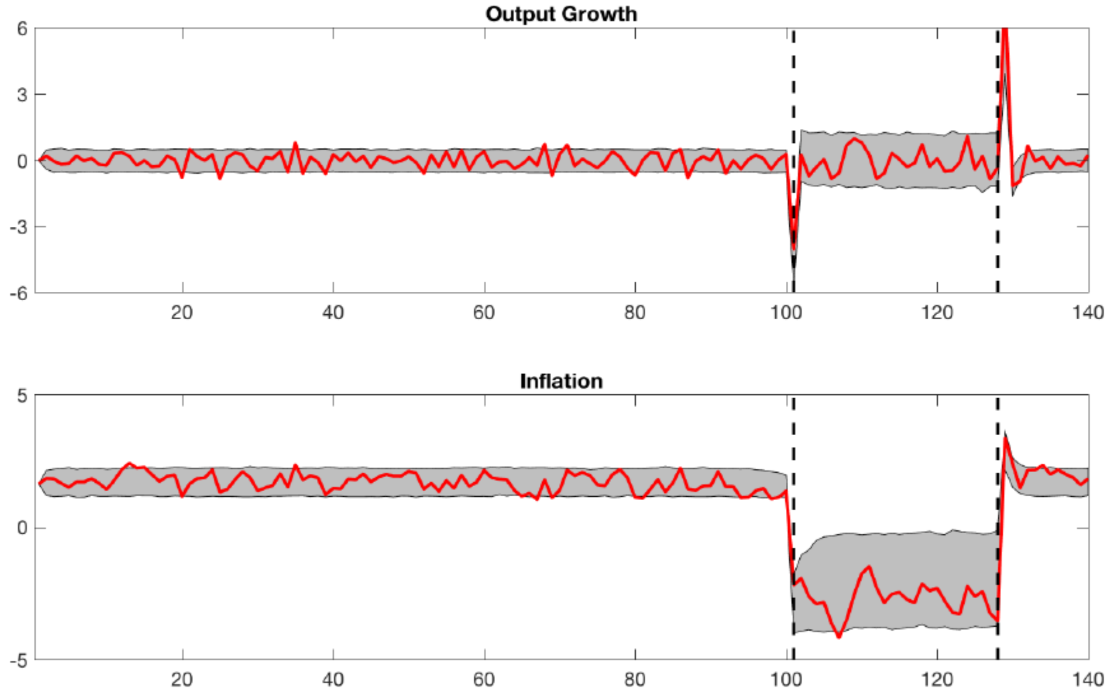
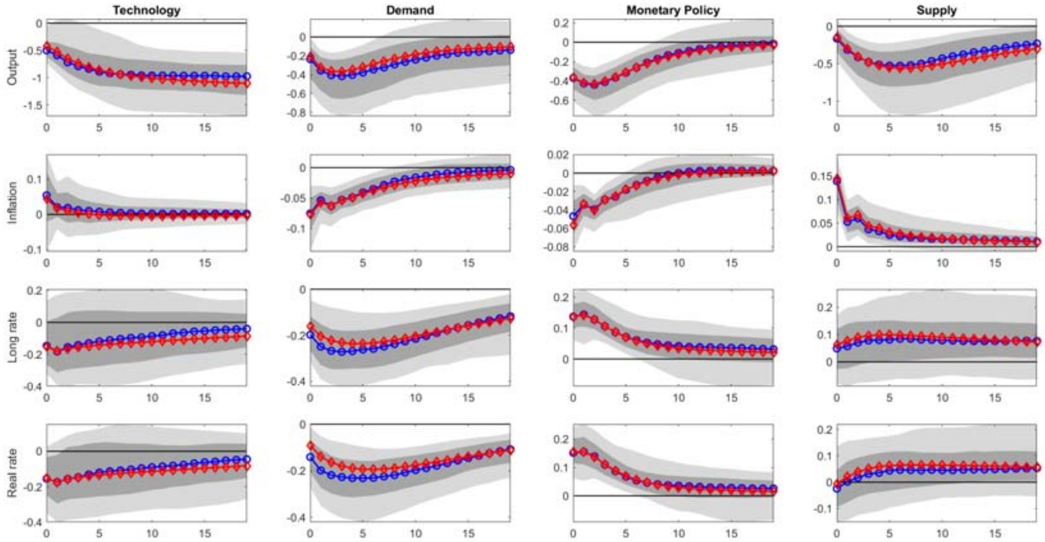


Figure 3. Macroeconomic Volatility and the ZLB: Model Simulations
Baseline Interest Rate Rule



blue: 2002Q1-2008Q4 red: 2009Q1-2015Q4

Figure 4a. Dynamic Responses: The Impact of the Binding ZLB
Short sample

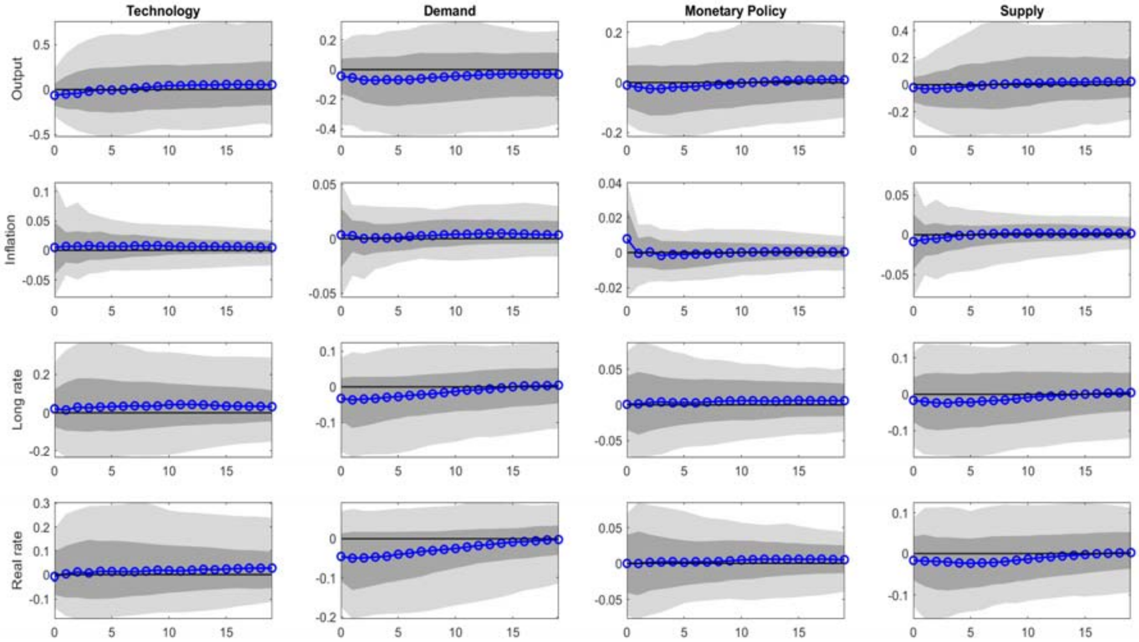
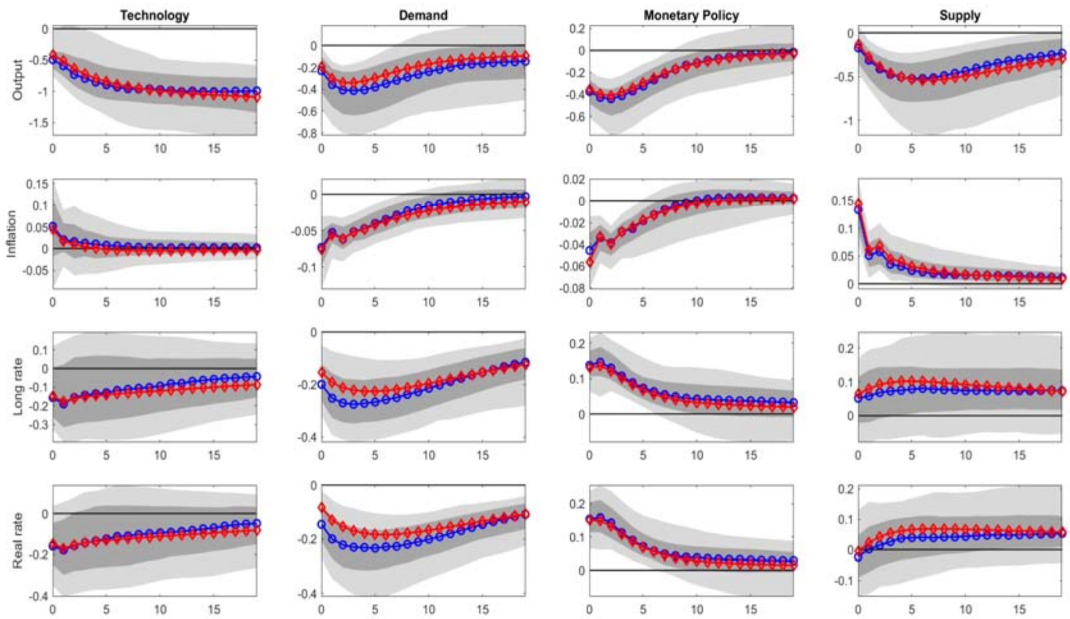


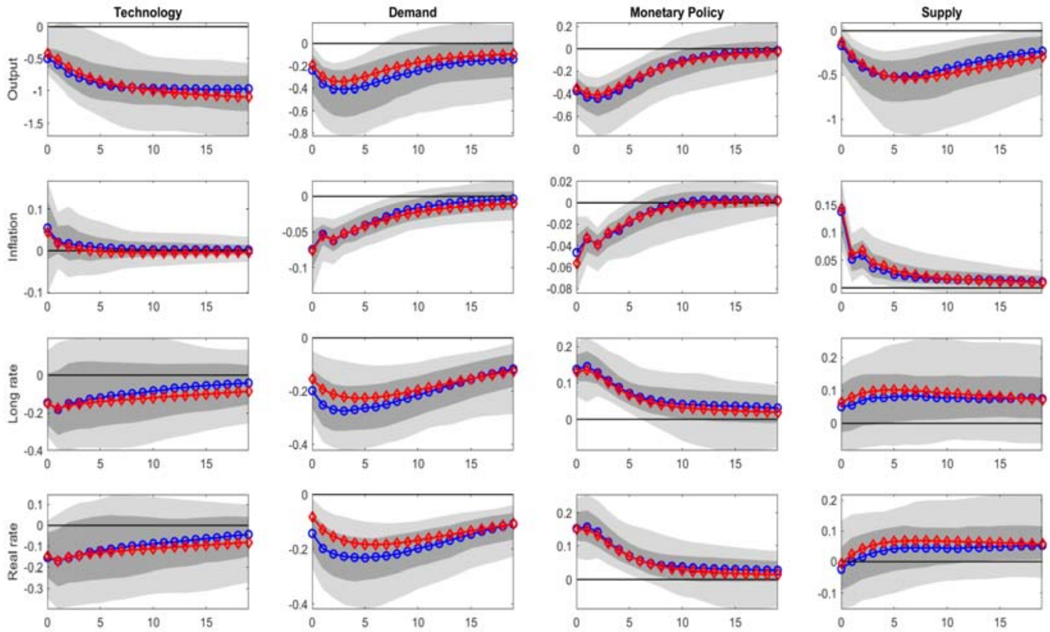
Figure 4B. Dynamic Response Differentials: The Effect of the Binding ZLB

Short sample



blue: 2002Q1-2007Q4 red: 2010Q1-2015Q4

Figure 4C. Dynamic Responses: The Effect of the Binding ZLB
Short sample excluding Great Recession



blue: 1984Q1-2008Q4 red: 2009Q1-2015Q4

Figure 4D. Dynamic Responses: The Effect of the Binding ZLB

Extended pre-ZLB sample

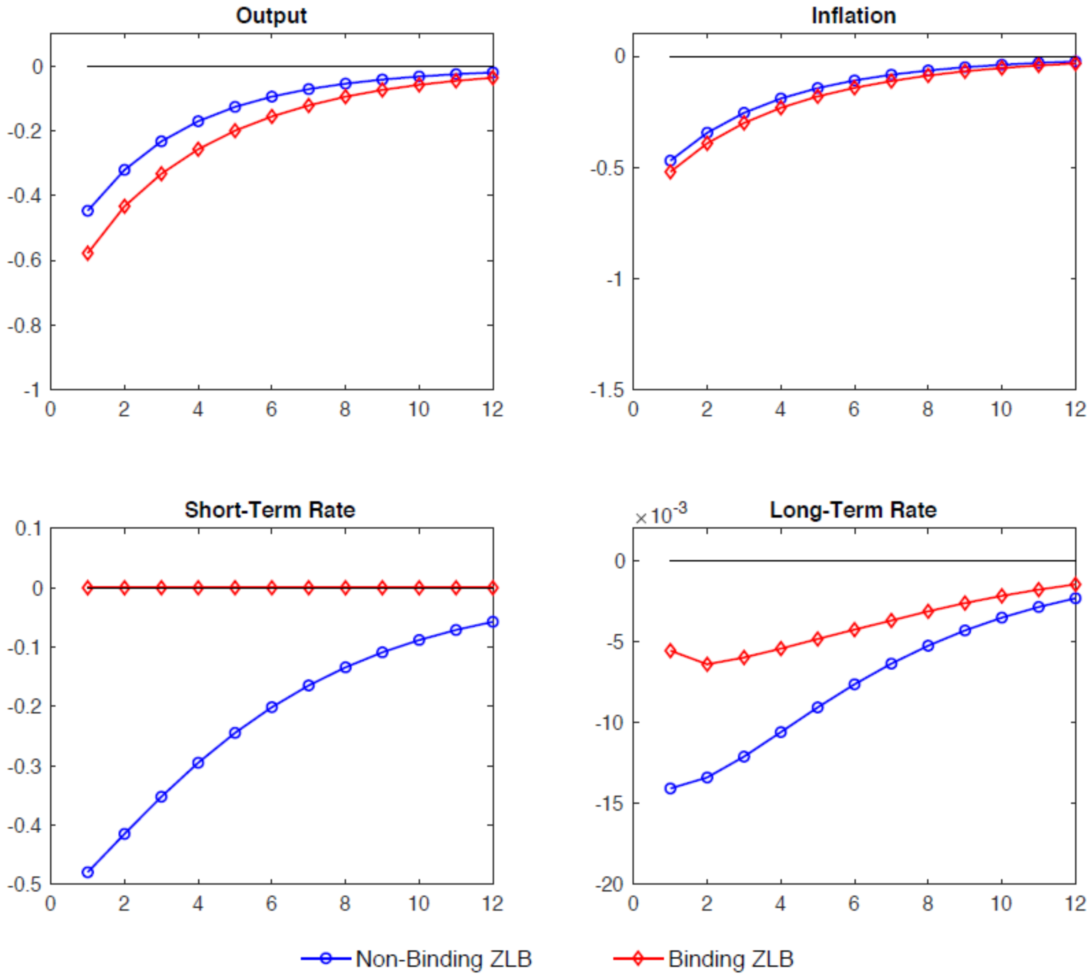


Figure 5. The Impact of a Binding ZLB on the Dynamic Effects of a Demand Shock
Shadow Rate Rule

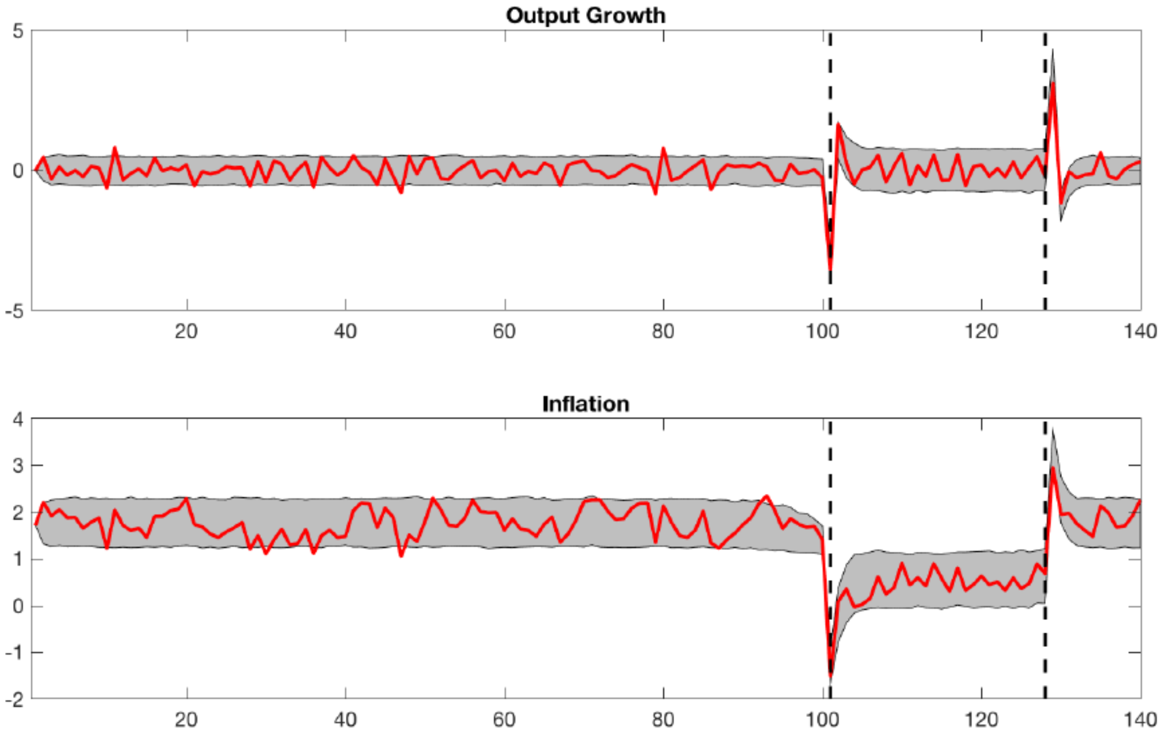


Figure 6. Macroeconomic Volatility and the ZLB: Model Simulations

Shadow Rate Rule