Balance Sheet Channel with Information-Trading Frictions in Secondary Markets

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Abstract

This paper develops a theory of the balance sheet channel that places a central emphasis on the liquidity of secondary markets for macro-contingent claims. We show that the presence of dispersed information and imperfect competition in secondary markets, interacted with financial constraints, results in mispricing and misallocation of aggregate risk, distorts aggregate investment, and exacerbates asset price and output volatility. The magnitude of balance sheet amplification effects becomes endogenously tied to the severity of market frictions, which likely vary over time and across economies. The laissez-faire equilibrium is constrained inefficient due to a novel externality originating from rent-extracting behavior of agents in secondary markets. Optimal corrective policy boosts secondary market liquidity through subsidies to trade in macro-contingent claims, which enhances aggregate risk-sharing and stabilizes the business cycle.

JEL: E32, E44, G10.

Keywords: balance sheet channel; secondary markets; risk-sharing; liquidity; information dispersion; trading frictions; mechanism design; financial regulation.

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1 Introduction

The global recession of the last decade has once again underscored the importance of financial market imperfections in the amplification and propagation of economic shocks. We witnessed how the concentration of aggregate (e.g., real-estate) risks on the balance sheets of leveraged agents, combined with the malfunctioning of secondary markets where such risks could be traded (e.g., MBS), allowed even small disturbances to be magnified into a full-blown financial crisis. Although the macroeconomics literature on the *balance sheet channel* has long recognized that concentrated risks may give rise to powerful feedback effects, it is less well understood what market frictions may prevent aggregate risks from being shared more widely in the population.[1] Answering this question is imperative not only for understanding the origins of economic fluctuations, but also for the design of optimal corrective policy.

In light of this, we develop a theory of the balance sheet channel, which places a central emphasis on the liquidity of secondary markets for macro-contingent claims. We show that dispersed information and imperfect competition in these markets –which we term *information-trading frictions*– can result in mispricing and misallocation of aggregate risk, distort aggregate investment, and exacerbate asset price and output volatility. Our theory links the strength of balance sheet amplification effects to the severity of information-trading frictions, which likely vary across time, markets and economies. We uncover a novel source of (constrained) inefficiency of laissez-faire equilibrium that originates from rent-extracting behavior of agents in secondary markets. We characterize the optimal corrective policy and find that, in contrast to the more conventional macro-prudential policies, it aims to boost secondary market liquidity, which enhances risk-sharing and stabilizes the business cycle.

We build upon a canonical framework of the balance sheet channel in the spirit of Kiyotaki and Moore (1997). Financially constrained entrepreneurs (borrowers) undertake productive long-term projects and finance them by issuing claims to investors (lenders). The cashflows of entrepreneurs' projects are exposed to an aggregate shock that *may* generate fluctuations in entrepreneurs' net worth, which in turn trigger liquidations of productive capital, destabilizing the prices of capital and aggregate output. Crucially, the extent of such net worth fluctuations depends on the type of claims with which entrepreneurs choose to finance their projects, i.e., on the allocation of aggregate risk between entrepreneurs and investors.

At the center of our model is the investors' desire for liquidity; namely, due to idiosyncratic

[1] Balance sheet channel broadly refers to feedback effects between the health of borrowers' balance sheets (e.g., net worth) and the general economic activity (e.g., asset prices, output). See the seminal papers by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), and some recent contributions by He and Krishnamurthy (2011) and Brunnermeier and Sannikov (2014). With a few exceptions, much of the literature on the balance sheet channel simply assumes that financial markets for trading aggregate risks are absent.

liquidity needs, investors value the option to re-trade their claims in secondary markets. These markets, however, are prone to information-trading frictions. We introduce an *information friction* by supposing that prior to trade in secondary markets investors observe dispersed private signals about the aggregate state of the economy. We introduce a *trading friction* by supposing that secondary markets are imperfectly competitive: an investor who wants to re-trade his claims can do so only with finitely many other investors. We treat these frictions as primitive features of our economy, and we motivate them further below. As we discuss next, the interaction of these information-trading frictions with entrepreneurial financial constraints are a crucial determinant of how aggregate risk is priced and allocated in the economy.

To fix ideas, we begin our analysis by considering a useful benchmark economy, in which information-trading frictions are shut down. As secondary markets are now frictionless, the claims issued by entrepreneurs are always priced fairly and, as a result, aggregate risk is always allocated efficiently among all agents, i.e., there is full risk-sharing. Even though overall economic activity may be depressed due to financial constraints, entrepreneurs manage to fully insulate their net worth from fluctuations through the issuance of state-contingent claims. This endogenously shuts down the balance sheet channel. Though stark, this benchmark provides a clear illustration of how the agents' ability to trade contingent claims in frictionless markets can mute the impact of aggregate shocks on economic activity. This point has also been made by Krishnamurthy (2003) and, more recently, by Di Tella (2017).

We then turn to the economy of our interest, in which secondary markets suffer from information-trading frictions. In this economy, the pricing and allocation of claims in secondary markets affects how investors value the claims issued by entrepreneurs ex-ante. In particular, "less liquid" claims are less attractive to issue as investors demand a compensation for holding such claims. One of this paper's contributions is to show that, in the presence of information-trading frictions, claims that are more contingent on *aggregate* states are effectively less liquid; moreover, the required compensation associated with such "illiquidity" is determined in general equilibrium. It is worth noting that typical asymmetric-information-based explanations for why the provision of state-contingent claims may be limited (e.g., Hölmstrom (1979), Townsend (1979), Myers and Majluf (1984)) do not really apply to aggregate states: first, atomistic agents cannot influence aggregate outcomes, ruling out moral-hazard-based theories; and, second, it is unlikely that sellers of claims have exclusive access to information about aggregate variables, ruling out adverse-selection-based theories.

We employ a mechanism design approach and allow each investor to design the optimal trading arrangement by which to re-sell his claims in secondary markets; that is, each investor decides the payments to collect from and the claims to allocate to buyers, i.e., other investors who want to buy claims. Due to information-trading frictions, the investor knows that as a seller he will be faced with finitely many buyers who have dispersed signals about the aggregate state. We show that state-contingent claims (and thus aggregate risks) are systematically mispriced and misallocated in secondary markets, despite the optimal design of trading arrangements. Intuitively, the interaction of information dispersion with trading frictions gives each buyer of claims some market power over his private information. This allows the buyers to extract rents from the sellers, but only when the traded claims are contingent on the aggregate state, as only then are the buyers' private signals useful for valuing them. By designing optimal trading arrangements, investors minimize but cannot eliminate these rents.

A key tension arises in equilibrium. On the one hand, risk-neutral entrepreneurs become averse to fluctuations in their net worth, as these may generate liquidations of productive capital precisely at times when its prices are depressed. As a result, entrepreneurs want to transfer aggregate risk to investors by issuing fewer claims against states in which capital prices are low. This benefit from risk-sharing depends on the aggregate supply of state-contingent claims by entrepreneurs, which determines the aggregate liquidations and, thus, the fluctuations in the price of capital. On the other hand, risk-neutral investors become averse to fluctuations in the value of the claims they hold, as they anticipate that state-contingent claims may be mispriced/misallocated in secondary markets. As a result, investors want to be compensated for absorbing aggregate risk. This cost to risk-sharing also depends on the aggregate supply of claims, since investors hold and re-trade diversified portfolios.

The equilibrium resolves the above tension by linking the extent of equilibrium risk-sharing, and thus of amplification effects, to the severity of information-trading frictions relative to the gains from risk-sharing, i.e., the anticipated severity of aggregate shocks and of the resulting liquidation costs. When the information-trading frictions are relatively severe, the equilibrium features no risk-sharing: entrepreneurs optimally finance their projects with non-contingent claims, as the claims prices are too distorted. Entrepreneurs thus willingly expose their net worth to large fluctuations, enabling the balance sheet channel to kick in. On the other hand, when information-trading frictions are not as severe, the equilibrium features partial risk-sharing: entrepreneurs optimally finance their projects with state-contingent claims but obtain incomplete insurance from investors as the latter still perceive its provision costly. In this case, the effects of the balance sheet channel are dampened though not fully eliminated. As information-trading frictions vanish, the equilibrium gradually converges to the benchmark economy with full risk-sharing, in which the balance sheet channel becomes inoperative.

These results are thus consistent with the often-expressed view that it is lack of liquidity in markets for macro-contingent claims that hinders aggregate risk-sharing and destabilizes the business cycle (Case, Shiller and Weiss, 1991; Shiller, 1994; Caballero, 2003; Mian, 2013). As we discuss next, by formalizing this view, our theory helps us think about the potential inefficiencies stemming from such illiquidity and appropriate policy responses.

We find that the laissez-faire equilibrium is constrained inefficient due to a novel externality that originates from the manner by which secondary markets price and allocate aggregate risk. To establish this, we solve the problem of a social planner who chooses the agents' allocations to maximize social welfare, but who is constrained by the same primitive frictions as the agents. We show that the laissez-faire allocations differ from those of the constrained planner for the following reason. When choosing his portfolio of claims and trading arrangements by which to re-sell them, each investor takes as given the claims and the trading arrangements of other investors; hence, the rents that he expects to extract from them when buying claims. As a result, each investor behaves in a way that minimizes the rents that others extract from him in secondary markets, which in turn distorts the allocation of aggregate risk in the economy. In equilibrium, however, these rents are ex-post transfers among ex-ante identical investors and thus do not generate a welfare loss. As the planner understands this, she is able to achieve a more efficient allocation of aggregate risk in the economy.

We show that the constrained efficient allocations can be decentralized through plausible interventions in secondary markets, such as subsidies to the sales of risky claims or government purchases and sales of risky claims. The upshot of these interventions is that, by correcting the mispricing and misallocation of state-contingent claims, they boost secondary market liquidity and, as a result, enhance aggregate risk-sharing and stabilize economic fluctuations. We note that such policy prescriptions differ markedly from those obtained from the more conventional models with financial constraints (e.g., Caballero and Krishnamurthy (2003); Lorenzoni (2008); Korinek (2011); Bianchi (2011)). In these models, pecuniary externalities arise either because capital prices enter directly into financial constraints or because capital price movements have distributional effects (Dávila and Korinek, 2017). Such externalities are intentionally not present in our setting, which allows us to clearly isolate the new inefficiencies and policy implications stemming from the presence of information-trading frictions in financial markets.

Even though our model is stylized and abstracts from a number of institutional details of real-world markets, it provides a useful framework for interpreting some of the observed dynamics in the run-up to the 2008-09 financial crisis.[2] A puzzling feature of this episode was the financial sector's heavy exposure to real-estate risks. During the years of rising real-estate prices and low uncertainty, banks were able to share part of these risks with the broader population through issuance/purchase of structured products and credit default swaps. In 2007, however, as uncertainty over real-estate prices and disagreements over real-estate linked assets rose, it became harder for banks to continue off-loading some of their real-estate exposures.

[2]See Baily, Litan and Johnson (2008) and Brunnermeier (2009) for an anatomy of these dynamics.

The inability to share real-estate risks effectively was in turn essential in the balance-sheet effects that unfolded following the Lehman's collapse in September 2008. Looking through the lens of our model, we can interpret these perverse developments as being triggered by an increase in the severity of information-trading frictions in markets for real-estate exposed securities, which in turn exacerbated the distortions in the pricing and allocation of real-estate risks. Furthermore, our model sheds light on a number of policies implemented in the midst of the crisis, whereby governments intervened to support trade in markets for real-estate exposed securities (e.g., TAF and TARP).[3] Our theory suggests that, beyond the direct stimulative effect on secondary market liquidity, these interventions may also help spread real-estate related risks more efficiently throughout the population, ameliorating balance sheet effects and their repercussions for the real economy.

The information-trading frictions, which are at the heart of this paper, are consistent with different strands of stylized evidence. For example, in terms of information frictions, Mankiw et al. (2003) find substantial heterogeneity in inflation forecasts among professional forecasters, economists and consumers, while Dovern et al. (2012) document similar findings in a cross-country study of surveys of professional forecasters. More recently, Coibion and Gorodnichenko (2012, 2015) argue that disagreements are pervasive across a variety of population subgroups and macroeconomic variables. As for trading frictions, note that in practice an entrepreneur can insure her net worth against an aggregate state in two ways. First, she could issue entrepreneur-specific liabilities that are contingent on that state, which would naturally trade in decentralized, frictional over-the-counter markets (Duffie, 2010; Rocheteau and Weill, 2011). Alternatively, she could issue non-contingent liabilities while purchasing/issuing standardized derivatives correlated with that aggregate state. Though in advanced economies some derivatives markets are well-developed, e.g., currency and commodity futures; many others still remain extremely thin, e.g., real-estate, GDP (Shiller, 1994).[4] Moreover, the theory developed here predicts that aggregate investment and sensitivity of business cycles to shocks depends, among other things, on the development of secondary markets in which the associated risks can be traded as well as the extent of disagreements in the economy.

This paper belongs to a large literature on the balance sheet channel, starting with the

[3]The Term Auction Facility (TAF), established in December 2007, allowed commercial banks to borrow from the FED against a broad set of collateral, including mortgage-backed securities. The Troubled Asset Relief Program (TARP) was instead initiated by the US Treasury in October 2008 and consisted, among other things, of government purchases of mortgage-backed securities from the banking sector. See Baily et al. (2008), Brunnermeier (2009) and Blanchard (2009) for detailed accounts of these interventions.

[4]In referring to the process of derivatives market formation, Shiller (1994) says that "the history of basic economic institutions is one of punctuated equilibrium, where basic economic institutions remain largely unchanged for long periods of time, only to be superseded by new institutions whose advent can only be attributed to innovation." Tufano (1989) provides a narrative of the process of financial innovation.

seminal works of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), and followed by Krishnamurthy (2003); Rampini and Viswanathan (2010); He and Krishnamurthy (2011); Brunnermeier and Sannikov (2014); Di Tella (2017); Bocola and Lorenzoni (2018); Jeenas (2018), among others. We contribute to this literature by developing a theory that places a central emphasis on the functioning of secondary markets. To the best of our knowledge, this is the first paper to provide a comprehensive framework that links the frictions inhibiting secondary market liquidity for macro-contingent claims to the severity of balance sheet amplification effects. In doing so, we uncover new inefficiencies that originate from rent-extracting activities in financial markets, which leads to a novel perspective on optimal corrective policy.

The role of information-trading frictions in financial markets for aggregate investment has also been explored by Albagli, Hellwig and Tsyvinski (2017), though in a different setting.[5] They embed the noisy REE model of Albagli, Hellwig and Tsyvinski (2011) into a general equilibrium setting and show that lack of information aggregation, resulting from noise-trading and limits to arbitrage, leads to incumbent shareholders seeking to extract rents from noise traders, which distorts aggregate investment. Due to the presence of noise traders, entrepreneurial claims in their setting can be over- or under-valued, and thus there can be over- or under-investment relative to first-best. In contrast to them, our focus is on how information-trading frictions introduce distortions to aggregate risk-sharing, and on the implications this has for the amplification and propagation of aggregate shocks. Furthermore, as all agents are rational in our setting, state-contingent claims are systematically under-valued; as a result, there is always under-investment relative to first-best and risk-sharing is always limited. Finally, the mechanism design approach we employ ensures that our results come from the primitive information-trading frictions, which facilitates our normative analysis.

This paper is related to the classical literature on mechanism design under asymmetric information (Myerson and Satterthwaite, 1983). It is well-known that in environments with correlated types (as in our case, due to dispersed signals), there exist trading mechanisms that are both efficient and able to extract the mechanism participants' full surplus (Cremer and McLean, 1988; McAfee, McMillan and Reny, 1989). We show, however, that this result does not hold whenever traders can threaten to leave the trading mechanism *after* learning their mechanism allocations (i.e., prices and quantities of claims). This generates a set of "ex-post" participation constraints, which are natural for our applications to financial markets.[6] As a

[5] See, also, Gorton and Ordonez (2014), Asriyan, Laeven and Martin (2019) and Kantorovitch (2020) for how information acquisition about the quality of collateral or firm fundamentals interacts with capital markets and the aggregate economy. See Hollifield and Zetlin-Jones (2017) and Broner, Martin and Ventura (2008) for how secondary markets foster maturity-/risk-transformation and mitigate problems due to weak enforcement institutions, respectively. Bond, Edmans and Goldstein (2012) provide an overview of the literature on real effects of financial markets.

[6] As we discuss in Section 3.2, ex-post implementation constraints can also be justified on grounds that

result, we show that the claims traded in the optimal mechanism can now be both mispriced and allocated inefficiently among agents, a feature that is critical for our main results.

This paper is also related to the literature on security design showing how claims can be mispriced/misallocated in the presence of asymmetric information (e.g., Gorton and Pennacchi (1990); Nachman and Noe (1994); DeMarzo and Duffie (1999); Biais and Mariotti (2005); Axelson (2007); Dang, Gorton and Holmström (2012); Daley, Green and Vanasco (2016)). We contribute to this literature on two fronts. First, we show that information dispersion among many agents, which is more plausible for aggregate states than adverse selection, generates mispricing and misallocation of aggregate risk but only if interacted with trading frictions. To this end, we employ a mechanism design approach since in settings with dispersed information the choice of trading protocols may be important. Second, our tractable characterization of the optimal mechanism allows us to embed it in a general equilibrium environment to study the inefficiencies stemming from how information dispersion distorts the pricing and allocation of aggregate risk and, as a result, the aggregate economic activity.

Closer in spirit to our paper is the recent work by Hartman-Glaser and Hébert (2019), who also study the effects of information frictions on aggregate risk-sharing. They consider a security design game, where banks can offer securities to households indexed to real-estate prices. They show that, if banks are better informed about the quality of the index, there can exist an equilibrium in which all banks optimally offer unindexed securities, due to households' fear that indexed securities are 'lemons.' Besides different mechanisms, our focus is not on whether there is indexation, but on how information frictions affect the pricing and allocation of aggregate risk, aggregate economic activity and the design of optimal corrective policy.

Finally, this paper follows a long tradition in economics of studying the aggregate implications of dispersed information (e.g., Lucas (1972); Lorenzoni (2009); Angeletos and La'O (2013); Gaballo (2017)). Relative to this literature, this paper emphasizes the interaction between information dispersion and trading frictions in financial markets, which also relates our work to the literature on asset pricing with heterogeneous information and limits to arbitrage (e.g., Grossman and Stiglitz (1980); Albagli et al. (2011)). In our model, it is the finiteness of traders within each trading arrangement, rather than the presence of noise traders, what prevents information aggregation and distorts claims prices; this in turn facilitates the mechanism design approach we employ and our normative analysis.

The paper is organized as follows. In Section 2, we present the model. In Sections 3 and 4, we characterize the equilibrium and its implications for risk-sharing, investment and output. In Section 5, we study the efficiency properties of the equilibrium and the implications for corrective policy. We conclude in Section 6. All proofs are relegated to the Appendix.

the mechanism designer has concerns for robustness.

2 The Model

There are three periods, indexed by $t \in \{0,1,2\}$, and two sets of agents, entrepreneurs and investors, each of unit mass. There are two goods, perishable consumption and durable capital.

Preferences and Endowments. Entrepreneurs are risk-neutral with lifetime utility $U^E = \mathbb{E}\{c_0^E + c_1^E + c_2^E\}$, where c_t^E is an entrepreneur's consumption in period t. Investors are also risk-neutral, but they may be subject to preference shocks at $t = 1$. Their lifetime utility is $U^I = \mathbb{E}\left\{c_0^I + \beta\left(c_1^I + c_2^I\right)\right\}$, where c_t^I is an investor's consumption in period t and where $\beta \in \{0,1\}$ denotes his preference type, distributed independently among investors with $\lambda \equiv \mathbb{P}(\beta = 0) \in \left[0, \frac{1}{2}\right]$. Thus, λ is also the fraction of investors who are shocked at $t = 1$. The sole purpose of the preference shocks is to generate gains from trade in secondary markets.

Entrepreneurs have access to long-term productive projects, but have no endowments to fund the projects on their own. They can finance them by issuing claims to investors, each of whom has a large endowment e of the consumption good at $t = 0$.

Technology. At $t = 0$, each entrepreneur creates k_0 units of capital by spending $\chi(k_0)$ units of the consumption good, where $\chi(\cdot)$ is increasing and convex with $\chi(0) = 0$. At $t = 1$, she receives ak_0 consumption goods and chooses continuation scale k_1 by liquidating or buying capital in a competitive capital goods market at price p. Finally, at $t = 2$, the entrepreneur receives Ak_1 consumption goods. The intermediate "cashflow" a depends on an aggregate state $s \in \{l,h\}$, where $a(l) < a(h)$ and $\pi(h) \equiv \mathbb{P}(s = h) \in (0,1)$, whereas the final cashflow A is for simplicity deterministic.

The units of capital liquidated by the entrepreneurial sector can be absorbed by a "traditional" sector, composed of a mass of competitive firms, owned by the investors. These firms are less productive than the entrepreneurs: by employing $\widehat{k}_1$ units of capital at $t = 1$, each firm produces $gA\widehat{k}_1$ units of consumption goods at $t = 2$, where $g \in (0,1)$.[7]

Financial Markets. Period $t = 0$ is divided into two stages. First, there is an *issuance stage*, where entrepreneurs issue claims to investors in a competitive market that prices each unit of consumption at date t and state s at $q_t(s)$. After the issuance stage, the investors learn their preference shocks and receive dispersed signals about the aggregate state. At this point, there is a *trading stage*, where investors can re-trade claims in a frictional secondary market.[8]

Agents consume at the end of each period, and the economy's timeline is depicted in Figure

[7] This technological assumption is standard in the literature on the balance sheet channel (Kiyotaki and Moore, 1997; Lorenzoni, 2008).

[8] That the issuance market is competitive is convenient, but not essential. What is crucial is that the claims issued by entrepreneurs are re-traded in a frictional market at some point before they mature.

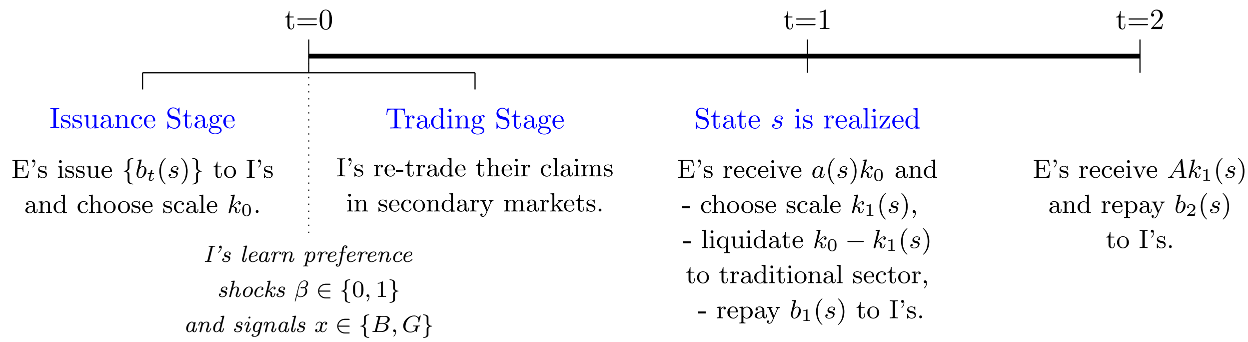

Figure 1: **Timeline**. E's denotes the entrepreneurs, whereas I's denotes the investors.

1. Our economy is affected by two types of frictions, which are at the heart of this paper and which we describe next.

Financial frictions. An entrepreneur can always walk away with her $t = 2$ output. Let $b_t(s)$ denote the claims issued by the entrepreneur for date t and state s. Then, any payments she can credibly promise to make must satisfy the following limited pledgeability constraint: $b_2(s) \leq 0$ for $s \in \{l, h\}$. This friction is standard in the literature on the balance sheet channel, as it creates scope for entrepreneurial net worth to matter for investment.

Information-trading frictions. After the issuance stage, each investor privately learns her preference type $\beta \in \{0, 1\}$ and signal $x \in \{B, G\}$ about the aggregate state s (information friction). Conditional on the aggregate state, the signals are distributed independently among investors, and $\mathbb{P}(s = h|x = G) > \mathbb{P}(s = h|x = B)$, i.e., investors with *G*ood (*B*ad) signals are optimistic (pessimistic) about the state. An investor who wants to re-trade his claims in secondary markets must post them in a *trading arrangement*, which is a mechanism that allocates these claims to and collects payments from a finite number $n \geq 1$ of randomly selected other investors, whom we refer to as *traders* (trading friction). Importantly, the format of trading arrangements is not arbitrary and is chosen optimally by each investor. These information-trading frictions are an innovation of this paper, which highlights the role of secondary financial markets for aggregate risk-sharing and investment.

To facilitate the exposition, in the main analysis, we impose directly that only the impatient investors (fraction λ) post their claims in trading arrangements, whereas the remaining investors (fraction $1-\lambda$) are traders; for convenience, we set $n\lambda = 1-\lambda$ in the main text, so that there is an exact match between trading arrangements and traders. A microfoundation for such sorting of investors is provided in Appendix C.[9] All trades are executed simultaneously.

[9] We suppose common knowledge of gains from trade within each trading arrangement, which by arguments akin to Milgrom and Stokey (1982) ensures that there is no trade between investors of same preference type.

2.1 Preliminaries

We will assume throughout that the investors' endowment of consumption goods is sufficiently large so that they effectively have "deep pockets."

Assumption 1 *The endowment satisfies:* $e > 2\chi(k)$ *for* $k > 0$ *s.t.* $\chi(k) = (a(h) + A)k$.

This will ensure that in equilibrium the patient investors' consumption is positive at all dates. It will in turn imply that the equilibrium risk-free interest rate equals to one, i.e., the claims prices satisfy:

$$q_1(s) = q_2(s) = q(s),\ \sum_s q(s) = 1. \tag{1}$$

Since in equilibrium the price of capital will be bounded by the discounted return to capital in the traditional and the entrepreneurial sectors, property (1) in turn implies that:

$$gA \leq p(s) \leq A\ \forall s. \tag{2}$$

To simplify the analysis, we impose the properties (1) and (2) in the agents' problems, and we verify them formally in Appendix B.

We will also make the following technological assumptions.

Assumption 2 *The economy's technology satisfies:*

(i) $\chi(k) < \sum_s \pi(s)a(s)k$ *for* k *s.t.* $\chi'(k) = \sum_s \pi(s)(a(s) + gA)$.

(ii) $\chi(k) > a(l)k$ *for* k *s.t.* $\chi'(k) = \frac{\pi(h)g}{\pi(h)g+1-\pi(h)}(a(h) + A) + \left(1 - \frac{\pi(h)g}{\pi(h)g+1-\pi(h)}\right)(a(l) + gA)$.

(iii) $\sum_s \pi(s)a(s) < a(l) + gA < a(h)$.

These parametric assumptions do not drive our main results, but they allow us to focus the equilibrium analysis on the economically interesting cases; namely, that in equilibrium capital liquidations never occur in state h, but that liquidations may occur in state l depending on distortions in the claims prices. More concretely, they guarantee that it is sub-optimal for entrepreneurs to liquidate capital with probability one (Assumption 2(i)); that, when issuing non-contingent claims, some liquidations (but only in state l) are optimal (Assumption 2(ii) and second inequality in Assumption 2(iii)); and that entrepreneurs can raise more funds by issuing non-contingent claims and liquidating capital than by issuing contingent claims and avoiding liquidations with probability one (first inequality in Assumption 2(iii)).

Finally, it is useful to define a notion of first-best scale of investment, k^{FB}, which would arise in a frictionless economy:

$$\chi'(k^{FB}) = \sum_s \pi(s)(a(s) + A). \tag{3}$$

As we will see, k^{FB} will provide an upper bound on the investment scale in our economy.

2.2 Entrepreneurs' Problem

All entrepreneurs are identical. The representative entrepreneur takes as given the prices of claims $\{q(s)\}$ and capital $\{p(s)\}$, and chooses the investment scale k_0, the continuation scales $\{k_1(s)\}$, and the claims $\{b_t(s)\}$ to maximize her lifetime welfare:

$$\sum_s q(s)\left(b_1(s) + b_2(s)\right) - \chi(k_0) + \sum_s \pi(s)\left[(a(s) + p(s))k_0 - p(s)k_1(s) - b_1(s) + Ak_1(s) - b_2(s)\right] \tag{P1}$$

subject to the following set of constraints:

$$\chi(k_0) \leq \sum_s q(s)\left(b_1(s) + b_2(s)\right) \tag{4}$$

$$p(s)k_1(s) + b_1(s) \leq (a(s) + p(s))k_0 \ \forall s, \tag{5}$$

$$b_2(s) \leq 0 \ \forall s, \tag{6}$$

$$0 \leq k_1(s) \ \forall s. \tag{7}$$

The first two constraints simply impose consumption non-negativity. Constraint (4) states that at $t = 0$ the entrepreneur's expenditures on investment cannot exceed the funds raised by issuing claims to investors. Constraint (5) states that at $t = 1$ the entrepreneur's expenditure on claims repayments plus net purchases of capital cannot exceed the output of her project. Constraint (6) is the limited pledgeability friction and constraint (7) requires that the continuation scale be non-negative. The entrepreneur's consumption non-negativity at $t = 2$ is implied by limited pledgeability and non-negativity of continuation scale.

2.3 Traditional Sector Firms' Problem

The problem of the traditional sector firms is simple. At $t = 1$ and state s, each firm in this sector takes the price of capital $p(s)$ as given and chooses to purchase $\widehat{k}_1(s) \geq 0$ units of capital to maximize its discounted profits:

$$\Pi(s) = gA\widehat{k}_1(s) - p(s)\widehat{k}_1(s). \tag{8}$$

Since in equilibrium the price of capital will be (weakly) greater than gA, these firms will only demand capital when $p(s) = gA$. The investors, who own these firms, have sufficient resources to fund these activities (see Appendix B). Finally, the investors' claims on this sector have value of zero, since the traditional sector firms earn zero discounted profits in each state.

2.4 Investors' Problem

At the issuance stage, all investors are identical. The representative investor takes as given the claims prices $\{q(s)\}$, and he chooses to purchase claims $\{d_t(s)\}$ and the trading arrangement in which to re-trade them, denoted by μ.[10] The set of feasible trading arrangements from which the investor chooses is denoted by $\mathcal{M}$ and will be described shortly.

At the trading stage, the investors learn their preferences and signals about the aggregate state. An impatient investor posts the claims $\{d_t(s)\}$ in the trading arrangement μ that he has designed, which gives him a net expected payoff $V(\{d_t(s)\}, \mu)$. A patient investor does not post his claims, but he participates as a trader in the trading arrangements of other investors, which gives him a net expected payoff of $\widetilde{W}$. Note that this payoff is an equilibrium object as it depends on the joint design of claims and trading arrangements of other investors.

Thereferore, the investor's problem is to choose $\{d_t(s)\}$ and μ to maximize lifetime welfare:

$$e - \sum_s q(s)\left(d_1(s) + d_2(s)\right) + \lambda V\left(\{d_t(s)\}, \mu\right) + (1-\lambda)\left(\widetilde{W} + \sum_s \pi(s)\left(d_1(s) + d_2(s)\right)\right), \tag{P2}$$

where recall that λ is the ex-ante probability that an investor becomes impatient. This problem is subject to two sets of constraints. First, the claims purchases must respect the investor's budget constraint at the issuance stage, i.e., $e \geq \sum_s q(s)(d_1(s) + d_2(s))$, and consumption non-negativity in periods 1 and 2, i.e., $d_t(s) \geq 0 \ \forall t, s$. Second, the trading arrangement μ must be in the feasible set $\mathcal{M}$, which we describe next.

Feasible Trading Arrangements

When designing a trading arrangement, the investor anticipates that, if impatient, he will have to sell his claims to privately informed traders (the patient investors). He designs an

[10] That the trading arrangement is chosen before the investor learns his private signal x avoids the unnecessary complications of analysing the mechanism design problem of a privately informed designer.

optimal mechanism, which solicits reports from the traders, and then allocates claims to and collects payments from them based on these reports. We focus on direct revelation mechanisms implementable as Bayesian Nash equilibria, in which each trader is willing to report his signal truthfully to the mechanism, given that other traders also report truthfully.

Because the risk-free rate between $t = 1$ and $t = 2$ equals one, for the design problem it suffices to keep track of the total claims held by the investor for state s, which we denote by $v(s) \equiv d_1(s) + d_2(s)$. Let $i \in \{1, ..., n\}$ index the traders matched with a given trading mechanism, and let θ^i denote report of trader i, who recall has a private signal x^i. The reports and the signals of traders other than i are for short denoted by θ^{-i} and x^{-i} respectively. Upon collecting the traders' reports $(\theta^1, ..., \theta^n)$, the mechanism proposes an allocation $\mathcal{A}^i(\theta^i, \theta^{-i}) \equiv (\omega^i(\theta^i, \theta^{-i}), \{v^i_s(\theta^i, \theta^{-i})\})$ to trader i, where $\omega^i(\theta^i, \theta^{-i})$ is the payment that trader i makes to the mechanism and $v^i_s(\theta^i, \theta^{-i})$ are the units of claims for state s that the mechanism transfers to the trader; when there is no trade with trader i, we say that the allocation is empty.[11] The trader can accept or reject this allocation, and his payoff from accepting it is:

$$U^i\left(x^i, \mathcal{A}^i\left(\theta^i, \theta^{-i}\right)\right) = \sum_s v^i_s\left(\theta^i, \theta^{-i}\right) \cdot \mathbb{P}\left(s|x^i, \mathcal{A}^i\left(\theta^i, \theta^{-i}\right)\right) - \omega^i\left(\theta^i, \theta^{-i}\right), \tag{9}$$

where $Prob\left(s|x^i, \mathcal{A}^i(\theta^i, \theta^{-i})\right)$ is trader i's belief that the state is s, given his signal and his allocation, where the latter may contain information about other traders' signals and, thus, about the aggregate state.

Naturally, the trading mechanism needs to satisfy a set of participation, incentive compatibility and feasibility constraints.

Participation (PC). We impose a form of "ex-post" participation constraint by supposing that trader i has the right to choose not to participate in the trading mechanism *after* learning his allocation, i.e., he participates if and only if the payoff in equation (9) is non-negative. Because the mechanism can always choose the empty allocation, it is without loss to focus on mechanisms which ensure that each trader participates in equilibrium:

$$U^i\left(x^i, \mathcal{A}^i\left(x^i, x^{-i}\right)\right) \geq 0 \;\forall i, x^i, x^{-i}. \tag{10}$$

This effectively implies that the designer cannot punish trader i were he to leave the mechanism upon learning his allocation $\mathcal{A}^i$, which might inform him of other traders' signals.

Incentive compatibility (IC). In order for a trader to report his signal truthfully, given that

[11]The implicit assumption that the mechanism does not condition the allocations on the signal of the selling investor economizes on notation, but it is not essential (see proof of Proposition 1 in Appendix A).

the other traders do so as well, it must be that:

$$\mathbb{E}\left\{U^i\left(x^i,\mathcal{A}^i\left(x^i,x^{-i}\right)\right)\middle|\, x^i\right\} \geq \mathbb{E}\left\{\max\left\{0,U^i\left(x^i,\mathcal{A}^i\left(\theta^i,x^{-i}\right)\right)\right\}|x^i\right\} \;\; \forall i,x^i,\theta^i. \tag{11}$$

Note that the IC's take into account that each trader can potentially deviate to both misreport his signal and then choose not to trade.

Feasibility (FC). The claims that the mechanism allocates to the traders cannot exceed the claims available to the investor:

$$\sum_i v_s^i(\theta^i,\theta^{-i}) \leq v\left(s\right) \;\; \forall s,\theta^i,\theta^{-i}. \tag{12}$$

Assumption 1 will ensure that in equilibrium the mechanism allocations are consistent with the investors' consumption non-negativity (see Appendix B). If out of equilibrium an investor cannot afford to participate in the mechanism, we assume that he gets the empty allocation.

A trading arrangement is thus defined by $\mu = \{\mathcal{A}^i\left(x^i,x^{-i}\right)\}_{i,x^i,x^{-i}}$, and the set of feasible trading arrangements $\mathcal{M}$ consists of all such μ's satisfying (PC), (IC) and (FC). The expected payoff to the investor who trades his claims in the trading arrangement μ is:

$$V\left(\{v(s)\},\mu\right) = \mathbb{E}\left\{\sum_i \omega^i(x^i,x^{-i})\right\}, \tag{13}$$

whereas the ex-ante expected payoff to a trader from participating in this arrangement is:

$$W\left(\{v(s)\},\mu\right) = \mathbb{E}\left\{U^i(x^i,\mathcal{A}^i(x^i,x^{-i}))\right\}. \tag{14}$$

Importantly, note that when an investor designs the trading arrangement by which to trade his claims, he takes as given trading arrangements and the claims of other investors and, thus, her payoff $\widetilde{W}$ from participating in them. Thus, the investor's optimal trading arrangement maximizes $V(\{v(s)\},\mu)$. In equilibrium, of course, all investors will choose identical claims and trading arrangements, so that $\widetilde{W} = W$.

2.5 Equilibrium Notion

We are ready to define an equilibrium of our economy. In what follows, when necessary to avoid confusion, we will make use of capitalized letters in order to distinguish aggregate quantities from the individual ones: e.g., K_0 denotes the aggregate investment scale. We will focus on symmetric equilibria, in which the two coincide, i.e., $k_0 = K_0$, $k_1(s) = K_1(s)$,

$b_t(s) = B_t(s)$, $d_t(s) = D_t(s)$ $\forall t, s$.

Definition 1 *An equilibrium consists of prices* $\{q(s), p(s)\}$*, a trading arrangement* $\mu \in \mathcal{M}$*, and allocations* $\{k_0, K_0, k_1(s), K_1(s), \widehat{k}_1(s), \widehat{K}_1(s), b_t(s), B_t(s), d_t(s), D_t(s)\}$ *such that:*

1. *Given the prices* $\{q(s), p(s)\}$*, the allocations* $\{k_0, k_1(s), \widehat{k}_1(s), b_t(s), d_t(s)\}$ *and the trading arrangement* μ *solve the agents' problems P1 and P2,*

2. *The markets for claims and capital clear:* $B_t(s) = D_t(s)$ *and* $K_1(s) + \widehat{K}_1(s) = K_0$ $\forall t, s$.

3 Equilibrium Characterization

We now characterize the equilibrium of our economy. We first study the entrepreneurs' optimal investment, financing, and continuation decisions, for given asset prices. Then, we study the investors' optimal choice of trading arrangements. Finally, we combine these results to determine the equilibrium pricing of claims and capital.

3.1 Optimal Investment, Financing and Continuation

In this section, we characterize the solution to the entrepreneurs' problem. We solve this problem backwards.

At $t = 2$ and state s, the entrepreneur simply consumes her final output net of repayment, $Ak_1(s) - b_2(s)$, which recall is non-negative since $k_1(s) \geq 0$ and $b_2(s) \leq 0$.

At $t = 1$ and state s, the entrepreneur has net worth $(a(s) + p(s))k_0 - b_1(s)$, and she must decide the continuation scale, $k_1(s)$, which will in turn determine her consumption.

Lemma 1 *The entrepreneur's optimal continuation scale at* $t = 1$ *and state* s *is given by:*

$$k_1(s) \begin{cases} = \frac{(a(s)+p(s))k_0 - b_1(s)}{p(s)} & \text{if } p(s) < A, \\ \in \left[0, \frac{(a(s)+p(s))k_0 - b_1(s)}{p(s)}\right] & \text{if } p(s) = A. \end{cases} \tag{15}$$

The entrepreneur's return to a unit of capital at this date is A. So, she prefers to purchase capital rather than consume if $p(s) < A$, and she is indifferent otherwise (i.e., if $p(s) = A$).

At $t = 0$, the entrepreneur chooses the investment scale k_0 and the claims $\{b_t(s)\}$. Since the entrepreneur is (weakly) constrained at $t = 1$, she will neither consume at $t = 0$ nor save for consumption at $t = 2$, as stated formally in the following lemma.

Lemma 2 *The entrepreneur optimally sets* $\chi(k_0) = \sum_s q(s)b_1(s)$ *and* $b_2(s) = 0$ $\forall s$.

To see this, suppose to the contrary that the entrepreneur saves, $b_2(s) < 0$. Then she can do better by increasing $b_2(s)$ and reducing $b_1(s)$, since she could thus reduce capital liquidations at $t = 1$, given by $k_0 - k_1(s)$, while keeping k_0 unchanged (Lemma 1). By analogous reasoning, the entrepreneur can do better by reducing consumption at $t = 0$ together with $b_1(s)$.

Using Lemmas 1 and 2, the entrepreneur's problem simplifies to the maximization of her expected net worth weighted by her marginal value of funds in each state, $\frac{A}{p(s)}$:

$$\max_{k_0,\{b_1(s)\}} \sum_s \pi(s)\frac{A}{p(s)}\left((a(s)+p(s))k_0 - b_1(s)\right) \tag{16}$$

subject to $\chi(k_0) = \sum_s q(s)b_1(s)$ and $b_1(s) \leq (a(s)+p(s))k_0$ $\forall s$. Thus, entrepreneurs attach a higher value to a unit of net worth in states with lower prices of capital.

Consider the entrepreneur's choice of financing, for a given scale k_0. When deciding how to raise funds, the entrepreneur compares across the two states the ratio $\frac{\pi(s)A/p(s)}{q(s)}$, which captures how many expected 'utils-per-dollar' she loses by borrowing through a claim for state s. If $\frac{\pi(h)}{q(h)}\frac{A}{p(h)} < \frac{\pi(l)}{q(l)}\frac{A}{p(l)}$, borrowing against the high state is less costly; as a result, the entrepreneur exhausts her borrowing capacity in that state and borrows the remainder against the low state, and vice versa. If $\frac{\pi(h)}{q(h)}\frac{A}{p(h)} = \frac{\pi(l)}{q(l)}\frac{A}{p(l)}$, she is indifferent to borrowing against the high vs. the low state. This is formalized in the following lemma.

Lemma 3 *The entrepreneur's optimal choice of claims is such that, for $s', s'' \in \{l, h\}$:*

(i) If $\frac{\pi(s')}{q(s')}\frac{A}{p(s')} < \frac{\pi(s'')}{q(s'')}\frac{A}{p(s'')}$, then $b_1(s') = (a(s')+p(s'))k_0$;

(ii) If $\frac{\pi(s')}{q(s')}\frac{A}{p(s')} = \frac{\pi(s'')}{q(s'')}\frac{A}{p(s'')}$, then the entrepreneur is indifferent to the choice of claims.

Equipped with the entrepreneur's optimal financing decision, we can now determine the optimal investment scale by maximizing (16) with respect to k_0.

Lemma 4 *The entrepreneur's optimal investment scale at $t = 0$ is given by:*

$$\chi'(k_0) = \sum_s q(s)(a(s)+p(s)). \tag{17}$$

Thus, the entrepreneur invests until the marginal cost of creating an additional unit of capital equals the market value of cashflows produced by that unit plus its resale.

3.2 Optimal Trading Arrangements

We now study the investor's choice of trading arrangement, which maximizes the value of claims given in (13) subject to (PC), (IC), and (FC) given in (10)-(12).

Let $v(s)$ be the claims held by an investor for state s, and let μ^* denote an optimal trading arrangement by which these claims are traded. The following proposition summarizes the key properties of optimal trading arrangements.

Proposition 1 *The optimal trading arrangement for claims $\{v(s)\}$ has the following features:*

(i) The value of the claims is given by:

$$V(\{v(s)\}, \mu^*) = \sum_s \pi(s)v(s) - \zeta|v(h) - v(l)|, \tag{18}$$

where $\zeta = \begin{cases} \zeta^+ & \text{if } v(h) \geq v(l) \\ \zeta^- & \text{if } v(h) < v(l) \end{cases}$, *and* $\zeta^+ \in (0, \pi(h))\,, \zeta^- \in (0, 1 - \pi(h))$.

(ii) The ex-ante net expected payoff of a trader participating in it is:

$$0 \leq W(\{v(s)\}, \mu^*) \leq n^{-1}\zeta|v(h) - v(l)|, \tag{19}$$

where the last inequality is an equality if and only if the trading arrangement allocates the claims efficiently.

(iii) ζ^+, ζ^- are scalars that depend only on the trading friction, n, the distribution over the signals, $\{\mathbb{P}(x|s)\}_{x,s}$, and over the state, $\{\pi(s)\}_s$. In particular, ζ^+, ζ^- decline to zero either as n grows large or as information dispersion vanishes, i.e., signals become uninformative or perfectly informative, or uncertainty about the state vanishes.

It is worth noting that the above three properties of the optimal mechanism are proved using arguments that do not rely on the particular binary signal structure that we have assumed; this approach is illustrative of the general economic forces that underlie the mispricing/misallocation of state-contingent claims, which we discuss next.

First, state-contingent claims are *mispriced* in trading arrangements, despite their optimal design. As a result, the value of contingent claims is less than actuarially fair. If to the contrary the claims were priced fairly, it would have to be that the mechanism allocates the claims to traders with probability one and that the traders break even. But then, there is a profitable deviation for a trader to misreport his signal: the traders who are more optimistic about the value of the claims have an incentive to mimic those who are more pessimistic; because the latter break even, the former must earn rents. The distortion $\zeta|v(h) - v(l)|$ captures the extent to which the traders are able to engage in rent-extraction.

Second, state-contingent claims may be *misallocated* in trading arrangements, i.e., the mechanism may not allocate all the claims to the traders, which is inefficient since the impatient investors do not value future consumption. This is reflected in the feature that the traders' aggregate payoff nW can be lower than the loss of the mechanism, $\zeta|v(h) - v(l)|$. The reason is that it may be optimal for the mechanism to exclude pessimistic traders in order to reduce the rents earned by the optimistic ones.

Third, the mispricing/misallocation of state-contingent claims arises due to the interaction of information dispersion and trading frictions. Intuitively, the extent to which a trader can extract rents from the mechanism hinges on how relevant is his signal for valuing contingent claims, conditional on the information set of the mechanism (i.e., what it learns from other traders' reports). As either information dispersion or trading frictions vanish, a single trader's contribution to valuing the mechanism's claims vanishes as well.

In other words, it is as if the mechanism splits the claims into a "safe tranche" (i.e., claim that pays $\min_s\{v(s)\}$ in both states) and "risky tranche" (i.e., claim that pays $v(s) - \min_s\{v(s)\}$ in state s). Whereas the mechanism values the safe tranche at fair value, $\min_s\{v(s)\}$, and sells it to traders with probability one, it values the risky tranche below its expected value, $\sum_s \pi(s)(v(s) - \min_s\{v(s)\})$, and may retain it inefficiently rather sell it to traders.

We use the binary signal structure to provide further characterization of the optimal mechanism, which as we discuss next facilitates comparative statics on the distortion term ζ (see complementary Lemma A.3). To illustrate the workings of the mechanism, consider the case where private signals are binary-symmetric, i.e., $\mathbb{P}(x = G|s = h) = \mathbb{P}(x = B|s = l) = \phi \in (\frac{1}{2}, 1)$. When ϕ is small, the optimal mechanism allocates the claims efficiently, always to the optimists (if there are any) and sometimes to the pessimists (if there are no optimists), but it leaves information rents to the optimists to discourage them from misreporting their signals. As shown in Figure 2, when ϕ is small, the distortion term ζ^+ rises with the gap between the optimistic and pessimistic valuations, which increases with ϕ. Instead, when ϕ is large, the mechanism allocates the risky tranche inefficiently, only to optimists (if there are any), which discourages these traders from misreporting their signals, without leaving them any rents. As shown in Figure 2, when ϕ is large, the distortion term ζ^+ falls with ϕ; here, since optimists do not earn rents, ζ^+ captures the probability of no trade, which decreases with ϕ.

The results in Proposition 1 provide a sharp contrast to those in Cremer and McLean (1988) and McAfee et al. (1989). In a related environment, they show that traders are neither able to extract rents from the mechanism nor is the allocation of claims in the mechanism inefficient. In contrast to this paper, they consider mechanisms in which the traders effectively have enough commitment power to enter into ex-ante lotteries whose payoffs depend on the entire profile of reported signals, despite the fact that these lotteries may ex-post collect large payments from traders who end up receiving very few claims. But, because we allow the

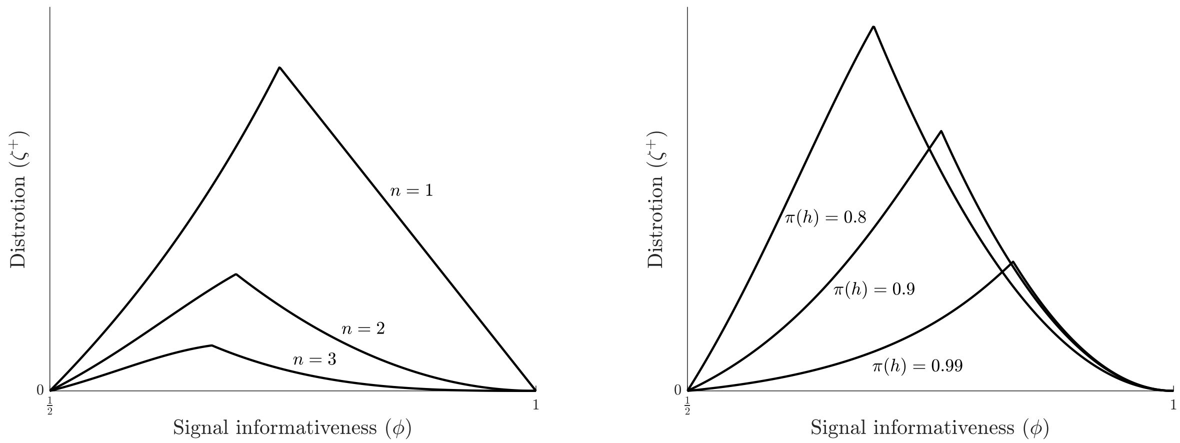

Figure 2: Illustrates how ζ^+ depends on the trading friction, n, the signal informativeness, ϕ, and the distribution over the state, $\pi(h)$. Unless stated otherwise, the figure assumes that $\pi(h) = 0.8$ and $n = 2$.

traders to costlessly leave the mechanism *after* learning their allocations, such lotteries are ruled out in our setting, which is enough to generate mispricing/misallocation of claims in secondary markets. We conjecture that this result would continue to hold in broader settings where some (but not full) committment to participate in such lotteries is present.

Lastly, we note that one could justify ex-post rather than ex-interim implementation constraints on grounds that these represent a more robust (i.e., less dependent on the details of the environment) solution concept (see, e.g., Hagerty and Rogerson (1987); Green and Laffont (1987); Lopomo (2001); Bergemann and Morris (2005)). For instance, in the spirit of Hagerty and Rogerson (1987), we could require that the allocations of the mechanism satisfy a form of "no-regret property," whereby each trader must find it optimal to report his signal truthfully and to participate in the mechanism no matter his allocation. But this approach would make both the incentive compatibility and the participation constraints hold ex-post, which would again prevent full surplus extraction and lead to mispricing/misallocation of claims.

3.3 Equilibrium Prices of Claims and Capital

Now that we have determined the value of claims in secondary markets, we can study the determination of equilibrium claims prices at the issuance stage. The following corollary shows how the mispricing/misallocation of contingent claims in secondary markets distorts the claims prices faced by the entrepreneurs.

Corollary 1 *In equilibrium, the prices of state-contingent claims satisfy:*

$$q(l) = 1 - q(h),\ q(h) \begin{cases} = \pi(h) - \lambda\zeta^{+} & \text{if } B_1(h) > B_1(l), \\ \in [\pi(h) - \lambda\zeta^{+}, \pi(h) + \lambda\zeta^{-}] & \text{if } B_1(h) = B_1(l), \\ = \pi(h) + \lambda\zeta^{-} & \text{if } B_1(h) < B_1(l). \end{cases} \tag{20}$$

When the aggregate supply of claims by entrepreneurs satisfies $B_1(h) > B_1(l)$, the market discounts an additional unit of a claim for the high state by $\lambda\zeta^{+}$, since with probability λ each investor will need to re-trade such a claim and face a discount ζ^{+} (Proposition 1). Analogously, when $B_1(h) < B_1(l)$, the market discounts each additional unit of a claim for the low state by $\lambda\zeta^{-}$. Finally, the investors are happy to hold a non-contingent portfolio when $q(h) \in [\pi(h) - \lambda\zeta^{+}, \pi(h) + \lambda\zeta^{-}]$, since the resale value of an additional unit of a claim for the high (low) state is weakly lower than $q(h)$ $(1 - q(h))$.

To complete the determination of asset prices, we use the entrepreneurs' optimality conditions together with the traditional sector firms' demand for capital to pin down the market clearing prices of capital.

Corollary 2 *The equilibrium price of capital in state s satifies:*

$$p(s) \begin{cases} = A & \text{if } B_1(s) < a(s)K_0, \\ \in [gA, A] & \text{if } B_1(s) = a(s)K_0, \\ = gA & \text{if } B_1(s) > a(s)K_0. \end{cases} \tag{21}$$

When the entrepreneurial sector's promised repayments exceed its cashflows, i.e., the entrepreneurs' net worth is low, the entrepreneurs have to liquidate capital; thus, $K_1(s) < K_0$. As a result, the price of capital becomes depressed, as capital must be absorbed by the traditional sector, which is less productive than entrepreneurs. When the repayments are lower than cashflows, i.e., the entrepreneurs' net worth is high, then the price must be A, since if it were lower there would be excess demand for capital from the entrepreneurs; thus, $K_1(s) = K_0$. Finally, when repayments are just equal to the cashflows, the price must be in the interval $[gA, A]$, so that in equilibrium capital is not traded; thus, $K_1(s) = K_0$.

We note a key tension that arises in equilibrium. On the one hand, even though entrepreneurs are risk-neutral, they become averse to fluctuations in their net worth, as due to balance sheet effects these may generate capital liquidations precisely at times when capital prices are depressed (Corollary 2). As a result, entrepreneurs want to transfer aggregate risk to investors by issuing claims against states in which capital prices are high (Lemma 3). On the other hand, even though investors are risk-neutral, they become averse to fluctuations in the value of their tradable claims, as the presence of information-trading frictions generates mispricing/misallocation of contingent claims in secondary markets (Proposition 1). As a result, investors prefer to hold portfolios that are less exposed to aggregate risk (Corollary 1). In the following sections, we investigate how this tension gets resolved in equilibrium, and whether there is scope for corrective policy.

4 Equilibrium Risk-Sharing, Investment and Output

We now use the results in Sections 3.1-3.3 to study the implications of information-trading frictions for equilibrium risk-sharing, investment and output.

We begin by considering a benchmark, where information-trading frictions are shut down. This benchmark starkly illustrates how risk-sharing between entrepreneurs and investors can eliminate the balance sheet channel and stabilize economic activity. We will then see how the interaction of financial constraints with information-trading frictions alters this picture.[12]

4.1 Benchmark: Economy without Information-Trading Frictions

Let us suppose that the parameters are such that $\lambda\zeta = 0$, so that the claims prices are undistorted and given by $q(s) = \pi(s)\ \forall s$. By Corollary 1 and Proposition 1, this would arise for instance if dispersion in the private signals were to vanish.

The first implication is that full risk-sharing must be attained in equilibrium. That is, the entrepreneurs' marginal value of funds, $\frac{A}{p(s)}$, and thus the prices of capital must be equalized across states. If this were not the case, then entrepreneurs would exhaust their borrowing capacity against the state with the higher price of capital (Lemma 3), resulting in more liquidations and a lower price of capital in that state (Lemma 1 and Corollary 2), a contradiction.

Second, entrepreneurs will only borrow against their intermediate cashflows and avoid capital liquidations, i.e., $B_1(s) \leq a(s)K_0$ and $K_1(s) = K_0\ \forall s$. If they were to borrow more, they would have to start liquidating capital (Lemma 1), which would depress its price to gA (Corollary 2), making investment less desirable than K_0 (Lemma 4 and Assumption 2(i)).

Third, the economy's final output gets fully insulated from the shock to intermediate cashflows, i.e., $Y_2(s) = AK_0\ \forall s$. This is despite the fact that the shock directly affects the economy's output at $t = 1$, i.e., $Y_1(s) = a(s)K_0$ in state s. This idea that risk-sharing can

[12] An alternative benchmark, where information-trading frictions are present but the entrepreneurs' output is fully pledgeable, is presented in Appendix D. The equilibrium of that economy is not very interesting: although ex-ante investment may be depressed relative to first-best, the equilibrium always features full risk-sharing between entrepreneurs and investors and, thus, there are no balance sheet amplification effects.

mute the impact of aggregate shocks on the borrowers' net worth and thereby eliminate the balance sheet channel has also been noted by Krishnamurthy (2003) and Di Tella (2017).

Finally, although the balance sheet channel is eliminated, aggregate investment and output may be depressed relative to first-best due to limited pledgeability. This occurs whenever the funding required to invest at the first-best scale exceeds the entrepreneurs' intermediate cashflows, i.e., $\chi(k^{FB}) > \sum_s \pi(s)a(s)k^{FB}$ as defined in equation (3).[13] This last feature is not very surprising and is common to models with financial frictions.

4.2 Economy with Information-Trading Frictions

We now return to the economy with information-trading frictions. It will be convenient to divide the parameter space into two regions since as we will see, depending on the parameters, the economy will either be in a "no risk-sharing" or a "partial risk-sharing" regime. To this end, it is useful to define a claims price $\bar{q}$, at which an entrepreneur is indifferent to issuing an additional claim for state h, when the equilibrium aggregate supply of claims is non-contingent:

$$\bar{q} \equiv \min\left\{\frac{\pi(h)g}{\pi(h)g + 1 - \pi(h)}, \frac{\chi'(\bar{k}) - (a(l) + gA)}{(a(h) + A) - (a(l) + gA)}\right\}, \tag{22}$$

where $\bar{k} > 0$ is such that $\chi(\bar{k}) = (a(l) + gA)\bar{k}$. We will explain the intuition behind the expression for $\bar{q}$ shortly. As we shall see we can think of $\pi(h) - \bar{q} > 0$ as an equilibrium "insurance premium" that entrepreneurs are willing to pay to insure net worth fluctuations.

The following proposition shows that when information-trading frictions are severe relative to the insurance premium, the equilibrium features no risk-sharing whatsoever.

Proposition 2 (No Risk-Sharing) *Suppose that $\lambda\zeta^+ > \pi(h) - \bar{q}$. Then, an equilibrium exists and in it:*

(i) Aggregate supply of claims is non-contingent, $B_1(h) = B_1(l) = B_1$;

(ii) Capital is liquidated if and only if the state is low;

(iii) Asset prices satisfy $q(h) = \bar{q}$ and $p(l) = gA < A = p(h)$.

The first result follows from the observation that, when the aggregate supply of claims is non-contingent, i.e., $B_1(h) = B_1(l) = B_1$, investors demand a premium $\lambda\zeta^+$ for holding an additional claim for the high state (Proposition 1), while entrepreneurs are only willing to pay

[13] If $\chi(k^{FB}) > \sum_s \pi(s)a(s)k^{FB}$, then entrepreneurs invest all of their intermediate cashflows, $\chi(K_0) = \sum_s \pi(s)a(s)K_0$, and the price of capital is $p = \chi'(K_0) - \sum_s \pi(s)a(s) \in (gA, A)$ by Assumption 2(i). But, if $\chi(k^{FB}) \leq \sum_s \pi(s)a(s)k^{FB}$, then entrepreneurs invest at the first-best scale and the price of capital is A.

a premium of $\pi(h) - \bar{q}$. The claims price $q(h)$ is then such that entrepreneurs are willing to remain uninsured. Given this, Assumptions 2(ii)-(iii) imply that entrepreneurs find it optimal to liquidate capital in the low state, but have excess net worth at $t = 1$ in the high state, i.e., $B_1 < a(h)K_0$. This in turn implies the statement about capital prices (Corollary 2).

To understand how we arrive at the expression for $\bar{q}$, note that there are two possibilities. If in equilibrium entrepreneurs do not exhaust borrowing capacity in the low state, i.e., $B_1 < (a(l) + gA)K_0$, then an entrepreneur must be indifferent to issuing an additional claim for the high state and using the proceeds to reduce borrowing against the low state by $q(h)$. This can be shown to change the entrepreneur's expected utility by $u_1 \equiv (1 - \pi(h))\, g^{-1} q(h) + \pi(h)(q(h) - 1)$. Setting u_1 to zero and solving for $q(h)$ yields the first term in the definition of $\bar{q}$. But, if in equilibrium entrepreneurs do exhaust the borrowing capacity in the low state, i.e., $B_1 = (a(l) + gA)K_0$ and $K_0 = \bar{k}$, it must be that $u_1 \leq 0$ and $q(h) \leq \frac{\pi(h)g}{\pi(h)g+1-\pi(h)}$. But then, an entrepreneur must be indifferent to issuing an additional claim for the high state and increasing the investment scale by $q(h)(\chi'(\bar{k}) - (a(l) + gA))^{-1}$. This can be shown to change the entrepreneur's expected utility by $u_2 \equiv \pi(h)\left(q(h)\frac{(a(h)+A)-(a(l)+gA)}{\chi'(\bar{k})-(a(l)+gA)} - 1\right)$. Setting u_2 to zero and solving for $q(h)$ yields the second term in the definition of $\bar{q}$.

Because in this economy there is no risk-sharing at all, the shock to intermediate cashflows now does propagate to the economy's final output through capital liquidations in the low state, i.e., $Y_2(l) = AK_0 - (1 - g)A(K_0 - K_1(l)) < AK_0 = Y_2(h)$. Moreover, the shock is now amplified through the decline in the capital price, since $K_0 - K_1(l)|_{p(l)=gA} = \frac{B_1 - a(l)K_0}{gA} > \frac{B_1 - a(l)K_0}{A} = K_0 - K_1(l)|_{p(l)=A}$. This idea that fluctuations in borrowers' net worth can amplify and propagate aggregate shocks goes back to the classic works of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997), and has more recently been explored by Brunnermeier and Sannikov (2014), among others. In the literature, however, the limits to aggregate risk-sharing are often imposed ad-hoc rather than derived from first principles. In Section 5, we will show that understanding the source of the underlying distortion has important policy implications.

We next show that, when information-trading frictions are not severe relative to the insurance premium, the equilibrium features partial risk-sharing, which endogenously bounds economic fluctuations.

Proposition 3 (Partial Risk-Sharing) *Suppose that $\lambda\zeta^+ < \pi(h) - \bar{q}$. Then, an equilibrium exists and in it:*

(i) Aggregate supply of claims is contingent, $B_1(h) > B_1(l)$;

(ii) Capital may be liquidated, but only if the state is low;

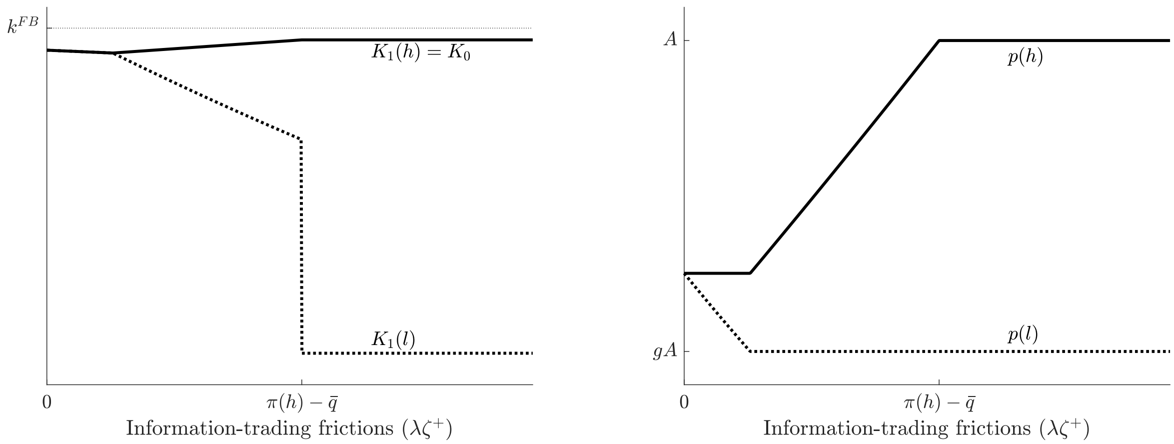

Figure 3: Illustrates the equilibrium effect of information-trading frictions. The left panel depicts the equilibrium investment scale K_0, and the continuation scale, $K_1(s)$, whereas the right panel depicts the price of capital, $p(s)$. For comparative statics, it is useful to relax the exact matching assumption (see Appendix C). We use the following values for the remaining parameters: $\pi(h) = 0.8$, $a(l) = 0$, $a(h) = 1$, $A = 1.1$, $g = 0.83$, and $\chi(k) = \chi k^{\omega}$ with $\omega = 2.2$ and $\chi = 3$.

(iii) Asset prices satisfy $q(h) = \pi(h) - \lambda\zeta^+$ *and* $\gamma p(h) \leq p(l) < p(h)$ *with* $\gamma \equiv \frac{\pi(h)-\lambda\zeta^+}{1-\pi(h)+\lambda\zeta^+}\frac{1-\pi(h)}{\pi(h)}$, *where* $p(l) = \gamma p(h)$ *if* $B_1(l) < (a(l) + gA)K_0$.

Since the entrepreneurs' willingness to insure net worth fluctuations (if they were uninsured) is now greater than the investors' cost of providing insurance, the equilibrium aggregate supply of claims must be state-contingent. However, because insurance provision is still privately costly to investors due to the mispricing/misallocation of claims in secondary markets, full risk-sharing cannot be attained, i.e., there is partial risk-sharing.

Because of partial risk-sharing, the fluctuations in the entrepreneurs' net worth, in their marginal value of funds and in capital prices are endogenously diminished relative to the economy with no risk-sharing. As a result, the magnitude of economic fluctuations becomes tied to the severity of information-trading frictions. Indeed, as $\lambda\zeta^+$ goes to zero, the term γ which controls the volatility in capital prices (and thus of the entrepreneurs' marginal value of funds) across the two states goes to one, and the equilibrium converges to the benchmark economy of Section 4.1.

Finally, the equilibrium investment scale always remains depressed below first-best, independently of the equilibrium region, since:

$$\chi'(K_0) = \sum_s q(s)(a(s) + p(s)) < \sum_s \pi(s)(a(s) + A) = \chi'(k^{FB}), \tag{23}$$

where the inequality follows from the fact that $q(h) < \pi(h)$ and $p(l) < p(h) \leq A$ (see Propositions 2 and 3). As we discuss below, however, K_0 is actually non-monotonic in the severity of information-trading frictions and can be above that of the benchmark economy.

Propositions 2 and 3 establish existence of an equilibrium. Moreover, it is straightforward to show that the equilibrium is unique in terms of the aggregate supply of claims, investment and continuations scales, as well as the prices of claims and capital. However, we refrain from stating that the equilibrium is unique since there may be multiple ways to allocate claims in the optimal mechanism, without affecting the equilibrium objects of interest.

Figure 3 provides an illustration of the results in Propositions 2 and 3 by showing how information-trading frictions, $\lambda\zeta^+$, affect the aggregate investment and continuation scales (left panel), and the prices of capital (right panel). The changes in information-trading frictions should be interpreted as induced by changes in λ, n or signal distribution $\{\mathbb{P}(x|s)\}$, but not $\pi(h)$ (see Proposition 1(iii) and Figure 2), as these are the primitives that enter the equilibrium conditions only through $\lambda\zeta^+$ and $\lambda\zeta^-$ in Corollary 1.

Let us start with the "partial risk-sharing" region, i.e., $\lambda\zeta^+ < \pi(h) - \bar{q}$. Initially, when information-trading frictions are small, entrepreneurs borrow only against their (contingent) intermediate cashflows to avoid capital liquidations. As a result, as $\lambda\zeta^+$ rises, the market value of entrepreneurial claims falls and so does aggregate investment. Eventually, as $\lambda\zeta^+$ rises further, entrepreneurs increase borrowing against the low state and start liquidating capital in order to boost investment ex-ante. Indeed, because of the possibility to create less contingent claims through liquidations, the investment scale actually rises above that of the benchmark economy without information-trading frictions (i.e., $\lambda\zeta^+ = 0$)! In this region, the gap in capital prices across the two states, which is needed to keep entrepreneurs indifferent to borrowing against high vs. low state (as captured by γ), increases with $\lambda\zeta^+$. Finally, note that in the "no risk-sharing" region, as entrepreneurs borrow with non-contingent claims, the equilibrium allocations no longer depend on the magnitude of information-trading frictions.

We conclude this section by providing the conditions on the primitives under which the "no risk-sharing" region is more likely to emerge.

Corollary 3 *Assume that $\chi(k) = \chi k^\omega$ for some $\chi > 0$ and $\omega > 1$, and that the private signals are binary-symmetric, i.e., $\mathbb{P}(x = G|s = h) = \mathbb{P}(x = B|s = l) = \phi \in (\frac{1}{2}, 1)$. Then, the equilibrium is more likely to be in the "no risk-sharing" region when, all else equal:*

- *(i) λ is large, n is small, and ϕ is intermediate, i.e., the secondary markets for contingent claims are sufficiently relevant and frictional;*
- *(ii) g, ω, and $\frac{a(l)+gA}{a(h)+A}$ are large, i.e., the benefits from transferring aggregate risk to investors are not too great.*

The effect of $\pi(h)$, however, is ambiguous.

We have established in Propositions 2 and 3 that the equilibrium features "no risk-sharing" whenever $\lambda\zeta^+$ is greater than $\pi(h) - \bar{q}$. The proof of the corollary then relies on the following observations. First, while $\pi(h) - \bar{q}$ is independent of the parameters (λ, n, ϕ), larger values for λ, smaller values for n and intermediate values for ϕ make the distortion term $\lambda\zeta^+$ larger. Intuitively, this is when secondary market liquidity is more important and the mispricing/misallocation of contingent claims in them is greater. Second, while $\lambda\zeta^+$ is independent of the parameters $(a(l), a(h), A, \omega, g)$, larger values for g, ω, and $\frac{a(l)+gA}{a(h)+A}$ make the threshold $\bar{q}$ larger. Intuitively, this is when the potential falls in asset prices and in net worth in the low state absent risk-sharing become smaller relative to the costs of additional investments. Combining these two observations, we obtain the conditions (i) and (ii) on the primitives under which the equilibrium is more likely to be in the "no risk-sharing" regime.

4.3 Takeaways

To the best of our knowledge, this is the first model to put a central emphasis on the role of liquidity in secondary markets, where macro-contingent claims can be traded, for aggregate fluctuations. We showed how the presence of information-trading frictions in secondary markets, and their interaction with financial constraints, can distort aggregate risk-sharing and aggregate investment, and exacerbate asset price and output volatility. The magnitude of these effects will depend on a number of primitives of the environment. Namely, we should expect less risk-sharing and greater amplification of shocks when: (i) secondary markets for contingent claims are more relevant, i.e., re-trading claims is important; (ii) these markets are more frictional, i.e., there are more disagreements about how to value contingent claims and competition is lower; and (iii) the gains from transferring aggregate risks to investors are lower, i.e., fluctuations in asset prices and net worth are expected to be smaller.

These results can help interpret some of the observed dynamics in the run-up to the 2008-09 financial crisis (Baily et al., 2008; Brunnermeier, 2009). A puzzling feature of this episode was the financial sector's heavy exposure to real-estate risks, despite the availability of various structured products (e.g., MBS) and credit default swaps (CDS) that could in principle help share these risks with the broader population. Although pooling of many mortgages together helped ameliorate risks associated with individual borrower defaults, and thus adverse selection problems stemming from it, this process did not eliminate the downside risk from housing price declines. During the years of rising asset prices, banks were able to share some (though not all) of these risks through issuance/purchase of structured products and CDS. In 2007, however, things changed dramatically when uncertainty over asset prices suddenly increased, and it became harder for banks to continue off-loading some of their real-estate exposures to investors. The inability to share real-estate risks effectively was in turn essential in the balance sheet effects that unfolded following the Lehman's collapse in September 2008. Note that these dynamics are consistent with the model's key ingredients: (i) the secondary markets for real-estate contingent claims were important; (ii) these claims were traded in frictional over-the-counter markets, where disagreements about how to value real-estate contingent securities became larger in 2007; and (iii) it has been argued that the risks of large declines in aggregate housing prices were underestimated ex-ante, making the gains from risk-transfer appear smaller. Even though our model is stylized and abstracts from a number of other frictions and institutional details, it does provide a comprehensive framework for thinking about the build-up to and the subsequent unfolding of the crisis. As we discuss in Section 5.2, the model also sheds light on several government interventions implemented in the midst of the crisis.

Our theory resonates well with (and formalizes) the views of Case et al. (1991) and Shiller (1994), who have long argued that lack of liquidity in markets for real-estate contingent claims is what inhibits risk-sharing and has a destabilizing effect on the business cycle.[14] Indeed, through the lens of our model, some of Robert Shiller's subsequent work, consisting of creation of improved real-estate indices and promotion of centralized platforms for real-estate derivatives, can be understood as a way to ameliorate the effects of information-trading frictions in markets, by boosting information aggregation and competition in them.

Our theory relies on the interaction of financial frictions, which are by now standard in the literature; with information-trading frictions, which are an innovation of this paper. Whereas the former is behind generating gains from aggregate risk-sharing, the latter is key in distorting it. The microfoundations for these distortions have taught us several lessons. First, that the "liquidity" of a given entrepreneur's claims is an equilibrium object that depends on the aggregate supply of state-contingent claims in the economy: e.g., it is cheaper for an entrepreneur to issue a claim against the high state when investors are endowed with relatively few claims against that state, and vice versa.[15] Second, that information-trading frictions not only distort the allocation of aggregate risk between entrepreneurs and investors, but also its allocation among investors, as some impatient investors inefficiently retain risky claims. Third, that the magnitude of these distortions depends on the extent of information frictions and of competition in secondary markets, which are likely to vary over time and across economies with differing financial market development. Finally, as we show next, that mispricing/misallocation of aggregate risk originates from information-trading frictions in sec-

[14]Caballero (2003) and Mian (2013), respectively, express related views on the role of markets for indexed government bonds in emerging economies and for indexed household mortgages in the US.

[15]Note that in our economy the aggregate supply of claims consists only of those on the entrepreneurial sector; in a richer setting, investors may also hold claims on their future labor income or other financial income.

ondary markets also has important distributional consequences, suggesting that some market participants may naturally oppose (uncompensated) reforms aimed at alleviating their effects.

Proposition 4 (Rent-Extraction) *In equilibrium, the investors' lifetime welfare satisfies:*

$$U^I = e + (1-\lambda)W(\{B_t(s)\}, \mu^*) \leq e + \lambda\zeta^+(B_1(h) - B_1(l)), \tag{24}$$

where the last inequality is an equality if and only if the optimal trading arrangements allocate the claims efficiently. In particular, whereas the entrepreneurs' welfare weakly decreases, the investors' welfare can increase in the severity of information-trading frictions.

Proposition 4 states that, in equilibrium, investors extract rents from entrepreneurs, and that the extent to which they can do so also depends on the aggregate supply of claims. It is clear that investors cannot extract rents from entrepreneurs when the information-trading frictions are so severe that the aggregate supply is non-contingent (i.e., $\lambda\zeta^+ > \pi(h) - \bar{q}$), as non-contingent claims are always priced fairly. Instead, when information frictions are not too severe (i.e., $\lambda\zeta^+ < \pi(h) - \bar{q}$), the aggregate supply is state-contingent, and investors demand compensation for the potential mispricing/misallocation of these claims in secondary markets. However, in equilibrium, the mispricing of these claims ex-post does not generate a welfare loss to investors ex-ante; hence, the compensation received from entrepreneurs for the mispricing (though not for the misallocation) of claims is a pure rent earned by investors. These observations will play an important role in the policy analysis of the next section.

5 Constrained Efficiency and Policy

In this section, we study the efficiency properties of the equilibrium. To do so, we first solve the problem of a social planner, who chooses allocations to maximize the lifetime welfare of entrepreneurs, subject to the information and incentive constraints faced by the agents, and subject to delivering investors the same lifetime welfare U as in the laissez-faire equilibrium; thus, we search for Pareto-improvement upon the equilibrium. Second, we derive our main inefficiency result by comparing the planner's solution to the laissez-faire equilibrium. Finally, we show how to decentralize the constrained efficient allocations through government interventions in secondary markets.

5.1 Constrained Efficient Allocations and Equilibrium Inefficiency

The planner chooses an investment scale k_0, continuation scales $\{k_1(s)\}$, transfers to the investors at the issuance stage, T_0, and at $t \in \{1, 2\}$, $\{T_t(s)\}$, and allocates $\{k_0 - k_1(s)\}$ units of capital to the traditional sector, and the remaining goods each period to the entrepreneurs. In addition, the planner designs optimal trading arrangements, μ, where investors can re-trade their claims (rights) to the transfers $\{T_t(s)\}$. The planner faces the same set of pledgeability and information-trading frictions as the agents. When necessary, in order to avoid confusion, we will use the superscript SP to denote the planner's allocations.

Formally, the planner's problem is to maximize the expected welfare of entrepreneurs:

$$\max_{k_0,T_0,\{k_1(s),T_t(s)\},\mu} e - \chi(k_0) - T_0 + \sum_s \pi(s)\left[(a(s)k_0 - T_1(s) + gA\left(k_0 - k_1(s)\right) + Ak_1(s) - T_2(s)\right] \tag{P3}$$

subject to the following set of constraints:

$$0 \leq T_0 + \chi(k_0) \leq e, \tag{25}$$
$$0 \leq T_1(s) \leq a(s)k_0 \ \forall s, \tag{26}$$
$$0 \leq T_2(s) \leq gA(k_0 - k_1(s)) \ \forall s, \tag{27}$$
$$0 \leq k_1(s) \leq k_0 \ \forall s, \tag{28}$$
$$\mu \in \mathcal{M}, \tag{29}$$
$$U \leq U^I = T_0 + \lambda V\left(\{T_t(s)\},\mu\right) + (1-\lambda)\left(W\left(\{T_t(s)\},\mu\right) + \sum_s \pi(s)(T_1(s) + T_2(s))\right). \tag{30}$$

Constraints (25)-(27) are the economy's resource constraints combined with the limited pledgeability friction, which states that at $t = 2$ the planner can allocate to investors only the output produced by the traditional sector as entrepreneurs can always divert their output. Constraint (28) imposes feasibility on the economy's allocation of capital at $t = 1$.

Constraint (29) states that the trading arrangements that the planner designs on investors' behalf must be feasible, i.e., she is also subject to the information-trading frictions when reallocating the claims to the transfers $\{T_t(s)\}$ among investors. Let us unbundle what this entails. In particular, recall that each trading arrangement (and without loss assume that all of the planner's arrangements are identical) is a direct revelation mechanism, which solicits reports from n traders and then allocates claims to and collects payments from them based on these reports. Let $\mathcal{A}^{SP}\left(\theta^i,\theta^{-i}\right) \equiv \left(\omega^{SP}\left(\theta^i,\theta^{-i}\right), \left\{v_s^{SP}\left(\theta^i,\theta^{-i}\right)\right\}\right)$ denote the allocation of trader i who has observed signal x^i and reports θ^i to the mechanism when other traders report θ^{-i}, where $\omega^{SP}(\theta^i,\theta^{-i})$ is the payment that trader i makes to the planner's mechanism and $v_s^{SP}(\theta^i,\theta^{-i})$ are the units of claims for state s that the mechanism transfers to the trader.[16]

[16] We drop superscript i from the planner's mechanism allocations, as it is without loss to focus on allocations that depend on the traders' reports but not on their identity.

This trader's payoff from accepting his allocation is:

$$U(x^i, \mathcal{A}^{SP}(\theta^i, \theta^{-i})) = \sum_s v_s^{SP}(\theta^i, \theta^{-i}) \cdot \mathbb{P}(s|x^i, \mathcal{A}^{SP}(\theta^i, \theta^{-i})) - \omega^{SP}(\theta^i, \theta^{-i}). \tag{31}$$

First, the planner needs to ensure that traders are willing to participate in the mechanism after learning their equilibrium allocations:

$$U(x^i, \mathcal{A}^{SP}(x^i, x^{-i})) \geq 0 \ \forall x^i, x^{-i}. \tag{32}$$

Second, it must be incentive compatible for each trader to report his signal truthfully to the planner's mechanism, given that other traders do so as well:

$$\mathbb{E}\{U(x^i, \mathcal{A}^{SP}(x^i, x^{-i}))|x^i\} \geq \mathbb{E}\left\{\max\{0, U(x^i, \mathcal{A}^{SP}(\theta^i, x^{-i}))\}|x^i\right\} \ \forall x^i, \theta^i. \tag{33}$$

Finally, the planner's allocation of claims within each mechanism cannot exceed the total claims available in it:

$$\sum_i v_s^{SP}(\theta^i, \theta^{-i}) \leq T_1(s) + T_2(s) \ \forall s, \theta^i, \theta^{-i}. \tag{34}$$

Observe that the constraints (32)-(34) are essentially the same as the constraints (10)-(12) in the investor's problem (P2); the only difference is that the planner's trading arrangements reallocate claims to the transfers $\{T_t(s)\}$ rather than those issued by the entrepreneurs.[17]

Constraint (30) states that the planner must deliver each investor an ex-ante welfare of at least U. Here, $V\left(\{T_t(s)\}, \mu^{SP}\right)$ denotes the ex-ante expected payoff to the investor from posting the claims $\{T_t(s)\}$ in the trading arrangement μ^{SP} designed by the planner (which occurs with probability λ), whereas $W\left(\{T_t(s)\}, \mu^{SP}\right)$ denotes the ex-ante net expected payoff to the investor from participating as a trader in the planner's trading arrangement (which occurs with probability $1-\lambda$). Given the claims to the transfers $\{T_t(s)\}$ and the trading arrangement $\mu^{SP} = \left\{\mathcal{A}^{SP}(x^i, x^{-i})\right\}_{x^i, x^{-i}}$, these payoffs are computed as follows:

$$V\left(\{T_t(s)\}, \mu^{SP}\right) = \mathbb{E}\left\{\sum_i \omega^{SP}(x^i, x^{-i})\right\} \tag{35}$$

and

$$W\left(\{T_t(s)\}, \mu^{SP}\right) = \mathbb{E}\left\{U(x^i, \mathcal{A}^{SP}(x^i, x^{-i}))\right\}. \tag{36}$$

[17]We again omit the constraint that traders have sufficient resources to participate in the trading arrangements. Assumption 1 will ensure that this is the case at the social optimum (see Appendix B).

We note the key difference between the planner's and the investor's problems. As we discussed in Section 2.4, when choosing $\{d_t(s)\}$ and μ in problem P2, each investor takes as given the equilibrium claims (i.e., $\{B_t(s)\}$) and trading arrangements (i.e., μ^*) of other investors, and thus his payoff $\widetilde{W} = W(\{B_t(s)\}, \mu^*)$. In sharp contrast, the planner designs the claims and the trading arrangements on behalf of all investors; as a result, she internalizes that by adjusting $\{T_t(s)\}$ and μ^{SP} she affects the payoff $W(\{T_t(s)\}, \mu^{SP})$ as well. As we will see, the dependence of W on the joint design of claims and trading arrangements will be the source of externality that underlies the inefficiency of the laissez-faire equilibrium.

We now proceed to characterize the solution to the planner's problem, and we begin by studying the implications of information-trading frictions, as captured by the constraints (29) and (30), for the planner's problem (P3). The following lemma shows that the ex-ante welfare of the investors depends on how the claims are allocated, rather than on how they are priced in trading arrangements.

Lemma 5 *Fix a feasible trading arrangement μ^{SP} and transfers $\{T_t(s)\}$. Then:*

$$\lambda V\left(\{T_t(s)\}, \mu^{SP}\right) + (1-\lambda) W\left(\{T_t(s)\}, \mu^{SP}\right) = \lambda \sum_s \pi(s) \mathbb{E}\left\{\sum_i v_s^{SP}\left(x^i, x^{-i}\right)|s\right\}. \quad (37)$$

Since the trading arrangements and the claims traded in them are identical for all investors, the payments (i.e., the ω^{SP}'s in the mechanism allocations) that an investor expects to receive from his arrangement are equal to those he expects to make when participating as a trader in the arrangements of other investors; hence, these payments net out. The investors' welfare, however, does depend on how the claims are allocated in the arrangements (i.e., the v_s^{SP}'s in the mechanism allocations). In particular, as we see from equation (37), the welfare is higher when the investors expect that the trading arrangements will allocate more claims to them as traders, since in that case they do value consumption at future dates. But then, using the constraint (34), it follows that the investors' welfare is highest when the arrangements allocate all of the claims to the transfers $\{T_t(s)\}$ to the traders, as stated in the next corollary.

Corollary 4 *Fix a feasible trading arrangement μ^{SP} and transfers $\{T_t(s)\}$. Then:*

$$\lambda V\left(\{T_t(s)\}, \mu^{SP}\right) + (1-\lambda) W\left(\{T_t(s)\}, \mu^{SP}\right) \leq \lambda \sum_s \pi(s)(T_1(s) + T_2(s)), \quad (38)$$

with equality if and only if μ^{SP} allocates the claims to the transfers efficiently.

Note that, for given transfers $\{T_t(s)\}$, the choice of trading arrangement only enters the planner's problem through the constraints (29) and (30). Therefore, by Corollary 4, if it were possible for the planner to design a feasible trading arrangement that is also able to allocate the claims to the transfers efficiently, then such a trading arrangement must be socially optimal. The next lemma states that such a design is indeed possible.

Lemma 6 *Fix transfers $\{T_t(s)\}$, then there exists a feasible trading arrangement μ^{SP} that allocates the claims to these transfers efficiently. Such a trading arrangement is optimal for the social planner, and it implies investor welfare of:*

$$U^{I,SP} = T_0 + \sum_s \pi(s) \left(T_1(s) + T_2(s)\right). \tag{39}$$

From the constraint (30) in the planner's problem and Corollary 4, it is clear that the right-hand side of equation (39) is the upper bound on the investors' welfare, and that this upper bound is attained by μ^{SP} if it existed. That such a trading arrangement exists is proved through the following example. Suppose that μ^{SP} allocates a share n^{-1} of the claims to the transfers to each of the n traders in the arrangement; that is, $v_s^{SP}(x^i, x^{-i}) = n^{-1} \cdot (T_1(s) + T_2(s))\ \forall s, x^i, x^{-i}$. By construction, μ^{SP} satisfies the constraints given by (34) and it also allocates the claims efficiently. Next, suppose that μ^{SP} collects from each trader a payment equal to the expected value of the claims allocated to him, conditional on the worst possible report of that trader and the actual reports of other traders; that is, $\omega^{SP}(x^i, x^{-i}) = n^{-1} \cdot \min_x \mathbb{E}\{T_1(s) + T_2(s) | x^i = x, x^{-i}\}\ \forall x^i, x^{-i}$. By construction, the proposed allocations satisfy the participation constraints given by (32), as each trader expects to extract non-negative rents from the trading arrangement conditional on knowing his allocation; and the incentive compatibility constraints given by (33), as neither the payment collected from a trader nor the allocation of claims to him depend on his actual report. Thus, the proposed trading arrangement μ^{SP} is feasible and it allocates the claims to the transfers efficiently.

Now that we know how the planner optimally allocates a given set of transfers $\{T_t(s)\}$ through trading arrangements, we can study its implications for the planner's optimal investment and allocation of capital. We first make some useful definitions. Let $\widetilde{k}_U > 0$ be the largest investment scale the planner can achieve with no capital liquidations and by delivering investors welfare U, i.e., $\chi(\widetilde{k}_U) = \sum_s \pi(s) a(s) \widetilde{k}_U + e - U$. Next, we define the planner's effective value of capital at $t = 1$ as:

$$p_U^{SP} = \begin{cases} A & \text{if } \sum_s \pi(s)a(s) + A \leq \chi'(\widetilde{k}_U) \\ \chi'(\widetilde{k}_U) - \sum_s \pi(s)a(s) & \text{if } \sum_s \pi(s)a(s) + gA \leq \chi'(\widetilde{k}_U) < \sum_s \pi(s)a(s) + A \\ gA & \text{if } \chi'(\widetilde{k}_U) < \sum_s \pi(s)a(s) + gA \end{cases}. \tag{40}$$

The following lemma completes the characterization of the planner's problem.

Lemma 7 *The social planner's investment scale is given by:*

$$\chi'(k_0^{SP}) = \sum_s \pi(s)a(s) + p_U^{SP}, \tag{41}$$

and she liquidates capital if and only if $p_U^{SP} = gA$, in which case her continuation scale is:

$$k_1^{SP}(h) = k_1^{SP}(l) = k_0^{SP} - \frac{U - \left(e - \chi(k_0^{SP}) + \sum_s \pi(s)a(s)k_0^{SP}\right)}{gA}. \tag{42}$$

By allocating the claims efficiently in trading arrangements, the planner effectively faces undistorted "claims prices" when allocating consumption goods between entrepreneurs and investors across states (Lemma 6). As a result, she finds it optimal to attain full risk-sharing between entrepreneurs and investors by equalizing the effective value of capital across states.

Despite attaining full risk-sharing, however, the planner's ex-ante investment scale may still be depressed (relative to first-best) and she may choose to liquidate capital prematurely. The reason is that the planner cannot extract the final output from entrepreneurs but she must still deliver investors a welfare of U, which increases with the rents that investors extract from entrepreneurs at equilibrium (see Proposition 4). As a result, when the intermediate cashflows are insufficient to deliver to investors a lifetime welfare of U, the planner has two options: she can reduce ex-ante investment (increase T_0) and/or increase ex-post capital liquidations (increase $\{T_2(s)\}$) to transfer goods to investors. From Lemma 7, we see that the planner under-invests relative to first-best when $\chi'(\widetilde{k}_U) < \sum_s \pi(s)a(s) + A$, and she liquidates capital when $\chi'(\widetilde{k}_U) < \sum_s \pi(s)a(s) + gA$, which are more likely to occur for higher values of U.[18]

We are now ready to utilize the above characterization of the planner's problem, in conjunction with the results in Sections 3 and 4, to state our main normative result.

Proposition 5 (Inefficiency) *The equilibrium of the economy with information-trading frictions is generally constrained inefficient.*[19] *In sharp contrast, the equilibrium of the benchmark economy is constrained efficient.*

Consider first the equilibrium of the benchmark economy. In that economy, the investors do not earn rents and their lifetime welfare is $U = e$. By inspection of the planner's problem for $U = e$, we immediately see that the allocations of the benchmark economy coincide with the planner's and are thus constrained efficient. Hence, we can conclude that any inefficiency

[18]Observe that Assumption 2(i) ensures that when $U = e$ the planner does not liquidate capital, since $\chi'(\widetilde{k}_U)|_{U=e} \geq \sum_s \pi(s)a(s) + gA$.

[19]The only scenario in which equilibrium could be constrained efficient is when both the allocation of claims in markets is efficient, and the supply of claims satisfies $B_1(s) = a(s)K_0 \ \forall s$, despite mispricing (see proof).

of the laissez-faire equilibrium must be due to information-trading frictions, which introduce mispricing/misallocation of aggregate risk.

To identify the source of inefficiency introduced by information-trading frictions, we compare the planner's allocations with those of the laissez-faire equilibrium. Recall that, when designing claims and trading arrangements in problem (P2), each investor tries to minimize the mispricing of his claims due to rent-extraction by traders in his trading arrangement, but he takes as given the rents she will extract from other investors; that is, the investor maximizes the payoff $V(\{d_t(s)\}, \mu)$ while taking as given the equilibrium payoff $\widetilde{W}$, since the latter is determined in equilibrium by the joint design of claims and trading arrangements of other investors, i.e., $\widetilde{W} = W(\{B_t(s)\}, \mu^*)$. The planner instead designs claims and trading arrangements on behalf of all investors. Thus, when solving problem (P3), she internalizes that, although the investors' payoff $V(\{T_t(s)\}, \mu^{SP})$ is lower when the claims are more state-contingent and the trading arrangements are more efficient, this decline is offset by the resulting increase in the investors' payoff $W(\{T_t(s)\}, \mu^{SP})$. As we have seen, once the planner internalizes this effect, she opts for trading arrangements that are efficient (Lemma 6) and, conditional on such arrangements, she finds it optimal to achieve full aggregate risk-sharing between entrepreneurs and investors, though the level of her ex-ante investment and continuation scales may still be depressed relative to first-best due to both limited pledgeability and need to deliver the rents $U - e$ that investors earned at equilibrium (Lemma 7).

5.2 Policy Implications

In this section, we explore the implications of our theory for optimal corrective policy. From the previous analysis, it should be clear that in order to decentralize the planner's allocations, it suffices to search for policies that incentivize investors: (i) to allocate the claims efficiently in secondary markets, and (ii) to price them in an undistorted manner ex-ante; while respecting the primitive frictions faced by the agents in the laissez-faire equilibrium. In what follows, we propose government interventions in secondary markets that achieve these objectives.

Proposition 6 *The constrained efficient allocations can be decentralized through a subsidy,* $S(\{d_t(s)\}, \mu) = nW(\{d_t(s)\}, \mu)$*, given to a seller who chooses* $(\{d_t(s)\}, \mu)$*, financed with lump sum taxes on the sellers of claims. To ensure that the intervention is Pareto improving, the government needs to make a lump sum transfer* $U - e$ *from entrepreneurs to investors ex-ante.*

Given the subsidy, the privately optimal arrangement can now be shown to maximize the expected payoff $V(\{d_t(s)\}, \mu) + nW(\{d(s)\}, \mu)$ instead of just $V(\{d_t(s)\}, \mu)$. The investor will therefore find it optimal to choose the efficient trading arrangement μ^{SP}, since:

$$\begin{aligned}\max_{\mu \in \mathcal{M}} V(\{d_t(s)\}, \mu) + nW(\{d_t(s)\}, \mu) &= V(\{d_t(s)\}, \mu^{SP}) + nW(\{d_t(s)\}, \mu^{SP}) \\ &= \sum_s \pi(s)(d_1(s) + d_2(s)), \end{aligned} \tag{43}$$

where the last equality holds as the losses of an efficient trading arrangement must equal the rents of the traders. Moreover, as the investors will now value the claims (in expectation) at $\sum_s \pi(s)(d_1(s) + d_2(s))$, independently of whether they sell or keep them, the intervention will also ensure that in equilibrium the claims prices are undistorted, i.e., $q(s) = \pi(s)\ \forall s$. The government finances the intervention through a lump sum tax on all sellers equal to $S(\{B_t(s)\}, \mu^{SP})$, where $\{B_t(s)\}$ are the aggregate claims supplied by entrepreneurs after the intervention. Therefore, as the subsidy is fully financed by the sellers, and as it corrects the mispricing/misallocation of claims in secondary markets, this intervention also eliminates the rents that the investors were able to extract from entrepreneurs in the laissez-faire equilibrium (see Proposition 4). Hence, to deliver these rents back to them, the government makes a transfer of $U - e$ from entrepreneurs to investors at the issuance stage ex-ante.

Albeit abstract, the policy described in Proposition 6 is conceptually very clear: the constructed subsidy simply makes each seller internalize the effects of his choice of claims and trading arrangements on the buyers of claims. Nevertheless, as we show next, one can also decentralize the planner's allocations through policies with a closer "real-world" counterpart.

Proposition 7 *The constrained efficient allocations can be decentralized through government purchases and sales of risky claims within each trading arrangement, financed with lump sum taxes on the sellers of claims. To ensure that the intervention is Pareto improving, the government needs to make a lump sum transfer $U - e$ from entrepreneurs to investors ex-ante.*

An alternative to subsidizing the joint design of claims and arrangements, as in Proposition 6, is to intervene in secondary markets through purchases and sales of claims. In the proof, we show that such an intervention takes the following form: the government enters each trading arrangement and offers to purchase its claims $\{d_t(s)\}$ at some price $\bar{\omega}(\{d_t(s)\})$ to then resell these claims to the traders in that same arrangement at price $\underline{\omega}(\{d_t(s)\}) = \min_x \mathbb{E}\{d_1(s) + d_2(s) | \forall i\ x^i = x\} \leq \bar{\omega}(\{d_t(s)\})$. By reselling the claims at their lowest possible valuation, $\underline{\omega}$, the government ensures that it always allocates the purchased claims to the traders. By selecting the appropriate "price floor" of the arrangement, $\bar{\omega}$, the government ensures that the seller always allocates the claims either to the traders (at prices above $\bar{\omega}$) or to the government (at price $\bar{\omega}$), and that she receives the fair value for them. The intervention thus corrects both the mispricing and the misallocation of claims in secondary markets, thereby also eliminating the distortion in the pricing of claims ex-ante, i.e., $q(s) = \pi(s)\ \forall s$. The price difference, $\bar{\omega} - \underline{\omega}$, is effectively a subsidy that the government extends to each seller of claims in secondary markets when the price of his claims would otherwise be too low, and it is zero when the claims in the arrangement are non-contingent. As with the intervention in Proposition 6, the government finances these subsidies through lump sum taxes levied on the sellers of claims in secondary markets, and it transfers $U - e$ from entrepreneurs to investors at the issuance stage ex-ante.

An interesting and appealing feature of the above asset purchase/sale scheme is that it is more likely to be triggered and is larger in bad aggregate states of nature, since in those states the number of pessimists in the market is larger, depressing the prices of contingent claims. Moreover, this intervention is broadly consistent with a number of policies implemented in the midst of the global financial crisis, whereby governments stepped-in to support trade in securities exposed to real-estate risks, which were perceived to be undervalued by the market, through programs such as TAF and TARP (Baily et al., 2008; Blanchard, 2009; Brunnermeier, 2009). Our model suggests that, beyond directly boosting liquidity in secondary markets, anticipation of such policies can help spread real-estate related risks more evenly throughout the population, ameliorating balance sheet effects and their repercussions for the real economy.

Finally, we find it useful to relate our normative findings to the recently growing literature on pecuniary externalities in models with financial constraints (e.g., Caballero and Krishnamurthy (2003); Lorenzoni (2008); Korinek (2011); Bianchi (2011)). An often-drawn conclusion from this literature is that, because borrowers do not internalize their contribution to balance sheet effects ex-post, they tend to borrow excessively ex-ante, which in turn justifies the by-now conventional macro-prudential policies such as leverage limits or capital requirements. Dávila and Korinek (2017) disect the pecuniary externalities in this class of models and show that they arise either because capital prices enter directly into financial constraints, or because movements in capital prices redistribute wealth between borrowers and creditors. These pecuniary externalities are intentionally not present in our setting,[20] which has allowed us to clearly isolate the novel externality originating from the pricing and allocation of aggregate risk in secondary markets. Moreover, as we show next, this novel externality can itself be a source of “inefficient credit booms”.

Proposition 8 *The laissez-faire equilibrium can feature excessive borrowing and investment ex-ante relative to second-best, i.e., the decentralized equilibrium under the optimal corrective policy of either Proposition 6 or Proposition 7.*

This result can be understood by comparing the determinants of the investment scale in the

[20]First, the financial constraint faced by entrepreneurs is $b_2(s) \leq 0\ \forall s$, which does not feature capital prices. Second, the traditional sector firms make zero profits and, thus, movements in equilibrium prices of capital do not generate wealth transfers between entrepreneurs and investors.

laissez-faire equilibrium (equation (23)) with that of the planner (equation (41)). There are two differences. On the one hand, the distortion in equilibrium claims prices has a depressing effect on K_0, as $q(h) < \pi(h)$. On the other hand, the resulting distortion to risk-sharing leads to fluctuations in capital prices, $p(h) \geq p_U^{SP} \geq p(l)$, which we show can push K_0 above k_0^{SP}. Intuitively, even though the planner faces a higher average return to investment (due to better risk-sharing), the marginal return to investment can be higher at equilibrium because the additional investments are liquidated only in the low state.

The takeaway from the above result is that policymakers should exercise extra caution when attempting to moderate seemingly excessive booms that are anticipated to lead to balance sheet effects and severe economic contractions. Although the conventional macro-prudential policies and the policy interventions proposed in Propositions 6 and 7 may both moderate credit expansions and stabilize output, only the latter policies are optimal when misallocation of aggregate risk originates from the secondary market frictions emphasized in this paper.[21] Therefore, consistent with the theory of the second-best, understanding the underlying source of equilibrium inefficiency is crucial for thinking about the design of optimal corrective policy.

6 Concluding Remarks

This paper develops a theory of the balance sheet channel that places a central emphasis on the liquidity of secondary markets for macro-contingent claims. We show that the presence of dispersed information and imperfect competition in these markets, interacted with financial constraints, results in mispricing and misallocation of aggregate risk, distorts aggregate investment, and exacerbates asset price and output volatility. Our theory implies that the magnitude of balance sheet amplification effects should be tied to the severity of these information-trading frictions, which are likely to vary over time, markets, and economies. The laissez-faire equilibrium is constrained inefficient due to a novel externality originating from rent-extracting behavior of agents in secondary markets. Optimal corrective policy boosts secondary market liquidity by subsidizing trade in state-contingent claims, which in turn enhances aggregate risk-sharing and stabilizes the business cycle. Our theory is therefore consistent with a number of calls for policy makers to enhance risk-sharing through interventions in markets for macro-contingent claims (Case et al., 1991; Shiller, 1994; Caballero, 2003; Mian, 2013). By formalizing the source of illiquidity in these markets, our theory sheds light on the type of policy interventions that can achieve this objective.

[21] A related point has been recently made by Kurlat (2018), who shows that policy implications of canonical models with financial constraints are reversed if the markets for capital suffer from an adverse selection problem à la Akerlof (1970).

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A Proofs for Sections 3-5

Proofs of Lemmas 1 - 4. See text. ■

Proof of Proposition 1. In what follows, we first establish the properties (i)-(iii) stated in Proposition 1 for a generalized investor signal structure, in order to illustrate the broad economic forces that generate mispricing/misallocation of claims in secondary markets. We will then compute the allocations of the optimal mechanism explicitly for the binary signal structure assumed in the paper.

Let $X = \{x_1, ..., x_M\}$ denote the set of the traders' signals, where $M \geq 2$ and where $\mathbb{P}(s = h|x^i = x)$ is increasing in x, i.e., traders with higher signals are more optimistic about the aggregate state. We will also allow the mechanism to have access to some external imperfectly informative signal y about the aggregate state (e.g., public information, seller's signal), which takes values in some set $Y = \{y_1, ..., y_N\} \subset \mathbb{R}$, has the property that $\mathbb{P}(s = h|y) \in (0, 1)$ is increasing in y, and is conditionally independent of the traders' signals. The case of no external signal is captured by assuming that Y is a singleton. The mechanism design problem is in the text, except that the allocations of trader i can also be conditioned on $y \in Y$, i.e., $\mathcal{A}^i(x^i, x^{-i}, y) = (\omega^i(x^i, x^{-i}, y), \{v^i_s(x^i, x^{-i}, y)\})$.

Part (i). Any mechanism μ for trading claims $\{v(s)\}$ that satisfies the participation and the feasibility constraints (PC) and (FC), given by (10) and (12), must also satisfy $V(\{v(s)\}, \mu) \leq \mathbb{E}\{v(s)\}$. Thus, $\mathbb{E}\{v(s)\}$ is the upper bound on the value of the mechanism. If a mechanism μ reaches this upper bound, i.e., $V(\{v(s)\}, \mu) = \mathbb{E}\{v(s)\}$, then we say that mechanism μ achieves *full surplus extraction.* Such a mechanism must clearly be optimal.

When the investor's portfolio of claims is non-contingent, i.e., $v(h) = v(l)$, the optimal mechanism clearly achieves full surplus extraction. For example, consider the mechanism with the following allocations: $v^i_s(x^i, x^{-i}, y) = \omega^i(x^i, x^{-i}, y) = n^{-1}v(l)$ for all s, i, x^i, x^{-i}, and y. Thus, non-contingent claims must be traded fairly and allocated efficiently w.p.1. We are therefore left to study the case where $v(h) \neq v(l)$. We will do so in two steps.

First, we show that full surplus extraction is impossible in any mechanism that satisfies the participation, incentive compatibility and feasiblity constraints (10)-(12).

Lemma A.1 *Let μ^* be an optimal mechanism for trading claims $\{v(s)\}$, and assume that $v(h) \neq v(l)$. Then, $V(\{v(s)\}, \mu^*) < \mathbb{E}\{v(s)\}$.*

Proof. Suppose to the contrary that $V(\{v(s)\}, \mu^*) = \mathbb{E}\{v(s)\}$, and consider the associated allocations $\mathcal{A}^i(x^i, x^{-i}, y) = (\omega^i(x^i, x^{-i}, y), \{v^i_s(x^i, x^{-i}, y)\})$. Also, suppose that $v(h) > v(l)$; the argument for $v(h) < v(l)$ is analogous.

First, full surplus extraction implies that $U^i(x^i, \mathcal{A}^i(x^i, x^{-i}, y)) = 0$ for all i, x^i, x^{-i}, y and $\sum_{i \in I} v^i_s(x^i, x^{-i}, y) = v(s)$ for all x^i, x^{-i}, y, s, i.e., in the optimal mechanism the traders' rents must be zero and the claims must be allocated efficiently. To see this, from (10) and (12):

$$\begin{aligned}
V(\{v(s)\},\mu^*) &= \mathbb{E}\left\{\sum_{i\in I}\omega^i\left(x^i,x^{-i},y\right)\right\} \\
&\le \mathbb{E}\left\{\sum_{i\in I}\sum_s v_s^i\left(x^i,x^{-i},y\right)\cdot\mathbb{P}\left(s|x^i,\mathcal{A}^i\left(x^i,x^{-i},y\right)\right)\right\} \\
&= \mathbb{E}\left\{\mathbb{E}\left\{\sum_{i\in I}\sum_s v_s^i\left(x^i,x^{-i},y\right)\cdot\mathbb{P}\left(s|x^i,\mathcal{A}^i\left(x^i,x^{-i},y\right)\right)|x^i,x^{-i},y\right\}\right\} \\
&= \mathbb{E}\left\{\mathbb{E}\left\{\sum_{i\in I}\sum_s v_s^i\left(x^i,x^{-i},y\right)\cdot\mathbb{P}\left(s|x^i,x^{-i},y\right)|x^i,x^{-i},y\right\}\right\} \\
&\le \mathbb{E}\left\{\mathbb{E}\left\{\sum_s v\left(s\right)\cdot\mathbb{P}\left(s|x^i,x^{-i},y\right)|x^i,x^{-i},y\right\}\right\} \\
&= \mathbb{E}\left\{v\left(s\right)\right\},
\end{aligned} \tag{44}$$

where the first inequality is strict if $U^i\left(x^i,\mathcal{A}^i\left(x^i,x^{-i},y\right)\right)>0$ for some i,x^i,x^{-i},y, and the second inequality is strict if $\sum_{i\in I}v_s^i\left(x^i,x^{-i},y\right)<v\left(s\right)$ for some x^i,x^{-i},y,s.

Second, we find a profitable deviation for a trader to misreport his signal. Since by the argument above $U^i\left(x^i,\mathcal{A}^i\left(x^i,x^{-i},y\right)\right)=0$ for all i,x^i,x^{-i},y, incentive compatibility requires that $U^i\left(x^i,\mathcal{A}^i\left(\theta^i,x^{-i},y\right)\right)\le 0$ for all i,x^i,θ^i,x^{-i},y. To reach a contradiction, we will now show that $U^i\left(x^i,\mathcal{A}^i\left(\theta^i,x^{-i},y\right)\right)>0$ for some i,x^i,θ^i,x^{-i},y. Note that:

$$\begin{aligned}
U^i\left(x^i,\mathcal{A}^i\left(\theta^i,x^{-i},y\right)\right) &= \sum_s v_s^i\left(\theta^i,x^{-i},y\right)\cdot\mathbb{P}\left(s|x^i,\mathcal{A}^i\left(\theta^i,x^{-i},y\right)\right)-\omega^i\left(\theta^i,x^{-i},y\right) \\
&= \sum_s v_s^i\left(\theta^i,x^{-i},y\right)\cdot\left(\mathbb{P}\left(s|x^i,\mathcal{A}^i\left(\theta^i,x^{-i},y\right)\right)-\mathbb{P}\left(s|\theta^i,\mathcal{A}^i\left(\theta^i,x^{-i},y\right)\right)\right)
\end{aligned}$$

for all i,x^i,θ^i,x^{-i},y, since $U^i\left(\theta^i,\mathcal{A}^i\left(\theta^i,x^{-i},y\right)\right)=0$. Observe that (i) $\mathbb{P}\left(h|x^i,\mathcal{A}^i\left(\theta^i,x^{-i},y\right)\right)\gtreqless\mathbb{P}\left(h|\theta^i,\mathcal{A}^i\left(\theta^i,x^{-i},y\right)\right)$ whenever $x^i\gtreqless\theta^i$ since the allocation cannot perfectly reveal the state, and (ii) for all x^i,x^{-i},y, we have that $v_h^i\left(x^i,x^{-i},y\right)>v_l^i\left(x^i,x^{-i},y\right)$ for some i because, as we have argued above, $\sum_i v_h^i\left(x^i,x^{-i},y\right)=v\left(h\right)>v\left(l\right)=\sum_i v_l^i\left(x^i,x^{-i},y\right)$ for all x^i,x^{-i},y. From (ii), there exist i and $\widehat{x}<x_M$ such that $v_h^i\left(\widehat{x},x^{-i},y\right)>v_l^i\left(\widehat{x},x^{-i},y\right)$ for some x^{-i},y; in other words, the mechanism must allocate more claims in the high than in the low state to some trader with signal other than highest (e.g., in the events that no trader has received signal x_M). Combining with (i), we have that $U^i\left(x_M,\mathcal{A}^i\left(\widehat{x},x^{-i},y\right)\right)>0$, and it is sub-optimal for trader i who has received signal x_M to report his signal truthfully. ■

Next, we show that the losses of the optimal mechanism must be proportional to the contingency of the investor's portfolio of claims.

Lemma A.2 *Let μ^* be an optimal mechanism for trading claims $\{v(s)\}$. Then,*

$$V(\{v(s)\},\mu^*) = \sum_s \pi(s)v(s) - \zeta|v(h) - v(l)|,$$

where $\zeta = \begin{cases} \zeta^+ & \text{if } v(h) \geq v(l) \\ \zeta^- & \text{if } v(h) < v(l) \end{cases}$, *and* $\zeta^+ \in (0, \pi(h))$, $\zeta^- \in (0, 1 - \pi(h))$ *are scalars.*

Proof. Suppose that $v(h) > v(l)$; the argument for $v(h) < v(l)$ is analogous. Denote by $\mathcal{A}^i(x^i, x^{-i}, y) = (\omega^i(x^i, x^{-i}, y), \{v_s^i(x^i, x^{-i}, y)\})$ the allocations of the optimal mechanism.

Define $\tilde{v}_s^i(x^i, x^{-i}, y) \equiv \frac{v_s^i(x^i,x^{-i},y) - n^{-1}v(l)}{v(h)-v(l)}$ and $\tilde{\omega}^i(x^i, x^{-i}, y) = \frac{\omega^i(x^i,x^{-i},y) - n^{-1}v(l)}{v(h)-v(l)}$ for all i, x^i, x^{-i}, y, and consider the modified allocations $\tilde{\mathcal{A}}^i(x^i, x^{-i}, y) = (\tilde{\omega}^i(x^i, x^{-i}, y), \{\tilde{v}_s^i(x^i, x^{-i}, y)\})$. Then the design problem can be re-written as follows:

$$V(\{v(s)\}, \mu^*) = v(l) + (v(h) - v(l)) \cdot \max_{\{\tilde{v}_s^i, \tilde{\omega}^i\}} \mathbb{E}\left\{\sum_{i\in I} \tilde{\omega}^i\left(x^i, x^{-i}, y\right)\right\}$$

subject to:

$$\sum_s \tilde{v}_s^i\left(x^i, x^{-i}, y\right) \cdot \mathbb{P}\left(s|x^i, \tilde{\mathcal{A}}^i\left(x^i, x^{-i}, y\right)\right) \geq \tilde{\omega}^i\left(x^i, x^{-i}, y\right) \text{ for all } i, x^i, x^{-i}, y,$$

$$\mathbb{E}\left\{U^i(x^i, \tilde{\mathcal{A}}^i(x^i, x^{-i}, y))|x^i\right\} \geq \mathbb{E}\left\{\max\left\{0,\ U^i(x^i, \tilde{\mathcal{A}}^i(\theta^i, x^{-i}, y)\right\}|x^i\right\} \text{ for all } i, x^i, \theta^i,$$

and

$$\sum_{i\in I} \tilde{v}_s^i\left(x^i, x^{-i}, y\right) \leq \tilde{v}(s) \text{ for all } x^i, x^{-i}, y, s,$$

where $\tilde{v}(l) = 0$ and $\tilde{v}(h) = 1$. It follows that:

$$V(\{v(s)\}, \mu^*) = v(l) + (v(h) - v(l)) \cdot V(\{\tilde{v}(s)\}, \tilde{\mu}^*)$$

where $\tilde{\mu}^*$ is the optimal mechanism for trading the claims $\{\tilde{v}(s)\}$. Since this mechanism also satisfies the constraints (10), (11) and (12), full surplus extraction is also not possible in this mechanism (see Lemma A.1), and thus $V(\{\tilde{v}(s)\}, \tilde{\mu}^*) < \pi(h)$. Also, $V(\{\tilde{v}(s), \tilde{\mu}^*\}) > 0$ because the mechanism designer always has the option to trade the claims at the expected valuation of the trader with the lowest signal, which is strictly positive. We define $\zeta^+ \equiv \pi(h) - V(\{\tilde{v}(s)\}, \tilde{\mu}^*) \in (0, \pi(h))$. Analogous arguments imply that $\zeta^- \in (0, 1 - \pi(h))$. Note that by construction the scalars ζ^+, ζ^- depend only on the trading friction, n, the distribution over the signals $\{\mathbb{P}(x, y|s)\}_{x,y,s}$, and the distribution over the state, $\{\pi(s)\}_s$ ■

Part (ii). The traders' participation constraint (10) implies that their ex-ante expected payoff $\overline{W(\{v(s)\}, \mu^*)}$ is non-negative. It is also clear that the traders' expected payoff cannot exceed the expected losses of the mechanism. That the mechanism may trade the claims with probability less than one and, thus, allocate them inefficiently is shown in Lemma A.3 below.

Part (iii). That the scalars ζ^+ and ζ^- monotonically decline to zero as n grows large follows from two simple observations. First, the mechanism designer always has the option to disregard a trader's signal and set his allocations to zero; thus, the value of the mechanism must be non-decreasing in the number of traders. Second, it is straightforward to construct a mechanism in which the traders' rents go to zero as n grows to ∞; hence, this must also hold in the optimal mechanism. To this end, consider a mechanism, which allocates to each trader $n^{-1}v(s)$ units of claims for state s, and asks each trader to pay the expected value of the claims allocated to him, conditional on (i) the trader having received the 'worst' possible signal and (ii) the other traders' signals, i.e., $\omega^i(x^i, x^{-i}, y) = \min_x \sum_s \mathbb{P}(s|x^i = x, x^{-i}, y)n^{-1}v(s)$. Clearly, this mechanism satisfies the (PC), (IC), and (FC) constraints, and the expected value of the mechanism increases monotonically to $\sum_s \pi(s)v(s)$ with n.

Next, let us study what happens when information dispersion vanishes. Suppose first that the private signals become uninformative, i.e., $\mathbb{P}(s|x^i)$ goes to $\pi(s)$ $\forall s, x^i$, holding the distribution over the state, $\{\pi(s)\}_s$, and over the external signal, $\{\mathbb{P}(y|s)\}_{y,s}$, fixed. Then, the following mechanism satisfies (PC), (IC) and (FC) constraints and it clearly achieves full surplus extraction asymptotically: $v^i_s(x^i, x^{-i}, y) = n^{-1}v(s)$ and $\omega^i(x^i, x^{-i}, y) = \min_x \mathbb{E}\{n^{-1}v(s)|x^i = x\} \to \mathbb{E}\{n^{-1}v(s)\}$ as signals become uninformative.

Suppose second that the private signals become perfectly informative, i.e., $\mathbb{P}(s|x^i)$ goes to either 0 or 1 $\forall s, x^i$, again holding the distribution over the state and the external signal fixed. Let $\Psi = \{x \in X : \mathbb{P}(h|x) \to 1\}$ denote the set of signals that become perfectly informative of the high state, and consider the following mechanism: $v^i_s(x^i, x^{-i}, y) = \mathbb{1}_{\{x^i\in\Psi\}} \cdot n^{-1}v(s) + \mathbb{1}_{\{x^i\notin\Psi\}} \cdot n^{-1}v(l)$ and $\omega^i(x^i, x^{-i}, y) = \mathbb{1}_{\{x^i\in\Psi\}} \cdot \min_{x\in\Psi} \mathbb{E}\{n^{-1}v(s)|x^i = x\} + \mathbb{1}_{\{x^i\notin\Psi\}} \cdot n^{-1}v(l)$. Observe, the trader has no incentive to misreport his signal and, moreover, $\mathbb{E}\{\sum_i \omega^i(x^i, x^{-i}, y)\} \to \mathbb{E}\{v(s)\}$ as signals become perfectly informative.

Finally, suppose that $\pi(h)$ goes either to 1 or 0, holding the distribution over signals, $\{\mathbb{P}(x|s)\}$, fixed. It is clear that ζ^+ (ζ^-) goes to zero as $\pi(h) \to 0$ ($\pi(h) \to 1$), since $\zeta^+ \in (0, \pi(h))$ and $\zeta^- \in (0, 1 - \pi(h))$ by Part (i). Next, consider the following feasible mechanism: $v^i_s(x^i, x^{-i}, y) = n^{-1}v(s)$ and $\omega^i(x^i, x^{-i}, y) = \min_x \mathbb{E}\{n^{-1}v(s)|x^i = x, x^{-i}, y\}$. The expected revenues of this mechanism clearly go to $\mathbb{E}\{v(s)\}$ when $\pi(h) \to 1$ and $v(h) > v(l)$ (i.e., ζ^+ is the relevant discount), or when $\pi(h) \to 0$ and $v(h) < v(l)$ (i.e., ζ^- is the relevant discount).

We now return to the binary signal structure assumed in the main analysis (and without an external signal), in order to provide an explicit characterization of the optimal mechanism:

Lemma A.3 *Consider the mechanism design problem of Section 2 with a binary signal structure, i.e., $\forall i$ $x^i \in X = \{B, G\}$. Then, the allocations of the optimal mechanism satisfy $v^i_h, v^i_l, \omega^i \geq 0$. The mechanism allocates the claims efficiently if and only if $\Delta \geq 0$, where:*

$$\Delta \equiv \begin{cases} \mathbb{P}(h|x^i = B\ \forall i) - \mathbb{P}(h|x^i = G, x^j = B\ \forall j \neq i)\mathbb{P}(x^i = G|x^j = B\ \forall j \neq i) & \text{if } v(h) > v(l) \\ \mathbb{P}(l|x^i = G\ \forall i) - \mathbb{P}(l|x^i = B, x^j = G\ \forall j \neq i)\mathbb{P}(x^i = B|x^j = G\ \forall j \neq i) & \text{if } v(h) < v(l). \end{cases} \tag{45}$$

and the expected revenues of the mechanism are given by (18)*, where:*

$$\zeta^+ = \begin{cases} \mathbb{P}\left(x^i = G, x^j = B\ \forall j \neq i\right)\left[\mathbb{P}\left(h|x^i = G, x^j = B\ \forall j \neq i\right) - \mathbb{P}\left(h|x^i = B\ \forall i\right)\right] & \text{if } \Delta \geq 0 \\ \mathbb{P}\left(x^i = B\ \forall i\right)\mathbb{P}\left(h|x^i = B\ \forall i\right) & \text{if } \Delta < 0 \end{cases}, \tag{46}$$

$$\zeta^- = \begin{cases} \mathbb{P}\left(x^i = B, x^j = G\ \forall j \neq i\right)\left[\mathbb{P}\left(l|x^i = B, x^j = G\ \forall j \neq i\right) - \mathbb{P}\left(l|x^i = G\ \forall i\right)\right] & \text{if } \Delta \geq 0 \\ \mathbb{P}\left(x^i = G\ \forall i\right)\mathbb{P}\left(l|x^i = G\ \forall i\right) & \text{if } \Delta < 0 \end{cases}. \tag{47}$$

Proof. Suppose that $v(h) > v(l)$; the proof for $v(h) < v(l)$ is analogous. We solve the relaxed problem in which the (IC) constraint for the trader who has received signal B and the (PC) constraint of the trader who has received signal G are slack, and then we verify that this is the case at the optimum.

In the relaxed problem, the (PC) constraint of trader with signal B must be binding, i.e.,

$$\omega^i(B, x^{-i}) = \sum_s v_s^i(B, x^{-i})\mathbb{P}(s|B, \mathcal{A}^i(B, x^{-i})) \tag{48}$$

for all i, x^{-i}, since by assumption this trader prefers to report his signal truthfully and the mechanism will therefore extract his full surplus. It is also clear that the (IC) constraint of trader with signal G must be binding, i.e.,

$$\mathbb{E}\{\omega^i(G, x^{-i})|G\} = \mathbb{E}\left\{\sum_s v_s^i(G, x^{-i})\mathbb{P}(s|G, \mathcal{A}^i(G, x^{-i})) - \max\{0, U^i(G, \mathcal{A}^i(B, x^{-i}))\}|G\right\} \tag{49}$$

for all i, since if this trader were to strictly prefer to report his signal truthfully, then the mechanism could increase its revenues by asking this trader to make larger payments for some realizations of other traders' signals. From (48), the payoff to trader with signal G from reporting that he has signal B is:

$$U^i(G, \mathcal{A}^i(B, x^{-i})) = \left(v_h^i(B, x^{-i}) - v_l^i(B, x^{-i})\right)\left(\mathbb{P}(h|G, \mathcal{A}^i(B, x^{-i})) - \mathbb{P}(h|B, \mathcal{A}^i(B, x^{-i}))\right), \tag{50}$$

where $\mathbb{P}(h|G, \mathcal{A}^i(B, x^{-i})) > \mathbb{P}(h|B, \mathcal{A}^i(B, x^{-i}))$ for all i, x^{-i}. The following inequality is straightforward to establish:

$$\begin{aligned} &\mathbb{E}\left\{\max\left\{0, U^i\left(G, \mathcal{A}^i\left(B, x^{-i}\right)\right)\right\}|G\right\} \geq \\ &\mathbb{E}\left\{\max\left\{0, \left(v_h^i\left(B, x^{-i}\right) - v_l^i\left(B, x^{-i}\right)\right)\right\}\left(\mathbb{P}\left(h|G, x^{-i}\right) - \mathbb{P}\left(h|B, x^{-i}\right)\right)|G\right\} \end{aligned} \tag{51}$$

for all i. Intuitively, it states that the rents of the trader with G signal are minimized if the allocations of the trader who reports to have received signal B are fully revealing of other traders' signals x^{-i}. Let us conjecture and then verify that at the optimum (51) holds with equality for all i. The relaxed problem then reduces to finding the allocations of claims $\{v_s^i\}$

to solve the following:

$$\max_{\{v_s^i\}} \quad \mathbb{E}\left\{\sum_i v_s^i(x^i,x^{-i})\right\} - \sum_i \mathbb{P}(x^i=G)\mathbb{E}\left\{\max\left\{0,\left(v_h^i\left(B,x^{-i}\right)-v_l^i\left(B,x^{-i}\right)\right)\right\}\left(\mathbb{P}\left(h|G,x^{-i}\right)-\mathbb{P}\left(h|B,x^{-i}\right)\right)|G\right\} \tag{52}$$

subject to the feasibility constraint $\sum_i v_s^i(x^i,x^i) \leq v(s)$ for all s, x^i, x^{-i}.

Thus, we see that the value of the mechanism increases in the claims it allocates to traders, but it decreases in the rents it leaves to traders with G signals. By inspection, we see that since selling claims to traders who reported signal G does not generate rents, it is optimal to set: $v_s^i(G,x^{-i}) = m(x^1,...,x^n)^{-1}v(s)$, where $m(x^1,...,x^n)$ is the number of traders in the mechanism who have reported signal G, when there is at least one such trader. This implies that traders with B signals receive the empty allocation when there is at least one trader in the mechanism with signal G.

We next determine the allocations, $v_s^i(x^i,x^{-i})$, whenever all traders in the mechanism report signal B. For this case, since selling equal units of claims in both states does not entail rents, it is optimal to set $v_h^i(x^i,x^{-i})|_{x^i=B,\ x^j=B\ \forall j\neq i} \geq v_l^i(x^i,x^{-i})|_{x^i=B,\ x^j=B\ \forall j\neq i} = n^{-1}v(l)$. Maximization of (52) with respect to $v_h^i(x^i,x^{-i})|_{x^i=B,\ x^j=B\ \forall j\neq i} \in [n^{-1}v(l), n^{-1}v(h)]$ yields:

$$v_h^i(x^i,x^{-i})|_{x^i=B,\ x^j=B\ \forall j\neq i} = \begin{cases} n^{-1}v(h) & \text{if } \Delta \geq 0 \\ n^{-1}v(l) & \text{if } \Delta < 0. \end{cases} \tag{53}$$

Thus, the mechanism allocates these claims to the traders iff the expected payoff from doing so exceeds the rents this allocation generates for traders with G signals. Note that, given the computed allocations, when a trader's allocation is non-empty, he perfectly infers the other traders' signals from it. In particular,

$$U^i\left(G,\mathcal{A}^i\left(B,x^{-i}\right)\right) = \begin{cases} n^{-1}\left(v\left(h\right)-v\left(l\right)\right)\left(\mathbb{P}\left(h|G,x^j=B\forall j\neq i\right)-\mathbb{P}\left(h|B,x^j=B\forall j\neq i\right)\right) & \text{if } \Delta \geq 0 \\ 0 & \text{if } \Delta < 0 \end{cases} \tag{54}$$

and our conjecture that (51) holds with equality is verified.

We have thus characterized the allocations of claims $\{v_s^i\}$, which by (48) also pin down the payments collected from the traders with B signals. Let us now characterize the payments collected from the traders with G signals. To this end, suppose that the mechanism sets:

$$\omega^i\left(G,x^{-i}\right) = \sum_s v_s^i\left(G,x^{-i}\right)\mathbb{P}\left(s|G,x^{-i}\right) - U^i\left(G,\mathcal{A}^i\left(B,x^{-i}\right)\right) \tag{55}$$

for all i, x^{-i}. By construction, these payments satisfy the (IC) constraint of the trader with G signal (see equation (49)), and clearly they also satisfy the (PC) constraints of this trader.

To complete the characterization, we are left to verify that the (IC) constraint of the trader with signal B is satisfied. To this end, note that: (i) $U^i(B,\mathcal{A}^i(G,x^{-i})) = \sum_s v(s)\mathbb{P}(s|B,x^{-i}) - \omega^i(G,x^{-i}) \leq 0$ when $\Delta \geq 0$ and all other traders have received signal B (with strict inequality if $n > 1$); (ii) $U^i(B, \mathcal{A}^i(G, x^{-i})) = \sum_s v(s)\mathbb{P}(s|B, x^{-i}) - \omega^i(G, x^{-i}) < 0$ when $\Delta < 0$ and all other traders have received signal B, since then we have $\omega^i(G, x^{-i}) = \sum_s v(s)\mathbb{P}(s|G, x^{-i})$; (iii) $U^i(B, \mathcal{A}^i(G, x^{-i})) = m(x^1, ..., x^n)^{-1} \sum_s v(s)\mathbb{P}(s|B, x^{-i}) - \omega^i(G, x^{-i}) < 0$ when some other traders have received signal G, since then $\omega^i(G, x^{-i}) = m(x^1, ..., x^n)^{-1} \sum_s v(s)\mathbb{P}(s|G, x^{-i})$. Thus, this trader's the ex-post payoff from reporting a G signal is always weakly negative, which establishes the result.

Finally, if $\Delta < 0$, then the mechanism's revenues are given by:

$$V(\{v(s)\}, \mu^*) = \sum_s \pi(s)v(s) - \underbrace{\mathbb{P}(x^i = B\ \forall i)\mathbb{P}(h|x^i = B\ \forall i)}_{\zeta^+ \equiv}(v(h) - v(l)), \tag{56}$$

since the mechanism does not allocate the risky portion of the claim to traders with B-signals, but it allocates all the claims and extracts full surplus from traders with G signals (if there are any). But, if $\Delta \geq 0$, then:

$$V(\{v(s)\}, \mu^*) = \sum_s \pi(s)v(s) - \underbrace{\mathbb{P}\left(x^i = G, x^j = B\ \forall j \neq i\right)\left[\mathbb{P}\left(h|x^i = G, x^j = B\ \forall j \neq i\right) - \mathbb{P}\left(h|x^i = B\ \forall i\right)\right]}_{\zeta^+ \equiv}(v(h) - v(l)), \tag{57}$$

since the mechanism must give information rents to each trader with G signal to discourage him from reporting to have received a B signal. ■

Thus, Lemma A.3 shows that the claims allocations and the payments for all traders are non-negative in the optimal mechanism. Though we conjecture that this result holds for general investor signal structure, we do not have a proof of that. In Appendix B, we will use these properties of the optimal mechanism to verify the assertions made in Section 2.1 that, at the proposed claims prices (see property (1)), the investors' consumptions remain non-negative at all times.

The lemma also provides the conditions under which the optimal trading arrangements allocate the claims inefficiently in secondary markets. In particular, the optimal mechanism trades with probability less than one whenever the loss generated from excluding the pessimistic traders from trade is smaller than the resulting reduction in the rents earned by the more optimistic traders. For instance, when $n = 1$ and $v(h) > v(l)$, this occurs if and only if $\mathbb{P}(h|x^i = B) < \mathbb{P}(x^i = G)\mathbb{P}(h|x^i = G)$, i.e., when signals are very informative about the state and/or there is a high probability of a trader having received a Good signal.

For the binary-symmetric parameterization of the signals, the expression for ζ^+ that is depicted in Figure 2 becomes:

$$\zeta^+ = \min\left\{(1-\phi)^n, \phi(1-\phi)^{n-1} - \frac{\phi(1-\phi)^{n-1}\pi(h) + (1-\phi)\phi^{n-1}(1-\pi(h))}{(1-\phi)^n \pi(h) + \phi^n(1-\pi(h))}(1-\phi)^n\right\} \cdot \pi(h), \tag{58}$$

where the condition $\Delta \geq 0$ is equivalent to the first term in the min operator being greater than the second term. Consistent with the properties (i)-(iii) of the proposition, we see that

ζ^+ goes to zero as n grows large, as ϕ goes to $\frac{1}{2}$ or 1, and as $\pi(h)$ goes to 0 or 1. ■

Proof of Corollary 1. In equilibrium, the claims issued by the entrepreneurs must be held by the investors. Thus, for all s, $d_1(s) = B_1(s)$ and $d_2(s) = 0$, which implies that $v(s) = B_1(s)$. The investor's net expected payoff from purchasing claims $\{v(s)\}$ at the issuance stage is:

$$\Gamma(\{v(s)\}) = -\sum_s q(s)v(s) + \lambda V(\{v(s)\}, \mu^*) + (1-\lambda)\sum_s \pi(s)v(s). \tag{59}$$

From Proposition 1: (i) when $q(h) = \pi(h) - \lambda\zeta^+$, then $\Gamma(\{v(s)\}) = 0$ if $v(h) \geq v(l)$ and $\Gamma(\{v(s)\}) < 0$ otherwise; (ii) when $q(h) = \pi(h) + \lambda\zeta^-$, then $\Gamma(\{v(s)\}) = 0$ if $v(h) \leq v(l)$ and $\Gamma(\{v(s)\}) < 0$ otherwise; (iii) when $q(h) \in (\pi(h) - \lambda\zeta^+, \pi(h) + \lambda\zeta^-)$, then $\Gamma(\{v(s)\}) = 0$ if $v(h) = v(l)$ and $\Gamma(\{v(s)\}) < 0$ otherwise. It thus follows that the claims prices given in Corollary 1 are part of equilibrium.

Next, note that these are the only prices consistent with equilibrium. From Proposition 1: (i) if $v(h) > v(l)$, then $\Gamma(\{v(s)\})$ is decreasing in $q(h)$ and equal to zero when $q(h) = \pi(h) - \lambda\zeta^+$. Therefore, if $q(h)$ were higher, the investors would not want to hold the claims $\{v(s)\}$, but if $q(h)$ were lower, there would be excess demand. By an analogous argument, if $v(h) < v(l)$, then it must be that $q(h) = \pi(h) + \lambda\zeta^-$. Finally, if $v(h) = v(l)$, then $\Gamma(\{v(s)\}) = 0$ for any $q(h)$. However, if $q(h) \notin [\pi(h) - \lambda\zeta^+, \pi(h) + \lambda\zeta^-]$, then the investor would deviate and form a contingent portfolio: if $q(h) < \pi(h) - \lambda\zeta^+$, then $\Gamma(\{v(s)\}) > 0$ for $v(h) > v(l)$; if $q(h) > \pi(h) + \lambda\zeta^-$, then $\Gamma(\{v(s)\}) > 0$ for $v(h) < v(l)$. ■

Proof of Corollary 2. See text. ■

The next two lemmas will be used in the proofs of Propositions 2 and 3.

Lemma A.4 *In equilibrium, $B_1(h) \geq B_1(l)$ and $q(h) < \pi(h)$.*

Proof. Suppose to the contrary that $B_1(h) < B_1(l)$. Then, we must have $p(h) \geq p(l)$ and $q(h) = \pi(h) + \lambda\zeta^-$ (Corollaries 1 and 2). But, Lemma 3 implies that $\frac{\pi(h)}{q(h)}\frac{A}{p(h)} \geq \frac{1-\pi(h)}{1-q(h)}\frac{A}{p(l)}$ or, equivalently, that:

$$p(h) \leq \frac{\pi(h)}{q(h)} \cdot \frac{1-q(h)}{1-\pi(h)} \cdot p(l) < p(l), \tag{60}$$

a contradiction.

Suppose to the contrary that $q(h) \geq \pi(h)$. By the argument above and Corollary 1, it must be that $B_1(h) = B_1(l) \leq (a(l)+gA)K_0$. By Assumption 2(iii), it must be that $B_1(h) < a(h)K_0$ and, thus, $p(h) = A$ (Corollary 2). Since also $B_1(h) < (a(h) + p(h))K_0$, we must have $\frac{\pi(h)}{q(h)}\frac{A}{p(h)} \geq \frac{1-\pi(h)}{1-q(h)}\frac{A}{p(l)}$ (Lemma 3). But, because $q(h) \geq \pi(h)$ and $p(h) = A$, this is only possible if $q(h) = \pi(h)$ and $p(l) = A$, which in turn implies that $B_1(l) \leq a(l)K_0$ (Corollary 2). By Lemma 4, the entrepreneurs' investment scale satisfies, $\chi'(K_0) = \sum_s \pi(s)a(s) + A$, which implies that $\chi(K_0) > a(l)K_0 \geq \sum_s q(s)B_1(s)$ by Assumption 2(ii), a contradiction. ■

Lemma A.5 *In equilibrium, $p(h) > p(l)$ and, thus, $B_1(l) \geq a(l)K_0$ and $B_1(h) \leq a(h)K_0$.*

Proof. Suppose to the contrary that $p(l) \geq p(h)$. By Lemma A.4, $q(h) < \pi(h)$ and, thus, it must be $\frac{\pi(h)}{q(h)} \frac{A}{p(h)} > \frac{1-\pi(h)}{1-q(h)} \frac{A}{p(l)}$. But then we must have $B_1(h) \geq B_1(l) = (a(l)+gA)K_0$ (Lemmas 3 and A.4) and, thus, $p(l) = gA$ (Corollary 2). Since $p(h) \geq gA$, we must thus also have $p(h) = gA$. By Lemma 4, the entrepreneurs' investment satisfies $\chi'(K_0) = \sum_s q(s)a(s) + gA$. Since $q(h) < \pi(h)$, we have $\chi(K_0) < \sum_s \pi(s)a(s)K_0 < (a(l)+gA)K_0$ by Assumptions 2(i)&(iii). But we also have $\chi(K_0) = \sum_s q(s)B_1(s) \geq (a(l) + gA)K_0$, a contradiction. Finally, by Corollary 2, if $B_1(l) < a(l)K_0$, then $p(l) = A$; and if $B_1(h) > a(h)K_0$, then $p(h) = gA$. These contradict the previous result that $p(h) > p(l)$. ■

Proof of Proposition 2. Assume that $\bar{q} > \pi(h) - \lambda\zeta^+$.

We first show that $B_1(h) = B_1(l)$. Suppose to the contrary that $B_1(h) > B_1(l)$ (recall $B_1(h) \geq B_1(l)$ by Lemma A.4), then $q(h) = \pi(h) - \lambda\zeta^+$ (Corollary 1). Since $B_1(h) < (a(h) + p(h))K_0$ (Lemma A.5), the entrepreneurs' optimal financing decision implies that $\frac{\pi(h)}{q(h)} \frac{A}{p(h)} \geq \frac{1-\pi(h)}{1-q(h)} \frac{A}{p(l)}$ or, equivalently,

$$p(l) \geq \frac{\pi(h) - \lambda\zeta^+}{\pi(h)} \frac{1 - \pi(h)}{1 - \pi(h) + \lambda\zeta^+} p(h), \tag{61}$$

where the inequality is an equality whenever the borrowing constraint is slack in state l, i.e., if $B_1(l) < (a(l) + gA)K_0$ (Lemma 3). If $B_1(l) < (a(l) + gA)K_0$, then (61) holds with equality and, using the definition of $\bar{q}$ in (22) and the assumption that $\bar{q} > \pi(h) - \lambda\zeta^+$, we have that $p(l) < gp(h) \leq gA$, a contradiction. Instead, if $B_1(l) = (a(l) + gA)K_0$, then entrepreneurs liquidate capital in state l and, thus, $p(l) = gA$ (Corollary 2). By Lemma 4, the entrepreneurs' investment scale satisfies:

$$\chi'(K_0) = q(h)(a(h) + p(h)) + (1 - q(h))(a(l) + gA). \tag{62}$$

Since also $\chi(K_0) = \sum_s q(s)B_1(s) > (a(l) + gA)K_0$, it follows that K_0 is greater than $\bar{k} > 0$ such that $\chi(\bar{k}) = (a(l) + gA)\bar{k}$. Thus,

$$\begin{aligned} \chi'(\bar{k}) &< q(h)(a(h) + p(h)) + (1 - q(h))(a(l) + gA) \\ &< \bar{q}(a(h) + A) + (1 - \bar{q})(a(l) + gA) \\ &\leq \chi'(\bar{k}), \end{aligned} \tag{63}$$

where the last inequality follows from the definition of $\bar{q}$ in (22), a contradiction.

Next, we show that $B_1(l) > a(l)K_0$ and capital gets liquidated in state l. Suppose to the contrary that $B_1(l) \leq a(l)K_0$. Because the claims are non-contingent, $B_1(h) = B_1(l) \leq a(l)K_0$, it must be that $p(h) = A$ and $p(l) = \frac{q(h)}{\pi(h)} \frac{1-\pi(h)}{1-q(h)} A$ (Corollary 2 and Lemma 3), and also that the investment scale satisfies $\chi(K_0) = B_1(l) \leq a(l)K_0$ (Lemma 2). But, by Assumption 2(ii),

$$\begin{aligned} \chi'(K_0) &< \frac{\pi(h)g}{\pi(h)g + 1 - \pi(h)}(a(h) + A) + \left(1 - \frac{\pi(h)g}{\pi(h)g + 1 - \pi(h)}\right)(a(l) + gA) \\ &\leq q(h)(a(h) + A) + (1 - q(h))(a(l) + p(l)) \end{aligned} \tag{64}$$

where the second inequality follows from the fact that $p(l) = \frac{q(h)}{\pi(h)} \frac{1-\pi(h)}{1-q(h)} A$ and that $p(l) \geq gA$. But then the investment scale K_0 is sub-optimal (Lemma 4), a contradiction.

Next, we show that $q(h) = \bar{q}$. Since $B_1(l) > a(l)K_0$, we have $p(l) = gA$. Since the claims are non-contingent, $B_1(h) = B_1(l)$, and since $a(l) + gA < a(h)$ by Assumption 2(iii), it must be that $B_1(h) < a(h)K_0$ and, thus, $p(h) = A$. By Lemma 3, the entrepreneurs' optimal financing decision implies that $\frac{\pi(h)}{q(h)} \frac{A}{p(h)} \geq \frac{1-\pi(h)}{1-q(h)} \frac{A}{p(l)}$, where if $B_1(l) < (a(l) + gA)K_0$, then $\frac{\pi(h)}{q(h)} \frac{A}{p(h)} = \frac{1-\pi(h)}{1-q(h)} \frac{A}{p(l)}$. Using the fact that $p(l) = gA$ and $p(h) = A$, we have that $q(h) = \frac{\pi(h)g}{\pi(h)g+1-\pi(h)}$. Since $\chi(K_0) = \sum_s q(s)B_1(s) < (a(l) + gA)K_0$, it must be that $K_0 < \bar{k}$. By Lemma 4, this is an equilibrium if and only if:

$$\chi'(\bar{k}) > \frac{\pi(h)g}{\pi(h)g + 1 - \pi(h)}(a(h) + A) + \left(1 - \frac{\pi(h)g}{\pi(h)g + 1 - \pi(h)}\right)(a(l) + gA), \tag{65}$$

which holds if and only if also $\bar{q} = \frac{\pi(h)g}{\pi(h)g+1-\pi(h)}$ (see definition of $\bar{q}$); thus, $q(h) = \bar{q}$. On the other hand, if $B_1(l) = (a(l) + gA)K_0$, then $\frac{\pi(h)}{q(h)} \frac{A}{p(h)} \geq \frac{1-\pi(h)}{1-q(h)} \frac{A}{p(l)}$. Using the fact that $p(l) = gA$ and $p(h) = A$, we have $q(h) \leq \frac{\pi(h)g}{\pi(h)g+1-\pi(h)}$. We also have that $K_0 = \bar{k}$ and, therefore, by Lemma 4 it must be that:

$$\chi'(\bar{k}) = q(h)(a(h) + A) + (1 - q(h))(a(l) + gA), \tag{66}$$

which holds if and only if $q(h) = \bar{q}$ (see definition of $\bar{q}$).

Finally, $p(l) = gA$ since capital gets liquidated in state l, and $p(h) = A$ since $B_1(h) = B_1(l) \leq (a(l) + gA)K_0$ and since $a(l) + gA < a(h)$ (Assumption 2(iii)); thus, $B_1(h) < a(h)K_0$.

As for equilibrium existence, we have shown that shown that, in equilibrium, asset prices satisfy $q(h) = \bar{q}$, $p(h) = A$ and $p(l) = gA$; entrepreneurs fund investment with non-contingent claims $\chi(K_0) = B_1(h) = B_1(l)$, liquidate capital only in state l according to equation (1), and they invest ex-ante at scale:

$$\chi'(K_0) = \bar{q}(a(h) + A) + (1 - \bar{q})(a(l) + gA). \tag{67}$$

Hence, an equilibrium exists by continuity of $\chi(\cdot)$ and $\chi'(\cdot)$, and convexity of the former. ■

Proof of Proposition 3. Assume that $\bar{q} < \pi(h) - \lambda\zeta^+$.

We first show that $B_1(h) > B_1(l)$ (recall $B_1(h) \geq B_1(l)$ by Lemma A.4). Suppose to the contrary that $B_1(h) = B_1(l)$. If $B_1(l) < (a(l) + gA)K_0$, then by Assumption 2(iii) we also have $B_1(h) < a(h)K_0$ and, thus, $p(h) = A$. Following the same arguments as in the proof of Proposition 2, we can show that then $B_1(l) > a(l)K_0$, capital gets liquidated in state l and, thus, $p(l) = gA$. The entrepreneurs' optimal financing decision implies $\frac{\pi(h)}{q(h)} \frac{A}{p(h)} = \frac{1-\pi(h)}{1-q(h)} \frac{A}{p(l)}$, which in turn implies that $q(h) = \frac{\pi(h)g}{\pi(h)g+1-\pi(h)}$. Since $K_0 < \bar{k}$ defined by $\chi(\bar{k}) = (a(l) + gA)\bar{k}$, using Lemma 4 and the definition of $\bar{q}$ we have that $\bar{q} = \frac{\pi(h)g}{\pi(h)g+1-\pi(h)}$ and, thus, $q(h) < \pi(h) - \lambda\zeta^+$, which contradicts Corollary 1. Instead, if $B_1(l) = (a(l) + gA)K_0$, then again $p(l) = gA < A = p(h)$ but $K_0 = \bar{k}$. By Lemma 3, $\frac{\pi(h)}{q(h)} \frac{A}{p(h)} \geq \frac{1-\pi(h)}{1-q(h)} \frac{A}{p(l)}$ and, thus, $q(h) \leq \frac{\pi(h)g}{\pi(h)g+1-\pi(h)}$. By Lemma 4, we must have $q(h) = \frac{\chi'(\bar{k})-(a(l)+gA)}{(a(h)+A)-(a(l)+gA)} = \bar{q} < \pi(h) - \lambda\zeta^+$, again contradicting Corollary 1.

Since $B_1(h) > B_1(l)$, we have $q(h) = \pi(h) - \lambda\zeta^+$.

Since $B_1(h) \leq a(h)K_0$ (Lemma A.5), it must be that $\frac{\pi(h)}{q(h)}\frac{A}{p(h)} \geq \frac{1-\pi(h)}{1-q(h)}\frac{A}{p(l)}$ or, equivalently, that $\gamma p(h) \leq p(l)$, where the inequality is an equality if the borrowing constraint in the low state is slack. That $p(l) < p(h)$ is also shown in Lemma A.5.

Finally, we find the necessary and sufficient conditions for capital to be liquidated in the low state. Note that, by Lemma A.5, $B_1(h) \leq a(h)K_0$ and, thus, capital is not liquidated in state h. If capital is to be liquidated in the low state, then it must be that $B_1(l) > a(l)K_0$ and $p(l) = gA$. By Lemmas 3 and A.5, and the assumption that $\bar{q} < \pi(h) - \lambda\zeta^+$, it must be that $B_1(h) = a(h)K_0$ and $p(h) < A$; otherwise, the entrepreneurs would strictly prefer to borrow against the high state. By Lemma 4, the optimal investment scale satisfies:

$$\chi'(K_0) = q(h)(a(h) + p(h)) + (1 - q(h))\,(a(l) + gA), \tag{68}$$

where $p(h) \leq \gamma^{-1}p(l) = \gamma^{-1}gA$, with equality if $B_1(l) < (a(l) + gA)K_0$. Thus, capital gets liquidated in the low state if and only if at the capital price $p(h) = \gamma^{-1}gA$, the investment scale satisfies $\chi(K_0) > \sum_s q(s)a(s)K_0$, i.e., at the lowest possible equilibrium prices of capital, the entrepreneurs still want to borrow more than their intermediate cashflows.

As for equilibrium existence, we have shown that $a(l)K_0 \leq B_1(l) < B_1(h) \leq a(h)K_0$ (thus, there are no liquidations in in state h), $q(h) = \pi(h) - \lambda\zeta^+$, and $p(l) \geq \gamma p(h)$ with equality when $B_1(l) < (a(l) + gA)K_0$. It follows that the equilibrium falls into one of four cases:

Case 1. If $\chi(k) \leq \sum_s q(s)\,a(s)\,k$ for $k > 0$ s.t. $\chi'(k) = q(h)\,(a(h) + A) + (1 - q(h))\,(a(l) + \gamma A)$, then:

$$\chi'(K_0) = q(h)\,(a(h) + A) + (1 - q(h))\,(a(l) + \gamma A)\,; \tag{69}$$

$$p(h) = A;\ p(l) = \gamma A. \tag{70}$$

Case 2. If $\chi(k) > \sum_s q(s)\,a(s)\,k$ for $k > 0$ s.t. $\chi'(k) = q(h)\,(a(h) + A) + (1 - q(h))\,(a(l) + \gamma A)$, but $\chi(k) \leq \sum_s q(s)\,a(s)\,k$ for $k > 0$ s.t. $\chi'(k) = q(h)\,(a(h) + \gamma^{-1}gA) + (1 - q(h))\,(a(l) + gA)$, then:

$$\chi(K_0) = \sum_s q(s)\,a(s)\,K_0; \tag{71}$$

$$p(h) = (q(h) + (1 - q(h))\,\gamma)^{-1} \cdot \left(\chi'(K_0) - \sum_s q(s)\,a(s)\right);\ p(l) = \gamma p(h)\,. \tag{72}$$

Case 3. If $\chi(k) > \sum_s q(s)\,a(s)\,k$ for $k > 0$ s.t. $\chi'(k) = q(h)\,(a(h) + \gamma^{-1}gA) + (1 - q(h))\,(a(l) + gA)$, but $\chi(k) \leq (q(h)\,a(h) + (1 - q(h))\,(a(l) + gA))\,k$ for $k > 0$ s.t. $\chi'(k) = q(h)\,(a(h) + \gamma^{-1}gA) + (1 - q(h))\,(a(l) + gA)$, then:

$$\chi'(K_0) = q(h)\left(a(h) + \gamma^{-1}gA\right) + (1 - q(h))\,(a(l) + gA)\,; \tag{73}$$

$$p(h) = \gamma^{-1}gA;\ p(l) = gA. \tag{74}$$

Case 4. Finally, if $\chi(k) > (q(h)\,a(h) + (1 - q(h))\,(a(l) + gA))\,k$ for $k > 0$ s.t. $\chi'(k) = q(h)\,(a(h) + \gamma^{-1}gA) + (1 - q(h))\,(a(l) + gA)$, then:

$$\chi(K_0) = (q(h)\,a(h) + (1 - q(h))\,(a(l) + gA))\,K_0, \tag{75}$$

$$p(h) = q(h)^{-1} \cdot \left(\chi'(K_0) - \sum_s q(s)\, a(s) - (1 - q(h))\, gA \right);\ p(l) = gA. \tag{76}$$

Hence, an equilibrium exists by continuity of $\chi(\cdot)$ and $\chi'(\cdot)$, and convexity of the former. ■

Proof of Corollary 3. From Propositions 2 and 3, the equilibrium features "no risk-sharing" if and only if $\lambda\zeta^+ > \pi(h) - \bar{q}$. We next make two observations.

First, from Proposition 1, the distortion term ζ^+ depends only on the primitives $(n, \phi, \pi(h))$; moreover, all else equal, ζ^+ is decreasing in n, and it is non-monotonic in ϕ (i.e., increasing at first and then eventually decreasing; see equation (58) in the proof of Proposition 1 and Figure 2 for an illustration).

Second, from the definition of $\bar{q}$ in equation (22), we have that:

$$\bar{q} = \min \left\{ \frac{\pi(h)g}{\pi(h)g + 1 - \pi(h)}, \frac{(\omega - 1)(a(l) + gA)}{(a(h) + A) - (a(l) + gA)} \right\}, \tag{77}$$

which depends only on the primitives $(a(l), a(h), A, \omega, g, \pi(h))$; moreover, $\bar{q}$ is increasing in g, ω and $\frac{a(l)+gA}{a(h)+A}$.

Combining these two observations, we conclude that the equilibrium is more likely to be in the "no risk-sharing" region if, all else equal, (i) λ is large, n is small, and ϕ is intermediate; and (ii) g, ω and $\frac{a(l)+gA}{a(h)+A}$ are large.

Finally, the effect of $\pi(h)$ is more subtle, as it affects both the severity of information-trading frictions (see Proposition 1 and Figure 2) and the entrepreneurs' desire to insure net worth fluctuations. Indeed, one can construct numerical examples in which larger values of $\pi(h)$ put the equilibrium either into the "no risk-sharing" or the "partial risk-sharing" region. ■

Proof of Proposition 4. First, that the entrepreneurs' lifetime welfare decreases with the severity of information-trading frictions is straightforward, as they simply face a higher cost to insuring net worth fluctuations. Second, in equilibrium, the investors hold the claims issued by the entrepreneurs, i.e., $v(s) = B_1(s)$ for all s. Since $B_1(h) \geq B_1(l)$ (see Propositions 2 and 3), using Proposition 1 and Corollary 1, the investors' lifetime welfare is:

$$\begin{aligned} U^I &= e - \sum_s q(s)B_1(s) + \lambda \left(\sum_s \pi(s)B_1(s) - \zeta^+ (B_1(h) - B_1(l)) \right) + (1 - \lambda) \left(\widetilde{W} + \sum_s \pi(s)B_1(s) \right) \\ &= e + (1 - \lambda)W(\{B_t(s)\}, \mu^*) \\ &= e + \lambda n W(\{B_t(s)\}, \mu^*) \\ &\leq e + \lambda\zeta^+ (B_1(h) - B_1(l)), \end{aligned} \tag{78}$$

with equality if and only if the trading arrangements allocate the claims efficiently. The second equality follows from the fact that $B_1(h) > B_1(l)$ implies that $q(h) = \pi(h) - \lambda\zeta^+$ (Corollary 1), and that $\widetilde{W} = W(\{B_t(s)\}, \mu^*)$. The third equality follows from $n\lambda = 1 - \lambda$. The last inequality follows from the fact that the traders' rents are bounded by the losses of the mechanism (Proposition 1). Also, since $W(\{B_t(s)\}, \mu^*) \geq 0$, it must be that $U^I \geq e$. ■

Proof of Lemma 5. From equation (31) and equations (35)-(36), we have:

$$\lambda V\left(\{T_t(s)\},\mu^{SP}\right)+(1-\lambda)W\left(\{T_t(s)\},\mu^{SP}\right)=$$

$$=\lambda\mathbb{E}\left\{\sum_i\omega^{SP}\left(x^i,x^{-i}\right)\right\}+(1-\lambda)\mathbb{E}\left\{U\left(x^i,\mathcal{A}^{SP}(x^i,x^{-i})\right)\right\}$$

$$=\lambda\mathbb{E}\left\{\sum_{i=1}^{n}\omega^{SP}\left(x^i,x^{-i}\right)\right\}+(1-\lambda)\mathbb{E}\left\{\sum_s v_s^{SP}\left(x^i,x^{-i}\right)\cdot\mathbb{P}\left(s|x^i,\mathcal{A}^{SP}\left(x^i,x^{-i}\right)\right)-\omega^{SP}\left(x^i,x^{-i}\right)\right\}$$

$$=(1-\lambda)\mathbb{E}\left\{\sum_s v_s^{SP}\left(x^i,x^{-i}\right)\cdot\mathbb{P}\left(s|x^i,\mathcal{A}^{SP}\left(x^i,x^{-i}\right)\right)\right\}, \tag{79}$$

where the last equality follows by symmetry: $\lambda\mathbb{E}\left\{\sum_{i=1}^n\omega^{SP}\left(x^i,x^{-i}\right)\right\}=\lambda n\mathbb{E}\left\{\omega^{SP}\left(x^i,x^{-i}\right)\right\}=(1-\lambda)\mathbb{E}\left\{\omega^{SP}\left(x^i,x^{-i}\right)\right\}$. Next, by Bayes' rule:

$$\mathbb{E}\left\{\sum_s v_s^{SP}\left(x^i,x^{-i}\right)\cdot\mathbb{P}\left(s|x^i,\mathcal{A}^{SP}\left(x^i,x^{-i}\right)\right)\right\}=\sum_s\sum_{x^i,\mathcal{A}^{SP}(x^i,x^{-i})}v_s^{SP}\left(x^i,x^{-i}\right)\cdot\mathbb{P}\left(x^i,\mathcal{A}^{SP}\left(x^i,x^{-i}\right)|s\right)\cdot\pi(s)$$

$$=\sum_s\pi(s)\,\mathbb{E}\left\{v_s^{SP}\left(x^i,x^{-i}\right)|s\right\}. \tag{80}$$

Finally, by symmetry $(1-\lambda)\sum_s\pi(s)\,\mathbb{E}\left\{v_s^{SP}\left(x^i,x^{-i}\right)|s\right\}=\lambda n\sum_s\pi(s)\,\mathbb{E}\left\{v_s^{SP}\left(x^i,x^{-i}\right)|s\right\}=\lambda\sum_s\pi(s)\,\mathbb{E}\left\{\sum_i v_s^{SP}\left(x^i,x^{-i}\right)|s\right\}$, which establishes the result. ■

Proof of Corollary 4. Follows from Lemma 5 and the constraint (34) in the planner's mechanism design problem. ■

Proof of Lemma 6. That the right-hand side of equation (39) is the upper bound on the investors' welfare follows by Corollary 4. Consider the trading arrangement μ^{SP} proposed in the text, which consists of allocations: $\mathcal{A}^{SP}\left(x^i,x^{-i}\right)=\left(\omega^{SP}\left(x^i,x^{-i}\right),\left\{v_s^{SP}\left(x^i,x^{-i}\right)\right\}\right)$ for all x^i,x^{-i}. Note that these allocations reveal to traders each others' reports. To show that μ^{SP} is feasible, we need to show that the constraints (32), (33) and (34) in the planner's mechanism design problem are satisfied. To this end, observe that:

$$U\left(x^i,\mathcal{A}^{SP}\left(x^i,x^{-i}\right)\right)=\sum_s v_s^{SP}\left(x^i,x^{-i}\right)\cdot\mathbb{P}\left(s|x^i,x^{-i}\right)-\omega^{SP}\left(x^i,x^{-i}\right) \tag{81}$$

$$=n^{-1}\cdot\left(\sum_s\mathbb{P}\left(s|x^i,x^{-i}\right)\cdot\left(T_1(s)+T_2(s)\right)-\min_x\sum_s\mathbb{P}\left(s|x^i=x,x^{-i}\right)\cdot\left(T_1(s)+T_2(s)\right)\right)$$

$$\geq 0, \tag{82}$$

and, thus, the (PC) constraint (32) holds. The (IC) constraint (33) holds trivially since a trader's allocation does not depend on his own report. Finally, the (FC) constraint (34) holds trivially since the mechanism allocates all the claims that it has available to the traders. ■

Proof of Lemma 7. Let U denote the investors' lifetime welfare in the laissez-faire equilibrium. By inspection of problem (P3), it is weakly optimal for the planner to set $T_0=e-\chi(k_0^{SP})$

and $T_2(s) = gA(k_0 - k_1(s))\ \forall s$, as this minimizes the transfers $\{T_1(s)\}$ and, thus, the inefficient capital liquidations. Using Lemma 6, the planner's problem therefore reduces to:

$$\max_{k_0, \{k_1(s), T_1(s)\}} \sum_s \pi(s) \left[(a(s)k_0 - T_1(s) + Ak_1(s) \right]$$

subject to

$$0 \leq T_1(s) \leq a(s)k_0, \tag{83}$$

$$0 \leq k_1(s) \leq k_0, \tag{84}$$

$$U \leq e - \chi(k_0) + \sum_s \pi(s) \left(T_1(s) + gA(k_0 - k_1(s)) \right). \tag{85}$$

It is without loss of generality to assume that (i) $k_1(s) = k_1$ for all s, and (ii) $T_1(h) \geq T_1(l)$ with equality if $T_1(l) < a(l)k_0$. Furthermore, it must be that $k_1 = k_0$ if $T_1(h) < a(h)k_0$, since it is cheaper for the planner to finance investment with intermediate cashflows than by liquidating capital inefficiently. Finally, constraint (85) must hold with equality, as otherwise the planner can reduce transfers to the investors and increase the entrepreneurs' welfare.

We now show that it is optimal to set $T_1(l) = a(l)k_0$. Suppose to the contrary that $T_1(s) = T_1 < a(l)k_0$. Then, since $k_1 = k_0$, from constraint (85) the planner's investment scale is $\chi(k_0) = e - U + T_1 < a(l)k_0$, as $U \geq e$ by Proposition 4. Consider next a small increase in the transfer of dT_1 and the investment scale of $dk_0 = \frac{dT_1}{\chi'(k_0)}$, so that constraint (85) is still satisfied. Then, the change in the entrepreneurs' welfare is:

$$\sum_s \pi(s)(a(s) + A)dk_0 - dT_1 = \left(\frac{\sum_s \pi(s)(a(s) + A)}{\chi'(k_0)} - 1 \right) dT_1, \tag{86}$$

which is positive by Assumption 2(ii) as $\chi(k_0) < a(l)k_0$, a contradiction.

We now use the above results to show that k_0 and k_1 are given by equations (41) and (42).

Suppose that $p_U^{SP} = A$, as defined in equation (40). It is sufficient to show that the first-best investment scale is indeed feasible without inefficient liquidations. But this follows immediately since $k_0^{FB} \leq \widetilde{k}_U$ and, thus, the first-best scale can be implemented with some transfer $a(l)k_0^{FB} < T_1(h) \leq a(h)k_0^{FB}$.

Suppose that $gA < p_U^{SP} < A$. It is straightforward that $T_1(h) = a(h)k_0$, as otherwise the planner can always increase the entrepreneurs' welfare by increasing the investment scale and the transfer in the high state at the same time. Assume to the contrary that $k_1 < k_0$, and consider a small increase in the continuation scale of dk_1 and a reduction in the investment scale of $dk_0 = \frac{gA}{\sum \pi(s)a(s) + gA - \chi'(k_0)} dk_1 < 0$, so that constraint (85) is still satisfied. Observe that this is feasible: because $T_1(s) = a(s)k_0$ and $k_1 < k_0$, it follows that $k_0 > \widetilde{k}_U$, which together with $p_U^{SP} > gA$ implies that $\chi'(k_0) > \chi'(\widetilde{k}_U) > \sum \pi(s)(a(s) + gA)$. The entrepreneurs' welfare clearly increases, since it is equal to Ak_1 and k_1 has increased, a contradiction. Since we have established that $T_1(h) = a(h)k_0$ and $k_1 = k_0$, constraint (85) implies that $k_0 = \widetilde{k}_U$.

Suppose that $p_U^{SP} = gA$. It is again straightforward to show that $T_1(h) = a(h)k_0$ by the same reasoning as above. Assume to the contrary that $k_1 = k_0$. From constraint (85), it follows that $k_0 = \widetilde{k}_U$, which together with $p_U^{SP} = gA$ implies $\chi'(k_0) < \sum_s \pi(s)(a(s) + gA)$.

Consider a small increase in the investment scale of dk_0 and an increase in the continuation scale of $dk_1 = \frac{\sum_s \pi(s)a(s)+gA-\chi'(k_0)}{gA} dk_0 > 0$, so that constraint (85) is still satisfied. Since the entrepreneurs' welfare is Ak_1, it increases, a contradiction. Therefore, we have established that $T_1(h) = a(h)k_0$ and $k_1 < k_0$. Maximization of the entrepreneurs' welfare w.r.t. k_0 subject to constraint (85) implies $\chi'(k_0) = \sum_s \pi(s)a(s) + gA$. Given k_0, the expression for k_1 follows directly from constraint (85). ■

Proof of Proposition 5. First, clearly the equilibrium is constrained inefficient if the trading arrangements in it allocate the claims inefficiently among investors (see explicit conditions in Lemma A.3). Second, if the equilibrium features no risk-sharing, then $U^I = e$ by Proposition 4. By inspection of the planner's problem, setting $U = e$, we see that her investment and continuation scales coincide with those of the benchmark economy, which are clearly different from equilibrium. Thus, the equilibrium is again constrained inefficient. Finally, suppose that in equilibrium both the claims are allocated efficiently and there is partial risk-sharing. In this case, we know that in equilibrium $B_1(l) \geq a(l)K_0$ and $B_1(h) \in (a(l)K_0, a(h)K_0]$ (see Proposition 3 and Lemmas A.4 and A.5). There are thus three cases to consider.

Case 1. Suppose that in equilibrium $B_1(l) = a(l)K_0 < B_1(h) < a(h)K_0$, and thus there are no capital liquidations. Since the allocation of claims is efficient, from Proposition 4, it follows that $U = e + \lambda\zeta^+(B_1(h) - B_1(l))$ and, since $q(h) = \pi(h) - \lambda\zeta^+$, we have:

$$\chi(K_0) = \sum_s q(s)B_1(s)K_0 = \sum_s \pi(s)B_1(s) + e - U. \tag{87}$$

Therefore, $K_0 < \widetilde{k}_U$. The only scenario in which the planner chooses scale below $\widetilde{k}_U$ is when $k_0^{SP} = k^{FB}$ (see Lemma 7). But the scale in the laissez-faire equilibrium is below the first-best scale k^{FB}. Thus, in this case, the equilibrium is constrained inefficient.

Case 2. Suppose that in equilibrium $B_1(s) = a(s)K_0$ $\forall s$, and thus there are no capital liquidations. By reasoning analogous to Case 1, it must be that:

$$\chi(K_0) = \sum_s q(s)B_1(s)K_0 = \sum_s \pi(s)a(s)K_0 + e - U, \tag{88}$$

which implies that $K_0 = \widetilde{k}_U$. For this to be consistent with equilibrium, it must be that $p(l) = \gamma p(h)$, $p(h) \in [\gamma^{-1}gA, A]$, and using the expression for γ:

$$\begin{aligned} \chi'(\widetilde{k}_U) &= \sum_s q(s)(a(s) + p(s)) \\ &= \sum_s \pi(s)a(s) - \lambda\zeta^+(a(h) - a(l)) + \frac{\pi(h) - \lambda\zeta^+}{\pi(h)} p(h). \end{aligned} \tag{89}$$

Plugging the bounds for $p(h)$, we conclude that this is an equilibrium if and only if:

$$\frac{1 - \pi(h) + \lambda\zeta^+}{1 - \pi(h)} gA - \lambda\zeta^+(a(h) - a(l)) \leq \chi'(\widetilde{k}_U) - \sum_s \pi(s)a(s) \leq \frac{\pi(h) - \lambda\zeta^+}{\pi(h)} A - \lambda\zeta^+(a(h) - a(l)). \tag{90}$$

On the other hand, from Lemma 7, the planner sets the investment scale to $\widetilde{k}_U$ and does not liquidate capital if and only if:

$$gA \leq \chi'(\widetilde{k}_U) - \sum_s \pi(s)a(s) \leq A. \tag{91}$$

By inspection of (90) and (91), we conclude that, in this case, the equilibrium may or may not be constrained efficient.

Case 3. Suppose that in equilibrium $B_1(l) > a(l)K_0$ and $B_1(h) = a(h)K_0$, and thus capital is liquidated in the low state. By reasoning analogous to Case 1, it must be that:

$$\chi(K_0) = \sum_s q(s)B_1(s)K_0 > \sum_s \pi(s)a(s)K_0 + e - U, \tag{92}$$

which implies that $K_0 > \widetilde{k}_U$. For this to be consistent with equilibrium, it must be that $p(l) = gA$, $p(h) = \gamma^{-1}gA$, and

$$\begin{aligned}\chi'(K_0) &= \sum_s q(s)\left(a(s) + p(s)\right) \\ &= \sum_s \pi(s)a(s) - \lambda\zeta^+(a(h) - a(l)) + \frac{1 - \pi(h) + \lambda\zeta^+}{1 - \pi(h)}gA \\ &> \chi'(\widetilde{k}_U).\end{aligned} \tag{93}$$

On the other hand, from Lemma 7, the planner would choose the same investment scale, i.e., $k_0^{SP} = K_0$ if and only if:

$$\chi'(K_0) = \sum_s \pi(s)a(s) + gA. \tag{94}$$

Combining equations (93) and (94) implies $(1 - \pi(h))(a(h) - a(l)) = gA$. Thus, in this case, the equilibrium is generically constrained inefficient. ■

Proof of Proposition 6. Let U denote the investors' welfare in the laissez-faire equilibrium without intervention. Consider the following intervention. The planner enters a trading arrangement μ for claims $\{d_t(s)\}$, she observes whether the seller is impatient (just like the traders do), and intervenes if and only if that is the case, so as to discourage patient investors from selling claims. In particular, the planner gives the seller a subsidy $S\left(\{d_t\left(s\right)\}, \mu\right) = nW(\{d_t\left(s\right)\}, \mu)$. The planner will finance the intervention with a lump sum tax $T_f = S(\{B_t(s)\}, \mu^{SP})$ on all sellers of claims, where $\{B_t(s)\}$ is the equilibrium aggregate supply of claims after the intervention with $B_1(s) = T_1(s) + T_2(s)$ and $B_2(s) = 0$ $\forall s$, and where the planner's allocations $\{T_t(s)\}$ are given in the proof of Lemma 7. The planner also makes a transfer $T_u = U - e$ from entrepreneurs to investors ex-ante.

Given the above intervention, the sorting of investors into sellers and buyers will remain unchanged, and the representative investor's problem will become:

$$U^I = \max_{\{d_t(s)\},\mu} U - \sum_s q(s)(d_1(s) + d_2(s)) \tag{95}$$

$$+ \lambda \left(V(\{d_t(s)\}, \mu) + S(\{d_t(s)\}, \mu) - S\left(\{B_t(s)\}, \mu^{SP}\right)\right) \tag{96}$$

$$+ (1-\lambda)\left(\widetilde{W} + \sum_s \pi(s)(d_1(s) + d_2(s))\right), \tag{97}$$

subject to the budget constraint at the issuance stage, i.e., $U \geq \sum_s \pi(s)(d_1(s) + d_2(s))$, and consumption non-negativity in periods 1 and 2, i.e., $d_t(s) \geq 0\ \forall t, s$; and subject to the trading arrangement μ being in the feasible set $\mathcal{M}$.

We immediately see that the seller's choice of trading arrangement only enters the term $V(\{d_t(s)\}, \mu) + S(\{d_t(s)\}, \mu)$. Since $V(\{d_t(s)\}, \mu) \leq \sum_s \pi(s)(d_1(s) + d_2(s))$ for all feasible μ, it is optimal for the investor to set $\mu = \mu^{SP}$, in which case:

$$V\left(\{d_t(s)\}, \mu^{SP}\right) + S\left(\{d_t(s)\}, \mu^{SP}\right) = \sum_s \pi(s)(d_1(s) + d_2(s)). \tag{98}$$

Thus, the investors' expected value of the claims is $\sum_s \pi(s)(d_1(s) + d_2(s))$, independently of whether they sell or keep them. But then, it is straightforward to show that in equilibrium $d_t(s) = b_t(s) = B_t(s)\ \forall t, s$ and $q(s) = \pi(s)\ \forall s$, and that the investment and continuation scales coincide with those of the planner. That the investors' lifetime welfare is U then follows from the observation that $\lambda S(\{B_t(s)\}, \mu^{SP}) = \lambda n W(\{B_t(s)\}, \mu^{SP}) = (1-\lambda)\widetilde{W}$. ∎

Proof of Proposition 7. Let U denote the investors' welfare in the laissez-faire equilibrium without intervention. Consider the following intervention. The planner enters a trading arrangement μ for claims $\{d_t(s)\}$, she observes whether the seller is impatient (just like the traders do), and intervenes if and only if that is the case, so as to discourage patient investors from selling claims. Let us focus on the case where $v(h) = d_1(h) + d_2(h) > d_1(l) + d_2(l) = v(l)$; the proof for the case $v(h) < v(l)$ is analogous; and, if $v(h) = v(l)$, then there is no need for intervention. Our candidate intervention consists of the planner offering to buy the mechanism's claims at some price $\bar{\omega}(\{v(s)\})$ and then re-sell these claims to the traders at some price $\underline{\omega}(\{v(s)\})$.

First, observe that setting $\underline{\omega}(\{v(s)\}) = \min_x \mathbb{E}\{v(s) | \forall i\ x^i = x\}$ ensures that, if the government were to purchase claims from the mechanism, then it would allocate them to the traders with probability 1, as this is the lowest possible valuation of the traders.

Second, observe that if the planner were to set $\bar{\omega}(\{v(s)\}) = \underline{\omega}(\{v(s)\})$, then the allocation of claims and their pricing in the mechanism would remain unchanged. Instead, if the planner were to set $\bar{\omega}(\{v(s)\}) = \max_x \mathbb{E}\{v(s) | \forall i\ x^i = x\}$, then the mechanism would sell the claims to the government with probability 1, but this would generate over-pricing of claims. We will next show that there exists a price $\bar{\omega}(\{v(s)\}) \in (\min_x \mathbb{E}\{v(s) | \forall i\ x^i = x\},\ \max_x \mathbb{E}\{v(s) | \forall i\ x^i = x\})$, such that after the intervention the optimal mechanism sells its claims (either to the government or the traders) with probability 1, and the claims are priced fairly. To construct the price $\bar{\omega}(\{v(s)\})$, we will next utilize the prices received by the optimal mechanism in the absence of any intervention, as characterized in Lemma A.3, where recall that there are two possible cases, depending on the sign of $\Delta \geq 0$.

Case (a). When $\Delta \geq 0$, the allocations of the optimal mechanism are given by:

$$\omega^i\left(B,x^{-i}\right)=\begin{cases}0 & \text{if } \exists j\neq i\ x^j=G\\ n^{-1}\mathbb{E}\left\{v\left(s\right)|x^i=B,x^{-i}\right\} & \text{if } \forall j\neq i\ x^j=B\end{cases},\ v_s^i\left(B,x^{-i}\right)=\begin{cases}0 & \text{if } \exists j\neq i\ x^j=G\\ n^{-1}v\left(s\right) & \text{if } \forall j\neq i\ x^j=B\end{cases};$$

and

$$\omega^i\left(G,x^{-i}\right)=\begin{cases}m\left(x^1,...,x^n\right)^{-1}\mathbb{E}\left\{v\left(s\right)|x^i=G,x^{-i}\right\} & \text{if } \exists j\neq i\ x^j=G\\ \mathbb{E}\left\{v\left(s\right)|x^i=G,x^{-i}\right\}-\mathcal{R}n^{-1}\left(v\left(h\right)-v\left(l\right)\right) & \text{if } \forall j\neq i\ x^j=B\end{cases},\ v_s^i\left(G,x^{-i}\right)=m\left(x^1,...,x^n\right)^{-1}v\left(s\right)$$

where $\mathcal{R}$ is the rent earned by the optimistic trader, and it is given by:

$$\mathcal{R}=\mathbb{P}\left(h|x^i=G,\forall j\neq i\ x^j=B\right)-\mathbb{P}\left(h|x^i=B,\forall j\neq i\ x^j=B\right). \tag{99}$$

Case (b). When $\Delta<0$, the allocations of the optimal mchanism are given by:

$$\omega^i\left(B,x^{-i}\right)=\begin{cases}0 & \text{if } \exists j\neq i\ x^j=G\\ n^{-1}v\left(l\right) & \text{if } \forall j\neq i\ x^j=B\end{cases},\ \text{and } v_s^i\left(B,x^{-i}\right)=\begin{cases}0 & \text{if } \exists j\neq i\ x^j=G\\ n^{-1}v\left(l\right) & \text{if } \forall j\neq i\ x^j=B\end{cases};$$

and

$$\omega^i\left(G,x^{-i}\right)=m\left(x^1,...,x^n\right)^{-1}\mathbb{E}\left\{v\left(s\right)|x^i=G,x^{-i}\right\},\ \text{and } v_s^i\left(G,x^{-i}\right)=m\left(x^1,...,x^n\right)^{-1}v\left(s\right).$$

Next, define $\bar{\omega}\left(\{v\left(s\right)\}\right)$ as follows:

$$\begin{aligned}\mathbb{E}\left\{v\left(s\right)|\forall j\neq i\ x^j=B\right\}&=\frac{\mathbb{P}\left(\forall i\ x^i=B\right)}{\mathbb{P}\left(\exists i\ \forall j\neq i\ x^j=B\right)}\bar{\omega}\left(\{v\left(s\right)\}\right)+\\ &+\left(1-\frac{\mathbb{P}\left(\forall i\ x^i=B\right)}{\mathbb{P}\left(\exists i\ \forall j\neq i\ x^j=B\right)}\right)\max\left\{\bar{\omega}\left(\{v\left(s\right)\}\right),\omega^i\left(G,x^{-i}\right)|_{\Delta\geq 0,\ \forall j\neq i\ x^j=B}\right\},\end{aligned} \tag{100}$$

where $\omega^i\left(G,x^{-i}\right)|_{\Delta\geq 0,\ \forall j\neq i\ x^j=B}$ is the payment of the trader who receives signal G, when $\Delta\geq 0$ and all other traders received signal B. It is straightforward to verify that $\underline{\omega}\left(\{v\left(s\right)\}\right)<\bar{\omega}\left(\{v\left(s\right)\}\right)<\mathbb{E}\left\{v\left(s\right)|x^i=G,\ \forall j\neq i\ x^j=B\right\}$.

Now, consider the following candidate mechanism, given the intervention:

(i) allocate all the claims to the traders with G signals at prices as in Case (a) if there are at least two such traders;

(ii) allocate all the claims to the government if all traders received signal B; and

(iii) allocate all the claims to the government if only one trader received signal G and $\bar{\omega}\left(\{v\left(s\right)\}\right)>\omega^i\left(G,x^{-i}\right)|_{\Delta\geq 0,\ \forall j\neq i\ x^j=B}$; otherwise, allocate all the claims to the only trader with signal G for a price $\omega^i\left(G,x^{-i}\right)|_{\Delta\geq 0,\ \forall j\neq i\ x^j=B}$.

Thus, the mechanism behaves as in Case (a), as long as its revenues from allocating the claims to traders exceed $\bar{\omega}(\{v(s)\})$. Otherwise, the mechanism allocates the claims to the government at price $\bar{\omega}(\{v(s)\})$. And, as we mentioned above, all the traders within the mechanism find it optimal to purchase the claims from the government (if the government has purchased any from the mechanism) at price $\underline{\omega}(\{v(s)\})$, in which case each trader receives share n^{-1} of the claims by paying $n^{-1}\underline{\omega}(\{v(s)\})$ to the government.

To see that the candidate mechanism is optimal, first, note that the payment $\bar{\omega}(\{v(s)\})$ in equation (100) is defined precisely so that the expected revenues of the proposed mechanism are equal to the expected value of the claims. Hence, the mechanism designer cannot do better by never allocating the claims to the government (the revenues in either Case (a) or (b) are strictly lower). Second, the mechanism is clearly better off allocating the claims to the government rather than the traders when all the traders received signal B, since the mechanism at least gets the implicit subsidy from government; and in this case it is also clearly sub-optimal to not allocate the claims to the government nor the traders. Third, note that for all realizations of signals such that at least two traders receive signal G, the proposed mechanism extracts the traders' full surplus; thus, it cannot do better for those realization of signals. Finally, if only one trader received signal G and the price $\bar{\omega}(\{v(s)\})$ exceeds $\omega^i(G, x^{-i})|_{\Delta\geq 0,\ \forall j\neq i\ x^j=B}$, then the mechanism is also better off allocating the claims to the government at price $\bar{\omega}(\{v(s)\})$, since otherwise the maximal payment it can extract from this trader is $\omega^i(G, x^{-i})|_{\Delta\geq 0,\ \forall j\neq i\ x^j=B}$; and vice versa.

The rest of the proof follows the same steps as the proof of Proposition 6. First, since now the investors value the claims $\{d_t(s))\}$ at $\sum_s \pi(s)(d_1(s) + d_2(s))$, independently of whether they sell or keep them, the equilibrium claims prices will be undistorted; hence, in the post-intervention equilibrium $q(s) = \pi(s)\ \forall s$. Second, let Ω denote the set of signal realizations in which according to the intervention above the government purchases the claims from a given mechanism. Given the equilibrium asset supply $\{B_t(s)\}$ where $B_1(s) = T_1(s) + T_2(s)$ and $B_2(s) = 0\ \forall s$ and where $\{T_t(s)\}$ are as given in the proof of Lemma 7, the lump sum taxes that the government needs to collect from all the sellers in order to fund the intervention are $T_f(s) = \mathbb{P}(\Omega|s)\,(\bar{\omega}(\{B_t(s)\}) - \underline{\omega}(\{B_t(s)\}))\ \forall s$. Finally, the government also makes a transfer $T_u = U - e$ from entrepreneurs to investors ex-ante to make sure that the latter's lifetime welfare is U. ■

Proof of Proposition 8. Suppose that $\lambda\zeta^+ > \pi(h) - \bar{q}$, so that the laissez-faire equilibrium is in the "no risk-sharing" region (see Proposition 2). By Proposition 4, the equilibrium welfare of the investors is $U^I = e$. By Lemma 7, when the investors' lifetime welfare is $U = e$, the constrained efficient investment scale (and thus borrowing) is given by k_0^{SP} such that $\chi(k_0^{SP}) = \sum_s \pi(s)a(s)k_0^{SP}$, just as in the benchmark economy analyzed in Section 4.1, since $\chi'(k_0^{SP}) = \sum_s \pi(s)a(s) + p_e^{SP}$ with $p_e^{SP} \in (gA, A)$. But that the equilibrium investment scale K_0 can be above k_0^{SP} is illustrated in Figure 3, since all we need to do is compare the investment scale when $\lambda\zeta^+ > \pi(h) - \bar{q}$ to the investment scale when $\lambda\zeta^+ = 0$. Under the parameterization $\chi(k) = \chi k^\omega$, this is equivalent to the following inequalities holding:

$$\frac{\sum_s \pi(s)(a(s) + gA)}{\omega \sum_s \pi(s)a(s)} < 1 < \frac{\pi(h)(a(h) + A) + (1 - \pi(h))g^{-1}(a(l) + gA)}{(\pi(h) + (1 - \pi(h))g^{-1})\,\omega \sum_s \pi(s)a(s)}. \tag{101}$$

The first inequality states that $\chi'(k_0^{SP}) > \sum_s \pi(s)(a(s) + gA)$ or that $p_e^{SP} > gA$, i.e., that the planner's return to an additional unit of investment beyond k_0^{SP} is lower than the interest rate (as required by Assumption 2(i)). The second inequality states that the equilibrium return to increasing investment by a unit beyond k_0^{SP} is greater than the riskless interest rate. Note that though the equilibrium has a higher marginal cost of investment due to premature liquidations (see denominator in the last term), it also has a higher marginal benefit of investment in the high state and possibly in the low state, again due to liquidations (see numerator). ■

B Verification of Consumption Non-Negativity

In Section 2.1, we conjectured that the claims prices $q_t(s) = q(s)$ and $\sum_s q(s) = 1$ are part of equilibrium. We now verify these conjectures. To do so, it suffices to show that, at the conjectured prices, every investor's consumption is non-negative in every period.

We begin with $t = 0$. We need to show that all investors' consumptions are non-negative after they have purchased the claims from the entrepreneurs and participated in the trading arrangements. In equilibrium, the entrepreneurs' claims satisfy $B_1(s) < (a(h) + A)k$ for all s, for k such that $\chi(k) = (a(h) + A)k$. Therefore, the consumption of an impatient investor at $t = 0$ is:

$$c_0^{I,\beta=0} = e - \sum_s q(s)B_1(s) + \sum \omega, \tag{102}$$

where $\sum \omega$ denote the (possibly random) payments that he receives in the trading arrangement. Since $\omega \geq 0$ (by Lemma A.3) and $e > 2(a(h) + A)k$ (by Assumption 1), it follows that $c_0^{I,\beta=0} > 0$. On the other hand, the consumption of a patient investor at $t = 0$ is:

$$c_0^{I,\beta=1} = e - \sum_s q(s)B_1(s) - \omega, \tag{103}$$

where ω denotes the (possibly random) payment that he makes in the trading arrangement that he participates in. From Lemma A.3 and the traders' participation constraints (PC), we have that $\omega \leq B_1(h) < (a(h) + A)k$. Thus, $c_0^{I,\beta=1} > 0$ since $e > 2(a(h) + A)k$.

We next consider $t = 1$. Let us first look at the investors' resources prior to financing the operations of the traditional sector firms. The impatient investors' resources at $t = 1$ and state s are given by $B_1(s) - \sum v_s$, where $\sum v_s$ denotes the (possibly random) allocation of claims of the mechanism. From the feasibility constraint (FC), we have $B_1(s) \geq \sum v_s$. The patient investors' resources at $t = 1$ and state s are given by $B_1(s) + v_s$, where v_s is the (possibly random) allocation of claims that this investor receives in the mechanism he participates in. Since by Lemma A.3, $v_s \geq 0$, we must also have $B_1(s) + v_s \geq 0$. Next, observe that the aggregate resources of the investors at $t = 1$ are given by $B_1(s)$. There are two scenarios in equilibrium. Either (i) there are no capital liquidations, or (ii) there are liquidations and the resources required to operate the traditional sector firms are: $p(s)\widehat{K}_1(s) = B_1(s) - a(s)K_0$. But, in both cases, the investors' resources $B_1(s)$ are more than sufficient to finance these firms' operations. Moreover, every investor (impatient or patient) is willing to do so at an interest rate of one, since the investors do not discount consumption between $t = 1$ and $t = 2$.

The investors' consumption at $t = 2$ is trivially non-negative, since they (if they finance these firms) receive positive repayments from the traditional sector firms.

Finally, in the planner's allocation, each investor receives a transfer $T_0 = e - \chi(k_0^{SP})$ at the issuance stage; by Assumption 1 and the fact that $\chi(k_0^{SP}) < (a(h) + A)k_0^{SP}$, we have that

$T_0 > \chi(k)$ for $k > 0$ s.t. $\chi(k) = (a(h) + A)k$. Moreover, at the trading stage, each patient investor makes a payment of $0 < \omega^{SP}(x^i, x^{-i}) = n^{-1} \min_x \mathbb{E}\{T_1(s) + T_2(s)|x^i = x, x^{-i}\}$ to the planner's trading arrangement; but $\min_x \mathbb{E}\{T_1(s)+T_2(s)|x^i = x, x^{-i}\} \leq (a(h)+A)k$ for $k > 0$ s.t. $\chi(k) = (a(h) + A)k$. Hence, $T_0 > \omega^{SP}(x^i, x^{-i})$ and each patient investor's consumption at $t = 0$ is non-negative; each impatient investor's consumption is trivially non-negative since he receives payments from trading arrangements. Moreover, the investors' consumptions at $t \in \{1, 2\}$ are non-negative since $T_t(s) \geq 0$ and each investor has a non-negative allocation of claims to these transfers. It is also straightforward to check that the investors' consumptions are also non-negative after the policy interventions described in Propositions 6 and 7, which implement the constrained efficient allocations.

C Sorting in Secondary Markets

We now provide a microfoundation for the sorting of investors in secondary markets; namely, that an investor posts his claims in a trading arrangement if and only if he is impatient. To do so, we will generate common knowledge of gains from trade within each trading arrangement; that is, we suppose that the traders within a given trading arrangement observe the preference type β of the investor who posts his claims for sale in that arrangement. We then use arguments akin to Milgrom and Stokey (1982) to show that there cannot be trade between investors based solely on information heterogeneity.

For concreteness, we specify the following timing for the sorting decision. First, each investor purchases the claims $\{d_t(s)\}$ and designs the trading arrangement μ in which to sell them. Second, he learns his type (β, x). Finally, he decides whether to post his claims in the arrangement μ, and whether to become a trader in the trading arrangements designed by other investors. The timing assumption that the mechanism is designed before the types are learned is inessential for the arguments that follow.

We make the following indifference-breaking assumptions. First, an investor posts his claims in a trading arrangement only if his net expected payoff from doing so is strictly positive. Second, an investor becomes a trader if and only if he is willing to make a strictly positive payment for some portfolio of claims. This has two immediate implications: (i) an investor is a trader only if he is patient, and (ii) an investor posts his claims in the trading arrangements if he is impatient. Therefore, we are only left to prove that in equilibrium no patient investor would post his claims in a trading arrangement.

Let $\widetilde{V}(x)$ denote the net expected payoff to a patient investor from posting his claims in the trading arrangement if he has received signal x, then

$$\widetilde{V}(x) = \mathbb{E}\left\{\sum_{i \in I} \omega^i - \sum_{i \in I} v_s^i | x\right\}, \tag{104}$$

where $\{\omega^i, v_h^i, v_l^i\}_{i \in I}$ denotes the allocations of trader $i \in I$ in the arrangement and I is the (possibly random) set of traders participating in the trading arrangement. Thus, this investor's net expected payoff from posting his claims is given by the expected payments he receives from the traders minus the expected value of the claims that he transfers to them. For now, we leave the matching process by which traders are matched with trading arrangements unspecified, as it is not essential for our arguments.

Let $P = \{x : \widetilde{V}(x) > 0\}$ denote the equilibrium set of patient investors who post their claims for sale. Thus, in equilibrium, if the traders are matched with a mechanism of a patient investor, they know that this investor has signal $x \in P$. As a result, the traders' participation constraints imply that the traders' (aggregate) expected payoff from participating in the trading arrangement of a patient investor must be non-negative, i.e.

$$\mathbb{E}\left\{\sum_{i\in I} v_s^i - \sum_{i\in I} \omega^i | x \in P\right\} \geq 0. \tag{105}$$

But then, we must have that $P = \emptyset$, since otherwise the payoff in (105) would be strictly negative, a contradiction.

Thus, we have shown that a fraction λ of investors post their claims in trading arrangements, whereas a fraction $1-\lambda$ of investors becomes traders in those trading arrangements. In the text, we had assumed that parameters satisfy $\lambda n = 1-\lambda$ and that there is an exact matching of traders with arrangements: each trading arrangement is matched with n randomly selected traders. For the comparative statics illustrated in Figure 3, it is useful to generalize this matching process slightly in order to avoid the integer problem. To this end, let $\underline{n} \equiv \lfloor \frac{1-\lambda}{\lambda} \rfloor$, $\bar{n} \equiv \lceil \frac{1-\lambda}{\lambda} \rceil$ and $\alpha \equiv \frac{\frac{1-\lambda}{\lambda} - \underline{n}}{\bar{n} - \underline{n}}$, and suppose that a fraction α of randomly selected trading arrangements is matched with $\bar{n}$ randomly selected traders and the rest with $\underline{n}$ traders. This matching process ensures that all trading arrangements and traders are matched and, when $\underline{n} = \bar{n}$, then as in our baseline case each trading arrangement is matched with exactly $n = \frac{1-\lambda}{\lambda}$ traders. Furthermore, as λ increases continuously from 0 to $\frac{1}{2}$, the number of traders in each trading arrangement decreases gradually to one.

D Alternative Benchmark

In this section, we consider an alternative benchmar benchmark economy, in which information-trading frictions are present but entrepreneurs' output is fully pledgeable. In what follows, we sketch the main results, though a formal results are available upon request.

The manner by which investors re-trade claims in secondary markets and price them ex-ante is essentially unchanged, and given by Proposition 1 and Corollary 1. The only exception is that now the entrepreneurs' claims may also pay off in the final period:

$$q(l) = 1 - q(h),\ q(h) \begin{cases} = \pi(h) - \lambda\zeta^+ & if\ B_1(h) + B_2(h) > B_1(l) + B_2(l) \\ \in [\pi(h) - \lambda\zeta^+, \pi(h) + \lambda\zeta^-] & if\ B_1(h) + B_2(h) = B_1(l) + B_2(l) \\ = \pi(h) + \lambda\zeta^- & if\ B_1(h) + B_2(h) < B_1(l) + B_2(l) \end{cases}. \tag{106}$$

Clearly, in equilibrium entrepreneurs will never liquidate capital, i.e., $K_1(s) = K_0\ \forall s$, and as a result capital prices satisfy $p(s) = A\ \forall s$. Intuitively, the reason is that entrepreneurs can always promise investors to delay payments until $t = 2$ rather than liquidate capital prematurely. As a result, the entrepreneurs' marginal value of funds at $t = 1$ is equalized across states and there is therefore full risk-sharing. In turn, the entrepreneur's optimal investment scale satisfies:

$$\chi(K_0) = \sum_s q(s)(a(s) + A), \tag{107}$$

where $\chi(K_0) = \sum_{t,s} q(s)B_t(s)$, just as in Lemma 4. Thus, to characterize the equilibrium, we are left to determine $\{B_t(s)\}$ and $q(h)$, which will also pin down the ex-ante investment scale K_0. There are three cases to consider:

Case 1. Suppose that $\chi(k^{FB}) \leq (a(l) + A)k^{FB}$. This condition states that entrepreneurs can fund the first-best scale investment with non-contingent claims. Then, in equilibrium, entrepreneurs' investment scale is undistorted, i.e., $K_0 = k^{FB}$, and entrepreneurs fund it with non-contingent claims, i.e., $B_1(s) = B_1 = \min\{a(l)k^{FB}, \chi(k^{FB})\}$ and $B_2(s) = B_2 = \chi(k^{FB}) - B_1 < Ak^{FB}$ $\forall s$. Finally, the equilibrium claims prices are actuarialy fair, i.e., $q(s) = \pi(s)$ $\forall s$.

Case 2. Suppose that $\chi(k^{FB}) > (a(l)+A)k^{FB}$, but that $\chi'(k) \geq \sum_s q(s)(a(s)+A)$ for $q(h) = \pi(h) - \lambda\zeta^+$ and k s.t. $\chi(k) = (a(l) + A)k$. This condition states that, though entrepreneurs cannot fund the first-best scale with non-contingent claims, they would rather limit ex-ante investment than issue contingent claims at distorted claims prices. Then, in equilibrium, entrepreneurs' investment scale is depressed below first-best, i.e., $K_0 < k^{FB}$ s.t. $\chi(K_0) = (a(l) + A)K_0$, and entrepreneurs fund it with non-contingent claims, i.e., $B_1(s) = B_1 = a(l)k_0$ and $B_2(s) = B_2 = Ak_0$ $\forall s$. Finally, the equilibrium claims prices are such that entrepreneurs are indifferent to borrowing an additional unit against state h: $q(h) = \frac{\chi'(K_0)-a(l)-A}{a(h)-a(l)}$ from (107).

Case 3. Suppose that $\chi(k^{FB}) > (a(l) + A)k^{FB}$, and that $\chi'(k) < \sum_s q(s)(a(s) + A)$ for $q(h) = \pi(h) - \lambda\zeta^+$ and k s.t. $\chi(k) = (a(l) + A)k$. This condition states that, though entrepreneurs cannot fund the first-best scale with non-contingent claims, they would like to expand investment by issuing contingent claims at distorted claims prices. Then, in equilibrium, entrepreneurs' investment scale is depressed below first-best, i.e., $K_0 < k^{FB}$ such that (107) holds, and entrepreneurs fund it with contingent claims, i.e., $a(l)K_0 = B_1(l) < B_1(h) = q(h)^{-1} \cdot (\chi(K_0) - (1 - q(h))a(l)K_0 - AK_0) \in (a(l)K_0, a(h)K_0)$, and $B_2(l) = B_2(h) = AK_0$. Finally, the equilibrium claims prices satisfy $q(h) = \pi(h) - \lambda\zeta^+$.

This completes the equilibrium characterization. We now characterize the solution to the planner's problem and compare it with the laissez-faire allocations. This analysis follows closely in the footsteps of the analysis in Section 5. First, it is straightforward that Lemmas 5-6 and Corollary 4 continue to hold in this benchmark economy. Thus, the only change in the planner's allocations from those found in Section 5 is the ex-ante investment scale and the ex-post allocation of capital. Now, clearly, the planner also will never liquidate capital prematurely, since she can simply transfer goods to the investors in the final period, i.e., $k_1^{SP}(s) = k_0^{SP}$ for all s. And, the final observation is that the planner always chooses to invest at first-best scale ex-ante, i.e., $k_0 = k^{FB}$. The latter is intuitive. As project cashflows are fully pledgeable, the planner no longer faces a tradeoff between delivering utility to investors and the efficiency of investment: no matter what feasible level of welfare U the planner promises to investors, she always maximizes social welfare by investing in capital until the net present value of additional unit of investment is equal to zero, evaluated at undistorted claims prices.

We can now compare the laissez-faire allocations with those of the planner. By inspection, we can immediately see that in *Case 1* and *Case 2* the laissez-faire allocations are constrained inefficient as they do not coincide with those of the planner. Instead, in *Case 1*, the laissez-faire allocations are constrained efficient. Here, entrepreneurs' claims are non-contingent and thus are not mispriced/misallocated in secondary markets; and entrepreneurs never liquidate capital and their ex-ante investment scale is at first-best.