



# Vacancy posting, job separation and unemployment fluctuations

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## ABSTRACT

What is the relative importance of hiring and separation in driving unemployment fluctuations? This paper presents a framework to decompose the moments of unemployment and study the respective contributions of vacancy posting, a measure of firms' hiring efforts, and separation. Separation accounts for about 40% of unemployment's variance, compared to 60% for vacancy posting, and contributes to about 60% of unemployment steepness asymmetry, the fact that unemployment increases faster than it decreases. Further, while vacancy posting is, on average, the most important contributor of unemployment fluctuations, the opposite is true around business cycle turning points, when separation is responsible for most of unemployment movements.

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## 1. Introduction

At the beginning of a recession, does unemployment go up because of less hiring, more job loss or both? What is the most effective policy to mitigate that increase, a firing tax, a hiring subsidy or a combination of both? And why does unemployment increase faster than it goes down?

The answers to these questions depend for a large part on the determinants of unemployment fluctuations. Going back at least to *Darby et al. (1986)*, a strand of the literature has aimed to understand the determinants of unemployment fluctuations by studying the flows of workers in and out of unemployment.<sup>1</sup> In recent work, *Shimer (2007)* studies workers' transition rates – the job finding rate (JF) and the job separation rate (JS) – and concludes that the job separation rate contributes much less to unemployment fluctuations than the job finding rate. This influential conclusion led many researchers to treat the job separation rate as acyclical and only study the firm's hiring decision.<sup>2</sup> However, this interpretation relies on the implicit assumption that JF and JS are two independent determinants of unemployment.

Granger-causality tests performed in this paper show that JS Granger-causes JF, casting some doubt on the independence of JF and complicating the interpretation of a decomposition between the job finding rate and the job

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<sup>1</sup> See, among others, *Shimer (2007)*, *Petrongolo and Pissarides (2008)*, *Elsby et al. (2009b)*, *Fujita and Ramey (2009)*, *Elsby et al. (2009a)* and *Nekarda (2009)*.

<sup>2</sup> See, among others, *Blanchard and Galí (2010)*, *Gertler and Trigari (2009)* and *Hall (2005)*.

separation rate.<sup>3</sup> If JF depends on JS, a flow rates decomposition may give a biased picture of the relative importance of hiring and job separation as driving forces of cyclical unemployment. In particular, the contribution of separation is likely to be underestimated relative to the contribution of hiring. JF is the ratio of new hires to the stock of unemployed. As a result, an increase in JS with no change in hiring will increase unemployment and mechanically lower JF. In that case, a decomposition between JF and JS will attribute the higher unemployment to a low JF, i.e. little hiring, even though the true cause was an increase in job separation.

The first contribution of this paper is to propose an alternative decomposition – between vacancy posting and JS – that can better assess the relative importance of recruiting effort and job separation. By using a measure of vacancy posting, I can model the flow of new jobs with a matching function and isolate the fluctuations in the job finding rate caused solely by changes in firms' recruiting efforts. I find that job separation and vacancy posting respectively account for about 40 and 60 percent of unemployment's variance. In contrast, Shimer's (2007) decomposition between JF and JS attributes only 25 percent of unemployment's variance to separation.

The second contribution of this paper is to go beyond the literature's focus on second moments and to study the determinants of unemployment's higher order moments. For instance, while the asymmetric behavior of the unemployment rate is a well documented fact, there is a little work studying the reasons behind this asymmetry.<sup>4</sup> I find that the steepness asymmetry of unemployment – the fact that increases are steeper than decreases – is due in large part to the behavior of job separation. After generalizing Fujita and Ramey's (2009) variance decomposition to higher-order moments, I find that job separation accounts for more than 60 percent of the skewness of first-differenced unemployment, which captures the extent of steepness asymmetry. Studying fourth moments also yields interesting insights. While unemployment has a mild (i.e. almost Gaussian) kurtosis, vacancy posting and job separation have very different kurtoses with opposite signs. Vacancy posting presents a large negative excess kurtosis, but job separation presents a positive excess kurtosis. I find that both margins contribute roughly equally to unemployment's fourth moment. The kurtosis of unemployment is almost normal because it is an average of the low kurtosis of vacancy posting and of the high kurtosis of job separation. A high kurtosis distribution such as that of job separation has a sharper peak and longer, fatter tails, i.e. extreme values are drawn more often than with a normal distribution. On the other hand, a low kurtosis distribution such as that of vacancy posting has a more rounded peak and shorter thinner tails, i.e. fewer extreme values. Thus, this result suggests that vacancy posting is driving unemployment during normal times but that job separation is responsible for rare but violent fluctuations in unemployment. To explore this idea further, I depart from an average decomposition and analyze the relative contributions of job separation and vacancy posting at business cycles turning points. I find that job separation is responsible for almost *all* of the movements in unemployment during the first two quarters after unemployment reaches a low or a high, and that vacancy posting does not become the main contributor until a year later.

The remainder of the paper is organized as follows: Section 2 presents an empirical framework to decompose the determinants of unemployment fluctuations; Section 3 assesses the contributions of vacancy posting and job separation to unemployment's second, third and fourth moments, Section 4 studies the behavior of the hazard rates at business cycles turning points; and Section 5 offers some concluding remarks.

## 2. Empirical framework

This section presents a framework to quantify the contributions of vacancy posting and job separation to unemployment fluctuations.

### 2.1. Steady-state unemployment

Denote  $u_{t+\tau}$  the unemployment rate at instant  $t+\tau \in \mathbb{R}_+$  with  $t \in \mathbb{N}$  and  $\tau \in [0, 1[$ . Assume that during a “period  $t$ ” of one month – i.e.  $\tau \in [0, 1[$  – all unemployed workers find a job according to a Poisson process with constant arrival rate  $f_t$  and all employed workers lose their job according to a Poisson process with constant arrival rate  $s_t$ . Assuming no movements in and out of the labor force, the unemployment rate satisfies

$$\frac{du_{t+\tau}}{d\tau} = s_t(1-u_{t+\tau}) - f_t u_{t+\tau} \quad (1)$$

As argued by Shimer (2007), the magnitudes of the two hazard rates are such that the half-life of a deviation of unemployment from its steady state value is about a month. As a result, at a quarterly frequency, the unemployment rate

<sup>3</sup> Fujita and Ramey (2009) also emphasize that the dynamic interactions between JS and JF may lead to erroneous conclusions about the contributions of JS and JF. While Fujita and Ramey (2009) conclusion is based on a non-structural VAR, the present paper follows a more structural approach by modeling the relationship between JS and JF.

<sup>4</sup> A large literature has documented that increases in cyclical unemployment are steeper than decreases. See, among others, Neftci (1984), DeLong and Summers (1986), Sichel (1993) and Mc Kay and Reis (2008).

is very well approximated by its steady-state value  $u_t^{ss}$  so that

$$u_t \simeq \frac{s_t}{s_t + f_t} \equiv u_t^{ss} \tag{2}$$

### 2.2. Modeling $f_t$ with a matching function

The job finding rate is defined as the ratio of new hires to the stock of unemployed, so that the job finding rate can be written as  $f_t = m_t/U_t$  with  $m_t$  the number of new matches at instant  $t$  and  $U_t$  the number of unemployed. By modeling  $m_t$  with a constant returns to scale Cobb–Douglas matching function, a specification widely used in the search and matching literature (see e.g., Pissarides, 2001), I can express  $m_t$  as

$$m_t = m_0 U_t^\sigma V_t^{1-\sigma}$$

with  $m_0$  a positive constant and  $V_t$  the number of job openings.<sup>5</sup>

In this context, the job finding rate  $f_t$  can be modeled as

$$\ln f_t = (1-\sigma) \ln \frac{v_t}{u_t} + m_0 + \zeta_t \tag{3}$$

with  $v_t$  the number of job openings per member of the labor force.

### 2.3. A decomposition of unemployment

Writing the steady-state approximation for unemployment (2) and modeling the job finding rate with a matching function, I can write<sup>6</sup>

$$u_t^{ss} \equiv \frac{s_t}{s_t + f_t} \simeq \frac{s_t}{s_t + m_0 \left(\frac{v_t}{u_t^{ss}}\right)^{1-\sigma}} \tag{4}$$

This approximation relies on the implicit assumption that movements in  $s_t$  have an effect on steady-state unemployment (which is the case by definition) as well as on the job finding rate  $f_t$  within the time period, so that the quarterly average of the monthly job finding rate reflects the influence of the job separation rate. Fortunately, since the half-life of a deviation of unemployment from its steady state is about a month in the US, approximation (4) is reasonable at a quarterly (and a fortiori yearly) frequency.<sup>7</sup> Nonetheless, to make sure that the steady-state assumption does not affect the results, Appendix presents a more general framework that allows unemployment to depart temporarily from its steady-state value. Since this generalization has little effect on the results (consistent with the rapid convergence of unemployment towards its steady-state), in the main text, I maintain the assumption that unemployment is always in steady-state.

Log-linearizing (4) with  $u_t^{ss}$  around some value  $u$  gives

$$d \ln u_t^{ss} = (1-u)[d \ln s_t - (1-\sigma)(d \ln v_t - d \ln u_t^{ss})] + \zeta_t \tag{5}$$

with  $\zeta_t$  the sum of successive approximation errors due to the first-order log-linearization, the use of a matching function to model  $f_t$ , and the fact that steady-state unemployment is used inside the matching function.

Rearranging (5) gives

$$d \ln u_t^{ss} = \frac{1-u}{1-(1-\sigma)(1-u)} d \ln s_t - \frac{(1-\sigma)(1-u)}{1-(1-\sigma)(1-u)} d \ln v_t + \eta_t \tag{6}$$

or

$$du_t^{ss} = du_t^{js} + du_t^v + \eta_t \tag{7}$$

with  $du_t^{js} = d \ln u_t^{ss}$ ,  $du_t^{js} = ((1-u)/(1-(1-\sigma)(1-u))) d \ln s_t$ ,  $du_t^v = -((1-\sigma)(1-u)/(1-(1-\sigma)(1-u))) d \ln v_t$  and  $\eta_t = ((1-u)/(1-(1-\sigma)(1-u)))\zeta_t$ .  $du_t^{js}$  can be interpreted as movements in unemployment due to changes in the job separation rate, and  $du_t^v$  can be interpreted as movements in unemployment solely due to changes in vacancy posting.

Approximating  $s_t$  and  $v_t$  around their trend values  $\underline{s}_t$  and  $\underline{v}_t$  so that  $d \ln u_t^{ss} = \ln(u_t^{ss}/\underline{u}_t^{ss})$ , (6) can be rewritten as

$$\ln\left(\frac{u_t^{ss}}{\underline{u}_t^{ss}}\right) = \frac{1-\underline{u}_t^{ss}}{1-(1-\sigma)(1-\underline{u}_t^{ss})} \ln\left(\frac{s_t}{\underline{s}_t}\right) - \frac{(1-\sigma)(1-\underline{u}_t^{ss})}{1-(1-\sigma)(1-\underline{u}_t^{ss})} \ln\left(\frac{v_t}{\underline{v}_t}\right) + \eta_t. \tag{8}$$

<sup>5</sup> I assume a constant returns to scale specification because this is a standard assumption in the search literature. However, the paper's approach goes through with a decreasing or increasing returns to scale matching function.

<sup>6</sup> It is important to note that (4) is only an approximation and does not define steady-state unemployment. Steady-state unemployment is still determined from (2). I only use a matching function to approximate  $f$  and isolate movements due to changes in vacancy posting.

<sup>7</sup> As a robustness check, the quantitative exercises were also conducted at a yearly frequency, and the results were unchanged.

Alternatively, approximating  $s_t$  and  $v_t$  around their past value, I can write a log-decomposition in first-difference with  $d \ln u_t = \Delta \ln u_t^{ss} = \ln(u_t^{ss}/u_{t-1}^{ss})$  and

$$\Delta \ln u_t^{ss} = \frac{1-u_{t-1}^{ss}}{1-(1-\sigma)(1-u_{t-1}^{ss})} \Delta \ln s_t - \frac{(1-\sigma)(1-u_{t-1}^{ss})}{1-(1-\sigma)(1-u_{t-1}^{ss})} \Delta \ln v_t + \eta_t. \quad (9)$$

#### 2.4. A variance decomposition

Using (8) or (9), it is possible to assess the separate contributions of job separation and vacancy posting to unemployment's variance by noting as Fujita and Ramey (2009) that

$$\text{Var}(du_t^{ss}) = \text{Cov}(du_t^{ss}, du_t^v) + \text{Cov}(du_t^{ss}, du_t^{js}) + \text{Cov}(du_t^{ss}, \eta_t) \quad (10)$$

or

$$1 = \beta^v + \beta^{js} + \beta^\eta \quad (11)$$

so that  $\beta^v = \text{Cov}(du_t^{ss}, du_t^v) / \text{Var}(du_t^{ss})$  and  $\beta^{js} = \text{Cov}(du_t^{ss}, du_t^{js}) / \text{Var}(du_t^{ss})$  measure the contributions of vacancy posting and job separation to unemployment's variance. I can track the accuracy of my successive approximations by looking at  $\beta^\eta$ , the contribution of the residual  $\eta_t$ . The residual is the sum of successive approximation errors: the first-order log-linearization approximation, the use of a matching function to model  $f_t$ , and the fact that steady-state unemployment is used inside the matching function. While the first approximation error has been shown to be very small (Fujita and Ramey, 2009), the last two approximations are less standard. To evaluate the bias that they introduce, I can compare the results of variance decompositions between JF and JS with and without the use of a matching function. Log-linearizing (2) gives

$$d \ln u_t^{ss} = (1-u)[d \ln s_t - d \ln f_t] + \varepsilon_t \quad (12)$$

or

$$d \ln u_t^{ss} = (1-u)[d \ln s_t - d \ln \hat{f}_t] + \xi_t \quad (13)$$

with  $\hat{f}_t = m_0(v_t/u_t^{ss})^{1-\sigma}$  as in (5). By comparing the contributions of  $\hat{f}_t$  and  $f_t$  (or the contributions of  $\xi_t$  and  $\varepsilon_t$ ), one can identify the bias introduced by modeling JF with a matching function.

#### 2.5. Higher-order moments

In this section, I generalize the previous variance decomposition to higher-order moments and notably to the concept of skewness and kurtosis.

Denote the mean of  $X$  as  $\mu = E(X)$  and its  $n$ th moment  $\alpha_n \equiv E(X-\mu)^n / (E(X-\mu)^2)^{n/2}$  for  $n \in \mathbb{N}$ . We saw in (6) that changes in (log) unemployment can be written as a sum of components. So, let us assume that  $X-\mu$  can be written as a sum of  $N$  terms so that  $X-\mu = \sum_{i=1}^N (X_i-\mu_i)$  with  $\mu_i = EX_i$ . By noting that

$$(X-\mu)^n = \sum_i (X-\mu)^{n-1} (X_i-\mu_i)$$

I have

$$E(X-\mu)^n = \sum_i E(X-\mu)^{n-1} (X_i-\mu_i)$$

so that the  $n$ th moment of  $X$  can be written

$$\alpha_n = \frac{E(X-\mu)^n}{(E(X-\mu)^2)^{n/2}} = \sum_i \frac{E(X-\mu)^{n-1} (X_i-\mu_i)}{(E(X-\mu)^2)^{n/2}}. \quad (14)$$

Dividing (14) by  $\alpha_n$ , I obtain a generalized version of (11) for any  $n$ th moment of  $X$

$$1 = \sum_i \frac{E(X-\mu)^{n-1} (X_i-\mu_i)}{E(X-\mu)^n} \quad (15)$$

and one can interpret  $\gamma_i = E(X-\mu)^{n-1} (X_i-\mu_i) / E(X-\mu)^n$  as a measure of the contribution of  $X_i$  to  $X$ 's  $n$ th moment. Indeed,  $E(X-\mu)^{n-1} (X_i-\mu_i)$  captures the fraction of  $E(X-\mu)^n$  that is due to movements in  $X_i$ .

For example, with (15), it is possible to estimate the contributions of vacancy posting and job separation to the steepness asymmetry of unemployment, the fact that increases are steeper than decreases. As defined by Sichel (1993), steepness asymmetry is captured by the skewness of first-difference log-unemployment. Using the first-difference log-decomposition (9), I can interpret  $E(du_t^{ss})^2 du_t^{js} / E(du_t^{ss})^3$  and  $E(du_t^{ss})^2 du_t^v / E(du_t^{ss})^3$  as the contributions of job separation and vacancy posting to the skewness of first-differenced unemployment.

Similarly, one can study the fourth moment (kurtosis) of the level of unemployment and its hazard rates. Using the log-decomposition in level (8),  $E(du_t^{ss})^3 du_t^{ss} / E(du_t^{ss})^4$  and  $E(du_t^v)^3 du_t^v / E(du_t^v)^4$  give the contributions of job separation and vacancy posting to the kurtosis of unemployment.

### 3. Empirical result

I now apply the decomposition framework to US data. The unemployment rate is constructed by the BLS from the Current Population Survey (CPS), and measures of  $s_t$  and  $f_t$ , the unemployment inflow and outflow rates, are obtained following Shimer (2007).<sup>8</sup> More difficult is the choice of a measure for vacancy posting  $v_t$ . There are two standard measures of job openings; the Help-Wanted advertising Index (HWI) and the Job Openings and Labor Turnover Survey (JOLTS). The Help-Wanted Index is constructed by the Conference Board and measures the number of help-wanted advertisements in 51 major newspapers. This index is only a proxy for vacancy posting but has the advantage of dating back to 1951, thus providing a long time series. However, this “print” HWI index has become increasingly unrepresentative as advertising over the internet has become more prevalent. In fact, the Conference Board stopped publishing its print HWI in May 2008 and publishes instead since 2005 a measure of online help wanted advertising.<sup>9</sup> To obtain a consistent measure of Help-Wanted advertising over 1951–2008, I proceed as in Barnichon (2010) and construct a “composite” index, that combines information on “print” and “online” advertising. JOLTS is produced by the BLS and contains monthly data on job openings from 16,000 establishments since December 2000. Since JOLTS provide a more direct, and arguably better, measure of vacancy posting than HWI, Fig. 1 presents the different measures of vacancy posting over 2000–2008, and shows that the two composite indexes track each other remarkably well over the last 10 years.

#### 3.1. Estimating a Cobb–Douglas matching function

I estimate a Cobb–Douglas matching function

$$\ln f_t = (1-\sigma) \ln \frac{v_t}{u_t} + m_0 + \zeta_t \quad (16)$$

after detrending all variables with an HP-filter.<sup>10</sup> I first estimate (16) with monthly data and using the composite HWI index from 1951:M01 to 2008:M12. Table 1 presents the result. The elasticity  $\sigma$  is precisely estimated at 0.59, and apart from JF’s high-frequency movements, a matching function does a very good job at capturing movements in the job finding rate. Indeed, after taking quarterly averages, Fig. 2 shows that a matching function tracks the empirical job finding rate very closely. To make sure that the results are not biased by the strong low-frequency movements in HWI before 1977 that are unrelated to the labor market, I estimate (16) with the composite print-online help-wanted index over 1977:M01–2008:M12 only. The estimated  $\sigma$  is unchanged at 0.59. Finally, I use JOLTS data only over 2000:M12–2008:M12 and find a slightly lower  $\sigma$  at 0.57. These estimates lie in the middle of the plausible range reported by Petrongolo and Pissarides (2001).

A legitimate concern with this regression exercise is that Eq. (16) may be subject to an endogeneity bias. The use of a monthly frequency and the fact that  $u_t$  denotes the beginning of period unemployment rate should minimize the problem, but it is still important to verify that there is no significant bias. To do so, I estimate (16) using lagged values of  $v_t/u_t$  as instruments.<sup>11</sup> Encouragingly, Table 1 shows that the endogeneity bias is likely to be small as the coefficient is little changed at 0.58.<sup>12</sup>

The robustness of the results over different measures of vacancies and over different sample periods is promising and suggests that a matching function provides a good model of the job finding rate. For the rest of the paper, I will use the composite HWI measure of vacancy posting with a matching function elasticity  $\sigma = 0.59$  but the results do not rely on this specific choice.

<sup>8</sup> Specifically, the unemployment outflow rate is estimated from the share of short-term (less than 5 weeks) unemployed with  $f_t = -\ln(1-F_t)$  and  $F_t = 1 - (u_{t+1} - u_{t+1}^{5wks})/u_t$ . The separation rate is then estimated by solving (1) over  $[t, t+1]$  and finding  $s_t$  such that the solution  $u_{t+\tau}$  equals  $u_{t+1}$  for  $\tau = 1$ .

<sup>9</sup> Another problem with the HWI is that it is subject to low-frequency fluctuations that are related only tangentially to the labor market; notably, the decline in print advertising in the 1990s and the 1960s newspaper consolidation that may have increased advertising in surviving newspapers. Fortunately, detrending all series with a low frequency trend (since I am only focusing on business cycle fluctuations) should remove the effect of such secular shifts.

<sup>10</sup> Davis et al. (2010) study the behavior of vacancies and hirings in JOLTS and find that one in six hires occur outside of the matching function framework, i.e. without a prior vacancy. Regression (16) could then be subject to an omitted variable bias. Denoting  $z_t$  the fraction of hires outside the matching function framework, total hires equals  $m_t/(1-z_t)$  so that I can write  $\ln f_t = -\ln(1-z_t) + (1-\sigma) \ln(v_t/u_t) + c + \varepsilon_t$ . In that case, the absolute value of the omitted variable bias is given by  $|\text{cov}(\ln(1-z_t), \ln(v_t/u_t)) / \text{var}(\ln(v_t/u_t))| = |\text{corr}(\ln(1-z_t), \ln(v_t/u_t))| \cdot \text{var}(\ln(1-z_t))$ . Assuming the worse case scenario in which  $|\text{corr}(\ln(1-z_t), \ln(v_t/u_t))| = 1$  and (roughly) estimating the standard-deviation of  $z_t$  from DFH, Fig. 10 to be at most 0.04, I get a maximal bias for  $\sigma$  of  $1 \cdot \text{var}(\ln(1-z_t)) \approx 1 \cdot \text{var}(z_t) = 0.04^2 = 0.016$ , suggesting that the omitted variable bias is small.

<sup>11</sup> Such instruments are valid if the residual is not serially correlated. The Durbin–Watson statistics for regression (1) in Table 1 is 1.83. To verify that serial correlation is definitely not an issue, I performed a GMM regression over 1951–1990 for which the Durbin–Watson statistics is 2.02. Results are unchanged.

<sup>12</sup> An issue that was brushed aside is the timing of the measurement of unemployment, vacancy and the job finding rate. The Appendix presents a more rigorous way to address these measurement issues and shows that estimates of  $\sigma$  are unchanged by these timing considerations.

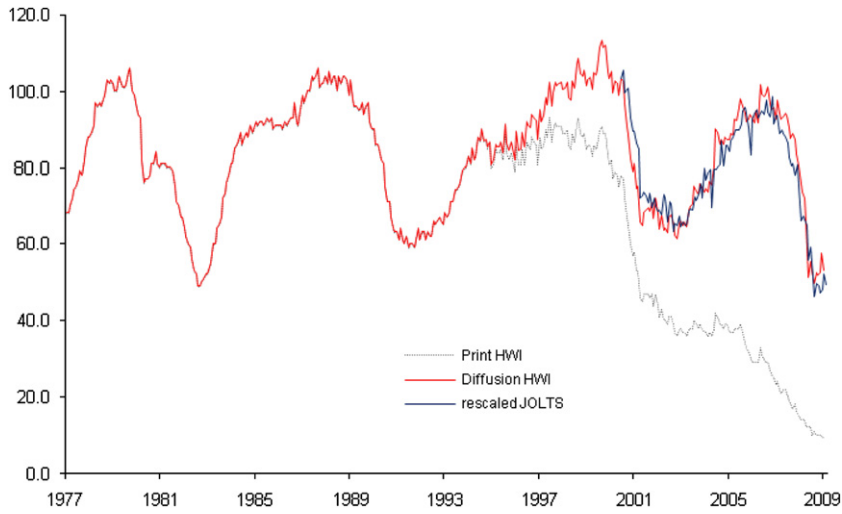


Fig. 1. Different measures of vacancy posting, 1951M01–2009M09.

**Table 1**  
Estimating the matching function from Shimer's job finding rate.

Dependent variable:	$f$	$f$	$f$	$f$
Sample	1951:M1–2008:M12	1977:M1–2008:M12	2000:M12–2008:M12	1951:M1–2008:M12
Regression	(1)	(2)	(3)	(4)
	Composite HWI index	Composite HWI index	JOLTS	Composite HWI index
Estimation	OLS	OLS	OLS	GMM
$\sigma$	0.59*** (0.01)	0.59*** (0.01)	0.57*** (0.02)	0.58*** (0.02)
$R^2$	0.81	0.81	0.73	–

Notes: Standard-errors are reported in parentheses. For Eq. (4), three lags used for instruments.  
\*\*\* Indicates significance at the 1% level.

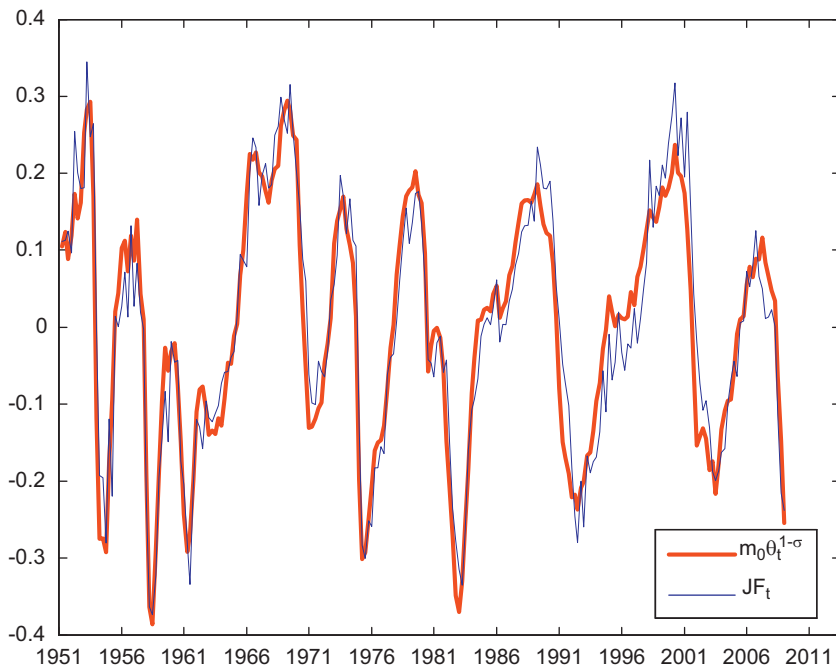


Fig. 2. Empirical and model job finding rate.

**Table 2**  
Contribution of V and JS to unemployment variance, 1951–2008.

Decomposition	Variance			Variance		
	$\beta^{JS}$	$\beta^V$	$\beta^\eta$	$\beta^{d(JS)}$	$\beta^{d(V)}$	$\beta^\eta$
Decomposition: V and JS	39.3%	55.4%	5.3%	63.4%	34.8%	1.9%
Decomposition: JF and JS	$\beta^{JS}$	$\beta^{JF}$	$\beta^\eta$	$\beta^{d(JS)}$	$\beta^{d(JF)}$	$\beta^\eta$
Model JF	24.2%	71.8%	4.0%	39.6%	59.2%	1.2%
Empirical JF	24.4%	75.9%	−0.3%	39.6%	59.6%	0.8%

Notes: “Model JF” indicates that JF is modelled using a matching function (with a matching elasticity  $\sigma = 0.59$ ). “Empirical JF” indicates that Shimer’s (2007) estimate of JF is used.

### 3.2. Variance decomposition

Table 2 presents the results of different variance decomposition exercises. The first three columns reports the results of a decomposition in level using (8), and the last three columns report the results of a decomposition in first-difference using (9). The first row studies the contributions of job separation and vacancy posting. Looking first at the decomposition in level, the contribution of job separation stands at about 40 percent, and the contribution of vacancy posting at about 55 percent. The successive approximations naturally generate a non-zero contribution of the error component, but the contribution of the residual remains small at about 5 percent. This confirms that the matching function does a good job at approximating the job finding rate. To evaluate the bias introduced by a matching function, the second and third rows of Table 2 present two variance decompositions between JF and JS; one in which JF is the empirical job finding rate (following (12)) and the other in which JF is modeled with a matching function (following (13)). Comparing rows 2 and 3, we can see that modeling JF with a matching function increases the contribution of the residual and biases the contribution of JF downward by about 4 percent. As a result, the contribution of vacancy posting is likely to be underestimated and is probably closer to 60 than 55 percent, and a reasonable split between job separation and vacancy posting is 40/60.<sup>13</sup> Finally, as shown in the last three columns of Table 2, a first-differenced decomposition gives a stronger role to job separation with a 60/40 split, with a small contribution from the residual.

Thus, job separation plays a major role with respect to unemployment’s variance, and modeling the job separation probability as acyclical will lead researchers to understate the volatility of unemployment.<sup>14</sup>

### 3.3. Comparison with previous findings

The important role played by job separation in explaining the variance of unemployment contrasts with the much smaller role identified by Shimer (2007) using a decomposition between JF and JS. Given the influence of Shimer’s (2007) results, in this section, I contrast the results of a decomposition between vacancy and JS with the results from a decomposition between JF and JS.

#### 3.3.1. A decomposition between JF and JS

Shimer (2007), Elsby et al. (2009b), and Fujita and Ramey (2009) study the contributions of the job finding rate and the job separation rate from (12) or

$$d \ln u_t^{JS} = (1-u)[d \ln s_t - d \ln f_t] + \varepsilon_t = du_t^{JS} + du_t^{JF} + \varepsilon_t \quad (17)$$

The third row of Table 2 presents the result of such a decomposition and shows that the contribution of the job separation rate is about 25 percent (for a decomposition in level), i.e. much lower than the 40 percent obtained from a decomposition between vacancy posting and separation. Shimer’s (2007) results led many researchers to treat the job separation rate as acyclical and only study the firm’s hiring decision. However, to interpret a decomposition based on (17) in terms of hiring versus separation, the job finding rate and the job separation rate must be two independent determinants of unemployment. In a standard MP model, this is indeed the case.<sup>15</sup> However, this needs not be the case

<sup>13</sup> In the Appendix, I extend the decomposition by using CPS data from the BLS on the reasons for unemployment (layoffs, quits or labor force entrants) over 1968–2004 as used in Elsby et al. (2009b). I find that layoffs contribute to 45 percent of unemployment fluctuations but quits, being procyclical and highly negatively correlated with layoffs, lower the contribution of JS by 10 percentage points, a point made qualitatively by Elsby et al. (2009b).

<sup>14</sup> Shimer (2005) shows that the Mortensen and Pissarides (1994) model with a constant job separation rate lacks an amplification mechanism because it generates less than 10 percent of the observed business cycle fluctuations in unemployment given labor productivity shocks of plausible magnitude.

<sup>15</sup> In the standard Mortensen and Pissarides (1994) model, the job creation and job destruction condition can be written  $(1-R)/(r+\rho) = c/\theta f(\theta)$  and  $R - b/p - c\theta + \rho/(r+\rho) \int_0^R (x-R) dG(x) = 0$  with  $p$  aggregate productivity,  $R$  the reservation idiosyncratic productivity level below which the match is destroyed,  $G(x)$  the cdf of idiosyncratic productivity shocks,  $r$  the risk-free rate,  $b$  unemployment benefits,  $c$  the vacancy posting cost,  $\theta = v/u$  aggregate



in the data, and interpreting a decomposition between JF and JS in terms of hiring versus separation is difficult if JF and JS are not independent.

### 3.3.2. The relationship between JF and JS

Empirically, a simple way to study the relationship between JF and JS is to run Granger-causality tests between the two series. Table 3 presents the results using specifications with one to three lags.<sup>16</sup> In all cases, I can strongly reject that JS does not Granger-cause JF. Granger-causality running from JF to JS is weaker, and with one lag, I can accept the null that JF does not Granger-cause JS.

Another way to assess the extent of the relationship between JF and JS is proposed by Fujita and Ramey (2009). Fujita and Ramey (2009) extend the JF–JS variance decomposition (17) by allowing for some dynamic interactions between JF and JS. Indeed, a static variance decomposition may overstate or understate the true contributions of hiring and separation, because it ignores the dynamic interaction between JS and JF. For example, if a high separation rate leads to a low job finding rate next period, one may attribute the high unemployment next period to a low job finding rate and little hiring, even though a high separation rate was the true cause. Specifically, Fujita and Ramey (2009) estimate a (non-structural) bivariate VAR  $\left(\frac{du_t^{js}}{du_t^{jf}}\right)$  to capture the dynamic interactions between JF and JS. Using the standard moving average representation, they can write

$$\begin{cases} du_t^{js} = \varepsilon_t^{js} + \sum_{k=1}^{\infty} (a_k \varepsilon_{t-k}^{js} + b_k \varepsilon_{t-k}^{jf}) \\ du_t^{jf} = \varepsilon_t^{jf} + \sum_{k=1}^{\infty} (c_k \varepsilon_{t-k}^{js} + d_k \varepsilon_{t-k}^{jf}) \end{cases}$$

with  $\varepsilon_t^{js}$  and  $\varepsilon_t^{jf}$  variations in  $d \ln s_t$  and  $d \ln f_t$  that are uncorrelated over time, but can be correlated contemporaneously. Proceeding as in (10), they can write

$$\text{var}(du_t^{ss}) = \text{var}(\varepsilon_t^{js}) + \text{var}(\varepsilon_t^{jf}) + 2 \text{cov}(\varepsilon_t^{js}, \varepsilon_t^{jf}) + A_t^{jf} + A_t^{js} \quad (18)$$

and the contribution of job separation is given by<sup>17</sup>

$$\beta^{js} = \frac{\text{var}(\varepsilon_t^{js}) + \text{cov}(\varepsilon_t^{js}, \varepsilon_t^{jf}) + A_t^{js}}{\text{var}(d \ln u_t^{ss})} \quad (19)$$

with

$$A_t^{js} = \sum_{k=1}^{\infty} ((a_k + c_k) \text{var}(\varepsilon_{t-k}^{js}) + (a_k + c_k)(b_k + d_k) \text{cov}(\varepsilon_{t-k}^{js}, \varepsilon_{t-k}^{jf})).$$

Once dynamic interactions between the separation rate and job finding rate are incorporated into the decomposition, Fujita and Ramey (2009) find that the contribution of JS increases by about 10 percent (the contribution of JF correspondingly declines).<sup>18</sup> Their result indicates that endogeneity running from the separation rate to the job finding rate is quantitatively significant but that endogeneity running in the other direction is small, a result in line with the Granger-causality tests.

### 3.3.3. The endogeneity of JF

Although little emphasized in the literature, the existence of a structural link between JS and JF is not surprising when one considers that the job finding rate is, by definition, the ratio of new hires to the stock of unemployed  $f_t = m_t/u_t$ . An increase in JS with no change in hiring ( $m_t$ ) will increase unemployment and mechanically lower JF.

To restate this argument more rigorously, hiring  $m_t$  can be modeled as a function of unemployment and vacancies using a matching function as in Section 2.2. With  $m_t = m_0 V_t^{1-\sigma} U_t^\sigma$ , the job finding rate can be written as  $f_t = m_0 (v_t/u_t)^{1-\sigma}$  so that

$$d \ln f_t = (1-\sigma)[d \ln v_t - d \ln u_t]$$

(footnote continued)

labor market tightness, and  $\rho$  the arrival rate of idiosyncratic productivity shocks. (For clarity of presentation, I simplified the model to no unemployment benefits and I gave all the bargaining power to the firm.). Given these two equations, aggregate productivity  $p$  uniquely determines  $\theta$  and  $R$ . Since the job finding rate is given by  $f = m_0 \theta^{1-\sigma}$ , a function of  $\theta$  only, and the (endogenous) job separation rate by  $s = \rho G(R)$ , a function of  $R$  only, one can identify the respective contributions of hiring and separation by studying the relative contributions of  $f$  and  $s$ .

<sup>16</sup> With Granger-causality tests, the results can be sensitive to the number of lags used in the regressions as well as the detrending method. I report the results with one to three lags, as the Akaike information criteria equally favors those three specifications. The results in Table 1 were obtained after including a quadratic trend in the regressions. Removing the trend gives similar results.

<sup>17</sup> The contribution of JF is  $\beta^{jf} = (\text{var}(\varepsilon_t^{jf}) + \text{cov}(\varepsilon_t^{js}, \varepsilon_t^{jf}) + A_t^{jf}) / \text{var}(d \ln u_t^{ss})$  with  $A_t^{jf} = \sum_{k=1}^{\infty} ((b_k + d_k) \text{var}(\varepsilon_{t-k}^{jf}) + (a_k + c_k)(b_k + d_k) \text{cov}(\varepsilon_{t-k}^{js}, \varepsilon_{t-k}^{jf}))$ .

<sup>18</sup> The magnitude of Fujita and Ramey's (2009) upward revision to JS is sensitive to the data source and type of decomposition. The result reported here refers to a decomposition in level and using Shimer's data for JF and JS as in the present paper.



**Table 3**  
Granger causality tests, 1951:Q1–2008:Q4.

Hypothesis test	p-Value in parenthesis (1 lag)	p-Value in parenthesis (2 lags)	p-Value in parenthesis (3 lags)
JS does not Granger-cause JF?	No (0.01)	No ( $6 \times 10^{-13}$ )	No ( $2 \times 10^{-12}$ )
JF does not Granger-cause JS?	Yes (0.49)	No ( $1 \times 10^{-3}$ )	No ( $2 \times 10^{-4}$ )

which, using the steady-state approximation, gives

$$d \ln f_t = (1-\sigma)[d \ln v_t - (1-u)(d \ln s_t - d \ln f_t)]$$

or

$$d \ln f_t = \underbrace{\frac{1-\sigma}{1-(1-u)(1-\sigma)} d \ln v_t}_{\text{Changes in firms' recruiting effort}} + \underbrace{\frac{1-u}{1-(1-u)(1-\sigma)} d \ln s_t}_{\text{Endogenous movements in JF}}. \tag{20}$$

Eq. (20) highlights the link between JF and JS and the fact that JF can move endogenously with JS. Changes in JF are driven by changes in firms recruiting effort, but also by changes in the job separation rate. As a result, a decomposition between JF and JS based on (17) will be biased against JS. Using a measure of vacancy posting, one can isolate the fluctuations in JF solely caused by changes in firms' recruiting intensity, but not by fluctuations in JS. Combining (20) with the baseline JF–JS decomposition (17) gives the present paper's decomposition

$$d \ln u_t^{ss} = \frac{1-u}{1-(1-\sigma)(1-u)} d \ln s_t - \frac{(1-\sigma)(1-u)}{1-(1-\sigma)(1-u)} d \ln v_t + \eta_t. \tag{21}$$

Comparing (17) and (21), one can see that, in a decomposition between JF and JS, the contribution of JS is biased downward by a factor  $1/(1-(1-\sigma)(1-u))$ . As a back-of-the-envelope calculation, with  $\sigma \simeq 0.6$  and  $u \simeq 0.05$ , the endogeneity of JF biases the contribution of separation downward by 60 percent (from  $1/(1-(1-\sigma)(1-u)) \simeq 1.6$ ).

### 3.3.4. The benefits of a decomposition between V and JS

As we saw in the previous section, the baseline JS–JF decomposition (17) biases downward the contribution of separation. Moreover, while Fujita and Ramey's (2009) dynamic decomposition (18) highlights the importance of a possible link between JF and JS (by capturing the fact that JS helps predict future values of JF), it provides little guidance in terms of the respective roles played by hiring and separation in driving unemployment fluctuations. Since  $cov(e_t^{js}, e_t^{jf}) \neq 0$ ,  $e_t^{js}$  and  $e_t^{jf}$  are not independent and  $e_t^{jf}$  may still be driven by  $e_t^{js}$  as in (20). Moreover, while Fujita and Ramey (2009) split the contribution of the covariance terms (current and lagged) equally between JS and JF, this split is arbitrary, and it is difficult to interpret these covariance terms and draw conclusions regarding the importance of hiring and separation.

Compared to the baseline JS–JF decomposition (17) and Fujita and Ramey's (2009) dynamic decomposition (18), decomposition (21) studies the contribution of observable variables that more directly correspond to firms' control variables. As such, it avoids the pitfalls of decompositions (17) and (18) and allows for a better assessment of the relative importance of recruiting effort and separation in driving unemployment fluctuations, and thus can better inform the development of models of unemployment.<sup>19</sup>

### 3.4. Higher-order moments

I now turn to the higher-order moments of unemployment, and I study (i) the determinants of the steepness asymmetry in unemployment, i.e., the skewness of first-difference unemployment, and (ii) the determinants of the kurtosis of unemployment.<sup>20</sup>

First, looking at steepness asymmetry, Table 4 shows that over 1955–2008, the first-differenced of log-unemployment is heavily skewed with a skewness coefficient of 1.2.<sup>21</sup> Vacancy posting and job separation also present significant asymmetries in steepness with coefficients of –0.79 and 0.42. To study the determinants of unemployment's steepness

<sup>19</sup> Job separation comprises layoffs, which are decisions of the firm, and quits, which are decisions of the worker. Thus, to map our decomposition into models of unemployment and interpret our decomposition in terms of agents' control variables, one should distinguish between quits and layoffs. Although the main text reports the contributions of the total job separation rate and vacancy posting, the Appendix presents a decomposition that treats separately the three main decision variables of economic agents: vacancy posting, layoffs and quits.

<sup>20</sup> The other third and fourth moments of unemployment are not particularly interesting, and I do not report their decomposition. Specifically, the skewness of the level of unemployment and the skewness of its components (vacancy and JS) are small, and the kurtoses of first-difference unemployment, vacancy and JS are not significantly different from 3, i.e. normality.

<sup>21</sup> Over 1951–1954, unemployment experienced very large quarterly movements that dramatically increase the skewness coefficient (by 0.4) and confidence interval. Since the skewness estimate is otherwise stable over 1955–2008, I omit the 1951–1954 time period for clarity of exposition. Nonetheless, my results remain valid over 1951–2008.

**Table 4**  
Higher-order moments of unemployment and hazard rates, 1955–2008.

Moment	$u^{SS}$	$v$	$JS$
Skewness	1.21** (0.53)	−0.79** (0.24)	0.42** (0.09)
Kurtosis	2.66 (1.16)	2.06** (0.40)	3.54** (1.40)

Notes: All variables are expressed in log. All variables are detrended with an HP filter ( $\lambda = 10^5$ ). Newey–West standard errors are reported in parentheses and \*\* Indicates significance at the 5% level. The Skewness is measured with variables in first-difference while the Kurtosis is measured with variables in levels.

**Table 5**  
Contribution of V and JS to higher-order moments of unemployment, 1955–2008.

Decomposition	Skewness			Kurtosis		
	$\gamma^{d(JS)}$	$\gamma^{d(V)}$	$\gamma^{\eta}$	$\gamma^{JS}$	$\gamma^V$	$\gamma^{\eta}$
Decomposition: V and JS	62.5%	42.7%	−5.2%	44.0%	52.5%	3.5%
Decomposition: JF and JS	$\gamma^{d(JS)}$	$\gamma^{d(JF)}$	$\gamma^{\eta}$	$\gamma^{JS}$	$\gamma^{JF}$	$\gamma^{\eta}$
Model JF	38.8%	63.5%	−2.3%	27.4%	69.7%	3.0%
Empirical JF	38.8%	60.1%	1.1%	27.4%	73.1%	−0.4%

Notes: “Model JF” indicates that JF is modelled using a matching function (with a matching elasticity  $\sigma=0.59$ ). “Empirical JF” indicates that Shimer’s (2007) estimate of JF is used. The Skewness is measured with variables in first-difference while the Kurtosis is measured with variables in levels.

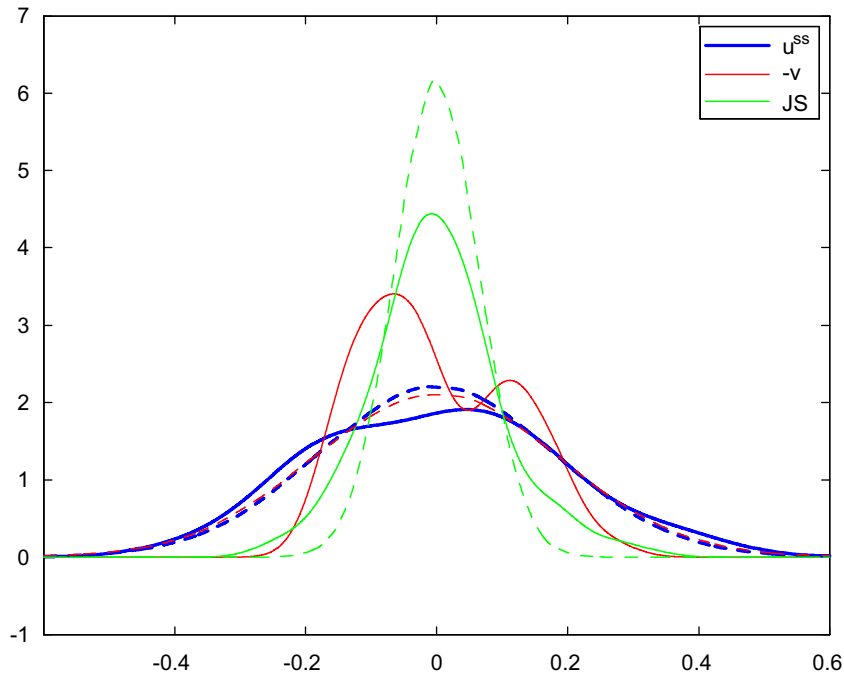
asymmetry, Table 5 shows the result of a first-difference log-decomposition (9).<sup>22</sup> While job separation contributes to less than half of unemployment’s variance, this is hardly the case with unemployment asymmetry since the job separation’s contribution stands at more than 62 percent. The contribution of the residual remains low and stands at around 5 percent. A comparison of the second and third row of Table 5 shows that the contribution of JF increases from 60 to 63 percent, indicating that the use of a matching function biases upwards the contribution of JF. As a result, the contribution of vacancy posting is likely to be overestimated, and a rough split between job separation and vacancy posting is 60/40. Thus, a model that would not consider fluctuations in the job separation rate would seriously downplay the asymmetric behavior of unemployment.

Turning to kurtosis, Table 4 presents a new fact pertaining to the fourth moment of unemployment and its hazard rates. While unemployment has a mild negative excess kurtosis (−0.34), vacancy posting and job separation have very different kurtosis with opposite signs. Vacancies present a large negative excess kurtosis (−0.94) but JS presents a positive excess kurtosis (0.54). Recall that a high kurtosis distribution such as that of JS has a sharper peak and longer, fatter tails, i.e. extreme values are drawn more often than with a normal distribution. This finding is not surprising if we think of job separation as capturing (among other things) bursts of layoffs. On the other hand, a low kurtosis distribution such as that of vacancies has a more rounded peak and shorter thinner tails, i.e. fewer extreme values. To visualize the distribution of steady-state unemployment, vacancy posting and the job separation rate, Fig. 3 plots the kernel density estimates of these variables using a Gaussian kernel with optimal bandwidth. The dashed lines represent the corresponding (i.e. same mean and variance) normal distributions. While unemployment’s distribution is very close to being normal, this is hardly the case for vacancy posting and job separation. Vacancy posting has almost a bimodal distribution with rapidly decreasing tails but the job separation rate has a small mass of points around the mean and very fat tails.

Looking at the contributions of each hazard rate, Table 5 shows that vacancy posting and job separation contribute in roughly equal proportion to unemployment’s fourth moment, with a slight advantage for vacancy posting. The mild negative kurtosis of unemployment despite the large negative kurtosis of vacancy posting is consistent with an interpretation of job separation influencing unemployment through rare but violent episodes of job separation. The contribution of the residual amounts to less than 4 percent, and the second row of Table 5 indicates that the use of a matching function biases the contribution of JF downward. As a result, the split between job separation and vacancy posting is roughly 45/55.

The fourth-moment decomposition suggests that vacancy posting drives unemployment during normal times but that job separation is responsible for rare but violent fluctuations in unemployment. To explore this idea further, the next

<sup>22</sup> I first detrend unemployment, vacancy and the hazard rates before studying the skewness of first-differenced variables as trends may bias the skewness coefficient.



**Fig. 3.** Kernel density estimates (Gaussian kernel) for steady-state unemployment, vacancy posting and the job separation rate. Dotted-lines represent the corresponding Normal distributions. All variables are logged and detrended with an HP-filter ( $\lambda = 10^5$ ).

section departs from an average decomposition and analyzes the relative contributions of JS and vacancy posting at business cycles turning points.

#### 4. The contributions of vacancy posting and job separation at business cycle turning points

In this section, I analyze the relative contributions of the job separation rate and vacancies around the turning points of unemployment fluctuations. After detrending unemployment using an HP-filter with  $\lambda = 10^5$ , I follow Mc Kay and Reis (2008) and identify highs and lows in unemployment using the algorithm of Bry and Boschan (1971).<sup>23</sup> Fig. 4 plots the average dynamics of the log-deviation from trend of steady-state unemployment, job separation, and vacancy posting in a window of 3 and 6 quarters before and after the highs and lows of unemployment. I plot  $du_t$  and its components  $du_t^{js}$  and  $du_t^v$  following (7) and (8), so that the components sum to the unemployment rate with  $du_t = du_t^{js} + du_t^v + \eta_t$ .<sup>24</sup> Thanks to this log-decomposition, Fig. 4 shows the contributions of job separation and vacancy posting to unemployment movements around a high or a low.

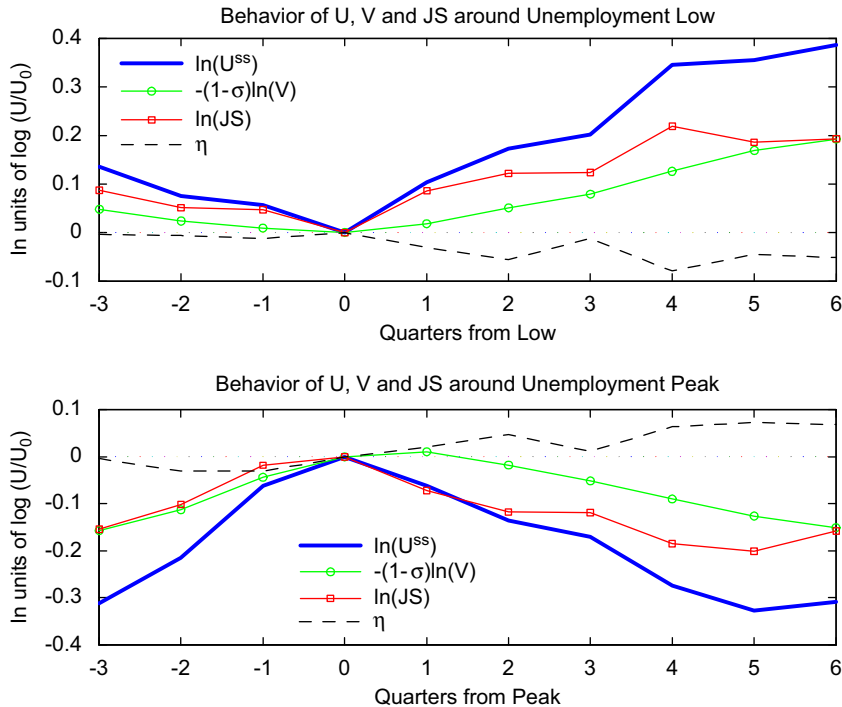
While the previous section showed that vacancy posting was, on average, the most important contributor of unemployment fluctuations, this is hardly the case at business cycle turning points. Around highs and lows, job separation is the prime determinant of movements in unemployment. Job separation accounts for more than 50 percent of unemployment movements for as much as 6 quarters after a high or a low, and for almost all of the initial response.<sup>25</sup> This result is consistent with the decomposition of unemployment's fourth moment in the previous section, which suggests that extreme values of unemployment are due to the job separation rate. Looking at the contribution of the residual, the approximation is relatively good three quarters before and after a turning point but deteriorates slightly thereafter. However, assigning all of the residual's contribution to vacancy posting (a worst case scenario for job separation) does not change the main conclusion; job separation still accounts for more than 50 percent of unemployment movements a year after a high or low.

Two other observations are worth noting. First, the asymmetric nature of unemployment is clearly apparent in Fig. 4 as unemployment increases faster than it decreases. This asymmetry can be linked to the asymmetric response of job

<sup>23</sup> See Mc Kay and Reis (2008) for a presentation of possible methods to identify the peaks and troughs of a series. All the results are robust to using the alternative methods reported by Mc Kay and Reis (2008).

<sup>24</sup> Specifically, for each quarter  $j$  around an unemployment turning point  $t_0$ , I plot  $\ln(u_{t_0+j}^{ss}/u_{t_0+j}^{ss}) - \ln(u_{t_0}^{ss}/u_{t_0}^{ss}) - (1 - u_{t_0+j}^{ss}) d \ln s_{t_0+j} - (1 - u_{t_0}^{ss}) d \ln s_{t_0}$ , and  $-[(1 - u_{t_0+j}^{ss})(1 - \sigma) d \ln v_{t_0+j} - (1 - u_{t_0}^{ss})(1 - \sigma) d \ln v_{t_0}]$ . According to (8), the first term is the sum of the last two so that, for each quarter  $j$  around a turning point  $t_0$ , Fig. 4 shows the contributions of  $V$  and  $JS$  to deviations of unemployment from its low or high.

<sup>25</sup> This result echoes Elsby et al.'s (2009b) finding that after unemployment lows, JS is responsible for the initial (i.e. during the first two quarters after a low) increase in unemployment after lows. Their conclusion is not as strong because they study a decomposition between JS and JF, that may underestimate the role of JS.



**Fig. 4.** Average business cycle dynamics for steady-state unemployment, the job separation rate, vacancies, and the residual near unemployment lows (upper-panel) and unemployment highs (lower panel), 1951–2008.

separation. Vacancy posting reacts slowly, and the slope of vacancy posting is much weaker than that of job separation in the first quarters after a turning point. Second, after unemployment highs, vacancies lag job separation by a quarter. This is in line with Fujita and Ramey (2009), who find that the job separation rate leads the job finding rate.

An implication of these last findings is that ignoring the job separation margin when modeling unemployment will lead researchers to underestimate the magnitude and speed of adjustments in unemployment around turning points.

## 5. Conclusion

While much progress has been achieved in understanding the functioning of the labor market, developing a successful theory of unemployment fluctuations remains a major goal for future research (e.g., Shimer, 2005). Previous decomposition frameworks based on the inflows and outflows of unemployment have aimed to inform modeling choices by identifying the contributions of hiring and separation in driving unemployment fluctuations. Although such decomposition frameworks have the advantage of being simple and general, their interpretation is difficult, because the inflow rate may mechanically drive some of the fluctuations of the outflow rate and thus lead to erroneous conclusions about the contributions of hiring and separation in driving unemployment fluctuations. By making one additional and relatively uncontroversial assumption – the existence of a matching function – and by using a measure of vacancy posting, the present paper proposes an alternative decomposition – between vacancy posting and separation – that can better assess the relative importance of recruiting effort and job separation.

I find that job separation and vacancy posting respectively account for about 40 and 60 percent of unemployment's variance. In contrast, Shimer's (2007) decomposition between the inflow rate and outflow rate attributes only 25 percent of unemployment's variance to separation. Moreover, job separation contributes to about 60 percent of unemployment steepness asymmetry, the fact unemployment increases faster than it decreases. Further, while vacancy posting is, on average, the more important contributor of unemployment fluctuations, the opposite is true around business cycle turning points, when job separation is responsible for most of unemployment movements.

A recent modeling trend treats the job separation rate as acyclical, but this paper's results caution against such an assumption. Assuming an acyclical job separation rate will lead researchers to (i) understate the volatility of unemployment, (ii) seriously downplay the asymmetric behavior of unemployment, and (iii) underestimate the magnitude and speed of adjustments in unemployment around business cycle turning points. Moreover, the dynamic interactions between the job separation rate and the job finding rate suggested by Granger causality tests are inconsistent with the standard MP model and suggest that the canonical model is incomplete. One extension would be to treat vacancy posting as a state variable. For example, an MP model with sunk cost in vacancy creation as in Fujita and Ramey (2007) could

explain why the job separation rate Granger-causes the job finding rate.<sup>26</sup> Another possibility would be time to build in vacancy posting.

This paper also highlights the importance of considering higher-order moments when developing and testing theoretical economic models. While researcher typically evaluate their models against second moments, this paper shows that, in the case of the unemployment rate, higher-order moments can provide valuable additional information.

By listing a number of stylized facts about the roles played by the recruiting and separation margins in driving unemployment fluctuations, the accounting framework proposed in this paper can inform modeling assumptions and help compare the performances of different models. Nonetheless, this framework does not identify the structural shocks driving unemployment fluctuations, and recruiting effort and separation need not be independent of each other as leads or lags of one variable could influence the value of the other variable. Such hypothetical links originate in the firm's employment adjustment problem, i.e. in the conditions governing the optimal choice between hiring and separation. Taking such links into accounts requires more structure and assumptions than built in the present framework, and I leave this task for future research.

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## Appendix A

### A.1. The timing of $u_t$ , $v_t$ , and $f_t$

An issue when using measures for unemployment, vacancy posting and job finding probability concerns the precise definition of each variable. In particular, while some variables are beginning or end of month values, others are monthly averages.

In the CPS, the BLS surveys the number of unemployed during the reference week, defined as the week including the 12th day of the month. The Help-Wanted Index  $v_t^{HWI}$  measures the total number of advertisements (print or online) from the 14th ( $t$ ) of the month to the 13th of next month ( $t+1$ ). JOLTS, on the other hand, indicates the number of job openings  $v_t^{JOLTS}$  on the last day of month  $t$ . Finally, Shimer's (2007) definition of  $F_t$  (Footnote 8) implies that  $F_t$  measures the average job finding probability between two unemployment measurement dates, i.e. between the week including the 12th of next month and the week including the 12th of the current month.

To be as consistent as possible with these measurement dates, the average job finding probability should depend on the average unemployment rate and the average number of posted vacancy between two reference weeks. Since  $u_t$  measures the unemployment rate during the first reference week, the correct measure of unemployment inside the matching function should be  $\frac{1}{2}(u_t + u_{t+1})$ . Since  $v_t^{HWI}$  already corresponds to an average over a period and  $v_t^{JOLTS}$  measures the number of job openings at a date roughly in between two reference weeks,  $v_t^{JOLTS}$  corresponds to  $v_t^{HWI}$  as those two measures would be equal if the number of job openings remained constant in between two reference weeks.

As a result, a more consistent regression to estimate a matching function would be

$$\ln f_t = (1-\sigma) \ln \frac{v_t}{\frac{1}{2}(u_t + u_{t+1})} + c + \zeta_t \quad (22)$$

after detrending all variables with an HP-filter. Of course, such a regression is clearly subject to an endogeneity bias as  $u_{t+1}$  is a function of  $f_t$ . Therefore, to estimate (22), I use GMM as in column (4) of Table 1. Encouragingly, the regression results are virtually identical to the ones obtained using (3).<sup>27</sup>

### A.2. The contributions of layoffs and quits

In this section, I study the separate contributions of layoffs and quits to unemployment's variance by using CPS data from the BLS on the reasons for unemployment (layoffs, quits or labor force entrants) over 1968–2004 as in Elsby et al. (2009b). Denoting  $u_t^l$ ,  $u_t^q$  and  $u_t^e$  the unemployment rates by reason respectively layoff, quit and labor force entrance, I have  $u_t = u_t^l + u_t^q + u_t^e$  and  $d \ln u_t = \omega_l d \ln u_t^l + \omega_q d \ln u_t^q + \omega_e d \ln u_t^e$ , with  $\omega_r$  the unemployment share of reason  $r$ ,  $u_t^l = s_t^l e_t / f_t^l$ ,  $u_t^q = s_t^q e_t / f_t^q$  and  $u_t^e = s_t^e i_t / f_t^e$  where  $e_t$  is the employment rate and  $i_t$  the labor force participation rate. Looking at Elsby et al. (2009b) decomposition, we can see that business cycle fluctuations in  $e_t$  and  $i_t$  are small compared to cyclical fluctuations in the hazard rates, and that fluctuations in  $s_t^e$  are small compared to movement in the other inflows rates (see Elsby et al., 2009b,

<sup>26</sup> Fujita and Ramey (2007) do not allow for an endogenous separation rate.

<sup>27</sup> The results are available upon request.

Figs. 9 and 11). As a result, I can write the following approximation

$$\begin{aligned} d \ln u_t^{ss} &= \omega_\lambda d \ln s_t^\lambda - \omega_\lambda d \ln f_t^\lambda + \omega_q d \ln s_t^q - \omega_q d \ln f_t^q + \omega_e d \ln s_t^e - \omega_e d \ln f_t^e + (\omega_e + \omega_q) d \ln e_t + \omega_e d \ln i_t \\ &\simeq \omega_\lambda d \ln s_t^\lambda + \omega_q d \ln s_t^q - \omega_\lambda d \ln f_t^\lambda - \omega_q d \ln f_t^q - \omega_e d \ln f_t^e \simeq \omega_\lambda d \ln s_t^\lambda + \omega_q d \ln s_t^q - d \ln f_t \end{aligned}$$

And using a matching function to model the job finding rate, I can write

$$\Delta \ln u_t^{ss} \simeq \frac{\omega_\lambda \Delta \ln s_t^\lambda + \omega_q \Delta \ln s_t^q - (1-\sigma) \Delta \ln(v_t)}{1-(1-\sigma)(1-u_{t-1}^{ss})}$$

and

$$\ln \frac{u_t^{ss}}{u_t^{ss}} \simeq \frac{\omega_\lambda \ln \ln \left( \frac{s_t}{s_t} \right) + \omega_q \Delta \ln \ln \left( \frac{s_t}{s_t} \right) - (1-\sigma) \Delta \ln \left( \frac{v_t}{u_t} \right)}{1-(1-\sigma)(1-u_t^{ss})}$$

Using this extended methodology, I find that layoffs contribute to 45 percent of unemployment fluctuations but quits, being procyclical, lower the contribution of JS by 10 percentage points. The contribution of vacancy posting is 63 percent, close to that reported in Table 2 despite the shorter time period.

### A.3. Non-steady-state decompositions of changes in unemployment

As in Shimer (2007), Elsby et al. (2009b) and Fujita and Ramey (2009), decomposition (7) relies on the steady-state assumption that  $u_t \simeq u_t^{ss}$ , an approximation that has been shown to be excellent for the US (e.g., Elsby et al., 2009a). However, compared to the existing literature, the present paper relies more heavily on the steady state assumption because decomposition (7) models the job finding rate using a matching function in which unemployment is approximated by steady-state unemployment (cf. (4)).

This section presents a method to generalize the level decomposition (8) and the first-difference decomposition (9) by taking into account out of steady state dynamics.

#### A.3.1. A non-steady-state decomposition in level

Solving forward the differential equation (1) governing unemployment fluctuations gives

$$u_t = \lambda_t u_t^{ss} + (1-\lambda_t) u_{t-1} \quad (23)$$

with  $\lambda_t = 1 - e^{-3(s_t + f_t)}$ , the quarterly rate of convergence to steady-state. Log-linearizing (23) around  $s_t = \underline{s}_t$ ,  $f_t = \underline{f}_t$  and  $u_{t-1} = \underline{u}_{t-1}$  (with “ $\underline{\quad}$ ” indicating the trend of a variable) gives

$$\begin{aligned} \ln u_t &\simeq \ln(\lambda_t u_t^{ss} + (1-\lambda_t) u_{t-1}) + \left( \lambda_t \frac{\partial u_t^{ss}}{\partial s_t} \Big|_{u_t^{ss}} + (u_t^{ss} - u_{t-1}) \frac{\partial \lambda_t}{\partial s_t} \Big|_{\lambda_t} \right) \frac{1}{u_t^{ss}} (s_t - \underline{s}_t) \\ &\quad + \left( \lambda_t \frac{\partial u_t^{ss}}{\partial f_t} \Big|_{u_t^{ss}} + (u_t^{ss} - u_{t-1}) \frac{\partial \lambda_t}{\partial f_t} \Big|_{\lambda_t} \right) \frac{1}{u_t^{ss}} (f_t - \underline{f}_t) + (1-\lambda_t) \frac{1}{u_t^{ss}} (u_{t-1} - \underline{u}_{t-1}) \end{aligned} \quad (24)$$

Assuming that  $u_t \simeq u_t^{ss} \forall t$  since deviations from steady-state are short lived and do not affect the trends, and using the fact that the trend is (by definition) slow moving so that  $u_t^{ss} \simeq u_{t-1}^{ss}$ , I can simplify (24) and write

$$d \ln u_t \simeq \lambda_t (1 - u_t^{ss}) d \ln s_t - \lambda_t (1 - u_t^{ss}) d \ln f_t + (1 - \lambda_t) d \ln u_{t-1} \quad (25)$$

with  $d \ln X_t = \ln X_t - \ln \underline{X}_t$  for any  $X_t$ . Expression (25) generalizes the equation used by Fujita and Ramey (2009) to decompose the fluctuations of unemployment around its trend. With fast convergence to steady-state (i.e.  $\lambda_t \simeq 1$ ), one gets the baseline equation used for the steady-state decomposition, i.e.

$$d \ln u_t = (1 - u_t^{ss}) (d \ln s_t - d \ln f_t)$$

since  $d \ln u_t^{ss} = (1 - u_t^{ss}) (d \ln s_t - d \ln f_t)$ .

Modeling the job finding rate with a matching function  $f_t = m_0 \left( \frac{v_t}{u_t} \right)^{1-\sigma}$  gives

$$d \ln u_t \simeq \lambda_t (1 - u_t^{ss}) (d \ln s_t - (1-\sigma) (\ln v_t - \ln u_t)) + (1 - \lambda_t) d \ln u_{t-1}$$

which after rearranging yields

$$d \ln u_t \simeq \frac{1}{1 - \lambda_t (1 - u_t^{ss}) (1 - \sigma)} [\lambda_t (1 - u_t^{ss}) (d \ln s_t - (1 - \sigma) \ln v_t) + (1 - \lambda_t) d \ln u_{t-1}]$$

**Table A1**  
Non-steady-state decomposition of unemployment variance, 1951–2008.

Decomposition	Variance				Variance			
	$\beta^{CS}$	$\beta^{CV}$	$\beta^{C^0}$	$\beta^\eta$	$\beta^{C^{d(JS)}}$	$\beta^{C^{d(V)}}$	$\beta^{C^0}$	$\beta^\eta$
Decomposition: V and JS	38.4%	59.2%	0.1%	2.4%	56.3%	40.5%	0.8%	2.3%

which generalizes (8), the decomposition in level used in the main text, to out of steady-state dynamics. Using the insight from [Elsby et al. \(2009a\)](#) and denoting

$$\begin{cases} C_t^V = \frac{1}{1-\lambda_t(1-u_t^{SS})(1-\sigma)}[-\lambda_t(1-u_t^{SS})(1-\sigma) d \ln v_t + (1-\lambda_t)C_{t-1}^V] \text{ with } C_0^V = 0 \\ C_t^{JS} = \frac{1}{1-\lambda_t(1-u_t^{SS})(1-\sigma)}[\lambda_t(1-u_t^{SS}) d \ln s_t + (1-\lambda_t)C_{t-1}^{JS}] \text{ with } C_0^{JS} = 0 \end{cases}$$

and  $C_t^0 = ((1-\lambda_t)/(1-\lambda_t(1-u_t^{SS})(1-\sigma)))C_{t-1}^0$  with  $C_0^0 = d \ln u_0$  the initial deviation from trend at time  $t=0$ , I can write

$$du_t = C_t^V + C_t^{JS} + C_t^0 + \eta_t \tag{26}$$

with  $du_t = d \ln u_t$  and  $\eta_t$  the approximation error. The cumulative contributions of contemporaneous and past variations in vacancy posting and job separation are then given by

$$\beta^V = \frac{Cov(du_t, C_t^V)}{Var(du_t)} \quad \text{and} \quad \beta^{JS} = \frac{Cov(du_t, C_t^{JS})}{Var(du_t)},$$

and the contributions of the initial deviation from steady-state and of the approximation error are given by  $\beta^{C^0} = Cov(du_t, C_t^0)/Var(du_t)$  and  $\beta^\eta = Cov(du_t, \eta_t)/Var(du_t)$

[Table A1](#) presents the result of a non-steady-state decomposition based on (26) that parallels [Table 2](#) based on the steady-state decomposition based on (8). The results are similar, and the split between vacancy and separation remains at about 60/40.

**A.3.2. A non-steady-state decomposition in first-difference**

To generalize the first-difference decomposition to out of steady-state dynamics, I follow [Elsby et al. \(2009a\)](#). A log-linear approximation to (23) around  $s_t = s_{t-1}$ ,  $f_t = f_{t-1}$  and  $u_{t-1} = u_{t-1}^{SS}$  gives

$$\ln u_t \approx \ln u_{t-1}^{SS} + \lambda_{t-1}(\ln u_t^{SS} - \ln u_{t-1}^{SS}) + (1-\lambda_{t-1})(\ln u_{t-1} - \ln u_{t-1}^{SS})$$

which after a little bit of algebra detailed in [Elsby et al. \(2009a\)](#) gives

$$d \ln u_t = \lambda_{t-1} \left[ (1-u_{t-1}^{SS})(d \ln s_t - d \ln f_t) + \frac{1-\lambda_{t-2}}{\lambda_{t-2}} d \ln u_{t-1} \right]$$

with  $d \ln X_t = \ln X_t - \ln X_{t-1}$ .

Modeling the job finding rate with a matching function  $f_t = m_0(v_t/u_t)^{1-\sigma}$  and rearranging gives

$$d \ln u_t = \frac{\lambda_{t-1}}{1-\lambda_{t-1}(1-u_{t-1}^{SS})(1-\sigma)} \left[ (1-u_{t-1}^{SS})(d \ln s_t - (1-\sigma) d \ln v_t) + \frac{1-\lambda_{t-2}}{\lambda_{t-2}} d \ln u_{t-1} \right].$$

I can then write

$$du_t = C_t^{d(V)} + C_t^{d(JS)} + C_t^0 + \eta_t \tag{27}$$

with

$$\begin{cases} C_t^{d(V)} = \frac{\lambda_{t-1}}{1-\lambda_{t-1}(1-u_{t-1}^{SS})(1-\sigma)} \left[ -(1-u_{t-1}^{SS})(1-\sigma) d \ln v_t + \frac{1-\lambda_{t-2}}{\lambda_{t-2}} C_{t-1}^{d(V)} \right] \text{ with } C_0^{d(V)} = 0 \\ C_t^{d(JS)} = \frac{\lambda_{t-1}}{1-\lambda_{t-1}(1-u_{t-1}^{SS})(1-\sigma)} \left[ (1-u_{t-1}^{SS}) d \ln s_t + \frac{1-\lambda_{t-2}}{\lambda_{t-2}} C_{t-1}^{d(JS)} \right] \text{ with } C_0^{d(JS)} = 0 \end{cases}$$

and

$$C_t^0 = \frac{\lambda_{t-1}}{1-\lambda_{t-1}(1-u_{t-1}^{SS})(1-\sigma)} \frac{1-\lambda_{t-2}}{\lambda_{t-2}} C_{t-1}^0$$

with  $C_0^0 = d \ln u_0$ . The cumulative contributions of contemporaneous and past past variations in vacancy posting and job separation are given by  $\beta^{d(V)} = Cov(du_t, C_t^{d(V)})/Var(du_t)$  and  $\beta^{d(JS)} = Cov(du_t, C_t^{d(JS)})/Var(du_t)$ , and the contributions of the initial deviation from steady-state and of the approximation error are given by  $\beta^{C^0} = Cov(du_t, C_t^0)/Var(du_t)$  and  $\beta^\eta = Cov(du_t, \eta_t)/Var(du_t)$ .



Table A1 presents the result of a non-steady-state decomposition based on (27) that parallels Table 2 based on the steady-state decomposition based on (9). The results are similar, with a rough split between vacancy and separation at about 40/60.

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